Name: Solutions Date: 02/08/2018

M20580 L.A. and D.E. Tutorial
Quiz 3

1. Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be a linear transformation given by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} 2018x_2 \\ -x_1 \end{bmatrix},$$

Find the standard matrix for T, i.e. find a matrix A such that $T(\mathbf{x}) = A\mathbf{x}$.

$$A = \begin{bmatrix} \underset{\substack{\uparrow \\ 1^{st}\text{ column}}}{T(\vec{e_1})} & \underset{\substack{\uparrow \\ 2^{nd}\text{ column}}}{T(\vec{e_2})} \end{bmatrix} \quad \text{where } \vec{e_1} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \ \vec{e_2} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$T(\vec{e_1}) = T\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 0 \\ -1 \end{bmatrix} \quad \text{and} \quad T(\vec{e_2}) = T\left(\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 2018 \\ 0 \end{bmatrix}$$

Thus, the standard matrix for T is $\boxed{A = \begin{bmatrix} 0 & 2018 \\ -1 & 0 \end{bmatrix}}$

2. Find the inverse of the matrix

$$B = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}.$$

$$\left[\underset{B}{\begin{matrix} 0 & 1 \\ 1 & 0 \end{matrix}} \ \middle| \ \underset{I}{\begin{matrix} 1 & 0 \\ 0 & 1 \end{matrix}}\right] \xrightarrow{R_1 \leftrightarrow R_2} \left[\underset{I}{\begin{matrix} 1 & 0 \\ 0 & 1 \end{matrix}} \ \middle| \ \underset{B^{-1}}{\begin{matrix} 0 & 1 \\ 1 & 0 \end{matrix}}\right]$$

Thus, $B^{-1} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

Check: $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$ ✓