We need to prove that cosine distance satisfies triangle inequality

$$\rho(x,y) = 1 - \frac{x^T y}{|x||y|}$$

Let prove:

$$\rho(A,C) \leq \rho(A,B) + \rho(B,C)$$

$$1 - \frac{A^T C}{|A||C|} \leq \cancel{\beta} 1 - \frac{|A^T \cdot B}{|A| \cdot |B|} + 1 - \frac{\cancel{B}^T C}{|B||C|}$$

Let take: $A = (1;0)$ $B = \left(\frac{\sqrt{2}}{2}; \frac{\sqrt{2}}{2}\right)$ $C = (0;1)$

$$1 + \frac{A^T C}{|A||C|} \geq \frac{B^T C}{|\cancel{B}||C|} + \frac{A^T B}{|A||B|}$$

$$\frac{A^T C}{|A||C|} = \frac{0}{\sqrt{1}\sqrt{1}} = 0 \qquad \frac{B^T C}{|B||C|} = \frac{\frac{\sqrt{2}}{2} + 0}{\sqrt{1}\sqrt{1}} = \frac{\sqrt{2}}{2}$$

$$\frac{A^T B}{|A||B|} = \frac{\frac{\sqrt{2}}{2} + 0}{\sqrt{1}\sqrt{1}} = \frac{\sqrt{2}}{2}$$

$1 + 0 \not\geq \sqrt{2}$

contradiction