PRACTICE QUIZ 12 SOLUTIONS

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Time: 12 min
Time to beat: ? min

Problem 1. Find the derivative of $f(x) = 4x^8 - \frac{5}{x^3} + 2x^{18}$.

$$f'(x) = \frac{15}{x^4} + 32x^7 + 36x^{17}$$

Problem 2. If $h(x) = 2f(x) - 4g(x)$ and $f(1) = 2$, $g(1) = 4$, $f'(1) = -1$, $g'(1) = 4$, then what is $h'(1)$?

We have $h'(x) = 2f'(x) - 4g'(x)$, so by substituting we have

$$h'(1) = 2f'(1) - 4g'(1) = -18$$

Problem 3. Find the derivative for $f(x) = \frac{x^2-6x-3}{\sqrt[5]{x^3}}$.

Write the function as $f(x) = x^{7/5} - 3x^{-3/5} - 6x^{2/5}$ so by the power rule the derivative is

$$f'(x) = \frac{7}{5}x^{2/5} - \frac{12}{5}x^{-3/5} + \frac{9}{5}x^{-8/5}$$

Problem 4. Using the fact that $\lim_{x\to 0} \frac{1-\cos(x)}{x^2} = 1/2$, find the limit $\lim_{x\to 0} \frac{1-\cos(2x)}{\sin 5x^2}$

Using the hint we get that

$$\lim_{x\to 0} \frac{1-\cos(2x)}{4x^2} = 1/2$$

so we need to have a $4x^2$ in the denominator. At the same time, to take care of the sine, we need a $5x^2$ in the numerator.

$$\lim_{x\to 0} \frac{1-\cos(2x)}{\sin 5x^2} = \lim_{x\to 0} \frac{1-\cos(2x)}{\sin 5x^2} \cdot \frac{5x^2}{5x^2} \frac{4x^2}{4x^2}$$

and if we group things appropriately this is

$$\lim_{x\to 0} \frac{1-\cos(2x)}{4x^2} \cdot \frac{5x^2}{\sin 5x^2} \cdot \frac{4x^2}{5x^2} = \frac{1}{2} \cdot 1 \cdot \frac{4}{5} = \frac{2}{5}$$

Problem 5. Find the equation of the tangent line to the curve $y = 2\tan x$ at $x = \frac{\pi}{4}$.

The derivative $y' = 2\sec^2 x$ evaluated at $\pi/4$ is

$$\frac{2}{\cos^2(\pi/4)} = \frac{2}{(\sqrt{2}/2)^2} = 4$$

and the original function evaluated at $\pi/4$ is 2, so putting all this together we get

$$y = 4\left(x - \frac{\pi}{4}\right) + 2 = 4x - \pi + 2$$