Quiz 9, Solutions

1. **Q1** One of the statements below holds for the series $\displaystyle\sum_{n=1}^{\infty} \frac{\sin(n)+\cos(n)}{n^2+1}$. Which one?

   (a) This series is absolutely convergent by Comparison Test.

   (b) This series is conditionally convergent.

   (c) This series converges by Alternating Series Test.

   (d) This series diverges by Ratio Test.

   (e) This series diverges because $\lim_{n\to\infty} \frac{\sin(n)+\cos(n)}{n^2+1}$ is not 0.

   Solution: We have $\left|\dfrac{\sin(n)+\cos(n)}{n^2+1}\right| \le \dfrac{2}{n^2}$ since $0 \le |\sin(n)+\cos(n)| \le 2$ and $|n^2+1| \ge n^2$. $\displaystyle\sum_{n=1}^{\infty} \frac{2}{n^2}$ converges by the p-series test with $p=2$. Thus, by the Comparison Test, $\displaystyle\sum_{n=1}^{\infty} \left|\frac{\sin(n)+\cos(n)}{n^2+1}\right|$ converges, and so $\displaystyle\sum_{n=1}^{\infty} \frac{\sin(n)+\cos(n)}{n^2+1}$ is *absolutely convergent.*

   Additional Questions: Why must we take the absolute value of the terms? Does absolute convergence imply convergence? Why do the Alternating Series Test, the Ratio Test, and the Divergence Test not apply?

2. **Q2** Compute the radius of convergence of the power series $\sum_{n=1}^{\infty} 3^n(x-1)^n$.
   Solution: By the ratio test, the radius of convergence of a power series $\sum_{n=1}^{\infty} c_n(x-a)^n$ is given by

   $$R = \lim_{n\to\infty} \frac{|c_n|}{|c_{n+1}|}.$$

   So, in our case,

   $$R = \lim_{n\to\infty} \frac{3^n}{3^{n+1}} = \frac{1}{3}.$$