WORKSHEET 4

ADRIAN PĂCURAR

Problem 1. Compute the following limits:

(a) $\lim_{x\to 2}\sqrt{\dfrac{2x^2+1}{3x-2}}$

(b) $\lim_{h\to 0}\dfrac{(x+h)^8-x^8}{h}$

(c) $\lim_{x\to -1}\dfrac{x^2+2x+1}{x^4-1}$

Problem 2. If

$$f(x)=\begin{cases} x^2 & \text{if } x \text{ is rational}\\ 0 & \text{if } x \text{ is irrational}\end{cases}$$

what is $\lim_{x\to 0} f(x)$?

Problem 3.

(a) Why is the following function discontinuous at $x = 1$? Sketch it's graph.

$$f(x) = \begin{cases} 1 - x^2 & \text{if } x < 1 \\ 1/x & \text{if } x \geq 1 \end{cases}$$

(b) Why is the following function discontinuous at $x = 0$? Sketch it's graph.

$$f(x) = \begin{cases} \cos x & \text{if } x < 0 \\ 0 & \text{if x} = 0 \\ 1 - x^2 & \text{if } x \geq 0 \end{cases}$$

Problem 4. Find the values a and b that make f continuous everywhere

$$f(x) = \begin{cases} \dfrac{x^2 - 4}{x - 2} & \text{if } x < 2 \\ ax^2 - bx + 3 & \text{if } 2 \leq x < 3 \\ 2x - a + b & \text{if } x \geq 3 \end{cases}$$

Problem 5. Find an equation to the tangent line to the curve $y = x^3 - 3x + 1$ at the point $(2, 3)$.

Problem 6. Find the derivative of $f(x) = \dfrac{2x + 1}{x + 3}$.

Problem 7. Using the limit definition, compute the derivatives of the following functions:

(a) $f(x) = \sqrt{x}$

(b) $g(x) = x^{-1}$

(c) $y = \dfrac{1}{x+1}$

(d) $f(x) = x^5$

(e) $f(x) = x^{5/2}$