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M20550 Calculus III Tutorial
Worksheet 9

1. Using the Fundamental Theorem of Line Integrals, evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$, where $\mathbf{F} = (y^2 \cos(xy^2) + 3x^2)\,\mathbf{i} + (2xy\cos(xy^2) + 2y)\,\mathbf{j}$ is a conservative vector field and C is any curve from the point $(-1, 0)$ to $(1, 0)$.

2. Use Green's Theorem to evaluate
$$\int_C \left(-\frac{y^3}{3} + \sin x\right)\, dx + \left(\frac{x^3}{3} + y\right)\, dy,$$
where C is the circle of radius 1 centered at $(0, 0)$ oriented counterclockwise when viewed from above.

3. A particle starts at the origin $(0, 0)$, moves along the x-axis to $(2, 0)$, then along the curve $y = \sqrt{4 - x^2}$ to the point $(0, 2)$, and then along the y-axis back to the origin. Find the work done on this particle by the force field $\mathbf{F}(x, y) = y^2\,\mathbf{i} + 2x(y + 1)\,\mathbf{j}$.

4. (a) Compute div $\mathbf{F}$, where $\mathbf{F} = \langle e^y, zy, xy^2\rangle$.

   (b) Is there a vector field $\mathbf{G}$ on $\mathbb{R}^3$ such that $\operatorname{curl}\mathbf{G} = \langle xyz, -y^2z, yz^2\rangle$? Why?

5. Write an equation of the tangent plane to the parametric surface
$$x = u^2 + 1, \quad y = v^3 + 1, \quad z = u + v,$$
at the point $(5, 2, 3)$.

6. Write the integral that computes the surface area of the surface S parametrized by $\mathbf{r}(u, v) = \langle u^2 \cos v, u^2 \sin v, v\rangle$, where $0 \le u \le 1$ and $0 \le v \le \pi$.

7. Compute the surface integral $\iint_S (x + y + z)\, dS$, where S is a surface given by $\mathbf{r}(u, v) = \langle u + v, u - v, 1 + 2u + v\rangle$ and $0 \le u \le 2,\ 0 \le v \le 1$.