WORKSHEET 10

ADRIAN PĂCURAR

Problem 1. Compute the following limits. Do NOT use L'Hopital's rule.

(a) $\lim_{x\to\infty} \sqrt{x} \sin \dfrac{1}{x}$

(b) $\lim_{x\to\infty} x \tan \dfrac{1}{x}$.

(c) $\lim_{x\to 0} \dfrac{\sin 4x}{\tan 5x}$

Problem 2. Sketch a (not necessarely continuous) function f satisfying all of the following conditions:

- $f(-x) = -f(x)$ and $\lim_{x \to \infty} f(x) = 0$
- $f'(2) = 0$ and $f'(0) = 1$
- $f'(x) < 0$ if $x > 2$ $f'(x) > 0$ if $0 < x < 2$
- $f''(x) < 0$ if $0 < x < 4$ $f''(x) > 0$ if $x > 4$

Problem 3. What would the graph look like if we require the function in Problem 2 to be continuous? Give a sketch.

Problem 4. Sketch the following functions (compute the necessary derivatives, find critical and inflection points, concavity, increasing/decreasing, asymptotes, etc.).

(a) $f(x) = e^{-x^2}$

(b) $f(x) = xe^{-x^2}$

Problem 5. Compute the following limits:

(a) $\lim_{x \to 0^+} \dfrac{\ln x}{x}$

(b) $\lim_{x \to 0} \dfrac{x 3^x}{3^x - 1}$

(c) $\lim_{x \to \infty} x^3 e^{-x^2}$

(d) $\lim_{x \to \infty} (x - \ln x)$

(e) $\lim_{x \to 0^+} x^{\sqrt{x}}$

(f) $\lim_{x \to \infty} x^{1/x}$