State space models

Sensor fusion & nonlinear filtering

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DISCRETE-TIME STATE SPACE MODELS

Discrete-time state space models

For a state vector, $\mathbf{x}_k$, and a measurement vector, $\mathbf{y}_k$, where k denotes a discrete time index, we have the following models,

Motion model: $$\mathbf{x}_k = f_{k-1}(\mathbf{x}_{k-1}, \mathbf{q}_{k-1}) \quad (1)$$

Measurement model: $$\mathbf{y}_k = h_k(\mathbf{x}_k, \mathbf{r}_k) \quad (2)$$

where $\mathbf{x}_0 \sim p(\mathbf{x}_0)$.

- We also assume that both the motion noise, $\mathbf{q}_{k-1}$, and the measurement noise, $\mathbf{r}_k$, are independent of all other noise vectors.
  - $\rightsquigarrow$ this ensures the Markov property (see the next video).

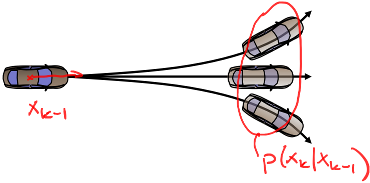
THE MOTION MODEL

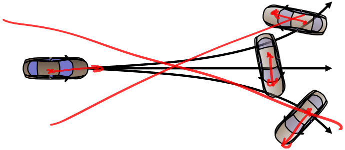
Motion / process model

- The system dynamics are described by (1),

$$\mathbf{x}_k = f_{k-1}(\mathbf{x}_{k-1}, \mathbf{q}_{k-1}), \Longleftrightarrow p(x_k | x_{k-1})$$

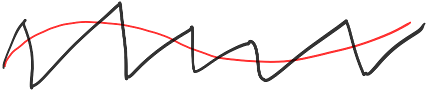
which we refer to as the motion / process model.





Note:

- It describes the state evolution, $p(\mathbf{x}_k | \mathbf{x}_{k-1})$, i.e., the distribution of $\mathbf{x}_k$ given $\mathbf{x}_{k-1}$.
- The motion model thus connects state over time and helps us to rule out unreasonable trajectories.



THE MEASUREMENT MODEL

Measurement model

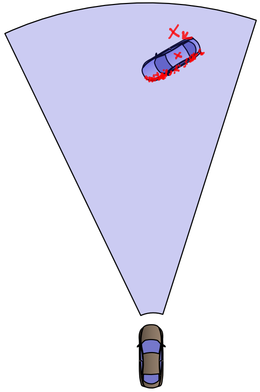
- How the measurements relate to the state vector is described by (2),

$$\mathbf{y}_k = h_k(\mathbf{x}_k, \mathbf{r}_k) \Leftrightarrow p(y_k|x_k)$$

and is called the measurement model or the sensor model.

Note:

- It describes the distribution of $\mathbf{y}_k$ given $\mathbf{x}_k$, $p(\mathbf{y}_k | \mathbf{x}_k)$, i.e., it defines the likelihood function.
- The measurement model relates data to the state vector and helps us to use data to learn about the states.



MODELS WITH INPUT VARIABLES

Known input signal

- The system may also have a known input signal, $\mathbf{u}_k$,

$$\begin{cases} \mathbf{x}_k &= f_{k-1}(\mathbf{x}_{k-1}, \mathbf{u}_k, \mathbf{q}_{k-1}) \\ \mathbf{y}_k &= h_k(\mathbf{x}_k, \mathbf{u}_k, \mathbf{r}_k). \end{cases} \iff \begin{cases} & p(\mathbf{x}_k | \mathbf{x}_{k-1}; \mathbf{u}_k), \\ & p(\mathbf{y}_k | \mathbf{x}_k; \mathbf{u}_k), \end{cases}$$

  The time index for $\mathbf{u}$ in the motion model can also be $k-1$.

- The input signal is often a control signal but it may also be an accurate measurement.

SELF-ASSESSMENT

An important benefit with having both a measurement and a motion model is that past data can provide information about the current state.

- True.
- False.