WORKSHEET 6

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Problem 1. Consider the function

$$f(x) = \begin{cases} ax^2 + bx & \text{if } x < 1 \\ \cos(x-1) & \text{if } x \geq 1 \end{cases}$$

(a) Which values for a and b which make f continuous but *not* differentiable. Using these values explain why f is continuous on any interval.

(b) Find values for a and b which make f continuous *and* differentiable. Using these values explain why f is differentiable on any interval.

(c) Is it possible to find a and b for which f is differentiable but not continuous? If so, find such a and b. Explain.

Problem 2. Compute the derivatives of the following functions:
(a) $f(x) = (3 + 4x - x^2)^{1/2}$

(b) $f(x) = \left(\dfrac{x}{1+x}\right)^5$

(c) $f(x) = 2x^2\sqrt{2-x}$

(d) $f(x) = (x^2 + 3)^4(2x^3 - 5)^3$

Problem 3. Compute the derivative of:

$$f(x) = \cos\left(x \sin(x \tan x)\right)$$

Problem 4. Use implicit differentiation to find the following:
(a) dy/dx for $2y^2 = y + x$

(b) dx/dy for $2y^2 = y + x$

(c) $y'(x)$ for $x^2 - xy + y^2 = 3$

(d) $x'(y)$ for $x^2 - xy + y^2 = 3$

Problem 5. Given that $y(t) = t^3$ and $x(t) = \sin t$, differentiate the equation $x^3y + xy^3 = t^2$ with respect to t. Don't simplify.

Problem 6. The radius of a circle is changing at the rate of $dr/dt = +2$ cm/s. How fast is the area changing when $r = 2$? Is this increasing or decreasing? What are the units?

Problem 7. A spherical balloon is filling up with air at the rate of 5 cm^3/s. How fast is the radius of the balloon changing when $r = 2$? Is this increasing or decreasing? Use the fact that the volume of a sphere in terms of its radius r is given by $V = \frac{4}{3}\pi r^3$.

Problem 8. The position of a particle is given by $s(t) = \frac{1}{5}x^5 - 2x^2 + x$, where t is measured in seconds. Is there ever a time during the first second where the particle is at rest?

Problem 9. The position of a particle is given by $s(t) = -\dfrac{\cos(\pi t)}{2\pi}$. For what (non-negative) values of t is the particle at rest?