PRACTICE QUIZ 16 SOLUTIONS

ADRIAN PĂCURAR

Time: 10 min
Time to beat: ? min

Problem 1. Find the slope of the tangent line to the curve $x = y^2 - 4y$ at the points where the curve crosses the y axis.

The intersection points are $(0,0)$ and $(0,4)$. We need to compute dy/dx at those points, and we can use implicit differentiation. Take the derivative of the equation with respect to x:

$$1 = 2y\frac{dy}{dx} - 4\frac{dy}{dx}$$

and solve for $\frac{dy}{dx}$ to get

$$\frac{dy}{dx} = \frac{1}{2y-4}$$

Now we can just plug in. At $(0,0)$ the slope is $-1/4$, while at $(0,4)$ the slope is $1/4$.

Problem 2. Find dy/dx given that $y = \frac{u^2-1}{u^2+1}$ and $u = \sqrt{x^2+2}$.

By chain rule, we have

$$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$$

so we need to compute the derivatives dy/du and du/dx. These are

$$\frac{dy}{du} = \frac{4u}{(u^2+1)^2} \qquad \text{and} \qquad \frac{du}{dx} = \frac{2x}{3(x^2+2)^{2/3}} = \frac{2x}{3u^2}$$

Thus

$$\frac{dy}{dx} = \frac{4u}{(u^2+1)^2}\frac{2x}{3u^2} = \frac{8x}{3u(u^2+1)^2}$$

and you can subsitute $\sqrt{x^2+2}$ for u in the above to express everything in terms of x.

Problem 3. A point moves along the curve $y = x^3 - 3x + 5$ so that $x = \frac{1}{2}\sqrt{t} + 3$, where t represents time. At what rate is y changing when $t = 4$?

We must find the value of dy/dt at $t = 4$. First $dy/dx = 3(x^2-1)$ and $dx/dt = \frac{1}{4\sqrt{t}}$. Hence by chain rule

$$\frac{dy}{dt} = \frac{dy}{dx}\frac{dx}{dt} = \frac{3(x^2-1)}{4\sqrt{t}}$$

When $t = 4$, $x = \frac{1}{2}\sqrt{4} + 3 = 4$, so $dy/dt = \frac{3(16-1)}{4(2)} = \frac{45}{8}$.

Problem 4. If $y = x^2 - 4x$ and $x = \sqrt{2t^2+1}$, find dy/dt when $t = \sqrt{2}$.

$$\frac{dy}{dx} = 2(x-2) \quad \text{and} \quad \frac{dx}{dt} = \frac{2t}{(2t^2+1)^{1/2}}$$

so by chain rule

$$\frac{dy}{dt} = \frac{dy}{dx}\frac{dx}{dt} = \frac{4t(x-2)}{(2t^2+1)^{1/2}}$$

When $t = \sqrt{2}$, $x = \sqrt{4}$ and

$$\frac{dy}{dt} = \frac{4\sqrt{2}(\sqrt{5}-2)}{\sqrt{5}} = \frac{4\sqrt{2}}{5}(5-2\sqrt{5})$$

after rationalizing the denominator.