FINITE MATH
EXAM 3 PRACTICE PROBLEMS

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Important study tip: All the examples that are in the notes or past quizzes are considered fair game for this exam. They should all be part of your review. The examples in this review are a bit more complex (in the sense that they involve more steps), and if you don't understand the basic counting techniques, go back to the lectures and make sure you can do the simpler stuff before trying these review problems.

1. Basic Statistics

Problem 1. The data below shows the number of children in each family of 45 students.

2	2	5	3	4	4	3	3	6	4	3	4	4	4	2
2	5	2	3	1	3	5	2	4	3	3	2	2	4	1
6	1	3	3	2	4	3	2	3	2	3	2	3	5	1

Construct a frequency and relative frequency table for the number of children per family. Calculate the mean, variance, and standard deviation for the data.

Problem 2. The U.S. Census Bureau reported the mean annual income of women by level of education as follows:

Education	Income
Less than 9th grade	10,695
High-school diploma	18,251
Associate degree	27,741
Bachelor's degree	36,259
Master's degree	49,000
Doctorate	60,621

Give a rough sketch of the frequency histogram for these categories.

Problem 3. Calculate the class average, and the standard deviation for the following grade distribution on an exam:

Grade	Frequency
96	2
91	3
85	9
80	13
75	11
70	10
60	8
50	4

Problem 4. An IT technician records the number of bad sectors per hard disk while performing data recovery. Here is a summary of the data:

Number of bad sectors	Frequency
0	480
1	395
2	255
3	25
4	5

What is the average number of defects, and the standard deviation for the number of bad sectors on hard disks that need data recovery? Use the sample statistics this time, as we are trying to infer something about ALL the hard disks from the sample of everything he worked on so far.

Problem 5. The class average on an exam was 73.25. The average of the 16 Freshmen in the class was 71.75. There were also 8 sophomores registered in the course. What was their average?

Problem 6. A data set has 8 data points: 4, 5, or 6. The mean of the set is 5.

a) If $\sigma = 0.5$, what is the data set?
b) If $\sigma = 1$, what is the data set?

2. Random Variables

Problem 7. A 4-sided die is rolled twice, and X is the maximum of the two values (or the common value if the they are the same). The outcome space for the two rolls is given by

$$S = \{(a, b) \mid a, b = 1, 2, 3, 4\}$$

and each of the 16 pairs of numbers is equally likely to occur. The random variable X (the max) can take values 1,2,3,4. Find the associated probabilities, the expected value of X, and the standard deviation.

Problem 8. A pair of six-sided dice is tossed, and X is the sum of the two numbers. Find the probability distribution of X, the expected value, and the standard deviation.

Problem 9. A bag contains 7 Red marbles and 11 White marbles. You select 5 marbles at random (without replacement), and X is the random variable counting the number of Red marbles you observe. Find the distribution of X, the expected value, and the standard deviation.

Problem 10. A bag contains N_1 Red marbles and N_2 White marbles, so a total of $N = N_1 + N_2$. You select n of these at random (without replacement). What is the probability that you end up with exactly k Red marbles? That is, find a general formula (based on the previous problem) for $P(X = k)$. Note that k cannot be larger than N_1, and $n - k$ cannot be larger than N_2. The distribution of X has a special name: it is the **hypergeometric distribution**.

Problem 11. A chip is selected at random from a bowl of 6 White, 3 Red, and 1 Blue. Let the random variable $X = 1$ if the outcome is a White chip, $X = 5$ if the outcome is Red, and $X = 10$ if the outcome is Blue. Find the distribution of X, and the expected value.

Problem 12. Five cards are drawn at random from a standard deck of cards. Let X be the number of face cards in the hand (kings, queens, jacks). What is the distribution of X?

Problem 13. Find the expected value and the standard deviation of the following probability distribution:

X	4	7	15	28
P(X)	0.3	0.2	0.1	0.4

Problem 14. A game consists of tossing a **fair coin** twice. A player tho throws the same face (2 Heads or 2 Tails) on both tosses wins \$15.

a) How much should the house charge for the game if they want to break even?
b) How much should the house charge in order to make an average profit of \$4 per game?

Problem 15. A game consists of tossing an **unfair coin** twice, with $P(H) = 0.3$. A player tho throws the same face (2 Heads or 2 Tails) on both tosses wins \$15.

a) How much should the house charge for the game if they want to break even?
b) How much should the house charge in order to make an average profit of \$4 per game?

Problem 16. A crate contains 20 apples, 5 of which are bad. If you select 3 apples at random, let X be the number of good apples you select. Find the probability distribution of X, and calculate its expected value.

Problem 17. A crate contains an unknown (but finite) number of apples, some of which may be bad. If you select 5 apples at random, let X be the number of good apples you select, and Y the number of bad apples you select. What is $E(X + Y)$?

Problem 18. Professor Dracula sends his grader to the supply room to get a solution manual. The grader does not know which of the seven keys unlock the room, so he tries them one at a time. Find the expected number of attempts it takes to unlock the supply room.

Problem 19. Professor Dracula gives a multiple choice exam. Each question has 4 possible answers. The exam is graded by giving 5 points for each correct answer, and subtracting 1 point for each incorrect answer. A student who hasn't studies randomly guesses at each question (no blanks).

a) What is the expected point value for a single question?
b) If the exam has 20 questions, what is the expected number of points (out of a maximum of 100)?
c) How would your answers change if a correct answer earns 5 points, an incorrect answer subtracts 1 point, but leaving a question blank has no penalty?

3. Binomial Distribution

Problem 20. Compute the probability distribution for the following binomial random variables. For each, also find the mean and the variance.

a) $n = 3, p = 0.5$
b) $n = 3, p = 0.1$
c) $n = 5, p = 0.7$

Problem 21. An unfair coin with bias $P(H) = 0.3$ is tossed 100 times. Let X be the number of Heads you observe, and Y be the number of Tails you observe out of the 100 trials. Find $E(X)$ and $E(Y)$, as well as $Var(X)$ and $Var(Y)$.

Problem 22. A 4-sided die is tossed 10 times. What is the probability of observing four 2's? Calculate the expected number of sixes you will see. Repeat your calculations, but this time using a 6-sided die.

Problem 23. A number is drawn at random from $S = \{1, 2, 3, \ldots, 10\}$.

a) What is the probability that the number is (strictly) less than 3?
b) If you draw 20 numbers at random from S (independently, so repetitions are allowed), how many times do you expect to get a number less than 3?
c) What is the probability that at most 4 of the 20 numbers are less than 3?
d) What is the probability that at least 5 of the 20 numbers are less than 3?

Problem 24. For the sample space $S = \{1, 2, 3\}$, the probabilities are given below:

x	1	2	3
P(x)	0.2	0.3	0.5

A random experiment is devised as follows. Let X and Y be two independently chosen numbers from S, and let the random variable Z be their sum (so $Z = X + Y$).

a) What are the possible values for Z?
b) Find the probability distribution of Z.
c) Find the expected value $\mu_Z = E(Z)$.
d) What is the probability that Z is strictly less than its mean? In other words, calculate $P(Z < \mu_Z)$.
e) If you repeat the experiemnt for 10 independent trials (i.e. you observe Z 10 times and record its value), how many times on average will your observation be less than μ_Z?
f) What is the probability that, out of the 10 trials, exactly 3 times out of 10 Z will be below its mean?