PRACTICE QUIZ 10 SOLUTIONS

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Time: 15 min
Time to beat: 5 min

Problem 1. Find dy/dx for $y = \frac{1}{x} + \frac{3}{x^2} + \frac{2}{x^3}$.

$$\frac{dy}{dx} = (x^{-1} + 3x^{-2} + 2x^{-3})' = -x^{-2} - 6x^{-3} - 6x^{-4}$$

Problem 2. Find dy/dx for for $y = \frac{2}{\sqrt{x}} + \frac{6}{\sqrt[3]{x}} - \frac{2}{\sqrt{x^3}}$.

$$\frac{dy}{dx} = (2x^{-1/2} + 6x^{-1/3} - 2x^{-3/2})' = -x^{-3/2} - 2x^{-4/3} + 3x^{-5/2}$$

Problem 3. Find the derivative of $f(x) = \frac{2}{x^{1/2}} + \frac{6}{x^{1/3}} - \frac{2}{x^{3/2}} - \frac{4}{x^{3/4}}$.

$$f'(x) = -x^{-3/2} - 2x^{-4/3} + 3x^{-5/2} + 3x^{-7/2}$$

Problem 4. Without using chain rule, find the derivative of $f(x) = (2x+1)^5$. (Hint: the product rule for more than one function is $(f_1 f_2 \cdots f_n)' = f_1' f_2 \cdots f_n + f_1 f_2' \cdots f_n + \ldots\ldots + f_1 f_2 \cdots f_n'$).

Since we are not allowed to use chain rule, we could multipy out and differentiate, but that would be very time consuming. Instead, interpret $f(x)$ as a product of five terms:

$$(2x+1)(2x+1)\cdots(2x+1)$$

Hence its derivative is:

$$(2x+1)'(2x+1)\cdots(2x+1) + \ldots\ldots + (2x+1)(2x+1)\cdots(2x+1)'$$

Now since $(2x+1)' = 2$, each term in the sum is exactly $2(2x+1)^4$, and we have 5 terms in the sum. Hence the answer is $5 \cdot 2(2x+1)^4 = 10(2x+1)^4$.

Problem 5. Without using the chain rule, find the derivative of $f(x) = e^{5x}$.

Similar to the problem above, interpret $f(x) = e^{x+x+x+x+x} = e^x e^x \ldots e^x$ five times.

$$f'(x) = (e^x)' e^x e^x e^x e^x + \ldots\ldots e^x e^x e^x e^x (e^x)'$$

and since the derivative $(e^x)' = e^x$, we actually get a sum of five identical terms

$$f'(x) = e^x \cdots e^x + \ldots\ldots + e^x \cdots e^x = e^{5x} + \ldots + e^{5x} = 5e^{5x}$$