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Partial Credit

You must show your work on the partial credit problems to receive credit!

11.(12 pts.)

11a. Find ALL equations of the vertical asymptotes of the curve $y = \dfrac{x^2-9}{x^2-x-6}$

(Remark: Your answers should be in the form: $x = c$.)

$$\frac{x^2-9}{x^2-x-6} = \frac{\cancel{(x-3)}(x+3)}{\cancel{(x-3)}(x+2)} = \frac{x+3}{x+2} \quad \text{when } x \neq 3.$$

$x = -2$ is the only vertical asymptotes

(at $x = 3$, the function is undefined)

11b. Find ALL values of x for which the graph of $g(x) = 16x + \dfrac{1}{x^2}$ has a horizontal tangent line.

$g(x)$ has horizontal tangent line when $g'(x) = 0$.

$$16 - 2x^{-3} = 0 \Rightarrow 16 - \frac{2}{x^3} = 0$$

$$\Rightarrow \frac{2}{x^3} = 16 \Rightarrow x^3 = \frac{2}{16} = \frac{1}{8}$$

$$\Rightarrow x = \sqrt[3]{\frac{1}{8}} = \frac{1}{2}$$

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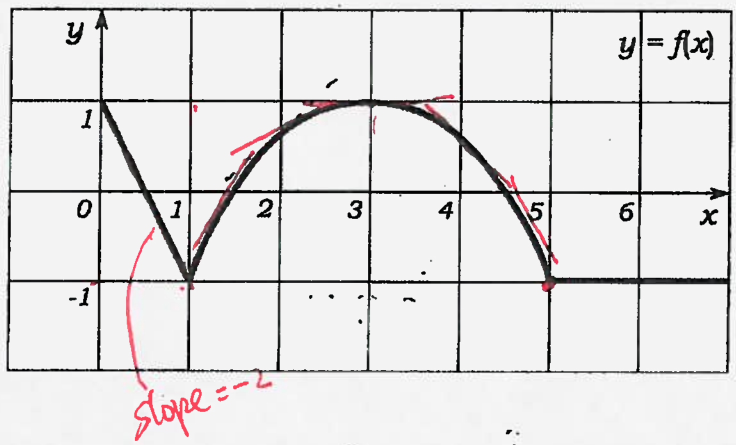
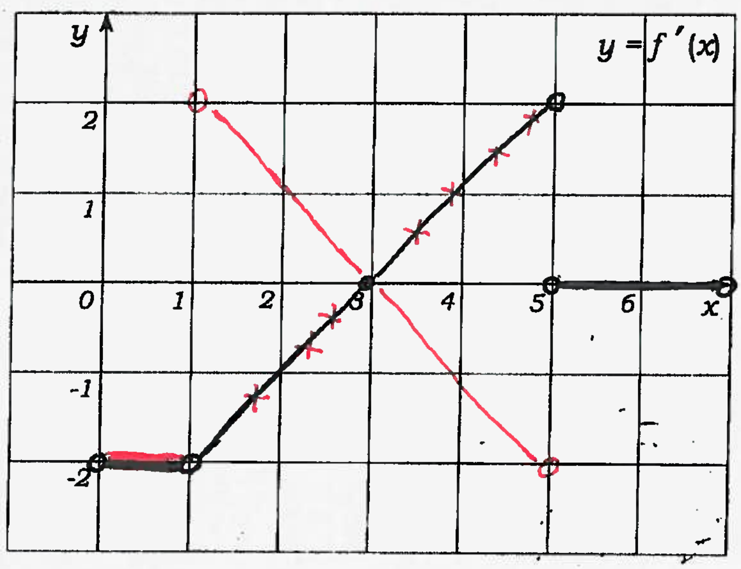
12.(12 pts.) The graph of the function $f(x)$ for $0 \le x \le 7$ is given below.



12a. Sketch the graph of the **derivative** $f'(x)$ of the function $f(x)$ in the axes given below for $0 \le x \le 7$.



12b. For what values of x in the interval $0 < x < 6$ is $f'(x)$ undefined?

Answer: $x = 1, \quad 5.$

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13.(12 pts.) The position function of a ball thrown upward , measured from ground level, is given by the function

$s=0$

$$s(t) = -5t^2 + 4t + 1.$$

13a. Find the time at which the ball hits the ground.

$S(t) = 0 \Rightarrow -5t^2 + 4t + 1 = 0$

$\Rightarrow (-5t - 1)(t - 1) = 0$

$\Rightarrow t = -1/5$ (rejected) , 1.

So $\boxed{t = 1}$

13b. Find the instantaneous **velocity** at time t.

$v = S'(t) = -10t + 4$

13c. Find the instantaneous rate of change of the **velocity** at time t.

$v'(t) = -10$

13d. Find the velocity at the moment when the ball hits the ground.

$v(1) = -10 + 4 = -6$

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14.(12 pts.) Consider the function

$$f(x) = \frac{1}{x}.$$

14a. Write down the average rate of change of $f(x)$ over the interval $2 \le x \le 2+h$. You may assume that $h \neq 0$.

$$\frac{f(2+h) - f(2)}{h} = \frac{\frac{1}{2+h} - \frac{1}{2}}{h} = \frac{\frac{2-(2+h)}{2(2+h)}}{h}$$

$$= \frac{\cancel{2} - \cancel{2} - h}{2(2+h)} \times \frac{1}{h} = \frac{-\cancel{h}}{2(2+h)} \times \frac{1}{\cancel{h}}$$

$$= \frac{-1}{2(2+h)}$$

14b. Using Part (a) above and limits (only), find the **slope** of the curve $y = \frac{1}{x}$ at $x = 2$.

The slope of $\frac{1}{x}$ at $x=2$ is

$$\lim_{h \to 0} \frac{f(2+h) - f(2)}{h} = \lim_{h \to 0} \frac{-1}{2(2+h)}$$

$$f'(2) \qquad = \frac{-1}{2(2)} = \boxed{-\frac{1}{4}}$$

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Partial Credit

You must show your work on the partial credit problems to receive credit!

13.(12 pts.)

13a. Solve for x that satisfies the equation:

$\log_{10}(3x+2) - \log_{10}(x-1) = 1$

$$\Rightarrow \log_{10}\left(\frac{3x+2}{x-1}\right) = 1 \quad \Rightarrow \frac{3x+2}{x-1} = 10^1$$

$$\Rightarrow 3x+2 = 10(x-1)$$

$$\Rightarrow 3x+2 = 10x-10$$

$$\Rightarrow 3x - 10x = -10-2$$

$$\Rightarrow -7x = -12 \Rightarrow x = 12/7.$$

13b. (Not related to above.)

If $f'(a) = \lim_{h\to 0} \frac{3^{5+h} - 3^5}{h}$ then $f(x) \stackrel{?}{=}$ 3^x and $a \stackrel{?}{=}$ 5.

$$= \lim_{h\to 0} \frac{f(a+h) - f(a)}{h}$$

$$a = 5 \; ; \; f(x) = 3^x$$

Note: This is not a unique answer. For example $f(x) = 3^x + 6$. or $3^x - 7$ .... etc. if $a = 5$.

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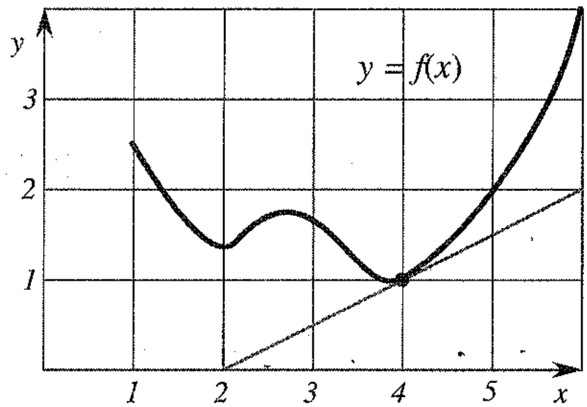
14.(12 pts.)



14a. The figure above describes the graph of $y = f(x)$ and its tangent line at $x = 4$. Answer the problems below:

i. $f(4) \stackrel{?}{=}$ $\underline{\quad 1 \quad}$ and $f'(4) \stackrel{?}{=}$ $\underline{\ 2/4 = 1/2\ }$.

ii. Find the equation of the tangent line at $x = 4$. Give your answer in slope-intercept form.

$$y - 1 = \frac{1}{2}(x-4) \Rightarrow y - 1 = \frac{1}{2}x - 2$$

$$\Rightarrow y = \frac{1}{2}x - 1$$

14b. (Not related to above.)

Find the equations of the tangent lines to the graph of $f(x) = 4x^3$ such that they are parallel to the line $y - 12x = 8$.

$f'(x) = 12x^2$, $y = 12x + 8.$

Set $f'(x) = 12 \Rightarrow 12x^2 =$

$\Rightarrow x^2 = 1 \Rightarrow x = \pm 1.$

$x = 1 \Rightarrow y = f(1) = 4.$ One tangent is $y - 4 = 12(x-1)$

i.e. $y = 12x - 8$

$x = -1 \Rightarrow y = f(-1) = -4.$ The other tangent is

$y + 4 = 12(x+1) \Rightarrow y = 12x + 8.$

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15.(12 pts.) Consider the function $f(x) = x^2 + 2x$.

15a. Compute the average rate of change of $f(x)$ over the interval $2 \le x \le 2+h$. You may assume that $h \neq 0$ and simplify your answer.

$$\frac{f(2+h) - f(2)}{h} = \frac{(2+h)^2 + 2(2+h) - (2^2 + 2(2))}{h}$$

$$= \frac{\cancel{4} + 4h + h^2 + \cancel{4} + 2h - \cancel{8}}{h} = \frac{6h + h^2}{h}$$

$$= \frac{\cancel{h}(6+h)}{\cancel{h}}$$

$$= 6 + h.$$

15b. Using Part (a) above and limits (only), find the **slope** of the curve $y = x^2 + 2x$ at $x = 2$.

The slope of $y = x^2 + 2x$ at $x = 2$ is

$$\left.\frac{dy}{dx}\right|_{x=2} = \lim_{h \to 0} \frac{f(2+h) - f(2)}{h}$$

$$= \lim_{h \to 0} (6+h) = 6$$