Name: SOLUTIONS Date: 04/26/2018

M20580 L.A. and D.E. Tutorial
Quiz 10

1. Find the solution of the initial value problem

$$y'' + y' = 6y, \qquad y(0) = 1,\ y'(0) = -8$$

> ***Solution:*** $y'' + y' = 6y \iff y'' + y' - 6y = 0$. The characteristic equation is $r^2 + r - 6 = 0$, which is equivalent to $(r+3)(r-2) = 0$. So, all the roots are $r = -3, 2$. The general solution is $y(t) = c_1e^{-3t} + c_2e^{2t}$.
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> Now, we use the initial conditions $y(0) = 1,\ y'(0) = -8$ to find c_1 and c_2. Note, $y'(t) = -3c_1e^{-3t} + 2c_2e^{2t}$. So, we have the system of linear equations
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> $$\begin{aligned} c_1 + c_2 &= 1 \\ -3c_1 + 2c_2 &= -8 \end{aligned}$$
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> Solving for c_1 and c_2 in the equations above we obtain $c_1 = 2$ and $c_2 = -1$.
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> In conclusion, the solution to the initial value problem is
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> $$y(t) = 2e^{-3t} - e^{2t}$$

2. Solve the differential equation

$$(2xy + 3) + (x^2 - 2)\frac{dy}{dx} = 0.$$

> ***Solution:*** This is an exact equation: $M = 2xy + 3$ and $N = x^2 - 2$. $M_y = 2x = N_x$. To solve this exact equation, we want to find $\psi(x, y)$ satisfies $\psi_x = 2xy + 3$ and $\psi_y = x^2 - 2$.
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> First $\psi_x = 2xy + 3 \implies \psi = \int(2xy + 3)\,dx = x^2y + 3x + h(y)$. Then $\psi_y = \frac{\partial}{\partial y}(x^2y + 3x + h(y)) = x^2 + h'(y)$. And $x^2 + h'(y)$ must equal $x^2 - 2$ from above. Thus, $h'(y) = -2$ and so $h(y) = -2y + C$. It suffices to choose $\psi(x, y) = x^2y + 3x - 2y$.
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> And the solutions take the form
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> $$x^2y + 3x - 2y = c$$