Empirical Project Part 3

For the Empirical Project Part 3, the series from part 1 - log(IP Index) [lIP], first difference of log(IP Index) [*gIP*], interest rate [IR], and first difference of interest rate [*dIR*] - are chosen to build a one-step-ahead Autoregressive distributed lag (ARDL) prediction model.

$$y_t = \alpha + \gamma_1 y_{t-1} + \cdots + \gamma_P y_{t-P} + \beta_0 x_t + \beta_1 x_{t-1} + \cdots + \beta_R x_{t-R} + e_t$$

First, we identified the impact propensity (IP) and long -run propensity (LRP) for the general ARDL model above. Calculating the long-run multiplier, we assumed the stability condition for the ARDL model holds, i.e., $y^* = y_t = y_{t-1} = ... = y_{t-P}$ and $x^* = x_t = x_{t-1} = ... = x_{t-R}$. Then, the equation becomes:

$$y^* = \alpha + \gamma_1 y^* + \cdots + \gamma_P y^* + \beta_0 x^* + \cdots + \beta_R x^*$$

$$y^* = \alpha + (\sum_{i=1}^{P} \gamma_i) \times y^* + (\sum_{i=0}^{R} \beta_i) \times x^*$$

$$y^*(1 - \sum_{i=1}^{P} \gamma_i) = \alpha + (\sum_{i=0}^{R} \beta_i) \times x^*$$

$$y^* = \frac{\alpha}{(1 - \sum_{i=1}^{P} \gamma_i)} + \frac{\sum_{i=0}^{R} \beta_i}{(1 - \sum_{i=1}^{P} \gamma_i)} \times x^* \quad (***)$$

where $\frac{\sum_{i=0}^{R} \beta_i}{(1 - \sum_{i=1}^{P} \gamma_i)}$ is the LRP of the variable x, which is the first difference of interest rate.

Since $y^* = y_t = y_{t-1} = ... = y_{t-P}$ and $x^* = x_t = x_{t-1} = ... = x_{t-R}$, we may take ARDL(2,2) to compute LRP, while to find IP it is enough to regress y on x at time t to capture short-term impact of x on y. Then, the model becomes:

$$\text{LRP:} \quad y^* = \frac{\alpha}{(1 - \gamma_1 - \gamma_2)} + \frac{\beta_0 + \beta_1 + \beta_2}{(1 - \gamma_1 - \gamma_2)} \times x^*$$

$$\text{IP:} \quad y_t = \alpha + \beta_0 x_t$$

Second, the ARDL(6,6) model was estimated using the OLS method. The ARDL(6,6) model is

$$y_t = \alpha + \gamma_1 y_{t-1} + \gamma_2 y_{t-2} + \gamma_3 y_{t-3} + \gamma_4 y_{t-4} + \gamma_5 y_{t-5} + \gamma_6 y_{t-6} + \beta_0 x_t + \beta_1 x_{t-1} + \beta_2 x_{t-2} + \beta_3 x_{t-3} + \beta_4 x_{t-4} + $$
$$+ \beta_5 x_{t-5} + \beta_6 x_{t-6} + \varepsilon_t$$

Impact propensity for the general ARDL model is 0.6900, while long-run propensity is 1.3732

See the ARDL(6,6) OLS regression results below.

Figure 1. Statistical summary of ARDL(6,6) OLS regression.

```
Call:
lm(formula = gIP ~ gIP_1 + gIP_2 + gIP_3 + gIP_4 + gIP_5 + gIP_6 +
    dIR + dIR_1 + dIR_2 + dIR_3 + dIR_4 + dIR_5 + dIR_6, data = dataset)

Residuals:
    Min      1Q  Median      3Q     Max
-8.8246 -1.6213 -0.0387  1.6686  7.2965

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  0.62084    0.22796   2.723 0.007148 **
gIP_1       -0.25134    0.07554  -3.327 0.001078 **
gIP_2       -0.21805    0.07619  -2.862 0.004749 **
gIP_3       -0.15300    0.07616  -2.009 0.046156 *
gIP_4        0.26672    0.07562   3.527 0.000543 ***
gIP_5        0.08242    0.07600   1.085 0.279666
gIP_6       -0.22794    0.07484  -3.046 0.002699 **
dIR          0.65808    0.27606   2.384 0.018257 *
dIR_1        0.86826    0.27821   3.121 0.002125 **
dIR_2        0.54659    0.28344   1.928 0.055502 .
dIR_3       -0.53052    0.28099  -1.888 0.060757 .
dIR_4       -0.39777    0.28651  -1.388 0.166887
dIR_5       -0.63549    0.28415  -2.236 0.026647 *
dIR_6       -0.18698    0.28232  -0.662 0.508699
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 2.717 on 167 degrees of freedom
  (7 observations deleted due to missingness)
Multiple R-squared:  0.4223,    Adjusted R-squared:  0.3774
F-statistic: 9.392 on 13 and 167 DF,  p-value: 1.913e-14
```

From the results above, we calculated IP = 0.65808, the coefficient of the first difference of interest rate at time t. The LRP is 0.21461. It was calculated using the formula (***).

In the estimated ARDL model we hypothesize negative long-run propensity for the monetary policy, i.e., interest rate. Referring to economic theory, the decrease of an interest rate creates an incentive to firms to take loans and increase their production in the near future. Since, the change in interest rate has a lagging effect on the Industrial production index, we expect dIR lagging variables to be negatively correlated with gIP variable at time t. In the figure 1, we can notice that the lagging variables for dIR beginning from the third lag is negatively correlated with current Industrial Production growth rate, justifying our hypothesis on lagging effect of the monetary policy on Industrial Production. However, theoretically, the short-term effect of monetary policy on growth rate of Industrial production at time t is undefined.

Fourth, the serial correlation test for ARDL(6,6) disturbances were done using the AR(2) serial correlation test, where

H0: disturbances are not serially correlated
H1: disturbances are serially correlated

See the serial correlation test results below.

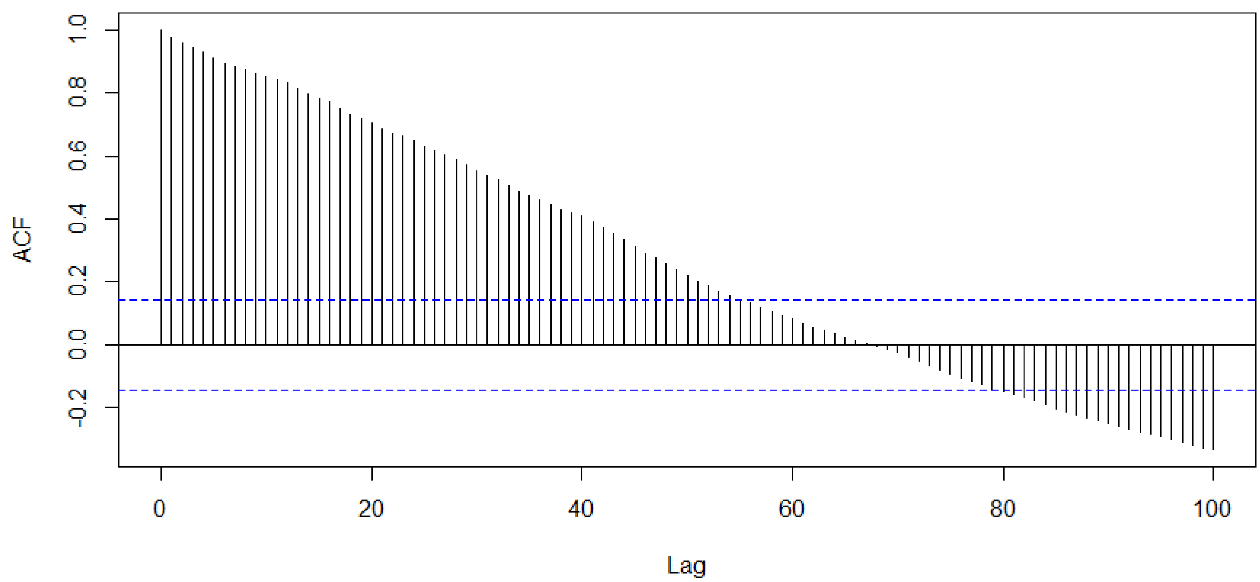
Figure 2. AR(2) serial correlation test for ARDL(6,6) model.

```
Model 1: u_hat ~ gIP_1 + gIP_2 + gIP_3 + gIP_4 + gIP_5 + gIP_6 + dIR +
    dIR_1 + dIR_2 + dIR_3 + dIR_4 + dIR_5 + dIR_6
Model 2: u_hat ~ gIP_1 + gIP_2 + gIP_3 + gIP_4 + gIP_5 + gIP_6 + dIR +
    dIR_1 + dIR_2 + dIR_3 + dIR_4 + dIR_5 + dIR_6 + u_hat_1 +
    u_hat_2
  Res.Df    RSS Df Sum of Sq      F Pr(>F)
1    165 1226.3
2    163 1212.8  2    13.555 0.9109 0.4042
```

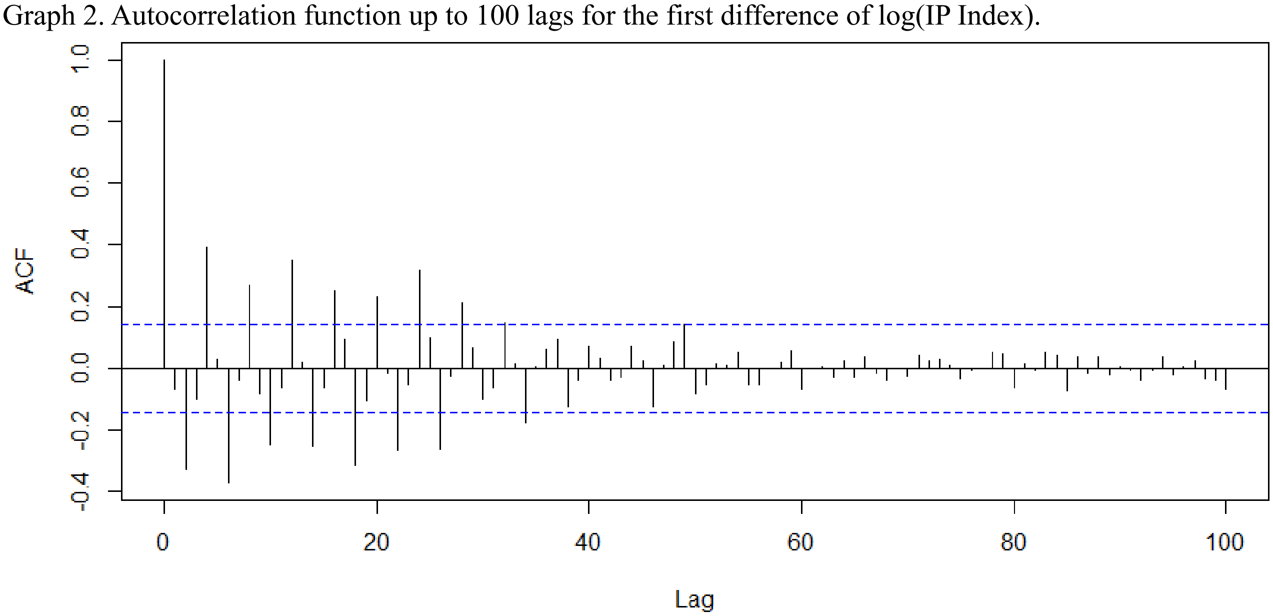
According to the findings above, the p-value for two way ANOVA test is 0.4042, which is greater than 0.05; thus, we fail to reject the Null hypothesis, implying that disturbances are not serially correlated.

Fifth, we plotted an autocorrelation function via the ACF command to see if the time series correlated with its past values and compare the findings with the ones from EP#2. Notice, in the ACF plot, the correlation coefficient is in the y-axis, whereas the number of lags is shown in the x-axis.

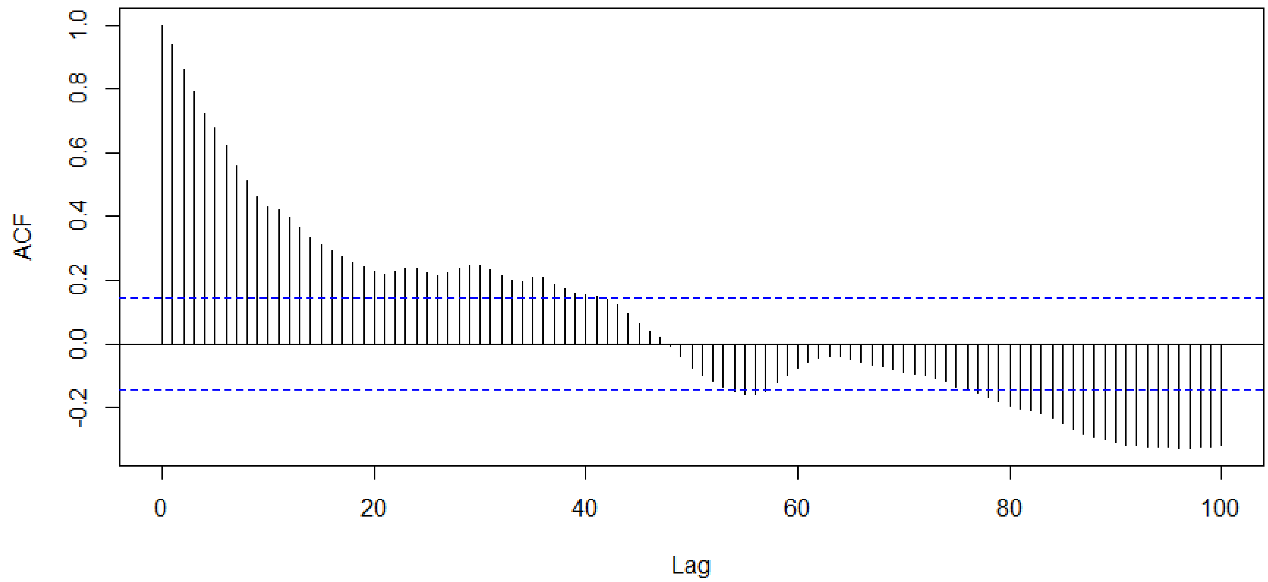
Graph 1. Autocorrelation function up to 100 lags for the log(IP Index).



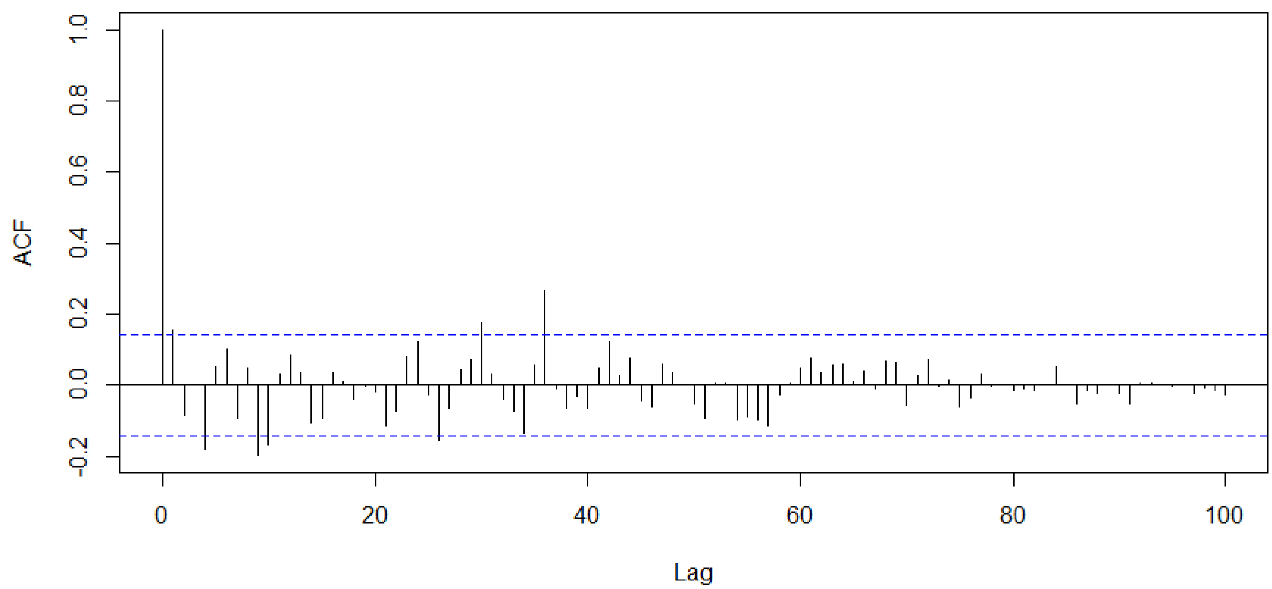
Graph 2. Autocorrelation function up to 100 lags for the first difference of log(IP Index).



Graph 3. Autocorrelation function up to 100 lags for the interest rate.



Graph 4. Autocorrelation function up to 100 lags of the first difference of interest rate.



From the plots above, we can notice that autocorrelation coefficients for both differenced series from the previous empirical project converge to zero confidence interval denoted by the x-axis as the number of lags increases. However, the same cannot be stated regarding the variables that are not differenced, i.e., log(IP Index), interest rate. The graphs 1 and 3 look like continuous functions that go from 1 to -1 as the number of lags increases.

Sixth, the unit root test for the four variables - log(IP Index) [lIP], first difference of log(IP Index) [*gIP*], interest rate [IR], and first difference of interest rate [*dIR*] - were conducted using augmented Dickey-Fuller test with 4 lags.

See the test results below.

Figure 3. Unit root test for the log(IP Index).

```
Residuals:
   Min     1Q Median     3Q    Max 
-5.960 -1.067 -0.079  1.388  7.960 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept)  5.93642    1.70876   3.474 0.000645 ***
z.lag.1     -0.14829    0.04169  -3.557 0.000482 ***
tt           0.05459    0.01447   3.773 0.000220 ***
z.diff.lag1 -0.01444    0.07412  -0.195 0.845723    
z.diff.lag2 -0.23219    0.07266  -3.195 0.001655 ** 
z.diff.lag3 -0.07636    0.07008  -1.090 0.277335    
z.diff.lag4  0.41070    0.06949   5.911 1.73e-08 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 2.202 on 176 degrees of freedom
Multiple R-squared:  0.4044,	Adjusted R-squared:  0.3841 
F-statistic: 19.92 on 6 and 176 DF,  p-value: < 2.2e-16


Value of test-statistic is: -3.5568 6.4858 7.2918 

Critical values for test statistics: 
      1pct  5pct 10pct
tau3 -3.99 -3.43 -3.13
phi2  6.22  4.75  4.07
phi3  8.43  6.49  5.47
```

Figure 4. Unit root test for the interest rate.

```
Residuals:
     Min       1Q   Median       3Q      Max 
-2.37704 -0.33946 -0.03175  0.31488  2.24379 

Coefficients:
             Estimate Std. Error t value Pr(>|t|)   
(Intercept)  0.389812   0.178644   2.182  0.03043 * 
z.lag.1     -0.078729   0.029998  -2.624  0.00944 **
tt          -0.002449   0.001294  -1.893  0.06004 . 
z.diff.lag1  0.232536   0.073462   3.165  0.00183 **
z.diff.lag2 -0.119418   0.073834  -1.617  0.10758   
z.diff.lag3  0.105825   0.073084   1.448  0.14940   
z.diff.lag4 -0.168917   0.072145  -2.341  0.02033 * 
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 0.7737 on 176 degrees of freedom
Multiple R-squared:  0.1223,	Adjusted R-squared:  0.09237 
F-statistic: 4.087 on 6 and 176 DF,  p-value: 0.0007277


Value of test-statistic is: -2.6244 2.436 3.6017 

Critical values for test statistics: 
      1pct  5pct 10pct
tau3 -3.99 -3.43 -3.13
phi2  6.22  4.75  4.07
phi3  8.43  6.49  5.47
```

Figure 5. Unit root test for the first difference of log(IP Index).

```
Residuals:
     Min       1Q   Median       3Q      Max
-14.8234  -1.8510   0.0892   1.9489   8.3192

Coefficients:
             Estimate Std. Error t value Pr(>|t|)
(Intercept)  0.128619   0.475225   0.271  0.78698
z.lag.1     -1.170621   0.207752  -5.635 6.89e-08 ***
tt           0.003830   0.004366   0.877  0.38162
z.diff.lag1  0.061369   0.187489   0.327  0.74381
z.diff.lag2 -0.191492   0.150180  -1.275  0.20397
z.diff.lag3 -0.320355   0.110687  -2.894  0.00428 **
z.diff.lag4 -0.023065   0.075958  -0.304  0.76175
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 3.078 on 175 degrees of freedom
Multiple R-squared:  0.6326,    Adjusted R-squared:   0.62
F-statistic: 50.23 on 6 and 175 DF,  p-value: < 2.2e-16


Value of test-statistic is: -5.6347 10.612 15.9171

Critical values for test statistics:
      1pct  5pct 10pct
tau3 -3.99 -3.43 -3.13
phi2  6.22  4.75  4.07
phi3  8.43  6.49  5.47
```

Figure 6. Unit root test for the first difference of interest rate.

```
Residuals:
     Min       1Q   Median       3Q      Max
-2.32350 -0.32561  0.00765  0.25920  2.44954

Coefficients:
              Estimate Std. Error t value Pr(>|t|)
(Intercept)  0.0596537  0.1196868   0.498   0.6188
z.lag.1     -0.9218743  0.1541649  -5.980 1.23e-08 ***
tt          -0.0007235  0.0011008  -0.657   0.5119
z.diff.lag1  0.1482927  0.1312575   1.130   0.2601
z.diff.lag2  0.0043553  0.1156306   0.038   0.9700
z.diff.lag3  0.0858102  0.0925406   0.927   0.3551
z.diff.lag4 -0.1317997  0.0724989  -1.818   0.0708 .
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 0.7757 on 175 degrees of freedom
Multiple R-squared:  0.4617,    Adjusted R-squared:  0.4432
F-statistic: 25.01 on 6 and 175 DF,  p-value: < 2.2e-16


Value of test-statistic is: -5.9798 11.9201 17.8796

Critical values for test statistics:
      1pct  5pct 10pct
tau3 -3.99 -3.43 -3.13
phi2  6.22  4.75  4.07
phi3  8.43  6.49  5.47
```

From the results above, test statistics for the log(IP Index) and interest rates are -3.557 and -2.624, respectively, which are less than the α=0.05 critical value -3.43. Thus, we fail to reject the null hypothesis, implying that there is a unit root in log(IP Index) and interest rate series. We might state the series are non-stationary. On the other hand, test statistics for the differenced variables are greater than the critical value, implying that we reject the null hypothesis and therefore, there is no unit root in these series.

It is important to note that the unit root in time series implies the serial correlation, but not vice versa. Thus, we donated serial correlation in the previous Empirical Project for gIP and dIR variables, but rejected the presence of unit root.

As the unit root test was done, one-step-ahead predictions were generated. To begin with, the data were divided into two parts such that n=0.75T and m=T-n, where T=total number of observations: first n=141 observations are used to reestimate the model and last m=47 are used to generate one-step-ahead predictions.

Model 1: ARDL(6,6) for the first difference of log(IP Index) and first difference of interest rate.

$$\widehat{gIP} = \alpha + \gamma_1 gIP\_1 + \gamma_2 gIP\_2 + \gamma_{3^3} gIP\_3 + \gamma_4 gIP\_4 + \gamma_5 gIP\_5 + \gamma_6 gIP\_6 + \beta_1 dIR\_1 + \beta_2 dIR\_2 + \beta_3 dIR\_3 + \beta_4 dIR\_4 + \beta_5 dIR\_5 + \beta_6 dIR\_6 + u_t$$

Model 2: AR(4) for the first difference of log(IP Index)

$$\widehat{gIP} = \beta_0 + \beta_1 gIP\_1 + \beta_2 gIP\_2 + \beta_3 gIP\_3 + \beta_4 gIP\_4 + u_t$$

Model 3: AR(1) for the first difference of log(IP Index)

$$\widehat{gIP} = \beta_0 + \beta_1 gIP\_1 + u_t$$

To compare each of the four models out-of-sample root mean squared error (RMSE) and mean absolute error (MAE) for each model were calculated.

Table 1. RMSE and MAE values.

Model	**RMSE value**	**MAE value**
Model 1: ARDL(6,6)	0.03326386	0.02755496
Model 1: gIP AR(4)	0.03947916	0.03263953
Model 2: gIP AR(1)	0.04545096	0.03912331

It is important to note that the models with the lowest RMSE and MAE are more precise for forecasting the explanatory variable, that is, these models have lower error terms between predicted and actual values. Consequently, the ARDL(6,6) model is more appropriate to forecast the growth rate of the industrial production index and is more precise than the AR(4) model by 18.68% and 18.45% concerning RMSE and MAE, respectively.