TEST 1 PRACTICE PROBLEMS

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Problem 1. Find the limit $\lim_{x\to 4}\left(\dfrac{1}{x-4}-\dfrac{8}{x^2-16}\right)$

Problem 2. Find the limit $\lim_{x\to 0}\sqrt[5]{x}\cos^5(x)$.

Problem 3. The limit $\lim_{t\to 0}\dfrac{(2+t)^5-2^5}{t}$ represents the derivative of some function $f(x)$ at a point $x=a$. What is $f(x)$ and what is a?

Problem 4. Find the limit $\lim_{h\to 0}\dfrac{e^x(e^h-1)}{h}$.

Problem 5. Suppose the point $(2,2)$ is another point on the tangent line to the graph of $f(x)$ at $(1,1)$. Answer the following:

(a) What is the equation of the tangent line?

(b) What is $f'(1)$?

(c) Find the instantaneous rate of change for the function $g(x)=\dfrac{f(x)}{f(x)+1}$ when $x=1$.

Problem 6. The point $(2,4)$ is another point on the tangent line to the graph of $f(x)$ at $(1,1)$. What is $g'(1)$ for $g(x)=[f(x)]^2$?

Problem 7. Given that $2\leq f(x)\leq 2+\sin(x)$ for all $x\in\mathbb{R}$, find the limit $\lim_{x\to\pi}f(x)$.

Problem 8. It is a known fact that $\lim_{x\to 0}\dfrac{\sin x}{x}=1$. Compute the following:

(a) $\lim_{x\to 0}\dfrac{x}{\sin x}$.

(b) $\lim_{x\to 0}\dfrac{\sin x}{x^2}$.

(c) $\lim_{x\to 0^+}\dfrac{\sin x}{x^2}$.

(d) $\lim_{x\to 0}\dfrac{\sin(2x)}{2x}$.

(e) $\lim_{x\to 0}\dfrac{\sin(5x)\cos x}{x}$.

Problem 9. Compute the following limits:

(a) $\lim_{x\to 0^-} x^{1/4}$

(b) $\lim_{x\to\infty} \dfrac{x^4 - 3x^3 + 5x + 1}{x^5 + 12x + 8}$

(c) $\lim_{x\to\infty} \dfrac{x^4 - 6x + 8}{5x^3 + 8x^4}$

(d) $\lim_{x\to\infty} \dfrac{x^8 + e^x + 1}{5x^8 + 3e^x + 12x^2 + 5}$

(e) $\lim_{x\to\infty} \dfrac{5\ln x + 12}{7\ln x + \cos x + 6}$

(f) $\lim_{x\to\infty} \dfrac{5 + e^{-x} + 2e^{-2x}}{7 + 2e^{-x} + 3e^{-2x}}$

(g) $\lim_{x\to-\infty} \dfrac{1 + 2e^x + 3e^{2x}}{4 + 5e^x + 6e^{2x} + e^{-x}}$

(h) $\lim_{x\to\infty} \dfrac{\sqrt{x^2+1}}{x^4+1}$

(i) $\lim_{x\to\infty} \dfrac{\sqrt{2x^2+8}}{x+5}$

(j) $\lim_{x\to-\infty} \dfrac{\sqrt{\pi x^6 + 23x + 8}}{2x^3 + x^2 + 1}$

Problem 10. Using the linerization of x^3 at $x = 8$, estimate the value of 8.1^3.

Problem 11. We know that $e^0 = 1$. Use linearization to approximate the value of $e^{0.1}$.

Problem 12. Use linear approximation to estimate the value of $(12.2)^2$.

Problem 13. Given any two real numbers $a < b$, what is the average rate of change of f from $x = a$ to $x = b$ if $f(x) = \dfrac{2x+5}{4+\pi}$? What is the instantaneous rate of change of f?

Problem 14. What is the instantaneous rate of change of $f(x) = \dfrac{x^2 + x - 2}{x - 1}$ when $x \neq 1$?

Problem 15. Suppose $f(x) = 3x^2 + 8$ and $h(x) = \dfrac{1}{2}x^3 + 2x + 1$. Find the instantaneous rate of change of the graph of $g(x) = \dfrac{f(x)}{3} + 2h(x)$ at $x = 1$.

Problem 16. You are told that a parabola which "opens up" has roots $x = a$ and $x = b$, where $a < b$. At what x value is the minimum of the parabola attained?