

Estimating Derivative of a Function

Recall there are three ways to estimate the derivative (or the rate of change) of a given function $f(x)$ when we ONLY know a table of values of $f(x)$. We have the:

Forward Difference formula**Backward Difference formula****Central Difference formula**

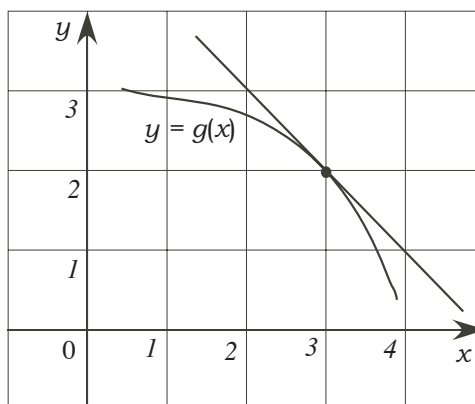
Consider a particle moving on a straight line. Its displacement $s(t)$ from a fixed point O on the straight line the table of values below

t	0	0.5	1.0	2.0	2.5	3.0	3.5
$s(t)$	-4.0	-2.0	-1.0	0	1.2	1.8	2.2

a. Give all possible estimates for the instantaneous velocity at $t = 0.5$. State which estimate you apply.

b. Give all possible estimates for the instantaneous velocity at $t = 3.5$. How does your conclusion differ from Part (a).

c. Give all possible estimates for the instantaneous velocity at $t = 0$. How does your conclusion differ from Parts (a) and (b).



1. The graph of the function $g(x)$ is given above.

(a) What is the value of $g(3)$? Answer: _____

(b) What is the **instantaneous** rate of change of $g(x)$ at $x = 3$? Answer: _____

(c) Find the slope of the graph of $f(x) = \frac{e^{g(x)-2}}{x+1}$ at $x = 3$.

2. Consider a particle P moving **counterclockwise** around the ellipse

$$x^2 + 4y^2 = 5.$$

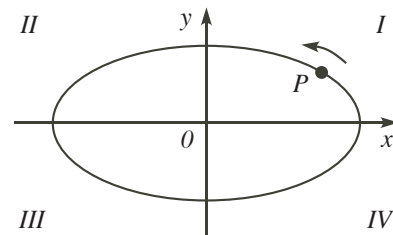
Fill in below the sign (> 0 or < 0) of $\frac{dx}{dt}$ and $\frac{dy}{dt}$ in quadrants of the xy -plane.

2a. $\frac{dx}{dt}$ _____ and $\frac{dy}{dt}$ _____ when P is in Quadrant I.

2b. $\frac{dx}{dt}$ _____ and $\frac{dy}{dt}$ _____ when P is in Quadrant II.

2c. $\frac{dx}{dt}$ _____ and $\frac{dy}{dt}$ _____ when P is in Quadrant III.

2d. $\frac{dx}{dt}$ _____ and $\frac{dy}{dt}$ _____ when P is in Quadrant IV.



2e. For the same Particle P above, find $\frac{dx}{dt}$ at $(1, -1)$ if $\frac{dy}{dt} = 2$ units per second.

3. A huge spherical snowball is melting such that its radius is **reducing** at a constant rate of 2 cm per minute. At what rate is the volume changing at the instant when the radius of the snowball is 10 cm? (You may leave your answers in terms of π and use the formula $V = \frac{4}{3}\pi r^3$.)

4. Consider the curve given by $x^2y^2 - x^4 = 6e^{y-2} - 3$.

4a. Find $\frac{dy}{dx}$.

4b. Find the equation of the tangent line to the curve given by $x^2y^2 - x^4 = 6e^{y-2} - 3$ at the point $(-1, 2)$.