PRACTICE QUIZ 4 SOLUTION

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Time: 15 min
Time to beat: 7 min

Problem 1. If $f(x) = x^2 - 4x + 6$, show that $f(2-h) = f(2+h)$.

$$f(2-h) = 4 - 4h + h^2 - 8 + 4h + 6 = h^2 + 2$$
$$f(2+h) = 4 + 4h + h^2 - 8 - 4h + 6 = h^2 + 2$$

They are equal.

Problem 2. If $f(x) = \frac{x-1}{x+1}$, show that $f(1/x) = -f(x)$ and $f(-1/x) = -\frac{1}{f(x)}$.

$$f(1/x) = \frac{\frac{1}{x} - \frac{x}{x}}{\frac{1}{x} + \frac{x}{x}} = \frac{1-x}{x} \cdot \frac{x}{1+x} = \frac{1-x}{1+x} = -\frac{x-1}{x+1} = -f(x)$$
$$f(-1/x) = \frac{-\frac{1}{x} - \frac{x}{x}}{-\frac{1}{x} + \frac{x}{x}} = \frac{-1-x}{x} \cdot \frac{x}{-1+x} = -\frac{x+1}{x-1} = -\frac{1}{f(x)}$$

Problem 3. If $f(x) = x^2 - x$, show that $f(x+1) = f(-x)$.

$$f(x+1) = x^2 + 2x + 1 - (x+1) = x^2 + x = (-x)^2 - (-x) = f(-x)$$

Problem 4. If $y = f(x) = \frac{5x+3}{4x-5}$, show that $x = f(y)$.

$$f(y) = \frac{5y+3}{4y-5} = \frac{5\frac{5x+3}{4x-5} + 3}{4\frac{5x+3}{4x-5} - 5} = \frac{25x + 15 + 12x - 15}{4x-5} \cdot \frac{4x-5}{20x + 12 - 20x + 25} = \frac{(25+12)x}{12+25} = x$$