Name: Date: 03/29/2018

M20580 L.A. and D.E. Tutorial
Worksheet 8
Sections 6.1, 6.2, 6.3, 6.4

1.

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

Denote $\alpha_1 = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}$, $\alpha_2 = \begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \end{bmatrix}$, $\alpha_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix}$, these are the three columns of A. Let $W = Span\{\alpha_1, \alpha_2, \alpha_3\}$.

(a) Find a basis for $W^\perp$.

(b) Check your answer in (a), i.e. each vector in your basis for $W^\perp$ is perpendicular to every α_i $(i = 1, 2, 3)$.

(c) Use Gram-Schmidt process to find an orthogonal basis for $W = Span\{\alpha_1, \alpha_2, \alpha_3\}$. **You need not normalize your basis.**

(d) Let $\beta = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$. Find the orthogonal projection of β onto W, i.e. $proj_W \beta$, using the orthogonal basis you've found in (c).

(e) Find the orthogonal projection of β onto $W^{\perp}$, i.e. $proj_{W^{\perp}} \beta$, using an orthogonal basis for $W^{\perp}$.

We know that $proj_{W^{\perp}} \beta = \beta - proj_W \beta$, so you may solve this part using (d). But I suggest you to calculate $proj_{W^{\perp}} \beta$ by again using orthogonal projection formula, so that you can practice the formula again.

(f) Using your results of (d) and (e), check that $\beta = proj_W \beta + proj_{W^{\perp}} \beta$. Thus, we get a decomposition of β into two parts, one part is in W, the other part is in $W^{\perp}$.

2. Find a **least squares solution** to the system

$$\begin{bmatrix} 1 & 1 & 3 \\ 2 & -1 & 1 \\ 1 & 1 & -2 \\ 3 & 0 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 2 \end{bmatrix}.$$

Note that the columns $\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3$ of the coefficient matrix A form an **orthogonal** basis for $\operatorname{Col} A$.

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3. Let

$$A = \begin{bmatrix} 2 & 3 \\ 2 & 4 \\ 1 & 1 \end{bmatrix}.$$

Use Gram-Schmidt process to find a orthogonal basis for Col A, and use the orthogonal basis you get to find the QR factorization of A.