Name: Date: 02/01/2018

M20580 L.A. and D.E. Tutorial

Quiz 2

1. Describe all solutions of $A\mathbf{x} = \mathbf{b}$ in parametric form, where

$$A = \begin{bmatrix} 1 & 0 & 3 & 1 \\ 2 & 0 & 6 & 2 \\ 1 & 0 & 0 & 1 \\ 2 & 0 & 0 & 2 \end{bmatrix}, \qquad \mathbf{b} = \begin{bmatrix} 3 \\ 6 \\ 2 \\ 4 \end{bmatrix}$$

Solution:

Use the augmented matrix

$$\begin{bmatrix} 1 & 0 & 3 & 1 & 3 \\ 2 & 0 & 6 & 2 & 6 \\ 1 & 0 & 0 & 1 & 2 \\ 2 & 0 & 0 & 2 & 4 \end{bmatrix} R2 = R2 - 2R1$$

$$\begin{bmatrix} 1 & 0 & 3 & 1 & 3 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 2 \\ 2 & 0 & 0 & 2 & 4 \end{bmatrix} R4 = R4 - 2R3$$

$$\begin{bmatrix} 1 & 0 & 3 & 1 & 3 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \text{Swap R2 and R3}$$

$$\begin{bmatrix} 1 & 0 & 3 & 1 & 3 \\ 1 & 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} R2 = R2 - R1$$

$$\begin{bmatrix} 1 & 0 & 3 & 1 & 3 \\ 0 & 0 & -3 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} R2 = R2/-3 \text{ and } R1 = R1 + R2$$

$$\begin{bmatrix} 1 & 0 & 0 & 1 & 2 \\ 0 & 0 & 1 & 0 & 1/3 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$Thus$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ 1/3 \\ 0 \end{bmatrix} + s \begin{bmatrix} -1 \\ 0 \\ 0 \\ 1 \end{bmatrix} + t \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

2. Do the columns of B span all of $\mathbb{R}^3$? (You must show work to get full credit)

$$B = \begin{bmatrix} 1 & 0 & 2 \\ 3 & -3 & 3 \\ 1 & 2 & 4 \end{bmatrix}$$

Solution:

$$\begin{bmatrix} 1 & 0 & 2 \\ 3 & -3 & 3 \\ 1 & 2 & 4 \end{bmatrix} R2 = R2 - 3R1$$

$$\begin{bmatrix} 1 & 0 & 2 \\ 0 & -3 & -3 \\ 1 & 2 & 4 \end{bmatrix} R3 = R3 - R1$$

$$\begin{bmatrix} 1 & 0 & 2 \\ 0 & -3 & -3 \\ 0 & 2 & 2 \end{bmatrix} R2 = R2/-3 \text{ and } R3 = R3/2$$

$$\begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix} R3 = R3 - R2$$

$$\begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

There is a free variable thus the last column is a linear combination of the first two. Thus the columns of B can't span all of $\mathbb{R}^3$.