

Name:

Date:

Worksheet 7, Math 10560

1. Find the sum of the following series:

$$\sum_{n=0}^{\infty} \frac{3^n + (-2)^n}{5^n}.$$

2. Find the sum of the following series:

$$\sum_{n=4}^{\infty} \left[\frac{2n}{n+4} - \frac{2(n+1)}{n+5} \right].$$

Name:

Date:

3. State the divergence test:

What can you say about the following series using the divergence test?

$$(I) \sum_{n=1}^{\infty} \frac{5^n}{n}$$

$$(II) \sum_{n=1}^{\infty} \frac{5 \cdot (-1)^n}{n}$$

$$(III) \sum_{n=1}^{\infty} \frac{(-1)^n(5n+1)}{n}$$

Name:

Date:

4. Consider the following sequences.

$$(I) \left\{ (-1)^n \frac{n^3 + 2n - 1}{4n^3 + 1} \right\}_{n=1}^{\infty} \quad (II) \left\{ (-1)^n \frac{n^2 - 1}{5^n} \right\}_{n=1}^{\infty} \quad (III) \left\{ (-1)^n e^{(1/n)} \right\}_{n=1}^{\infty}$$

Determine which ones converge and find $\lim_{n \rightarrow \infty} a_n$ for those sequences $\{a_n\}$ that converge.

5. A tank initially contains 100 liters of salt water with 1.5 kilogram of dissolved salt. A well mixed salt water solution containing 3 kilograms of salt per 100 liters is pumped into the tank at a rate of 10 liters per minute. The salt water in the tank is kept thoroughly mixed and is drained at a rate of 5 liters per minute.

(a) Let $y = y(t)$ be the amount of salt in the tank at time t . Give a differential equation relating $\frac{dy}{dt}$ to y .

Name:

Date:

(b) Give a formula for the amount of salt in the tank at time t .

(c) At a given time t , the concentration is given by $C(t) = y(t)/V(t)$, where $V(t)$ is the volume in the tank at time t . Find the limit (algebraically) of $C(t)$ as t goes to infinity. Interpret your answer.

(d) Suppose the tank has total capacity of 200L. What is the concentration in the tank at the moment when the tank begins to overflow?

Name:

Date:

6. **Extra Problem:** Write the following numbers as single fractions. Hint: Write each decimal as a geometric series or a sum of geometric series.

a. $0.\overline{1} = 0.1111111111 \dots$

b. $0.\overline{12} = 0.1212121212 \dots$

c. $0.\overline{123} = 0.123123123123 \dots$