10350 Tutorial Week 06 - Set 01 Name ______________________

Section ______________

1. A differentiable function $g(x)$ is such that

$$g(2) = -2, \qquad g'(2) = 3, \qquad g(3) = 10 \qquad \text{and} \qquad g'(3) = -4$$

1a. If $C(x) = [g(x)]^4$ find $C'(2) \stackrel{?}{=}$ $\underline{\quad -96 \quad}$

$$\begin{aligned} C'(x) &= 4 \cdot [g(x)]^3 \cdot g'(x) \\ C'(2) &= 4 \cdot [g(2)]^3 \cdot g'(2) \\ &= 4 \cdot (-2)^3 \cdot 3 \\ &= -96 \end{aligned}$$

1b. If $P(x) = x \cdot e^{g(x)}$ find $P'(3) \stackrel{?}{=}$ $\underline{\quad -11e^{10} \quad}$

$$\begin{aligned} P'(x) &= e^{g(x)} + x \cdot e^{g(x)} \cdot g'(x) \\ P'(3) &= e^{g(3)} + 3 \cdot e^{g(3)} \cdot g'(3) \\ &= e^{10} + 3 \cdot e^{10} \cdot (-4) \\ &= -11e^{10} \end{aligned}$$

1c. If $Q(x) = \dfrac{3}{(g(x)+1)^4}$ find $Q'(2) \stackrel{?}{=}$ $\underline{\quad 36 \quad}$

$$\begin{aligned} Q'(x) &= \left(\frac{3}{(g(x)+1)^4}\right)' \\ &= (3 \cdot (g(x)+1)^{-4})' \\ &= 3 \cdot (-4) \cdot (g(x)+1)^{-5} \cdot g'(x) \\ &= 3 \cdot (-4) \cdot \tfrac{g'(x)}{(g(x)+1)^5} \\ Q'(2) &= 3 \cdot (-4) \cdot \tfrac{g'(2)}{(g(2)+1)^5} \\ &= -12 \cdot \tfrac{3}{(-2+1)^5} = 36 \end{aligned}$$

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2. Assuming the limit $\lim_{x\to 0}\frac{\sin x}{x}=1$, find the values of the following limits showing your steps VERY CLEARLY.

2a. $\lim_{x\to 0}\frac{\sin(5x)}{7x} =$

$$\begin{aligned}\lim_{x\to 0}\frac{\sin(5x)}{7x} &= \lim_{x\to 0}\frac{\sin(5x)}{x}\cdot\frac{1}{7} = \lim_{x\to 0}\frac{\sin(5x)}{5x}\cdot\frac{5}{1}\cdot\frac{1}{7}\\ &= \lim_{x\to 0}\frac{\sin(5x)}{5x}\cdot\frac{5}{7} = \frac{5}{7}\end{aligned}$$

2b. $\lim_{x\to 0}\frac{\sin(2x)}{\sin(3x)} =$

$$\begin{aligned}\lim_{x\to 0}\frac{\sin(2x)}{\sin(3x)} &= \lim_{x\to 0}\frac{\sin(2x)}{1}\cdot\frac{1}{\sin(3x)} = \lim_{x\to 0}\frac{\sin(2x)}{2x}\cdot 2x\cdot\frac{3x}{\sin(3x)}\cdot\frac{1}{3x}\\ &= \lim_{x\to 0}\frac{\sin(2x)}{2x}\cdot\frac{3x}{\sin(3x)}\cdot\frac{2x}{3x} = 1\cdot 1\cdot\frac{2}{3} = \frac{2}{3}\end{aligned}$$

2c. $\lim_{x\to 0}\frac{\tan(6x)}{\tan x} =$

$$\begin{aligned}\lim_{x\to 0}\frac{\tan(6x)}{\tan x} &= \lim_{x\to 0}\frac{\sin(6x)}{\cos(6x)}\cdot\frac{\cos x}{\sin x} = \lim_{x\to 0}\frac{\sin(6x)}{1}\cdot\frac{1}{\sin x}\cdot\frac{\cos x}{\cos(6x)}\\ &= \lim_{x\to 0}\frac{\sin(6x)}{6x}\cdot 6x\cdot\frac{x}{\sin x}\cdot\frac{1}{x}\cdot\frac{\cos x}{\cos(6x)} = \lim_{x\to 0}\frac{\sin(6x)}{6x}\cdot\frac{6x}{x}\cdot\frac{x}{\sin x}\cdot\frac{\cos x}{\cos(6x)}\\ &= 1\cdot 6\cdot 1\cdot 1 = 6\end{aligned}$$

2d. $\lim_{x\to 0}\frac{x^2}{\sin 5x} =$

$$\begin{aligned}\frac{x^2}{\sin 5x} &= \lim_{x\to 0} x\cdot\frac{x}{\sin 5x} = \lim_{x\to 0} x\cdot\frac{5x}{\sin 5x}\cdot\frac{1}{5}\\ &= 0\cdot 1\cdot\frac{1}{5} = 0\end{aligned}$$

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3. Consider the function

$$f(x) = \begin{cases} \dfrac{\sin(x-1)}{(x-1)} + 2 & x \neq 1 \\ \\ -1 & x = 1 \end{cases}$$

3a. Using limits describe the kind of discontinuity at $x = 1$.

$$\lim_{x\to 1} f(x) = \lim_{x\to 1} \frac{\sin(x-1)}{(x-1)} + 2$$

$$\stackrel{z\,=\,x\,-\,1}{=} \lim_{z\to 0} \frac{\sin z}{z} + 2$$

$$= 3$$

So, we have

$$\lim_{x\to 1} f(x) = 3, \qquad f(1) = -1$$

Thus, removable discontinuity.

3b. Is it possible to redefine $f(1)$ so that $f(x)$ is continuous for all x?

Yes, redefine $f(1) = 3$.

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Find the derivatives of the following functions:

4. $f(x) = (2x^2 + \pi)^4$

$$\begin{aligned} f'(x) &= 4(2x^2+\pi)^3 \cdot 4x \\ &= 16x(2x^2+\pi)^3 \end{aligned}$$

5. $g(x) = e^{x^2+2x}$

$$\begin{aligned} g'(x) &= e^{x^2+2x} \cdot (2x+2) \\ &= (2x+2)e^{x^2+2x} \end{aligned}$$

6. $h(x) = x\cos(2x)$

$$\begin{aligned} h'(x) &= \cos(2x) + x \cdot (-\sin(2x)) \cdot 2 \\ &= \cos(2x) - 2x\sin(2x) \end{aligned}$$

7. $y = \dfrac{e^{2x}-1}{e^{2x}+1}$. Simplify the expression you get.

$$\begin{aligned} y' &= \frac{2 \cdot e^{2x} \cdot (e^{2x}+1) - (e^{2x}-1) \cdot 2 \cdot e^{2x}}{(e^{2x}+1)^2} \\ &= \frac{2e^{4x} + 2e^{2x} - 2e^{4x} + 2e^{2x}}{(e^{2x}+1)^2} \\ &= \frac{4e^{2x}}{(e^{2x}+1)^2} \end{aligned}$$