PRACTICE QUIZ 17 SOLUTIONS

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Time: 10 min
Time to beat: ? min

Problem 1. Find $\frac{dy}{dx}$ given that $x^2y - xy^2 + x^2 + y^2 = 0$.

We take derivative with respect to x and get

$$2xy + x^2\frac{dy}{dx} - y^2 - x(2y)\frac{dy}{dx} + 2x + 2y\frac{dy}{dx} = 0$$

and after moving everythign without $\frac{dy}{dx}$ to the RHS, this is

$$x^2\frac{dy}{dx} - 2xy\frac{dy}{dx} + 2y\frac{dy}{dx} = y^2 - 2x - 2xy$$

and now factor $\frac{dy}{dx}$ to get

$$\frac{dy}{dx}\left(x^2 - 2xy + 2y\right) = y^2 - 2x - 2xy$$

Finally,

$$\frac{dy}{dx} = \frac{y^2 - 2x - 2xy}{x^2 - 2xy + 2y}$$

Problem 2. Find $\frac{dy}{dx}$ if $x^2 + y^2 = xy + 3$.

Taking the derivative w.r.t x gives us

$$2x + 2y\frac{dy}{dx} = y + x\frac{dy}{dx}$$

which we rewrite as

$$2y\frac{dy}{dx} - x\frac{dy}{dx} = y - 2x$$

$$\frac{dy}{dx}(2y - x) = y - 2x$$

$$\frac{dy}{dx} = \frac{y - 2x}{2y - x}$$

Problem 3. Given that $x^3y + xy^3 = 2$, find $\frac{dy}{dx}$ at the point $(1, 1)$.

Taking the derivative gives

$$3x^2y + x^3\frac{dy}{dx} + y^3 + x(3y^2)\frac{dy}{dx} = 0$$

and substituting $x = 1$ and $y = 1$ allows us to get

$$3 + \frac{dy}{dx} + 1 + 3\frac{dy}{dx} = 0$$

so $\frac{dy}{dx} = -1$.

Problem 4. Find the second derivative y'' for $x + xy + y = 2$.

First we need the first derivative:

$$1 + y + x\frac{dy}{dx} + \frac{dy}{dx} = 0$$
$$\frac{dy}{dx}(x + 1) = -y - 1$$
$$\frac{dy}{dx} = y' = \frac{-y - 1}{x + 1}$$

and now we take another second derivative using quotient rule, then substitute the formula we just found for y' where appropriate

$$y'' = \frac{-y'(x + 1) - (-y - 1)(1)}{(x + 1)^2} = \frac{-\left(\frac{-y-1}{x+1}\right)(x + 1) + y + 1}{(x + 1)^2} = \frac{2y + 2}{(x + 1)^2}$$