

Name: _____

June 30, 2017

FINITE MATH: EXAM 1

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- The Honor Code is in effect for this exam. All work must be your own.
- Please turn off all cellphones or any other electronic devices.
- Calculators are NOT allowed.
- There are **80 points available** for you to try. You may choose to attempt any of the problems, or all the problems. There is no penalty for getting a wrong answer.
- The exam will be **graded out of 60**. You can NOT get more than 60 points on this exam.
- You are allowed a single-sided 8 by 11 formula sheet for the exam; the formula sheet must be handwritten. You must **turn in your formula sheet with your exam**.
- The exam lasts **1 hour and 15 minutes**.

Problem 1. Consider the Venn diagram on the right.

a) (2pt) Find $n(U)$.

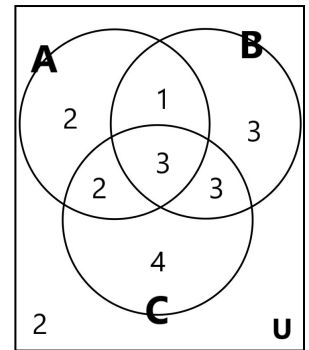
b) (2pt) Find $n(A \cap B)$.

c) (3pt) Find $n(A' \cup B')$.

d) (2pt) Compute the size of $(B \cup C) \setminus A$.

e) (2pt) Compute the size of $(A' \cup C')' \cap B$.

f) (2pt) Compute the size of $(A \cup B \cup C)'$.

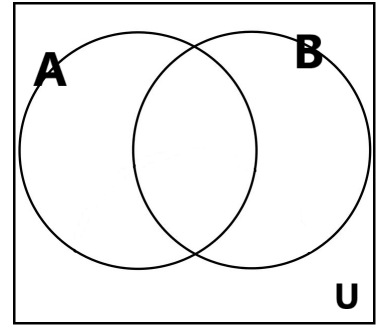


Problem 2. (1pt) How many subsets of size 5 does the set $\{1, 2, 3, \dots, 13\}$ have?

Problem 3. (4pt) Suppose we have the universal set $U = \{1, 2, 3, \dots, 11, 12\}$, and consider the following sets:

$$A = \{1, 2, 3, 4, 9\} \quad \text{and} \quad B = \{3, 4, 5, 7, 9, 11\}$$

Fill in the Venn diagram shown on the right with the correct **number of elements** for each region.



Problem 4. In our finite math class, we have 7 women and 8 men.

a) (1pt) In how many ways can we arrange everyone in a **single row** for a class photo?

b) (3pt) In how many ways can we arrange the students in **two rows** for a class photo, if all the women are in the front row, and all the men are in the back row?

Problem 5. A multiple choice exam consists of 25 TRUE/FALSE questions.

a) (1pt) In how many ways can the exam be answered students must answer every question (no questions can be left unanswered)?

b) (1pt) In how many ways can the exam be answered if students have the option of leaving questions blank.

c) (2pt) In how many ways can the exam be answered if a student marks exactly 10 of the answers as TRUE, and the remaining answers as FALSE?

Problem 6. A coin is flipped 10 times in a row, and the result on each flip is recorded.

- a) (1pt) How many possible outcomes are there **for a single flip**?

- b) (1pt) How many possible outcomes are there for the 10 flips in a row?

- c) (2pt) After 10 flips, in how many ways can you get exactly 5 Heads and 5 Tails?

- d) (2pt) After 10 flips, in how many ways can you get **at least one** Tail?

Problem 7. Dana decides to plant either a rose bush or a small tree in her back yard. Home Depot has 4 varieties of rose bushes, and 7 varieties of small trees.

- a) (1pt) In how many ways can she select **a single plant**?

- b) (1pt) In how many ways can she select **one plant of each type** (so she ends up with 2 plants total)?

Problem 8. (2pt) A classroom is split into two separate groups (they don't necessarily have equal sizes). The teacher selects two students, one from each group, and the number of ways in which he can select the students is 35. How many students are in each group?

Problem 9. (2pt) In how many different ways can you rearrange the letters of the word CALCULUS?

Problem 10. A child forms **3-letter words** using letters from the word CALCULUS. Letters may be repeated.

- a) (1pt) How many different words can be formed if letters may be repeated?
- b) (1pt) How many different words begin with 'C'?
- c) (1pt) How many different words begin with a vowel?
- d) (1pt) How many different words end with a vowel?
- e) (3pt) How many different words begin with a vowel or end with a vowel, **but not both**?
- f) (1pt) How many 3-letter words can be formed if letters may NOT be repeated?

Problem 11. (3pt) In how many different ways can 7 people be seated at a **round table**, if any rotation of the table is considered to be the same arrangement?

Problem 12. (3pt) In how many ways can 8 different pairs of shoes be put on display **in a single row** if the pairs are to appear in the correct order (left shoe before right shoe, not the other way around)?

Problem 13. (3pt) Use the Binomial Theorem and Pascal's triangle to expand the product $(x + y)^6$. **Compute the coefficients** (don't leave them in "choose" form).

Problem 14. (3pt) A 4-digit PIN number is formed using the digits $\{1, 2, 3, 4, 5\}$. How many numbers can be formed if the first 2 digits may be repeated, but the last 2 digits must be unique?

Problem 15. (3pt) A die is rolled five times, and the **sum** of the two numbers is recorded. In how many ways can one get a sum of 6?

Problem 16. Suppose you select **six cards** from a standard deck of 52 cards.

- a) (1pt) How many different 6-card samples are possible?
- b) (1pt) How many different 6-card samples contain the four Queens?
- c) (3pt) How many different 6-card samples contain 3 Hearts and 3 Black cards?
- d) (3pt) How many different 6-card samples contain at least one red card?

Problem 17. (3pt) Billy has 12 baseball cards, and Scottie only has 2. Billy owes Scottie \$5, but instead Scottie would prefer to trade 2 of his own cards for 4 of Billy's cards. In how many ways can the trade be made?

Problem 18. (3pt) How many positive factors/divisors does the number 3,888 have? It may be useful to know that the prime factorization of 3,888 is $2^4 \cdot 3^5$.

Problem 19. (3pt) A child is making 3-letter words using letters from the English alphabet. Letters can be repeated. How many 3-letter words will contain the letter 'A' exactly once?

Problem 20. (2pt) How many different phone numbers are there that begin with area code 574? A standard number has the format (574) xxx-xxxx, and any of the digits $\{0, 1, 2, \dots, 9\}$ may be used for the rest of the number (digits may be repeated).