Name: ______________________________

Instructor: __________________________

Math 10560, Final Review

May 20, 3000

- The Honor Code is in effect for this examination. All work is to be your own.
- No calculators.
- The exam lasts for 50 min.
- Be sure that your name is on every page in case pages become detached.
- Be sure that you have all 0 pages of the test.

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Multiple Choice

1.(6 pts.) What can be said about the integrals

$$(i) \int_0^1 \frac{e^x}{x^2}\,dx;$$

$$(ii) \int_1^\infty \frac{\cos^2 x}{x^2}\,dx?$$

(a) both (i) and (ii) converge

(b) (i) diverges and (ii) converges

(c) (i) converges and (ii) diverges

(d) both (i) and (ii) diverge

(e) neither integral (i) nor (ii) is improper

Solution:

$$\frac{e^x}{x^2} \geq \frac{1}{x^2}$$

which diverges by the p-test for series for intergrals since $2 \geq 1$, thus i diverges.

$$\frac{\cos^2(x)}{x^2} \leq \frac{1}{x^2}$$

which converges by the p-test for series for intergrals since $2 \geq 1$, thus ii converges.

2.(6 pts.) The point $\left(2, \frac{7\pi}{3}\right)$ in polar coordinates corresponds to which point below in Cartesian coordinates?

(a) $(\sqrt{3}, 1)$

(b) Since $\frac{7\pi}{3} > 2\pi$, there is no such point

(c) $(1, \sqrt{3})$

(d) $(-1, \sqrt{3})$

(e) $(-\sqrt{3}, 1)$

Solution:

$$x = r\cos(\theta) = 2\cos(7\pi/3) = 2\cos(\pi/3) = 1$$
$$y = r\sin(\theta) = 2\sin(7\pi/3) = 2\sin(\pi/3) = \sqrt{3}$$

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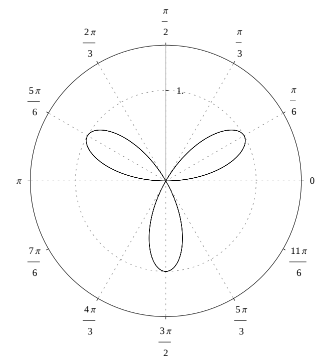
3.(6 pts.) Which integral below gives the area inside the polar curve $r = \sin(3\theta)$?



(a) $\dfrac{1}{2}\displaystyle\int_0^{\pi} \sqrt{\sin^2(3\theta) + 9\cos^2(3\theta)}\ d\theta$

(b) $\dfrac{1}{2}\displaystyle\int_{\pi/6}^{\pi/3} \sin^2(3\theta)\ d\theta$

(c) $\dfrac{1}{2}\displaystyle\int_0^{\pi} \sin^2(3\theta)\ d\theta$

(d) $\dfrac{1}{2}\displaystyle\int_0^{2\pi} \sin^2(3\theta)\ d\theta$

(e) $\dfrac{1}{2}\displaystyle\int_0^{2\pi} \sqrt{\sin^2(3\theta) + 9\cos^2(3\theta)}\ d\theta$

Solution: If θ runs from 0 to π then the curve is drawn out. Thus the bounds of the integral are 0 and π, then using the formula

$$\int_0^{\pi} \frac{1}{2} r^2 d\theta$$

the answer is c

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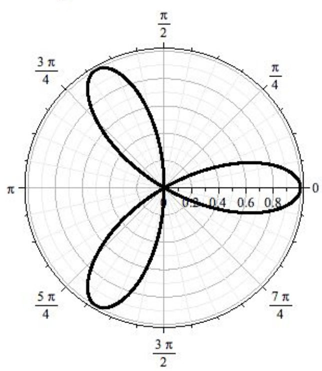
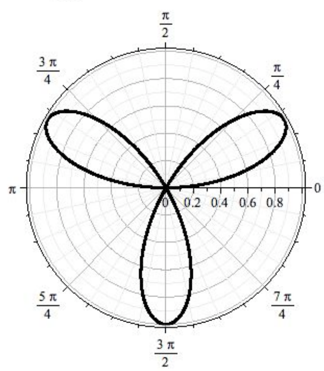
4.(6 pts.) Which of the following gives the graph of the curve described by the polar equation

$$r = \cos(3\theta).$$

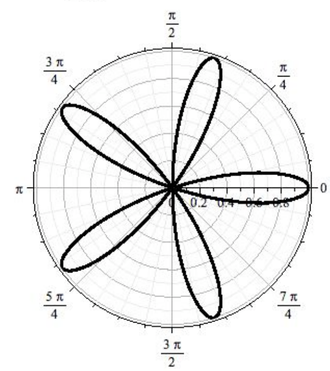
I

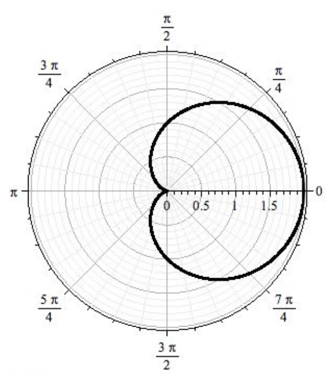


II

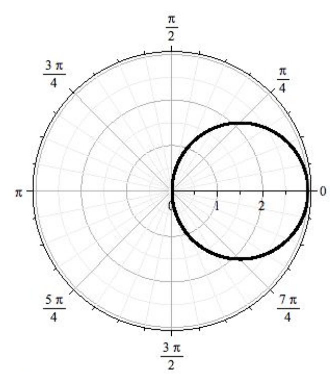


III





IV



V

(a) V (b) IV (c) II (d) I (e) III

Solution: When $\theta = 0$, $r = 1$, thus that eliminates II, IV and V. Then at $\theta = \pi/3$, $r = -1$, thus I must be the correct graph.

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5.(6 pts.) The function $f(x) = x + \sqrt{x}$ is one-to-one. Find the tangent line to the inverse function $f^{-1}(x)$ at the point $x = 2$.

(a) $y - 2 = \frac{3}{2}(x - 1)$

(b) $y - 2 - \sqrt{2} = \frac{3}{2}(x - 2)$

(c) $y - 2 - \sqrt{2} = \frac{2}{3}(x - 2)$

(d) $y - 1 = \frac{2}{3}(x - 2)$

(e) $y - 1 = \frac{3}{2}(x - 2)$

Solution: $f^{-1}(2) = 1$, since $1 + \sqrt{1} = 2$, and $f'(x) = 1 + \dfrac{1}{2}x^{-1/2}$, thus

$$\left(f^{-1}\right)'(2) = \frac{1}{f'(f^{-1}(2))} = \frac{1}{f'(1)} = \frac{2}{3}$$

Thus the answer is clearly d

6.(6 pts.) Compute the integral

$$\int_0^1 4\tan^{-1}(x)dx\ .$$

(a) $\pi - \ln 4$

(b) $2\pi - \ln 2$

(c) $\dfrac{\pi}{\ln 2}$

(d) $\pi - 1$

(e) 0

Solution: Let $u = \arctan(x)$ and $dv = 4dx$, thus $du = \dfrac{dx}{1+x^2}$ and $v = 4x$, thus by Integration by Parts

$$\begin{aligned}\int_0^{\frac{\pi}{4}} \tan^2 x\ \sec^4 x\ dx =& 4x\arctan(x)|_0^1 - \int_0^1 \frac{4x}{1+x^2}dx \\ =& \pi - \int_1^2 \frac{2}{u}du \\ =& \pi - 2\ln(u)|_1^2 \\ =& \pi - 2\ln(2) \\ =& \pi - \ln(4)\end{aligned}$$

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7.(6 pts.) Find $\displaystyle\int_0^{\frac{\pi}{4}} \tan^2 x \ \sec^4 x \ dx$.

(a) $\frac{2}{5}$ (b) $\frac{2}{3}$ (c) $\frac{8}{15}$ (d) $\frac{2}{15}$ (e) 1

Solution: Let $u = tan(x)$, thus $du = \sec^(x)$

$$\int_0^{\frac{\pi}{4}} \tan^2 x \ \sec^4 x \ dx = \int_0^{\frac{\pi}{4}} \tan^2 x \ (1+\tan^2) \sec^2 x \ dx = \int_0^1 u^2(1+u^2)du = \int_0^1 u^2 + u^4 du$$

Thus

$$\int_0^1 u^2 + u^4 du = \frac{1}{3}u^3 + \frac{1}{5}u^5|_0^1 = \frac{1}{3} + \frac{1}{5} = \frac{8}{15}$$

8.(6 pts.) Which equation below is the partial fraction decomposition of the rational function

$$\frac{5x^2 - 10x - 8}{(x-2)(x^2+4)}.$$

(a) $\frac{-1}{x-2} + \frac{6x+2}{x^2+4}$ (b) $\frac{-1}{x-2} + \frac{x+2}{x^2+4}$

(c) $\frac{5}{x-2} + \frac{x+1}{x^2+4}$ (d) $\frac{5}{x-2} + \frac{6x+1}{x^2+4}$

(e) $\frac{-1}{x-2} + \frac{2}{x^2+4}$

Solution:

$$\frac{5x^2 - 10x - 8}{(x-2)(x^2+4)} = \frac{A}{x-2} + \frac{Bx+C}{x^2+4}$$

Thus

$$5x^2 - 10x - 8 = A(x^2+4) + (Bx+C)(x-2)$$

When $x = 2$, $-8 = 8A$, thus $A = -1$, when $x = 0$, $-8 = 4A - 2C = -4 - 2C$, thus $C = 2$.

When $x = 3$, $7 = (-1)(13) + (3B+2)(1)$, thus $20 = 3B + 2$, thus $B = 6$.

Thus the answer is a

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9.(6 pts.) The length of the curve $y = \dfrac{x^3}{6} + \dfrac{1}{2x}$, $\quad \dfrac{1}{2} \le x \le 1$, is given by:

(a) $\dfrac{1}{2}\displaystyle\int_{1/2}^{1} \sqrt{1 + (x + x^{-1})^2}dx$

(b) $\dfrac{1}{2}\displaystyle\int_{1/2}^{1} \sqrt{(x^2 + x^{-2})}dx$

(c) $\dfrac{1}{2}\displaystyle\int_{1/2}^{1} (x^2 + x^{-2})dx$

(d) $\dfrac{1}{2}\displaystyle\int_{1/2}^{1} \sqrt{1 + (x^2 + x^{-2})^2}dx$

(e) $\dfrac{1}{2}\displaystyle\int_{1/2}^{1} (x + x^{-1})dx$

Solution:

$$y' = \frac{x^2}{2} - \frac{1}{2x^2}$$

$$\int_{1/2}^{1} \sqrt{1 + (\frac{x^2}{2} - \frac{1}{2x^2})^2}dx = \int_{1/2}^{1} \sqrt{(\frac{x^2}{2} + \frac{1}{2x^2})^2}dx = \frac{1}{2}\int_{1/2}^{1} (x^2 + x^{-2})dx$$

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10.(6 pts.) Find the area enclosed by the following cycloid and the x-axis:

$$x(t) = t - \sin t \qquad y(t) = 1 - \cos t \qquad 0 \leq t \leq 2\pi.$$

(a) 2π (b) π (c) $\dfrac{\pi^2}{3}$

(d) 3π (e) π^2

Solution:

$$\int_0^{2\pi} (1 - \cos(t))^2 dt = \int_0^{2\pi} 1 - 2\cos(t) + \cos^2(t) dt = \int_0^{2\pi} 1 - 2\cos(t) + (\frac{1}{2} + \frac{1}{2}\cos(2t)) dt$$

Since $\sin(0) = \sin(2\pi) = 0$

$$\int_0^{2\pi} (1 - \cos(t))^2 dt = \int_0^{2\pi} \frac{3}{2} dt = 3\pi$$

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11.(6 pts.) Let C be a constant. Which of the following is a solution to the differential equation $y' = x + \frac{1}{x}y$?

(a) $y = C$ (b) $y = x + C$ (c) $y = \frac{x+C}{x}$

(d) $y = x(x + C)$ (e) $y = Cx^2$

Solution:

$$y' - \frac{1}{x}y = x$$

Then $\int -\frac{1}{x} = \ln(x^{-1})$,thus integral factor is $\frac{1}{x}$

$$\left(\frac{y}{x}\right)' = 1$$

$$\frac{y}{x} = x + C$$

$$y = x(x + C)$$

12.(6 pts.) Use Simpson's rule with step size $\Delta x = 1$ to appoximate the integral $\int_0^4 f(x)dx$ where a table of values for the function $f(x)$ is given below.

x	0	1	2	3	4
$f(x)$	2	1	2	3	5

(a) 9 (b) 11 (c) 9.5 (d) 8 (e) 10.4

Solution:

$$\frac{1}{3}\left(f(0) + 4f(1) + 2f(2) + 4f(3) + f(4)\right) = \frac{1}{3}\left(2 + 4 + 4 + 12 + 5\right) = 9$$

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13.(6 pts.) Which one of the following statements is TRUE?

(a) $\sum_{n=1}^{\infty} \frac{1}{((\sin n)^2+1)\,n}$ is divergent by ratio test.

(b) $\sum_{n=1}^{\infty} \frac{1}{((\sin n)^2+1)\,n}$ is absolutely convergent by root test.

(c) $\sum_{n=1}^{\infty} \frac{1}{((\sin n)^2+1)\,n}$ is divergent by comparison test.

(d) $\sum_{n=1}^{\infty} \frac{1}{((\sin n)^2+1)\,n}$ is absolutely convergent by ratio test.

(e) none of the above

Solution: Compare to $\frac{1}{2n}$ for c. Check to see why the others are false.

14.(6 pts.) Which of the following statements is TRUE?

(a) $\sum_{n=1}^{\infty} \frac{(-1)^n(\sqrt{n}+1)}{n}$ diverges.

(b) $\sum_{n=1}^{\infty} \frac{(-1)^n(\sqrt{n}+1)}{n}$ converges conditionally.

(c) $\sum_{n=1}^{\infty} \frac{(-1)^n 3^n}{5^n}$ diverges by divergence test.

(d) $\sum_{n=1}^{\infty} \frac{(-1)^n(\sqrt{n}+1)}{n}$ converges absolutely.

(e) $\sum_{n=1}^{\infty} \frac{(-1)^n 3^n}{5^n}$ converges conditionally.

Solution: Limit comparision test with $\frac{1}{\sqrt{n}}$ shows that b doesn't converge absolutely. But the sequences converges to 0 and it is decreasing thus passes AST. Thus the answer is b. Check to see why the others are false.

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15.(6 pts.) Which line below is the tangent line to the parameterized curve $x = t - \cos t$, $y = t + \sin t$ when $t = 0$?

(a) $x = -1$, a vertical tangent

(b) $y = \dfrac{1 + \cos t}{1 + \sin t}\,(x + 1)$

(c) $y = 2x + 2$

(d) $y = \dfrac{\pi}{2}x + \dfrac{\pi}{2}$

(e) $y = \dfrac{t + \sin t}{t - \cos t}\,(x + 1)$

Solution:

$$\frac{dx}{dt}|_{t=0} = 1 + \sin(t)|_{t=0} = 1$$

$$\frac{dy}{dt}|_{t=0} = 1 + \cos(t)|_{t=0} = 2$$

Thus

$$\frac{dy}{dx}|_{t=0} = 2$$

Thus tangent line is $y - 0 = 2(x - (-1))$ or $y = 2x + 2$.

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Instructor: ANSWERS

Math 10560, Final Review
May 20, 3000

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- No calculators.
- The exam lasts for 50 min.
- Be sure that your name is on every page in case pages become detached.
- Be sure that you have all 0 pages of the test.

PLEASE MARK YOUR ANSWERS WITH AN X, not a circle!

1.	(a)	(•)	(c)	(d)	(e)
2.	(a)	(b)	(•)	(d)	(e)
3.	(a)	(b)	(•)	(d)	(e)
4.	(a)	(b)	(c)	(•)	(e)
5.	(a)	(b)	(c)	(•)	(e)
6.	(•)	(b)	(c)	(d)	(e)
7.	(a)	(b)	(•)	(d)	(e)
8.	(•)	(b)	(c)	(d)	(e)
9.	(a)	(b)	(•)	(d)	(e)
10.	(a)	(b)	(c)	(•)	(e)
11.	(a)	(b)	(c)	(•)	(e)
12.	(•)	(b)	(c)	(d)	(e)
13.	(a)	(b)	(•)	(d)	(e)
14.	(a)	(•)	(c)	(d)	(e)
15.	(a)	(b)	(•)	(d)	(e)