10350 ALG Gateway 01 Name _____ Section _____

1. Simplify the following expression giving your answer in the form $ka^m b^n$ where k, n, and m are constants to be determined.

$\frac{a^2}{b^3} \cdot \sqrt{\frac{b^5}{16a}} \stackrel{?}{=}$

Solution:

$$\begin{aligned}\frac{a^2}{b^3} \cdot \sqrt{\frac{b^5}{16a}} &= \frac{a^2 b^{5/2}}{b^3 \cdot 4a^{1/2}} \\ &= \frac{1}{4} a^{3/2} b^{-1/2}\end{aligned}$$

Alternate solution:

$$\begin{aligned}\frac{a^2}{b^3} \cdot \sqrt{\frac{b^5}{16a}} &= \sqrt{\frac{a^4 b^5}{b^6 \cdot 16a}} \\ &= \sqrt{\frac{a^3 b^{-1}}{16}} \\ &= \frac{1}{4} a^{3/2} b^{-1/2}\end{aligned}$$

2. Write the following quadratic function in the form $a(x+h)^2 + k$ where a, h and k are constants to be found.

$2x^2 - 8x + 1 \stackrel{?}{=}$

Solution:

$$\begin{aligned}2x^2 - 8x + 1 &= 2(x^2 - 4x) + 1 \\ &= 2(x^2 - 4x + (-2)^2) + 1 - 2(-2) \\ &= 2(x-2)^2 + 1 - 8 \\ &= 2(x-2)^2 - 7\end{aligned}$$

3. Let $f(x) = \dfrac{1}{x}$ and $g(x) = \dfrac{x}{2x-1}$. Find and simplify the composite function $g(f(x))$.

$g(f(x)) \stackrel{?}{=}$

Solution:

$$g(f(x)) = \frac{\dfrac{1}{x}}{2\left(\dfrac{1}{x}\right) - 1} = \frac{\dfrac{1}{x}}{2\left(\dfrac{1}{x}\right) - 1} \cdot \left(\frac{x}{x}\right) = \frac{1}{2-x}$$

Alternate solution:

$$\begin{aligned} g(f(x)) &= \frac{\dfrac{1}{x}}{2\left(\dfrac{1}{x}\right) - 1} \\ &= \frac{\dfrac{1}{x}}{\dfrac{2-x}{x}} \\ &= \frac{1}{x} \cdot \frac{x}{2-x} \\ &= \frac{1}{2-x} \end{aligned}$$

4. Factor the following expression completely:

$1 - 81x^4 \stackrel{?}{=}$

Solution:

$$1 - 81x^4 = (1 - 9x^2)(1 + 9x^2) = (1 - 3x)(1 + 3x)(1 + 9x^2)$$