Name: Date: 02/01/2018

M20580 L.A. and D.E. Tutorial
Worksheet 3
Sections 1.5, 1.7–1.9

1. Determine if the system has a non-trival solution. If yes, describe all solutions in parametric form.

$$(a)\quad \begin{aligned} x_1 - 2x_2 + x_3 &= 0 \\ 2x_1 + 4x_2 + x_3 &= 0 \\ 3x_1 + 2x_2 + x_3 &= 0 \end{aligned} \qquad (b)\quad \begin{aligned} x_1 - 2x_2 &= 0 \\ 2x_1 + 4x_2 + x_3 &= 0 \\ 3x_1 + 2x_2 + x_3 &= 0 \end{aligned} \qquad (c)\quad x_1 + x_2 + x_3 = 0$$

$$(a) \quad \begin{bmatrix} 1 & -2 & 1 \\ 2 & 4 & 1 \\ 3 & 2 & 1 \end{bmatrix} \text{ becomes } \begin{bmatrix} 1 & -2 & 1 \\ 0 & 8 & -1 \\ 0 & 8 & -2 \end{bmatrix} \text{ becomes } \begin{bmatrix} 1 & -2 & 1 \\ 0 & 8 & -1 \\ 0 & 0 & -1 \end{bmatrix}$$

Thus (a) only has a trivial solution.

$$(b) \quad \begin{bmatrix} 1 & -2 & 0 \\ 2 & 4 & 1 \\ 3 & 2 & 1 \end{bmatrix} \text{ becomes } \begin{bmatrix} 1 & -2 & 0 \\ 0 & 8 & 1 \\ 0 & 8 & 1 \end{bmatrix} \text{ becomes } \begin{bmatrix} 1 & -2 & 0 \\ 0 & 8 & 1 \\ 0 & 0 & 0 \end{bmatrix} \text{ becomes } \begin{bmatrix} 1 & 0 & 1/4 \\ 0 & 1 & 1/8 \\ 0 & 0 & 0 \end{bmatrix}$$

Thus (b) has a non-trival solution.

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = s \begin{bmatrix} -1/4 \\ -1/8 \\ 1 \end{bmatrix}$$

And (c) has a non-trival solution.

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = s \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} + t \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$$

2. Describe all solutions of $A\mathbf{x} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$, where

$$A = \begin{bmatrix} 1 & -2 & 0 \\ 2 & 4 & 1 \\ 3 & 2 & 1 \end{bmatrix}.$$

$$(b) \quad \begin{bmatrix} 1 & -2 & 0 & 1 \\ 2 & 4 & 1 & 2 \\ 3 & 2 & 1 & 3 \end{bmatrix} \text{ becomes } \begin{bmatrix} 1 & -2 & 0 & 1 \\ 0 & 8 & 1 & 0 \\ 0 & 8 & 1 & 0 \end{bmatrix} \text{ becomes } \begin{bmatrix} 1 & -2 & 0 & 1 \\ 0 & 8 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$
$$\text{becomes } \begin{bmatrix} 1 & 0 & 1/4 & 1 \\ 0 & 1 & 1/8 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Thus

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + s \begin{bmatrix} -1/4 \\ -1/8 \\ 1 \end{bmatrix}$$

3. Let $A = \begin{bmatrix} 1 & 3 & 0 & 3 \\ -1 & -1 & -1 & 1 \\ 0 & -4 & 2 & -8 \\ 2 & 0 & 3 & -1 \end{bmatrix}$.

(a) How many rows of A contain a pivot position? Does the equation $A\mathbf{x}=\mathbf{b}$ have a solution for each $\mathbf{b}$ in $\mathbb{R}^4$?

$$\begin{bmatrix} 1 & 3 & 0 & 3 \\ -1 & -1 & -1 & 1 \\ 0 & -4 & 2 & -8 \\ 2 & 0 & 3 & -1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 3 & 0 & 3 \\ 0 & 2 & -1 & 4 \\ 0 & -4 & 2 & -8 \\ 0 & -6 & 3 & -7 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 3 & 0 & 3 \\ 0 & 2 & -1 & 4 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 3 & 0 & 3 \\ 0 & 2 & -1 & 4 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Thus there are 3 pivot points and there is not a solution for each b in $\mathbb{R}^4$.

(b) Do the columns of A span $\mathbb{R}^4$?

Thus the columns of A do not span $\mathbb{R}^4$ based on the calculations above.

4. Determine if the vectors are linearly independent.

$$(a) \quad \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 3 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 4 \end{bmatrix} \qquad (b) \quad \begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ 7 \\ 5 \end{bmatrix}, \begin{bmatrix} 3 \\ 10 \\ 7 \end{bmatrix}$$

Clearly the vectors of (a) are all linearly independent. For (b)

$$\begin{bmatrix} 1 & 2 & 3 \\ 3 & 7 & 10 \\ 2 & 5 & 7 \end{bmatrix} \to \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix} \to \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

Thus the vectors of (b) are NOT linearly independent.