Worksheet 9, Math 10560

1. (a) Give a definition of conditional convergence.

(b) Which series below conditionally converges (justify your answer)?

$$i) \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{\sqrt{n}}$$

ii)
$$\sum_{n=1}^{\infty} \frac{(-1)^n 7^n}{\sqrt{n}}$$

iii)
$$\sum_{n=1}^{\infty} (-1)^{n-1}$$

iv)
$$\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^2}$$

2. (a) Say whether or not the following series are convergent and how you arrived at your conclusion including which test you used and how it applies.

$$i) \sum_{n=1}^{\infty} \frac{e^n}{n^2 + e^n}$$

$$ii) \sum_{n=2}^{\infty} \frac{\cos(n^n)}{n^4 + 1}$$

iii)
$$\sum_{n=1}^{\infty} \frac{(-1)^n 2^n}{(n-1)!}$$

iv)
$$\sum_{n=1}^{\infty} \left(\frac{n^2 + n}{2n^2 + 1} \right)^n$$

3. Find the radius of convergence and interval of convergence of the following power series $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}(x-2)^n}{\sqrt{n}}.$

Some Extra Old Exam Questions

- 4. Find $\sum_{n=1}^{\infty} \frac{2^{2n}}{5^{n-1}}$.
 - (a) 20
 - (b) $\frac{4}{5}$
 - (c) $\frac{5}{4}$
 - (d) 4
 - (e) 5

5. Test the following series for absolute convergence, conditional convergence or divergence:

$$(1)\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$$

$$(2)\sum_{n=1}^{\infty} \frac{(-1)^n}{(1.2)^n};$$

$$(1)\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}; \qquad (2)\sum_{n=1}^{\infty} \frac{(-1)^n}{(1.2)^n}; \qquad (3)\sum_{n=1}^{\infty} \frac{(-1)^n}{n^{1.2}}.$$

- (a) (1) converges conditionally, (2) and (3) converge absolutely
- (b) (1) and (2) converge conditionally, (3) converges absolutely
- (c) (1) and (2) converge absolutely, (3) converges conditionally
- (d) (1) and (3) converge absolutely, (2) converges conditionally
- (e) (1) converges absolutely, (2) and (3) converge conditionally.

6. Find the center a and the radius of convergence R for

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n^2 + 2}} \left(\frac{x+1}{2}\right)^n.$$

- (a) a = -1, R = 2
- (b) $a = -\frac{1}{2}$, R = 2
- (c) a = 1, R = 2
- (d) a = -1, R = 1
- (e) a = -1, $R = \frac{1}{2}$

7. Consider the following series

(I)
$$\sum_{n=1}^{\infty} \frac{n \cdot 3^n}{(n+1)!}$$
 (II)
$$\sum_{n=1}^{\infty} \left(\frac{e^n}{2e^n + 1}\right)^n.$$

Which of the following statements is true?

- (a) They both converge.
- (b) They both diverge.
- (c) (I) converges and (II) diverges.
- (d) (I) diverges and (II) converges.
- (e) The root test is inconclusive when applied to (II).