WORKSHEET 13

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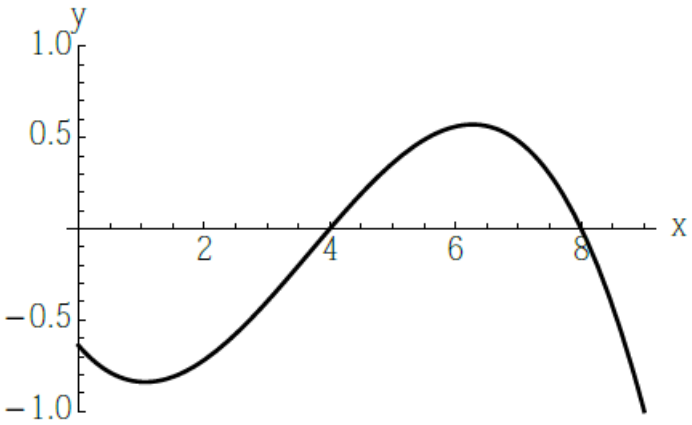
Problem 1. The following shows the graph of a function $f(x)$:



a) Give a rough sketch of the graph of the function $g(x) = \int_0^x f(t)dt$ for $x \in [0, 10]$.

b) Calculate the derivative of $g(x) = \int_0^{x^2} f(t)dt$.

Problem 2. Calculate the derivative of the following function:

$$g(x) = \int_x^{\sin^2 x} \frac{t^2}{1+2e^t} dt$$

Problem 3. Find the general indefinite integral.

a) $\int (\cos x - \sin x) dx$

b) $\int x^{-3/4} dx$

c) $\int 2xe^{x^2} dx$

d) $\int \frac{2x}{x^2+1} dx$

Problem 4. Evaluate the definite integral.

a) $\int_0^2 (x^3 + 6x + 1)dx$

b) $\int_0^1 \frac{1}{1+x^2} dx$

c) $\int_0^1 (1 + x^2)^2 dx$

d) $\int_0^4 \sqrt{2t} dt$

e) $\int_0^\pi \sin\theta d\theta$

f) $\int_{-2016}^{2016} x e^{x^2} dx$

g) $\int_1^2 \left(x + \frac{1}{x}\right)^2 dx$

Problem 5. Suppose the function f satisfies the following:

$$\int_{-2}^{2} f(x)dx = 1 \qquad \int_{2}^{5} f(x)dx = 2 \qquad \int_{-2}^{-1} f(x)dx = 3$$

What is the value of $\int_{-1}^{5} f(x)dx$?

Problem 6. Suppose that an integrable function f satisfies $\int_{2}^{7} |f(x)|dx = 0$. What can you say about the value of the integral $\int_{2}^{7} f(x)dx$?

Problem 7. Suppose that an integrable function f satisfies $\int_{0}^{1} [f(x)]^2 dx = 0$. What can you say about the value of the integral $\int_{0}^{1} f(x)dx$?