Bayes' rule – a first example

Sensor fusion & nonlinear filtering

Lars Hammarstrand

BAYES' RULE: A FIRST EXAMPLE

Selecting fruit from an urn

- An urn is selected at random (prob. $1/2$, $1/2$).
  From that urn we pick a fruit.

- If fruit is orange, what is probability that we chose the red urn?

PROBABILITY THEORY

- Bayesian statistics is simple! We only need two rules:

Conditional probability (product rule)

$$\Pr\{y, \theta\} = \Pr\{y|\theta\} \Pr\{\theta\}$$

The law of total probability (sum rule)

$$\Pr\{y\} = \sum_{\theta} \Pr\{y, \theta\} \qquad \text{discrete variables}$$

$$p(y) = \int_{\theta} p(y, \theta)\, d\theta \qquad \text{continuous variables}$$

PROBABILITY THEORY – BAYES' RULE

- Bayes' rule is a consequence of conditional probability,

$$p(y|\theta)p(\theta) = p(\theta|y)p(y).$$

Bayes' rule

$$p(\theta|y) = \frac{p(y|\theta)p(\theta)}{p(y)}$$

- Usage of Bayes' rule: express a relation of interest, $p(\theta|y)$, in terms of the relation that we know $p(y|\theta)$.
- Note that $p(y) = \int_\theta p(y|\theta)p(\theta)\, d\theta$.

BAYES' RULE: A FIRST EXAMPLE

- Let $\theta \in \{r, b\}$ be color of urn, and $y \in \{o, a\}$ be the fruit.
- Question: If fruit is orange, what is probability that we chose the red urn?

$$\Pr\{\theta=r \mid y=o\} = \frac{\Pr\{y=o \mid \theta=r\}\Pr\{\theta=r\}}{\Pr\{y=o\}} = \frac{\frac{3}{4}\cdot\frac{1}{2}}{\frac{1}{2}} = \frac{3}{4}$$

$$\Pr\{y=o\} = \Pr\{y=o, \theta=r\} + \Pr\{y=o, \theta=b\} = \frac{3}{4}\cdot\frac{1}{2} + \frac{1}{4}\cdot\frac{1}{2} = \frac{1}{2}$$

BAYES' RULE: A FIRST EXAMPLE

- Let $\theta \in \{r, b\}$ be color of urn, and $y \in \{o, a\}$ be the fruit.
- Question: If fruit is orange, what is probability that we chose the red urn?
- Bayes' rule gives

$$\Pr\{\theta = r | y = o\} = \frac{\Pr\{y = o | \theta = r\} \Pr\{\theta = r\}}{\Pr\{y = o\}}$$

where $\Pr\{\theta = r\} = 1/2$, $\Pr\{y = o | \theta = r\} = 3/4$ and

$$\begin{aligned}\Pr\{y = o\} &= \Pr\{y = o, \theta = r\} + \Pr\{y = o, \theta = b\} \\ &= \frac{3}{4}\frac{1}{2} + \frac{1}{4}\frac{1}{2} = \frac{1}{2}.\end{aligned}$$

- Thus, $\Pr\{\theta = r | y = o\} = \frac{3}{4}$.