

M20580 L.A. and D.E. Tutorial
Quiz 9

1. Find an integrating factor for the following differential equation (Note, you DON'T need to solve it!)

$$y' = t^3 - 2ty$$

$$\mu(t) = e^{\int 2t dt} = \boxed{e^{t^2}} \Rightarrow y' + 2ty = t^3$$

2. Solve the differential equation $y' = y^2 e^t$ subject to the initial condition $y(1) = \frac{1}{e}$.

$$\frac{dy}{dt} = y^2 e^t \quad (\text{separable eq'n})$$

$$\Rightarrow \frac{dy}{y^2} = e^t dt \Rightarrow \int y^{-2} dy = \int e^t dt \Rightarrow -\frac{1}{y} = e^t + C$$

$$\Rightarrow y = \frac{-1}{e^t + C}$$

$$y(1) = \frac{1}{e} \Rightarrow \frac{1}{e} = \frac{-1}{e + C} \Rightarrow e + C = -e \Rightarrow C = -2e$$

$$\text{Thus, } \boxed{y = \frac{-1}{e^t - 2e}}$$

$$\text{or } \boxed{y = \frac{1}{2e - e^t}}$$