Name: Date: 10/05/2017

M20550 Calculus III Tutorial
Worksheet 5

1. Let $f(x, y, z) = x^2 - yz$. If $\mathbf{v} = \langle 1, 1, 0\rangle$, find the directional derivative of f in the direction of $\mathbf{v}$ at the point $(1, 2, 3)$. At what rate is f changing at the given point as we move in the direction of $\mathbf{v}$? Is f increasing or decreasing in this instance?

2. Find the tangent plane and the normal line to the surface $x^2y + xz^2 = 2y^2z$ at the point $P = (1, 1, 1)$.

3. Write an equation of the tangent line to the curve of intersection between the two surfaces defined by $z = x^2 + y^2$ and $x^2 + 2y^2 + z^2 = 7$ at the point $(-1, 1, 2)$.

   Hint: Think about the geometry of the gradient vectors. You don't have to parametrize the curve to do this problem.

4. Find the local maximum and the local minimum value(s) and saddle point(s) of the function $z = x^3 + y^3 - 3xy + 1$.

5. Identify the absolute maximum and absolute minimum values attained by $g(x, y) = x^2y - 2x^2$ within the triangle T bounded by the points $P(0, 0)$, $Q(2, 0)$, and $R(0, 4)$.

6. Identify the absolute maximum and absolute minimum values attained by $z = 4x^2 - y^2 + 1$ on the region $R = \{(x, y) \,|\, 4x^2 + y^2 \leq 16\}$.

7. Find the absolute maximum of $f(x, y, z) = xyz$ subject to the constraint $x^2 + 2y^2 + 3z^2 = 9$, assuming that x, y, and z are nonnegative.

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Optional/Review Problems:

8. (Chain Rule) Find $\dfrac{dz}{dt}$ when $t = 2$, where $z = x^2 + y^2 - 2xy$, $x = \ln(t-1)$ and $y = e^{-t}$.

9. (Chain Rule) Let $r = r(x, y)$, $x = x(s, t)$, and $y = y(t)$. Find $\dfrac{\partial r}{\partial t}$ at $(s, t) = (1, 0)$, given

$$\begin{array}{lll} x(1,0) = 2, & x_s(1,0) = -1, & x_t(1,0) = 7, \\ y(0) = 3, & y(1) = 0 & y'(0) = 4, \\ r(2,3) = -1, & r_x(2,3) = 3, & r_y(2,3) = 5, \\ r_x(1,0) = 6, & r_y(1,0) = -2, & \end{array}$$

10. (Chain Rule) If $h = x^2 + y^2 + z^2$ and $y \cos z + z \cos x = 0$, find $\dfrac{\partial h}{\partial x}$ assuming that x and y are the independent variables.

11. (Chain Rule) A cylinder containing an incompressible fluid is being squeezed from both ends. If the length of the cylinder is *decreasing* at a rate of 3m/s, calculate the rate at which the radius is changing when the radius is 2m and the length is 1m. (Note: An incompressible fluid is a fluid whose volume does not change.)

12. (Gradient) Let $f(x, y) = \ln(xy)$. Find the maximum rate of change of f at $(1, 2)$ and the direction in which it occurs.

13. (Gradient) Find all points on the surface $z = x^2 - y^3$ where the tangent plane is parallel to the plane $x + 3y + z = 0$.

14. (Gradient) Find all the critical points of $f(x, y) = y^3 + 3x^2y - 6x^2 - 6y^2 + 2$.

15. (Gradient) Find **all** points at which the direction of fastest change of the function $f(x, y) = x^2 + y^2 - 2x - 4y$ is $\mathbf{i} + \mathbf{j}$.