Name: Date: 04/05/2018

M20580 L.A. and D.E. Tutorial
Worksheet 9
Sections 6.5, 1.1, 1.2

1. Let

$$A = \begin{bmatrix} 2 & 3 \\ 2 & 4 \\ 1 & 1 \end{bmatrix}.$$

   (a) Use Gram-Schmidt process to find a orthogonal basis for Col A, and use the orthogonal basis you get to find the QR factorization of A.

(b) Use the QR factorization you found in part(a) to find the least-squares solution of $A\mathbf{x} = \mathbf{b}$, where $b = (7, 3, 1)$ (as a column vector).

2. Solve the initial value problem

$$\frac{dA}{dt} = 0.05A + 15, \quad A(0) = 0.$$

3. The partial differential equation

$$\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = 0$$

is called *Laplace's equation*, and any solution to this equation is called a *harmonic function*. Determine whether the given function is harmonic.

(a) $f(x, y) = x^2 + y^2$.

(b) $f(x, y) = x^2 - y^2$.

(c) (optional) $f(x, y) = e^x \cos y$ and $g(x, y) = e^x \sin y$.

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4. Consider the differential equation $\dfrac{dy}{dx} = y(y-1)(y-2)$, and $\phi(x)$ a solution for various initial conditions.

   (a) Give a rough sketch of the direction field.

   (b) Without using any integrals, find the general solution $\phi(x)$ for the initial condition $\phi(0) = 1$.

   (c) Compute $\lim_{x\to\infty} \phi(x)$ for any solution $\phi(x)$ satisfying the given initial condtion:

   - $\phi(-1) = 0.5$
   - $\phi(1) = 1.5$
   - $\phi(5) = 3$