QUIZ 2 SOLUTIONS

ADRIAN PĂCURAR

Time: 15 minutes

Problem 1. If $f(1) = 3$, $f(2) = 4$, $g(1) = 2$, $g(3) = 2$, what is $(f \circ g)(1) - (g \circ f)(1)$?

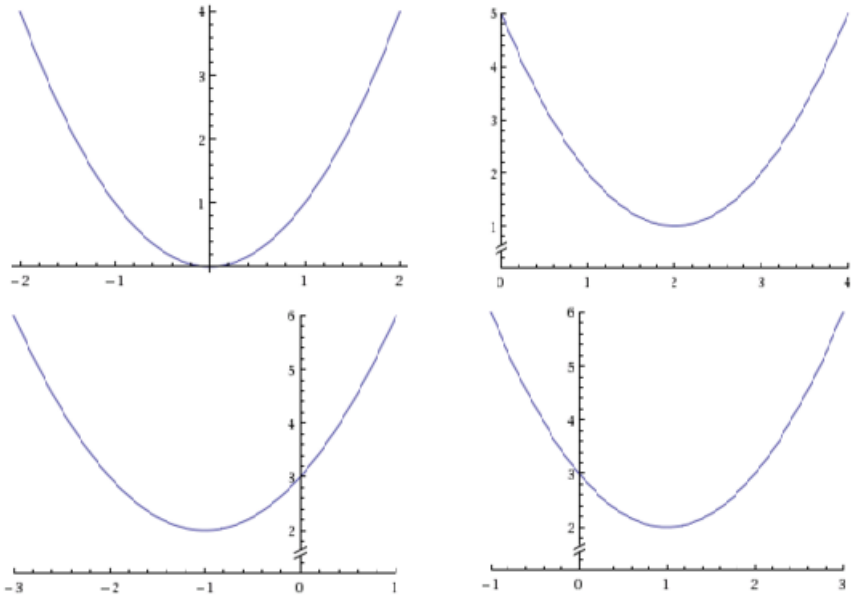
(a) 2 (b) -2 (c) 8 (d) 0 (e) π

This is a composition of functions:

$$(f \circ g)(1) - (g \circ f)(1) = f(g(1)) - g(f(1)) = f(2) - g(3) = 4 - 2 = 2$$

so the correct answer is (a).

Problem 2. Which of the following graphs most closely resemble the curve $y = (x-1)^2 + 2$?



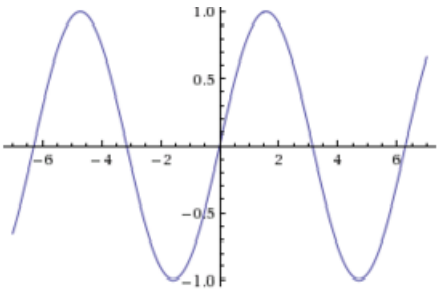
We have a right shift of 1 unit, and an up shift of 2 units. The correct answer is the last graph (lower right).

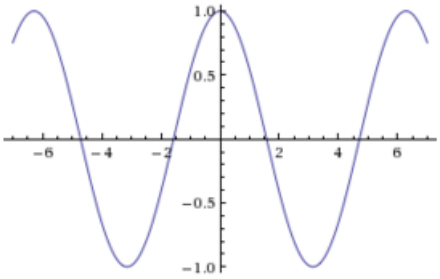
Problem 3. Which of the following functions has a graph which can be obtained from $y = x^3$ by shifting to the RIGHT 1 unit, then reflecting about the x-axis?

(a) $y = -x^3 + 1$ (b) $y = -(x-1)^3$ (c) $y = (-x-1)^3$ (d) $y = -x^3 - 1$

First, if we shift to the right 1 unit, we are looking at $(x-1)^3$. But then we reflect this about x, which can be done by adding a minus sign in front, giving us $-(x-1)^3$, which is option (b).

Problem 4. Recall that the graphs of $\sin x$ and $\cos x$ are related by a horizontal shift, as you can see in the diagrams below. Which of the given identities is true?





(a) $\sin x = -\cos(x)$ (b) $\sin x = \cos(x - \frac{\pi}{2})$ (c) $\sin x = \cos(x + \frac{\pi}{2})$

(d) $\sin x = \cos(x - \pi)$ (e) $\sin x = \cos(x) - \pi$

The graph of $\sin x$ can be obtained by shifting $\cos x$ to the RIGHT $\pi/2$ units, so we are subtracting $\pi/2$ from the variable. In other words

$$\sin x = \cos(x - \frac{\pi}{2})$$

which is option (b).

Problem 5. Compute $\log_4(16)$.

(a) 0 (b) 1 (c) 2 (d) 3 (e) 4

Since we need to raise 4 to the second power to get 16, the correct answer is (c) 2.

Problem 6. Using the properties of logarithms

$$\log(a \cdot b) = \log a + \log b \qquad c \cdot \log b = \log(b^c),$$

compute $\ln(e^e \cdot e^3) - e$.

(a) $3 - e$ (b) 3 (c) $2e$ (d) $-e$ (e) 0

$$\ln(e^e \cdot e^3) - e = \ln(e^e) + \ln(e^3) - e = e + 3 - e = 3$$

Problem 7. (BONUS) Compute $\log_8(2)$.

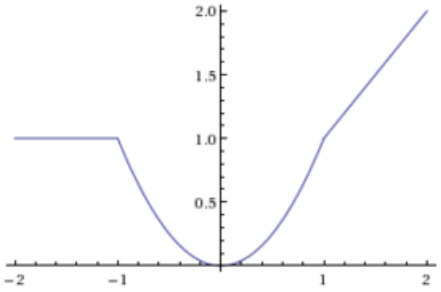
(a) 2 (b) 1 (c) $\frac{1}{2}$ (d) $\frac{1}{3}$ (e) 3

Since we need to raise 8 to the power 1/3 in order to obtain 2 (the cubic root of 8 is 2), the correct answer is (d) 1/3.

Problem 8. Sketch the graph of the piecewise function:

$$f(x) = \begin{cases} 1 & x < -1 \\ x^2 & -1 \le x \le 1 \\ x & x > 1 \end{cases}$$

The graph is shown below for $-2 \le x \le 2$:



Problem 9. Sketch the graph of the piecewise function:

$$g(x) = \begin{cases} -1 & x < -1 \\ x^3 & -1 \le x \le 1 \\ -x & x > 1 \end{cases}$$

The graph is shown below for $-2 \le x \le 2$: