An introduction to Bayesian statistics

Sensor fusion & nonlinear filtering

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WHAT IS BAYESIAN STATISTICS?

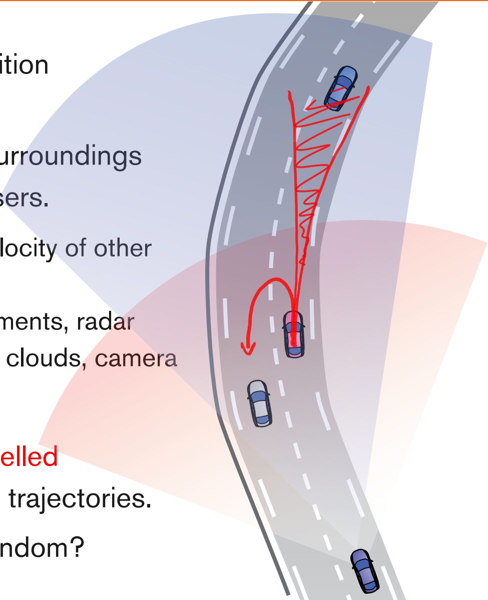
- A statistical inference framework.
- Can be used for estimation, classification, detection, model selection, etc.
- Key characteristic: unknown quantities are described as random.

APPLICATIONS OF BAYESIAN STATISTICS

- A medical application: analyze the disease of a patient.
  - Quantity of interest: the disease, θ.
  - Observations: blood samples, temperature, comments by patient, etc.
- In Bayesian statistics θ is described as random
  $\rightsquigarrow$ we can make statements like: “based on our observations, patient has disease X with 97% probability”.
- Possible concern: is the disease random?

APPLICATIONS OF BAYESIAN STATISTICS

- Self-driving vehicles rely on the ability to position surrounding vehicles.
- This enables the system safely navigate its surroundings without causing accidents with other road users.
  - Quantity of interest: relative position and velocity of other vehicles at the current time.
  - Observations: wheel speeds, INS measurements, radar detections (distance and angle), Lidar point clouds, camera images, etc.
- Bayesian statistics: vehicle motions are modelled statistically $\rightsquigarrow$ helps us to rule out unrealistic trajectories.
- Possible concern: are the vehicle motions random?



COMPARISON: BAYES VS FREQUENTIST

- There are two main strategies to decision making: Bayesian and frequentist statistics.
- In **frequenstist statistics**, the quantities of interest are described as **unknown and deterministic**.

Bayes vs Frequentist

We wish to estimate the height of the Eiffel tower. Is the height random or not?

- **Frequentist perspective:** the tower has a certain height and is therefore not random.
- **Bayesian perspective:** we describe our uncertainties in the height stochastically $\Rightarrow$ height is described as random!

OVERVIEW OF THE BAYESIAN STRATEGY

Suppose we wish to estimate θ given measurements y.

Key steps in a Bayesian method:

1. **Modeling.** Model what we know about θ (using a prior $p(\theta)$) and the how the measurements y relate to θ (using a density $p(y|\theta)$).
2. **Measurement update.** Combine what we knew before (the prior) with our measurement (with $p(y|\theta)$, also called the likelihood) to summarize what we know about θ ($p(\theta|y)$).
3. **Decision making.** Given what we know about θ (described by $p(\theta|y)$) and a loss function, we compute *an optimal decision*.

SELF-ASSESSMENT QUESTIONS

Which of the following statements are correct:

- Bayesian methods can be used to solve many types of decision making problems including estimation, detection and classification.
- We can model the height of the Eiffel tower as random only if we think that there are many similar towers with different heights.
- In Bayesian statistics we describe what we know about θ (the quantity of interest) before observing any measurements.

Check all that apply.