Math 10550 Exam 3 Practice Problems

1. What is the geometric idea behind Newton's method?
2. Use Newton's method with initial approximation $x_1 = \frac{3}{2}$ to find the first four approximations to the positive root of $f(x) = x^2 - 2$, i.e. find x_2, x_3, and x_4.
3. Show that $\arctan(x) = x^2$ has exactly two real solutions ($x = 0$ and some other positive number) by showing that the derivative of $f(x) = \arctan(x) - x^2$ has exactly one real root and this root is positive. The value $x = \frac{\pi}{4}$ is not far from the positive solution, use Newton's method (with a calculator) to approximate the root to within 4 decimal places.
4. A fence 8 feet tall runs parallel to a tall building at a distance of 4 feet from the building. What is the length of the shortest ladder that will reach from the ground over the fence to the wall of the building? [Hint: Use similar triangles.]
5. The best size medium box for UPS shipping has to have dimensions subject to the following restriction

$$l + 2w + 2h = 130,$$

where l is the length, w is the width, and h is the height. Assume $h = w$ and find the length and width which give the largest volume subject to this restriction.

6. A piece of wire 12cm long is cut into two pieces. One piece is bent into a square and the other is bent into an equilateral triangle. How should the wire be cut so that the total area enclosed is
   (a) a maximum?
   (b) a minimum?

   Hint 1: The function you want to maximize/minimize should be defined on the closed interval [0, 12].
   Hint 2: Begin by finding an equation for the area of an equilateral triangle given the length of a side.
7. Show that $F(x) = \ln|\sec(x) + \tan(x)|$ is an antiderivative of $f(x) = \sec(x)$.
8. Approximate $\int_0^\pi \sin(x)\,dx$:
   (a) by a left-hand Riemann sum with 3 rectangles;
   (b) by a right-hand Riemann sum with 4 rectangles;
   (c) using midpoints and 3 rectangles.
9. Write a limit defining $\int_1^3 xe^x\,dx$, your answer should only involve i's' and n's. Do not attempt to calculate this limit.
10. In this question you will need to use the following summation formulas:

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2} \qquad \sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6} \qquad \sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4}.$$

Use the definition of the definite integral to compute each of the following, afterward compute each integral using the Fundamental Theorem of Calculus, Part II to check your answers:

   (a) $\int_0^2 x\,dx$
   (b) $\int_0^2 x^2\,dx$
   (c) $\int_0^2 x^3\,dx$
   (d) $\int_1^2 (2x^3 - x^2 + 5x - 4)\,dx$

11. Find an antiderivative of $f(x) = \frac{1}{x^4}$ and apply the Fundamental Theorem of Calculus, Part II for the definite integral $\int_{-1}^{1} \frac{dx}{x^4}$. Explain why your answer does not make sense and why the Fundamental Theorem of Calculus should not be applied in this situation.

12. Compute $g'(x)$ for each of the following:

    (a) $g(x) = \int_{x+2}^{0} \sin^2(t)\,dt$

    (b) $g(x) = \int_{\cos(x)}^{e^x} \frac{x^2}{\cos(x)+\sin(x)}\,dx$

    (c) $g(x) = \int_{\arctan(\ln(x))}^{0} \frac{x}{x^3+1}\,dx$

13. Suppose that $f(x)$ is a continuous, **odd** function satisfying the following:

$$\int_{-3}^{5} f(x)\,dx = \pi \qquad\qquad \int_{5}^{8} f(x)\,dx = e.$$

    Find $\int_3^8 f(x)\,dx$.

14. Compute the following indefinite integrals:

    (a) $\int \frac{2x^3+4}{x^4+4x^2}\,dx$

    (b) $\int e^{2x}\sqrt{4-e^{2x}}\,dx$

    (c) $\int \frac{\arctan(x)}{x^2+1}\,dx$

15. Define $\ln(x) = \int_1^x \frac{1}{t}\,dt$.

    (a) Use the substitution $u = \frac{t}{x}$ to show that $\int_x^{xy} \frac{1}{t}\,dt = \int_1^y \frac{1}{u}\,du$ and conclude by observing the well-known logarithm identity $\ln(xy) = \ln(x) + \ln(y)$.

    (b) Use the substitution $u = \sqrt[n]{t}$ to show that $\int_1^{x^n} \frac{1}{t}\,dt = n\int_1^x \frac{1}{u}\,du$ and conclude by observing the well-known logarithm identity $\ln(x^n) = n\ln(x)$.

16. Compute the following definite integrals in two ways, by apply the results of Exercise 14 along with Part II of the Fundamental Theorem of Calculus and by making appropriate substitutions in the bounds:

    (a) $\int_0^1 \frac{2x^3+4}{x^4+4x^2}\,dx$

    (b) $\int_0^{\ln(2)} e^{2x}\sqrt{4-e^{2x}}\,dx$

    (c) $\int_{-1}^{1} \frac{\arctan(x)}{x^2+1}\,dx$