Quiz 7 Solutions

1. Consider the sequence
$$a_n = \left(1 + \frac{\pi}{n}\right)^n$$
for $n \geq 1$. What can you say about the convergence/divergence of the sequence?

Solution: Recall that the formula for continuously compounded interest formula can be obtained by taking the limit
$$\lim_{n\to\infty} \left(1 + \frac{r}{n}\right)^{nt} = e^{rt}$$
While in practice this is used for r values between 0 and 1 (a sensible range for interest rates), the formula holds true for all $r \in \mathbb{R}$. Setting $r = \pi$ and $t = 1$, we get that the limit of our sequence is
$$\lim_{n\to\infty} \left(1 + \frac{\pi}{n}\right)^{n\cdot 1} = e^{\pi\cdot 1} = e^{\pi}$$

2. Determine if the following series is convergent or divergent. If the series converges, find its sum.
$$\sum_{n=2}^{\infty} \frac{(-1)^n 2^n}{5^{n+2}}.$$

Solution: Begin by bringing the exponent of everything in the summand back to the common value n. We do this by factoring 5^2 from the denominator, then use the properties of summation:
$$\begin{aligned}
\sum_{n=2}^{\infty} \frac{(-1)^n 2^n}{5^{n+2}} &= \sum_{n=2}^{\infty} \frac{1}{5^2} \cdot \frac{(-1)^n 2^n}{5^n} \\
&= \frac{1}{25} \sum_{n=2}^{\infty} \frac{(-1)^n 2^n}{5^n} \\
&= \frac{1}{25} \sum_{n=2}^{\infty} \left(\frac{-2}{5}\right)^n
\end{aligned}$$
which is a geometric sum with common ration $r = -2/5$. Since $|r| = 2/5 < 1$, we know this is convergent, and the sum will converge to:
$$\begin{aligned}
\sum_{n=2}^{\infty} \frac{(-1)^n 2^n}{5^{n+2}} &= \frac{1}{25} \cdot \frac{\text{first term of the sum}}{1 - \text{common ratio } r} \\
&= \frac{1}{25} \cdot \frac{\left(\frac{-2}{5}\right)^2}{1 - \frac{-2}{5}} \quad \text{(first term is } (-2/5)^2 \text{ since sum starts at } n = 2) \\
&= \frac{1}{25} \cdot \frac{4}{25} \cdot \frac{1}{1 + 2/5} = \frac{1}{25} \cdot \frac{4}{25} \cdot \frac{5}{7} = \frac{4}{125 \cdot 7} \\
&= \frac{4}{875}
\end{aligned}$$