10350 Quiz 01 Name ______________________ Section ______________

1. The position function $s(t)$ of a particle travelling a straight line is given by $s(t) = \cos^2(t)$. Find the **acceleration** of the particle at any time t.

$$
\begin{aligned}
v(t) = s'(t) &= 2\cos t \cdot \frac{d}{dt}[\cos t] && \text{Apply Chain Rule} \\
&= 2\cos t \cdot (-\sin t) \\
&= -2\sin(t)\cos(t) \\
a(t) = v'(t) = s''(t) &= \frac{d}{dt}[-2\sin t \cos t] \\
&= 2\sin t \cdot (-\sin t) + (-2\cos t)(\cos t) && \text{Product Rule} \\
&= \boxed{2(\sin^2 t - \cos^2 t)}
\end{aligned}
$$

Note: If we use trig identities we can write the above as $v(t) = -\sin(2t)$ and $a(t) = -2\cos(2t)$. Either form is an acceptable answer.

2. Consider the **piece-wise defined** function $f(x) = \begin{cases} x\cos\left(\dfrac{1}{x}\right) & -\infty < x < 0 \\ 1 & x = 0 \\ \dfrac{\sin(5x)}{6x} & 0 < x < +\infty \end{cases}$

(a) Find the values of the following limits. **Justify your answer.** You are required to use $\lim_{x\to 0} \frac{\sin x}{x} = 1$ where applicable and not L'Hopital's Rule.

$\lim_{x\to 0^-} f(x) \stackrel{?}{=}$ ________

Solution:

$$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} x\cos\left(\frac{1}{x}\right)$$

We can use Squeeze Theorem to find this limit. We know $-1 \le \cos\left(\frac{1}{x}\right) \le 1$ for all $x < 0$. Multiplying the inequality by $x < 0$ we obtain $-x \ge x\cos\left(\frac{1}{x}\right) \ge x$, or equivalently, $x \le x\cos\left(\frac{1}{x}\right) \le -x$. We have

$$\lim_{x\to 0^-} x = \lim_{x\to 0^-} -x = 0$$

And by Squeeze Theorem, it follows, $\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} x\cos\left(\frac{1}{x}\right)$.

Note that Squeeze Theorem is useful for evaluating trigonometric limits of this form (you saw an example in class where $f(x) = x\sin(1/x)$, because the inequality $-1 \le \sin\theta \le 1$ makes it relatively easy to find two nice functions to "squeeze" $f(x)$ between.

$\lim_{x \to 0^+} f(x) \stackrel{?}{=}$ ________.

Solution:

$$\begin{aligned}
\lim_{x \to 0^+} f(x) &= \lim_{x \to 0^+} \frac{\sin(5x)}{6x} \\
&= \lim_{x \to 0^+} \frac{\sin(5x)}{6x} \cdot \frac{5}{5} \\
&= \lim_{x \to 0^+} \frac{\sin(5x)}{5x} \cdot \frac{5}{6} \\
&= \frac{5}{6} \cdot \lim_{x \to 0^+} \frac{\sin(5x)}{5x} \\
&= \frac{5}{6} \cdot 1 \\
&= \frac{5}{6}
\end{aligned}$$

(b) Which of the choices below **BEST** describes the behavior of $f(x)$ at $x = 0$? Circle one:

Removable Discontinuity	$\boxed{\text{Jump Discontinuity}}$
Right Continuous	Continuous

This is an example of a **jump discontinuity** because $\lim_{x \to 0^-} f(x)$ and $\lim_{x \to 0^+} f(x)$ both exist and are finite, but $\lim_{x \to 0^-} f(x) = 0 \neq \frac{5}{6} = \lim_{x \to 0^+} f(x)$. We cannot make this function continuous by simply redefining $f(0)$.

Also note that the function is *not* right continuous, because $\lim_{x \to 0^+} f(x) = \frac{5}{6} \neq f(0) = 1$.