The Kalman filter components

Sensor fusion & nonlinear filtering

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COMPONENTS IN THE KALMAN FILTER

$y_k = H_k x_k + r_k \Rightarrow E\{y_k | y_{1:k-1}\} = E\{H_k x_k + r_k | y_{1:k-1}\} = H_k \hat{x}_{k|k-1}$

A few remarks:

- It holds that $p(\mathbf{y}_k | \mathbf{y}_{1:k-1}) = \mathcal{N}(\mathbf{y}_k; \mathbf{H}_k \hat{\mathbf{x}}_{k|k-1}, \mathbf{S}_k)$, which means that $\mathbf{S}_k$ is the predicted covariance of $\mathbf{y}_k$.

- The innovation $\mathbf{v}_k = \mathbf{y}_k - \mathbf{H}_k \hat{\mathbf{x}}_{k|k-1}$ captures the new information in $\mathbf{y}_k$.

- In $\hat{\mathbf{x}}_{k|k} = \hat{\mathbf{x}}_{k|k-1} + \mathbf{K}_k \mathbf{v}_k$, the Kalman gain $\mathbf{K}_k$ determines how much we should trust the new information.

SELF ASSESSMENT

Recall that $\hat{\mathbf{x}}_{k|k} = \hat{\mathbf{x}}_{k|k-1} + \mathbf{K}_k \mathbf{v}_k$ and $\mathbf{v}_k = \mathbf{y}_k - \mathbf{H}_k \hat{\mathbf{x}}_{k|k-1}$.

Suppose $y_k = x_k + r_k$, such that $H_k = 1$, and $r_k \sim \mathcal{N}(0, R)$ (they are all scalar). Check all statements that apply:

- If $R = \infty$ then $K_k \approx 0$.
- If $R = 0$ then $K_k = \infty$.
- If $R = 1$ then $K_k = 0$.
- If $R = 0$ then $K_k = 1$.