

1. Let $f(x) = \frac{1}{x}$. Assuming that $h \neq 0$, find and simplify $\frac{f(x+h) - f(x)}{h}$.

$$\begin{aligned}\frac{f(x+h) - f(x)}{h} &= \frac{\frac{1}{x+h} - \frac{1}{x}}{h} \\ &= \frac{1}{h} \left(\frac{x}{x(x+h)} - \frac{x+h}{x(x+h)} \right) \\ &= \frac{1}{h} \left(\frac{-h}{x^2 + xh} \right) \\ &= \frac{-1}{x^2 + xh} \text{ (accepted solution)} \\ &= \frac{-1}{x(x+h)} \text{ (accepted solution)}\end{aligned}$$

2. Simplify the following expression as far as possible. Your answer should have no negative exponents:

(a)

$$\frac{y^4(x^3y^{-2})^2}{2x^{-1}} = \frac{1}{2}(y^4x^6y^{-4}x^1) = \frac{x^7}{2}$$

(b)

$$\begin{aligned} \frac{(x^2 + 4)^2(3) - 2x(x^2 + 4)(3x - 5)}{(x^2 + 4)^4} &= \frac{3(x^2 + 4) - 2x(3x - 5)}{(x^2 + 4)^3} \\ &= \frac{3x^2 + 12 - 6x^2 + 10x}{(x^2 + 4)^3} \\ &= \frac{-3x^2 + 10x + 12}{(x^2 + 4)^3} \end{aligned}$$

3. Find ALL the zeroes of $f(x) = 2x^2 - x - 3$ **exactly**.

$$f(x) = 2x^2 - x - 3 = (2x - 3)(x + 1) \implies f(x) = 0 \text{ when } x = \frac{3}{2}, -1$$

4. Solve $4^{x-2} = 8$. Be sure your answer is simplified.

$$\begin{aligned} 4^{x-2} &= 8 \\ (2^2)^{(x-2)} &= 2^3 \\ 2^{2x-4} &= 2^3 \\ 2x - 4 &= 3 \\ 2x &= 7 \\ x &= \frac{7}{2} \end{aligned}$$

5. Find and simplify the expression $\frac{g(n+1)}{g(n)}$ if $g(n) = \frac{2^n x^{2n-1}}{n^3}$.

$$\begin{aligned} \frac{g(n+1)}{g(n)} &= \frac{\frac{2^{n+1} x^{2(n+1)-1}}{(n+1)^3}}{\frac{2^n x^{2n-1}}{n^3}} \\ &= \left(\frac{2^{n+1} x^{2(n+1)-1}}{(n+1)^3} \right) \left(\frac{n^3}{2^n x^{2n-1}} \right) \\ &= \frac{2^{n+1} x^{2n+2-1} n^3}{(n+1)^3 2^n x^{2n-1}} \\ &= \frac{2^{(n+1)-n} x^{2n+1-(2n-1)} n^3}{(n+1)^3} \\ &= \frac{2x^2 n^3}{(n+1)^3} \end{aligned}$$