PRACTICE QUIZ 7 SOLUTIONS

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Time: 14 min
Time to beat: 5 min 30 sec

Problem 1. If $f(5) = 4$, $f'(5) = -6$, $g(4) = 5$, and $g'(4) = 8$, find $F'(4)$ if $F(x) = f(g(x))$.

For $F(x) = f(g(x))$, the inside function is $g(x)$ and outside function is $f(u)$. By chain rule, $F'(x) = f'(u) \cdot g'(x) = f'(g(x)) \cdot g'(x)$, and at $x = 4$ this is $F'(4) = f'(g(4)) \cdot g'(4) = f'(5) \cdot 8 = (-6)(8) = -48$.

Problem 2. Find the derivative for $f(x) = (x^2 - x + 1)^9(x^3 - 3x^2 + 1)^{12}$.

First we use the product rule, and get

$$\left((x^2 - x + 1)^9\right)' (x^3 - 3x^2 + 1)^{12} + (x^2 - x + 1)^9 \left((x^3 - 3x^2 + 1)^{12}\right)'$$

The derivative of the first piece is done using chain rule, and it is equal to

$$9(2x - 1)(x^2 - x + 1)^8$$

and similarly the second piece derivative is

$$12(3x^2 - 6x)(x^3 - 3x^2 + 1)^{11}$$

Put together, we have

$$9(2x - 1)(x^2 - x + 1)^8(x^3 - 3x^2 + 1)^{12} + 12(3x^2 - 6x)(x^2 - x + 1)^9(x^3 - 3x^2 + 1)^{11}$$

which could be simplified further if you wanted to by factoring out the common terms.

Problem 3. Find the derivative for $f(x) = \cos^3(4x)$.

Write the function as

$$[\cos(4x)]^3$$

and take derivative using chain rule (outer function is u^3) to get:

$$3\cos^2(4x) \cdot (-\sin(4x)) \cdot 4 = -12\cos^2(4x)\sin(4x)$$

Problem 4. Find the derivative for $f(x) = \tan^3(x) + \tan(x^3)$.

This is by chain rule

$$3\tan^2(x) \cdot \sec^2(x) + \sec^2(x^3) \cdot 3x^2 = 3\sec^2 x \tan^2 x + 3x^2\sec^2(x^3)$$

Problem 5. Find the equation of the tangent line to the curve $y = \frac{3}{\sqrt{16-6x}}$ at $x = 2$.

First we compute the derivative using chain rule:

$$\left(3(16-6x)^{-1/2}\right)' = 3\frac{-1}{2}(16-6x)^{-3/2} \cdot (-6)$$

which at $x = 2$ equals $9/8$. Since $f(2) = 3/2$, our tangent line equation is

$$y - \frac{3}{2} = \frac{9}{8}(x-2)$$