# ELEMENTS OF CALCULUS: EXAM 2 REVIEW

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**Problem 1.** Suppose $f(x) = x^2+1$ and $g(x) = -x^3$. Compute the derivative of $h(x) = \dfrac{f(x)}{g(x)}$.

**Problem 2.** Let $m(x) = x^2$. What is the tangent line to the graph of $f(x) = \dfrac{m(x)+x}{mx+1}$ when $x = 0$?

**Problem 3.** Let $f(x) = x - 1$. For what $x$ values, if any, does the function $g(x) = \dfrac{f(x)+1}{f(x)}$ have a horizontal tangent? What about a vertical tangent?

**Problem 4.** Let $f(x) = x - 1$. For what $x$ values, if any, does the function $g(x) = \dfrac{f(x)+x^2+1}{f(x)+1}$ have a horizontal tangent? What about a vertical tangent?

**Problem 5.** Let $f(x) = x^2$. At what point does the line passing through $(2, 2)$ and $(3, 3)$ intersect the graph of $f(x)$?

**Problem 6.** Find the equation to the tangent AND normal lines to the graph of $f(x) = x^3$ at the point $(1, 1)$.

**Problem 7.** Find the instantaneous rate of change of $y = (x^2+3)(2x^3-5)$ at $x = 0$.

**Problem 8.** Compute the derivatives for the following:

(a) $(x^2+x)(x+1)$

(b) $\dfrac{x^2-3x-10}{x+2}$

(c) $f(x) = (3+4x-x^2)^{1/2}$

(d) $f(x) = \left(\dfrac{x}{1+x}\right)^5$

(e) $f(x) = 2x^2\sqrt{2-x}$

(f) $f(x) = (x^2+3)^4(2x^3-5)^3$

**Problem 9.** Compute the derivative of the function

$$r(x) = \left[ \left( \frac{x+5}{x^2-1} \right)^3 + \left( (x+3)^2 + 9 \right)^4 \right]^{1/2}$$

**Problem 10.** Use implicit differentiation to find the following:

(a) $dy/dx$ for $2y^2 = y + x$

(b) $dx/dy$ for $2y^2 = y + x$

(c) $y'(x)$ for $x^2 - xy + y^2 = 3$

(d) $x'(y)$ for $x^2 - xy + y^2 = 3$

**Problem 11.** Given that $y(t) = t^3$ and $x(t) = 1 - t^2$, differentiate the equation $x^3y + xy^3 = t^2$ with respect to $t$. Don't simplify.

**Problem 12.** Find the tangent line to the graph of $x^2(x^2+y^2) = y^2$ at the point $(\sqrt{2}/2, \sqrt{2}/2)$.

**Problem 13.** Given $x^2 - y^2 = 36$, find $\dfrac{d^2y}{dx^2}$.

**Problem 14.** The radius of a circle is changing at the rate of $dr/dt = +2$ cm/s. How fast is the area changing when $r = 2$ cm? Is this increasing or decreasing? What are the units?

**Problem 15.** A spherical balloon is filling up with air at the rate of 5 $\text{cm}^3$/s. How fast is the radius of the balloon changing when $r = 2$? Is this increasing or decreasing? Use the fact that the volume of a sphere in terms of its radius $r$ is given by $V = \frac{4}{3}\pi r^3$.

**Problem 16.** The variables $x$ and $y$ are both functions of $t$ and are related by the equation $y = x^2 + 3$. Find $dy/dt$ when $x = 1$, given that $dx/dt = 2$ when $x = 1$.

**Problem 17.** A pebble is dropped into a calm pond, causing a ripple in the form of a circle. The radius $r$ of the ripple is increasing at a constant rate of 1 ft/sec. When the radius is 4 feet, at what rate is the total area enclosed by the ripple changing?

**Problem 18.** All edges of a cube are expanding at the same rate, causing the volume of the cube to increase at a rate of 5 $\text{cm}^3$/sec. At what rate are the sides of the cube increasing when the side length is 10 cm? What is the rate of the change of the cube's surface area at that time?

**Problem 19.** The position of a particle is given by $s(t) = \frac{1}{5}x^5 - 2x^2 + x$, where $t$ is measured in seconds. Is there ever a time during the first second where the particle is at rest?

**Problem 20.** The position of a particle is given by $s(t) = \pi x^2 - x + 18$. For what (non-negative) value(s) of $t$ is the particle at rest?

**Problem 21.** Consider the function $f(x) = (x-1)^{1/3} - \dfrac{x}{12}$.

(a) Find all critical points of $f(x)$.

(b) Find the absolute maximum and absolute minimum of $f(x)$ for $0 \leq x \leq 28$.

(c) Find intervals where $f$ is increasing/decreasing.

(d) Find intervals where $f$ is concave up/down.

**Problem 22.** Find the absolute extrema of the function on the closed interval:

a) $f(x) = x^3 - \dfrac{3}{2}x^2$ on $[-1, 2]$.

b) $g(t) = \dfrac{t^2}{t^2+3}$ on $[-1, 1]$.

c) $h(t) = \dfrac{t}{t+3}$ on $[-1, 6]$.