

Name: Solution

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M20580 L.A. and D.E. Tutorial  
Quiz 4

1. Let  $A, B, C$  be  $3 \times 3$  matrices with  $\det(A) = -2$ ,  $\det(B) = \frac{1}{3}$ , and  $\det(C) = \frac{1}{5}$ . What is  $\det(3A^{-1}B^3C^T)$ ?

$$\det(3A^{-1}B^3C^T) = 3^3 \cdot (-2)^{-1} \cdot \left(\frac{1}{3}\right)^3 \cdot \frac{1}{5} = -\frac{1}{2} \cdot \frac{1}{5} = -\frac{1}{10}$$

2. Consider the linear system of equations:

$$x_1 + 2x_2 + x_3 = 2$$

$$-x_2 + 2x_3 = 1$$

$$3x_1 + x_3 = 0$$

Use **Cramer's Rule** to find  $x_3$ . **Caution!** You must use Cramer's Rule and show all your work to receive full credit.

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 0 & -1 & 2 \\ 3 & 0 & 1 \end{bmatrix}, \quad \vec{b} = \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}$$

$$\begin{aligned} \det(A) &= \begin{vmatrix} 1 & 2 & 1 \\ 0 & -1 & 2 \\ 3 & 0 & 1 \end{vmatrix} = 1 \cdot \begin{vmatrix} -1 & 2 \\ 0 & 1 \end{vmatrix} + 3 \cdot \begin{vmatrix} 2 & 1 \\ -1 & 2 \end{vmatrix} \\ &= -1 + 3 \cdot 5 \\ &= 14 \end{aligned}$$

$$\det(A_3(\vec{b})) = \begin{vmatrix} 1 & 2 & 2 \\ 0 & -1 & 1 \\ 3 & 0 & 0 \end{vmatrix} = 3 \cdot \begin{vmatrix} 2 & 2 \\ -1 & 1 \end{vmatrix} = 3 \cdot 4 = 12$$

$$x_3 = \frac{\det(A_3(\vec{b}))}{\det(A)} = \frac{12}{14} = \frac{6}{7}$$