Quiz 6 Solutions

1. Compute
$$\int_0^4 \frac{1}{x-2}\,dx.$$

Solution: Recall that $\int_a^b f(x)\,dx$ exists only if $\int_a^b |f(x)|\,dx$ exists. Observe,
$$\int_0^4 \frac{1}{|x-2|}\,dx = \int_0^2 \frac{1}{|x-2|}\,dx + \int_2^4 \frac{1}{|x-2|}\,dx\ .$$

Let's take the second of these
$$\begin{aligned}\int_2^4 \frac{1}{|x-2|}\,dx &= \lim_{a\to 2}\int_a^4 \frac{1}{x-2}\,dx\\ &= \lim_{a\to 2}\ln(x-2)\Big|_a^4\\ &= \lim_{a\to 2}\ln\left(\frac{4}{a}\right)\\ &= \infty\end{aligned}$$

2. Use Euler's method with step size 0.5 to estimate $y(2)$ where $y(x)$ is the solution to the initial value problem
$$y' = x(y-x), \qquad y(1) = 2.$$

Solution:

Let's make a table using
$$y(x+\Delta x) \approx y(x) + y'(x)\Delta x\ .$$

x	$\approx y$	$\approx y'$
1	2	1
1.5	2.5	1.5
2	3.25	N/A

Therefore $y(2) \approx 3.25$.