Quiz 1, Solutions

1. **Q1** Evaluate the integral

$$\int_0^{\frac{\pi}{2}} \sin^7 x \cos^3 x dx$$

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Sol The idea in here is to use u substitution, but before that we need to change the integral a bit. Recall that $\cos^2 x = 1 - \sin^2 x$, then:

$$\int_0^{\frac{\pi}{2}} \sin^7 x \cos^3 x dx = \int_0^{\frac{\pi}{2}} (\sin^7 x)(1 - \sin^2 x) \cos x dx$$

Now the substitution is clear: let $u = \sin x$, then $du = \cos x dx$. Also, $x = 0 \to u = 0$, $x = \frac{\pi}{2} \to u = 1$ and we have:

$$= \int_0^1 u^7(1 - u^2)du = \int_0^1 \left(u^7 - u^9\right) du = \left[\frac{u^8}{8} - \frac{u^{10}}{10}\right]_0^1 = \frac{1}{8} - \frac{1}{10}$$

2. **Q2** Evaluate

$$\int_2^{e^3} \ln x dx$$

Sol We do integration by parts:

- $u = \ln x \Rightarrow du = \frac{1}{x}dx$
- $dv = dx \Rightarrow v = x$

Hence, we get:

$$\int_2^{e^3} \ln x dx = [u \cdot v]|_2^{e^3} - \int_2^{e^3} v du = [x \ln x]|_2^{e^3} - \int_2^{e^3} x \cdot \frac{1}{x} dx = e^3 \ln(e^3) - 2\ln(2) - \int_2^{e^3} dx$$
$$= 3e^3 - 2\ln 2 - x|_2^{e^3} = 3e^3 - 2\ln 2 - [e^3 - 2] = 2e^3 - 2\ln 2 + 2$$

Discuss the following with the person next to you for five minutes for both questions (and if you do not finish, please finish the exercise at home):

(a) In solving this question, what formulas/techniques are used from the material covered in class?

(b) What formulas/techniques are used from material learned prior to calculus 2 (calculus 1, precalculus, algebra and trigonometry)?

(c) How do you think the material used in this question will be used later in the course?

(d) Which of the following problem solving techniques would have helped in solving this problem (and how so?):
1. Drawing a picture.
2. Breaking the problem up into less difficult steps.
3. Writing down all of the details.
4. Writing down the formulas you think might be helpful and filling in details.

(e) If asked to solve this problem again, how would you change your approach?

(f) If you were to study for this quiz again, how would you change your approach?