

Name: _____

July 7, 2017

FINITE MATH: QUIZ 5 SOLUTION

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Problem 1. A bag contains 7 red marbles, 5 white marbles, and 8 yellow marbles.

- a) (2pt) Select 3 marbles at random. What is the probability of picking at least one yellow marble?

At least one means 1, 2, or 3, and all of these are disjoint scenarios. Note that we have $7 + 5 = 12$ non-yellow marbles, and 20 total marbles. The

probability we seek is
$$\frac{\binom{8}{1}\binom{12}{2} + \binom{8}{2}\binom{12}{1} + \binom{8}{3}\binom{12}{0}}{\binom{20}{3}}$$

- b) (2pt) Select 3 marbles at random. What is the probability that they are either all red or all yellow?

Either all red or all yellow are disjoint scenarios. The probability we

seek is
$$\frac{\binom{7}{3} + \binom{8}{3}}{\binom{20}{3}}$$

- c) (2pt) Select **six** marbles at random. What is the probability of picking exactly 3 red and 3 yellow?

We want 3 red and 3 yellow at the same time. The probability we seek

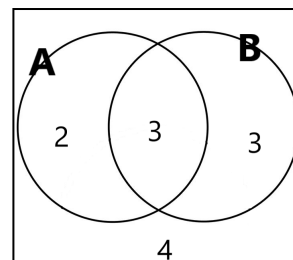
is
$$\frac{\binom{7}{3} \cdot \binom{8}{3}}{\binom{20}{6}}.$$
 Notice the difference from part (b) of the problem.

Problem 2. (2pt) Roll a six-sided die and observe the number that comes up. What is the probability of rolling a number less than 5? (Note: 5 is not less than 5).

We can roll a 1, 2, 3, or 4. Each happens with probability $1/6$, so

$$P(\text{less than 5}) = P(\{1, 2, 3, 4\}) = P(1) + P(2) + P(3) + P(4) = \frac{4}{6} = \frac{2}{3}$$

Problem 3. (2pt) The Venn Diagram on the right shows the number of students enrolled in either Art or Biology. You randomly select a student from the group. Let A be the event that the student is taking Art, and B the event that the student is taking Biology. What is $P(A' \cap B')$?



Using DeMorgan, we are looking for $P(A \cup B)'$. There are 4 outcomes in this event, and 12 outcomes total in the sample space. The probability

$$\text{is } P(A' \cap B') = \frac{4}{12} = \frac{1}{3}$$

Problem 4. (BONUS + 1pt) Using digits from $\{1, 2, 3, 4, 5\}$, create a 3-digit number without repeating the same digit. What is the probability that the 3-digit number you obtain is smaller than 400? You may assume equally likely outcomes.

Since we can't repeat digits, there are $5 \cdot 4 \cdot 3$ possible outcomes for the 3-digit number. The numbers that are smaller than 400 must have the beginning digit be $\{1, 2, 3\}$, and this can be done in $3 \cdot 4 \cdot 3$ ways. Each of these outcomes is equally likely, so the probability is

$$P(\text{less than 400}) = \frac{3 \cdot 4 \cdot 3}{5 \cdot 4 \cdot 3} = \frac{3}{5}$$