Exam Date & Time: 02-Dec-2024 (09:30 AM - 12:30 PM)

MANIPAL ACADEMY OF HIGHER EDUCATION

THIRD SEMESTER B.TECH. ((COMPUTER SCIENCE & ENGINEERING)/INFORMATION TECHNOLOGY)) EXAMINATIONS - NOVEMBER / DECEMBER 2024
SUBJECT: MAT 2126/MAT_2126- ENGINEERING MATHEMATICS - III

ENGINEERING MATHEMATICS - III [MAT 2126]

Marks: 50 **Duration: 180 mins.**

A

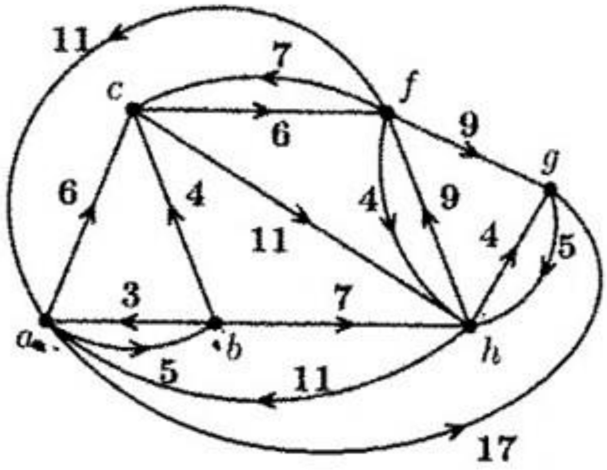
Answer all the questions.

1A) Draw any two self-complementary graphs with 5 vertices and find the diameter of those graphs. (3)

1B) Prove that a graph G is a tree if and only if between every pair of vertices there exist a unique path. (3)

1C) i) Suppose two graphs, G and H, are Eulerian. If an arbitrary vertex of G is made adjacent to an arbitrary vertex in H, is the new graph Eulerian? Give reason. (4)
ii) Show that $(\neg p \wedge (\neg q \wedge r)) \vee (q \wedge r) \vee (p \wedge r) \Leftrightarrow r$.

2A) Using Dijkstra's algorithm, find the shortest path from **b** to all other vertices for the network given below. (3)



2B) Let $G = \mathbb{R} - \{-1\}$ ($\mathbb{R}$ is the set of real numbers). Define $*$ on G as $a * b = a + b + ab$. Then show that $(G, *)$ forms a group. Is G Abelian? Justify your answer. (3)

2C) Show that $Z_5 = \{0,1,2,3,4\}$ forms a cyclic group under the operation addition modulo 5. List all the generators of Z_5. (4)

3A) Prove that a nonempty subset H of a group (G, $*$) is a subgroup of G if and only if $a * b^{-1} \in H$ for every $a, b \in H$. (3)

3B) If repetition is not allowed, how many four digitnumbers can be formed from the digits 1, 2, 3, 5, 7, 8. How many of the numbers are lesser than 4000? (3)

3C) How many ways are there to distribute 27 identical jellybeans among three children (4)
a) Without restrictions
b) With each child getting at least 5 beans

4A) For any a, b, c, d in a lattice $(A, \leq)$, if $a \leq b$ and $c \leq d$ then show that $a \vee c \leq b \vee d$ and $a \wedge c \leq b \wedge d$. (3)

4B) Let $E(x_1, x_2, x_3) = \overline{(x_1 \wedge x_2)} \vee (\overline{(x_1 \wedge x_3)} \vee x_2)$ be a Boolean expression over the two-valued Boolean algebra. Write $E(x_1, x_2, x_3)$ in both DNF and CNF. (3)

4C) Find the following permutations for n = 5 with initial permutation 12345. (4)
i) 90th permutation in lexicographical order
ii) 90th permutation in reverse lexicographical order
iii) 100th permutation in Fike's order

5A) Consider the following premises: (3)

If there was a ball game, then travelling was difficult.
If they arrived on time, then travelling was not difficult.
They arrived on time.

Using propositional inference theory, show that there was no ball game.

5B) Prove that $(\exists x)(P(x) \wedge Q(x)) \Rightarrow (\exists x)\, P(x) \wedge (\exists x)\, Q(x)$. (3)

5C) Let $(L, \leq)$ be a distributive lattice. Prove that for any elements $x, y \in L$, if $a \vee x = a \vee y$ and $a \wedge x = a \wedge y$ for some $a \in L$, then $x = y$. (4)

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