QUIZ 4 SOLUTION

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Time: 15 minutes

Problem 1. Evaluate the limit $\lim_{t\to 0} \dfrac{\sqrt{1+t}-\sqrt{1-t}}{t}$

(a) $\dfrac{1}{2\sqrt{1+t}}$ (b) $\dfrac{1}{2\sqrt{1-t}}$ (c) $\dfrac{1}{2}$ (d) 1 (e) 2

$$\lim_{t\to 0} \frac{\sqrt{1+t}-\sqrt{1-t}}{t} = \lim_{t\to 0} \frac{\sqrt{1+t}^2-\sqrt{1-t}^2}{t(\sqrt{1+t}+\sqrt{1-t})} = \lim_{t\to 0} \frac{2t}{t(\sqrt{1+t}+\sqrt{1-t})} = 1$$

Problem 2. Is there a number x that is exactly 1 more than it's cube? (Hint: set up and equation, and figure out if it has any solutions using the Intermediate Value Theorem)

(a) Yes (b) No

Since the number x needs to be 1 more than it's cube x^3, if we subtract 1 from x the two should be equal. Hence we get the equation

$$x^3 = x - 1$$

which we rearrange as

$$x^3 - x + 1 = 0$$

Then the original question is the same as trying to determine if the function $f(x) = x^3 - x + 1$ has any zeroes. Notice

$$f(0) = 1 \quad \text{and} \quad f(-2) = -5$$

and since f is a polynomial, it is continuous. Hence by the IVT, there exists (at least) a zero on the interval $(-2, 0)$, so the correct answer is Yes, such a number exists.

Problem 3. What is the equation of the tangent line to the curve $y = \sqrt{x}$ at the point $(1, 1)$?

(a) $y = \dfrac{1}{2}x - \dfrac{1}{2}$ (b) $y = 2x - 2$ (c) $y = \dfrac{1}{2}x + \dfrac{1}{2}$ (d) $y + 1 = \dfrac{1}{2}(x + 1)$

Rewrite $y = x^{1/2}$ and apply Power Rule to take the derivative

$$y' = \frac{1}{2}x^{1/2-1} = \frac{1}{2}x^{-1/2} = \frac{1}{2\sqrt{x}}$$

At $(1,1)$, the derivative is equal to $y'(1) = \frac{1}{2}$, and we are given the point $(1,1)$, so now we can write the equation of the line using the point-slope form. This is

$$y - 1 = \frac{1}{2}(x-1)$$

A little algebra shows this is the same as

$$y = \frac{1}{2}x + \frac{1}{2}$$

so the correct answer is (c).

Problem 4. Compute the limit $\lim_{h\to 0} \frac{e^{x+h} - e^x}{h}$. (Hint: definition of derivative).

(a) ∞ (b) 0 (c) x (d) e^x (e) e

We know that for a differentiable function $f(x)$, the derivative is given by

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

In our case, we recognize that the function at hand is $f(x) = e^x$, and we know the derivative of e^x is e^x, so the limit in question can be interpreted as the derivative of e^x, i.e.

$$\lim_{h\to 0} \frac{e^{x+h} - e^x}{h} = e^x$$

so the correct answer is (d).