**Topics: Normal distribution, Functions of Random Variables**

* The time required for servicing transmissions is normally distributed with = 45 minutes and = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
* 0.3875
* 0.2676
* 0.5
* 0.6987

**ANSWER: B**

Normally Distributed with, = 45 Min

,  *=* 8 Min

Let X be the amount of time required to complete the repair.

To finish in one hour you must have X ≤ 50 so the question is to find Pr(X > 50).

Thus, the question can be answered by using the normal table to find

Pr (X ≤ 50) = Pr(Z ≤ (50 - 45)/8.0) = Pr(Z ≤ 0.625)=73.4% Probability that the service manager will not meet his demand will be 100-73.4 = 26.6% or 0.2676.

* The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean = 38 and Standard deviation =6. For each statement below, please specify True/False. If false, briefly explain why.
* More employees at the processing center are older than 44 than between 38 and 44.

**ANSWER:**

To find the probability of employees greater than age of 44 is Pr(X>44).

Pr(X > 44) = 1 - Pr(X ≤ 44).

Z = (X - )/ = (X - 38)/6.

Thus, by using normal table to find,

Pr(X ≤ 44) = Pr(Z ≤ (44 - 38)/6) = Pr(Z ≤ 1) = 84.1345 %

Probability that the employee will be greater than age of 44 is,

100-84.1345 = 15.86 %

So, the probability of number of employees between 38-44 years of age is,

Pr(X<44) - 0.5 = 84.1345 - 0.5= 34.1345 %

By this, the given statement is **TRUE.**

* A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

**ANSWER:**

Probability of employees less than age of 30 is, Pr(X<30).

Z = (X - )/ = (30 - 38)/6.

Thus, by using normal table to find,

Pr(X ≤ 30) = Pr(Z ≤ (30 - 38)/6) = Pr(Z ≤ -1.333) = 9.12 %

So, the number of employees with the probability of 0.912, them being under age 30 = 0.0912\*400 = 36.48 % (or 36 employees)

By this, the given statement is **TRUE.**

* If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

**ANSWER:**

We know that, if X ∼ N (µ1, σ1^2) and Y ∼ N (µ2, σ2^2) are the two independent random variables, then X + Y ∼ N (µ1 + µ2, σ1^2 + σ2^2) and X − Y ∼ N (µ1 − µ2, σ1^2 + σ2^2).

Similarly if Z = aX + bY , where X and Y are as defined above, i.e., Z is linear combination of X and Y , then Z ∼ N(aµ1 + bµ2, a^2σ1^2 + b^2σ2^2 ).

Therefore,

2X1~ N (2 u,4 σ^2) and

X1+X2 ~ N (µ + µ, σ^2 + σ^2) ~ N (2 u, 2σ^2)

2X1-(X1+X2) = N (4µ,6 σ^2)

* Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
* 90.5, 105.9
* 80.2, 119.8
* 22, 78
* 48.5, 151.5
* 90.1, 109.9

**ANSWER: D**

We need to find out the values of a and b, which are symmetric about the mean, such that the probability of random variable taking a value between them is 0.99, we have to work out in reverse order.

The Probability of getting value between a and b should be 0.99

So, the Probability of going wrong, or the Probability outside the a and b area is 0.01 (i.e., 1- 0.99). The Probability towards left from a = -0.005 (i.e., 0.01/2).

The Probability towards right from b = +0.005 (i.e., 0.01/2).

Since we have the probabilities of a and b, we need to calculate X, the random variable at a and b which has got these probabilities.

By finding the Standard Normal Variable Z (Z Value), we can calculate the X values,

Z = (X- μ) / σ

For Probability 0.005 the Z Value is -2.57 (from Z Table),

Z \* σ + μ = X

Z (-0.005) \*20+100 = -(-2.57) \*20+100 = 151.4 Z (+0.005) \*20+100 = (-2.57) \*20+100

= 48.6

By this, we can say that option D (48.5, 151.5) is correct.

* Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
* Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.

**ANSWER:** 603.68

* Specify the 5th percentile of profit (in Rupees) for the company

**ANSWER:** 476.33

* Which of the two divisions has a larger probability of making a loss in a given year?

**ANSWER:** 0.1400