power calculations

1. Thrust Power

The primary requirement for the main engine thrust is that it must be able to **counteract the gravitational force** acting on the rocket when it is fully fueled. The maximum total mass of the rocket is:

$$m_{\text{total}} = m_{\text{dry}} + m_{\text{fuel}} = 38,000 \text{ kg} + 410,000 \text{ kg} = 448,000 \text{ kg}$$

The corresponding gravitational force is:

$$F_{\text{gravity}} = m_{\text{total}} \cdot |g| = 448,000 \cdot 9.81 \approx 4.395 \times 10^6 \text{ N}$$

To hover, the **thrust force must match or exceed** this value. I set:

$$\text{thrust\_power} = 5 \times 10^6 \text{ N}$$

This provides a margin above the gravitational force, allowing the rocket to:

- Hover at partial throttle (around 88% throttle at full mass)
- Accelerate upward for landing burns when needed
- Simulate throttle control dynamically based on mass depletion

This is physically realistic and matches thrust values for heavy-lift first stages like Falcon 9, which uses 7-8 MN at sea level.

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2. Cold Gas Thrust Power

Cold gas thrusters provide torque for attitude control. To determine the required **torque**, I estimate the moment of inertia of the rocket as a tall vertical cylinder (a reasonable approximation):

$$I \approx \frac{1}{2} M R^2$$

Assuming
$M \approx 4.48 \times 10^5 \text{ kg}$
and
$R \approx 3 \text{ m}$,
we get:

$$I \approx \frac{1}{2} \cdot 4.48 \times 10^5 \cdot 9 = 2.016 \times 10^6 \text{ kg·m}^2$$

To control the rocket orientation reasonably, we may want an angular acceleration on the order of
$\alpha \sim 0.5 \text{ deg/s}^2$,
which in radians is:

$$\alpha \approx \frac{0.5 \cdot \pi}{180} \approx 8.73 \times 10^{-3} \text{ rad/s}^2$$

The required torque is:

$$\tau = I \cdot \alpha \approx 2.016 \times 10^6 \cdot 8.73 \times 10^{-3} \approx 17,600 \text{ N·m}$$

Assuming the cold gas jets are mounted 3 meters from the center, the required force is:

$$F = \frac{\tau}{r} = \frac{17,600}{3} \approx 5,867 \text{ N}$$

Thus, I chose:

$$\text{cold\_gas\_thrust\_power} = 5000 \text{ N}$$

This provides sufficient angular control authority for realistic reorientation maneuvers without overcorrecting.

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3. Fuel Consumption Rate

Rocket engines consume fuel at a rate governed by their **specific impulse (Isp)**:

$$\dot{m} = \frac{F}{I_{\text{sp}} \cdot g_0}$$

For a Merlin-class engine with
$I_{\text{sp}} \approx 300\,\text{s}$,
and using the chosen thrust:

$$\dot{m} = \frac{5 \times 10^6}{300 \cdot 9.81} \approx 1,698\,\text{kg/s}$$

Hence, I set:

$$\text{fuel\_consumption\_rate} = 1700\,\text{kg/(s} \cdot \text{throttle)}$$

This ensures:

- The fuel depletes over a realistic timescale (e.g., full burn over ~230 s)
- The mass changes over time are consistent with what you'd expect for orbital-class rockets

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4. Time Step

I used a time step of:

$$\text{time\_step} = 0.1\,\text{s}$$

This value balances two needs:

- It is small enough for accurate integration using **Verlet integration**, which is second-order accurate.
- It is large enough to keep the simulation performant over up to 1000 steps (~100 s of simulation time).

The rocket descends from approximately 2300 m with a vertical speed of -240 m/s, so it would reach the ground in:

$$t_{\text{fall}} \approx \frac{2300}{240} \approx 9.6\,\text{s}$$

Thus, 0.1 s time steps allow for ~100 points of resolution during descent, which is adequate for resolving dynamics like thrust changes, drag, and attitude control.

Summary

Each parameter was derived to be consistent with the physics of:

- A 400,000+ kg launch vehicle
- Realistic thrust-to-weight ratios
- Proper fuel burn timing based on ISP
- Angular control based on inertial torque needs
- A numerically stable integration scheme