FINITE MATH
EXAM 2 PRACTICE PROBLEMS

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Important study tip: All the examples that are in the notes or past quizzes are considered fair game for this exam. They should all be part of your review. The examples in this review are a bit more complex (in the sense that they involve more steps), and if you don't understand the basic counting techniques, go back to the lectures and make sure you can do the simpler stuff before trying these review problems.

1. Basic Probability

Problem 1. The probability of randomly selecting a female from a group of 100 people is 0.6. What is the number of females in the group?

Problem 2. Consider the sample space $S = \{1, 2, 3, 4, 5, 6\}$ for an unbalanced six-sided die with probabilities given below:

Outcome e	1	2	3	4	5	6
P(e)	0.1	0.1	0.3	0.15	0.15	0.2

You roll two of these unbalanced dice, and observe the sum of the numbers that come up.

a) What is the probability that the sum of the two numbers is 1?
b) What is the probability that the sum of the two numbers is 2?
c) What is the probability that the sum of the two numbers is 4?
d) What is the probability that the sum of the two numbers is 7?

Problem 3. In a state lottery, four digits are drawn at random (repeats are possible) from among $\{0, 1, 2, \ldots, 9\}$. Suppose you win if any permutation of your selected integers is drawn. Give the probability of winning if you select:

a) $6, 7, 8, 9$ b) $6, 7, 8, 8$ c) $7, 7, 8, 8$ d) $7, 8, 8, 8$

Problem 4. A sample space of an experiment is $\{A, B, C\}$. Find the probability of each simple outcome if P(A) = P(B) and P(C) = 3P(A).

Problem 5. You draw a single card from a standard deck. Find the following probabilities:

a) Drawing an Ace.

b) Drawing a King or a Queen.

c) Drawing a Diamond.

d) Drawing a black card.

Problem 6. Pick a subset at random out of the subsets of $\{1,2,3,4,5\}$. What is the probability that the subset has size 2?

Problem 7. For the sample space $S = \{1,2,3,\ldots,11\}$, each outcome is equally likely to happen. Compute the probability of the following events:

$$A = \{1,2,3,4,5\} \qquad B = \{3,4,5,7,9\} \qquad C = \{1,3,5,7,9,11\}$$

Can you find a pair of events from A, B, C that are independent? Why or why not?

Problem 8. Suppose a college has 3 different history courses, 4 literature courses, and 2 sociology courses. A student doesn't know what to choose, so he picks e courses at random.

a) What is the probability that are all literature? What about all 3 history?
b) What is the probability that he ends up with a course of each type?
c) What is the probability that all 3 are sociology?

Problem 9. Two students are selected at random from a class of 14 males and 5 females.

a) Find the probability that both students are male.
b) If the two students are to be assigned the role of class president and class treasurer, find the probability that both students are male.

Problem 10. A crate contains 20 total apples, 4 of which are spoiled. If 4 apples are selected at random, find the possibility of each of the following:

a) All 4 are good.
b) All 4 are bad.
c) Exactly 2 are good.
d) At least one is bad.
e) At most 3 are good.

How do the last 2 parts of the problem relate to each other?

Problem 11. A multiple choice quiz consists of 5 True/False questions. David didn't study, so he decides to guess. He also may choose to leave answers blank. If David answers each question at random (or chooses not to answer), find the probability of each of the following:

a) The student scores 100% on the quiz.
b) The student scores 0% on the quiz.
c) The student scores 40% on the quiz.
d) The student scores passes the quiz (scores 60% or more).
e) The student fails the quiz (scores less than 60%)

Repeat the question, but this time the student answers everything without leaving any questions blank. Compare the probabilities. What's the better strategy? Answer everything or skip questions?

Problem 12. Kacey has seven cards numbered $\{1,2,3,4,5,6,7\}$. She creates 3-digit numbers by randomly picking 3 of the 7 cards, and placing them in some order. Find the following probabilities:

a) The number is 524.
b) The number is larger than 300.
c) The number is smaller than 300.
d) The number is smaller than 320.
e) The number is smaller than 325.
f) The number is bigger than 325.

Problem 13. A coin is flipped 3 times. What is the probability that the coin come up with exactly one H? At least one H?

Problem 14. Pick 8 people at random out of a group of 11 men and 9 women. What is the the probability that at least one is female?

Problem 15. The gene for brown eyes is dominant (C) over the gene for blue eyes (c). So if a person has genotype CC or Cc, the eye color is brown, but if the person is cc, they will have blue eyes. Two parents of genotype Cc and cc have a child. What is the probability that the child has blue eyes?

2. Conditional Probability

Problem 16. Given that $P(E) = 0.4$, $P(F) = 0.8$, and $P(E \cap F) = 0.3$, compute the probabilities $P(E|F)$ and $P(F|E)$.

Problem 17. A course is split into two sections, Section 1 (30 students total) and Section 2 (26 students total). During an exam, 24 from Section 1 pass, and 11 from Section 2 pass.

a) If one of the exams (from the sections combined) is selected at random, what is the probability that it's a Pass? What about Fail?
b) A randomly selected exam is known to be from Section 2. What is the probability that it's a Pass?
c) A randomly selected exam is known to be a Pass. What is the probability that it came from Section 1?

Problem 18. A mathematics professor assigns two problems for homework and knows that the probability of a student solving the first problem is 0.70, the probability of solving the second is 0.50, and the probability of solving both is 0.15.

a) Mike has solved the second problem. What is the probability he also solves the first problem?
b) Julia has solved the first problem. What is the probability she also solves the second problem?
c) Are the events independent?

Problem 19. A survey was one on students' coffee preferences. The results are shown below:

	No Coffee	Regular	Decaf	Total
Female	29	147	67	243
Male	18	196	45	259
Total	47	343	112	502

A student is selected at random out of the 502 total. Find the probability of the following events:

a) The student does not drink coffee.
b) The student is male.
c) The student is female who prefers regular coffee.
d) The student prefers decaf, given that the student is male.
e) The student is male, given that the student prefers decaf.
f) The student is female, given that the student prefers regular coffee or does not drink coffee.

Problem 20. JD has 3 Red and 7 White marbles in his left pocket, and 2 Red and 1 White in his right pocket. He transfers a marble at random from his left pocket to his right pocket. After the transfer, what is the probability of picking a White marble from his right pocket?

Problem 21. Four cards are to be dealt at successively at random and without replacement from an ordinary deck of cards. What is the probability of receiving, in order, a spade, a heart, a diamond, and a club?

Problem 22. A small grocery store has 5 cartons of milk left in stock, 2 of which are sour. If you are going to buy the **third** carton of milk sold that day at random, compute the probability of selecting a sour carton of milk.

Problem 23. A single card is drawn at random from each of six well-shuffled decks of cards. What is the probability that all six cards are different?

Problem 24. A group of 5 people contains a couple. If they are all to be seated at random, what is the probability that the couple ends up sitting together?

Problem 25. A drawer contains 4 Black, 6 Brown, and 8 White socks. Two socks are selected at random from the drawer.

a) Find the probability that both socks are the same color.
b) Find the probability that both socks are Black, given that they are of the same color.

Problem 26. A bag contains 8 Red and 7 White marbles. A second bag contains an unknown number of Red marbles, and 9 White marbles. A marble is drawn at random from each bag, and the probability of getting two marbles of the same color is 151/300. How many red marbles are in the second bag?

Problem 27. Consider all possible 3-digit numbers can be formed using digits from $\{1, 2, 3, 4, 5\}$ (repetitions are ok). Pick one such 3-digit number at random. What is the probability that there are no repeating digits in the number?

Problem 28. Dea, frustrated with her finite math homework, hits a fair coin with a hammer and skewes the probabilities from 50/50 to $P(H) = 0.3$. She then throws the coin at a wall 5 times in a row. What is the probability of getting exactly 3 Heads?

Problem 29. A basketball player makes 80% of his free throws. In practice, the coach tells him to take five shots. What is the probability that he makes the first 3 and misses the last 2? Also, what is the probability of exactly 3 successes out of the 5?

Problem 30. Consider the set $\{1, 2, 3, 4, 5\}$. You select a subset at random. Given that the subset you select has size 3, what is the probability that it contains 5?

Problem 31. A sample set contains 100 equally likely outcomes. Events E and F satisfy:

$$n(E \setminus F) = 20 \qquad n(F \setminus E) = 30 \qquad n(E \cap F) = 30$$

Are the events independent?

Problem 32. A manufacturer has 3 machines that produce spoons. Machine 1 produces 20% of the spoons, and 4% of its spoons are bent. Machine 2 produce 30% of the spoons, and 7% of its spoons are bent. Machine 3 produce 50% of the spoons, and 15% of its spoons are bent. You randomly select a spoon, and notice that it is bent. What is the probability that it came from:

a) Machine 1   b) Machine 2   c) Machine 3

Problem 33. A mythology class is composed of 12 sophomores, 23 juniors, and 17 seniors. On the first exam, 2 sophomores, 5 juniors, and 7 seniors earnned A's. Find the probability that a randomly chosen student who received an A is a junior.

Problem 34. An insurance company sells different policies: 60% for autos, 40% for homeowners, and 20% for both types. Consider the events:

- A = the people with only an auto policy
- H = the people with only a homeowner policy
- B = the people with both types of policy
- C = the people with other types of policy

a) Find $P(A), P(H), P(B)$, and $P(D)$.
b) Given that a randomly chosen person has an auto policy, what is the probability that the person also has a homeowner policy?

Problem 35. A hand of 13 cards are to be dealt at random and without replacement from a standard deck. Find the probability that there are at least 3 Queens in the hand, given that the hand contains at least 2 Queens.

Problem 36. A bag contains 4 marbles numbered 1,2,3, and 4. One marble is to be drawn at random from the bag. Define the following events:

$$A = \{1,2\} \qquad B = \{1,3\} \qquad C = \{1,4\}$$

a) Are A and B independent? What about A and C? B and C?
b) Are events A, B, and C independent?

In part a), the answer will be yes for all 3 pairs of events. However, in part b), the answer will be no. Something seems to be lacking for the complete independence of A,B, and C. If all were true, A, B, C would be called **mutually independent**. In this case, they are just **pairwise independent**.

Problem 37. Three inspectors look at a critical component of a rocket. Their probabilities of detecting a defect are different: 0.99, 0.98, 0.96. Assuming independence, find the following:

(1) What is the probability that at least one of the inspectors finds a defect?
(2) What is the probability that only one of the inspectorsfinds a defect?

Problem 38. If $P(A) = 0.8$, $P(B) = 0.5$, and $P(A \cup B) = 0.9$, are the events independent?

Problem 39. Suppose $P(A) = 0.6$ and $P(A \cap B) = 0.1$. If the events are known to be independent, what is $P(B)$?

Problem 40. Consider 3 fair six-sided dice that have colours instead of numbers. Die A has Orance on one face. Die B has Orange on two faces. Die C has Orange on 3 faces. If all three dice are rolled, find the probability that exactly 2 of the three dice come up orange.

Problem 41. There is a new diagnostic test for a disease that occurs in about 0.05% of the population. The test is not perfect, but it will detect a person with the disease 99% of the time. It will, however, say that a person without the disease has the disease 3% of the time. A randomly selected person from the population is tested, and it comes back positive. What is the probability that this person has the disease?