

Name: _____

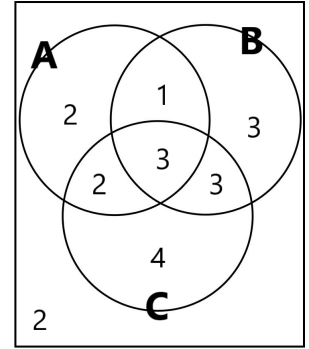
July 14, 2017

FINITE MATH: EXAM 2

ADRIAN PĂCURAR

- The Honor Code is in effect for this exam. All work must be your own.
- Please turn off all cellphones or any other electronic devices.
- Calculators are NOT allowed.
- You are NOT required to compute the answers, though simplifying may help for certain problems.
- There are **80 points available** for you to try. You may choose to attempt any of the problems, or all the problems. There is no penalty for getting a wrong answer.
- The exam will be **graded out of 70**. You can NOT get more than 70 points on this exam.
- You are allowed a single-sided 8 by 11 formula sheet for the exam; the formula sheet must be handwritten. You must **turn in your formula sheet with your exam**.
- The exam lasts **1 hour and 20 minutes**.

Problem 1. Consider the Venn diagram on the right of a sample space S with **equally likely outcomes**. For each region, the diagram lists the number of outcomes inside that region.



a) (2pt) Find $P(A)$. Simplify the fraction to lowest terms.

b) (2pt) Find $P(B)$. Simplify the fraction to lowest terms.

d) (2pt) Find $P(A \cap B)$. Simplify the fraction to lowest terms.

e) (2pt) Are the events A and B independent?

f) (1pt) Compute $P(A \cup B \cup C)'$.

Problem 2. A coin is flipped 10 times in a row, and the resulting H/T sequence is recorded.

a) (1pt) How many possible outcomes are there in the sample space?

b) (2pt) What is the probability of getting exactly 5 Heads?

c) (2pt) What is the probability of getting at least one Tail?

Problem 3. A classroom is split into two separate groups, G_1 and G_2 . There are 3 men and 5 women in G_1 , and there are 3 men and 13 women in G_2 . The teacher picks one of the two groups at random, with $P(G_1) = 0.3$ and $P(G_2) = 0.7$, then randomly selects a student from that group.

a) (3pt) What is the probability that the chosen student is a woman?

b) (1pt) Given that the selected group is G_1 , what is the probability that the student is a woman?

c) (4pt) The selected student is a woman. What is the probability that she is from the first group?

Problem 4. (3pt) You have 5 people, $\{A, B, C, D, E\}$, seated in a **single row** at random. What is the probability that person E ends up sitting next to person A? For example, some different such arrangements are BAEDC, BEADC, CDAEB, CDEAB, etc...

Problem 5. (2pt) The sample space $S = \{1, 2, 3, 4\}$ contains equally likely outcomes. Consider the events $A = \{1, 2\}$ and $B = \{1, 3\}$. Are A and B independent?

Problem 6. The probabilities for an unbalanced six-sided die are given below:

Outcome	1	2	3	4	5	6
Probability	0.1	0.1	0.3	0.2	0.1	0.2

You **roll two** of these unbalanced dice, and observe the sum of the numbers that come up. For example, (2,1) and (1,2) are different outcomes for which the sum is 3.

- a) (2pt) What is the probability that the sum of the two numbers is 1?

- b) (2pt) What is the probability that the sum of the two numbers is 2?

- c) (3pt) What is the probability that the sum of the two numbers is 4?

- d) (3pt) What is the probability that the sum of the two numbers is 7?

Problem 7. A child forms **3-letter words** by picking letters from $\{A, B, C, D, E\}$ at random. **Letters may be repeated.**

- a) (1pt) What is the probability that the word starts with A? Simplify your answer.

- b) (1pt) What is the probability that “DA” appears somewhere in the word? (the other letter can be anything)

- c) (1pt) What is the probability that the word begins with a consonant, has a vowel for the middle letter, and ends with consonant?

Problem 8. A mathematics professor assigns two problems for homework and knows that the probability of a student solving the first problem is 0.50, the probability of solving the second is 0.60, and the probability of solving both is 0.30.

- a) (1pt) Are the events independent?

- b) (2pt) A randomly chosen student has solved the second problem. What is the probability he also solves the first problem?

- c) (2pt) A randomly chosen student has solved the first problem. What is the probability she also solves the second problem?

Problem 9. (4pt) A child has 1 Red and 4 White marbles in his left pocket, and 2 Red and 2 White marbles in his right pocket. He transfers a marble (at random) from his left pocket to his right pocket. After the transfer, he picks (at random) a marble from his right pocket. What is the probability of picking a White marble from his right pocket?

Problem 10. (2pt) A crate contains 20 total apples, 4 of which are spoiled. You select 4 apples at random. What is the probability that at least one apple is bad?

Problem 11. (2pt) A child has eight cards numbered $\{2, 3, 4, 5, \dots, 9\}$. He creates **4-digit** numbers by randomly picking 4 of the 8 cards, and arranging them in some order. What is the probability that the number he obtains is smaller than 5000?

Problem 12. (4pt) An **unfair coin** has probability of Heads equal to $P(H) = 0.6$. If you toss the coin 10 times, what is the probability of getting exactly 5 Tails?

Problem 13. (2pt) A sample space contains 100 equally likely outcomes. Given that

$$n(E \setminus F) = 40 \quad n(F \setminus E) = 10 \quad n(E \cap F) = 10$$

are the events E and F independent?

Problem 14. Three inspectors look at a critical component of a rocket. Their probabilities for finding a defect are 0.95, 0.90, and 0.80. Each inspector operates independently of the rest.

a) (2pt) What is the probability that **all three** inspectors find a defect?

b) (3pt) What is the probability that **none** of the inspectors find a defect?

c) (3pt) What is the probability that **only one** of the inspectors finds a defect (and the other two don't)?

Problem 15. (4pt) A bag contains 3 Red and 2 White marbles. A second bag contains an unknown number of Red marbles, and 1 White marbles



A marble is drawn at random from each bag, and the probability of getting two marbles of the same color is $8/15$. How many red marbles are in the second bag?

Problem 16. (3pt) Suppose $P(A) = 0.6$ and $P(A \cap B) = 0.3$. If the events A and B are known to be independent, what is $P(B)$?

Problem 17. (3pt) A single card is drawn at random from six different decks (so you end up with six cards total). What is the probability that all six cards are different?

Problem 18. (2pt) A small grocery store has 7 cartons of milk left, 2 of which are sour. If you are going to buy the **second** carton of milk sold that day at random, what is the probability of selecting a sour carton of milk?