Quiz 5, Solutions

1. **Q1** Calculate:

$$\int e^{2x}\sin(x)dx$$

Solution

We use the integration by parts $\int udv = uv - \int vdu$. We set $u = e^{2x}$, $du = 2e^{2x}$ with $v = -\cos(x)$ and $dv = \sin(x)$. Then

$$\int e^{2x}\sin(x)dx = -e^{2x}\cos(x) + 2\int e^{2x}\cos(x).$$

Let us use the integration by parts again with $u = e^{2x}, du = 2e^{2x}, v = \sin(x)$ and $dv = \cos(x)$. We can continue

$$\begin{aligned}\int e^{2x}\sin(x)dx &= -e^{2x}\cos(x) + 2\int e^{2x}\cos(x)\\ \text{(integration by parts)} &= -e^{2x}\cos(x) + 2\left(e^{2x}\sin(x) - 2\int e^{2x}\sin(x)dx\right)\\ &= -e^{2x}\cos(x) + 2e^{2x}\sin(x) - 4\int e^{2x}\sin(x)dx.\end{aligned}$$

We can bring the summand $4\int e^{2x}\sin(x)dx$ to the left hand side as follows

$$5\int e^{2x}\sin(x)dx = -e^{2x}\cos(x) + 2e^{2x}\sin(x).$$

Finally,

$$\int e^{2x}\sin(x)dx = \frac{1}{5}\left(-e^{2x}\cos(x) + 2e^{2x}\sin(x)\right) + C.$$

2. **Q2** Calculate:

$$\int_0^{\pi/2}\cos^3(x)\sin^3(x)dx$$

Solution

$$\int_0^{\pi/2}\cos^3(x)\sin^3(x)dx = \int_0^{\pi/2}\cos^3(x)(\sin^2 x)\sin x dx = \int_0^{\pi/2}\cos^3(x)(1-\cos^2 x)\sin x dx.$$

We've used $\sin^2 x + \cos^2 x = 1$ in the last step. We can use the substitution $u = \cos x$, then $du = -\sin x$. Note that $x = 0$ gives us $u = \cos(0) = 1$ and $x = \frac{\pi}{2}$ gives us $u = \cos(\frac{\pi}{2})$. Now

$$\begin{aligned}\int_0^{\pi/2}\cos^3(x)\sin^3(x)dx &= -\int_1^0 u^3(1-u^2)du\\ &= \int_1^0 u^5 - u^3 du\\ &= \left[\frac{u^6}{6} - \frac{u^4}{4}\right]_1^0\\ &= \frac{1}{12}.\end{aligned}$$