WORKSHEET 3

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Problem 1. Compute the following limits:

(a) $\lim_{x\to 4} \dfrac{x-4}{x^2-8x+3}$

(b) $\lim_{x\to 3^-} \dfrac{x-4}{x^2-9}$

(c) $\lim_{x\to 3^-} \dfrac{\sqrt{5x}(x-3)}{|x-3|}$

Problem 2.

(a) For which values of x is the following function continuous? (Justify your answer)

$$f(x) = \frac{|\cos x| + \sqrt{x-2}}{(x^2-9)(x^2+4)}$$

(b) Find a value c that makes the following function continuous everywhere:

$$f(x) = \begin{cases} \frac{\sin(x)\cos(x)}{x} & \text{if } x \neq \frac{\pi}{4} \\ c & \text{if } x = \frac{\pi}{4} \end{cases}$$

(c) Suppose we have a continuous function $f(x)$ that satisfies $f(-1) = -1$ and $f(1) = 1$. Can this function have two zeroes inside the interval $(-1, 1)$? Justify. What can you say about the number of zeroes such a function can have inside $(-1, 1)$?

Problem 3. Suppose that $\lim_{x\to 1} f(x) = 7$, $\lim_{x\to 1} g(x) = 4$, and $\lim_{x\to 1} h(x) = -\infty$. Compute the limit

$$\lim_{x\to 1} \left(f(x) + \frac{1}{g(x) - h(x)} \right)$$

Problem 4. Suppose that $\dfrac{\sqrt{x^2+9} - 3}{2x^2} \leq f(x) \leq \frac{1}{12}$ for all $x \neq 0$. Compute $\lim_{x\to 0} f(x)$.

Problem 5. Argue without solving for x that there are at least two solutions to the equation $-x^4 + 3x + 2 = 0$. (Hint: use continuity of polynomials and IVT).

Problem 6. Compute the following limits:

(a) $\lim_{x\to\pi^+} \dfrac{\sqrt[3]{\cos x}}{x-\pi}$

(b) $\lim_{x\to\infty} \cos\left(\dfrac{1}{x}\right)$

(c) $\lim_{x\to 0} x^2 \cos\left(\frac{1}{x^3-x}\right)$