A primer in statistics – Conditional, Joint and marginal distributions

Sensor fusion & nonlinear filtering

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CONDITIONAL DISTRIBUTIONS

- Conditional distributions are indispensable components in sensor fusion, filtering and Bayesian estimation in general.

Conditional distribution (product rule)

- Let x and z be two random variables with the joint pdf $p(x, z)$.
- The *conditional density function*, $p(z|x)$, is defined through

$$p(x, z) = p(z|x)p(x),$$

and if $p(x) \neq 0$ this implies that

$$p(z|x) = \frac{p(x, z)}{p(x)}.$$

$\rightsquigarrow p(z|x=x') = \frac{p(x', z)}{p(x')} \propto p(x', z)$

- Interpretation: $p(z|x)$ describes the distribution of z given that x is known.

CONDITIONAL DISTRIBUTIONS

Example: Candy problem

- Every day Sara decides how many pieces of candy she can have for an after lunch snack.
- With 40% probability she tosses a coin, heads means 1 piece and tails means 0 pieces
- With 60% probability she throws a dice (number on the dice = number of candies).

- If z denotes number of candies she eats

$$\Pr\{z = i | \text{Sara tosses a coin}\} = \begin{cases} 0.5 & \text{if } i = 0, 1 \\ 0 & \text{otherwise} \end{cases}$$

$$\Pr\{z = i | \text{Sara throws a dice}\} = \begin{cases} 1/6 & \text{if } i = 1, 2, \ldots, 6 \\ 0 & \text{otherwise} \end{cases}$$

LAW OF TOTAL PROBABILITY

- Many important results in non-linear filtering is obtained from the law of total probability.

Law of total probability (sum rule)

- If x takes values in a set S_x, the law of total probability states that

$$\text{Discrete:} \qquad \Pr\{z\} = \sum_{x \in S_x} \Pr\{x, z\} = \sum_{x \in S_x} \Pr\{z \middle| x\} \Pr\{x\}$$

$$\text{Continuous:} \qquad p(z) = \int_{x \in S_x} p(x, z)\, dx = \int_{x \in S_x} p(z \middle| x) p(x)\, dx$$

LAW OF TOTAL PROBABILITY

Example: Candy pmf

- To calculate the pmf for the number of candies we use

$$\Pr\{z\} = \sum_{x \in S_x} \underbrace{\Pr\{z|x\} \Pr\{x\}}_{\Pr\{z,x\}},$$

where x is either 'Sara tosses a coin' or 'Sara throws a dice'.

40% 60%

- Hint: First calculate the joint probability of $\Pr\{z, x\}$

$\propto \Pr\{z | x = \text{"coin"}\}$

$\propto \Pr\{x | z = 5\}$

$\Pr\{z, x\}$	z=0	z=1	z=2	z=3	z=4	z=5	z=6	$\Pr\{x\}$
x = coin	0.2	0.2	0	0	0	0	0	0.4
x = dice	0	0.1	0.1	0.1	0.1	0.1	0.1	0.6
$\Pr\{z\}$	0.2	0.3	0.1	0.1	0.1	0.1	0.1	

$\propto \Pr\{z | x = \text{"dice"}\}$