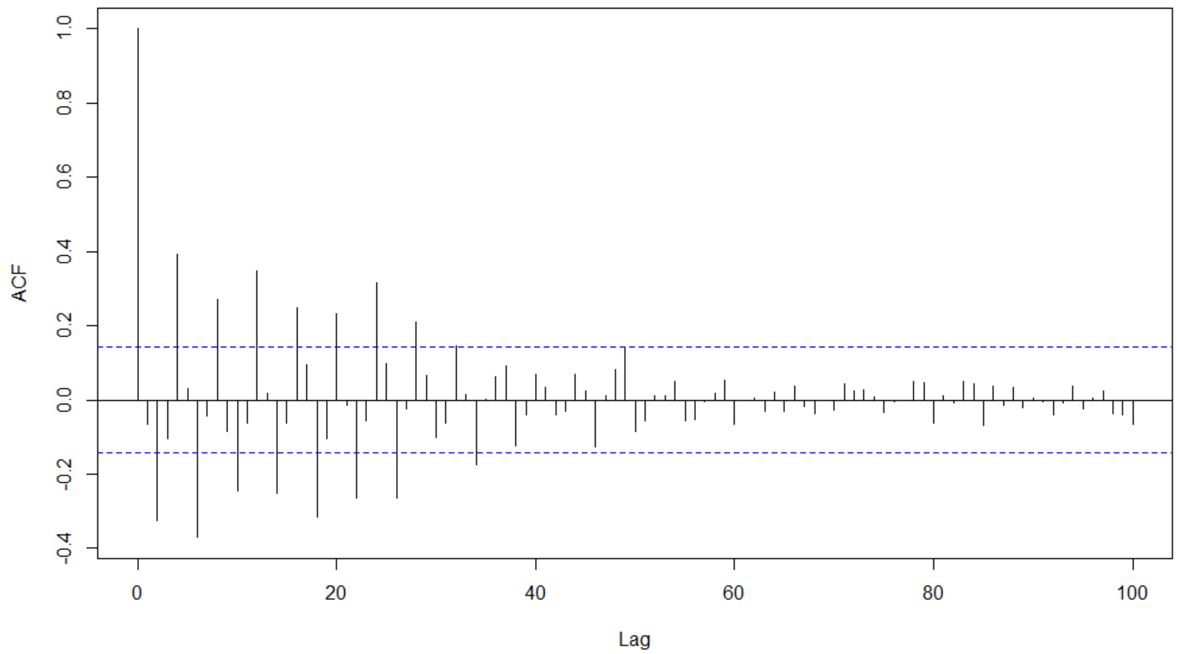
Empirical Project Part 2

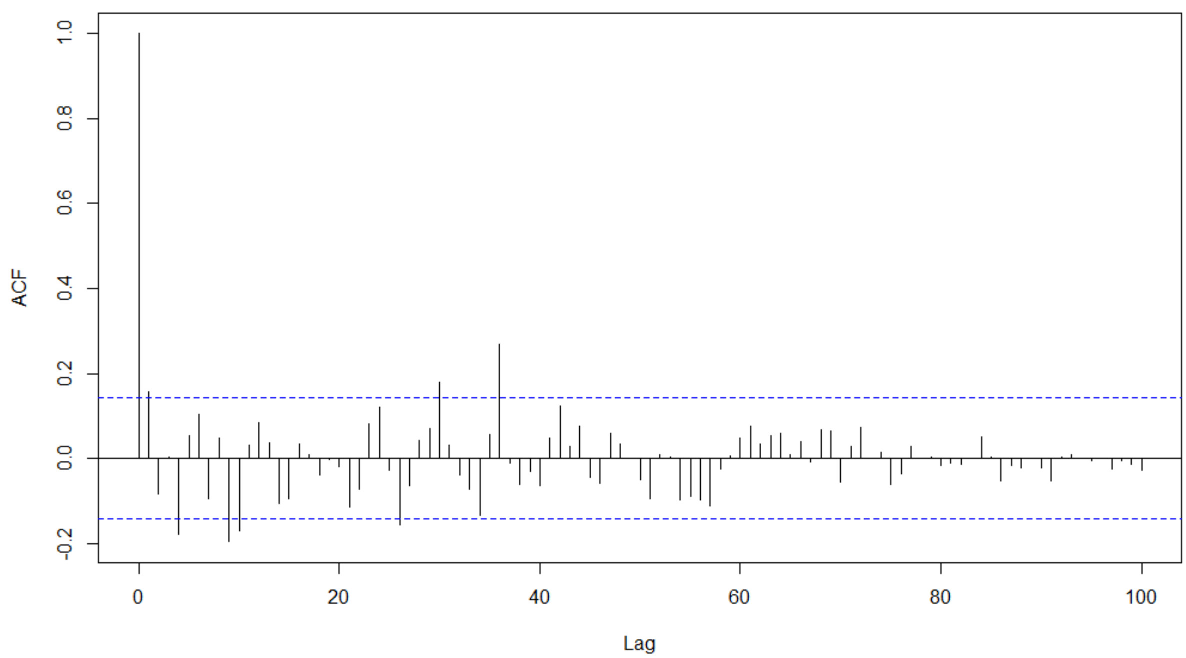
For the Empirical Project Part 2, the series from part 1 - first difference of log(IP Index) [*gIP*] and first difference of interest rate [*dIR*] - are chosen to build a one-step-ahead prediction model.

First, we plot an autocorrelation function via the ACF command to see if the time series correlated with its past values. Notice, in the ACF plot, the correlation coefficient is in the y-axis, whereas the number of lags is shown in the x-axis.

Graph 1. Autocorrelation function up to 100 lags for the first difference of log(IP Index).

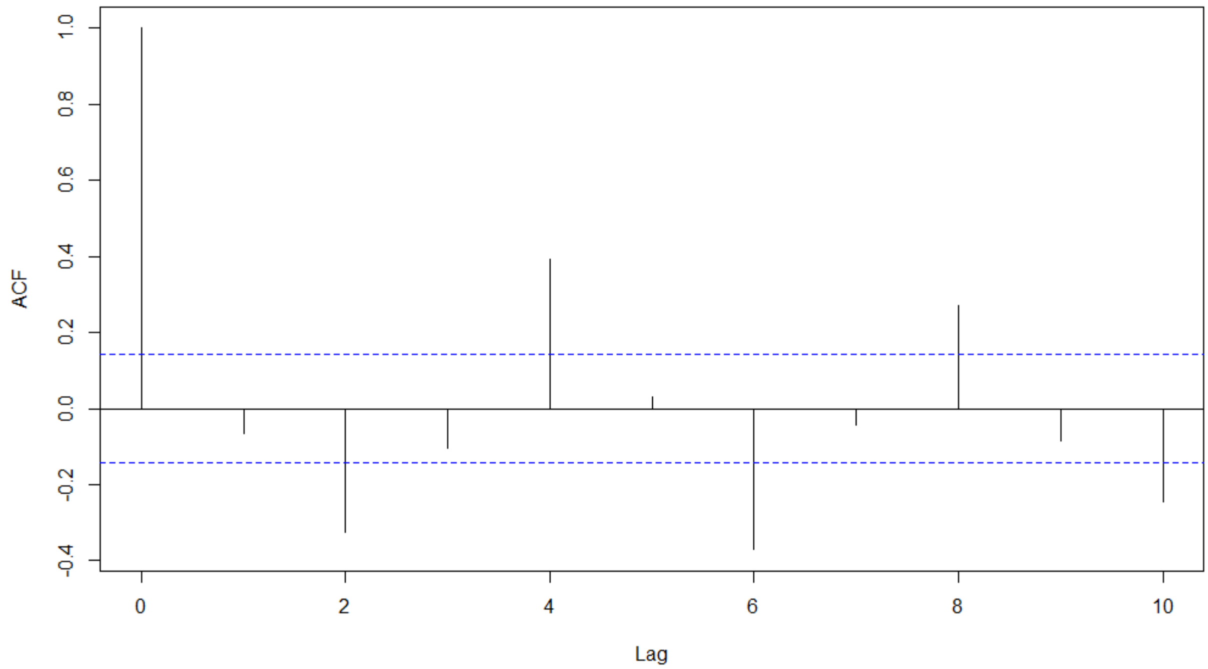


Graph 2. Autocorrelation function up to 100 lags of the first difference of interest rate.

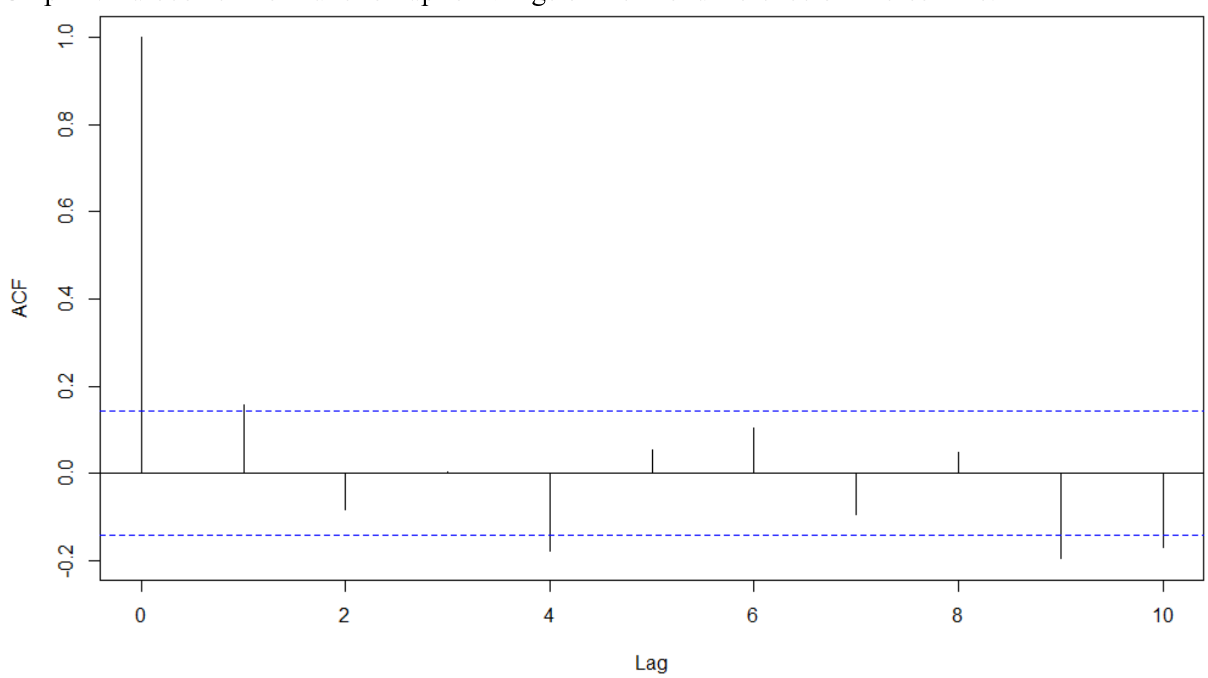


From the plots above, we can notice that both series converge to the zero confidence interval denoted by the x-axis as the number of lags increases. To be precise, for the growth rate of IP Index, even numbers of the first 28 lags, i.e., AR(2), AR(4),..., AR(28), are beyond the confidence interval denoted by the blue dashes, that is, they are statistically significant to be included in the model. Similarly, for the first difference of the interest rate, we distinguish AR(1), AR(4), AR(9), AR(10), AR(26), AR(30), and AR(36) for their statistical significance. Seemingly, we can highlight the AR(36) for the highest statistical significance. However, since the prediction model is suggested to be up to 10 lags, we shall compare their statistical significance up to 10 lags.

Graph 3. Autocorrelation function up to 10 lags of the first difference of log(IP Index).



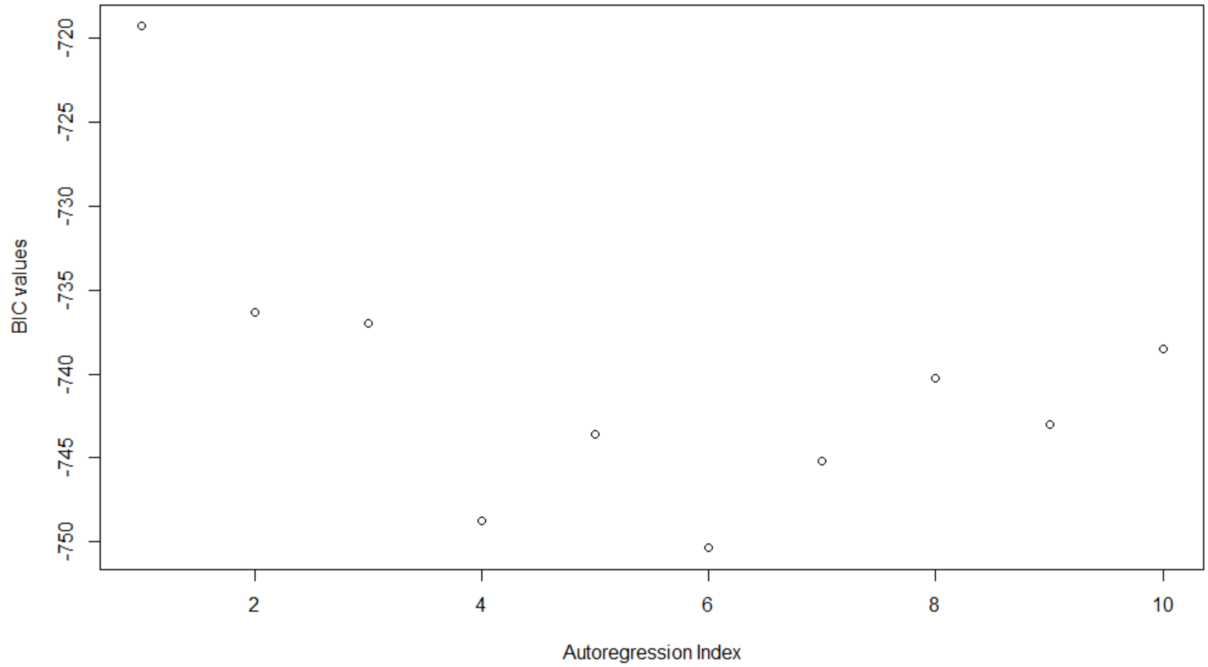
Graph 4. Autocorrelation function up to 10 lags of the first difference of interest rate.



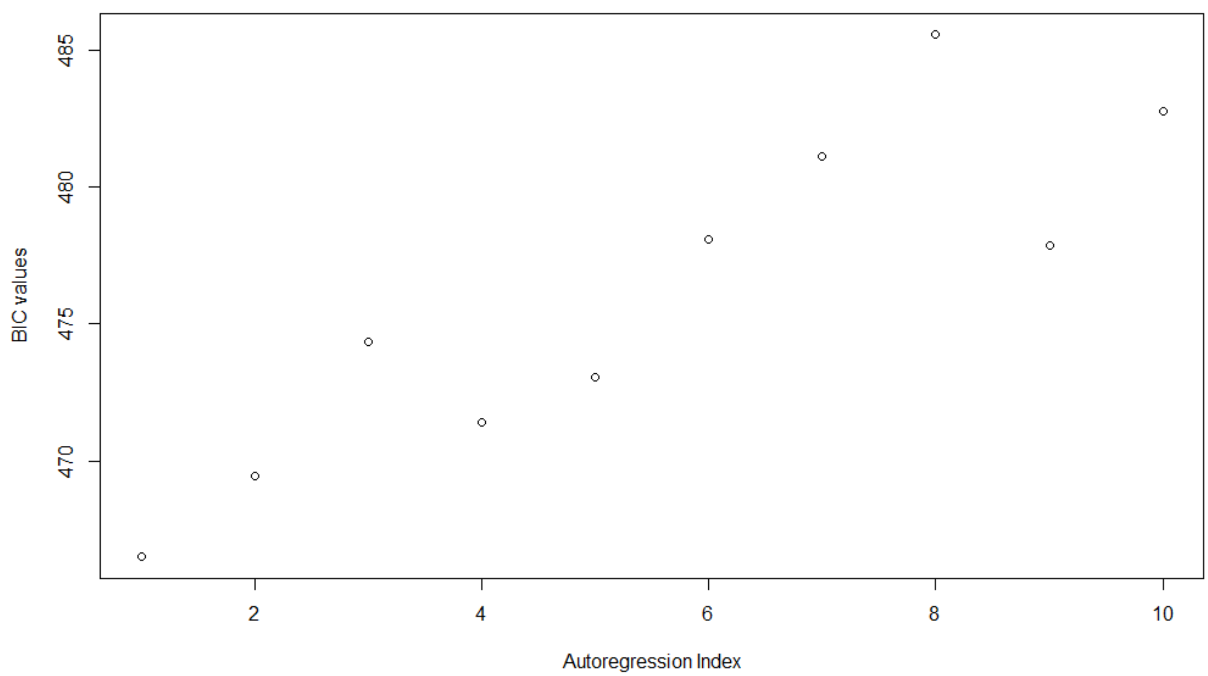
Comparing the autocorrelation function plots up to 10 lags above, we denote AR(4) and AR(9) for the *gIP* and *dIR* variables, respectively.

Second, for each of the two series - *gIP* and *dIR* - autoregression functions up to 10 lags were introduced in two variations: ARIMA and via lm command.

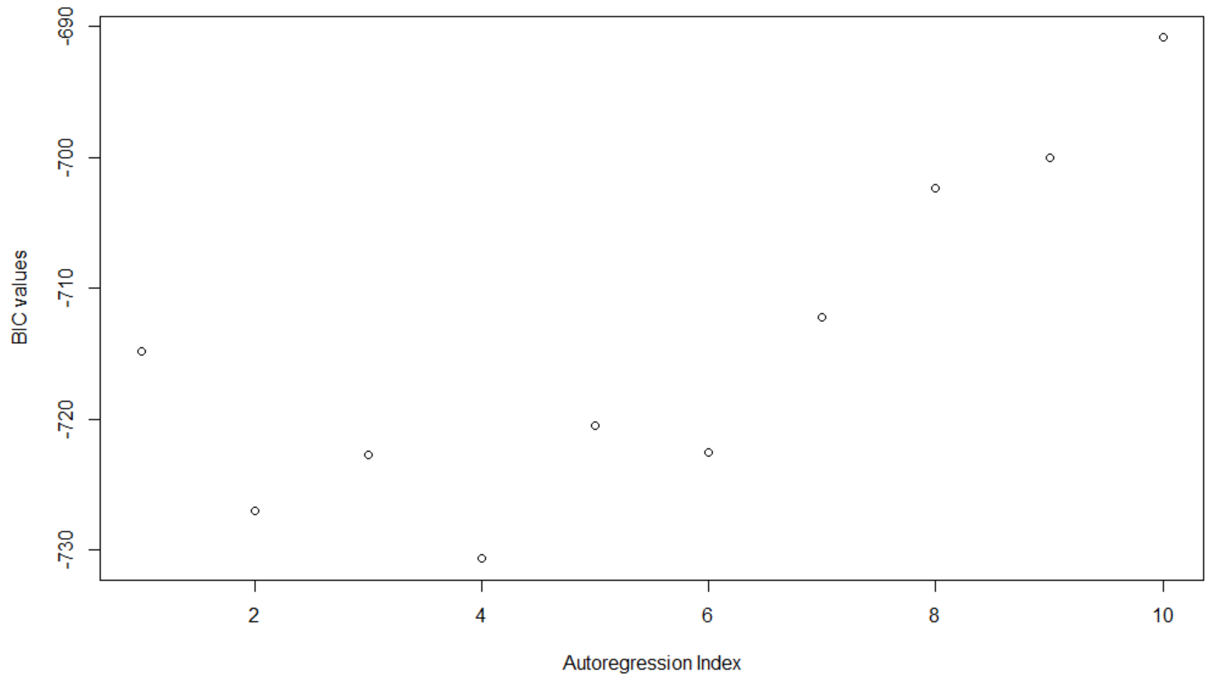
Graph 5. Bayesian Information Criterion (BIC) for *gIP* AR functions using ARIMA method.



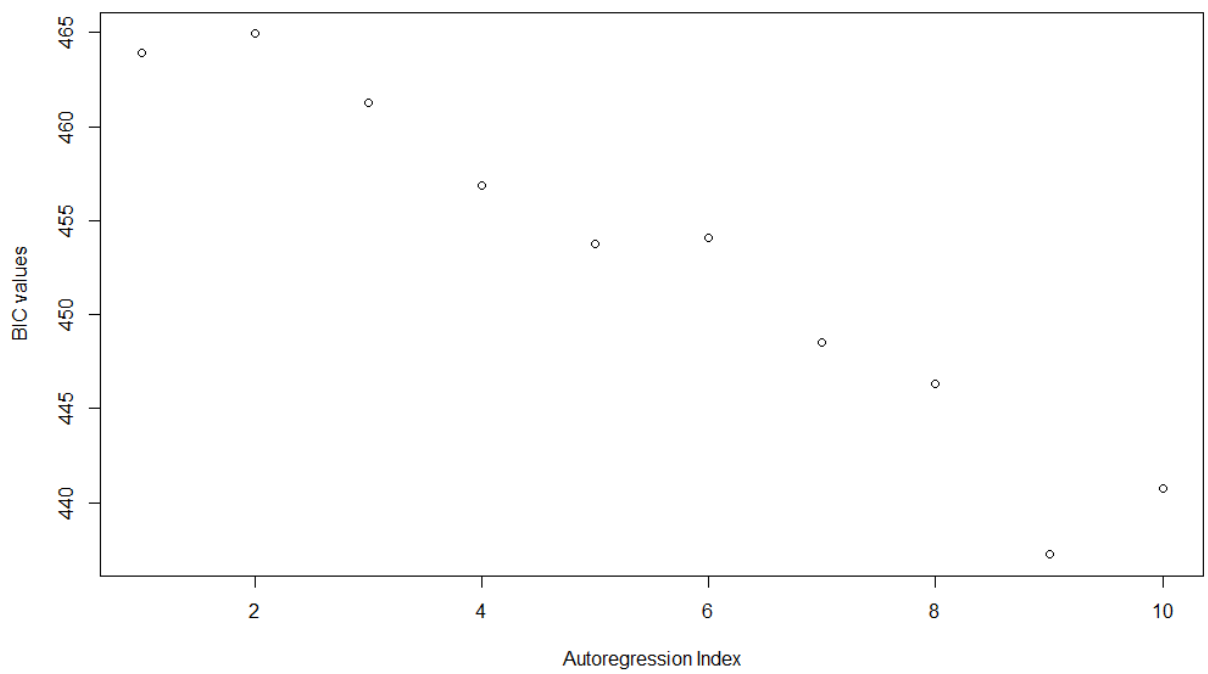
Graph 6. Bayesian Information Criterion (BIC) for *dIR* AR functions using ARIMA method.



Graph 7. Bayesian Information Criterion (BIC) for *gIP* AR functions using lm command.



Graph 8. Bayesian Information Criterion (BIC) for *dIR* AR functions using lm command.



Comparing the Bayesian Information Criterion (BIC) values in both methods, we choose the autoregression functions with the lowest BIC, that is, AR(6) and AR(1) using ARIMA, as well as AR(4) and AR(9) using lm command for *gIP* and dIR series, respectively. Since the BIC values for both methods differ, we take the model that corresponds to the autocorrelation plot drawn earlier, which are AR(4) and AR(9) functions built using the lm command. All things considered, for the further steps, we used four models: AR(1) and AR (4) for the *gIP* series, as well as AR(1) and AR(9) for the *dIR* series.

Next, serial correlation tests using AR(2) models were done to examine if the disturbances from the AR models chosen in the previous step are serially correlated. Since the strict exogeneity assumption is usually violated in the autoregression dynamic model, explanatory variables were included in the serial correlation test. The test is estimated using an ANOVA test, F-distribution.

See the AR(2) serial correlation test results below.

Figure 1. AR(2) serial correlation test for gIP AR(4) model.

```
Model 1: gIP_u_hat ~ gIP_1 + gIP_2 + gIP_3 + gIP_4
Model 2: gIP_u_hat ~ gIP_1 + gIP_2 + gIP_3 + gIP_4 + gIP_u_hat_1 + gIP_u_hat_2
  Res.Df     RSS Df Sum of Sq      F   Pr(>F)
1    176 0.16635
2    174 0.15555  2  0.010802 6.0414 0.002906 **
```

Figure 2. AR(2) serial correlation test for gIP AR(1) model.

```
Model 1: gIP_u_hat ~ gIP_1
Model 2: gIP_u_hat ~ gIP_1 + gIP_u_hat_1 + gIP_u_hat_2
  Res.Df     RSS Df Sum of Sq      F    Pr(>F)
1    182 0.21415
2    180 0.18404  2  0.030113 14.726 1.194e-06 ***
```

Figure 3. AR(2) serial correlation test for dIR AR(9) model.

```
Model 1: dIR_u_hat ~ dIR_1 + dIR_2 + dIR_3 + dIR_4 + dIR_5 + dIR_6 + dIR_7 +
    dIR_8 + dIR_9
Model 2: dIR_u_hat ~ dIR_1 + dIR_2 + dIR_3 + dIR_4 + dIR_5 + dIR_6 + dIR_7 +
    dIR_8 + dIR_9 + dIR_u_hat_1 + dIR_u_hat_2
  Res.Df    RSS Df Sum of Sq      F Pr(>F)
1    166 82.737
2    164 82.528  2   0.20927 0.2079 0.8125
```

Figure 4. AR(2) serial correlation test for dIR AR(1) model.

```
Model 1: dIR_u_hat ~ dIR_1
Model 2: dIR_u_hat ~ dIR_1 + dIR_u_hat_1 + dIR_u_hat_2
  Res.Df    RSS Df Sum of Sq      F Pr(>F)
1    182 116.84
2    180 114.71  2    2.1305 1.6716 0.1909
```

From the AR(2) serial correlation test results, it can be denoted that two lagged residuals in both AR(4) and AR(1) models for *gIP* are statistically significant at a 1% significance level, that is, at least one lagged residual is serially correlated. For the *dIR* series, unlikely to the *gIP*, p-values in both AR(9) and AR(1) are greater than 0.15 indicating that the lagged residuals in both autoregression functions are not statistically significant and thus are not serially correlated.

As the serial correlation test is done, one-step-ahead predictions were generated. To begin with, the data were divided into two parts such that n=0.75T and m=T-n, where T=total number of observations: first n=141 observations are used to reestimate the model and last m=47 are used to generate one-step-ahead predictions.

Model 1: AR(4) for the first difference of log(IP Index)

$$\widehat{gIP} = \beta_0 + \beta_1 gIP\_1 + \beta_2 gIP\_2 + \beta_3 gIP\_3 + \beta_4 gIP\_4 + u_t$$

Model 2: AR(1) for the first difference of log(IP Index)

$\widehat{gIP} = \beta_0 + \beta_1 gIP\_1 + u_t$

Model 3: AR(9) for the first difference of interest rate

$\widehat{dIR} = \beta_0 + \beta_1 dIR\_1 + \beta_2 dIR\_2 + \beta_3 dIR\_3 + \beta_4 dIR\_4 + \beta_5 dIR\_5 + \beta_6 dIR\_6 + \beta_7 dIR\_7 + \beta_8 dIR\_8 + \beta_9 dIR\_9 + u_t$

Model 4: AR(1) for the first difference of interest rate

$\widehat{dIR} = \beta_0 + \beta_1 dIR\_1 + u_t$

To compare each of the four models out-of-sample root mean squared error (RMSE) and mean absolute error (MAE) for each model were calculated.

Table 1. RMSE and MAE values.

Model	**RMSE value**	**MAE value**
Model 1: gIP AR(4)	0.03947916	0.03263953
Model 2: gIP AR(1)	0.04545096	0.03912331
Model 3: dIR AR(9)	0.417411	0.259249
Model 4: dIR AR(1)	0.3724704	0.1885622

It is important to note that the models with the lowest RMSE and MAE are more precise for forecasting the explanatory variable, that is, these models have lower error terms between predicted and actual values. Consequently, AR(4) model is more appropriate to forecast the growth rate of industrial production index and is more precise than the AR(1) model by 15.12% and 19.86% concerning RMSE and MAE, respectively.Regarding the *dIR*, AR(1) model is more appropriate to forecast the first difference of interest rate and is more precise than AR(9) model by 12.07% and 37.49% concerning RMSE and MAE, respectively.