10350 ALG Gateway 02 Name ______________________________ Section __________

1. Rationalize the denominator of the following expression and **simplify** your result assuming that $x \neq 2$.

$$\frac{x-2}{\sqrt{2x+5}-3} \stackrel{?}{=}$$

After multiplying by the conjugate of the denominator, we get

$$\frac{x-2}{\sqrt{2x+5}-3} \cdot \frac{\sqrt{2x+5}+3}{\sqrt{2x+5}+3} = \frac{(x-2)(\sqrt{2x+5}+3)}{(2x+5)-9} = \frac{(x-2)(\sqrt{2x+5}+3)}{2x-4}$$

and after factoring a 2 from our new denominator, notice that $x-2$ cancels. Hence our answer is

$$\frac{\sqrt{2x+5}+3}{2}$$

2. Find the values of x at which the following two curves intersect:

$$y = \frac{2}{x}; \qquad\qquad y = 3 - x$$

We set the two curves equal to each other:

$$\frac{2}{x} = 3 - x$$

and multiply both sides by x to get

$$2 = 3x - x^2$$

We rearrange this to look pretty, and then factor

$$x^2 - 3x + 2 = 0$$

$$(x-2)(x-1) = 0$$

which means that the two curves intersect at $x = 1$ and $x = 2$.

3. Solve for x in the following equations:

3a. $\ln(x-1) = 2 + \ln(2x-3)$

We bring all logs on the left side and combine them:

$$\ln(x-1) - \ln(2x-3) = 2$$

$$\ln\frac{x-1}{2x-3} = 2$$

At this point we can get rid of the natural log by raising e to both sides:

$$\frac{x-1}{2x-3} = e^2$$

Now we are ready to solve for x:

$$x - 1 = e^2(2x-3)$$

$$x - 1 = 2e^2x - 3e^2$$

$$x - 2e^2x = 1 - 3e^2$$

$$x(1-2e^2) = 1 - 3e^2$$

$$x = \frac{1-3e^2}{1-2e^2}$$

3b. $3e^{x+1} = e^{4x+2}$

We can start by taking natural log of both sides

$$\ln(3e^{x+1}) = \ln e^{4x+2}$$

$$\ln 3 + \ln e^{x+1} = \ln e^{4x+2}$$

and since the natural log is the inverse of the exponential base e, we get

$$x + 1 + \ln 3 = 4x + 2$$

Rearranging things to get all x terms on one side, we have

$$3x = \ln 3 - 1$$

$$x = \frac{\ln 3 - 1}{3}$$