The Kalman filter

Sensor fusion & nonlinear filtering

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ANALYTICAL SOLUTION TO THE FILTERING PROBLEM

- The filtering equations

$$p(\mathbf{x}_k|\mathbf{y}_{1:k-1}) = \int p(\mathbf{x}_k|\mathbf{x}_{k-1})p(\mathbf{x}_{k-1}|\mathbf{y}_{1:k-1})\,d\mathbf{x}_{k-1} \leftarrow \text{prediction}$$

$$p(\mathbf{x}_k|\mathbf{y}_{1:k}) \propto p(\mathbf{y}_k|\mathbf{x}_k)p(\mathbf{x}_k|\mathbf{y}_{1:k-1}) \leftarrow \text{update}$$

  are applicable to all filtering problems.

- Unfortunately, there are very few examples where the posterior distribution has an analytical expression.

LINEAR AND GAUSSIAN STATE SPACE MODELS

[m>n]

$p(x_{1:m}, y_{1:n})$

$\Rightarrow$ marginal Gaussian

$\Rightarrow$ conditional Gaussian

Definition (Linear and Gaussian models)

- For state vector $\mathbf{x}_k$ and observation $\mathbf{y}_k$,

Transition matrix, process noise, measurement noise, Measurement model matrix

$$\mathbf{x}_k = \mathbf{A}_{k-1}\mathbf{x}_{k-1} + \mathbf{q}_{k-1}, \qquad \mathbf{q}_{k-1} \sim \mathcal{N}(\cancel{\bar{\mathbf{q}}_{k-1}}^{\,0}, \mathbf{Q}_{k-1})$$

$$\mathbf{y}_k = \mathbf{H}_k\mathbf{x}_k + \mathbf{r}_k, \qquad \mathbf{r}_k \sim \mathcal{N}(\cancel{\bar{\mathbf{r}}_k}_{\,0}, \mathbf{R}_k)$$

and $\mathbf{x}_0 \sim \mathcal{N}(\bar{\mathbf{x}}_0, \mathbf{P}_{0|0})$.

Note:

- $p(\mathbf{x}_m | \mathbf{y}_{1:n})$ is Gaussian for all m and n, i.e., for all filtering, smoothing and prediction problems.