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FINITE MATH: QUIZ 8 SOLUTION

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Problem 1. Consider the following length (in inches) of a **randomly selected sample** of 5 adult male Betta fish from a large population: 2.23 1.88 2.47 2.19 1.80.

a) (1pt) Calculate the mean.

$$\bar{x} = \frac{2.23 + 1.88 + 2.47 + 2.19 + 1.80}{5} = \frac{10.57}{5} = \boxed{2.114}$$

b) (2pt) Calculate the variance.

This is a sample from a population, so we compute the sample variance, which means we only divide by 4.

$$s^2 = \frac{(2.23 - 2.114)^2 + (1.88 - 2.114)^2 + \ldots (1.80 - 2.114)^2}{4} = \frac{0.29932}{4} = \boxed{0.07483}$$

c) (1pt) Calculate the standard deviation.

$$s = \sqrt{0.07483} \approx \boxed{0.27355}$$

Problem 2. A game involves tossing an unfair coin, with $P(T) = 0.8$. The house charges players x dollars for the opportunity to play the game. If player tosses Tails, he gets nothing and the house keeps his bet. If the toss is Heads, he gets his money back, plus an additional \$5.

a) (2pt) How much should the house charge per game to break even?

We analyze this problem from the house's perspective. If the player wins, the house has a total loss of \$5, which occurs with 0.2 probability. If the player loses, the house has a total gain of x (the payment/bet the player makes). Hence the expected value of this game to the house is

$$(-5)(0.2) + (x)(0.8)$$

and since we want the house to break even, we set this equal to zero:

$$(-5)(0.2) + (x)(0.8) = 0$$

Solving for x gives what the house should charge: $\boxed{x = 1.25}$

b) (3pt) How much should the house charge per game to make an average of \$1 dollar profit per game?

From the house's perspective, we want

$$(-5)(0.2) + (x)(0.8) = 1$$

Solving for x gives what the house should charge: $\boxed{x = 2.5}$

Problem 3. (2pt) A random variable X is known to have $E(X) = 4$ and $E(X^2) = 25$. Find the variance and the standard deviation of X.

The variance is given by

$$Var(X) = E(X^2) - E(X)^2 = 25 - 4^2 = 25 - 16 = \boxed{9}$$

To get the standard deviation, we take the square root, so $\boxed{\sigma = 3}$.