Worksheet 10,   Math 10560

1. Find a power series representation for the function

$$\frac{x^2}{1+x}$$

   in the interval $(-1, 1)$.

2. Find a power series representation for the function

$$\frac{x^2}{(1-x^3)^2}$$

   in the interval $(-1, 1)$. Hint: Differentiation of a power series may help.

3. a) Write down the Taylor series expansion for $f(x) = \arctan(x)$ about $x = 0$.

b) Compute the following sum. Hint: Use part (a).

$$\sum_{n=0}^{\infty} \frac{4(-1)^n}{2n+1}$$

4. Find the radius of convergence and interval of convergence of the following power series. Remember to check the endpoints of your interval.

$$\sum_{n=1}^{\infty} \frac{(-1)^n (4x-1)^n}{2^n \sqrt{n+1}}$$

5. a) Find the 6th Taylor Polynomial of $f(x) = \sin(x)$ about $a = \frac{\pi}{2}$.

b) Write down the Taylor series expansion of $\sin(x)$ about $\frac{\pi}{2}$. Write out at least the first four terms for each series required below in addition to the general formula for the nth term; for example

$$\cos(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots + \frac{(-1)^n x^{2n}}{(2n)!} + \cdots$$