The code with explanations is in the attached R markdown file.

- a) To maximise revenue with a single congestion charge that is applied across both peak and non-peak hours, the optimal price that should be set is: £8. At this price, the total revenue will be £1,349,008.70, and the total emissions are 49,323,716.96 g/km.
- b) To maximise revenue with a peak pricing strategy, the optimal price to charge, when non-peak base price is £7, is £9. With these prices, total revenue is: £1,165,356.52 and the total emissions will be: 33,741,788.19 g/km. Compared to (a), the revenue is 13.6% lower, and emissions are 31.6% lower.
- c) To minimise emissions, while maintaining a minimum revenue of £1,100,000 a day, the optimal price to be set is £14. With this, the revenue is £1,102,469.57, and emissions are 30,325,189.93 g/km. This results in a 5.4% decrease in revenue, and a 10.1% decrease in emissions compared to (b).

## Q2.

- a) Implementing price differentiation allows for revenue maximisation while eroding customer surplus on different levels of price. It may also achieve economies of scale, and allow increased quantities due to lower prices, which may also lead to another competitors markets as a result.
- b) Price differentiation might lead to inaccurately identified customer segments. This may lead to an inability to purchase a product/service, and further lead towards feelings of discrimination. It may also lead to certain customers, who previously paid a unified price, having to pay more, and losing all existing customer surplus.
- c) To model consumer choice, we use surplus, which is the product price subtracted from the customers willingness to pay. The customer then self-selects between two identical products, by seeing which product would give them a higher surplus.

## Q3.

Continuing the example from problem set 1, which was spectator seating at CitySport sports hall. The idea behind it was to introduce ticket fees for spectators. In this example, the aim is to find a better idea of ticket prices for spectators, and how to implement price optimisation/differentiation to improve revenue.

The first idea can be to implement a single price for all tickets, namely to charge students, alumni and family all the same; and to find the ideal price through surveys to figure out the maximum of all WTPs.

Another method to use is peak period pricing, where the sports hall should break the day into multiple time slots to find where the maximum number of spectators arrive, and set a lower base price for all other hours, and to find an optimal peak period price. This can be done again through

surveys, in this case finding the net utility (or surplus) of the spectator (willingness to pay – base price), at each time slot. After this, the revenue at each price point should be calculated, then the optimal price that maximises revenue should be selected. This revenue should then be compared to the revenue from a single price structure, and then the option that maximises revenue should be selected.