PRACTICE QUIZ 2 SOLUTIONS

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Allowed Time: 10 min
Time to beat: 3 min

Problem 1. Find the region where $f(x) = -\frac{x^2}{(x-1)^5}$ is continuous.

Since f is a rational function, it is continuous everywhere on its domain. The denominator is $(x-1)^5$ which is zero when $x = 1$, so the domain is all real numbers except $x = 1$. Thus the region of continuity is $x \neq 1$ or alternately $(-\infty, 1) \cup (1\infty)$. Sometimes we also write this as $\mathbb{R} \setminus \{1\}$.

Problem 2. If $f(x) = 5x^2 + 3x$, find the equation of the tangent line at $x = 1$.

We evaluate the derivative $f'(x) = 10x + 3$ at $x = 1$ to obtain a slope of 13. We also need the y-value of the function at $x = 1$, which is $f(1) = 8$. This gives us a point $(x_0, y_0) = (1, 8)$ with the slope of $m = 13$, so we can use the point slope form of a line $y - y_0 = m(x - x_0)$:

$$y - 8 = 13(x - 1)$$

which can also be written as $y = 13x - 5$.

Problem 3. The limit $\lim_{h\to 0} \frac{\sqrt{h+4}-\sqrt{4}}{h}$ represents the derivative of a function $f(x)$ at some point $x = a$. State $f(x)$ and a.

The formal definition of the derivative of a function $f(x)$ at $x = a$ is

$$f'(a) = \lim_{h\to 0} \frac{f(a+h) - f(a)}{h}$$

In our case the limit can be written as

$$\lim_{h\to 0} \frac{\sqrt{4+h} - \sqrt{4}}{h} = \lim_{h\to 0} \frac{f(a+h) - f(a)}{h}$$

and we actually have two options for our answer. We can interpret this as $f(x) = \sqrt{x}$ with $a = 4$, or we can interpret it as $f(x) = \sqrt{x+4}$ with $a = 0$. Either one is correct.

Problem 4. Match the given graph of the function with the graph of its derivative.

The correct answer is A.