PRACTICE QUIZ 6 SOLUTIONS

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Time: 10 min
Time to beat: 4 min

Problem 1. Evaluate $\lim_{x\to+\infty} \frac{14x^3-5x+27}{x^4+10}$

The denominator has higher degree than the numerator, so the limit is zero.

Problem 2. Evaluate $\lim_{x\to+\infty} \frac{3^x-3^{-x}}{3^x+3^{-x}}$

As $x \to +\infty$, the 3^{-x} in both the numerator and denominator goes to zero, so the limit is the same as

$$\lim_{x\to+\infty} \frac{3^x}{3^x} = 1$$

Problem 3. Evaluate $\lim_{x\to-\infty} \frac{3^x-3^{-x}}{3^x+3^{-x}}$

As $x \to -\infty$, the 3^x goes to zero, so the limit is

$$\lim_{x\to-\infty} \frac{-3^{-x}}{3^{-x}} = -1$$

Problem 4. Find $\lim_{x\to 0} \frac{\sqrt{x+3}-\sqrt{3}}{x}$

After multiplying and dividing by the conjugate $\sqrt{x+3} + \sqrt{3}$, we get

$$\lim_{x\to 0} \frac{x+3-3}{x(\sqrt{x+3}+\sqrt{3}} = \frac{1}{2\sqrt{3}}$$

Problem 5. Evaluate $\lim_{x\to+\infty}(x - \sqrt{x^2-1})$ (Hint: multiply and divide by $x + \sqrt{x^2-1}$)

Following the hint, we end up with

$$\lim_{x\to\infty} \frac{(x-\sqrt{x^2-1})(x+\sqrt{x^2-1})}{x+\sqrt{x^2-1}} = \lim_{x\to\infty} \frac{x^2-(x^2-1)}{x+\sqrt{x^2-1}} = \lim_{x\to\infty} \frac{1}{x+\sqrt{x^2-1}} = 0$$