Name: ______________________________ July 28, 2017

FINITE MATH: EXAM 3

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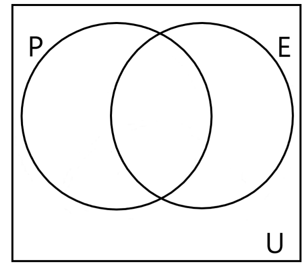
- The Honor Code is in effect for this exam. All work must be your own.
- Please turn off all cellphones or any other electronic devices.
- Calculators are allowed.
- You must **show your work** in order to receive credit (a single numerical answer without anything else will receive zero credit).
- There are **68 points available** for you to try. You may choose to attempt any of the problems, or all the problems. There is no penalty for getting a wrong answer.
- The exam will be **graded out of 60**. You can NOT get more than 60 points on this exam.
- You are allowed a two-sided 8 by 11 formula sheet for the exam; the formula sheet must be handwritten. You must turn in your formula sheet with your exam.
- The exam lasts **1 hour and 20 minutes**.

Problem 1. For the universal set $U = \{1, 2, 3, 4, \ldots, 13, 14\}$, let P be the subset of prime numbers, and let E be the subset of even numbers.



a) (2pt) List the elements of P and E.

b) (2pt) Fill the Venn diagram on the right with the number of elements for each region.

c) (1pt) How many subsets of size 3 does P have?

c) (1pt) How many subsets total does E have?

Problem 2. In our finite math class, we have 7 women and 6 men.

a) (1pt) In how many different ways can we seat **everyone** around a circular table for a class dinner, if any rotation of the table is considered the same arrangement? Do not compute.

b) (2pt) In how many ways can we seat all the men at a circular table, and all the women at a **separate** circular table? Do not compute.

Problem 3. (2pt) A child forms 4-letter words by choosing letters at random from among $\{A, B, C, D, E, U, V\}$. Assuming letters may be repeated, calculate the probability that the word starts with a vowel or ends with a vowel, **but not both**. Anything can go for the middle two letters.

Problem 4. (3pt) Three couples are to be seated in the front row at a piano concert, at random. Calculate the probability that each person ends up next to their significant other. For example, if $A \heartsuit B$ are a couple, and $E \heartsuit F$ are another, then ABEF and ABFE are two different arrangements in which both couples are together, where as in AEBF they are separated.

Problem 5. (3pt) A drawer contains 4 black, 6 brown, and an unknown number of white socks. Two socks are drawn at random, one at a time (without replacement). Given that the first sock was black, the probability of the second sock being white is now 1/4. How many white socks are in the drawer?

Problem 6. A classroom is split into two separate groups, G_1 and G_2. There are 2 men and 6 women in G_1, and there are 15 men and 5 women in G_2. The teacher picks one of the two groups at random, with $P(G_1) = 0.2$ and $P(G_2) = 0.8$, then randomly selects a student from that group.

a) (2pt) Calculate the probability that the chosen student is a man.

b) (1pt) Given that the selected group is G_1, calculate the probability that the student is a man.

c) (3pt) The selected student is a man. Calculate the probability that he is from the first group?

Problem 7. Suppose that $P(A) = 0.8$, $P(B) = 0.5$, and $P(A \cup B) = 0.9$.

a) (2pt) Determine whether the events A and B are mutually exclusive.

b) (2pt) Determine whether the events A and B are independent.

Problem 8. The probabilities for a sample space $S = \{1, 2, 3\}$ are given below:

Outcome	1	2	3
Probability	0.2	0.3	0.5

Let X and Y be two independent observations of a random selection from S, and $Z = X \cdot Y$ be the product of the two independent observations.

a) (1pt) What are the possible outcomes for Z?

b) (2pt) Calculate the probability distribution of Z. Give your result as a table (like the one above).

c) (2pt) Calculate $E(Z)$.

d) (2pt) Calculate the probability that Z is strictly less than its mean.

Problem 9. A movie survey about finds that the probability of a person liking *Ender's Game* is 70%, the probability of liking *The Giver* is 50%, and the probability of liking both is 30%. Let E be the event that a randomly chosen person likes *Ender's Game*, and let G be the event that a randomly chosen person likes *The Giver*.

a) (1pt) Are the events G and E independent?

b) (2pt) A randomly chosen person likes *Ender's Game*. What is the (conditional) probability that they also like *The Giver*?

c) (2pt) A randomly chosen person likes *The Giver*. What is the (conditional) probability that they **dislike** *Ender's Game*?

Problem 10. (3pt) An ordinary coin C_1, and a two-headed coin C_2 are placed in a bag. One of the coins is drawn at random from the bag, and flipped; it comes up Heads. What is the probability that the other side of the coin is also Heads?

Problem 11. (2pt) Out of 100 apples in a crate, 15 have gone bad. You randomly select 10 apples from the crate. Calculate the probability that you end up with at least 1 bad apple.

Problem 12. (2pt) Three inspectors look at a critical component of a rocket. Their probabilities for finding a defect are 0.90, 0.80, and 0.70, and each inspector operates independently of the others. What is the probability that none of the inspectors find a defect?

Problem 13. Sixteen observations of a **binomial** random variable X with parameter $n = 10$ and an unknown p value are given:

$$6 \quad 3 \quad 3 \quad 2 \quad 4 \quad 3 \quad 5 \quad 4 \quad 4 \quad 3 \quad 2 \quad 2 \quad 7 \quad 6 \quad 6 \quad 4$$

a) (1pt) Compute the sample mean.

b) (2pt) Using the sample mean (i.e. treat it as an approximation of the expected value of X), give an estimate for the value of p for this binomial random variable.

c) (2pt) Using the above, estimate the probability that X is at least 3, but at most 8.

Problem 14. (2pt) A child has six cards numbered $\{1, 2, 3, 4, 5, 6\}$. He creates **4-digit** numbers by randomly picking 4 of the 6 cards, and arranging them in some order. Calculate the probability that the number he obtains is smaller than 5200.

Problem 15. (4pt) A small grocery store has 10 cartons of milk left, 3 of which are sour. You need to buy 2 cartons of milk for a recipe. Find the expected value of the number of bad cartons you end up getting.

Problem 16. (2pt) An **unfair coin** has probability of Heads equal to $P(H) = 0.3$. If you toss the coin 28 times, calculate the probability of getting at least 7 Heads.

Problem 17. (3pt) You take a single card at random from 100 different (independent and well-shuffled) decks, so you end up with one hundred cards total. What is the expected number of $\heartsuit$ you end up with?

Problem 18. (2pt) A crate contains 1000 apples, and an unknown number of bad apples. You select 20 apples at random. Let X be the number of good apples you end up with, and Y be the number of bad apples you end up with. Calculate $E(X + Y)$.

Problem 19. (2pt) We learned that the Fibonacci sequence is given by $\{1, 1, 2, 3, 5, 8, 13, \ldots\}$, and that the same Fibonacci relation/formula can be applied to different starting values. Apply the recurrence formula with starting values -11 and 7, and give the first 11 terms of the resulting sequence (including -11 and 7).

Problem 20. (2pt) A game involves tossing an unfair coin with $P(H) = 0.8$. The house charges players x dollars for the opportunity to play the game. If a player tosses Heads, he gets nothing and the house keeps his bet. If the toss is Tails, his bet is returned, plus an additional \$10. How much should the house charge the player in order to make an average of \$2 profit per game?