10350 Tutorial Week 12 - Set 01 Name _____

Section _____

1. Sketch the graph of $g(x) = \dfrac{x^2}{x^2+1}$ by completing the steps below.

a. Find all x-intercepts and y-intercept of the graph of $g(x)$ whenever possible.

b. Find coordinates of all critical points, vertical asymptotes, and places where $g(x)$ are undefined.

$g'(x) = \frac{2x}{(x^2+1)^2}$

c. Determine where $g(x)$ is increasing and where it is decreasing.

d. Determine the concavity and coordinates of inflection points of $g(x)$.

$g''(x) = \frac{-6x^2+2}{(x^2+1)^3}$

e. Find all asymptotes and limit at infinity whenever applicable. Check for any symmetry.

f. Find the range of the function $g(x)$.

g. Sketch the graph below labeling all important features. Your picture should be large and clear.

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2a. Find the absolute (global) maximum and minimum of $f(x) = \dfrac{4}{(x^2-1)^2+1}$ on the interval $[0, 2]$.

Hint: $f(x) = 4[(x^2-1)^2+1]^{-1}$

2b. Using the steps below, find the global maximum and minimum of $f(x) = \dfrac{4}{(x^2-1)^2+1}$ on $[0, \infty)$.

Step 1: Find all critical points in the domain of $f(x)$ and the values of $f(x)$ there. Classify them using first derivative test.

Step 2: Find the values of $f(x)$ at the end-points (if any) of its domain. _____

Step 3: If end-point not included, or $\pm\infty$, find all limits of $f(x)$ towards end of interval.

Step 4: Give a schematic sketch (ignore concavity) of the graph of $f(x)$ clearly indicating where the global maximum and minimum are. State the global maximum and minimum of $f(x)$ on $[0, \infty)$ if any. Find the range of the function $f(x)$ for x in $[0, \infty)$.

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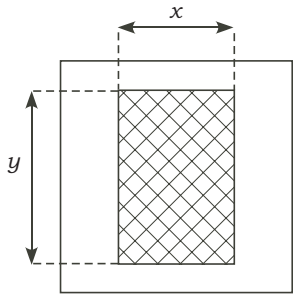
3. A graphic artist designing a poster for commercial use is instructed to have **one** inch margins top and bottom, and **two** inches along each side around the printed portion of the poster. It is further specified that the total area of the printed portion is 98 square inches. Suppose the width of the printed portion is x inches and the length of the poster is y inches. To cut paper cost, the total area of the poster (margin and printed portion) is to be minimized.

a. Write down the relation between x and y.



b. Find the function $A(x)$ you should minimize?

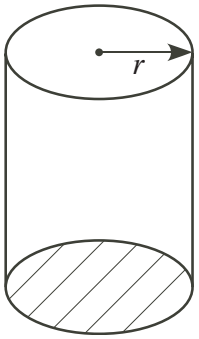
c. What is the range of x on which you should minimize $A(x)$? Is this a closed and bounded interval?

d. Find the value of x that minimizes the total area of the poster and give the area of the poster.

4. A cylindrical container with an **open** top is to be made with two different kind of material. The curve side is constructed with material of density 4 lb/ft^2 and the circular bottom is constructed with material of 5 lb/ft^2. Answer the questions below if the volume of the cylindrical container is 10π ft^3.

(The curved surface area of a cylinder is $2\pi rh$. Volume of a cylinder is $\pi r^2 h$)

(a) Write down the total weight of the container $W(r)$ in terms of its radius r.



(b) Write down the range of the possible values of r: _____.

(c) Using calculus, find the radius r that minimizes the weight W of the container. You must show appropriate checks to justify that your answer makes W minimum.