Quiz 2 Solutions

1. Simplify the expression

$$\log_5\left(\frac{5^{x^2+1}}{25^x}\right).$$

   Solution: The main idea here is to play with logarithms.

$$\begin{aligned}
\log_5\left(\frac{5^{x^2+1}}{25^x}\right) &= \log_5\left(\frac{5^{x^2+1}}{5^{2x}}\right) \\
&= \log_5\left(5^{x^2-2x+1}\right) \\
&= x^2 - 2x + 1 \\
&= (x-1)^2
\end{aligned}$$

2. A savings account with non-zero starting balance has a yearly interest rate of r. Let $y(t)$ be the balance of the savings account after t years, and suppose the compounding of interest on the account is such that $y(t)$ satisfies the condition $y'(t) = ry(t)$. For which value of r will your initial investment triple in 15 years? That is, solve for r in the equation $3P = Pe^{15r}$ where P is a nonzero number.

   Solution:

   Solving the given differential equation, we have $y(t) = Pe^{rt}$ where P is a constant. Since we want $y(15) = 3y(0)$ this means, $3P = Pe^{15r}$. Under the assumption that $y(0) = P \neq 0$ we can simplify this to $3 = e^{15r}$. Taking the log of both sides we have $\log(3) = 15r$, meaning $r = \log(3)/15$.