Names _____ Date _____

Math 10560 Worksheet 5
Show all your work to receive credit.

1. Consider the integral

$$\int_1^3 x^5\,dx.$$

(a) Estimate the integral using Simpson's Rule and n = 4. You do not need to simplify your answer.

(b) Estimate the error using the error bound for Simpson's Rule:

$$|E_S| \leq \frac{K(b-a)^5}{180n^4}, \quad K \geq |f^{(4)}(x)|.$$

2. Compute the integral $\int \frac{10}{(x-1)(x^2+9)}\,dx$ by completing the following steps:

   (a) Write $\frac{10}{(x-1)(x^2+9)}$ in partial fraction decomposition form (leaving A, B, C, etc. in the numerators).

   (b) Solve for the partial fraction coefficients A, B, C, etc. above.

   (c) Evaluate the integral $\int \frac{10}{(x-1)(x^2+9)}\,dx$.

3. Determine the following limit:

$$\lim_{x\to\infty} \frac{1}{x(e^{\frac{1}{x}} - 13)}$$

4. Evaluate the integral

$$\int_0^{\pi/4} x\sin(4x)dx.$$

5. A sample of a Cobalt-60 has an initial mass of 6 grams. Let $M(t)$ denote the mass of the sample after t days, $M(t)$ decreases at a rate that is proportional to the amount of the substance present at time t. That is

$$M'(t) = kM(t).$$

Cobalt-60 has a half-life of 1925 days.

(a) Give a formula for $M(t)$. (Solve for all unknown constants).

(b) How long (how many days) will it take for the sample to decrease from 6 grams to 1 gram?

6. Compute

$$\int \frac{1}{\sqrt{x^2 - 14x + 50}}\, dx$$

7. Compute

$$\int \sqrt{3 - 2x - x^2}\, dx.$$

Formula Sheet

$$\sin^2 x + \cos^2 x = 1$$

$$1 + \tan^2 x = \sec^2 x$$

$$\sin^2 x = \frac{1}{2}(1 - \cos 2x)$$

$$\cos^2 x = \frac{1}{2}(1 + \cos 2x)$$

$$\sin 2x = 2 \sin x \cos x$$

$$\sin x \cos y = \frac{1}{2}\big(\sin(x - y) + \sin(x + y)\big)$$

$$\sin x \sin y = \frac{1}{2}\big(\cos(x - y) - \cos(x + y)\big)$$

$$\cos x \cos y = \frac{1}{2}\big(\cos(x - y) + \cos(x + y)\big)$$

$$\int \sec \theta = \ln|\sec \theta + \tan \theta| + C$$