Name: Date: 02/08/2018

M20580 L.A. and D.E. Tutorial
Worksheet 4
Sections 1.8–1.9, 2.1–2.2

1. (a) Let $A = \begin{bmatrix} 1 & 3 & 2 \\ 2 & 1 & 2 \end{bmatrix}$, $\mathbf{u} = \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix}$ and define a transformation $T : \mathbb{R}^3 \longrightarrow \mathbb{R}^2$ by $T(\mathbf{x}) \doteq A\mathbf{x}$. Find $T(\mathbf{u})$, the image of $\mathbf{u}$ under the transformation T.

(b) Let $T : \mathbb{R}^3 \longrightarrow \mathbb{R}^2$ be a linear transformation. If

$$T(\mathbf{u}) = \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \quad T(\mathbf{v}) = \begin{bmatrix} 3 \\ 1 \end{bmatrix}, \quad T(\mathbf{w}) = \begin{bmatrix} 2 \\ 2 \end{bmatrix},$$

where $\mathbf{u}, \mathbf{v}, \mathbf{w} \in \mathbb{R}^3$. Find $T(\mathbf{x})$, where $\mathbf{x} = 2\mathbf{u} + 3\mathbf{v} - \mathbf{w}$.

2. (a) Suppose $T : \mathbb{R}^2 \to \mathbb{R}^3$ is a linear transformation such that

$$T\left(\begin{bmatrix}0\\1\end{bmatrix}\right) = \begin{bmatrix}1\\0\\1\end{bmatrix}, \qquad T\left(\begin{bmatrix}1\\1\end{bmatrix}\right) = \begin{bmatrix}1\\-1\\1\end{bmatrix}, \quad \text{and} \quad T\left(\begin{bmatrix}1\\0\end{bmatrix}\right) = \begin{bmatrix}0\\-1\\0\end{bmatrix}.$$

Find the *standard matrix* for T, i.e. find a matrix A such that $T(\mathbf{x}) = A\mathbf{x}$.

(b) Let $S : \mathbb{R}^3 \to \mathbb{R}^2$ be a linear transformation such that

$$S\left(\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}\right) = \begin{bmatrix}x_1 - 2x_3\\x_1 + x_2 + x_3\end{bmatrix},$$

Find $S\left(\begin{bmatrix}1\\0\\0\end{bmatrix}\right)$. Then find the standard matrix for S.

3. Let $A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$. Compute $(A+B)(A-B)^T$?

4. Which of the following equations involving 3×3-matrices A, B, C and I_3 (the identity matrix) *could* be *false* for some such matrices A, B, C ?

   (a) $(A+B)^2 = A^2 + 2AB + B^2$

   (b) $(A+B)C = AC + BC$

   (c) $(AB)C = A(BC)$

   (d) $A + B = B + A$

   (e) $(\mathrm{I}_3 + A)(\mathrm{I}_3 - A) = \mathrm{I}_3 - A^2$

5. Find the inverse of the matrix

$$Q = \begin{bmatrix} 2 & 0 & 5 \\ 0 & 1 & 0 \\ 3 & 0 & 7 \end{bmatrix}$$