Name: ______________________________ July 25, 2017

FINITE MATH: QUIZ 9 SOLUTION

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Problem 1. When dealing with the life events of two people (such as Kennedy and Lincoln), we estimated the probability of coincidence of a **single** life event to be 0.01 or 1%. Suppose we are looking at 1,000 independent life events in the lives of two people, and we are interested in the number of coincidences that occur.

a) (1pt) What is the expected number of coincidences for the 1,000 life events?

Notice that $X = \#$ of coincidences follows a binomial distribution with $n = 1000$ and $p = 0.01$. The expected value is given by $\boxed{\mu = E(X) = np = 1000(0.01) = 10}$.

b) (1pt) Calculate the probability of having exactly 10 coincidences.

$$\boxed{P(X = 10) = \binom{1000}{10}(0.01)^{10}(0.99)^{990} = bpdf(1000, 0.01, 10) \approx 0.1257}$$

c) (2pt) Calculate the probability of having at most 10 coincidences (inclusive).

$$\boxed{P(X \leq 10) = bcdf(1000, 0.01, 10) \approx 0.583}$$

d) (2pt) Calculate the probability of having at least 5 coincidences (inclusive).

$$\boxed{P(X \geq 5) = 1 - P(X \leq 4) = 1 - bcdf(1000, 0.01, 4) \approx 0.971}$$

e) (2pt) Calculate the probability of having between 5 and 10 coincidences (inclusive).

$$\boxed{P(5 \leq X \leq 10) = bcdf(1000, 0.01, 10) - bcdf(1000, 0.01, 4) \approx 0.554}$$

Problem 2. The Fibonacci sequence $\{1, 1, 2, 3, 5, 8, 13, 21, \ldots\}$ with starting values $F_1 = 1$ and $F_2 = 1$ (the first two values in the sequence) satisfies the relation

$$F_n = F_{n-1} + F_{n-2}$$

so to get the next number, you add up the previous two values (e.g. $13 = 5 + 8$).

a) (1pt) What sequence do you obtain of you use starting values 0 and 1? Write the first 10 terms (including 0 and 1).

It turns out that we get the same sequence back, except with a zero at the beginning! The first 10 terms are $\boxed{0, 1, 1, 2, 3, 5, 8, 13, 21, 34, \ldots}$.

b) (3pt) What sequence do you obtain if you use starting values 1 and 3? Write the first 10 terms (including 1 and 3).

This time, we obtain a different sequence. The first 10 terms are $\boxed{1, 3, 4, 7, 11, 18, 29, 47, 76, 123 \ldots}$. This is called the **Lucas sequence**, and will yield the same golden ratio as the Fibonacci sequence.