WORKSHEET 12

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Problem 1. Consider the function $f(x) = 1/x$ for $x \in [1, 2]$.

a) Break the interval $[1, 2]$ into n equal subintervals. What is the width of each subinterval?

b) Breaking the region below the graph into vertical strips, we can approximate the area of the function using rectangles. Draw a rough picture to illustrate this. What is the width of each rectangle?

c) For the i-th rectangle, give it's height if we are to use right endpoints for the corresponding subinterval. Also, give the height if we are to use the left endpoints for the corresponding subinterval. Give the area of the rectangle in both cases.

d) Using $\sum$ notation, write a formula for the approximate area under the curve in both cases (right and left endpoint approximations). Note: taking the limit as $n \to \infty$ for this formula, we obtain the exact area under the curve. We will see that this equals $\ln(2)$.

Problem 2. Following the process in Problem 1, find the expression for the area under the curve of the graph of f as a limit. You may use either left or right endpoints. Do not evaluate the limit.

a) $f(x) = \sqrt{x}$ for $x \in [0, 9]$.

b) $f(x) = \dfrac{2x}{x^2 + 1}$ for $x \in [1, 3]$.

c) $f(x) = \sqrt{\sin x}$ for $x \in [0, \pi]$.

Problem 3. The following limits represent the area under the graph of some function $f(x)$. What is the function? Also, give the corresponding interval of x values.

a) $\lim_{n\to\infty} \sum_{i=1}^{n} \dfrac{2}{n}\left(5+\dfrac{2i}{n}\right)^{10}$

b) $\lim_{n\to\infty} \sum_{i=1}^{n} \dfrac{1}{n}\ln(1+\frac{i}{n})$

c) $\lim_{n\to\infty} \sum_{i=1}^{n} \dfrac{4}{n}\left(\dfrac{4i}{n}-2\right)^{3}$

d) $\lim_{n\to\infty} \sum_{i=1}^{n} \dfrac{\pi}{n}\sin(\frac{\pi i}{n})$

e) $\lim_{n\to\infty} \sum_{i=1}^{n} \dfrac{\pi}{n}\cos(\frac{\pi i}{n}-\frac{\pi}{2})$

f) $\lim_{n\to\infty} \sum_{i=1}^{n} \dfrac{1}{n}e^{\frac{i}{n}}$

Problem 4.

a) Give an antiderivative $F(x)$ of the function $f(x) = 2x$.

b) We saw last tutorial that the net/total change in the antiderivative should equal to the area under the curve $f(x)$. What is the net change of your antiderivative from part (a) over the interval $[0, 2]$?

c) Set up the area as a limit, then evaluate the limit and compare your answer to what you obtained in part (b). You may use the following:

$$\sum_{i=0}^{n} 1 = 1 + 1 + \ldots + 1 = n$$

$$\sum_{i=0}^{n} i = 1 + 2 + \ldots + n = \frac{n(n+1)}{2}$$

$$\sum_{i=0}^{n} i^2 = 1 + 4 + 9 + \ldots + n^2 = \frac{n(n+1)(2n+1)}{6}$$