Name: Date: 01/25/2018

M20580 L.A. and D.E. Tutorial
Worksheet 2
Sections 1.1–1.3

1. (a) Find the general solution of the system of linear equations

$$\begin{array}{rcrcrcrcr} 2x_1 & - & 4x_2 & + & 5x_3 & + & x_4 & = & -3 \\ x_1 & - & 2x_2 & + & 2x_3 & + & x_4 & = & -1 \\ x_1 & - & 2x_2 & + & 3x_3 & & & = & -2 \end{array}$$

(b) If the linear system above has infinitely many solutions, give two solutions to the system.

2. *Recall*: Given a collection of vectors $\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_p$, a ***linear combination*** of $\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_p$ is a new vector of the form

$$\mathbf{w} = c_1\mathbf{v}_1 + c_2\mathbf{v}_2 + \cdots + c_p\mathbf{v}_p, \qquad \text{for } \textit{some} \text{ scalars } c_1, c_2 \ldots, c_p$$

Suppose

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} 4 \\ 7 \\ 1 \end{bmatrix}, \quad \mathbf{v}_3 = \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix}$$

(a) Give an example of a linear combination of $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$.

(b) Determine whether the vector $\mathbf{w} = \begin{bmatrix} 2 \\ 3 \\ 5 \end{bmatrix}$ can be written as a linear combination of the vectors $\mathbf{v}_1$, $\mathbf{v}_2$, and $\mathbf{v}_3$. If yes, find scalars a_1, a_2, a_3 such that $a_1\mathbf{v}_1 + a_2\mathbf{v}_2 + a_3\mathbf{v}_3 = \mathbf{w}$.

3. Fill in the blanks

Span$\{\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_p\}$ is the set of ______ linear combinations of the vectors __________

Let $\mathbf{v}_1 = \begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix}$, $\mathbf{v}_2 = \begin{bmatrix} -4 \\ 3 \\ 8 \end{bmatrix}$, $\mathbf{v}_3 = \begin{bmatrix} 2 \\ 5 \\ -4 \end{bmatrix}$.

(a) Give examples of two vectors that are in the set Span$\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$.

(b) How many vectors are there in Span$\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$?

(c) Determine whether the vector $\mathbf{w} = \begin{bmatrix} 3 \\ -7 \\ -3 \end{bmatrix}$ is in Span$\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$.

4. Let $\mathbf{v}_1 = \begin{bmatrix} 1 \\ 0 \\ -1 \\ 2 \end{bmatrix}$, $\mathbf{v}_2 = \begin{bmatrix} 2 \\ 3 \\ 1 \\ -2 \end{bmatrix}$, $\mathbf{v}_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 2 \end{bmatrix}$. Find the value of h such that $\mathbf{w} = \begin{bmatrix} 4 \\ 3 \\ h \\ 0 \end{bmatrix}$ is a linear combination of $\mathbf{v}_1$, $\mathbf{v}_2$, $\mathbf{v}_3$.