QUIZ 0 SOLUTIONS

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Time: 20 min

Problem 1. State the domain of the function $g(x) = \dfrac{x}{x^2 - 16}$.

The formula makes sense for all x values except when the denominator is equal to zero. This happens when $x^2 = 16$, so $x = \pm 4$. Hence the domain of the function is

$$(-\infty, -4) \cup (-4, 4) \cup (4, \infty).$$

We can also write this as $\mathbb{R} \setminus \{-4, 4\}$, i.e all real numbers except -4 and 4.

Problem 2. Find the inverse of the function $h(x) = 3 - x^5$.

We are looking at the function $y = 3 - x^5$. To determine the inverse of this, we interchange the variables

$$x = 3 - y^5$$

and then solve for y as follows:

$$y^5 = 3 - x$$

Taking the fifth root, we get

$$y = \sqrt[5]{3 - x} = (3 - x)^{1/5}$$

so the formula for the inverse function is

$$h^{-1}(x) = \sqrt[5]{3 - x}$$

Problem 3. Find all real solutions to the equation $e^{2x} + 3e^x - 10 = 0$.

First notice we can rewrite this as

$$(e^x)^2 + 3e^x - 10 = 0$$

so we set $y = e^x$ and solve

$$y^2 + 3y - 10 = 0$$

This factors as

$$(y + 5)(y - 2) = 0$$

so we get two possible solutions $y = -5$ and $y = 2$. Since we had $y = e^x$, we are looking at two equations

$$e^x = -5 \text{ and } e^x = 2$$

The first equation is impossible because the function e^x is always positive, we we only get one solution (from the second equation), namely $x = \ln 2$.

Problem 4. Evaluate $u(t-6)$ for $u(t) = t^2 + \dfrac{1}{t+5}$.

We have

$$u(t-6) = (t-6)^2 + \frac{1}{(t-6)+5} = t^2 - 12t + 36 + \frac{1}{t-1}$$

Problem 5. List the transformations necessary to change $f(x) = |x|$ into $g(x) = -|x| + 2$.
The first thing that happens is $|x|$ becomes $-|x|$ which is a reflection about the y-axis (note: if we had $|-x|$ it would be reflected about x). Then we add 2, which is a vertical shift up 2 units. So first we reflect about y-axis, then shift up 2 units.

Problem 6. Given $f(x) = \dfrac{2}{x}$ and $g(t) = t^3 + 1$, find $(f \circ g)(-1)$.

The notation means we are doing $f(g(-1))$, so first we are computing $g(-1)$ which is $(-1)^3 + 1 = 0$. Then we are taking $f(0)$ but that is $2/0$ which is undefined due to the zero in the denominator. So in fact $(f \circ g)(-1)$ is not defined.

Problem 7. Find all real solutions to the equation $\ln(t^2 - 3) = 0$.

First raise e to both sides to get rid of the natural log, then solve as usual:

$$t^2 - 3 = e^0$$
$$t^2 - 3 = 1$$
$$t^2 = 3$$
$$t = \pm\sqrt{3}$$

Problem 8. Find $f(2x)$ for $f(x) = x^4 - x^2$.

We replace x by $2x$ in the formula for f and get

$$f(2x) = (2x)^4 - (2x)^2 = 2^4x^4 - 2^2x^2 = 16x^4 - 4x^2$$

Problem 9. Given $v(x) = 3x - 1$ and $m(x) = x^2 + x$, find and simplify $(m \circ v)(x)$.

We know $(m \circ v)(x)$ means $m(v(x))$ so we have

$$m(v(x)) = m(3x-1) = (3x-1)^2 + (3x-1) = 9x^2 - 6x + 1 + 3x - 1$$

Simplifying, this is

$$9x^2 - 3x$$

Problem 10. Find the rule of the function g whose graph can be obtained from $f(x) = \sqrt{x}$ by stretching away from the x-axis by a factor of 2, and then reflecting in the y-axis.

A horizontal compression (shrinking along x-axis) would be $\sqrt{2x}$, while a stretch means we divide the variable by 2, so we have $\sqrt{x/2}$. If we want to reflect over y-axis, we put a negative sing in front of everything, so

$$g(x) = -\sqrt{\frac{x}{2}}$$