ELEMENTS OF CALCULUS: EXAM 1 REVIEW

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1. Precalculus

Problem 1. State the domain of the function

$$g(x) = \frac{x^2}{x^2 - 16}$$

Problem 2. Find the inverse of the function $h(x) = 3 - x^5$.

Problem 3. Find all real solutions to the equations:

a) $x^2 + 3x - 10 = 0$

b) $e^{2x} + 3e^x - 10 = 0$

c) $\ln(t^2 - 3) = 0$

d) $4^{x-2} = 8$.

Problem 4.

a) If $u(t) = t^2 + \frac{1}{t+5}$, what is $u(t-6)$?

b) If $f(x) = \frac{2}{x}$ and $g(t) = t^3 + 1$, compute $(f \circ g)(-1)$.

c) If $f(x) = x^4 - x^2$, find $f(2x)$.

d) If $u(x) = 3x - 1$ and $m(x) = x^2 + x$, find and simplify $(m \circ u)(x)$.

e) If $f(1) = 3$, $f(2) = 4$, $g(1) = 2$, $g(3) = 2$, compute $(f \circ g)(1) - (g \circ f)(1)$?

f) Sketch the graph of the piecewise function:

$$f(x) = \begin{cases} 1 & x < -1 \\ x^2 & -1 \le x \le 1 \\ x & x > 1 \end{cases}$$

g) Sketch the graph of the piecewise function:

$$g(x) = \begin{cases} -1 & x < -1 \\ x^3 & -1 \le x \le 1 \\ -x & x > 1 \end{cases}$$

h) Write the function $f(x) = |x + 1| + 2$ as a piecewise function.

Problem 5.

a) Graph the function $f(x) = -|x| + 2$.

b) Starting with the graph of $f(x) = \sqrt{x}$, what is the formula of the function obtained by a reflection about the y-axis, followed by a vertical strentch by a factor of 2?

c) What is the formula for the function whose graph which can be obtained from $y = x^3$ by shifting to the RIGHT 1 unit, then reflecting about the x-axis?

Problem 6. Consider the function $f(x) = x^2 + 6x + 11$. Complete the square and write it in the form $f(x) = (x+a)^2 + b$. What transformations would one perform on the graph of the basic parabola x^2 to obtain the graph of f?

Problem 7. Let $f(x) = \dfrac{1}{x}$. Find and simplify $\dfrac{f(x+h) - f(x)}{h}$ (assume $h \neq 0$).

2. Limits and Continuity

Problem 8. Compute the following limits:

a) $\lim_{x \to 4} \dfrac{x-4}{x^2 - 8x + 3}$

b) $\lim_{x \to 3^-} \dfrac{x-4}{x^2 - 9}$

c) $\lim_{x \to 3^-} \dfrac{\sqrt{5x}(x-3)}{|x-3|}$

d) Suppose that $\lim_{x \to 1} f(x) = 7$, $\lim_{x \to 1} g(x) = 4$, and $\lim_{x \to 1} h(x) = -\infty$. Compute the limit

$$\lim_{x \to 1} \left(f(x) + \frac{1}{g(x) - h(x)} \right)$$

e) $\lim_{x \to 2} \dfrac{x-2}{\sqrt{x^2 - 4}}$ (Hint: rationalize the denominator)

f) $\lim_{x \to +\infty} \dfrac{7x^9 - 4x^5 + 2x - 13}{-3x^9 + x^8 - 5x^2 + 2x}$

g) $\lim_{x \to 4} \left(\dfrac{1}{x-4} - \dfrac{8}{x^2 - 16} \right)$

h) $\lim_{x \to 0^-} x^{1/4}$

i) $\lim_{x \to \infty} \dfrac{x^4 - 3x^3 + 5x + 1}{x^5 + 12x + 8}$

j) $\lim_{x \to \infty} \dfrac{x^4 - 6x + 8}{5x^3 + 8x^4}$

k) $\lim_{x \to \infty} \dfrac{x^8 + e^x + 1}{5x^8 + 3e^x + 12x^2 + 5}$

l) $\lim_{x\to\infty} \dfrac{5\ln x + 12}{7\ln x + \cos x + 6}$

m) $\lim_{x\to\infty} \dfrac{5 + e^{-x} + 2e^{-2x}}{7 + 2e^{-x} + 3e^{-2x}}$

n) $\lim_{x\to-\infty} \dfrac{1 + 2e^{x} + 3e^{2x}}{4 + 5e^{x} + 6e^{2x} + e^{-x}}$

o) $\lim_{x\to\infty} \dfrac{\sqrt{x^2+1}}{x^4+1}$

p) $\lim_{x\to\infty} \dfrac{\sqrt{2x^2+8}}{x+5}$

q) $\lim_{x\to-\infty} \dfrac{\sqrt{\pi x^6 + 23x + 8}}{2x^3 + x^2 + 1}$

Problem 9. For which values of x is the following function continuous?

$$f(x) = \frac{|x| + \sqrt{x-2}}{(x^2-9)(x^2+4)}$$

Problem 10.

a) Suppose we have a continuous function $f(x)$ that satisfies $f(-1) = -1$ and $f(1) = 1$. Can this function have two zeroes inside the interval $(-1, 1)$? Justify. What can you say about the number of zeroes such a function can have inside $(-1, 1)$?

b) Argue without solving for x that there are at least two solutions to the equation $-x^4 + 3x + 2 = 0$. (Hint: use continuity of polynomials and IVT).

Problem 11. You are told that a parabola which "opens up" has roots $x = a$ and $x = b$ (where $a < b$). At what x value is the minimum of the parabola attained?

Problem 12. Find the limit $\lim_{x\to+\infty}(x - \sqrt{x^2-1})$.