10350 ALG Gateway 03 Name ______________________________ Section __________

1. Solve for all x at which the following curves intersect:

$$y = 3\sqrt{x-1}; \qquad y = x+1$$

Solution:

$$\begin{aligned} 3\sqrt{x-1} &= x+1 \\ 9(x-1) &= (x+1)^2 \\ 9x-9 &= x^2+2x+1 \\ 0 &= x^2-7x+10 \\ 0 &= (x-5)(x-2) \end{aligned}$$

2. Let $\ln(x) = a$ and $\ln(y) = b$. Express the following expression in terms of a and b simplifying your answer as far as possible.

$\ln\left(e^3 \cdot \sqrt{\dfrac{x^3}{y^4}}\right) \stackrel{?}{=}$

Solution:

$$\begin{aligned} \ln\left(e^3 \cdot \sqrt{\frac{x^3}{y^4}}\right) &= \ln(e^3) + \ln\left(\left(\frac{x^3}{y^4}\right)^{(1/2)}\right) \\ &= 3 + \frac{1}{2}\ln\left(\frac{x^3}{y^4}\right) \\ &= 3 + \frac{1}{2}\ln(x^3) - \frac{1}{2}\ln(y^4) \\ &= 3 + \frac{3}{2}\ln(x) - 2\ln(y) = 3 + \frac{3}{2}a - 2b \end{aligned}$$

3. Simplify the following expression giving your answer in the form $\dfrac{ax+b}{(cx+d)^k}$ where a, b, c, d and k are all constants.

$$\frac{(3x+5)^{5/2}\cdot 2-(2x-4)\cdot\dfrac{5}{2}(3x+5)^{3/2}\cdot 3}{(3x+5)^5}\stackrel{?}{=}$$

Solution:

$$\begin{aligned}
&\frac{(3x+5)^{5/2}\cdot 2-(2x-4)\cdot\dfrac{5}{2}(3x+5)^{3/2}\cdot 3}{(3x+5)^5}\\
&=\frac{(3x+5)\cdot 2-(2x-4)\cdot\dfrac{5}{2}\cdot 3}{(3x+5)^{7/2}}\\
&=\frac{6x+10-(15x-30)}{(3x+5)^{7/2}}\\
&=\frac{-9x+40}{(3x+5)^{7/2}}
\end{aligned}$$

4. Completely factor the expression below:

$9x^4 - 37x^2 + 4 \stackrel{?}{=}$
Solution:

$$\begin{aligned} 9x^4 - 37x^2 + 4 &= (9x^2 - 1)(x^2 - 4) \\ &= (3x - 1)(3x + 1)(x^2 - 4) \\ &= (3x - 1)(3x + 1)(x - 2)(x + 2) \end{aligned}$$

5. Express x in terms of y if they are related by:

$\dfrac{e^x - 1}{2e^x + 1} = y.$
Solution:

$$\begin{aligned} \frac{e^x - 1}{2e^x + 1} &= y \\ e^x - 1 &= 2ye^x + y \\ (1 - 2y)e^x &= 1 + y \\ e^x &= \frac{1 + y}{1 - 2y} \\ x &= \ln\left(\frac{1 + y}{1 - 2y}\right) = \ln(1 + y) - \ln(1 - 2y) \end{aligned}$$