QUIZ 12

ADRIAN PĂCURAR

Time: 20 minutes

You are allowed to use the following formulas for this quiz:

$$\sum_{i=0}^{n} 1 = 1 + 1 + \ldots + 1 = n$$

$$\sum_{i=0}^{n} i = 1 + 2 + \ldots + n = \frac{n(n+1)}{2}$$

$$\sum_{i=0}^{n} i^2 = 1 + 4 + 9 + \ldots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

Problem 1. Convert the following to $\sum$ notation.

- $\frac{1}{2} + \frac{1}{4} + \frac{1}{6} + \frac{1}{8} + \ldots + \frac{1}{20} =$

- $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \ldots + \frac{1}{256} =$

- $1 - x + x^2 - x^3 + x^4 - \ldots + x^{16} - x^{17} =$

Problem 2. Consider the function $f(x) = x^2$. We will use rectangular subintervals to find the area under the curve from $x = 0$ to $x = 1$.

i) If we break the interval $[0, 1]$ into n subintervals of equal length, what is the width of each subinterval?

(a) 1 (b) 0 (c) $n/2$ (d) $1/n$ (e) $1/n^2$

ii) Draw a picture of the graph of f in the given interval, and include a few subrectangles as we did in class to help you visualize the next step.

iii) For the first subinterval, if we take the right-endpoint, what is the height of the corresponding rectangle going to be?

(a) 0 (b) $f(0)$ (c) 1 (d) $f(1)$ (e) $f(\frac{1}{n})$

iv) For the second subinterval, if we take the right-endpoint, what is the height of the corresponding rectangle going to be?

(a) 1 (b) $f(1)$ (c) 2 (d) $f(\frac{1}{n})$ (e) $f(2 \cdot \frac{1}{n})$

v) For the i-th subinterval, if we take the right-endpoint, what is the height of the corresponding rectangle going to be?

(a) i (b) $f(i)$ (c) $1/n$ (d) $f(1/n)$ (e) $f(i \cdot \frac{1}{n})$

vi) Write a formula for the area of the i-th rectangle (your expression should depend on i and n).

vii) Using $\sum$ notation, write a formula for the total area of all the rectangles (i will go from 1 to n in the bounds).

viii) Using the formulas provided at the beginning of the quiz, compute the formula you just obtained. Your result should depend solely on n.

ix) Take the limit as $n \to \infty$ to get the area under the curve.