M20580 L.A. and D.E. Tutorial ${\bf Quiz}~5$

1. Given that $\mathcal{B} = \{1, t - 1, t^2 - t\}$ is a basis for \mathbb{P}_2 and $[\mathbf{p}]_{\mathcal{B}} = \begin{bmatrix} \alpha \\ \beta \\ \gamma \end{bmatrix}$ is the coordinate vector of the polynomial $\mathbf{p}(t) = t^2 + t + 6$ relative to \mathcal{B} , what is the value of α ?

2. The vector \vec{v} has coordinates $\begin{bmatrix} \vec{v} \end{bmatrix}_{\mathcal{B}} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ in the basis $\mathcal{B} = \{(1,0,-1),(2,2,2),(3,1,1)\}$. Find \vec{v} .

Solution: 1. Since

$$t^{2} + t + 6 = 8(1) + 2(t - 1) + 1(t^{2} - t), \tag{1}$$

we have $\alpha = 8$.

2. The components of \vec{v} serve as coefficients in the linear combination:

$$\vec{v} = 1(1,0,-1,) + 2(2,2,2) + 3(3,1,1) = (14,7,6).$$
 (2)