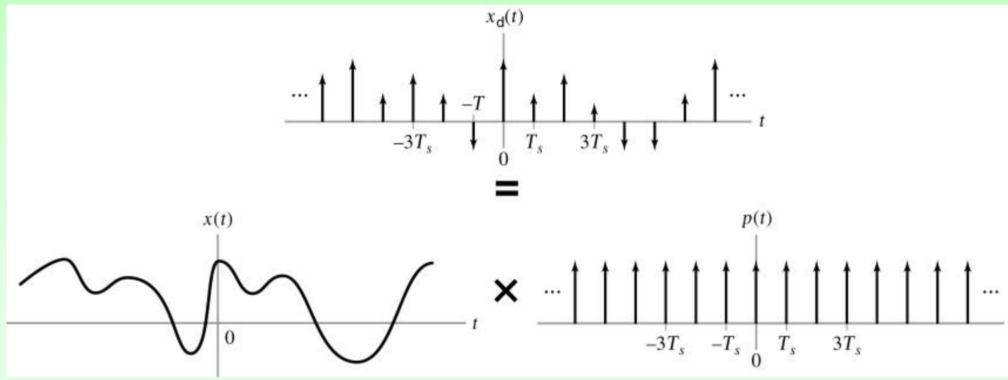
Image Interpolation

Lecturer: Sang Hwa Lee

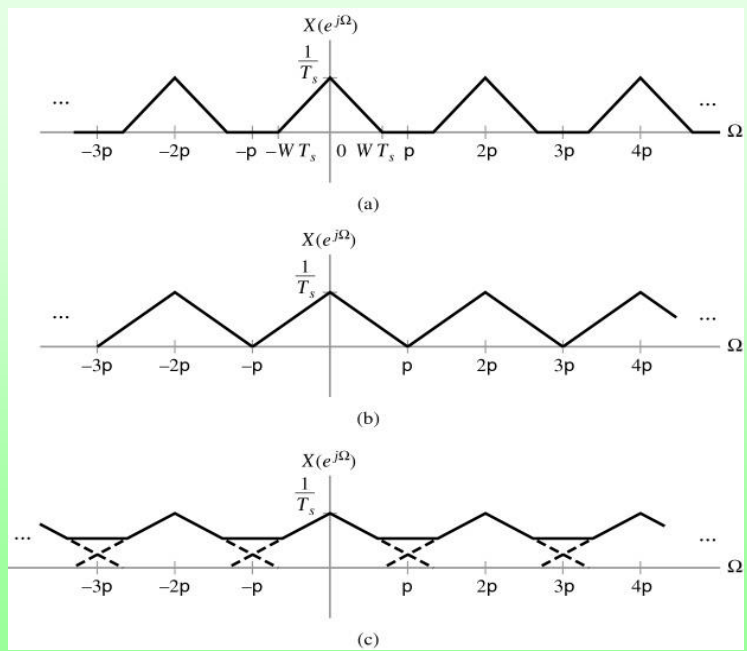
Introduction (I)

❑ Sampling

Sampling
in the time domain



Sampling
in the frequency domain

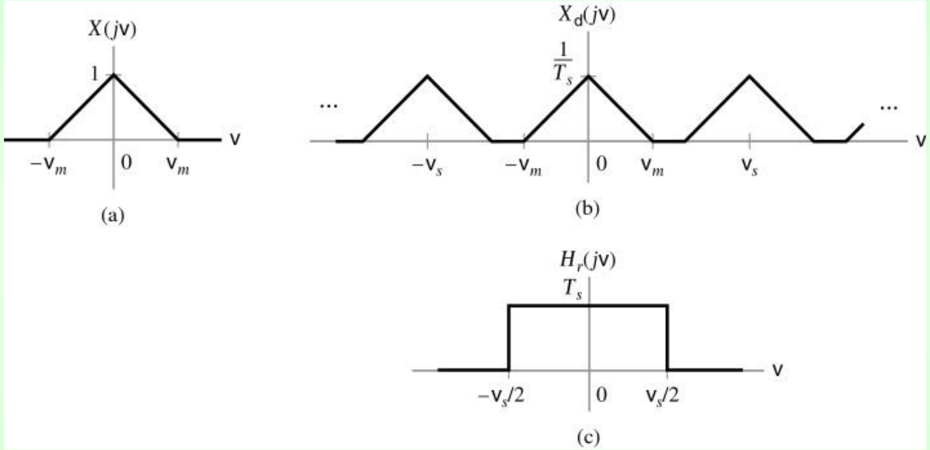


T=1/f

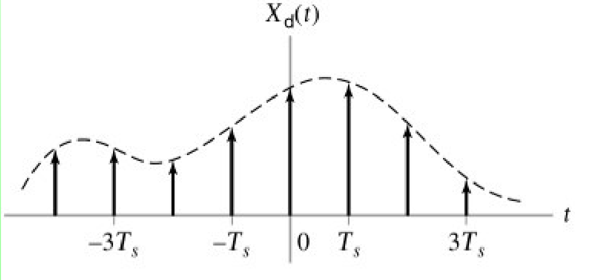
Introduction (II)

❑ Reconstruction

- Reconstruction in the frequency domain



- Reconstruction in the time domain



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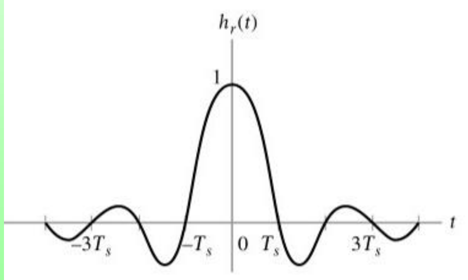

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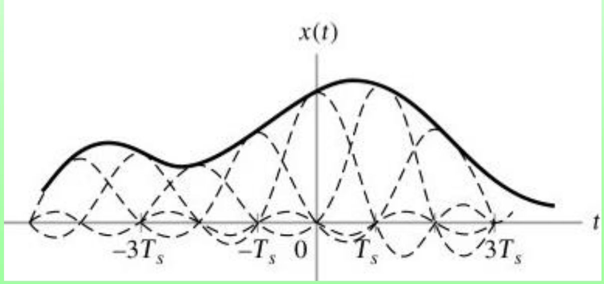
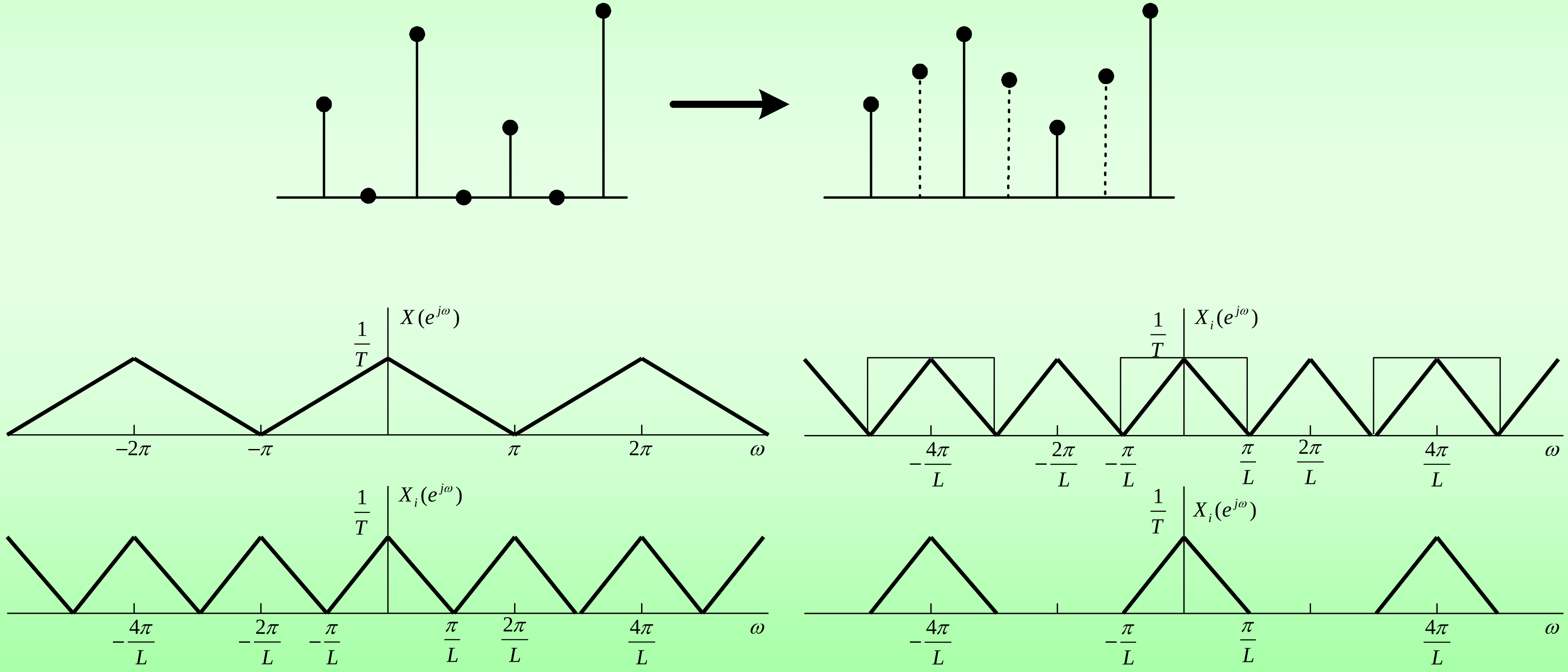


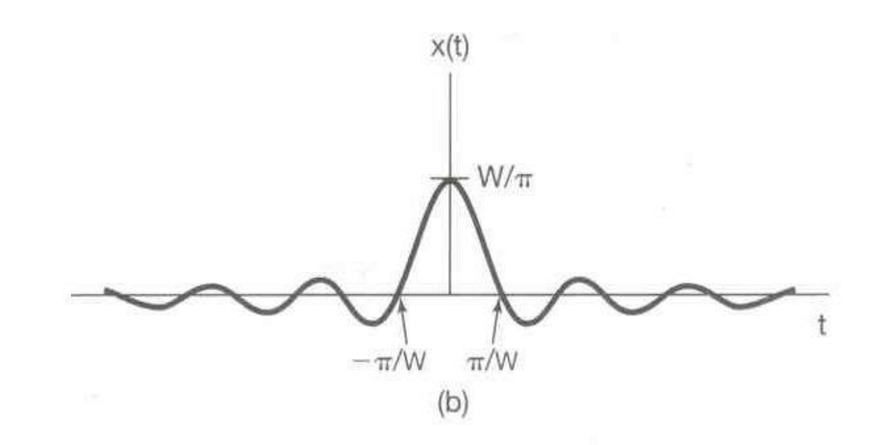
Interpolation (I)

❑ Interpolation in a view of DSP

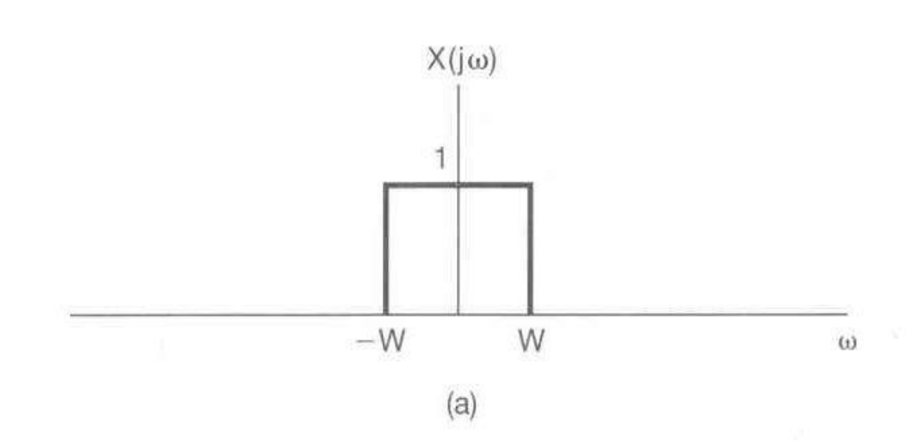


Interpolation (II)

- Ideal LPF (sinc/rectangular function)
  - Ideal interpolation when the signal is bandlimited.



Time domain (Sinc function)



Frequency domain (Rectangular function)

Interpolation (III)

- Why is the interpolation required ?
  - The sampled signal is not bandlimited.
    - An image usually consists of all frequency components.
  - Ideal low pass filter can't be implemented
    - Non-causal, infinite length (IIR)
- Interpolation problem
  - Design of FIR filter which approximate the ideal LPF
    - The interpolators approach sinc function in spatial domain.
  - Spatially adaptive filter design
    - Spatially varying frequency components
  - Anti-aliasing filtering
- Two aspects in interpolation
  - Restoration vs Enhancement

Interpolation (IV)

❑ What is the interpolation ?

- Convert discrete signals to continuous ones
- Filling with weighted sum of the neighboring samples
  - ➢ Weight function: interpolation filter

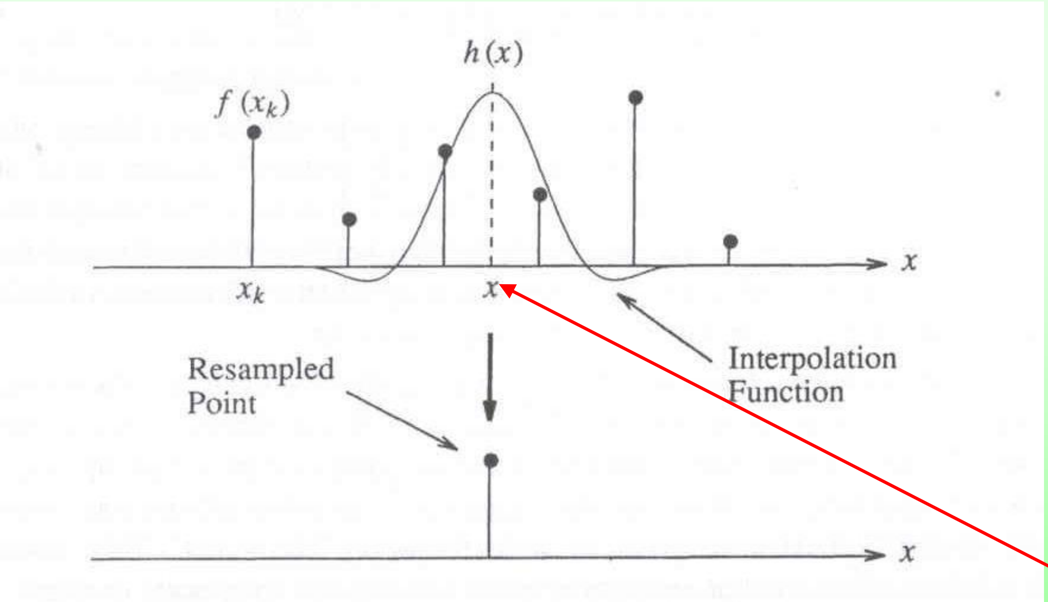


Figure 5.5: Interpolation of a single point.

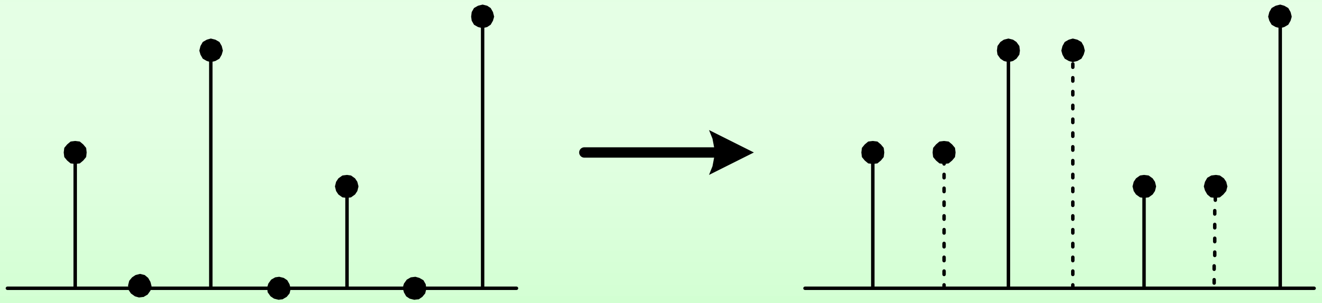
$$f(x) = \sum_{k=0}^{K-1} c_k h(x - x_k)$$

Interpolated site

Nearest neighbor (1)

❑ Nearest neighbor

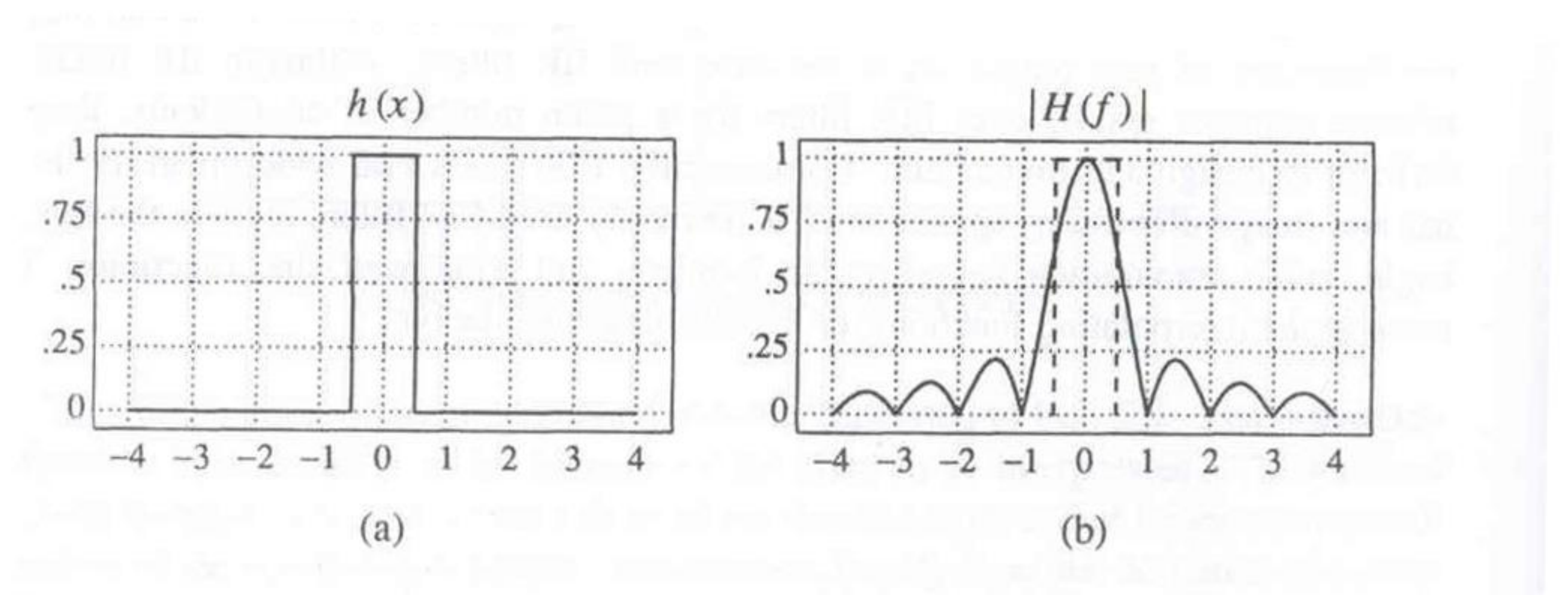
- zero order interpolator
- Assign the value of the nearest sample point



Nearest neighbor (2)

❑ Nearest neighbor

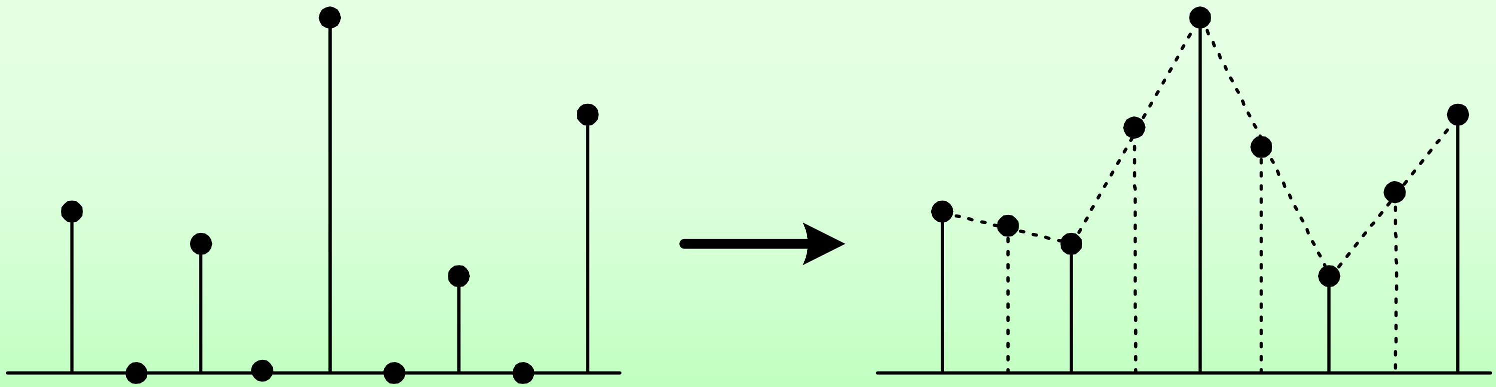
- prominent side lobes
- Poor low pass filter
- Blocky artifact



Linear interpolation (1)

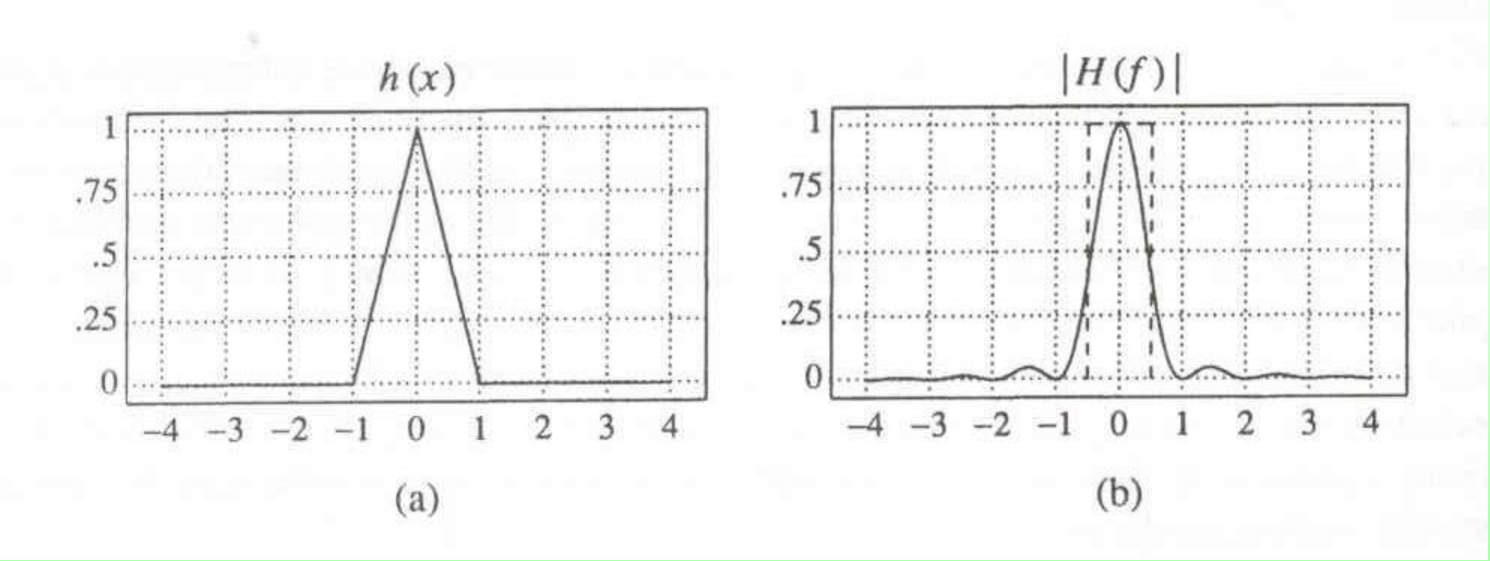
❑ Linear interpolation

- First order interpolator
- Passes a line through two consecutive points



Linear interpolation (2)

- Linear interpolation
  - Superior to nearest neighbor, but insufficient
  - Widely used since reasonably good results at moderate costs

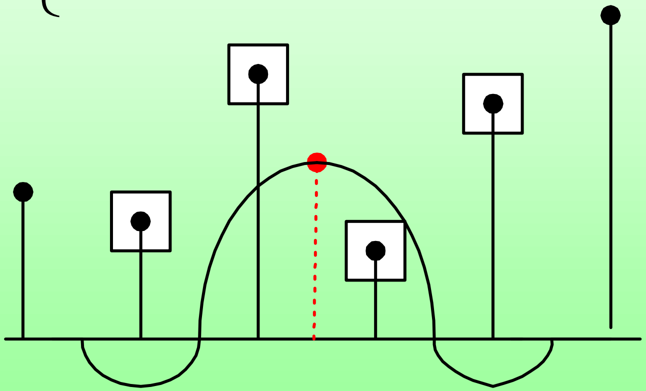


Cubic convolution (I)

Cubic convolution

- Third order interpolator
- Symmetry property of two points on each side
- kernel

$$h(x) = \begin{cases} a_{30}|x|^3 + a_{20}|x|^2 + a_{10}|x| + a_{00} & 0 \le |x| < 1 \\ a_{31}|x|^3 + a_{21}|x|^2 + a_{11}|x| + a_{01} & 1 \le |x| < 2 \\ 0 & 2 \le |x| \end{cases}$$



Cubic convolution (II)

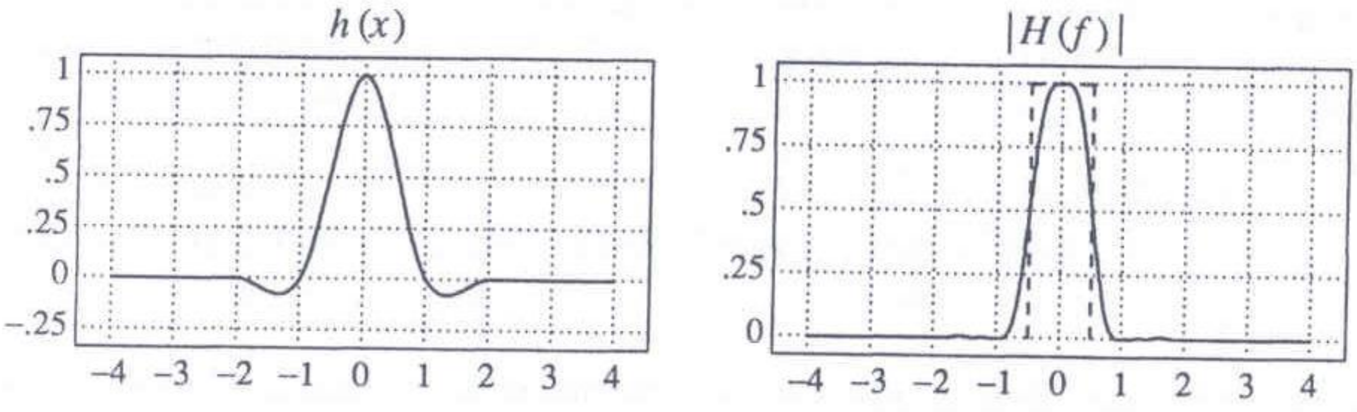
Cubic convolution

- Constraints
  - $h(0) = 1$ and $h(x) = 0$ for $|x| = 1$ and 2.
  - h must be continuous at $|x| = 0, 1,$ and $2.$
  - h must have a continuous first derivative at $|x| = 0, 1,$ and $2.$

$$h(x) = \begin{cases} (a+2)|x|^3 - (a+3)|x|^2 + 1, & 0 \le |x| < 1 \\ a|x|^3 - 5a|x|^2 + 8a|x| - 4a, & 1 \le |x| < 2 \\ 0 & , 2 \le |x| \end{cases}$$

Cubic convolution (III)

- Cubic convolution
  - $a = -1$ ： preferable if visually enhanced results
  - $a = -0.5$ ： mathematically precise



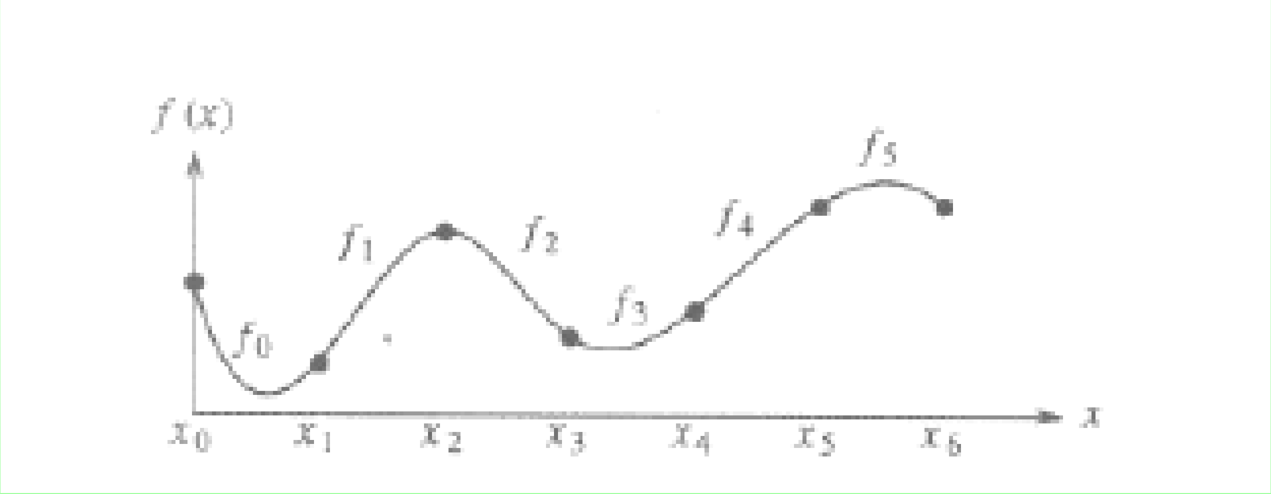
(a) (b)

$a = -1$

Cubic splines

❑ Cubic spline

- Piecewise continuous third-degree polynomials
- Curve fitting using polynomials
- N points → $(N\text{-}1)$th polynomials (# of variables: N)
- Polynomials, f_k are joined at each point x_k such that f_k, f_k' and f_k'' are continuous.



Cubic B splines

- Cubic B splines
  - B-spline: N convolutions of the box filter
  - Guarantee the positivity of the interpolated image
  - Undergoes considerable smoothing
  - Parzen window
  - Not interpolation filter
    - h(0)=4/6
    - An approximation function near the point

$$h(x) = \frac{1}{6}\begin{cases} 3|x|^2 - 6|x|^2 + 4 & 0 \le |x| < 1 \\ -|x|^3 + 6|x|^2 - 12|x| + 8 & 1 \le |x| < 2 \\ 0 & 2 \le |x| \end{cases}$$

Windowed sinc function (I)

- Windowed sinc function (in spatial domain)
  - Signal truncation: FIR
  - Sinc function: ideal LPF, IIR -> FIR filter (by window)
  - multiplication by window → convolution with spectrum of window
- Rectangular window
  - Truncation of sinc function
  - Bad performance in step edges
    - ringing artifacts (Gibbs phenomena)
    - Because of the convolution of sinc function in frequency domain, the stopband has non-zero ripples.
- We need some weighted windows to reduce the ripples

Windowed sinc function (II)

- Rectangular (zero-order)
- Bartlett (first order)
- Hanning (a=0.5)
- Hamming (a=0.54)

$$\begin{cases} a+(1-a)\cos\dfrac{2\pi x}{M-1} & |x|<\dfrac{M-1}{2} \\ 0 & otherwise \end{cases}$$

- Blackman window

$$\begin{cases} 0.42+0.5\cos\dfrac{2\pi x}{M-1}+0.08\cos\dfrac{4\pi x}{M-1}, & |x|<\dfrac{M-1}{2} \\ 0 & otherwise \end{cases}$$

- Kaiser window :

$$\frac{I_0[\beta\sqrt{1-(1-\frac{2n}{M-1})^2}]}{I_0[\beta]},\ 0\le n\le M-1$$

Windowed sinc function (III)

Lanczos window (I)

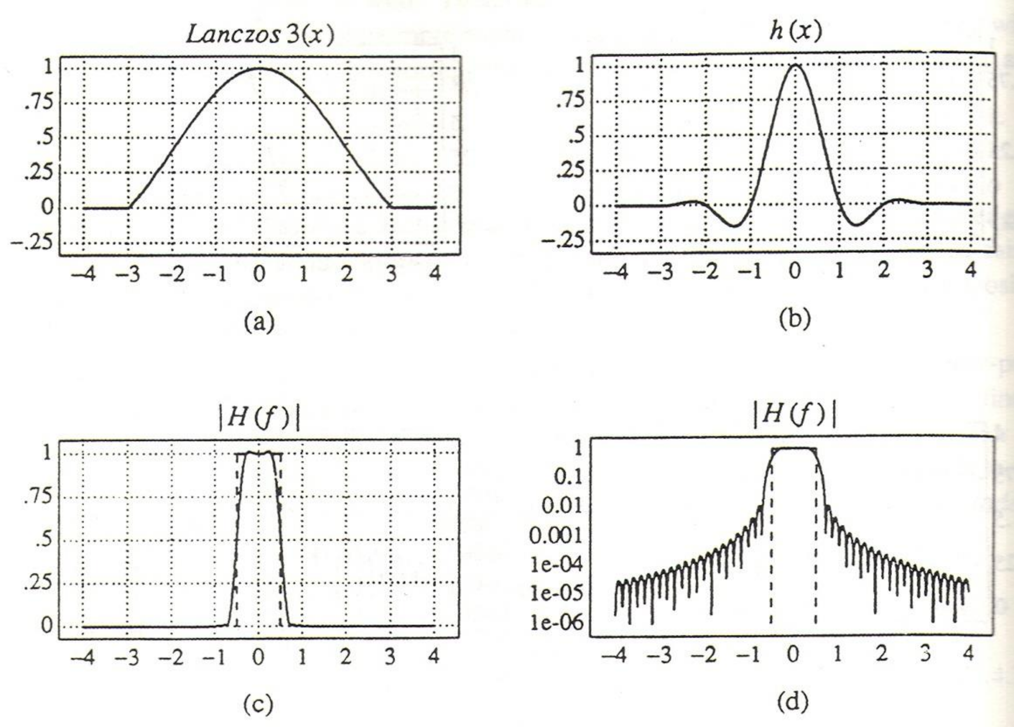
- Reducing ringing artifacts in the windowed sinc functions by constraints on the frequency response
  - Unity gain in passband
  - Zero gain in stopband
  - Linear transition
- Convolution of two box filters in frequency domain
  - Multiplication of two sinc functions in spatial domain
- Example: $\mathrm{sinc}(x)\mathrm{sinc}(x/2)\mathrm{Rect}(x/4)$
  - $\mathrm{Rect}(f)*\mathrm{Rect}(2f)*\mathrm{sinc}(4f)$

Windowed sinc function (IV)

❑ Lanczos window (II)

- N-lobed window function
  - ➢ Determine the extent of non-zero sinc function

$$LanczosN(x) = \begin{cases} \dfrac{\sin(\pi x / N)}{\pi x / N} & 0 \le |x| < N \\ 0 & |x| > N \end{cases}$$



Windowed sinc function (VI)

- Comparison of windowed sinc functions

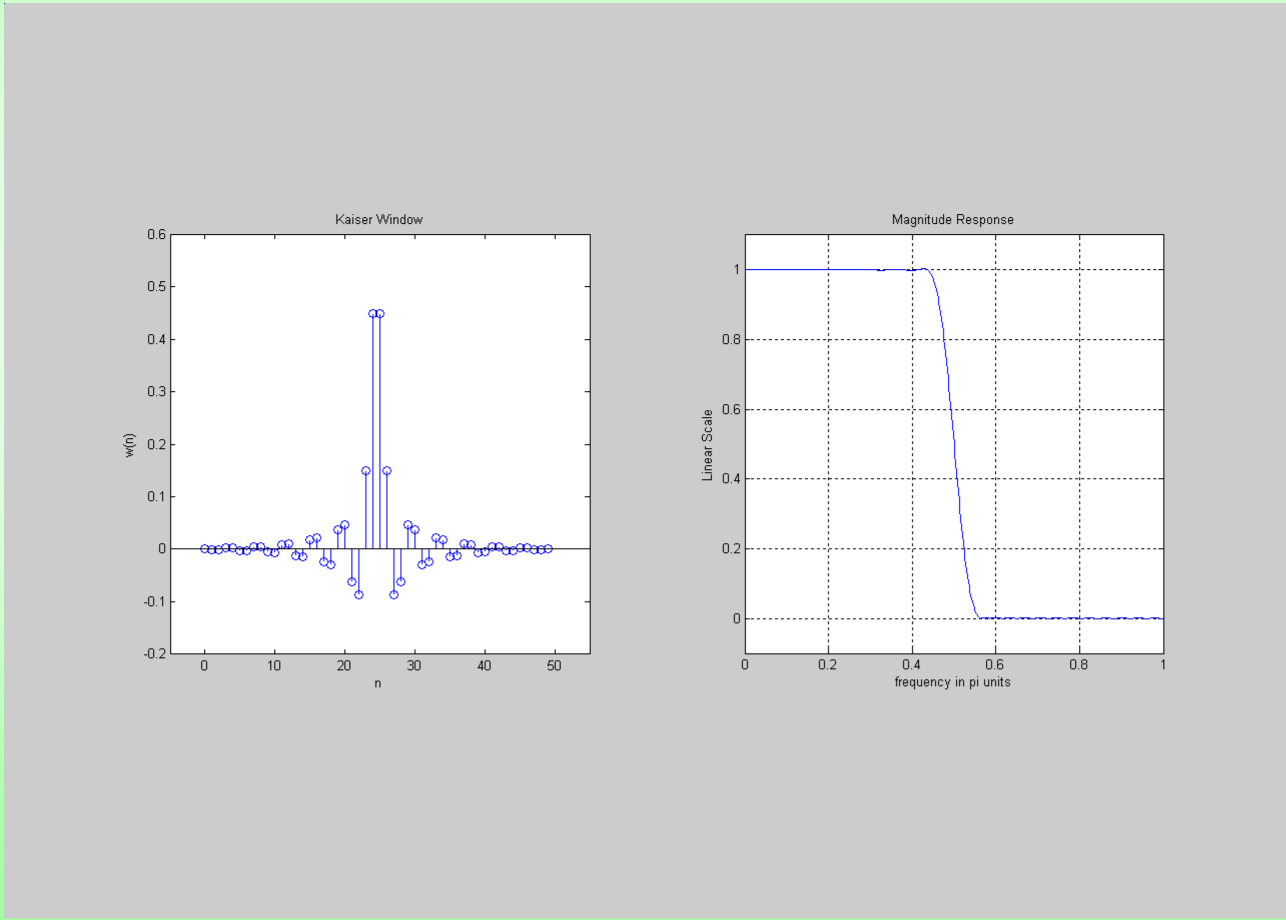
Blackman

Bartlett

Hamming

Hanning

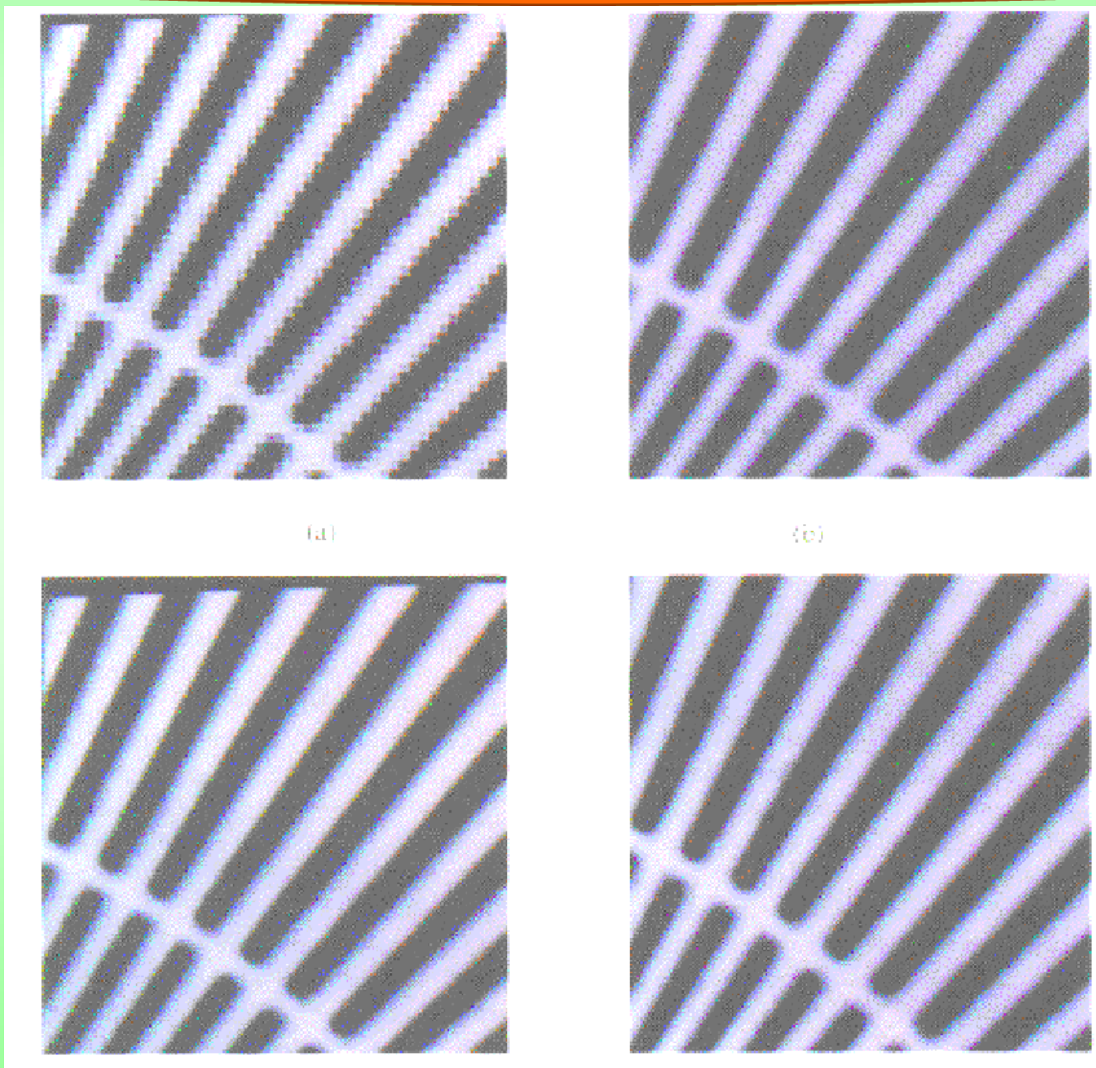
Kaiser



Comparisons of interpolation filters (I)

Nearest neighbor

Linear interpolation



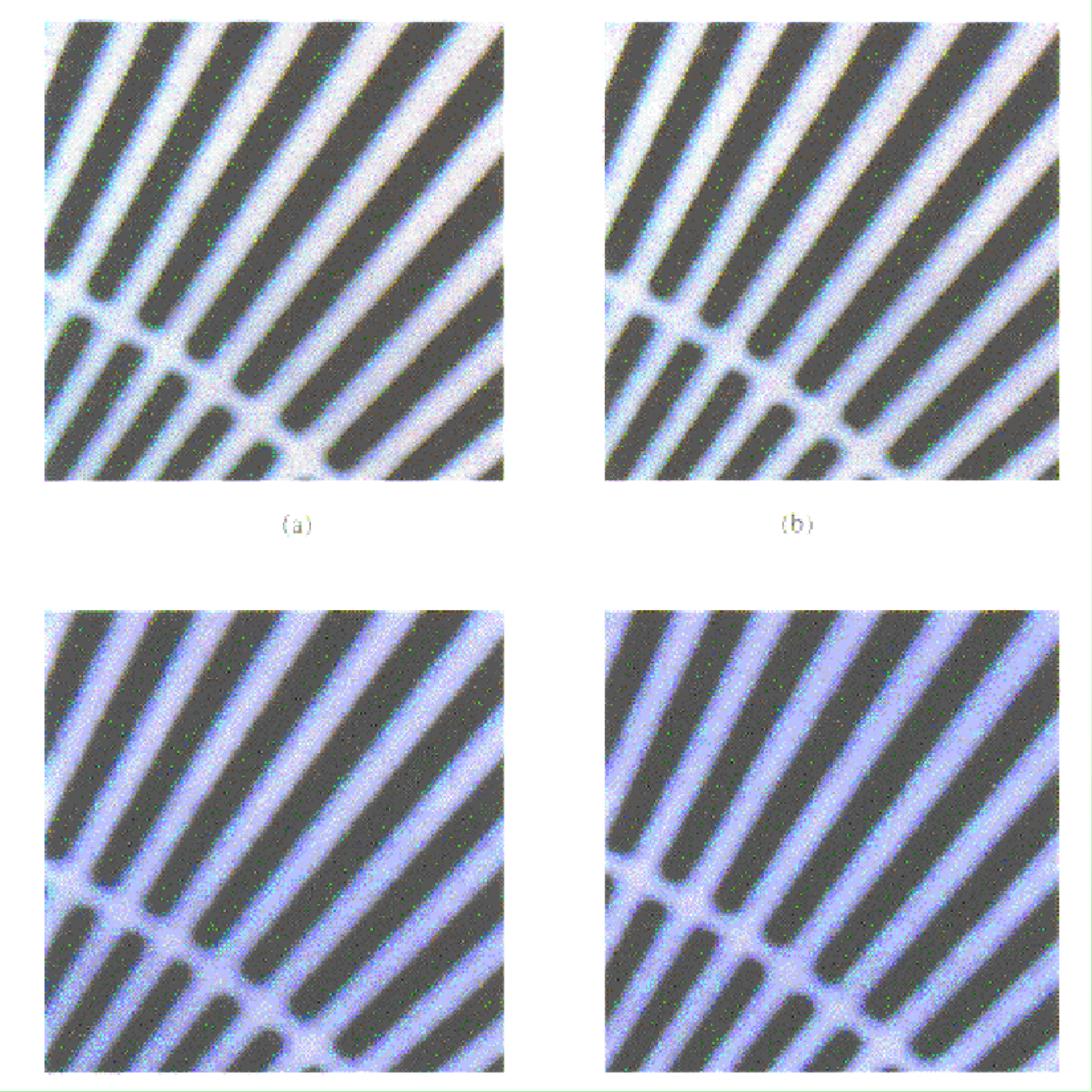
Cubic convo-
Lution (a=-1)

Cubic convo-
Lution (a=-0.5)

Comparisons of interpolation filters (II)

Cubic Spline

Lanczos2 window



Hamming

Exponential filter

Comparisons of interpolation filters (III)

Nearest neighbor

Bilinear

Cubic B spline

Cubic (a=-0.5)

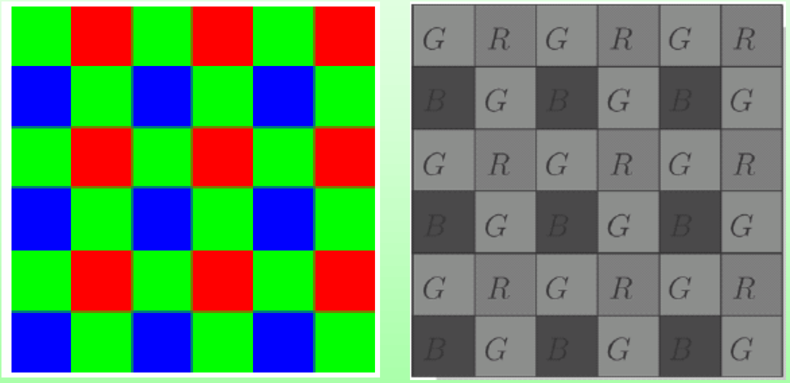
Lanczos window

Image Demosaic

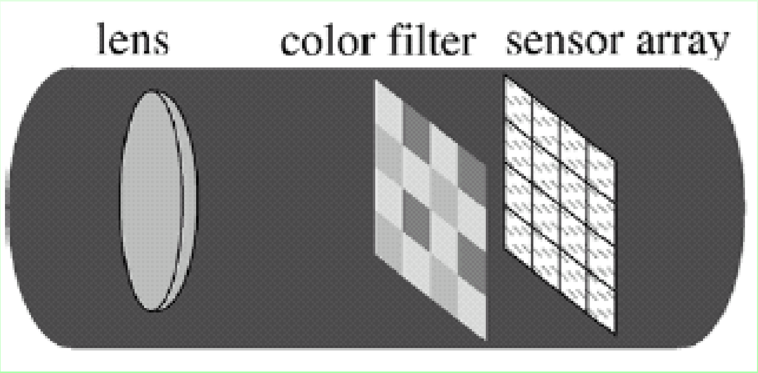
Digital Color Imaging

❑ Bayar CFA (color filter array) pattern in digital camera

- Each CCD cell takes only one color component via mosaiced CFA pattern.
- Color components (RGB) are mosaiced like tiles due to color filter array.
- Green is sampled at a higher rate than red and blue



Bayar CFA pattern



Color image acquisition

Demosaic (1)

- For full resolution color images
  - Interpolation is required for each color component.
  - Inverse process of mosaiced CFA pattern
  - Demosaicing
- Interpolating each color component, (R,G,B)
  - Independent interpolation
  - Using correlations between color components
- Artifacts generated from
  - Image contents (high frequency)
  - Interpolation algorithms
  - CFA pattern: zippering
  - More serous in color images than gray ones

Demosaic Examples (1)

- Some exemplary results (IEEE IP, March, 2005)
  - "Adaptive homogeneity-directed demosaicing algorithm"

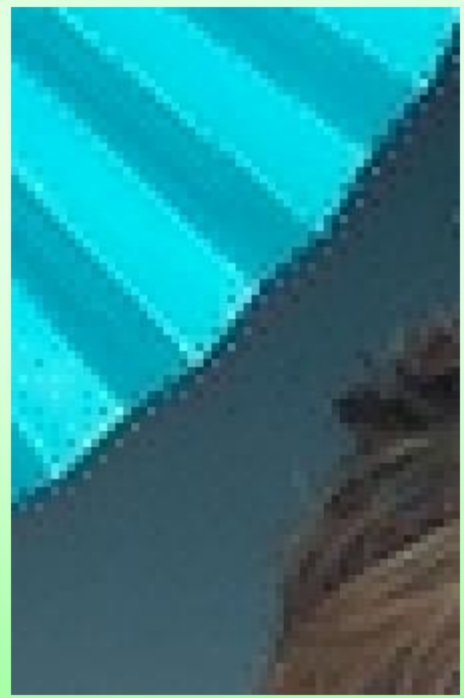
Horizontal interpolation

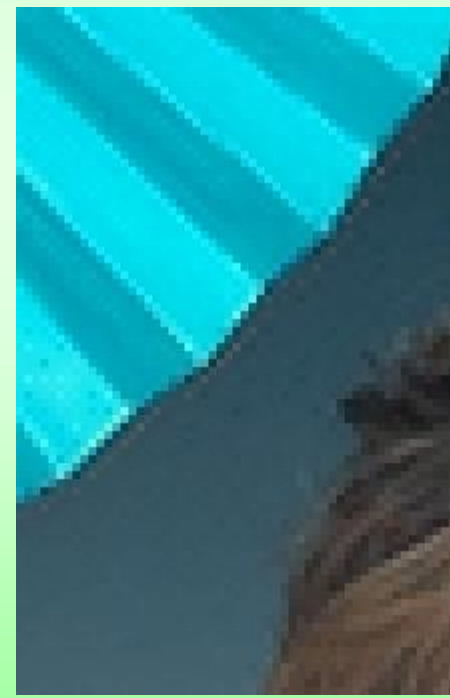
Vertical interpolation

Directional interpolation

Demosaic Examples (2)

- Some exemplary results
  - [1] “Color plane interpolation using alternating projections,” (IP, 2002.9)
  - [2] “Demosaicing using optimal recovery” (IP, 2005.2)





Result in [1]   Result in [2]

magnified part in [1]   in [2]