

M20580 L.A. and D.E. Tutorial
Worksheet 3
Sections 1.5, 1.7–1.9

1. Determine if the system has a non-trivial solution. If yes, describe all solutions in parametric form.

$$\begin{array}{lll} (a) & x_1 - 2x_2 + x_3 = 0 & (b) \quad x_1 - 2x_2 = 0 \\ & 2x_1 + 4x_2 + x_3 = 0 & 2x_1 + 4x_2 + x_3 = 0 \\ & 3x_1 + 2x_2 + x_3 = 0 & 3x_1 + 2x_2 + x_3 = 0 \end{array} \quad (c) \quad x_1 + x_2 + x_3 = 0$$

2. Describe all solutions of $A\mathbf{x} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$, where $A = \begin{bmatrix} 1 & -2 & 0 \\ 2 & 4 & 1 \\ 3 & 2 & 1 \end{bmatrix}$.

3. Let $A = \begin{bmatrix} 1 & 3 & 0 & 3 \\ -1 & -1 & -1 & 1 \\ 0 & -4 & 2 & -8 \\ 2 & 0 & 3 & -1 \end{bmatrix}$.

- (a) How many rows of A contain a pivot position? Does the equation $A\mathbf{x}=\mathbf{b}$ have a solution for each \mathbf{b} in \mathbb{R}^4 ?

- (b) Do the columns of A span \mathbb{R}^4 ?

4. Determine if the vectors are linearly independent.

$$(a) \quad \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad \begin{bmatrix} 0 \\ 2 \\ 0 \\ 0 \end{bmatrix}, \quad \begin{bmatrix} 0 \\ 0 \\ 3 \\ 0 \end{bmatrix}, \quad \begin{bmatrix} 0 \\ 0 \\ 0 \\ 4 \end{bmatrix}$$

$$(b) \quad \begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix}, \quad \begin{bmatrix} 2 \\ 7 \\ 5 \end{bmatrix}, \quad \begin{bmatrix} 3 \\ 10 \\ 7 \end{bmatrix}$$