Fourier-Enhanced Adaptive Manifold Latent Feature Analysis for Spatiotemporal Signal Recovery

No Institute Given

1 Computational Complexity Analysis

To efficiently incorporate spatiotemporal regularization while maintaining computational feasibility, we analyze the major operations in each training iteration:

- **Loss Computation:** Includes data fitting term, spatial proximity constraint, and temporal dynamic constraint.
- **Gradient Computation:** Involves calculating gradients with respect to latent factors P and E, incorporating both spatial and temporal dependencies.
- **SGD Update:** Performs iterative updates to P and E using stochastic gradient descent.

Loss Computation The total loss function consists of four terms:

$$\begin{aligned}\varepsilon(P,E) = &\underbrace{\beta\|J\circ(W-PE)\|_{\text{Huber}}}_{\text{Data fitting term}} + \underbrace{\lambda_{reg}\left(\|P\|_F^2+\|E\|_F^2\right)}_{\text{Tikhonov regularization}} \\ &+ \underbrace{\lambda_1 \,\text{trace}\left(P^T L_{rw} P\right)}_{\text{Spatial constraint}} + \underbrace{\lambda_2 \,\text{trace}\left(E^T C_{temp} E\right)}_{\text{Temporal constraint}}.\end{aligned} \tag{1}$$

The computational cost for each term is:

- Data fitting term: $O(NTd)$ for Huber loss over observed entries.
- Spatial regularization term: $O(KNd)$ from multiplication $L_{rw}P$, assuming K nonzero entries per row in L_{rw}.
- Temporal regularization term: $O(TSd)$ from multiplication $C_{temp}E$, where S is the number of frequency components.

Thus, the overall complexity for loss computation is:

$$O(NTd + KNd + TSd). \tag{2}$$

Gradient Computation and SGD Update For each observed entry, computing the gradient of the Huber loss has complexity $O(NTd)$, while spatial and temporal regularization gradients contribute $O(KNd)$ and $O(TSd)$, respectively.

Thus, the complexity per iteration is:

$$O(NTd + KNd + TSd). \tag{3}$$

Total Training Complexity The training process runs for T_{iter} iterations. Since each iteration involves updating latent matrices over all observed entries while incorporating spatiotemporal constraints, the total time complexity is:

$$O(T_{\text{iter}}(NTd + KNd + TSd)). \tag{4}$$

For sparse graphs where $K \ll N$ and frequency embedding scales $S \ll T$, the complexity simplifies to:

$$O(T_{\text{iter}} \cdot NTd). \tag{5}$$

Best- and Worst-Case Complexity Depending on the number of iterations T_{iter}, the complexity can be further categorized:

- **Best-case scenario (T_{iter} is small):** The complexity is approximately $O(NTd)$, suitable for small-scale WSNs.
- **Worst-case scenario (T_{iter} is large):** The complexity is $O(T_{\text{iter}}NTd)$, applicable for large-scale datasets but requires hyperparameter optimization to reduce computation time.

Strategies for Complexity Reduction To optimize computational efficiency, the following strategies can be applied:

- **Reduce T_{iter}:** Use adaptive learning rate strategies (e.g., Adam optimizer) to speed up convergence.
- **Reduce d:** Decrease the latent dimension to reduce matrix multiplication overhead.
- **Sparse Optimization of K:** Reduce the number of nonzero elements in the adjacency matrix L_{rw} to optimize computation.

1.1 Algorithm 1: ALFA-FE Computational Complexity Analysis

Algorithm 1 ALFA-FE Training Procedure

Input: Data Matrix W, Graph Laplacian L_{rw}, Temporal Encoding C_{temp}
Output: Latent Matrices P, E

Initialize P, E randomly
for $t = 1$ to T_{iter} **do**
  for $(i, j) \in Y_K$ **do**
    {Loop over observed entries}Compute $\Delta_{i,j}^{t-1} = w_{i,j} - p_{i,\cdot}e_{\cdot,j}$ **if** $|\Delta_{i,j}^{t-1}| \leq \delta$ **then**
      Update $p_{i,\cdot}$ and $e_{\cdot,j}$ using threshold-based rule
    else
      Update $p_{i,\cdot}$ and $e_{\cdot,j}$ using full gradient update
    end if
  end for
end for
Return optimized P, E

Since each iteration updates latent matrices for $|Y_K|$ known entries while incorporating graph and temporal regularization, the time complexity is:

$$O(T_{\text{iter}} \times |Y_K| \times d). \tag{6}$$

Table 1. Summary of Compared Models.

Model	Description
ST-LRMA[4]	Low-rank matrix approximation (LRMA) with global and local dependencies for spatio-temporal signal recovery.
BR-TVGS[5]	Batch reconstruction in LRMA for time-dependent graph signals.
LRDS[2]	LRMA with differential smoothness prior for time-varying graph signals.
RRImpu[3]	Optimal transport-based imputation using an end-to-end machine learning pipeline.
L^3F[7]	Latent feature analysis (LFA) with L_1- and L_2-norm constraints (no explicit spatio-temporal modeling).
TRSS[1]	Graph signal reconstruction with Sobolev smoothness for time-varying data.
LFA-STSR[6]	Spatio-temporal signal recovery based on LFA with structured regularization.
ALFA-FE	Manifold regularization and Huber-norm optimization for robust signal recovery.

2 Experiments

Baselines. The proposed ALFA-FE model is com-pared with seven related state-of-the-art models. Table 1 briefly describes these competitors.

References

1. Giraldo, J.H., Mahmood, A., Garcia-Garcia, B., Thanou, D., Bouwmans, T.: Reconstruction of time-varying graph signalsvia sobolev smoothness. IEEE Transactions on Signal and Information Processing over Networks **8**, 201–214 (2022)
2. Mao, X., Qiu, K., Li, T., Gu, Y.: Spatio-temporal signal recovery based on low rank and differential smoothness. IEEE Transactions on Signal Processing **66**(23), 6281–6296 (2018)
3. Muzellec, B., Josse, J., Boyer, C., Cuturi, M.: Missing data imputation using optimal transport. In: International Conference on Machine Learning. pp. 7130–7140. PMLR (2020)

4. Piao, X., Hu, Y., Sun, Y., Yin, B., Gao, J.: Correlated spatio-temporal data collection in wireless sensor networks based on low rank matrix approximation and optimized node sampling. Sensors **14**(12), 23137–23158 (2014)
5. Qiu, K., Mao, X., Shen, X., Wang, X., Li, T., Gu, Y.: Time-varying graph signal reconstruction. IEEE Journal of Selected Topics in Signal Processing **11**(6), 870–883 (2017)
6. Wu, D., Li, Z., Yu, Z., He, Y., Luo, X.: Robust low-rank latent feature analysis for spatiotemporal signal recovery. IEEE Transactions on Neural Networks and Learning Systems (2023)
7. Wu, D., Shang, M., Luo, X., Wang, Z.: An l 1-and-l 2-norm-oriented latent factor model for recommender systems. IEEE Transactions on Neural Networks and Learning Systems **33**(10), 5775–5788 (2021)