PRACTICE QUIZ 13 SOLUTIONS

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Time: 10 min
Time to beat: ? min

Problem 1. Find the limit $\lim_{x\to -3} \frac{x^3+27}{x+3}$. (Hint: $a^3+b^3=(a+b)(a^2-ab+b^2)$).

We can either use the hint and factor the numerator (and then cancel an $x+3$ factor), or use polynomial long division to see that the limit is equivalent to

$$\lim_{x\to -3}(x^2-3x+9)=27$$

Problem 2. Find the limit $\lim_{x\to 1} \frac{x^2-1}{\sqrt{x}-1}$.

Write the function as

$$\frac{(x+1)(x-1)}{\sqrt{x}-1}=\frac{(x+1)(\sqrt{x}+1)(\sqrt{x}-1)}{\sqrt{x}-1}=(x+1)(\sqrt{x}+1)$$

so the limit as $x\to 1$ is 4.

Problem 3. Find the left-sided limit $\lim_{x\to 1^-} \frac{x^2-1}{|x^3-x^2|}$.

Note that $x^3-x^2=x^2(x-1)$, so for $x<1$ the absolute value is

$$|x^3-x^2|=|x^2(x-1)|=x^2|x-1|=x^2(1-x)=-x^2(x-1)$$

Thus the limit is

$$\lim_{x\to 1^-}\frac{(x+1)(x-1)}{-x^2(x-1)}=\lim_{x\to 1^-}\frac{x+1}{-x^2}=-2$$

Problem 4. For the function

$$f(x)=\begin{cases} \frac{17}{5}-\frac{1}{5}x & \text{if } x<-3 \\ 5(x+3)^2-1 & \text{if } -3\le x<2 \\ 10x+105 & \text{if } x\ge 2 \end{cases}$$

determine if f is continuous at $x=-3$ and $x=2$.

For point $x=-3$, the limit from the left is 4, while the limit from the right is -1, so the limit does not exist, and f is not continuous at $x=-3$

For point $x=2$, the limit from the left is 124, while the limit from the right is 125, so again the limit does not exist, and f is not continuous at $x=2$.

Note: both cases are jump discontinuities as the one-sided limits are different.