PRACTICE QUIZ 1 SOLUTIONS

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Time: 10 min
Time to beat: 4 min

Problem 1. If $\lim_{x\to -1} f(x) = 2$, $\lim_{x\to -1} g(x) = 3$, $\lim_{x\to -1} h(x) = 1$, then what is $\lim_{x\to -1} \frac{f(x)h(x)}{g(x)-f(x)}$?

Since in the denominator we have

$$\lim_{x\to -1}(g(x)-f(x)) = \lim_{x\to -1} g(x) - \lim_{x\to -1} f(x) = 3-2 = 1 \neq 0$$

we don't get a zero in the denominator, so we can just plug in, and our limit is

$$\lim_{x\to -1} \frac{f(x)h(x)}{g(x)-f(x)} = \frac{\lim f(x)\cdot \lim h(x)}{\lim g(x) - \lim f(x)} = \frac{(2)(1)}{3-2} = 2$$

Problem 2. Find the limit $\lim_{x\to -1} \frac{x^2+3x+2}{x+1}$.

Factor $x^2+3x+2 = (x+1)(x+2)$, so

$$\lim_{x\to -1} \frac{x^2+3x+2}{x+1} = \lim_{x\to -1} \frac{(x+1)(x+2)}{x+1} = \lim_{x\to -1}(x+2) = -1+2 = 1$$

Problem 3. Find the limit $\lim_{x\to 1} \frac{x^2-6x+5}{x^3-1}$.
(Hint: Use the fact that $a^3 - b^3 = (a-b)(a^2+ab+b^2)$)

Factor the following (the second one uses the hint with $a = x$ and $b = 1$):

$$x^2 - 6x + 5 = (x-1)(x-5)$$

$$x^3 - 1 = (x-1)(x^2+x+1)$$

so our limit becomes

$$\lim_{x\to 1} \frac{x^2-6x+5}{x^3-1} = \lim_{x\to 1} \frac{(x-1)(x-5)}{(x-1)(x^2+x+1)} = \lim_{x\to 1} \frac{x-5}{x^2+x+1} = \frac{-4}{3}$$

Problem 4. Find the right-sided limit $\lim_{x\to 1^+} \frac{\sqrt{x}-1}{x^2-1}$.

Note: the reason we are only talking about the right-sided limit is because the square root function is undefined for negative values of x.

Factor the numerator as $(x+1)(x-1)$, and also multiply by the conjugate of the denominator. This gives us a new numerator of $(\sqrt{x}-1)(\sqrt{x}+1) = x-1$, so the limit is:

$$\lim_{x\to 1^+} \frac{x-1}{(x+1)(x-1)(\sqrt{x}+1)} = \lim_{x\to 1^+} \frac{1}{(x+1)(\sqrt{x}+1)} = \frac{1}{(2)(2)} = \frac{1}{4}$$