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FINITE MATH: QUIZ 6 SOLUTION

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Problem 1. Suppose you have **four bags** with marbles, and bag each contains exactly 5 red (R), 3 white (W), and 8 yellow (Y). You select **one marble from each bag**.

a) (2pt) What is the probability that you will draw R, W, Y, R, **in that order**?

Each draw is independent since we have four different bags. The probability of drawing RWYR is: $\boxed{\frac{5}{16}\cdot\frac{3}{16}\cdot\frac{8}{16}\cdot\frac{5}{16}}$

b) (2pt) How does your answer change if you only select the 4 marbles **from a single bag with replacement**, one at a time, and you want to draw R, W, Y, R, in that order?

Since we are drawing with replacement, previous results do not affect future draws. In other words, we still have independence. The answer is the same as in part (a). The probability of drawing RWYR is: $\boxed{\frac{5}{16}\cdot\frac{3}{16}\cdot\frac{8}{16}\cdot\frac{5}{16}}$

c) (2pt) How does your answer change if you only select the 4 marbles **from a single bag without replacement**, one at a time, and you want to draw R, W, Y, R, in that order?

This time, drawing without replacement means we lose the independence between the draws. The probability of drawing RWYR is: $\boxed{\frac{5}{16}\cdot\frac{3}{15}\cdot\frac{8}{14}\cdot\frac{4}{13}}$

Problem 2. (1pt) Two bags contain red (R) and white (W) marbles:

$$\underbrace{\boxed{3R \quad 5W}}_{Bag\ 1} \qquad\qquad \underbrace{\boxed{7R \quad 4W}}_{Bag\ 2}$$

You pick one of the two bags at random (each bag is equally likely to be picked), and select a marble. What is the probability that the marble is Red?

Getting red (event R) can occur in two ways. Either the red marble comes from the first bag (which has a 0.5 chance of being selected), or it comes from the second bag (which also has a 0.5 chance of being selected). Each of these are disjoint possibilities, so we use the addition principle. We have

$$\boxed{P(R) = P(B_1 \cap R) + P(B_2 \cap R) = \frac{1}{2}\cdot\frac{3}{8} + \frac{1}{2}\cdot\frac{7}{11}}$$

Problem 3. (2pt) Pick two numbers at random from among $\{1, 2, 3, 4\}$, and add them up. Given that at least one of the numbers is a 2, what is the probability that their sum is a 5?

We are picking two numbers, one at a time, and repeats are allowed. This is equivalent to rolling two four-sided dice and summing the numbers that come up. There are $4 \cdot 4 = 16$ possible outcomes (and they are equally likely). Of those, the outcomes where at least one of the numbers is a 2 are

$$(2,1)\ (2,2)\ (2,3)\ (2,4)\ (1,2)\ (3,2)\ (4,2)$$

This forms our new sample space (it is the conditioning event). Notice we didn't want to double-count the outcome $(2,2)$. Now, of those outcomes, there are 2 of them which add to 5: $(2,3)$ and $(3,2)$. Hence the probability we seek is $\boxed{\frac{2}{7}}$

Problem 4. (2pt) A sample space S contains 100 equally likely outcomes. Consider the events E and F satisfying

$$n(E \setminus F) = 20 \qquad n(F \setminus E) = 30 \qquad n(E \cap F) = 30$$

Determine if the events are independent or not.

Notice that $n(E) = 20 + 30 = 50$, and similarly $n(F) = 30 + 30 = 60$. This allows us to compute the probabilities $P(E) = 0.5$ and $P(F) = 0.6$. Also, $P(E \cap F) = 0.3$, which is also equal to $P(E) \cdot P(F) = (0.5)(0.6)$. Therefore $\boxed{\text{the events are independent}}$.