Visualization of Quandles in Mathematica

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Introduction

Quandles are a collection of algebras that have arisen as part of the classification of 3-dimensional knots.

Efficient research into the area has to overcome the following challenges:

- unintuitive or non-user-friendly interface for software that generate models;
- limited scope of supported formats (Mace4, UACalc);
- importing, visualization and analysis;
- dynamic editing and updating;
- storing, exporting and sharing.

Investing large amounts of resources (both human and computational) into the research process makes sharing of new findings among researchers a problem of primary importance.

Focus

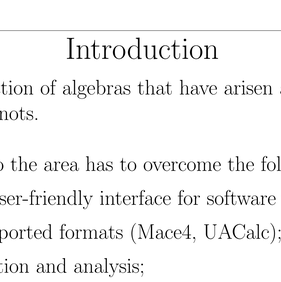
The focus of this project is to design, in Wolfram Mathematica, an interface for visualizing of the following features of finite quandles:

- operation tables;
- subquandles;
- cycle structure;
- right Cayley graphs.

Quandles

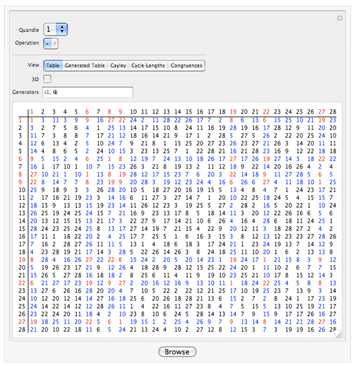
Quandles are a class of algebras that are **idempotent**, **right-cancellable** and **right self-distributive**. Properties: Involutory, connected, Latin, etc.

A **subquandle** Q' of a quandle Q is a subset of Q closed under the operations as defined in Q.

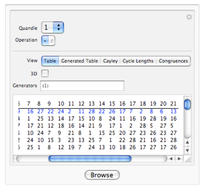


Operation Tables

Working with algebras over large sets presents QuandleViewer with the problem of how to efficiently display only portions of the object.



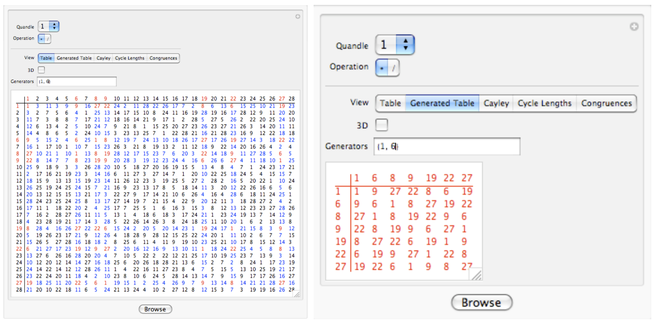
The figure above shows a problem associated with displaying an important but fairly large quandle. In this case Mathematica displays as much output on the screen as it can fit, leaving the rest of the data outside the visible area.



QuandleViewer deals with this problem by presenting only parts of the algebra at a time, leaving no data outside the area that can be manipulated.

Subquandles

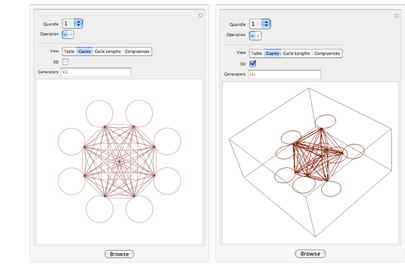
The problem of finding a subalgebra (subquandle) of an algebra (quandle) is generally a very computationally intensive task. Therefore, it is crucial to visualize the results in an intuitive way that would extract and retain only the relevant portions of data.



Presenting the data in this manner eases significantly the process of analyzing the results.

Cayley Graphs and Connectedness

The notion of connectedness in quandles is the property that every column of an operation table is a permutation of the elements of the quandle. This is visualized best by the right Cayley graph for the algebra.

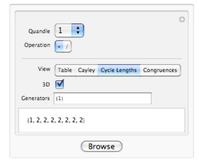


A quandle is connected if its right Cayley graph is connected.

Cycle Structure

For connected quandles, the right-cancellation property together with connectedness provide the cycle structure invariant. Starting with the elements in one column by the virtue of right translation (multiplying on the right one or multiple times with a fixed element), we form an image that happens to be another column in the same quandle, thereby preserving of the cycle structure.

- ▶ Right translation is a quandle homomorphism.
- ▶ All columns in a connected quandle have the same cycle structure.



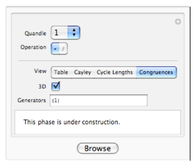
This invariant can be used to quickly determine if two connected quandles are identical up to isomorphism.

Future Work

At present ASC members are working in several different directions to extend the functionality of QuandleViewer:

- Latice of congruences viewer;
- Fish-eye View for large quandles;
- Internet connectivity;

  together with a Common format for representation;
- Various Export formats.



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