Sect 12.2 – Vectors

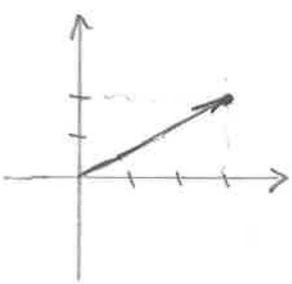
Def: A vector in $\mathbb{R}^2$ is an ordered pair of real #s

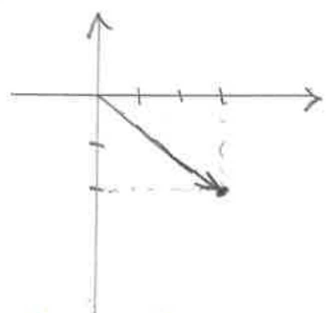
$$\vec{a} = \langle \underbrace{a_x}_{\text{x comp.}} , \underbrace{a_y}_{\text{y-comp.}} \rangle , \qquad a_x, a_y \in \mathbb{R}$$

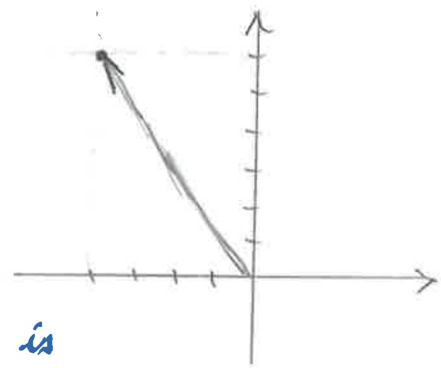
- used to represent quantities that have both a magnitude and a direction : - wind currents, - velocity, - displacement, - acceleration / force
- for 3-dim $\vec{b} = \langle b_x , b_y , b_z \rangle$
- can be extended to any # of dimensions (even infinite)

Def: A scalar is a real # (magnitude, no direction)

Ex: $\vec{u} = \langle 3, 2 \rangle$ $\qquad$ $\vec{v} = \langle 3, -2 \rangle$ $\qquad$ $\vec{w} = \langle -4, 6 \rangle$





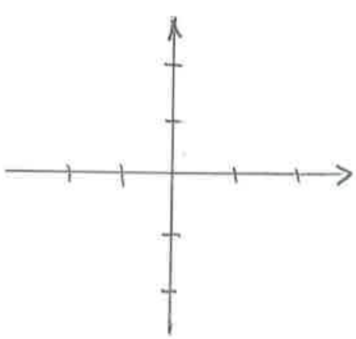


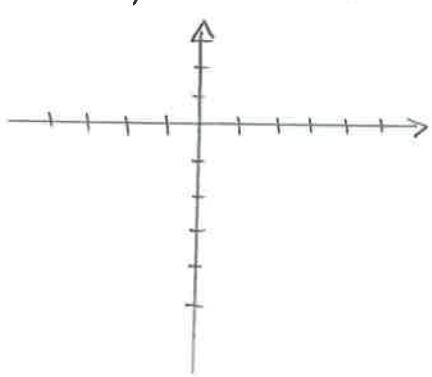
Def: The magnitude / length of a vector is

$$|\vec{a}| = \sqrt{a_x^2 + a_y^2 + a_z^2}$$

Ex: $|\vec{u}| = |\vec{v}| = \sqrt{3^2 + 2^2} = \sqrt{13}$, and $|\vec{w}| = \sqrt{(-4)^2 + 6^2} = \sqrt{52}$

$|\langle -1, 2 \rangle| = \ldots\ldots$ $\qquad$ $|\langle 3, -5 \rangle| = \ldots$





Operations with vectors

$\vec{a} = \langle a_x, a_y, a_z \rangle \qquad \vec{b} = \langle b_x, b_y, b_z \rangle \qquad \vec{c} = \langle c_x, c_y, c_z \rangle$

(1) Vector addition

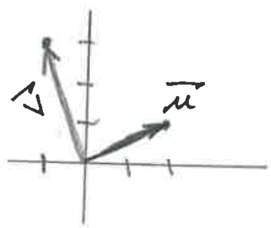
$\vec{a} + \vec{b} = \langle a_x + b_x, a_y + b_y, a_z + b_z \rangle$ (we add the x-comp, y-comp, etc separately)

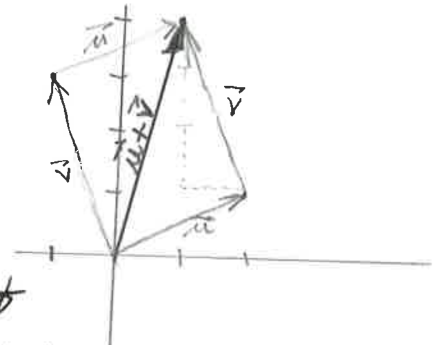
<u>Properties</u>: $\vec{a} + \vec{b} = \vec{b} + \vec{a}$ (commutative)

$(\vec{a} + \vec{b}) + \vec{c} = \vec{a} + (\vec{b} + \vec{c})$ (associative)

<u>Ex</u> $\vec{u} = \langle 2, 1 \rangle \qquad \vec{v} = \langle -1, 3 \rangle$

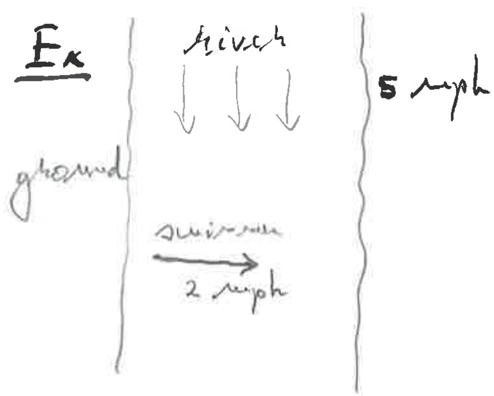
$\vec{u} + \vec{v} =$ . . . .





- 2 or more forces acting on an object can be replaced by a single force that is their sum.

<u>Ex</u>



Q: What is the velocity vector of the swimmer relative to the ground?

$\vec{v} = \langle 0, -5 \rangle + \langle 2, 0 \rangle = \boxed{\langle 2, -5 \rangle}$

The <u>speed</u> (rel to ground) is

$|\vec{v}| = \sqrt{4 + 25} = \sqrt{29} \approx \boxed{5.4 \text{ mph}}$

Rel to water, the speed is only $\boxed{2 \text{ mph}}$

(2) Scalar multiplication

$k\vec{v} = k \cdot \langle v_x, v_y \rangle = \langle k \cdot v_x, k \cdot v_y \rangle$ (mult. each comp. by the scalar k)

<u>Properties</u>

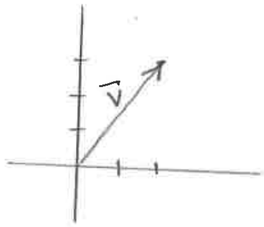
$a \cdot (\vec{v} + \vec{w}) = a\vec{v} + a\vec{w}$

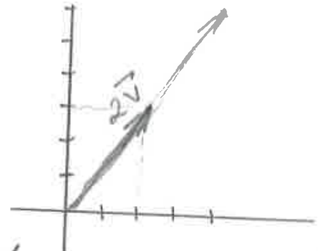
$(a + b)\vec{v} = a\vec{v} + b\vec{v}$ } distributivity

$(ab)\vec{v} = a(b\vec{v}) = b(a\vec{v})$

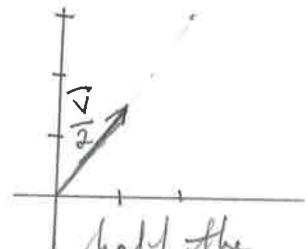
- mult. by a scalar changes the <u>length</u>
- if the scalar is negative, direction is flipped.

Ex $\vec{v} = \langle 2, 3 \rangle$ $\quad 2\vec{v} = \langle 4, 6 \rangle$ $\quad \frac{\vec{v}}{2} = \langle 1, \frac{3}{2} \rangle$ $\quad -\vec{v} = \langle -2, -3 \rangle$

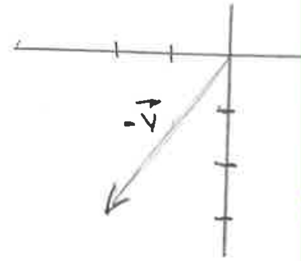




(double the length)



half the length



Ex: For $\vec{v} = \langle 2, 3 \rangle$, find a vector of the same dir, len 1.

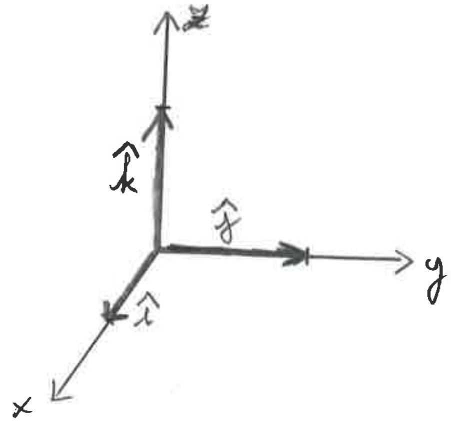
$|\vec{v}| = \sqrt{4+9} = \sqrt{13}$, so we want $\frac{1}{\sqrt{13}} \vec{v} = \left\langle \frac{2}{\sqrt{13}}, \frac{3}{\sqrt{13}} \right\rangle$

Unit Vectors

- vectors with <u>length 1</u> $\qquad \frac{\vec{v}}{|\vec{v}|} = \left\langle \frac{v_x}{|\vec{v}|}, \frac{v_y}{|\vec{v}|}, \frac{v_z}{|\vec{v}|} \right\rangle$
- if they point in the direction of the x, y, z-axis, they get special names: $\hat{\imath} = \langle 1, 0, 0 \rangle$

$$\hat{\jmath} = \langle 0, 1, 0 \rangle$$

$$\hat{k} = \langle 0, 0, 1 \rangle$$



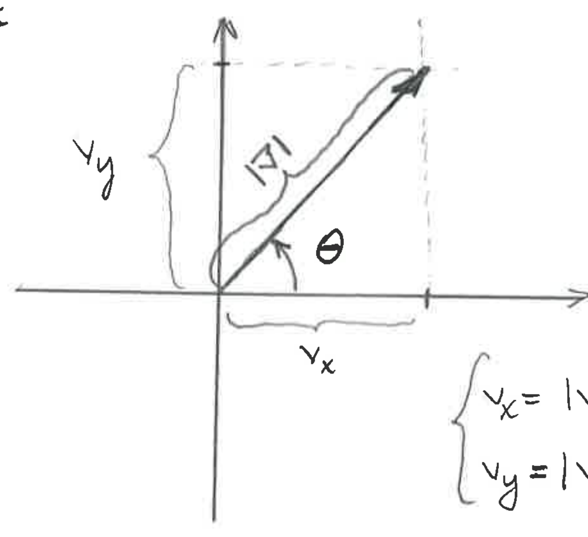
- called the <u>standard basis</u> (for $\mathbb{R}^3$)
- we can decompose any vector $\vec{v}$ as follows:

$$\vec{v} = \langle v_x, v_y, v_z \rangle = v_x \cdot \hat{\imath} + v_y \cdot \hat{\jmath} + v_z \cdot \hat{k}$$

Ex $\vec{a} = \langle 1, 2, -3 \rangle = \hat{\imath} + 2\hat{\jmath} - 3\hat{k}$

Polar Representation

$\vec{v} = \langle v_x, v_y \rangle$ can be represented by specifying its <u>length</u>, ~~ang~~ and the angle θ it makes with the positive x-axis.



$$\begin{cases} v_x = |v| \cos\theta \\ v_y = |v| \sin\theta \end{cases}$$