CALCULUS 2
EXAM 2 PRACTICE PROBLEMS

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Contents

1. Approximate Integrals

Problem 1. State the general formulas for the Midpoint Rule, Trapezoidal Rule, and Simpson's Rule, together with their corresponding error bounds.

Problem 2. Use the Midpoint Rule, Trapezoidal Rule, and Simpson's rule to approximate each of the following integrals with the specified value of n. Estimate the error in each case. Then compute the actual integral, and compare it to each of your estimates. Does the difference fall within the error bound? (you will need a calculator for this part in some cases)

a) $\int_0^\pi \sin(x)dx$ with $n = 4$.

b) $\int_0^4 x^3(x)dx$ with $n = 4$.

c) $\int_0^{16} \sqrt{x}dx$ with $n = 8$.

d) $\int_1^{4e+1} \ln(x)dx$ with $n = 4$.

Problem 3. Consider the error estimate for Simpson's rule. What can you say about the error with ANY number of subintervals when applying Simpson's rule to a polynomial of degree 1, 2, or 3? (Hint: what is the 4th derivative going to be? What value can you pick for K in that case?).

2. Improper Integrals

Problem 4. Compute the following integrals, or state if they are divergent:

a) $\displaystyle\int_0^1 x^{-1/2}dx$

b) $\displaystyle\int_{-1}^4 x^{-2}dx$

c) $\displaystyle\int_{-1}^\infty x^{-1/3}dx$

d) $\displaystyle\int_1^\infty x^{-1/3}dx$

e) $\displaystyle\int_{-1}^\infty x^{-3}dx$

f) $\displaystyle\int_1^\infty x^{-3}dx$

g) $\displaystyle\int_{-1}^\infty e^{-x}dx$

h) $\displaystyle\int_{-\infty}^0 xe^{x}dx$

i) $\displaystyle\int_{-\infty}^\infty \frac{3x^2}{x^6+1}dx$ (Hint: use the subsitution $u = x^3$, don't forget to change your bounds)

j) $\int_2^5 1/\sqrt{x-2}dx$

k) $\int_0^{\pi/2} \sec(x)dx$

l) $\int_0^2 1/(x^2-1)dx$ (Hint: use partial fractions)

m) $\int_0^1 r\ln(r)dr$

n) $\int_0^1 \frac{dx}{\sqrt{1-x^2}}$

Problem 5. Use the regular comparison test to determine if the following integrals converge or diverge. Do not compute the integrals:

a) $\int_1^\infty \frac{1+e^{-t}}{t}dt$

b) $\int_1^\infty \frac{1}{\sqrt[4]{1+x}}dx$

c) $\int_1^\infty \frac{1}{\sqrt[4]{1+x^2}}dx$

d) $\int_1^\infty \frac{1}{\sqrt{e^x+1}}dx$

e) $\int_1^\infty \frac{1}{(2x+1)^3}dx$

f) $\int_1^\infty \frac{x^3+1}{x^5+1}dx$

g) $\int_1^\infty \frac{1}{x(\ln x)^2}dx$

h) $\int_1^\infty \frac{\ln(x)}{x^3}dx$

i) $\int_1^\infty \frac{\ln(x)}{x^2}dx$

j) $\int_1^\infty \frac{dx}{\sqrt{x}+x\sqrt{x}}$

k) $\int_1^\infty \frac{1+\sin^2(x)}{\sqrt{x}}dr$

l) $\int_0^\pi \frac{\sin^2(x)}{\sqrt{x}}dx$

m) $\displaystyle\int_0^\infty \frac{\arctan(x)}{2+e^x}dx$

Problem 6. Evaluate the integral

$$\int_0^\infty \frac{1}{\sqrt{x}(1+x)}dx$$

Hint: this is improper due to the infinite discontinuity at $x = 0$, and the infinite region of integration. You need to split this into two integrals, one over $[0,1]$ and the second over $[1,\infty]$.

Problem 7. For which values of p do the following integrals converge?

$$\int_e^\infty \frac{1}{x(\ln x)^p}dx \quad \text{and} \quad \int_0^1 x^p \ln x dx$$

Problem 8. Draw the area represented by the integral $\int_0^1 \ln(r)dr$. Can you rewrite the same area in terms of an integral of some exponential function?

Problem 9. Is the following integral convergent or divergent?

$$\int_{-\infty}^\infty e^{-x^2}dx$$

Note: the antiderivative of e^{-x^2} does not have a closed formula given in terms of elementary functions. You will learn to compute its value if you take Calculus 3, it requires techniques from multivariable calculus. By the way, it turns out the integral equals $\sqrt{\pi}$.

Problem 10. Show that $\int_0^1 x^{-p}dx = 1 + \int_1^\infty y^{-1/p}dy$, hence proving that the following statements are equivalent (I briefly mentioned this duality in tutorial):

$$\int_1^\infty \frac{1}{x^p} = \begin{cases} \text{convergent if } p > 1 \\ \text{divergent if } p \le 1 \end{cases} \iff \int_0^1 \frac{1}{x^p} = \begin{cases} \text{convergent if } p < 1 \\ \text{divergent if } p \ge 1 \end{cases}$$

Hint: Draw a picture of the curve $y = x^{-p}$ and shade which area is represented by the first integral. Can you pinpoint a square inside the shaded region with area equal to 1? Then write an integral for the remaining area, but this time integrate over the y-axis.

3. Arc Length

Remember the formula for arc length can be expressed as an integral over x or over y, depending on the situation we're dealing with:

$$L = \int_{x_1}^{x_2} \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx = \int_{y_1}^{y_2} \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy$$

Problem 11. Compute the arc length of the following:

a) $y = \dfrac{x^3}{3} + \dfrac{1}{4x}$, $x \in [1, 2]$.

b) $x = \dfrac{y^4}{8} + \dfrac{1}{4y^2}$, $y \in [1, 2]$.

c) $y = \ln(\cos x)$, $x \in [0, \pi/3]$.

d) $y = \sqrt{x - x^2} + \arcsin(\sqrt{x})$ on its entire domain.

e) $x^2 = (y - 4)^3$ from $(1, 5)$ to $(8, 8)$. (Hint: implicit differentation)

Problem 12. Find the **arc length function** for the curve $y = 2x^{3/2}$ starting from $(1, 2)$.

Problem 13. Find the arc length function for the curve $y = \arcsin(x) + \sqrt{1 - x^2}$ starting from $(0, 1)$.

4. Direction Fields and Euler's Method

Problem 14. Differentiate the following functions implicitly and give a rough sketch of the direction field:

a) $x^2 + y^2 = r^2$.

b) $y = kx^2 + C$, sketch for $k = 1$.

c) $x^2 - y^2 = C$.

Problem 15. Give a rough sketch of the direction field for the following differential equations:

a) $y' = x^2 + y^2 - 1$.

b) $y' = x + y$.

c) $y' = 2 - y$ (Notice how this does not depend on the x-coodinate!)

d) $y' = x(2 - y)$.

e) $y' = x + y - 1$.

f) $y' = \sin(x)\sin(y)$.

g) $y' = y - 2x$.

5. Differential Equations

Problem 16. Solve the following differential equation:

a) $y' = 3x^2y^2$.

b) $y' = x\sqrt{y}$.

c) $xy \cdot y' = x^2 + 1$.

d) $y' + xe^y = 0$.

e) $\dfrac{dp}{dt} = t^2p - p + t^2 - 1$.

f) $\dfrac{dy}{dx} = xe^y$, $y(0) = 0$.

g) $\dfrac{dy}{dx} = \dfrac{x \sin x}{y}$ passing through $(0, -1)$.

h) $\dfrac{dP}{dt} = \sqrt{Pt}$, passing through $(1, 2)$.

i) $\dfrac{dy}{dx} = x + y$. (Hint: make the change of variable $u = x + y$)

j) $x\dfrac{dy}{dx} = y + xe^{y/x}$. (Hint: make the change of variable $v = y/x$)

Problem 17. Find the orthogonal trajectories of the following families of curves:

a) $x^2 + 2y^2 = k^2$.

b) $y^2 = kx^3$. (Hint: you must eliminate k entirely, see Example 5 on page 643)

c) $y = \dfrac{k}{x}$

d) $y = \dfrac{1}{x + k}$

Problem 18. An **integral equation** is an equation that contains an unknown function $y(x)$ and an integral that involves $y(x)$. Solve the given integral equations by turning them to differential equations first. (Hint: what happens if you take derivative with respect to x of the equation? Note: you can use an initial condition obtained from the original integral equation to get the particular solution rather than just a general solution).

a) $y(x) = 2 + \displaystyle\int_2^x (t - ty(t))\, dt$

b) $y(x) = 2 + \displaystyle\int_1^x \frac{dt}{ty(t)}$

c) $y(x) = 4 + \displaystyle\int_0^x 2t\sqrt{y(t)}dt$

Recall that for a **linear differential equation** of the form

$$y' + P(x) \cdot y = Q(x),$$

the solution will be given by

$$y(x) = \frac{1}{I(x)} \left(\int I(x)Q(x)dx + C \right)$$

where $I(x)$ is the integrating factor given by $I(x) = e^{\int P(x)dx}$ (Pick a particular integrating factor, not the most general one that has $+C$ at the end. In other words, pick the one with $C = 0$).

Problem 19. Solve the following linear differential equations.

a) $\dfrac{dy}{dx} + 3x^2y = 6x^2$.

b) $x^2y' + xy = 1$, $x > 0$, passing through $(1, 2)$.

c) $xy' + y = x \ln x$ passing through $(1, 0)$.

d) $xy' = y + x^2 \sin(x)$, $y(\pi) = 0$.

e) $xy'' + 2y' = 12x^2$ (Hint: make the substitution $u = y'$, find u first, then integrate to get y).

6. Sequences

Problem 20. Write the first few terms of the sequence given by the formula. State whether the sequence converges or diverges.

a) $a_n = n - 1$, $n \geq 1$

b) $a_i = i^2$, $i \geq 0$

c) $a_k = \sqrt{k+1}$, $n \geq 1$

d) $a_n = \dfrac{n}{n+2}$, $n \geq 1$

e) $a_n = \cos(n\pi)$, $n \geq 0$

f) $a_n = \cos\left(\dfrac{(2n+1)\pi}{2}\right)$, $n \geq 0$

g) $a_n = \sin(2n\pi)$, $n \geq 0$

Problem 21. Write the general formula for the following sequences. State whether they converge or diverge.

a) $\{1, 2, 3, 4, 5, \dots\}$

b) $\left\{0, \dfrac{1}{2}, \dfrac{2}{3}, \dfrac{3}{4}, \dots\right\}$

c) $\{1, -1, 1, -1, 1, -1, \dots\}$

d) $\left\{\frac{1}{2}, \frac{1}{4}, \frac{1}{6}, \frac{1}{8}, \dots\right\}$

e) $\{1, 3, 9, 27, 81, \dots\}$

f) $\left\{\frac{3}{5}, -\frac{4}{25}, \frac{5}{125}, -\frac{6}{625}, \frac{7}{3125}, \dots\right\}$

g) $\left\{4, -1, \frac{1}{4}, -\frac{1}{16}, \frac{1}{64}, \dots\right\}$

h) $\left\{-3, 2, -\frac{4}{3}, \frac{8}{9}, -\frac{16}{27}, \dots\right\}$

i) $\{5, 8, 11, 14, 17, \dots\}$

j) $\{1, 0, -1, 0, 1, 0, -1, 0, \dots\}$ (challenge: do not use trig functions!)

Problem 22. Discuss the convergence or divergence of the following sequences (if convergent, find the limit):

a) $a_n = \dfrac{3 + 5n^2}{n + n^2}$

b) $a_n = \dfrac{n^4}{n^3 - 2n + e^n}$

c) $a_n = \dfrac{3\sqrt{n}}{\sqrt{n} + 2}$

d) $a_n = e^{-1/\sqrt{n}}$

e) $a_n = \sin(n)$

f) $a_n = \left(1 + \dfrac{2}{n}\right)^n$

g) $a_n = \sqrt{\dfrac{1 + 4n^2}{1 + n^2}}$

h) $a_n = \dfrac{\ln n}{\ln(2n)}$

i) $a_n = \dfrac{\arctan(n)}{n}$

j) $a_n = n \sin(1/n)$

k) $a_n = \ln(n + 1) - \ln(n)$

l) $a_n = \sqrt[n]{n}$

m) $a_n = \{0, 1, 0, 0, 1, 0, 0, 0, 1, 0, 0, 0, 0, 1, \dots$

n) $a_1 = 1$, $a_{n+1} = 1 + \dfrac{1}{1 + a_n}$, $n \geq 2$. Note, this gives the continued fraction expansion

$$\sqrt{2} = 1 + \cfrac{1}{2 + \cfrac{1}{2 + \cfrac{1}{2 + \dots}}}$$

Indian mathematician Ramanujan did many contributions to the topic of continued fractions. The movie *The Man Who Knew Infinity (2015)* is about his life - very interesting to watch.

Problem 23. Find a closed formula for the sequence defined by the recursion

a) $a_1 = 1$, $a_{n+1} = a_n + 1$, $n \geq 2$.

b) $a_1 = 2$, $a_{n+1} = a_n + 2$, $n \geq 2$.

c) $a_1 = 1$, $a_{n+1} = 2a_n$, $n \geq 2$.

d) $a_1 = 1$, $a_{n+1} = na_n$, $n \geq 2$.

Problem 24. (Challenge problem) Let a and b be positive numbers with $a > b$. Let a_1 and b_1 be their arithmetic and geometric means, respectively:

$$a_1 = \frac{a+b}{2} \quad \text{and} \quad b_1 = \sqrt{ab}$$

Repeat this process so that, in general,

$$a_{n+1} = \frac{a_n + b_n}{2} \quad \text{and} \quad b_{n+1} = \sqrt{a_n b_n}$$

a) Show that $a_n > a_{n+1} > b_{n+1} > b_n$.
b) Show that both sequences are convergent.
c) Show that $\lim a_n = \lim b_n$.

7. Review of Sigma Notation

Sigma ($\sum$) notation is used as a way to write sums in more compact forms. Suppose we have a sequence of n numbers: $a_1, a_2, a_3, \dots, a_n$. The sum of these n terms is written as

$$\sum_{i=1}^{n} a_i = a_1 + a_2 + a_3 + \cdots + a_n$$

where i is the **index of summation**, a_i is the ith term of the sum, and the lower (starting) and upper (ending) bounds of the sum are 1 and n, respectively.

Note that the upper/lower bounds MUST be constants with respect to the index of summation i (it cannot depend on i). However, the lower bound doesn't need to start at 1. Any integer lower than or equal to the upper bound works. Here are a few examples (the first two are different ways of expressing the same sum in sigma notation):

$$\sum_{i=1}^{6} i = 1 + 2 + 3 + 4 + 5 + 6$$

$$\sum_{i=0}^{5}(i+1) = 1+2+3+4+5+6$$

$$\sum_{j=3}^{7} j^2 = 3^2+4^2+5^2+6^2+7^2$$

$$\sum_{j=1}^{5}\frac{1}{\sqrt{j}} = \frac{1}{\sqrt{1}}+\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{3}}+\frac{1}{\sqrt{4}}+\frac{1}{\sqrt{5}}$$

$$\sum_{k=1}^{n}\frac{1}{n}(k^2+1) = \frac{1}{n}(1^2+1)+\frac{1}{n}(2^2+1)+\frac{1}{n}(3^2+1)+\cdots+\frac{1}{n}(n^2+1)$$

$$\sum_{i=1}^{n} f(x_i)\Delta x = f(x_1)\Delta x + f(x_2)\Delta x + \cdots + f(x_n)\Delta x$$

Problem 25. Evaluate the following sums:

a) Consider the sequence $\{2, 4, 8, 10, \ldots\} = \{a_1, a_2, a_3, a_4, \ldots\}$. What is $\sum_{i=1}^{5} a_i$?

b) $\sum_{j=1}^{3}(2j-1)$

c) $\sum_{k=3}^{6}\frac{1}{2}k$

d) $\sum_{n=1}^{5}(2n+3)$ and $\sum_{n=1}^{5} 2n+3$ (What's the difference between them? Compute both.)

e) $\sum_{n=1}^{4} nx$

f) $\sum_{i=1}^{n}\frac{i+1}{n^2}$ for $n = 10, 100, 1000, 10000$. (Find the closed form, then use your calculator)

g) $\sum_{k=1}^{4}(k^2+1)$

h) $\sum_{i=0}^{4}(x^{i+1}-x^i)$. Simplify your result.

i) $\sum_{k=1}^{4}(k^2+1)$

j) $\sum_{i=0}^{4}(x^{i+1}-x^i)$. Simplify your result.

k) $\sum_{i=0}^{n}(2^{i+1}-2^i)$. Simplify your result.

l) Evaluate $\sum_{j=1}^{n}\frac{2j+1}{n^2}$. What is the limit as $n \to \infty$ of your result?

Problem 26. Use $\sum$ notation to write the following sums:

a) $3+6+9+12+\ldots$ for 28 terms.

b) $-3+6-12+24-48+\ldots$ for 35 terms.

c) $8.3+8.1+7.9+7.7+\ldots$ for n terms.

d) $7+14+21+28+35+\cdots+105$

e) $\dfrac{1}{5(1)}+\dfrac{1}{5(2)}+\dfrac{1}{5(3)}+\cdots+\dfrac{1}{5(11)}$

f) $\dfrac{9}{1+1}+\dfrac{9}{1+2}+\dfrac{9}{1+3}+\cdots+\dfrac{9}{1+14}$

g) $\left[7\left(\dfrac{1}{6}\right)+5\right]+\left[7\left(\dfrac{2}{6}\right)+5\right]+\cdots+\left[7\left(\dfrac{6}{6}\right)+5\right]$

h) $\left[1-\left(\dfrac{1}{4}\right)^2\right]+\left[1-\left(\dfrac{2}{4}\right)^2\right]+\cdots+\left[1-\left(\dfrac{4}{4}\right)^2\right]$

i) $\left[\left(\dfrac{2}{n}\right)^3-\dfrac{2}{n}\right]\left(\dfrac{2}{n}\right)+\cdots+\left[\left(\dfrac{2n}{n}\right)^3-\dfrac{2n}{n}\right]\left(\dfrac{2}{n}\right)$

j) $\left[2\left(1+\dfrac{3}{n}\right)^2\right]\left(\dfrac{3}{n}\right)+\cdots+\left[2\left(1+\dfrac{3n}{n}\right)^2\right]\left(\dfrac{3}{n}\right)$

8. Series

Problem 27. Find a closed formula for the sum of n terms. Use this formula to find the limit as $n \to \infty$. Note: each of these infinite sums represents a definite integral.

a) $\lim_{n\to\infty}\sum_{i=1}^{n}\frac{24i}{n^2}$

b) $\lim_{n\to\infty}\sum_{i=1}^{n}\left(\frac{3i}{n}\right)^2\left(\frac{3}{n}\right)$

Problem 28. Evaluate the following infinite series by converting them to a definite integral:

a) $\lim_{n\to\infty}\sum_{i=1}^{n}\frac{1}{n^3}(i-1)^2$

b) $\lim_{n\to\infty}\sum_{i=1}^{n}\left(1+\frac{2i}{n}\right)^2\left(\frac{2}{n}\right)$

c) $\lim_{n\to\infty}\sum_{i=1}^{n}\left(1+\frac{i}{n}\right)\left(\frac{2}{n}\right)$

d) $\lim_{n\to\infty}\sum_{i=1}^{n}\left(1+\frac{3i}{n}\right)^3\left(\frac{3}{n}\right)$

Problem 29. Find the sum of the following series, or argue that it is divergent:

a) $\sum_{n=0}^{\infty}x^n$ and $\sum_{n=1}^{\infty}x^n$ for $|x|<1$.

b) $\sum_{n=1}^{\infty}\dfrac{1}{n(n+1)}$ (Hint: telescoping sum)

c) $\sum_{n=1}^{\infty}\dfrac{n^2}{5n^3+1}$

d) $\dfrac{1}{2}+\dfrac{1}{4}+\dfrac{1}{8}+\dfrac{1}{16}+\dots$

e) $3-4+\dfrac{16}{3}-\dfrac{64}{9}+\dots$

f) $2+0.5+0.125+0.03125+\dots$

g) $\sum_{n=1}^{\infty}6\cdot(0.9)^{n-1}$

h) $\sum_{n=1}^{\infty} \dfrac{(-3)^{n-1}}{4^n}$

i) $\sum_{n=1}^{\infty} \dfrac{10^n}{(-9)^{n-1}}$

j) $\sum_{n=1}^{\infty} \dfrac{3^{n-1}}{\pi^n}$

k) $\sum_{k=1}^{\infty} \dfrac{3^k}{e^{k-1}+1}$

l) $\sum_{k=1}^{\infty} \dfrac{k(k+2)}{(k+3)^2}$

m) $\sum_{r=1}^{\infty} \dfrac{1+2^r}{3^r}$

n) $\sum_{n=1}^{\infty} \sqrt[n]{2}$

o) $\sum_{k=1}^{\infty} \ln\left(\dfrac{k}{k+1}\right)$

p) $\sum_{n=1}^{\infty} \left(\sin\dfrac{\pi}{4}\right)^n$

q) $\sum_{n=1}^{\infty} \dfrac{e^n}{n^4}$

r) $\sum_{n=1}^{\infty} \left(\dfrac{1}{e^n} + \dfrac{1}{n(n+1)}\right)$

s) $\sum_{n=2}^{\infty} \dfrac{2}{n^2-1}$

t) $\sum_{n=1}^{\infty} \dfrac{3}{n(n+3)}$

u) $\sum_{n=1}^{\infty} \left(\cos\dfrac{1}{n^2} - \cos\dfrac{1}{(n+1)^2}\right)$

v) $\sum_{n=1}^{\infty} \left(e^{1/n} - e^{1/(n+1)}\right)$

w) $\sum_{n=2}^{\infty} \dfrac{1}{n^3-n}$

Problem 30. For which values of x do the following series converge? Find the sum of the series for those values of x.

$$\sum_{n=0}^{\infty} e^{nx}, \qquad \sum_{n=0}^{\infty} \frac{2^{n+1}}{x^n}, \qquad \sum_{n=0}^{\infty} \frac{\sin^n(x)}{3^n}, \qquad \sum_{n=0}^{\infty} (x+2)^n, \qquad \sum_{n=0}^{\infty} \frac{(x-2)^n}{3^n}$$

9. Selected Answers and Hints

2a. $M = 2.052$, $T = 1.896$, $S = 2.00027$, $A = 2$ **2b.** $M = 62$, $T = 68$, $S = 64$, $A = 64$ **2c.** $M = 5.35$, $T = 5.265$, $S = 5.323$, $A = 16/3 = 5.3333$ **2d.** $M = 110.29$, $T = 103.83$, $S = 108.14$, $A = 108.555$

3. The fourth derivative will be zero, so Simpson's gives the EXACT value of the integral, not just an approximation.

4a. 2 **4b.** div **4c.** div **4d.** div **4e.** div **4f.** $1/2$ **4g.** e **4h.** -1 **4i.** π **4j.** $2\sqrt{3}$ **4k.** div **4l.** div **4m.** $-1/4$ **4n.** $\pi/2$

5a. div **5b.** div **5c.** div **5d.** conv **5e.** conv **5f.** conv **5g.** conv **5h.** conv **5i.** conv **5j.** conv **5k.** div **5l.** conv **5m.** conv

7. The first integral converges when $p > 1$. Make a substitution $u = \ln x$ to see why). The second integral converges for $p > -1$. The same substitution followed by integration by parts shows this.

8. The integral can be rewritten as $\int_0^\infty e^{-x} dx$.

9. Convergent. Write it as $\int \frac{1}{e^{x^2}} dx$, split it into three integrals over $(\infty, -1] \cup [-1, 1] \cup [1, +\infty)$. The first and the last are equal by symmetry and converge by comparison with $\int e^{-x} dx$. The second converges by comparison with a p-integral like $\int x^{-1/2} dx$.

16.a $y = \frac{-1}{x^3+C}$ **16.b** $y = (\frac{x^2}{4} + C)^2$ **16.c** $ydy = (x + 1/x)dx$ then integrate **16.e** write as $dp/dt = (p + 1)(t^2 - 1)$ then separate and integrate **16.f** $e^{-y}dy = xdx$ then integrate **16.g** $ydy = x \sin x dx$ needs integration by parts **16.h** $dP/dt = \sqrt{P}\sqrt{T}$ then separate and integrate

17. For each of these problems, you must first find dy/dx. Then take the opposite reciprogral of that, and use the usual method for solving the differential equation you obtain.

18. In each of these problems, set x to the lower bound of the integral. This gives you the initial condition, as the integral part will reduce to zero. Then take derivative of the equation with respect to x (use fundamental theorem to differentiate the integral), giving you a differential equation of the form $y' = f(x)$. This is separable, so use the usual method to solve it.

19.bcd divide by x^2 first to get y' alone

20a. $0, 1, 2, 3, \cdots \to \infty$ **20b.** $0, 1, 4, 9, 16, \cdots \to \infty$ **20c.** $\sqrt{2}, \sqrt{3}, \sqrt{4}, \cdots \to \infty$ **20d.** $\frac{1}{3}, \frac{2}{4}, \frac{3}{5}, \cdots \to 1$ **20e.** $1, -1, 1, -1, \ldots$ div **20f.** $0, 0, 0, 0, \cdots \to 0$ **20g.** $0, 0, 0, 0, \cdots \to 0$

21a. $a_n = n$ for $n \geq 1$ is div **21b.** $a_n = \frac{n}{n+1} \to 1$ for $n \geq 0$ **21c.** $a_n = (-1)^n$ for $n \geq 0$ is div **21d.** $a_n = \frac{1}{2n} to 0$ for $n \geq 1$ **21e.** $a_n = e^n$ for $n \geq 0$ is div **21f.** $a_n = (-1)^{n+1} \frac{n+2}{5^n} \to 0$ for $n \geq 1$ **21g.** $a_n = (-1)^{n+1}/4^n \to 0$ for $n \geq -1$ **21h.** $a_n = (-1)^{n+1} \frac{2^n}{3^{n-1}} \to 0$ for $n \geq 0$ **21i.** $a_n = 2 + 3n$ for $n \geq 1$ is div **21j.** $a_n = \frac{1}{2}\left((1)^n + (-1)^n\right)$ for $n \geq 0$ is div

22a. $a_n \to 5$ **22b.** $a_n \to \infty$ **22c.** $a_n \to 3$ **22d.** $a_n \to 1$ **22e.** div **22f.** $a_n \to e^2$ (think about compound interest) **22g.** $a_n \to 2$ **22h.** $a_n \to 1$ **22i.** $a_n \to 0$ **22j.** $a_n \to 1$ **22k.** $a_n \to 0$ **22l.** $a_n \to 1$ (take the log) **22m.** a_n div **22n.** $a_n \to \sqrt{2}$ by solving $L = 1 + \frac{1}{1+L}$

23a. $a_n = n$ for $n \geq 1$ **23b.** $a_n = 2n$ for $n \geq 1$ **23c.** $a_n = 2^{n-1}$ for $n \geq 1$ **23d.** $a_n = n!$ for $n \geq 1$

For the remaining problems, if you don't know how to solve them, it's a very good sign that you MUST come to the review session.