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FINITE MATH: QUIZ 2 SOLUTION

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Problem 1. Suppose you are counting **binary sequences** of length 6, using only digits 0 and 1. Repeating digits is allowed, so 001010, 100101, and 000000 are all valid.

a) (1pt) How many such binary sequences are possible?

We are creating sequences of length six, and for each position we have two choices. This gives a total of $\boxed{\underline{2}\cdot\underline{2}\cdot\underline{2}\cdot\underline{2}\cdot\underline{2}\cdot\underline{2} = 2^6 = 64}$ binary sequences.

b) (1pt) How many start with digit '0'? How many end with digit '0'?

For the number of sequences that start with zero, the first position only has ONE choice: the digit '0'. The remaining positions can be '0' or '1' (so 2 choices) for each, giving us $\boxed{\underline{1}\cdot\underline{2}\cdot\underline{2}\cdot\underline{2}\cdot\underline{2}\cdot\underline{2} = 2^5 = 32}$ binary sequences starting with zero. Similarly, the number of sequences that end with zero, we have $\boxed{\underline{2}\cdot\underline{2}\cdot\underline{2}\cdot\underline{2}\cdot\underline{2}\cdot\underline{1} = 2^5 = 32}$ of them.

c) (1pt) How many start with digit '0' AND end with digit '0'? (e.g. $\underline{0}1101\underline{0}$)

Both the first and last positions are '0' (1 choice), and the rest can be '0' or '1' (so 2 choices). $\boxed{\underline{1}\cdot\underline{2}\cdot\underline{2}\cdot\underline{2}\cdot\underline{2}\cdot\underline{1} = 2^4 = 16}$.

d) (2pt) How many start with '0' or end with '0' (or both)? (e.g. $\underline{0}10011$, $\underline{0}1101\underline{0}$, $11101\underline{0}$)

The easiest way to do this is to use the inclusion-exclusion principle, since we already computed everything we need for that. Let $A = \{\text{sequences that start with } 0\}$ and $B = \{\text{sequences that end with } 0\}$. Then the sequences that start AND end with zero is the intersection of the two sets, $A \cap B$.

If we are looking to count all the sequences that start or end with 0 (or both), then we are looking for the number of elements in the union $A \cup B$, which is given by $n(A \cup B) = n(A) + n(B) - n(A \cap B)$. We found in part (b) of the problem that $n(A) = 32$ and $n(B) = 32$. We also saw in part (c) that $n(A \cap B) = 16$. Then $\boxed{n(A \cup B) = 32 + 32 - 16 = 48}$.

Alternately, we can consider the following disjoint sequence configurations: starting with zero but ending with 1 ($0 * * * * 1$, so $2 \cdot 2 \cdot 2 \cdot 2 = 16$ total), starting with zero and ending with zero ($0 * * * * 0$, there are $2^4 = 16$ of them also), or starting with 1 and ending with 0 ($1 * * * * 0$, there are $2^4 = 16$ of these as well). By the addition principle, the number we are looking for is $\boxed{16 + 16 + 16 = 48}$.

Problem 2. Suppose we have a group of 3 men and 5 women, and we want to **arrange everyone in a single row**. Alexa is one of the women.

a) (1pt) In how many ways can we do this?

Since we are arranging the entire group of people in a row, it doesn't make any difference whether they are male or female (notice the problem didn't impose any requirements as to where the men or women should be located in our arrangement). We have 8 people total, so the number of possible arrangements is $\boxed{8! = 40,320}$.

b) (1pt) What if Alexa insists on being first or last? (Hint: you need to consider each possibility separately, as they are **disjoint** scenarios)

Alexa can either be first, or last, and these are disjoing scenarios. If she is first, then the number of possible arrangements becomes $\underline{\mathbf{1}} \cdot \underline{7} \cdot \underline{6} \cdot \underline{5} \cdot \underline{4} \cdot \underline{3} \cdot \underline{2} \cdot \underline{1} = 7! = 5040$. She might also sit last: $\underline{7} \cdot \underline{6} \cdot \underline{5} \cdot \underline{4} \cdot \underline{3} \cdot \underline{2} \cdot \underline{1} \cdot \underline{\mathbf{1}} = 7! = 5040$. By the addition princippe, the number of ways she can sit first or last is $\boxed{7! + 7! = 10,080}$.

c) (1pt) What if Alexa does NOT want to be first or last?

The easiest way to count this is to realize that this situation is the *complement* of the previous part of the problem. The number of ways is given by $\boxed{8! - (7! + 7!) = 40,320 - 10,080 = 30,240}$.

Problem 3. (2 pts) In how many ways can we permute the letters in ABRACADABRA?

This is a rearrangement/permutation problem with the following multiplicities: $A = 5, B = 2, R = 2, C = 1, D = 1$. The total number of rearrangements is given by $\boxed{\dfrac{11!}{5! \cdot 2! \cdot 2! \cdot 1! \cdot 1!} = 80,160}$.