Filtering, smoothing and prediction

Sensor fusion & nonlinear filtering

Lars Hammarstrand

WHAT IS FILTERING?

- *Filtering is about recursively estimating parameters of interest based on measurements.*

Notation

- Let $\mathbf{x}_k$ contain parameters of interest and $\mathbf{y}_k$ the measurements at time k. (Time is usually discrete.)

Objective

- Compute $p(\mathbf{x}_k | \mathbf{y}_{1:k})$ where $\mathbf{y}_{1:k} \triangleq \begin{bmatrix} \mathbf{y}_1 & \mathbf{y}_2 & \dots & \mathbf{y}_k \end{bmatrix}$ contains all data up to time k.

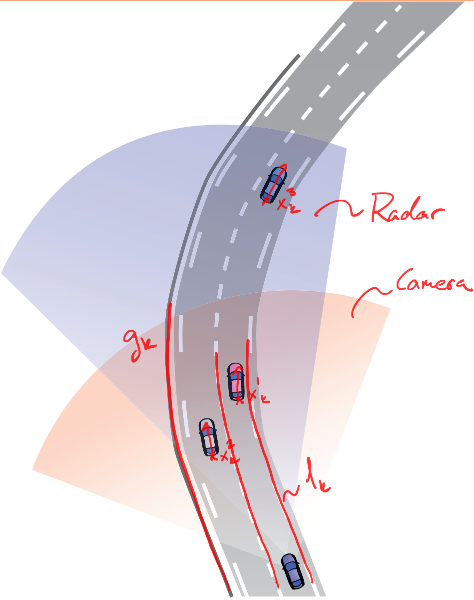
FILTERING IN AUTOMOTIVE APPLICATON

- Vehciles fuses / filters noisy observations from onboard sensor, i.e., radar, lidar and camera, to estimate the current traffic situation:

  $\mathbf{x}_k$: current relative position and velocity of other cars

  $\mathbf{l}_k$: current relative position, headning and shape of the current lane.

  $\mathbf{g}_k$: current relative position, heading and shape of the guard rails.



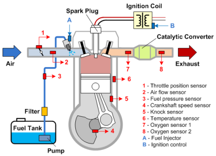
FILTERING IN OTHER APPLICATIONS

- Historically, positioning of airplanes and ships have been important examples.

  $\mathbf{x}_k$: positions and velocities of planes

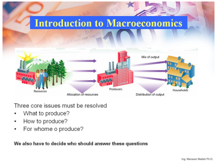
- Control of physical systems often require estimation of the interior state.

  $\mathbf{x}_k$: angle of crankshaft, pressure, etc.



- Often important to assess the states in many other types of systems, e.g., biological or economical.

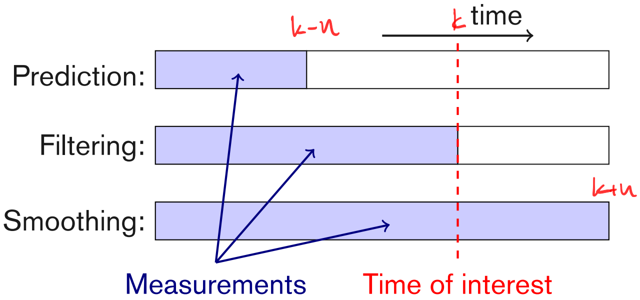
  $\mathbf{x}_k$: diffusion coefficients, spread of a disease or prices.



FILTERING, SMOOTHING AND PREDICTION

- Smoothing and prediction are closely related to filtering.

Smoothing ($n > 0$)	**Prediction ($n > 0$)**
• Compute $p(\mathbf{x}_k \mid \mathbf{y}_{1:k+n})$.	• Compute $p(\mathbf{x}_k \mid \mathbf{y}_{1:k-n})$.



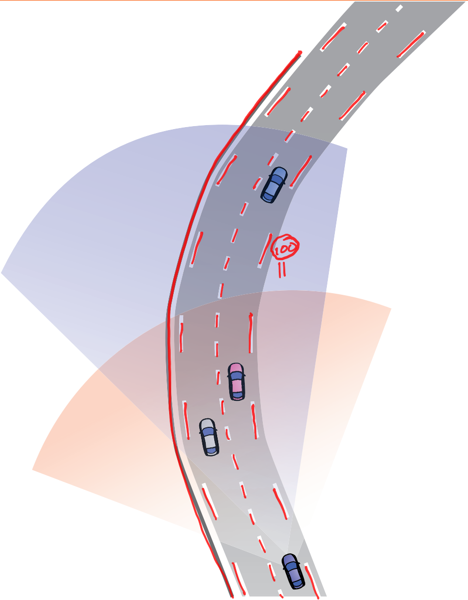
SMOOTHING IN AUTOMOTIVE APPLICATIONS

- Autonmous vehicles use detailed maps to position themselves and to navigate.
- Collect sensor data from many vehicles to jointly estimate their trajectories and the map:

l: global position, headning and shape of the all lanes.

g: global position, heading and shape of the guard rails.

s: global position of signs and its type.

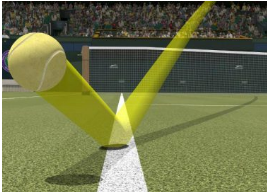


SMOOTHING IN OTHER APPLICATIONS

- Surveillance of, e.g., airports is important for safety reasons.

  $\mathbf{x}_k$: positions of people, bags, etc.

- Other examples:
  - **Communication systems:** having received a complete message you try to decode it.
  - **Sports:** determine where a ball bounced, if someone cheated...
  - **Medicine:** e.g., use sequences of arterial blood pressure to estimate the intracranial pressure.



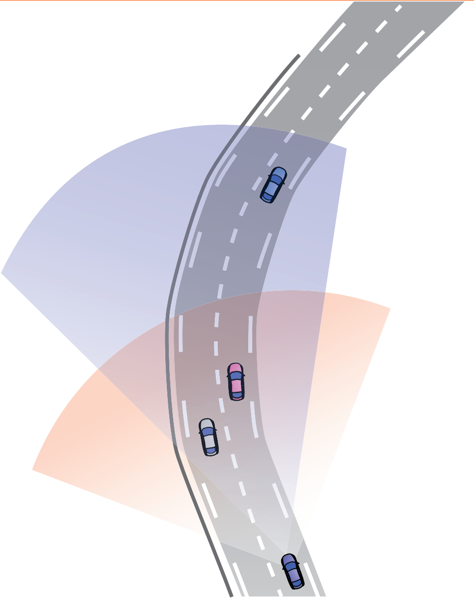
PREDICTIONS IN AUTOMOTIVE APPLICATON

- Vehciles make predicitons of the traffic situation in the near future when, e.g., planning for a safe path or assessing collision risks:

  $\mathbf{x}_{k+n}$: future relative position and velocity of other cars

  $\mathbf{l}_{k+n}$: future relative position, headning and shape of the current lane.

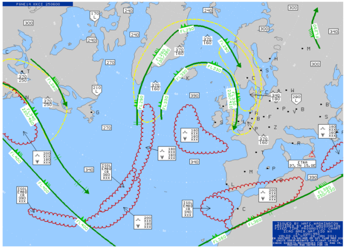
  $\mathbf{g}_{k+n}$: future relative position, heading and shape of the guard rails.



PREDICTION IN OTHER APPLICATIONS

- Weather predictions are important, e.g., to plan routes of airplanes.

  $\mathbf{x}_k$: winds, pressures, temperatures, etc.



- Other examples:
  - **Economy:** the management of companies relies on forecasts of, e.g., demand.
  - **Politics:** many decisions are based on predictions regarding population growth, the financial market, etc.

SELF-ASSESSMENT

Check all that apply.

- The prediction problem is about predicting future measurements given the current state vector.
- In smoothing we conditione on data observed after time k when we compute the distribution of $\mathbf{x}_k$.
- In filtering, smoothing and prediction, both the measurements and the state variables may vary with time.