QUIZ 3 SOLUTIONS

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Time: 15 minutes

Problem 1. Evaluate the limit $\lim_{x\to 2} \dfrac{x-2}{\sqrt{x^2-4}}$. (Hint: rationalize the denominator.)

(a) -2 (b) 2 (c) 0 (d) ∞ (e) $-\infty$

We multiply by $\frac{\sqrt{x^2-4}}{\sqrt{x^2-4}}$ to rationalize, giving us

$$\lim_{x\to 2} \frac{(x-2)\sqrt{x^2-4}}{x^2-4} = \lim_{x\to 2} \frac{(x-2)\sqrt{x^2-4}}{(x-2)(x+2)} = \lim_{x\to 2} \frac{\sqrt{x^2-4}}{x+2} = 0$$

so the correct answer is (c).

Problem 2. Evaluate $\lim_{x\to+\infty} \dfrac{7x^9-4x^5+2x-13}{-3x^9+x^8-5x^2+2x}$.

(a) $\frac{7}{3}$ (b) 0 (c) $\frac{-7}{3}$ (d) ∞ (e) $-\infty$

We look at the leading terms (the terms with the highest exponent in the numerator and denominator), so our limit is the same as

$$\lim_{x\to+\infty} \frac{7x^9}{-3x^9} = -\frac{7}{3}$$

so the correct answer is (c).

Problem 3. For which value of c is the function $f(x)$ continuous at $x = 4$?

$$f(x) = \begin{cases} c^2 - 2cx + 3x & x \leq 4 \\ \frac{cx}{-2} - x + 7 & x > 4 \end{cases}$$

(a) 7 (b) 1 (c) 3 (d) -1 (e) -3

We want the left and right limits be equal. To do so, we plug in $x = 4$ for the top and bottom branches (they are both polynomials so they are continuous, and the limits equal the function values), and set them equal to each other, giving us

$$c^2 - 8c + 12 = -2c - 4 + 7$$

which is the same as $c^2 - 6c + 9 = 0$. This factors as a perfect square $(c-3)^2 = 0$, so $c = 3$ is the unique solution that makes f continuous. So the correct answer is (c).

Problem 4. Compute the limit $\lim_{x\to-\infty} \dfrac{\sin x}{x}$. (Hint: Squeeze theorem).

(a) $-\infty$ (b) ∞ (c) 1 (d) -1 (e) 0

Notice that

$$\frac{-1}{x} \leq \frac{\sin x}{x} \leq \frac{+1}{x}$$

and that $\lim_{x\to-\infty} \frac{-1}{x} = \lim_{x\to-\infty} \frac{+1}{x} = 0$, so by Squeeze theorem

$$\lim_{x\to-\infty} \frac{\sin x}{x} = 0$$

and the correct answer is option (e).