Name: Date: 11/9/2017

M20550 Calculus III Tutorial
Worksheet 8

1. Compute $\iint_R \frac{1}{2}\, dA$ where R is the region bounded by $2x^2 + 2xy + y^2 = 8$ using the change of variables given by $x = u + v$ and $y = -2v$.

2. Let R be the parallelogram enclosed by the lines $x + 3y = 0$, $x + 3y = 2$, $x + y = 1$, and $x + y = 4$. Evaluate the following integral by making appropriate change of variables
$$\iint\limits_R \frac{x + 3y}{(x + y)^2}\, dA.$$

3. Evaluate the line integral $\int_C (z - 2xy)\, ds$ along the curve C given by $\mathbf{r}(t) = \langle \sin t, \cos t, t\rangle$, $0 \leq t \leq \frac{\pi}{2}$.

4. Find $\int_C 2xy^3\, ds$ where C is the upper half of the circle $x^2 + y^2 = 4$.

5. Calculate the line integral $\int_C (y^2 + x)\, dx + 4xy\, dy$ where C is the arc of $x = y^2$ from $(1, 1)$ to $(4, 2)$.

6. Compute $\int_C x^2\, ds$ where C is the intersection of the surface $x^2 + y^2 + z^2 = 4$ and the plane $z = \sqrt{3}$.

7. Determine whether or not the following vector fields are conservative:
   (a) $\mathbf{F} = (3 + 2xy)\,\mathbf{i} + (x^2 - 3y^2)\,\mathbf{j}$
   (b) $\mathbf{F} = \mathbf{i} + \sin z\ \mathbf{j} + y\cos z\ \mathbf{k}$

8. Compute $\oint_C \mathbf{F} \cdot d\mathbf{r}$ where $\mathbf{F} = \langle y + yz\sin xy, y + xz\sin xy, y - \cos xy\rangle$ and C is given as the path traced out by $\mathbf{r}(t) = \langle 0, 4\sin t, 3\cos t + 2\rangle$ from $t = 0$ to 4π, i.e. a circle traced around twice.