Math 10350 Review for Exam 1

1. Find the
$$\frac{dy}{dx}$$
 if (a) $y = \frac{3x^3 - \sqrt{x} + \pi x^2}{x^2}$, (b) $y = \frac{4^t + 1}{2^t}$. Hint: Divide first.

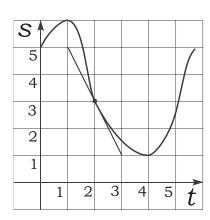
- **2.** Consider the function $f(x) = \sqrt{x+3}$.
- **2a.** Find the average rate of change of the function f(x) over the interval [1, 6].
- **2b.** Find the average rate of change of the function f(x) over the interval [1, 1+h]. Assuming that $h \neq 0$, completely simplify your answer.
- **2b.** Find the derivative of the function $f(x) = \sqrt{x+3}$ at x=1 using the limit definition of derivative. What other names does this value have?
- **2c.** Find the equation of the tangent line to the graph of $y = \sqrt{x+3}$ at x = 1.
- **3.** A differentiable function g(x) is such that

$$g(2) = -1,$$
 $g'(2) = 2,$ $f(2) = 2$ and $f'(2) = -4$

- (a) If $A(x) = 2g(x) + 3f(x) + e^2$ find $A'(2) \stackrel{?}{=}$ _____.
- (b) If $B(x) = g(x) \cdot (2e^x 3)$ find the slope of the graph of B(x) at x = 2.
- (c) If $C(x) = \frac{4x + g(x)}{f(x) + g(x)}$ find the instantaneous rate of change of C(x) at x = 2
- **4.** Solve the x in the following equations: (a) $\frac{e^t+3}{e^t-2}=5$; (b) $\log_2(3t^2)-\log_2(t^2-1)=2$
- **5.** An exponential function $y = a \cdot b^x$ passes through the points (2,9) and (4,1). (i) Find the values of a and b. Is this an exponential decay or growth function? (ii) For what value of x is y = 27?
- **6.** Find c so that f(x) given below is is continuous at x = 1/3.

$$f(x) = \begin{cases} \frac{9x^2 - 1}{3x^2 - 7x + 2} & x \neq 1/3, -2\\ c & x = 1/3 \end{cases}$$

- 7. The position s(t) (in feet) of a particle moving on a straight line at time t minutes is given by the graph below. The line is the tangent line to the graph of s(t) at t=2.
- **a.** Find the average velocity of the particle over the time duration $0 \le t \le 3$.
- **b.** Find the instantaneous velocity of the particle at t=2.
- **c.** What is the equation of the tangent line at t=3?
- **d.** Compare the velocities of the particle at t = 1, 2 and 5 minutes.
- **e.** Are there any time for which the velocity of the particle is momentarily zero?



- 8. If $\log_2 3 = x$ and $\log_2 5 = y$ express the following in terms of x and y: (a) $\log_2(45)$; (b) $\log_2(60)$; (c) $\log_2(9/10)$.
- **9.** Consider the curve given by $f(x) = x + \frac{1}{x}$.
- **a.** Find the points on the curve y = f(x) at which the tangent lines are horizontal.
- **b.** Find the equations of the tangent lines to the curve y = f(x) which are parallel to the line 3x 4y = 5.
- **10.** Consider the function

$$f(x) = \begin{cases} 3+x & -\infty < x < 5 \\ x^2 & 5 \le x < 11 \\ 5-3x & 11 \le x < +\infty \end{cases}$$

Compute the following limits

a.
$$\lim_{x \to 5^+} 2f(x) - 5$$

b.
$$\lim_{x \to 11^{-}} f(x) + \frac{2}{f(x)}$$

- **c.** $\lim_{x \to 7} (f(x))^2$
- **d.** If f(x) continuous? Explain with limits.
- 11. The graph of the function f(x) is given in Figure 1 below. Find exactly or state that it does not exist each of the following quantity. If it does not exist explain why.



(a)
$$\lim_{x \to 2} f(x)$$

(b) $\lim_{x \to 1^{-}} f(x)$

(c)
$$\lim_{x \to 1} f(x)$$

(c)
$$\lim_{x \to 1} f(x)$$

(d) $\lim_{x \to -1} f(x)$

(e)
$$\lim_{h \to 0} \frac{f(h) - f(0)}{h}$$

(f) $\lim_{h \to 0} \frac{f(2+h) + 2}{h}$

(f)
$$\lim_{h\to 0} \frac{f(2+h)+2}{h}$$

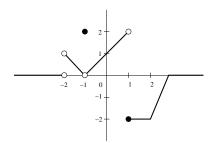


Figure 1

- **12.** If $f'(a) = \lim_{h \to 0} \frac{\frac{2}{3+h} \frac{2}{3}}{h}$, then $f(x) \stackrel{?}{=}$ ____ and the value of $a \stackrel{?}{=}$ ____.
- **13.** If $g'(a) = \lim_{h \to 0} \frac{\cos(\pi + h) + 1}{h}$, then $g(x) \stackrel{?}{=}$ and the value of $a \stackrel{?}{=}$.
- 14. In 20 years, the balance of a bank account grew from 1000 to 3000.
- 14a. If interest is compounded continuously, find the annual interest rate r. Write down a formula for the balance of the account t years after the account is set up. How long would it take for the account to double its balance. Answer: $r = \ln(3)/20$; double time = $20 \ln(2)/\ln(3)$ years.
- **14b.** If interest is compounded weekly, find the annual interest rate r. Write down a formula for the balance of the account t years after the account is set up. How long would it take for the account to double its balance.

Answer: $r = 52(3^{1/1040} - 1)$; double time = $20 \ln(2) / \ln(3)$ years.

Math 10350: Calculus A
Exam I Sample
September 16, 2018

Name: _		
Class Tin	ne:	

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- No calculators.
- The exam lasts for one hour and 15 minutes.
- Be sure that your name is on every page in case pages become detached.
- Be sure that you have all 10 pages of the test.

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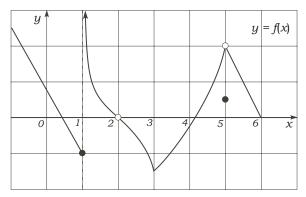
Good Luck!

PLE.	ASE MARK	YOUR ANSW	ERS WITH A	N X, not a cir	cle!
1.	a	b	$oxed{c}$	d	$oxed{e}$
2.	a	b	c	d	$oxed{e}$
3.	$\begin{bmatrix} \mathbf{a} \end{bmatrix}$	b	$oxed{c}$	d	$oxed{e}$
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8.	lacksquare	b	$oxed{c}$	d	$oxed{e}$
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Multiple Choice	
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Multiple Choice

1.(5 pts.)



The graph of y = f(x) is given above. Which one of the following statements is **FALSE**?

- (a) $\lim_{x\to 2} f(x)$ exists.
- (b) $\lim_{x \to 1^+} f(x)$ is infinite.
- (c) The slope of f(x) at x = 4 is positive.
- (d) f(x) has a removable discontinuity at x = 5.
- (e) $\lim_{h\to 0} \frac{f(3+h) f(3)}{h}$ exists.

2.(5 pts.) Find the average rate of change of the function

$$f(x) = \sqrt{x+2}$$

over the interval [-1, 2].

- (a) 1/6
- (b) 1/2
- (c) 1/4
- (d) 1/3
- (e) 1

3.(5 pts.) Treating x as a constant, find the following derivative:

$$\frac{d}{dy}\left(\frac{4x-y}{3x-2y}\right).$$

- $(a) \quad \frac{5x}{(3x-2y)^2}$
- $(b) \quad \frac{4-y}{3-2y}$
- (c) $\frac{1}{2}$
- $(d) \quad \frac{4x-1}{3x-2}$
- (e) $-\frac{5y}{(3x-2y)^2}$

4.(5 pts.) Find the value of the limit:

$$\lim_{h \to 0} \frac{(2+h)^5 - 2^5}{h}$$

- (a) 10
- (b) Does not exist.
- (c) 24
- (d) 80
- $(e) \quad 0$

5.(5 pts.) Find the equation of the tangent line to the graph of the function

$$f(x) = \sqrt[5]{x} + 4$$

at x = -1.

- (a) $y-3=\frac{x^{-4/5}}{5}(x+1)$
- (b) $y+1=\frac{1}{5}(x-3)$
- (c) $y-3=\frac{1}{5}(x+1)$
- (d) $y+3=\frac{1}{5}(x-1)$
- (e) $y+3=\frac{x^{-4/5}}{5}(x-1)$

6.(5 pts.) Consider the function

$$f(x) = \begin{cases} \frac{e^x - 1}{kx} & \text{if } x < 0\\ 2x + 5 & \text{if } x \ge 0 \end{cases}$$

Find the value of k, if it exists, so that f(x) is continuous at x = 0.

(a) $\frac{1}{2}$

(b) :

(c) $\frac{1}{5}$

- (d) Does not exist.
- (e) 5

7.(5 pts.) Find the derivative of the function

$$y = \frac{3x^5 - 4x^3 + x^2}{x^3}.$$

- (a) $\frac{15x^4 12x^2 + 2x}{3x^2}$
- (b) $6x^3 1$
- (c) $x^3 4x x^{-2}$
- (d) $\frac{15x^6 12x^4 + 2x^3}{3x^4}$
- (e) $6x x^{-2}$

8.(5 pts.) Find the value of the **left-hand** limit: $\lim_{x\to 2^-} \frac{|x-2|}{4-x^2}$

- (a) $\frac{1}{4}$
- (b) $+\infty$
- (c) $-\frac{1}{4}$
- (d) Does not exist.
- (e) -1

9.(5 pts.) Find the constant c so that the function

$$f(x) = \begin{cases} x^3 + 3x - 3c, & x > 1\\ (x - 1)^2 + c, & x \le 1. \end{cases}$$

is continuous.

- (a) -2
- (b) 0
- (c) 2
- (d) 1
- (e) -1

10.(5 pts.) Find the value of the limit:

$$\lim_{x \to 2} \frac{\sqrt{x+7} - 3}{x-2}$$

- (a) Does not exist.
- (b) 1/6
- (c) 1
- (d) 1/3
- (e) $+\infty$

Name:	
Class Time:	

Partial Credit

You must show your work on the partial credit problems to receive credit!

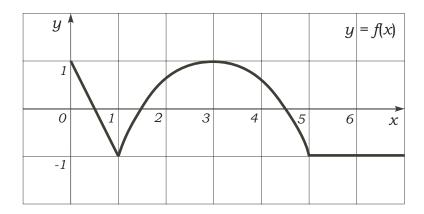
11.(12 pts.)

11a. Find ALL equations of the vertical asymptotes of the curve $y = \frac{x^2 - 9}{x^2 - x - 6}$ (Remark: Your answers should be in the form: x = c.)

11b. Find ALL values of x for which the graph of $g(x) = 16x + \frac{1}{x^2}$ has a horizontal tangent line.

Name:			
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12.(12 pts.) The graph of the function f(x) for $0 \le x \le 7$ is given below.



12a. Sketch the graph of the **derivative** f'(x) of the function f(x) in the axes given below for $0 \le x \le 7$.

y '						<i>y</i> =	f'(x)
2							
1							
0	1	2	3	4	5	6	X
-1							
-2							

12b. For what values of x in the interval 0 < x < 6 is f'(x) undefined?

Answer: _____

Name:	
Class Time:	

 ${\bf 13.} (12~{\rm pts.})$ The position function of a ball thrown upward , measured from ground level, is given by the function

$$s(t) = -5t^2 + 4t + 1.$$

13a. Find the time at which the ball hits the ground.

13b. Find the instantaneous velocity at time t.

13c. Find the instantaneous rate of change of the velocity at time t.

13d. Find the velocity at the moment when the ball hits the ground.

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Class Time:	

14.(12 pts.) Consider the function

$$f(x) = \frac{1}{x}.$$

14a. Write down the average rate of change of f(x) over the interval $2 \le x \le 2 + h$. You may assume that $h \ne 0$.

14b. Using Part (a) above and limits (only), find the **slope** of the curve $y = \frac{1}{x}$ at x = 2.

Math 10350: Calculus A Exam I Sample September 16, 2018

Name:		
Class Time:	ANSWERS	

- The Honor Code is in effect for this examination. All work is to be your own.
- No calculators.
- The exam lasts for one hour and 15 minutes.
- Be sure that your name is on every page in case pages become detached.
- Be sure that you have all 10 pages of the test.

Honor pledge. "As a member of the Notre Dame community, I will not participate in or tolerate academic dishonesty.":

Good Luck!

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4.	a	b	$oxed{c}$	•	e
5.	a	b	•	d	e
6.	a	b	•	d	e
7.	a	b	$lue{c}$	d	•
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9.	a	b	c	•	e
10.	a	•	$lue{c}$	$\boxed{\mathrm{d}}$	e

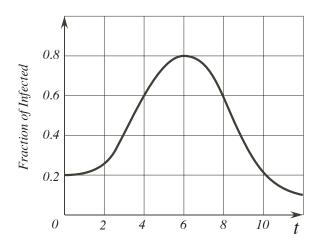
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Math 10350: Calculus A Exam I September 22, 2019			Name:		
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 The Honor Code is No calculators. The exam lasts for Be sure that your Be sure that you h Honor pledge. "As or tolerate academic seeds." 	one hour and 15 name is on every nave all 10 pages a member of the	of minutes. page in case pof the test.	ages become	detached.	pate in
		d Luck!			
PLEASE M	ARK YOUR AN	SWERS WITH		a circle!	
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5. a	b	$lue{c}$	$\boxed{\mathrm{d}}$	e	
6. a	b	$lue{c}$	$\boxed{\mathrm{d}}$	e	
7. a	b	$oxed{c}$	$oxed{d}$	e	
8. a	b	$lue{c}$	$\boxed{\mathrm{d}}$	e	
9. a	b	$lue{c}$	$oxed{d}$	e	
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Name: _______
Class Time: ______

Multiple Choice

1.(5 pts.)



The graph above shows the fraction of the population of a city infected by a flu virus over the a 12-month duration. What is average rate of change of the fraction of infected people over the duration of the first six months?

- (a) 0.6
- (b) 0.1
- (c) $\frac{5}{3}$
- (d) 10
- (e) -0.6

2.(5 pts.) Referring to the graph above, let v(t) be the **instantaneous** rate of change of the fraction of infected people in the city at time t where 0 < t < 12. Listing the values of v(2), v(6), and v(9) from the smallest to the biggest, we have:

- (a) v(6) < v(2) < v(9)
- (b) v(2) < v(6) < v(9)
- (c) v(6) < v(9) < v(2)
- (d) v(2) < v(9) < v(6)
- (e) v(9) < v(6) < v(2)

3.(5 pts.) Let f(x) be a differentiable function such that

$$f(0) = 2$$
 and $f'(0) = -3$

Find the slope of the tangent line to the graph of $P(x) = f(x) \cdot e^x$ at x = 0.

- (a) -1
- (b) -2
- (c) 2
- (d) 1
- (e) -3

4.(5 pts.) Let f(x) be a differentiable function such that

$$f(0) = 2$$
 and $f'(0) = -3$

Find the derivative of $Q(x) = \frac{x}{f(x) + 2}$ at x = 0.

- (a) $-\frac{1}{4}$
- (b) $\frac{1}{16}$
- $(c) \quad 0$
- (d) $\frac{1}{4}$
- (e) $-\frac{1}{3}$

5.(5 pts.) Find the equation of the tangent line to the graph of the function

$$f(x) = \frac{1}{x^4} + 3$$

at x = -1.

- (a) y + 1 = 4(x - 4)
- (b) y-4=4(x+1)
- (c) $y+4=-4x^{-5}(x-1)$
- (d) y+4=4(x-1)
- (e) $y-4=-4x^{-5}(x+1)$

6.(5 pts.) If $\log_3 x = -1$ and $\log_3 y = 4$, find the exact value of $\log_3\left(81x\sqrt{y}\right)$

- (a) -7
- (b) -8
- (c) -5
- (d) 7
- (e) 5

Name: ______ Class Time: ______

7.(5 pts.) Consider the function

$$f(x) = \begin{cases} \frac{x^2 + 3x - 4}{x^2 - 3x + 2} & \text{if } x < 1\\ kx + 1 & \text{if } x \ge 1 \end{cases}$$

Find the value of k, if it exists, so that f(x) is continuous at x = 1.

(a) -5

(b) 6

(c) 5

(d) -6

(e) Does not exist.

8.(5 pts.) On her 20th birthday, Joan decided to set up a certificate of deposit (CD) which will mature when she is 30 years old. If her initial deposit was \$2000 and interests is compounded **monthly** at a rate of 3% per year, how much will she receive when the certificate reaches maturity?

- (a) $2000 \left(1 + \frac{0.03}{12}\right)^{120}$
- (b) $2000 \left(1 + \frac{0.03}{12}\right)^{30}$
- (c) None of these.
- (d) $2000 (1 + 0.03)^{10}$
- (e) $2000 \left(1 + \frac{0.03}{12}\right)^{10}$

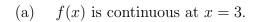
Name: _____

Class Time:

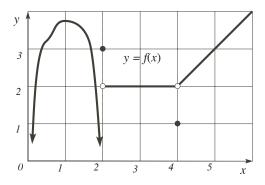
9.(5 pts.) Find $\frac{dy}{dx}$ if $y = \frac{2x^6 + 9x^2 - 1}{x^2}$.

- (a) $8x^4 + 2x^{-1}$
- (b) $\frac{12x^6 + 18x^3 x}{2x^3}$
- (c) $\frac{2}{5}x^5 + 9x + x^{-1}$
- (d) $8x^3 + 2x^{-3}$
- (e) $\frac{12x^5 + 18x}{2x}$

10.(5 pts.) Let f(x) be the function whose graph is shown below. Which of the following statements is **FALSE**?



- (b) f(x) is not continuous at x = 4.
- (c) $\lim_{x \to 4} f(x) = 1$.
- (d) $\lim_{x \to 2^{-}} f(x) = -\infty.$
- (e) f(2) = 3.



Name:	
Class Time:	

 $\mathbf{11.}(5 \text{ pts.})$ The temperature F (in Fahrenheit) of a hot liquid at time t hours is given by the equation

$$F(t) = 100(0.5)^t + 70.$$

Find the time at which the temperature is 95 $^o\mathrm{F}.$

- (a) 1 hour
- (b) ln(2) hours
- (c) 2 hours
- (d) 1/2 hour
- (e) $[\ln(0.25) \ln(0.5)]$ hours

12.(5 pts.) Let F(t) be the temperature of the hot liquid at time t hours as in the above problem. How fast is the temperature of the liquid changing at the moment when t = 1 hour?

- (a) $50 \, {}^{o}\mathrm{F/hr}$
- (b) $120 \, {}^{o}\text{F/hr}$
- (c) $120 \ln(0.5)$ °F/hr
- (d) $[50 \ln(0.5) + 70]$ °F/hr
- (e) $50 \ln(0.5) {}^{o}F/hr$

Name:	
Class Time:	_

Partial Credit

You must show your work on the partial credit problems to receive credit!

13.(12 pts.)

13a. Solve for x that satisfies the equation:

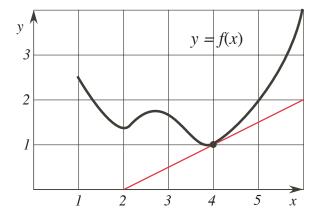
$$\log_{10}(3x+2) - \log_{10}(x-1) = 1$$

13b. (Not related to above.)

If
$$f'(a) = \lim_{h \to 0} \frac{3^{5+h} - 3^5}{h}$$
 then $f(x) \stackrel{?}{=}$ and $a \stackrel{?}{=}$.

Name: ______
Class Time: _____

14.(12 pts.)



14a. The figure above describes the graph of y = f(x) and its tangent line at x = 4. Answer the problems below:

i.
$$f(4) \stackrel{?}{=}$$
 _____ and $f'(4) \stackrel{?}{=}$ _____

ii. Find the equation of the tangent line at x=4. Give your answer in slope-intercept form.

14b. (Not related to above.)

Find the equations of the tangent lines to the graph of $f(x) = 4x^3$ such that they are parallel to the line y - 12x = 8.

Name:	
Class Time:	

$$f(x) = x^2 + 2x.$$

15.(12 pts.) Consider the function $f(x) = x^2 + 2x$. **15a.** Compute the average rate of change of f(x) over the interval $2 \le x \le 2 + h$. You may assume that $h \neq 0$ and simplify your answer.

15b. Using Part (a) above and limits (only), find the **slope** of the curve $y = x^2 + 2x$ at x = 2.

Math 10350: Calculus A Exam I September 22, 2019

Name:		
Class Time:	ANSWERS	

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PLE	ASE MARK	YOUR ANS	SWERS WITH	H AN X, not a	a circle!
1.	a	•	$oxed{c}$	$\boxed{\mathrm{d}}$	e
2.	a	b	$oxed{c}$	$\boxed{\mathrm{d}}$	•
3.	•	b	$oxed{c}$	$ \boxed{ \mathbf{d} } $	lacksquare
4.	a	b	$oxed{c}$	•	$oxed{e}$
5.	a	•	$oxed{c}$	d	$oxed{e}$
6.	a	b	$\boxed{\mathrm{c}}$	$\boxed{\mathrm{d}}$	•
7.	a	b	$oxed{c}$	•	$oxed{e}$
8.	•	b	$oxed{c}$	$\boxed{\mathrm{d}}$	$oxed{e}$
9.	a	b	$\boxed{\mathrm{c}}$	•	$oxed{e}$
10.	a	b	•	$\boxed{\mathrm{d}}$	e
11.	a	b	•	$\boxed{\mathrm{d}}$	lacksquare
12.	a	b	$oxed{c}$	$\boxed{\mathrm{d}}$	•

Please do NOT write in this box.			
Multiple Choice			
13.			
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