10350 Tutorial Week 16 - Set 01 Name ______________________________

1a. Perform the substitution $u = \cos^2 x - \sin x + 5$ for the integral $\displaystyle\int_0^{\pi/2} \frac{2\cos x \sin x + \cos x}{(\cos^2 x - \sin x + 5)^3}\, dx$.

Be sure to change the integration limits. Do NOT perform the integration. Fill in your answer below:

$$\int_{\_\_}^{\_\_} \underline{\hspace{6cm}}\, du$$

1b. Perform the integral you obtained in (a) to evaluate $\displaystyle\int_0^{\pi/2} \frac{2\cos x \sin x + \cos x}{(\cos^2 x - \sin x + 5)^3}\, dx$.

1c. If $g'(x) = \dfrac{2\cos x \sin x + \cos x}{(\cos^2 x - \sin x + 5)^3}$, find the total change of $g(x)$ over the interval $0 \leq x \leq \pi/2$.

2a. If the instantaneous rate of change of $f(x)$ is given by $\left(\dfrac{1}{\cos^2 x} + \dfrac{2}{\csc x}\right)$, find the total change of $f(x)$ over $0 \leq x \leq \pi/4$.

2b. Perform the following integral. If substitution is needed show all steps carefully.

$$\int \sec(3x)\tan(3x)\,dx \stackrel{?}{=}$$

3. Find the anti-derivative $F(x)$ of $f(x) = \sin x \cos^2 x$ such that $F(0) = 2/3$.

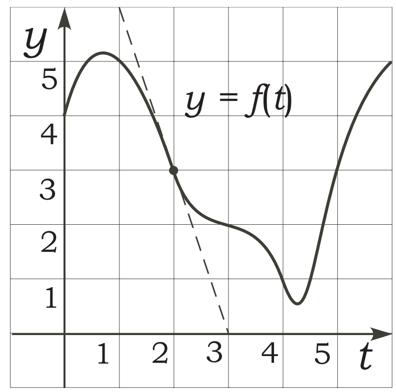
4. Find the derivatives of the following functions:

a. $f(x) = \displaystyle\int_e^x e^{\sin(2t)}\,dt$

b. $y = \displaystyle\int_{x^2}^1 \frac{t^2+1}{\ln(t^2+1)}\,dt$

5. Referring to the graph of $f(t)$ below, compute the following values or expressions. The dotted line the graph of the tangent line to curve at $t = 2$

a. Average rate of change of $f(t)$ over $[1, 5]$.



b. Find the linear approximation to the function $f(t)$ at $t = 2$. Estimate $f(1.9)$.

c. The instantaneous rate of change of $p(t) = tf(t)$ at $t = 2$

d. The slope to the graph of $Q(t) = \dfrac{f(t)}{t+1}$ at $t = 2$

6. Let $f(x) = \dfrac{3}{(2x+1)}$. Find all values $x = c$ in the interval $1 \leq x \leq 4$ that satisfy the Mean Value Theorem.

7. Find all critical points of the function $f(x) = x - 6 \cdot x^{2/3}$.

8 a. Solve the initial value problem: $\dfrac{dy}{dx} = e^{-x} + 3e^{x}$ such that $y(0) = 3$.

8 b. Find also x-intercept of the graph of the function $g(x) = y(x) - 1$ where y is the function you found in part (a).