FINITE MATH
EXAM 1 PRACTICE PROBLEMS

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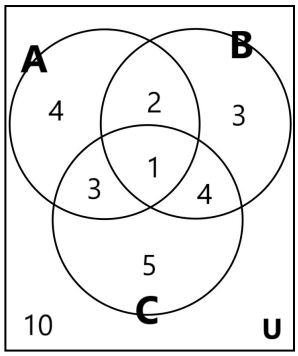
Contents

Important study tip: All the examples that are in the notes or past quizzes are considered fair game for this exam. They should all be part of your review. The examples in this review are a bit more complex (in the sense that they involve more steps), and if you don't understand the basic counting techniques, go back to the lectures and make sure you can do the simpler stuff before trying these review problems.

1. Set Theory

Problem 1. Consider the Venn diagram on the right.



a) Find $n(U)$.

b) Find $n(A \cap C)$.

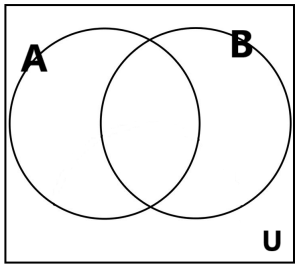
c) Find $n(A' \cup C')$.

d) Compute the size of $(A \cup B) \setminus C$.

e) Compute the size of $(A' \cup B')' \cap C$.

Problem 2. Each student in Liberal Arts at some college has a mathematics requirement (A) and a science requirement (B). A poll of 140 sophomores shows that:

- 60 completed A
- 45 completed B
- 20 completed both A and B



a) Fill each of the 4 regions of the Venn diagram with the appropriate number of students in each region.

b) Find the number of students who completed at least one of the two requirements.

c) Find the number of students who completed exactly one requirement.

d) Find the number of students who completed only the math requirement.

e) Find the number of students who completed neither requirement.

Problem 3. In a survey of 120 people, it was found that:

- 65 like *Newsweek*, 45 read *Time*, 42 read *Fortune*
- 20 read both *Newsweek* and *Time*
- 25 read both *Newsweek* and *Fortune*
- 15 read both *Time* and *Fortune*
- 8 read all three magazines.

a) Fill in the correct number of people in each of the eight regions of a Venn diagram representing this scenario.

b) Find the number of people who read at least one of the three magazines.

c) Find the number of people who read exactly one magazine.

d) How many people read *Time* or *Fortune*, but not *Newsweek*.

Problem 4. List all the subsets of $\{1, 2, 3, 4, 5\}$ of size 3. How many are there?

Problem 5. For the universe $U = \{1, 2, 3, \ldots, 11\}$, consider the following sets:

$$A = \{1, 2, 3, 4, 5\} \qquad B = \{3, 4, 5, 7, 9\} \qquad C = \{1, 3, 5, 7, 9, 11\}$$

Construct a Venn diagram and fill in the correct number of elements for each region.

2. Basic Counting Principles

Problem 6. Suppose a college has 3 different history courses, 4 literature courses, and 2 sociology courses.

a) In how many ways can a student choose one of each kind of course?
b) In how many ways can a student choose just one of the courses?

Problem 7. Suppose you have 3 hats (A, B, C), 2 coats (M, N), and 4 scarves (W, X, Y, Z). During winter, your typical outfit consists of one hat, one coat, and one scarf. However, if you're wearing hat A (which doesn't match any of your scarves), you will not wear a scarf that day. How many possible outfits can you make?

Problem 8. Dana decides to plant either a rose bush or a small tree in her back yard. Home Depot has 4 varieties of roses, and 7 varieties of trees. In how many ways can she select a single plant?

Problem 9. A multiple choice quiz consists of 6 True/False questions. Students may choose to leave answers blank. In how many ways can the quiz be answered?

Problem 10. How many different **binary strings** of length six are there? How many of those start with '0'? How many end with '0'? How many have '0' as their first and last digit? How many start or end with '0'?

Problem 11. A coin is flipped 3 times. How many possible outcomes are there? In how many outcomes does the coin come up with at least one H? In how many outcomes does the coin come up 'HT' in that order? (heads followed by tails).

Problem 12. The area codes for north/northwest suburbs of Chicago are 847 and 224. How many telephone numbers are possible given the format (area code) xxx-xxxx? Assume any digit may be used for the rest of the number.

Problem 13. Public companies whose stock can be traded on stock exchanges typically have an abbreviation for their name. For example, Microsoft is listed as MSFT, Apple is AAPL, etc. If the length of the abbreviation can be 3 or 4 letters, how many such abbreviations are possible? How many of these begin with A?

Problem 14. A teacher can select two students from two separate groups, with each group containing the same number of students, in 169 ways. How many students are in each group?

Problem 15. A 3-digit number is formed using digits from $\{0, 1, 2, 3, 4, 5\}$.

a) How many numbers can be formed using different digits.
b) How many numbers can be formed if digits may be repeated?
c) How many even numbers can be formed using different digits?
d) How many even numbers can be formed if digits may be repeated?
e) How many multiples of 5 can be formed if digits may be repeated?

Problem 16. A child formes 3-letter words using letters from INVOKED. Letters may be repeated.

a) How many different words are possible?
b) How many different words begin with K?
c) How many different words begin with a vowel?
d) How many different words have a vowel for the middle letter?
e) How many different words contain exactly one vowel?

Problem 17. In how many ways can 3 male betta fish and 2 female betta fish be put (separately) in 5 fish tanks (in a row) if the males can't sits next to other males.

Problem 18. In how many different arrangements can 4 people be seated around a round table if all we care about is the left/right neighbours (so any rotation of the table is considered the same).

Problem 19. Four different pairs of shoes (Left and Right) are arranged in a row on a shelf.

a) How many arrangements are possible if we don't care about having shoes of the same pair next to each other?
b) What if we want shoes from the same pair together, but don't care if they are put in the correct or reverse order?
c) What if we want the pairs to appear in the correct order?

Problem 20. In a group of 750 people, at least 2 have the same first and last initials. Why?

3. Permutations

Problem 21. How many arrangements of 3 people in a row are possible if there are 8 people to select from?

Problem 22. In how many ways can 5 people be seated in a row of 5 chairs? What if among the five there is a couple that must be seated together?

Problem 23. Pi Mu Epsilon is to select a president and a vice-president from a group of 8 math students. Find the total number of selections possible.

Problem 24. How many different words can be made by rearranging all the letters in the following words:

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Problem 25. How many 3-digit numbers can be formed using $\{2, 4, 6, 8\}$ if repetitions are not allowed? What if repetitions are allowed?

Problem 26. A coin is tossed 7 times. How many sequences of 3 heads and 4 tails are possible? What about 5 heads and 2 tails?

Problem 27. Starting at the origin $(0, 0)$ and going all the way to $(5, 3)$, how many possible paths are there if you can only travel in the positive x or positive y direction (so up or to the right), taking steps of length 1?

Problem 28. Aluel decides that instead of studying for a multiple choice Finite Math exam with 20 questions, she is going to use probability to beat the test. She figures that out of the 20 questions, half of them ought to be true, and the other half false, so she answers 10 true and 10 false. In how many ways can she answer the test?

Problem 29. The keypad to lock a computer lab has 5 buttons. An access code is a sequence of 3 or 4 buttons, pressed in the correct order. How many access are possible (a) if a button may be repeated, and (b) if a button may not be repeated?

4. Combinations

Problem 30. Prove algebraically that $\binom{n}{k} = \binom{n}{n-k}$.

Problem 31. Use the Binomial Theorem and Pascal's Triangle to find the expansion of $(a + b)^6$ and $(a + b)^7$.

Problem 32. Consider the expansion of $(x + z)^{20}$. What is the coefficient of x^2z^{18}?

Problem 33. A software company is going to hire 2 new employees out of 12 applicants. In how many ways can they do this? Tommy is one of the applicants. In how many ways can the company hire the 2 applicants if Tommy is one of the new employees?

Problem 34. Out of 7 math books, 9 science books, and 5 literature books, in how many ways can a student select 2 books of each type?

Problem 35. How many different committees can be formed from a group of 9 women and 11 men if a committee is composed of 3 women and 3 men?

Problem 36. A child has 3 pennies, 5 nickels, and 4 dimes. In how many ways can he select 2 coins of the same denomination?

Problem 37. In how many ways can JD invite 1 or more of 10 friends to a dinner party?

Problem 38. When you draw 3 lines such that no 2 are parallel and no 3 intersect at the same point, you will find that there are 3 points where the lines intersect (draw some examples). If you draw 4 lines where no 2 are parallel and no 3 intersect at the same point, determine the number of points of intersection.

Problem 39. How many subsets of $\{1, 2, \ldots, 10\}$ have at least 2 elements?

Problem 40. Find the number of poker hands that contain no red cards. What about the number of poker hands that contain at least 2 Aces?

5. Complex Counting Problems

Problem 41. A local police department has 3 empty jail cells (cell 1, 2, 3). Three shoplifters (A, B, C) are arrested one evening. The arresting officer can put each of them in any of the three available cells.

a) In how many ways can the officer do this?

b) In how many ways can this be done if each prisoner is to be alone in a cell?

c) In how many ways can this be done if all prisoners go in the same cell?

d) In how many ways can this be done if one of the cells contains 2 prisoners (so 2 prisoners go in one of the cells, 1 in another, and a cell which stays empty)?

Problem 42. A coin is flipped 4 times. How many possible outcomes are there? In how many outcomes does the coin come up with at least one H? In how many outcomes does the coin come up 'HT', in that order, at some point in the sequence of 4 flips? (heads followed by tails).

Problem 43. A 3-digit number is formed using digits from $\{1, 2, 3, 4, 5\}$. How many numbers can be formed if the first two digits may be repeated, but the third digit must be different from the first two?

Problem 44. A committee of 5 people is selected from a group of 6 men and 7 women. In how many ways can the committee be selected so it contains at least 1 man?

Problem 45. A die is rolled twice, and the sum of the two numbers is recorded. In how many ways can one get the sum to be equal to 6?

Problem 46. A die is rolled five times, and the numbers are written down in the order they appear. How many possible ordered samples are possible? How many of those samples have the last 2 numbers add up to 11?

Problem 47. An urn contains 10 tickets numbered $\{1, 2, 3, \ldots, 10\}$. Three tickets are selected one at a time and placed back in the urn. How many possible ordered samples are possible? What if we don't place the ticket back in the urn?

Problem 48. Among 9 orchids, 3 are white, 4 are purple, and 2 yellow. How many different color displays are possible if the orchids are placed in a row?

Problem 49. Cece wants to decorate her study desk with a vase of flowers. She goes to the campus flower shop and they only have 4 roses, 5 tulips, and 3 dahlias. She can choose any number of flowers from each species, but she certainly wants to choose at least one of each. In how many ways can she build the flower arrangement?

Problem 50. A tray contains 12 chocolate chip cookies and 16 brownies. In how many ways can a child select 4 cookies that include at least 1 chocolate chip cookie?

Problem 51. A box of candy hearts contains 52 hearts total, of which 19 are white, 10 are tan, 7 are pink, 3 are purple, 5 are yellow, 2 are orange, and 6 are green. You select 9 pieces of candy from the box.

a) In how many ways can you select the 9 candies?
b) In how many ways are 3 of the candies you selected white?
c) In how many ways are 3 white, 2 tan, 1 pink, 1 yellow, and 2 green?

Problem 52. How many positive divisors does the number $11^3 13^5$ have? (it may be helpful to realize 11 and 13 are prime numbers).

Problem 53. Billy has 43 baseball cards, and Scottie has 26. In how many ways can Billy trade 2 of his cards for 2 of Scottie's cards?

Problem 54. How many sequences of 3 different letters contain tle letter 'A' exactly once?

Problem 55. A Social Security number has 9 digits (the format is xxx-xx-xxxx) out of $\{0, 1, 2, 3, \ldots, 9\}$, and digits may be repeated. How many Social Security numbers are possible? Is this enough to cover the entire population of the U.S. (which is 341,400,000)? How many of those Social Security numbers have at least one repeated digit?