PRACTICE QUIZ 3 SOLUTIONS

ADRIAN PĂCURAR

Time: 15 min
Time to beat: 6 min

Problem 1. Given $f(x) = \frac{x-1}{x^2+2}$, find:
(a) $f(-1) = \frac{-1-1}{1+2} = -\frac{2}{3}$

(b) $f(2a) = \frac{2a-1}{4a^2+2}$

(c)
$$f(1/x) = \frac{\frac{1}{x} - 1}{\frac{1}{x^2} - 2} = \frac{1-x}{x} \cdot \frac{x^2}{1+2x^2} = \frac{x-x^2}{1+2x^2}$$

(d)
$$f(x+h) = \frac{x+h-1}{x^2+2hx+h^2+2}$$

Problem 2. Determine the domains of the functions
(a) $f(x) = \sqrt{4-x}$

Since the square root function only makes sense for nonnegative (≥ 0) values of the input, we need $4 - x \geq 0$, so $x \leq 4$, and the domain is the interval $(-\infty, 4]$.

(b) $f(x) = \sqrt{4-x^2}$

We need $4 - x^2 \geq 0$, or $x^2 \leq 4$. The domain is the interval $[-2, 2]$.

(c) $f(x) = \frac{1}{x^4-81}$

The denominator cannot be zero. Factor it as $(x^2+9)(x+3)(x-3)$ and set it equal to zero. The x^2+9 is never zero as any square is nonnegative, and we are adding a positive constant 9 so this is strictly positive. The zeros only happen when $x = \pm 3$, which are the holes in our domain. Hence function is defined for $x \neq \pm 3$.

Problem 3. If $f(x) = x^2 + 2x$, find $\frac{f(a+h)-f(a)}{h}$. Simplify your answer.

We have:
$$\frac{[(a+h)^2 + 2(a+h)] - (a^2+2a)}{h} = \frac{a^2 + 2ah + h^2 + 2a + 2h - a^2 - 2a}{h} = 2a + 2 + h$$

Problem 4. If $f(x) = 2^x$, show that

(a) $f(x+3) - f(x-1) = \frac{15}{2} f(x)$

Proof.

$$f(x+3) - f(x-1) = 2^{x+3} - 2^{x-1} = 2^x 2^3 - 2^x 2^{-1} = 2^x \left(2^3 - \frac{1}{2}\right) = \frac{15}{2} 2^x = \frac{15}{2} f(x)$$

□

(b) $\frac{f(x+3)}{f(x-1)} = f(4)$

Proof.

$$\frac{f(x+3)}{f(x-1)} = \frac{2^{x+3}}{2^{x-1}} = 2^{(x+3)-(x-1)} = 2^4 = f(4)$$

□