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M20580 L.A. and D.E. Tutorial
Quiz 8

1. The differential equation $\left(\frac{dy}{dx}\right)^2 + x^2y + x^3 - 5 = 0$ is (CIRCLE ONE)

(a) First order linear (b) First order nonlinear (c) Second order linear
(d) Second order nonlinear (e) None of the above

$\frac{dy}{dx} = y'$ is squared $\Rightarrow$ nonlinear
$\hookrightarrow$ highest derivative $\Rightarrow$ first order

2. Find the least-squares solution $\hat{\mathbf{x}}$ to $A\mathbf{x} = \mathbf{b}$ where $A = \begin{pmatrix} 1 & 1 \\ 0 & 1 \\ 1 & 0 \\ 1 & 1 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} 7 \\ -2 \\ 3 \\ 5 \end{pmatrix}$

We look at the system $A^TA\,x = A^Tb$, and solve.

$A^TA = \begin{pmatrix} 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 1 \\ 0 & 1 \\ 1 & 0 \\ 1 & 1 \end{pmatrix} = \begin{pmatrix} 3 & 2 \\ 2 & 3 \end{pmatrix}$ and $A^Tb = \begin{pmatrix} 15 \\ 10 \end{pmatrix}$

$$\left(\begin{array}{cc|c} 3 & 2 & 15 \\ 2 & 3 & 10 \end{array}\right) \longrightarrow \left(\begin{array}{cc|c} 1 & -1 & 5 \\ 2 & 3 & 10 \end{array}\right) \longrightarrow \left(\begin{array}{cc|c} 1 & -1 & 5 \\ 0 & 5 & 0 \end{array}\right)$$

$$\longrightarrow \left(\begin{array}{cc|c} 1 & -1 & 5 \\ 0 & 1 & 0 \end{array}\right) \longrightarrow \left(\begin{array}{cc|c} 1 & 0 & 5 \\ 0 & 1 & 0 \end{array}\right)$$

Hence $\boxed{\hat{x} = \begin{pmatrix} 5 \\ 0 \end{pmatrix}}$ is the least-squares solution.