Name: Date: 10/12/2017

M20580 L.A. and D.E. Tutorial
Worksheet 7
Sections 5.1–5.4

1. The vector $\begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$ is an eigenvector of the matrix $\begin{bmatrix} 3 & 6 & 7 \\ 3 & 3 & 7 \\ 5 & 6 & 5 \end{bmatrix}$. What is the corresponding eigenvalue?

2. Let the matrix
$$A = \begin{bmatrix} 1 & 0 & -4 \\ -6 & -1 & 12 \\ 0 & 0 & -1 \end{bmatrix}.$$
(a) Find all eigenvalues of A.

(b) Find a basis for each eigenspace corresponding to each eigenvalue which you found in part (a). **Make sure you indicate which eigenvalue each subspace basis corresponds to.**

(c) Give an invertible matrix P and a diagonal matrix D such that $A = PDP^{-1}$, or if none such exists, explain why. Note: You do **not** need to compute P^{-1}.

3. Which of the following is NOT an orthogonal set? (Using the standard inner product)

   1. $\begin{bmatrix}1\\1\\1\end{bmatrix}, \begin{bmatrix}1\\0\\-1\end{bmatrix}$
   2. $\begin{bmatrix}2\\3\end{bmatrix}, \begin{bmatrix}-6\\4\end{bmatrix}$
   3. $\begin{bmatrix}\cos(t)\\\sin(t)\end{bmatrix}, \begin{bmatrix}\sin(t)\\-\cos(t)\end{bmatrix}$
   4. $\begin{bmatrix}3\\0\\0\\2\end{bmatrix}, \begin{bmatrix}0\\1\\0\\1\end{bmatrix}$
   5. $\begin{bmatrix}0\\2\\0\end{bmatrix}, \begin{bmatrix}-1\\0\\3\end{bmatrix}$

4. Let A be the matrix
$$\begin{bmatrix}\cos(\theta) & \sin(\theta)\\-\sin(\theta) & \cos(\theta)\end{bmatrix}.$$
   Find two complex eigenvectors of A. What does the eigenvalue represent geometrically?