# PRACTICE QUIZ 18 SOLUTIONS

ADRIAN PĂCURAR

**Time: 10 min**
**Time to beat: ? min**

**Problem 1.** Find the critical points of $f(x) = x^4 + \frac{20}{3}x^3 - 12x^2$.

The derivative is $f'(x) = 4x^3 + 20x^2 - 24x$ and setting it equal to zero gives us

$$4x(x+6)(x-1) = 0$$

so the three critical points are $x = -6, 0, 1$. Since $f'$ is defined everywhere, we have no additional critical points.

**Problem 2.** Find the critical points of $f(x) = x + \frac{2}{3}\sqrt{3}\cos x$ in the interval $[0, \pi]$.

The derivative is $f'(x) = 1 - \frac{2}{3}\sqrt{3}\sin x$. We set it equal to zero and rearranging we see that we need to solve

$$\sin x = \frac{\sqrt{3}}{2}$$

which happens for $x = \pi/3$ as well as $x = 2\pi/3$ inside $[0, \pi]$.

**Problem 3.** Find the maximum and minimum value of $f(x) = x^3 + \frac{15}{2}x^2 + 3$ on $[-7, 2]$.

The derivative is $f'(x) = 3x^2 + 15x$ which has roots $x = -5, 0$. These are our critical points. Since we are restricted to an interval, we check all the critical points inside that interval (both happen to be inside), as well as the endpoints of the interval:

$$f(-7) = 27.5$$

$$f(-5) = 65.5$$

$$f(0) = 3$$

$$f(2) = 41$$

so our absolute min is 3 (at $x = 0$) and our absolute max is 65.5 (at $x = -5$).

**Problem 4.** Let $f(x) = x^3 - 4x^2 - 4x + 2$. Find all numbers $c \in (5, 10)$ that satisfy the conclusion of the Mean Value Theorem.

Notice $f$ is a polynomial, so it is continuous and differentiable on the entire real line, in particular on the given interval. Hence the MVT applies, and we are interested in finding $c \in (5, 10)$ that satisfies

$$f'(c) = \frac{f(10) - f(5)}{10 - 5} = \frac{562 - 7}{5} = 111$$

The derivative is $f'(x) = 3x^2 - 8x - 4$ and we set it equal to 111.

$$3x^2 - 8x - 4 = 111$$
$$3x^2 - 8x - 115 = 0$$

which factors as

$$(3x - 23)(x + 5)$$

so we get two solutions, $x = -5$ and $x = 23/3 \approx 7.6667$. However only the second one belongs to our interval $(5, 10)$, so the only $c$ that satisfies the MVT theorem on the interval is $c = 23/3$.