10350 Tutorial Week 02 - Set 01 Name ______________________

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1. Solve for x in following equations. Your answers should be simplified and have no log expressions.

1a. $16^{x+2} = 2^x$
Solution:

$$\begin{aligned} 16^{x+2} &= 2^x \\ (2^4)^{x+2} &= 2^x && \text{Write both sides as powers of 2.} \\ 2^{4(x+2)} &= 2^x \\ 4(x+2) &= x && \text{Exponents must be equal.} \\ 4x+8 &= x \\ 3x &= -8 \\ x &= -\frac{8}{3} \end{aligned}$$

1b. $27 \cdot 9^x = 3^{1-x}$
Solution:

$$\begin{aligned} 27 \cdot 9^x &= 3^{1-x} \\ 3^3 \cdot (3^2)^x &= 3^{1-x} \\ 3^{3+2x} &= 3^{1-x} \\ 3+2x &= 1-x \\ 3+3x &= 1 \\ 3x &= -2 \\ x &= -\frac{2}{3} \end{aligned}$$

1c. $\log_3(x^2) - \log_3(x^2-2) = 1$
Solution: Note that we must first apply the log rule $\log_a\left(\frac{M}{N}\right) = \log_a(M) - \log_a(N)$.

$$\begin{aligned} \log_3\left(\frac{x^2}{x^2-2}\right) &= 1 \\ \frac{x^2}{x^2-2} &= 3^1 \\ x^2 &= 3x^2 - 6 \\ 0 &= 2x^2 - 6 \\ 0 &= x^2 - 3 \\ x^2 &= 3 \\ x &= \pm\sqrt{3} \end{aligned}$$

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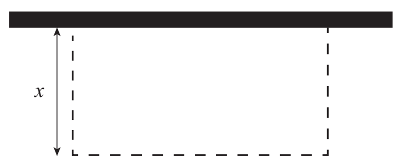
We need to check our answers by plugging them back into the original equation (remember that the log function is only defined over positive numbers).

$$\log_3\left[(-\sqrt{3})^2\right] - \log_3\left[(-\sqrt{3})^2 - 2\right] = \log_3 3 - \log_3 1 = 1 - 0 = 1$$
$$\log_3\left[(\sqrt{3})^2\right] - \log_3\left[(\sqrt{3})^2 - 2\right] = \log_3 3 - \log_3 1 = 1 - 0 = 1$$

Thus, both $x = \sqrt{3}$ and $x = -\sqrt{3}$ are valid solutions to our equation.

2. A gardener has 100 ft of fencing wishes to construct a rectangular enclosure with one side along a 200 ft building and the other three sides using fencing material as show below. Assume that all the fencing material are used in the construction.



2a. Let x be the length of the side of the enclosure as shown. Find the area of the enclosure in terms of x.

Solution: Let y be the length of the third side of the enclosure. Then we know:

$$A = xy$$
$$P = 2x + y = 100$$

From the second equation we obtain $y = 100 - 2x$. Substituting this back into the equation for area, we obtain $A = x(100 - 2x) = -2x^2 + 100x$.

2b. Find the dimensions (Length and Width) of the enclosure where the area enclosed is maximum. What is the maximum area.

Solution: Our equation for the area is a parabola which opens down. Thus the maximum occurs at the vertex of the parabola. We find this vertex by completing the square. We know that if the equation of a parabola is in the form $y = a(x - h)^2 + k$, then this parabola has vertex (x, k).

$$\begin{aligned} A &= -2x^2 + 100x \\ &= -2(x^2 - 50x) \\ &= -2(x^2 - 50x + (-25)^2) + -2(-25)^2 \\ &= -2(x - 25)^2 + 1250 \end{aligned}$$

Thus, the vertex of the parabola is $(25, 1250)$. This means that the value of x where the enclosed area is maximum is $x = 25\ ft$. Plugging in $x = 25$ into the equation for y, we obtain $y = 100 - 2x = 100 - 2(25) = 50\ ft$. Thus the dimensions where the area enclosed is maximum are $25\ ft \times 50\ ft$ and this maximum area is $1250\ ft^2$.

$$\boxed{\text{Dimensions: } = 25\ ft \times 50\ ft}$$

$$\boxed{\text{Maximum Area: } = 1250\ ft^2}$$

The Richter scale

$$\boxed{\text{Richter value } = \log_{10}\left(\frac{x}{A}\right),}$$

where A is the amplitude of the seismic wave of a reference earthquake and x is the amplitude of the seismic wave of the earthquake in question.

3. One of the worst earthquakes in history occured in Tokyo and registered 8.3 on the Richter scale. A more recent earthquake in California in 1989 registered 7.2. How much more severe was the earthquake in Tokyo in terms of the amplitude of its seismic wave?

Solution: We can solve for the amplitude of the seismic wave of each earthquake. Note that our answers are in terms of the constant A, the magnitude of the reference earthquake.

$$\begin{aligned}
\textbf{Tokyo Earthquake:}\quad 8.3 &= \log_{10}\left(\frac{x}{A}\right) \\
10^{8.3} &= \frac{x}{A} \\
x &= A \cdot 10^{8.3} \\
\textbf{Earthquake:}\quad 7.2 &= \log_{10}\left(\frac{x}{A}\right) \\
10^{7.2} &= \frac{x}{A} \\
x &= A \cdot 10^{7.2}
\end{aligned}$$

Now, we compare the magnitudes of the two earthquakes:

$$\frac{\text{Magnitude of Tokyo Earthquake}}{\text{Magnitude of California Earthquake}} = \frac{\cancel{A} \cdot 10^{8.3}}{\cancel{A} \cdot 10^{7.2}} = 10^{1.1} \approx 12.5893$$

Thus, the Tokyo earthquake was about 12.5893 times stronger than California Earthquake.