Name: Date: 02/22/2018

M20580 L.A. and D.E. Tutorial
Worksheet 5
Sections 3.1–3.3

1. Let A be an invertible matrix. Using properties of determinants, show that

$$\det(A^{-1}) = \frac{1}{\det(A)}$$

2. Find the determinant of the matrix:

$$A = \begin{bmatrix} 5 & -7 & 2 & 2 \\ 0 & 3 & 0 & -4 \\ -5 & -8 & 0 & -4 \\ 0 & 5 & 0 & -6 \end{bmatrix}$$

3. Let A and B be 4×4 matrices, with $\det(A) = 5$ and $\det(B) = -1$. Compute:

   (a) $\det(AB)$

   (b) $\det(5A)$

   (c) $\det(A^T BA)$

   (d) $\det(B^5)$

   (e) $\det(B^{-1}A)$

4. Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be a linear transformation given by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} 2 & 3 \\ 4 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}.$$

   Caclulate the area of the image of the parallelogram spanned by

$$b_1 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}, b_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

   under the linear transformation T.

5. Use Cramer's rule to compute the solutions of the following systems

$$\begin{aligned} x_1 + x_2 &= 3 \\ -3x_1 + 2x_3 &= 0 \\ x_2 - 2x_3 &= 2 \end{aligned}$$