10350 Tutorial Week 04 - Set 01 Name ______________________

Section ______________

1. The population $P(t)$ of a bacteria (in millions) is (roughly) given by $P(t) = t^2 + 1$ for $t \geq 0$ (in hours).

a. What is the initial population? Give units.
Solution:
$P(0) = 0^2 + 1 = 1$
1 million bacteria.

b. Find the average rate of change of the population over the time duration $[2, 5]$. Give units.
Solution:

$$\frac{P(5) - P(2)}{5 - 2} = \frac{5^2 + 1) - (2^2 + 1)}{3} = \frac{26 - 5}{3} = \frac{21}{3} = 7$$

7 million bacteria per hour

c. Find the average rate of change of the population over the time duration between 2 and t. Simplify for $t \neq 2$ and give units.
Solution:

$$\frac{P(t) - P(2)}{t - 2} = \frac{t^2 + 1) - (2^2 + 1)}{t - 2} = \frac{t^2 - 4}{t - 2} = \frac{(t - 2)(t + 2)}{t - 2} = t + 2$$

(t+2) million bacteria per hour

d. Using limits and Part (c), find the instantaneous rate of change of the population at the moment when $t = 2$ hour. Give units.
Solution:

$$P'(t) = \lim_{t \to 2} \frac{P(t) - P(2)}{t - 2} = \lim_{t \to 2} t + 2 = 2 + 2 = 4$$

4 million bacteria per hour

10350 Tutorial Week 04 - Set 02 Name ______________________

Section ______________

2. Determine the value of c such that the function $f(x)$ is continuous on the entire real line.

$$f(x) = \begin{cases} \dfrac{|x-3|}{x-3} & \text{if } x < 3 \\[2ex] cx + 5 & \text{if } x \geq 3 \end{cases}$$

Solution:

A function is continuous at $x = k$ if

$$\lim_{x \to k^-} f(x) = \lim_{x \to k^+} f(x) = f(k)$$

Therefore we need to determine for which value of c we have

$$\lim_{x \to 3^-} f(x) = \lim_{x \to 3^+} f(x) = f(3)$$

We have:

$$\lim_{x \to 3^-} f(x) = \lim_{x \to 3^-} \frac{|x-3|}{x-3} = \lim_{x \to 3^-} \frac{3-x}{x-3} = \lim_{x \to 3^-} (-1) = -1$$

$$f(3) = \lim_{x \to 3^+} f(x) = \lim_{x \to 3^+} cx + 5 = 3c + 5$$

Setting the limits equal, we solve:

$$3c + 5 = -1$$

$$c = -2$$

3. Consider the function $g(x) = \begin{cases} \dfrac{x^2+4x+3}{x+3} & \text{if } x \neq -3 \\[2ex] k & \text{if } x = -3 \end{cases}$

a. Find the value of k such that $g(x)$ is continuous at $x = -3$.
Solution:

$$\begin{aligned} \lim_{x \to 3^+} g(x) &= \lim_{x \to 3^-} g(x) \\ &= \lim_{x \to 3^-} \frac{x^2+4x+3}{x+3} \\ &= \lim_{x \to 3^-} \frac{(x+1)(x+3)}{x+3} \\ &= \lim_{x \to 3^-} x + 1 \\ &= -3 + 1 = -2 \end{aligned}$$

Setting $k = -2$, we have $g(3) = \lim_{x\to 3^+} = \lim_{x\to 3^-}$.

b. For what values of k will there be a **removable discontinuity** there?
Solution:
The discontinuity is removable if $\lim_{x\to 3^+} = \lim_{x\to 3^-} \neq g(3)$. Therefore any value $k \neq -2$ will give a removable discontinuity.