## Quiz 8, Solutions

(1) The series

$$\sum_{n=2}^{\infty} \frac{(-1)^{n+1}}{\sqrt{n}}$$

(a) diverges because it is a p-series with $p < 1$.
(b) diverges because $\lim_{n\to\infty} \dfrac{(-1)^{n+1}}{\sqrt{n}} \neq 0$.
(c) converges by the alternating series test.
(d) converges because it is a p-series with $p < 1$.
(e) diverges because the terms alternate.

**Sol:** Note that the given series is an alternating series. Hence we will try to use the alternating series test. For that let $b_n = \dfrac{1}{\sqrt{n}}$. Then we see that $b_n$ is positive and decreasing. Moreover,

$$\lim_{n\to\infty} \frac{1}{\sqrt{n}} = 0.$$

Hence, by the alternating series test the series $\sum_{n=2}^{\infty} \dfrac{(-1)^{n+1}}{\sqrt{n}}$ is convergent.

(2) Use Comparison Tests to determine which **one** of the following series is divergent.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{n}{n+1}\left(\frac{1}{2}\right)^n$
(b) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^{\frac{3}{2}}+1}$
(c) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2+8}$
(d) $\displaystyle\sum_{n=1}^{\infty} \frac{n^2-1}{n^3+100}$
(e) $\displaystyle\sum_{n=1}^{\infty} 7\left(\frac{5}{6}\right)^n$

**Sol:**

(a) Since $\displaystyle\sum_{n=1}^{\infty} \frac{n}{n+1}\left(\frac{1}{2}\right)^n < \sum_{n=1}^{\infty}\left(\frac{1}{2}\right)^n$ and the geometric series $\displaystyle\sum_{n=1}^{\infty}\left(\frac{1}{2}\right)^n$ converges,by the comparison test we know $\displaystyle\sum_{n=1}^{\infty} \frac{n}{n+1}\left(\frac{1}{2}\right)^n$ also converges.

(b) Since $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^{\frac{3}{2}}+1} < \sum_{n=1}^{\infty} \frac{1}{n^{\frac{3}{2}}}$ and by $p$ series test we know the right hand side coverges, the comparison test tells us that $\sum_{n=1}^{\infty} \dfrac{1}{n^{\frac{3}{2}}+1}$ also converges.

(c) Since $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2+8} < \sum_{n=1}^{\infty} \frac{1}{n^2}$ and by $p$ series test we know the right hand side coverges, the comparison test tells us that $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2+8}$ also converges.

(d) Since

$$\lim_{n\to\infty} \frac{\frac{n^2-1}{n^3+100}}{1/n} = \lim_{n\to\infty} \frac{n^3-n}{n^3+100} = 1$$

and $\sum_{n=1}^{\infty} \dfrac{1}{n}$ diverges, by the limit comparison test we know $\displaystyle\sum_{n=1}^{\infty} \frac{n^2-1}{n^3+100}$ diverges.

(e) It is a constant times a geometry series, hence it converges.