Math 10350 – Final Review

1. The graph of the function $f(x)$ is given in Figure 1 below. Find exactly or state that it does not exist each of the following quantity. If it does not exist explain why.

(a) $\lim_{x\to 2} f(x)$
(b) $\lim_{x\to 1^-} f(x)$
(c) $\lim_{x\to 1} f(x)$
(d) $\lim_{x\to -1} f(x)$
(e) $\lim_{h\to 0} \dfrac{f(h)-f(0)}{h}$
(f) $\lim_{h\to 0} \dfrac{f(2+h)+2}{h}$

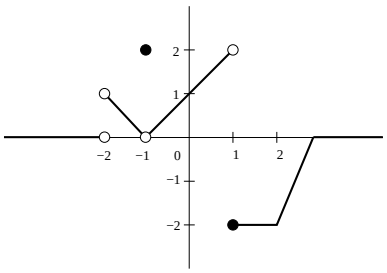


Figure 1

2. If $f'(a) = \lim_{h\to 0} \dfrac{\dfrac{2}{(3+h)^2} - \dfrac{2}{9}}{h}$, then $f(x) \stackrel{?}{=}$ _____ and the value of $a \stackrel{?}{=}$ ____.

What is the value of $\lim_{h\to 0} \dfrac{\dfrac{2}{(3+h)^2} - \dfrac{2}{9}}{h}$? (You shouldn't need to do too much.)

3. Find the values of (i) $\lim_{h\to 0} \dfrac{\cos(\pi+h)+1}{h}$, and (ii) $\lim_{h\to 0} \dfrac{e^{2h}-1}{h}$.

4. Find all horizontal and vertical asymptotes of the graph of $g(x) = \dfrac{x^2}{x^2-1}$. Determine also (a) the values of x for which $g'(x)$ is decreasing, and (b) the values of x for which $g(x)$ is concave up.

(Vert. asymp: $x=-1$; $x=1$; Hort. asymp: $y=1$, (a) Inc: $(-\infty,-1)\cup(-1,0)$; Dec: $(0,1)\cup(1,\infty)$, (b) Conc. down: $(-1,1)$; Conc. up: $(-\infty,-1)\cup(1,\infty)$)

5. Use Newton's method to find an approximation to $\sqrt[7]{100}$.

6. In a certain city the temperature (in ${}^0F$) t hours after 9a.m. was approximated by the function

$$T(t) = 50 + 14\sin\frac{\pi t}{12}.$$

Find the **average rate of change** of temperature during the period from 9a.m. to 9p.m.

7. A particle is moving on a straight line according to the velocity function:

$$v(t) = t(t-2)(t-4).$$

Find (a) the displacement and (b) the distance travelled by the particle in the time interval $1 \le t \le 6$.

8. Evaluate the following integrals

a. $\displaystyle\int \frac{x^2}{(x-1)^{20}}\,dx$ **d.** $\displaystyle\int_0^{\pi/6} \sin^2 u\cos u\,du$

9. (Review) Gravel is being dumped from a conveyor belt at the rate of $40\pi\ ft^3/min$ and its coarseness is such that it forms a pile in the shape of a cone whose height is always twice its diameter at the base. How fast is the height of the pile increasing when the pile is 5 feet high? (Answer: 25.6 ft/min)

10. Let $F(x) = \displaystyle\int_{\sqrt{x}}^{3} \frac{\cos t}{t}\,dt$. Find $F'(x)$.

11. Using limits find the derivative of $f(x) = \sqrt{x+1}$. Write down the linear approximation to $f(x)$ at $x = 3$. Estimate $\sqrt{3.8}$. Draw a graph to illustrate your estimation.

12. Two submarines at 1000 ft below sea level are travelling at 520 mph along straight-line courses that cross at right angles. How fast is the distance between the submarines closing when submarine A is 5 mi from the intersection point and submarine B is 12 mi from the intersection point.

13. The cross-section of a tunnel is a rectangle of height h meters surmounted by a semicircular roof section of radius r meters. If the cross-sectional area is 100 m^2, determine the dimensions of the cross-section which minimize the perimeter.

14. If $-4\cos(y) + 3xy^2 = x^7$ find $\dfrac{dy}{dx}$.

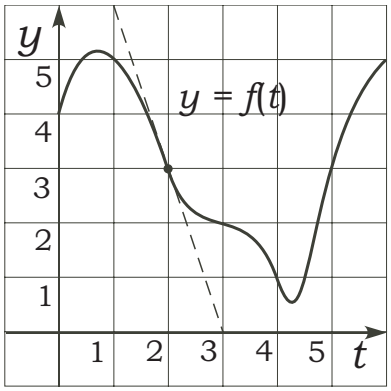
15. Water is flowing into a tank at a rate given by $r = f(t)$ (in m^3/min) whose graph is shown below (three identical ones). Let $V(t)$ denote the volume.The line is the tangent line to the graph of $f(t)$ at $t = 2$.

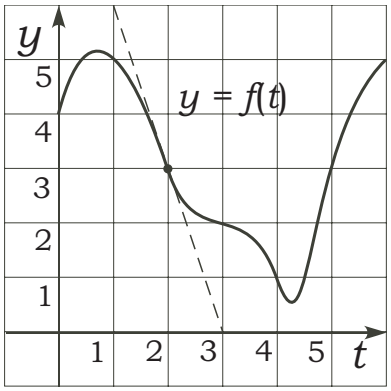
a. Estimate using (i) left end-point approximation, (ii) right end-point approximation, and (iii) mid-point rule with three equal sub-intervals, the total change in volume over $0 \le t \le 6$.

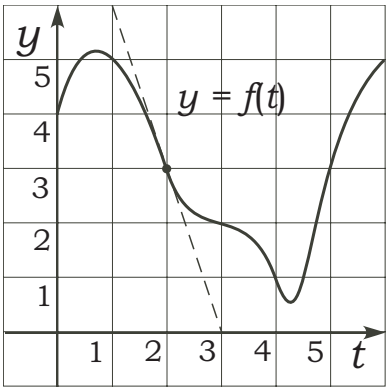
b. Find the $V'(2)$ and $V''(2)$.

c. If $V(3) = 25$ m^3, approximate the initial volume. Hint: Use one of the estimates in (a)

d. Is the amount of water in the tank always increasing?







Name: _____

Math. 10350: Calc A, Final Exam Class Time: _____
December 17, 2008

- Be sure that you have all 14 pages of the test.
- No calculators are to be used.
- The exam lasts for two hours.
- **When told to begin, remove this answer sheet and keep it under the rest of your test. When told to stop, hand in just this one page.**
- The Honor Code is in effect for this examination, including keeping your answer sheet under cover.

Good Luck!

PLEASE MARK YOUR ANSWERS WITH AN X, not a circle!

1.	[a]	[b]	[c]	[d]	[e]	14.	[a]	[b]	[c]	[d]	[e]
2.	[a]	[b]	[c]	[d]	[e]	15.	[a]	[b]	[c]	[d]	[e]
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Name: _____

Class Time: _____

Multiple Choice

1.(6 pts.) Suppose that $g(1) = 2$ and $g'(1) = 5$.

Find the slope of the graph of $P(x) = 2x^3 g(x)$ at $x = 1$.

(a) 2 (b) -2 (c) 12 (d) 30 (e) 22

2.(6 pts.) Suppose that $g(1) = 2$ and $g'(1) = 5$.

Find the derivative of $Q(x) = \dfrac{g(x)}{2x+1}$ at $x = 1$.

(a) $\dfrac{-19}{9}$ (b) $\dfrac{19}{3}$ (c) $\dfrac{11}{9}$

(d) $\dfrac{5}{2}$ (e) $\dfrac{-11}{9}$

Name: _____

Class Time: _____

3.(6 pts.) Find the derivative of $y = \tan^2(3\theta)$.

(a) $2\tan(3\theta)\sec^2(3\theta)$

(b) $6\tan(3\theta)\sec^2(3\theta)$

(c) $2\tan^2(3\theta)\sec(3\theta)$

(d) None of the given.

(e) $6\tan^2(3\theta)\sec(3\theta)$

4.(6 pts.) Evaluate the following limit:

$$\lim_{h\to 0}\frac{(x+h)^{-2}-x^{-2}}{h}$$

(a) 0 (b) $-\dfrac{2}{x}$ (c) $-2x^{-3}$

(d) Does not exist. (e) $-\dfrac{1}{x}$

Name: _____

Class Time: _____

5.(6 pts.) Find the value of k for which the function $f(x) = \begin{cases} \dfrac{x^2-3x+2}{x^2-4} & \text{if } x \text{ in } [0,2)\cup(2,\infty). \\ k & \text{if } x = 2. \end{cases}$ is a continuous for all $x \geq 0$.

(a) 0

(b) No such k.

(c) All real k except 2.

(d) 1

(e) $\dfrac{1}{4}$

6.(6 pts.) Find **all** horizontal and vertical asymptotes of the function:

$$g(x) = \frac{\sqrt{x^2 - 4\cos x + 4}}{x-2}.$$

(a) $x = 2$, $y = -1$ and $y = 1$.

(b) $x = 2$, $y = -2$ and $y = 2$.

(c) $x = 2$ and $y = 1$.

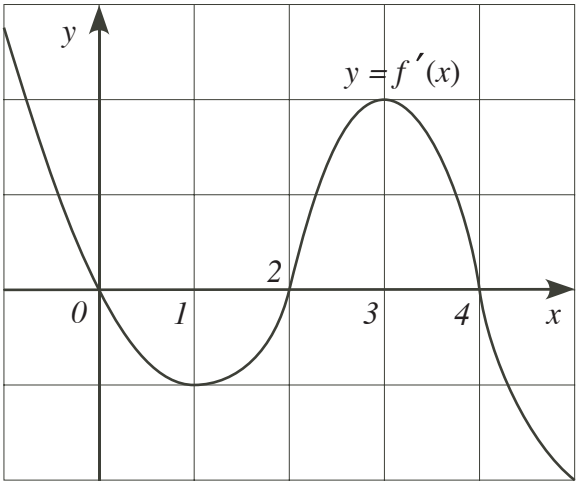
(d) $x = 2$ and $y = 2$.

(e) $x = 1$ and $y = -1$.

Name: _____

Class Time: _____

7.(6 pts.) The graph of the **derivative** $f'(x)$ of $f(x)$ for $-1 < x < 5$ is given below.



Derivative of $f(x)$

Find all local extrema of $f(x)$ for $-1 < x < 5$ and classify them?

(a) Local minimum at $x = 0$ and 4; local maximum at $x = 2$.

(b) Local minimum at $x = 1$; local maximum at $x = 3$.

(c) All local minimum at $x = 0, 2$ and 4.

(d) Local minimum at $x = 2$; local maximum at $x = 0$ and 4.

(e) Cannot be determined.

8.(6 pts.) Referring to the same graph of the **derivative** $f'(x)$ of $f(x)$ **ABOVE**, which of the following statements is **TRUE** about the graph of $f(x)$ for $-1 < x < 5$?

(a) The graph of $f(x)$ is concave downward on the intervals $(2, 4)$ ONLY.

(b) The graph of $f(x)$ is concave upward on the intervals $(1, 3)$ ONLY.

(c) The graph of $f(x)$ is concave upward on the intervals $(-1, 2)$ and $(4, 5)$.

(d) The graph of $f(x)$ is concave upward on the intervals $(-1, 1)$ and $(3, 5)$.

(e) The graph of $f(x)$ is concave downward on the intervals $(0, 2)$ and $(4, 5)$.

Name: _____

Class Time: _____

9.(6 pts.) The critical numbers of $f(x) = 5 + 8x + 6x^{2/3}$ are

(a) There are none. (b) -8 and 0 only. (c) $-\frac{1}{8}$ only.

(d) $-\frac{1}{8}$ and 0 only. (e) -8 only.

10.(6 pts.) What are the global maximum and global minimum values of the function $f(x) = x^3 - 12x$ for x in $[0, 3]$?

(a) The global maximum value is 0, the global minimum is -9.

(b) The global maximum value is 16, the global minimum is -9.

(c) The global maximum value is 0, the global minimum is -16.

(d) The global maximum value is 16, the global minimum is 0.

(e) The global maximum value is 9, the global minimum is 1.

Name: _____

Class Time: _____

11.(6 pts.) Evaluate the integral $\displaystyle\int_0^1 \frac{t+2}{\sqrt{t^2+4t+3}}\,dt$.

(a) 2

(b) $2\sqrt{8}-2\sqrt{3}$

(c) $\frac{1}{4}(3^{-3/2}-8^{-3/2})$

(d) 1

(e) $\sqrt{8}-\sqrt{3}$

12.(6 pts.) Find the **linear** approximation (tangent line approximation) of the function $f(x)=(2x+3)^5+3$ at $x=-1$.

(a) $f(x)\approx 10(2x+3)^4(x-1)-4$ for x near -1.

(b) $f(x)\approx 10(2x+3)^4(x+1)+4$ for x near -1.

(c) $f(x)\approx 10(x-4)-1$ for x near -1.

(d) $f(x)\approx 10(x+1)+4$ for x near -1.

(e) $f(x)\approx 10(x-1)-4$ for x near -1.

Name: _____

Class Time: _____

13.(6 pts.) Suppose that the derivative of $f(x)$ is given by

$$f'(x) = 2\sin x + 6x^2.$$

Find a formula for $f(x)$ if its graph passes through the point $(0, 5)$.

(a) $f(x) = -2\cos x + 2x^3 + 7$

(b) $f(x) = 2\cos x + 12x + 3$

(c) $f(x) = -2\cos x + 2x^3 + 5$

(d) $f(x) = 2\cos x + 2x^3 + 3$

(e) $f(x) = 2\cos x + 2x^3 + 5$

14.(6 pts.) The volume of a **spherical** balloon is growing at a constant rate of 1 cubic inch per second. How fast is the radius r growing when $r = 2$ inches? (Note that the volume of a sphere of radius r is $V = \frac{4\pi}{3}r^3$.)

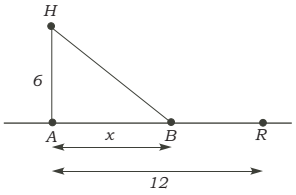
(a) $\frac{1}{8\pi}$ inch/sec (b) $(8\pi - 1)$ inch/sec (c) $\frac{1}{16\pi}$ inch/sec

(d) $\frac{1}{4\pi}$ inch/sec (e) $(4\pi - 1)$ inch/sec

Name: _____

Class Time: _____

15.(6 pts.) A house H is located in the woods, 6 miles from the nearest point, A, on a straight road. A restaurant, R, is located 12 miles down the road from A. Jack can ride his bike 2 miles per hour in the woods and 10 miles per hour along the road. He decides to ride the bike through the woods to some intermediate point B, x miles from A, and then ride along the road to R. Since he is starving, he wants to minimize his time. Which of the following is the function to be minimized? **Do not solve the rest of the problem!**



(a) $12 + 10x$

(b) $3 + \dfrac{x}{10}$

(c) $\dfrac{\sqrt{36 + x^2}}{2} + \dfrac{x}{10}$

(d) $2\sqrt{36 + x^2} + 10(12 - x)$

(e) $\dfrac{\sqrt{36 + x^2}}{2} + \dfrac{12 - x}{10}$

16.(6 pts.) The position function of a ball thrown upward, measured from **ground level**, is given by the function

$$s(t) = -5t^2 + 3t + 2.$$

Here s is in meters and t is in seconds. Find the velocity of the ball the moment it hits the ground.

(a) -7 m/sec

(b) -1 m/sec

(c) 1 m/sec

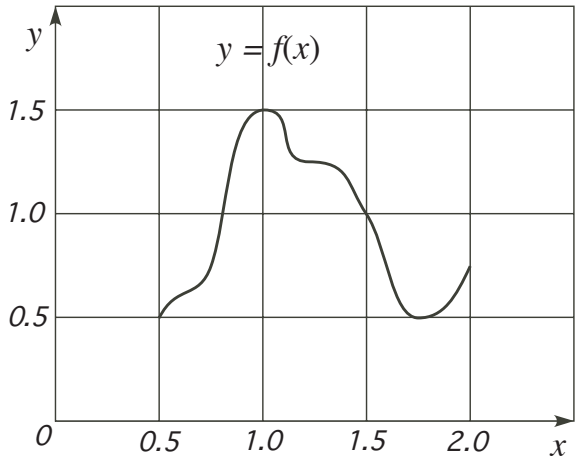
(d) -10 m/sec

(e) 0 m/sec

Name: _____

Class Time: _____

17.(6 pts.) Find the estimate for the area under the graph of $f(x)$ over $[0.5, 2.0]$, using **left-hand sum** with **three** equal subintervals.



(a) 1.0

(b) 3.0

(c) 4.0

(d) 2.0

(e) 1.5

18.(6 pts.) Find the **average rate of change** of the function $f(x) = \sec x \tan x$ for $[0, \pi/4]$.

(a) $\dfrac{4}{\pi}\left(\sqrt{2}-1\right)$

(b) $\dfrac{4\sqrt{2}}{\pi}$

(c) 1

(d) $\left(\sqrt{2}-1\right)$

(e) $\dfrac{4}{\pi}$

Name: _____

Class Time: _____

19.(6 pts.) Evaluate the integral

$$\int \frac{2x^5 - x^2 + 1}{x^4}\, dx$$

(a) $\dfrac{10x^4 - 2x}{4x^3} + C$

(b) $\dfrac{\dfrac{x^6}{3} - \dfrac{x^3}{3} + x + C}{\dfrac{x^5}{5} + C}$

(c) $\dfrac{5x}{3} - \dfrac{5x^{-2}}{3} + 5x + C$

(d) $2 + 2x^{-3} - 4x^{-5} + C$

(e) $x^2 + x^{-1} - \dfrac{x^{-3}}{3} + C$

20.(6 pts.) Let $f(x)$ be the function whose graph is shown below. Which of the following statements is **FALSE**?

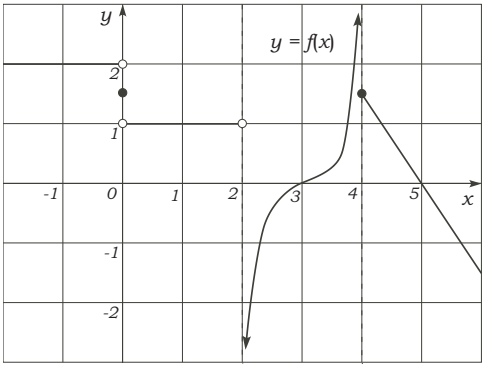
(a) $\lim_{x \to 2^+} f(x) = -\infty$.

(b) $f(x)$ is not continuous at $x = 0$.

(c) $\lim_{x \to 4^+} f(x)$ is finite.

(d) $\lim_{x \to 0} f(x)$ exists.

(e) $f(x)$ is continuous at $x = 3$.



(a) (b) (c) (d) (e)

Name: _____

Class Time: _____

21.(6 pts.) Given that

$$f''(x) = x(x-2)^2(x-4)^3.$$

Find all value of x at which the graph of $f(x)$ has an inflection point.

(a) $x = 0$ and 4 only.

(b) $x = 0$, 2 and 4.

(c) $x = 2$ only.

(d) $x = 0$ only.

(e) $x = 0$ and 2 only.

22.(6 pts.) Using implicit differentiation, find $\dfrac{dy}{dx}$ if $y^2 + xy - x^2 = 5$.

(a) $\dfrac{dy}{dx} = \dfrac{2x - y + 5}{2y + x}$

(b) $\dfrac{dy}{dx} = \dfrac{-2x + y}{2y + x}$

(c) $\dfrac{dy}{dx} = \dfrac{-y}{2y - x - 5}$

(d) $\dfrac{dy}{dx} = \dfrac{2x - y}{2y + x}$

(e) $\dfrac{dy}{dx} = \dfrac{1}{3}$

Name: _____

Class Time: _____

23.(6 pts.) The length (in mm) at time t (in seconds) of a straight metal rod being heated slowly is given by the function

$$L(t) = \sqrt{2t+1}$$

Using **calculus**, **estimate** the change in length of the rod over the time duration $4 \leq t \leq 4.5$

(a) $(2\sqrt{10} - 8)$ mm

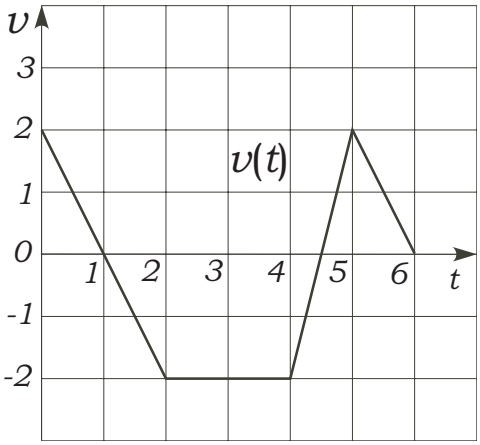
(b) Cannot be determined.

(c) 1/6 mm

(d) $(\sqrt{10} - 4)$ mm

(e) 1/12 mm

24.(6 pts.) The graph of $v(t)$ is given below:



Find $\int_0^4 v(t)dt$.

(a) 2

(b) -4

(c) 0

(d) -2

(e) 4

Name: _____

Class Time: _____

25.(6 pts.) Use Newton's method to estimate the solution of

$$x^3 + 2x + 5 = 0.$$

If the initial guess $x_1 = -1$, find the value of the second iterate x_2.

(a) $x_2 = -\frac{3}{5}$.

(b) $x_2 = -\frac{3}{2}$.

(c) $x_2 = -\frac{2}{5}$.

(d) $x_2 = -\frac{7}{2}$.

(e) $x_2 = -\frac{7}{5}$.

Name: _____

Math. 10350: Calc A, Final Exam Class Time: ANSWERS
December 17, 2008

- Be sure that you have all 14 pages of the test.
- No calculators are to be used.
- The exam lasts for two hours.
- **When told to begin, remove this answer sheet and keep it under the rest of your test. When told to stop, hand in just this one page.**
- The Honor Code is in effect for this examination, including keeping your answer sheet under cover.

Good Luck!

PLEASE MARK YOUR ANSWERS WITH AN X, not a circle!

1.	a	b	c	d	•	14.	a	b	•	d	e
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6.	•	b	c	d	e	19.	a	b	c	d	•
7.	a	b	c	•	e	20.	a	b	c	•	e
8.	a	•	c	d	e	21.	•	b	c	d	e
9.	a	b	c	•	e	22.	a	b	c	•	e
10.	a	b	•	d	e	23.	a	b	•	d	e
11.	a	b	c	d	•	24.	a	•	c	d	e
12.	a	b	c	•	e	25.	a	b	c	d	•
13.	•	b	c	d	e						

Name: _____

Math 10350: Calculus A, Final Exam Instructor: _____
December 15, 2007

- Be sure that you have all 14 pages of the test.
- No calculators are to be used.
- The exam lasts for one hour and 15 minutes.
- **When told to begin, remove this answer sheet and keep it under the rest of your test. When told to stop, hand in just this one page.**

Sign the pledge. "On my honor, I have neither given nor received unauthorized aid on this Exam":

_____

Good Luck!

PLEASE MARK YOUR ANSWERS WITH AN X, not a circle!

1.	[a]	[b]	[c]	[d]	[e]	14.	[a]	[b]	[c]	[d]	[e]
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Name: _____

Instructor: _____

Multiple Choice

1.(6 pts.) The graph of $g(x)$ and the tangent to the graph at $x = 2$ is given in Figure 1.

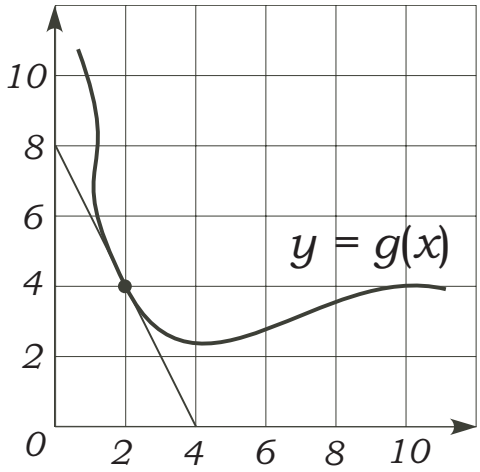

Figure 1.

Find the slope of $P(x) = x^2 g(x)$ at $x = 2$.

(a) -2 (b) -24 (c) 8 (d) -8 (e) 24

2.(6 pts.) Let $F(x) = \int_0^{x^2} f(t)\,dt$. Find $F'(x)$.

(a) $x^2 f'(x^2)$

(b) $f(x^2)$

(c) $f(x^2) - f(0)$

(d) $2x f'(x^2)$

(e) $2x f(x^2)$

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3.(6 pts.) Find the derivative of $y = \sin^3(2\theta)$.

(a) $6\sin^2(2\theta)\cos(2\theta)$ (b) None of the given. (c) $-6\sin(2\theta)\cos^2(2\theta)$

(d) $-6\sin^2(2\theta)\cos(2\theta)$ (e) $6\sin(2\theta)\cos^2(2\theta)$

4.(6 pts.) Compute the following limit:

$$\lim_{h\to 0}\frac{(x+h)^{10}-x^{10}}{h}$$

(a) $\dfrac{x^{11}}{11}$ (b) 0 (c) $10x^{11}$

(d) Does not exist. (e) $10x^9$

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5.(6 pts.)Consider the function

$$f(x) = \begin{cases} mx + 2 & \text{if } x \leq 1 \\ \dfrac{|x-1|}{x-1} & \text{if } x > 1 \end{cases}$$

Find the value of m so the $f(x)$ is continuous at $x = 1$.

(a) $m = 1$

(b) $m = -1$

(c) $m = -3$

(d) $m = 3$

(e) No such m exists.

6.(6 pts.) Find **all** horizontal and vertical asymptotes of the function:

$$R(x) = \frac{x^2 - x - 6}{x^2 + 3x + 2}.$$

(a) $x = -1$, $x = -2$ and $y = -1$.

(b) $x = 1$, $x = 2$ and $y = -3$.

(c) $x = -1$ and $y = 2$.

(d) $x = -1$, $x = -2$ and $y = 1$.

(e) $x = -1$ and $y = 1$.

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7.(6 pts.) The **deriative** $f'(x)$ of the function $f(x)$ is given below

$$f'(x) = \frac{x}{x^2 - 4} \qquad \leftarrow \textbf{Derivative of } f(x)$$

For what values of x is $f(x)$ **increasing**?

(a) $(-\infty, -2)$ only.

(b) $(-2, 0)$ only

(c) $(-2, 0) \cup (2, \infty)$

(d) $(-\infty, -2) \cup (0, 2)$

(e) All x except -2 and 2.

8.(6 pts.) Refering to the same function considered above with

$$f'(x) = \frac{x}{x^2 - 4} \qquad \leftarrow \textbf{Derivative of } f(x),$$

for what values of x is the graph of $f(x)$ **concave downward**?

(a) No values of x.

(b) All x except -2 and 2.

(c) $\left(-\infty, -1/\sqrt{2}\right) \cup \left(1/\sqrt{2}, \infty\right)$ only.

(d) $(-\infty, -2) \cup (2, \infty)$ only.

(e) $(-2, 2)$ only.

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9.(6 pts.) Which of the following statements is **TRUE** about the function

$$f(x) = x^3 - 3x^2.$$

(a) $f(x)$ is always increasing.

(b) $f(x)$ has a local minimum at $x = 3$ ONLY.

(c) $f(x)$ neither has a maximum nor minimum at $x = 0$.

(d) $f(x)$ has a local maximum at $x = 0$

(e) $f(x)$ has a local maximum at $x = 2$.

10.(6 pts.) Find the **absolute minimum** of the function $f(x) = x^3 - 3x^2 + 10$ for $-2 \leq x \leq 3$.

(a) -10

(b) $-\infty$

(c) -12

(d) 10

(e) 6

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11.(6 pts.) Evaluating the integral $\int_0^3 t\sqrt{2t^2+1}\,dt$.

(a) $\sqrt{3}$

(b) $\frac{1}{3}(19^{3/2}-1)$

(c) $\frac{1}{6}(19^{3/2}-1)$

(d) $4\sqrt{3}$

(e) $\frac{\sqrt{3}}{2}$

12.(6 pts.) Find the **linear** approximation (tangent line approximation) of the function $f(x) = \tan x + 3$ at $x = \pi$.

(You may use: $\tan\pi = 0$ and $\sec\pi = -1$.)

(a) $f(x) \approx (x-3) - \pi$ for x near π.

(b) $f(x) \approx (\sec^2 x)(x-\pi) + 3$ for x near π.

(c) $f(x) \approx (\sec^2 x)(x+\pi) - 3$ for x near π.

(d) $f(x) \approx (x+\pi) - 3$ for x near π.

(e) $f(x) \approx (x-\pi) + 3$ for x near π.

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13.(6 pts.) Voltage of a capacitor $V(t)$ (in volts) changes according to the rate

$$\frac{dV}{dt} = \cos t - \sin t; \quad V(0) = 20$$

where t is time in seconds. What is the voltage when $t = \pi$?

(a) $V(\pi) = 18$

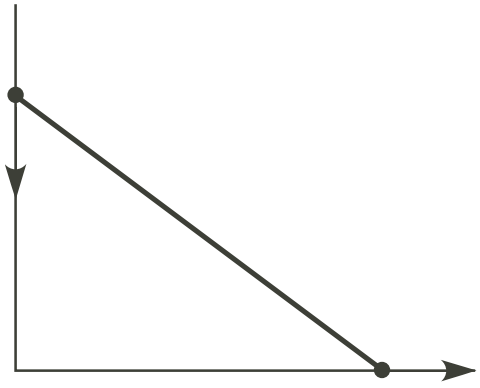
(b) $V(\pi) = 19$

(c) $V(\pi) = 22$

(d) $V(\pi) = 20$

(e) $V(\pi) = 21$

14.(6 pts.) Consider a 5 feet plank leaning against a vertical wall as shown. If the top of the plank is moving **down** at a rate of $\frac{1}{2}$ ft/sec, at what rate is the bottom of the plank moving when the top is 3 feet **above the ground.**



(a) 2 ft/sec

(b) $-\frac{3}{8}$ ft/sec

(c) $-\frac{2}{3}$ ft/sec

(d) $\frac{3}{8}$ ft/sec

(e) $\frac{2}{3}$ ft/sec

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15.(6 pts.) A manufacturer wishes to make a cylindrical can closed at **both ends** with volume 4π m^3. The material for the top and bottom costed \$ 2/m^2, and the material of the side costs \$ 1/m^2. Write the formula for the cost function $C(r)$ in terms of the radius of the cylinder r.

(a) $C(r) = 4\pi r^2 + \dfrac{6\pi}{r}$

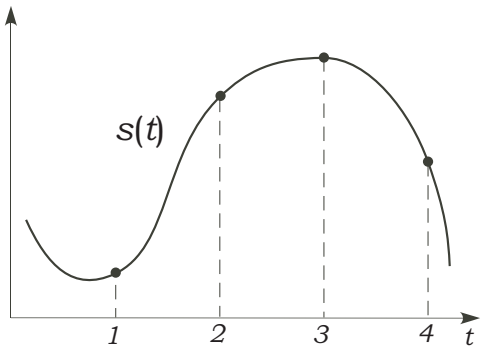
(b) $C(r) = 4\pi r^2 + \dfrac{8\pi}{r}$

(c) $C(r) = 4\pi r^2 + \dfrac{4\pi}{r}$

(d) $C(r) = 2\pi r^2 + \dfrac{8\pi}{r}$

(e) $C(r) = 4\pi r^2 + \dfrac{6\pi}{r^2}$

16.(6 pts.) Let O be a point on a straight line path. The graph of the position $s(t)$, measured from O, of a particle P moving on the straight path is given below. Let $v(t)$ be the velocity of P, and $a(t)$ be the acceleration of P.



Only ONE of the following statement is **TRUE**. Which is it?

(a) P returns to the origin twice in the duration $0 < t < 4$.

(b) $v(t)$ is **greatest** at $t = 3$.

(c) P is **decelerating** at $t = 1$.

(d) $v(2)$ and $a(2)$ are both **positive**.

(e) $v(4)$ and $a(4)$ are both **negative**.

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17.(6 pts.) Find the estimate for the area under the graph of $f(x) = x^2$ over $[1, 3]$, using **right-hand sum** with **four** equal subintervals.

(a) $\left[\left(\frac{3}{2}\right)^2 + 2^2 + \left(\frac{5}{2}\right)^2 + 3^2\right]$

(b) $\left[1^2 + \left(\frac{3}{2}\right)^2 + 2^2 + \left(\frac{5}{2}\right)^2\right]$

(c) $\frac{3}{2}\left[\left(\frac{3}{2}\right)^2 + 2^2 + \left(\frac{5}{2}\right)^2 + 3^2\right]$

(d) $\frac{1}{2}\left[\left(\frac{3}{2}\right)^2 + 2^2 + \left(\frac{5}{2}\right)^2 + 3^2\right]$

(e) $\frac{1}{2}\left[1^2 + \left(\frac{3}{2}\right)^2 + 2^2 + \left(\frac{5}{2}\right)^2\right]$

18.(6 pts.) Given that

$$\int_1^3 f(x)\,dx = 5 \qquad \int_2^3 f(x)\,dx = -1$$

Find the value of $\int_1^2 \big(2 - f(x)\big)\,dx$.

(a) -4

(b) 8

(c) -6

(d) 6

(e) -2

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19.(6 pts.) Evaluate the integral

$$\int \frac{x-2}{x^3}\,dx$$

(a) $\dfrac{x^2}{2} + x^{-2} + C$ (b) $\dfrac{\dfrac{x^2}{2} - 2x + C}{\dfrac{x^4}{4} + C}$ (c) $-x^{-1} - 2x + C$

(d) $-x^{-1} + x^{-2} + C$ (e) $-2x^{-2} + 6x^{-4} + C$

20.(6 pts.) Evaluate the limit

$$\lim_{x\to\infty} \left(x - \sqrt{x^2 + x}\right)$$

(a) $\dfrac{1}{2}$ (b) Does not exist. (c) $-\dfrac{1}{2}$

(d) 0 (e) $-\infty$

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21.(6 pts.) The graph of the function $y = f(x)$, and the size of the area enclosed by it and x-axis are shown below.

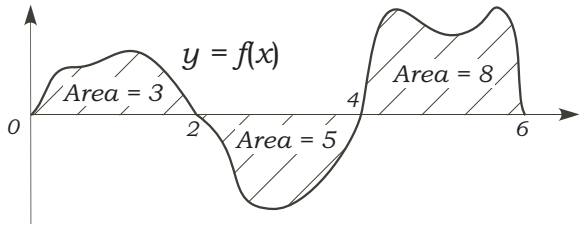

Figure 3.

Find the value of $\int_0^6 f(x)\,dx$.

(a) -16 (b) 6 (c) 16

(d) -6 (e) 0

22.(6 pts.) Find $\dfrac{dy}{dx}$ if $x^3 - xy + y^2 = 4$

(a) $\dfrac{dy}{dx} = \dfrac{3x^2}{2y - x}$ (b) $\dfrac{dy}{dx} = \dfrac{-3x^2}{2y - 1}$ (c) $\dfrac{dy}{dx} = \dfrac{y - 3x^2 + 4}{2y - x}$

(d) $\dfrac{dy}{dx} = \dfrac{-y - 3x^2}{2y - x}$ (e) $\dfrac{dy}{dx} = \dfrac{y - 3x^2}{2y - x}$

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23.(6 pts.) The critical numbers of $f(x) = 5x^{4/5} - 4x$ are

(a) There are none. (b) 0 only (c) 1 only

(d) 0 and 1 (e) -1 only

24.(6 pts.) Find the **coordinates** of the points on the curve $f(x) = x^3 + 11$ where the tangent lines are parallel to the line $12x - y + 5 = 0$

(a) $(-1, 10)$ and $(1, 12)$

(b) None exists.

(c) $(-2, 3)$ and $(2, 19)$

(d) $(2, 19)$ only.

(e) $(1, 12)$ only.

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25.(6 pts.) A student attempts to use Newton's method to estimate the solution of

$$x^3 + x + 1 = 0.$$

If her initial guess $x_0 = 0$. Find the values of the first two iterates x_1 and x_2.

(a) $x_1 = -1$ and $x_2 = -\frac{1}{2}$.

(b) $x_1 = 1$ and $x_2 = \frac{1}{4}$.

(c) $x_1 = 1$ and $x_2 = \frac{7}{4}$.

(d) $x_1 = -1$ and $x_2 = 2$.

(e) $x_1 = -1$ and $x_2 = -\frac{3}{4}$.

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Math 10350: Calculus A, Final Exam Instructor: ANSWERS
December 15, 2007

- Be sure that you have all 14 pages of the test.
- No calculators are to be used.
- The exam lasts for one hour and 15 minutes.
- **When told to begin, remove this answer sheet and keep it under the rest of your test. When told to stop, hand in just this one page.**

Sign the pledge. "On my honor, I have neither given nor received unauthorized aid on this Exam":

_____

Good Luck!

PLEASE MARK YOUR ANSWERS WITH AN X, not a circle!

1.	a	b	•	d	e	14.	a	b	c	•	e
2.	a	b	c	d	•	15.	a	•	c	d	e
3.	•	b	c	d	e	16.	a	b	c	d	•
4.	a	b	c	d	•	17.	a	b	c	•	e
5.	a	•	c	d	e	18.	•	b	c	d	e
6.	a	b	c	d	•	19.	a	b	c	•	e
7.	a	b	•	d	e	20.	a	b	•	d	e
8.	a	•	c	d	e	21.	a	•	c	d	e
9.	a	b	c	•	e	22.	a	b	c	d	•
10.	•	b	c	d	e	23.	a	b	c	•	e
11.	a	b	•	d	e	24.	a	b	•	d	e
12.	a	b	c	d	•	25.	a	b	c	d	•
13.	•	b	c	d	e						