WORKSHEET 2 SOLUTION

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Problem 1. Consider the function $f(x) = x^2 + 6x^2 + 11$.

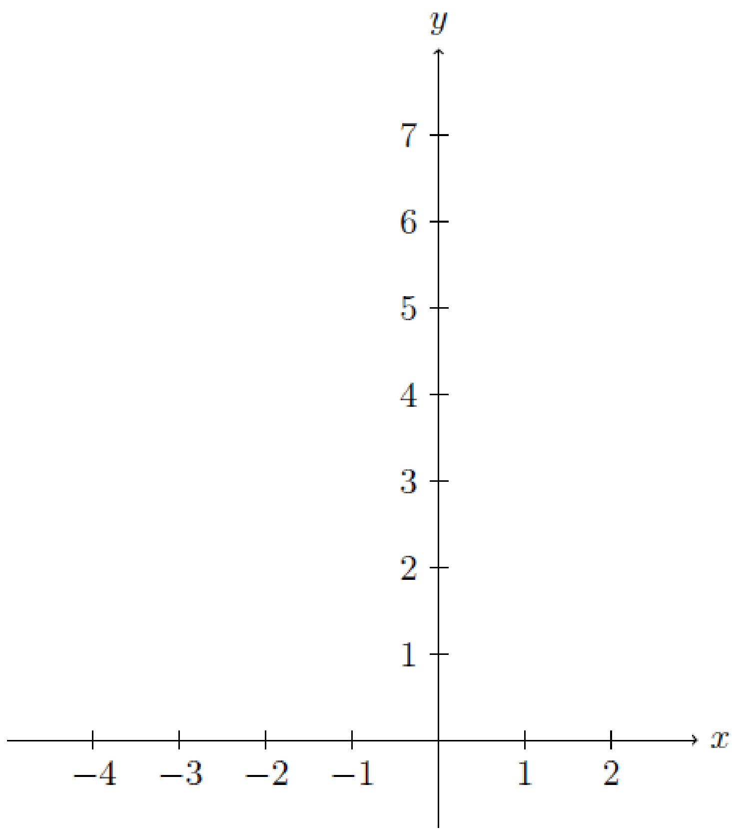
(a) Complete the square and write the function in the form $(x - a)^2 + b$.

The idea is to use the identity $(a + b)^2 = a^2 + 2ab + b^2$, and write $11 = 9 + 2$:

$$f(x) = x^2 + 6x + 11 = x^2 + 2 \cdot 3 \cdot x + 11 = x^2 + 2 \cdot 3 \cdot x + 3^2 + 2 = (x + 3)^2 + 2$$

which is the desired form.

(b) Sketch the graph of $g(x) = x^2$ and the function $f(x)$ from part (a).



(c) Explain what transformations would one perform on the graph of x^2 to obtain the graph of $f(x)$ from question (a).

First we are shifting to the LEFT 3 units, then shifting up 2 units.

(d) Consider the function $f(x) = x^2+6x+11$ as above. Let P be the point $(-3, 2)$. Compute the **slope of the secant line** between P and each of $Q(-4, 3)$ and $R(-2, 3)$.

$m_{PQ} =$

$$\frac{3-2}{-4-(-3)} = \frac{1}{-1} = -1$$

$m_{PR} =$

$$\frac{3-2}{-2-(-3)} = \frac{1}{1} = 1$$

(e) Draw the corresponding secants on the graph on the previous page, and **estimate the slope of the tangent** to the curve at the point P. Draw the tangent to the curve.

A good estimate for the slope of the tangent at P is $m = 0$. From the graph we can see that the tangent line is horizontal (zero slope). Alternately, one can estimate this by taking the average of $m_{PQ} = -1$ and $m_{PR} = 1$, which is zero.

(f) Write the equation of the tangent line at the point $x = -3$.

We already have a point $P(-3, 2)$, and we know the slope $m = 0$. So the equation for the line is $y - 2 = 0(x - (-3))$, which simplifies to $y = 2$.

Problem 2. Let $f(x) = \dfrac{1}{x}$. Assuming that $h \neq 0$, find and simplify $\dfrac{f(x+h) - f(x)}{h}$.

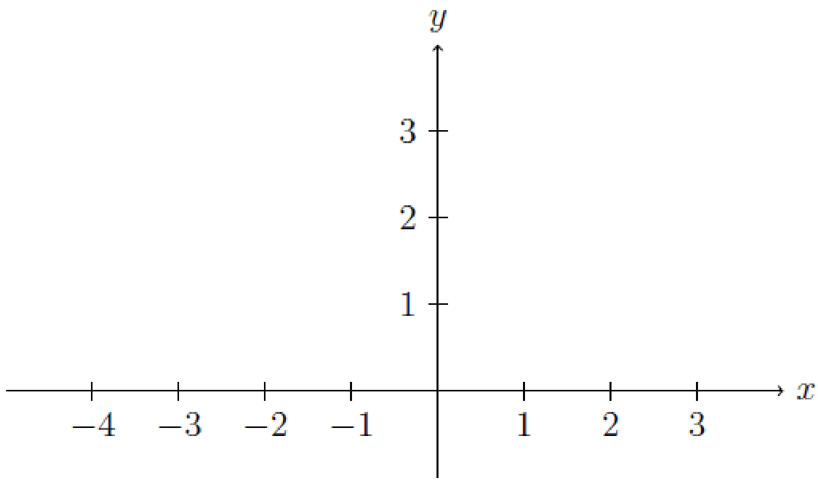
$$\frac{f(x+h)-f(x)}{h} = \frac{\frac{1}{x+h} - \frac{1}{x}}{h} = \frac{\frac{x-(x+h)}{x(x+h)}}{h} = \frac{1}{h} \cdot \frac{h}{x(x+h)} = \frac{1}{x(x+h)}$$

Problem 3. Consider the function $g(x) = |x+1| + 2$.

(a) Write the function as a piecewise function:

$$g(x) = \begin{cases} -x+1 & \text{when } x < -1 \\ \\ x+3 & \text{when } x \geq -1 \end{cases}$$

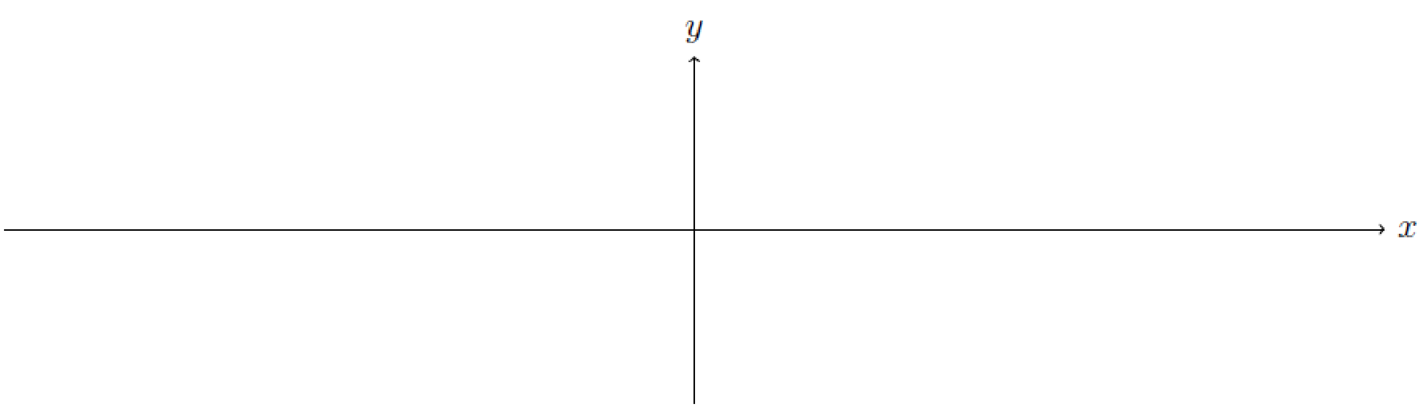
(b) Draw the graph of $g(x)$ on the axes below:



(c) Do you notice any geometric differences between the graphs of $f(x)$ from Problem 1 at the point $(-3, 2)$ and $g(x)$ (Problem 3) at the point $(-1, 2)$?

In problem 1, $f(x)$ had a "smooth" shape at $(-3, 2)$, while $g(x)$ has a sharp corner at $(-1, 2)$.

Problem 4. Plot and label the functions $f(x) = \sin x$ and $g(x) = \cos x$. Label the x-axis with the appropriate multiples of π. What is the domain and range of the sine and cosine functions?



The domain for both sine and cosine is $\mathbb{R}$ or $(-\infty, \infty)$, as both functions are defined for all real numbers. The range for both is $[-1, 1]$, as they only take values between -1 and 1 (inclusive).

Problem 5. Fill in the following table of values of the given trigonometric functions:

θ	0	π	$\frac{\pi}{2}$	$\frac{\pi}{3}$	$\frac{\pi}{4}$	$\frac{\pi}{6}$
$\cos(\theta)$						
$\sin(\theta)$						
$\tan(\theta)$						
$\sec(\theta)$						
$\csc(\theta)$						
$\cot(\theta)$						

Problem 6. Solve $4^{x-2} = 8$. Be sure your answer is simplified.

We begin by noting $4 = 2^2$ and $8 = 2^3$, so to have a common base which would allow us to take a logarithm, we rewrite the equation as

$$(2^2)^{x-2} = 2^3$$
$$2^{2x-4} = 2^3$$

Now we apply log base 2 to get

$$2x - 4 = 3$$
$$2x = 7$$

so our solution is

$$x = \frac{7}{2}$$