PRACTICE QUIZ 9 SOLUTIONS

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Time: 10 min
Time to beat: 3 min

Problem 1. Find the points of discontinuity of the piecewise function

$$f(x) = \begin{cases} \sin(x) & x < 0 \\ x & x \geq 0 \end{cases}$$

Since both branches $\sin(x)$ and x are continuous on the entire real line, the only potential problem is at $x = 0$. But $sin(0) = 0$, so the function is continuous at zero, and hence we have no points of discontinuity.

Problem 2. Identify the points of discontinuity and their type for $f(x) = \frac{x^4-1}{x^2-1}$.

Factor $f(x)$ as

$$f(x) = \frac{(x^2+1)(x+1)(x-1)}{(x+1)(x-1)}$$

which tells us we have removable discontinuities at $x = \pm 1$.

Problem 3. Using the limit definition, find the derivative of $f(x) = x^3$.

$$f'(x) = \lim_{h \to 0} \frac{(x+h)^3 - x^3}{h} = \lim_{h \to 0} \frac{x^3 + 3x^2h + 3xh^2 + h^3 - x^3}{h} = \lim_{h \to 0} \frac{3x^2 + 3xh + h^2}{1} = 3x^2$$

Problem 4. Using the limit definition, find the derivative of $f(x) = \frac{1}{x}$.

$$f'(x) = \lim_{h \to 0} \frac{\frac{1}{x+h} - \frac{1}{x}}{h} = \lim_{h \to 0} \frac{\frac{x-(x+h)}{x(x+h)}}{h} = \lim_{h \to 0} \frac{1}{h} \cdot \frac{-h}{x(x+h)} = \lim_{h \to 0} \frac{-1}{x(x+h)} = \frac{-1}{x^2}$$