## Worksheet 8, Math 10560

- 1. (a) State the comparison test.
  - (b) State the limit comparison test:
  - (c) For what values of p does the p-series  $\sum_{n=1}^{\infty} \frac{1}{n^p}$  converge/diverge?
  - (d) Use the comparison test or the limit comparison test to determine which of the following series are convergent? (Say which test you are using, which known series you are comparing to and show the work in making your conclusion of converges/diverges.)

i) 
$$\sum_{n=1}^{\infty} \frac{\ln n}{n} \left(\frac{2}{3}\right)^n$$

ii) 
$$\sum_{n=1}^{\infty} \frac{1}{(4n)^2 + n + 1}$$
.

iii) 
$$\sum_{n=1}^{\infty} \frac{n}{\sqrt[3]{3n^2 + 7}}$$

iv) 
$$\sum_{n=100}^{\infty} \sin^2\left(\frac{\pi}{n^2}\right)$$

- 2. (a) State the alternating series test:
  - (b) Can the alternating series test be used to show that a series diverges?
  - (c) Can you conclude that any of the series shown below converges using the alternating series test? (if so give details). For the series which the alternating series test does not apply, can you use any other test to conclude convergence or divergence?

$$i) \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{\sqrt{n}}$$

ii) 
$$\sum_{n=1}^{\infty} \frac{(-1)^{n-1}e^n}{\sqrt{n}}$$

iii) 
$$\sum_{n=14}^{\infty} \frac{(-1)^{n-1}}{\sqrt{n^3 + n}}$$

iv) 
$$\sum_{n=3}^{\infty} \frac{(-1)^n 2^n}{(n-1)!}$$

3. The series  $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}4}{n^2}$  is an alternating series which satisfies the conditions of the alternating series test. Use the Alternating Series Estimation Theorem to determine the smallest k so that the k-th partial sum is within  $\frac{1}{100}$  of the actual sum.