Name: Date: 03/29/2018

M20580 L.A. and D.E. Tutorial
Quiz 7

1. Use the Gram-Schmidt process to find an orthogonal basis for the span of the vectors

$$\mathbf{x}_1 = \begin{bmatrix}1\\1\\1\\1\end{bmatrix}, \quad \mathbf{x}_2 = \begin{bmatrix}1\\0\\0\\1\end{bmatrix}, \quad \mathbf{x}_3 = \begin{bmatrix}0\\2\\1\\-1\end{bmatrix}.$$

$$\alpha_1 = \vec{x}_1 = \begin{bmatrix}1\\1\\1\\1\end{bmatrix}$$

$$\alpha_2 = \vec{x}_2 - \frac{\vec{x}_2 \cdot \alpha_1}{\alpha_1 \cdot \alpha_1}\alpha_1 = \begin{bmatrix}1\\0\\0\\1\end{bmatrix} - \frac{2}{4}\begin{bmatrix}1\\1\\1\\1\end{bmatrix} = \begin{bmatrix}\frac{1}{2}\\-\frac{1}{2}\\-\frac{1}{2}\\\frac{1}{2}\end{bmatrix}$$

$$\alpha_3 = \vec{x}_3 - \frac{\vec{x}_3 \cdot \alpha_1}{\alpha_1 \cdot \alpha_1}\alpha_1 - \frac{\vec{x}_3 \cdot \alpha_2}{\alpha_2 \cdot \alpha_2}\alpha_2 = \begin{bmatrix}0\\2\\1\\-1\end{bmatrix} - \frac{2}{4}\begin{bmatrix}1\\1\\1\\1\end{bmatrix} - \frac{-2}{1}\begin{bmatrix}\frac{1}{2}\\-\frac{1}{2}\\-\frac{1}{2}\\\frac{1}{2}\end{bmatrix}$$

$$= \left[\frac{1}{2}, \frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}\right]^T$$

2. Find the orthogonal projection of $\mathbf{x}_1 = \begin{bmatrix}1\\2\\3\\4\end{bmatrix}$ onto the line spanned by $\mathbf{x}_2 = \begin{bmatrix}3\\2\\2\\-1\end{bmatrix}$. Thus, your result should be a multiple of $\mathbf{x}_2$.

$$\text{proj}_{\text{span}\{x_2\}} x_1 = \frac{x_1 \cdot x_2}{x_2 \cdot x_2} \cdot x_2$$

$$= \frac{1\cdot 3 + 2\cdot 2 + 3\cdot 2 + 4\cdot(-1)}{3\cdot 3 + 2\cdot 2 + 2\cdot 2 + (-1)\cdot(-1)} x_2$$

$$= \frac{9}{18} x_2 = \frac{1}{2} x_2 = \begin{bmatrix}\frac{3}{2}\\1\\1\\-\frac{1}{2}\end{bmatrix}$$