## WORKSHEET 6

## ADRIAN PĂCURAR

**Problem 1.** Consider the function

$$f(x) = \begin{cases} ax^2 + bx & \text{if } x < 1\\ \cos(x - 1) & \text{if } x \ge 1 \end{cases}$$

(a) Which values for a and b which make f continuous but not differentiable. Using these values explain why f is continuous on any interval.

(b) Find values for a and b which make f continuous and differentiable. Using these values explain why f is differentiable on any interval.

(c) Is it possible to find a and b for which f is differentiable but not continuous? If so, find such a and b. Explain.

**Problem 2.** Compute the derivatives of the following functions:

(a) 
$$f(x) = (3 + 4x - x^2)^{1/2}$$

(b) 
$$f(x) = \left(\frac{x}{1+x}\right)^5$$

(c) 
$$f(x) = 2x^2\sqrt{2-x}$$

(d) 
$$f(x) = (x^2 + 3)^4 (2x^3 - 5)^3$$

**Problem 3.** Compute the derivative of:

$$f(x) = \cos(x\sin(x\tan x))$$

**Problem 4.** Use implicit differentiation to find the following:

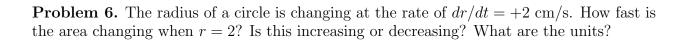
(a) 
$$dy/dx$$
 for  $2y^2 = y + x$ 

(b) 
$$dx/dy$$
 for  $2y^2 = y + x$ 

(c) 
$$y'(x)$$
 for  $x^2 - xy + y^2 = 3$ 

(d) 
$$x'(y)$$
 for  $x^2 - xy + y^2 = 3$ 

**Problem 5.** Given that  $y(t) = t^3$  and  $x(t) = \sin t$ , differentiate the equation  $x^3y + xy^3 = t^2$  with respect to t. Don't simplify.



**Problem 7.** A spherical balloon is filling up with air at the rate of 5 cm<sup>3</sup>/s. How fast is the radius of the balloon changing when r=2? Is this increasing or decreasing? Use the fact that the volume of a sphere in terms of its radius r is given by  $V=\frac{4}{3}\pi r^3$ .

**Problem 8.** The position of a particle is given by  $s(t) = \frac{1}{5}x^5 - 2x^2 + x$ , where t is measured in seconds. Is there ever a time during the first second where the particle is at rest?

**Problem 9.** The position of a particle is given by  $s(t) = -\frac{\cos(\pi t)}{2\pi}$ . For what (non-negative) values of t is the particle at rest?