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FINITE MATH: QUIZ 3 SOLUTION

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Problem 1. Consider the set $A = \{1, 2, 3, 4, 5, 6\}$.

a) (1pt) How many different subsets (of any size) of the set A are there?

In general, the number of elements of a set of size n is 2^n. In our case, we have a set of six elements, so the number of possible subsets is $\boxed{2^6 = 64}$.

b) (1pt) How many subsets of A have exactly 2 elements?

$$\boxed{\binom{6}{2} = 15}.$$

c) (1pt) How many subsets of A contain at least one element?

The long answer is: we want subsets of size 1, 2, 3, 4, 5 or 6, so by the addition principle, this is

$$\boxed{\binom{6}{1} + \binom{6}{2} + \binom{6}{3} + \binom{6}{4} + \binom{6}{5} + \binom{6}{6} = 6 + 15 + 20 + 15 + 6 + 1 = 63}$$

A better way is to take all the subsets, and subtract those of size zero:

$$\boxed{2^6 - \binom{6}{0} = 64 - 1 = 63}$$

Problem 2. (2pts) We saw in class the following product expansions:

$$\begin{aligned}
(x+y)^0 &= 1 \\
(x+y)^1 &= x + y \\
(x+y)^2 &= x^2 + 2xy + y^2 \\
(x+y)^3 &= x^3 + 3x^2y + 3xy^2 + y^3
\end{aligned}$$

Write out the product expansion of $(x+y)^5$.

We begin by constructing Pascal's triangle:

$$\begin{array}{ccccccc} & & & 1 & & & \\ & & 1 & & 1 & & \\ & 1 & & 2 & & 1 & \\ 1 & & 3 & & 3 & & 1 \\ 1 & 4 & & 6 & & 4 & 1 \\ 1 & 5 & 10 & & 10 & 5 & 1 \end{array}$$

We don't need more rows, because the first row corresponds to exponent zero, and so the last row is the one we look for (corresponding to exponent 5). Then the expansion we look for is

$$\boxed{x^5 + 5x^4y + 10x^3y^2 + 10x^2y^3 + 5xy^4 + y^5}$$

Problem 3. A standard deck of 52 cards has 4 suits, 13 ranks, and 2 colors.

a) (1pt) How many possible poker hands are there? A poker hand has 5 cards.

We don't have any restriction on what cards we can have, so

$$\boxed{\binom{52}{5} = \frac{52!}{5!\ 47!}}$$

b) (1pt) How many poker hands contain all four Aces?

We are forced to have all 4 Aces, so we only need one more card out of the remaining 48 in the deck:

$$\boxed{\binom{48}{1} = 48}$$

c) (1pt) How many poker hands contain 2 Spades and 3 Red cards?

There are 13 spades in the deck, and 26 red cards:

$$\boxed{\binom{13}{2} \cdot \binom{26}{3} = \frac{13!}{2!\ 11!} \cdot \frac{26!}{3!\ 23!}}$$

d) (BONUS: 2pts) How many poker hands contain at least one red card?

The easiest way to compute this is to take all the possible hands, and remove those with NO red cards (i.e. with all black cards). Since there are 52 total cards, and 26 black cards, this is

$$\boxed{\binom{52}{5} - \binom{26}{5} = \frac{52!}{5!\ 47!} - \frac{26!}{5!\ 21!}}$$