PRACTICE QUIZ 8 SOLUTIONS

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Time: 16 min
Time to beat: ? min

Problem 1. Find dy/dx by implicit differentiation for $5x^2y^2 + 4x^2 + 4y^5 = -4$.

I won't write out the entire detailed solution. You should get

$$y' = -\frac{5xy^2 + 4x}{5x^2y + 10y^4}$$

Problem 2. Find dy/dx by implicit differentiation for $3x \cos y + 5y \cos x = 1$.

You should get something equivalent to

$$y' = \frac{5y \sin x - 3 \cos y}{5 \cos x - 3x \sin y}$$

Problem 3. Find y'' by implicit differentiation if $2x^8 + 2xy + 5y^8 = 1$.

This problem is horrible, I wouldn't wish it upon my worst enemies. First, using implicit differentiation, you get

$$y' = -\frac{8x^7 + y}{20y^7 + x}$$

which you then differentiate using quotient rule and get

$$-\frac{(56x^6 + y')(20y^7 + x) - (140y^6y' + 1)(8x^7 + y)}{(20y^7 + x)^2)}$$

but then you have to substitute y' in the above expression and simplify:

$$-\frac{2(2080x^7y^7 + 11200x^6y^{14} + 4480x^{14}y^6 - xy + 20x^8 + 50y^8)}{(20y^7 + x)^3}$$

Yeah...

Problem 4. A particle moves according to $s(t) = t^3 - 9t^2 + 24t + 2$, $t \geq 0$, where t is measured in seconds and s in feet. What is the total distanc this particle traveled during the first 6 seconds? (Hint: this is not the same as the displacement!)

The velocity is $v(t) = s'(t) = 3t^2 - 18t + 24$. As the hint warns us, we need to decide when the particle moves forward ($v > 0$) and when it moves backward ($v < 0$). Factor v to get $v(t) = 3(t - 4)(t - 2)$, so it has zeroes at $t = 4$ and $t = 2$.

Plug in values in the intervals $[0, 2)$, $(2, 4)$, and $(4, 6)$ to get that the velocity is first positive (forward motion), then negative (backward motion), then positive (forward motion) again in these respective intervals (could also tell this by the fact that v is a parabola opening up, so if it has two zeroes, it must dip below the x axis and be negative between its zeroes).

In the interval $[0, 2]$, the distance it moves forward is $s(2) - s(0) = 20$. In the interval $[2, 4]$, the distance it moves backward is $s(2) - s(4) = 4$. In the interval $[4, 6]$, it moves forward again with a distance of $f(6) - f(4) = 20$.

So the total distance traveled is $20 + 4 + 20 = 44$.