Wave energy Part 1

Wave energy resource

OCEN4007 Ocean Renewable Energy

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1 Main questions

The main question we would like to answer through these series of lectures on wave energy is this: *Given a site with a supposedly good wave power resource, how do we harness this potential?* To answer this question, we may ask the following sub questions, which we hope to answer by the end of these lectures:

- What makes a good site?
- How do we calculate the wave energy resource at this site?
- What makes a good wave energy converter (WEC)?
- How do we calculate its power output?
- What practical aspects do we need to consider?

2 How waves are generated

Before discussing how to calculate the energy available in the waves, let us briefly consider how ocean waves are generated.

You might have seen ripples generated as a result of wind blowing over a puddle. You would have noticed that as the wind blew over the water surface, the ripples gathered energy and grew in size as they traveled from one side of the puddle to the other side. Ocean waves are formed in much the same way, at a larger scale. Wind generates ripples, which grow into waves. When the waves stop growing, we call this state a fully-developed sea. Waves that have travelled away from their generation area are called swells (see Fig. 1).

Swells can travel thousands of kilometres in the open sea with little loss of energy. Swells arriving at the southwestern coast of Australia, for example, may have originated from a storm thousands of kilometres away (see Fig. 2). As there is little land mass in the Southern Ocean, swells arriving at the southern coast of Australia travel almost uninterrupted. That is why you can find waves at the beach on days when it is not particularly windy.

Wind itself is in turn caused by the difference in atmospheric pressure on the earth surface. This difference is mainly caused by uneven heating of the earth surface by the sun. As energy is

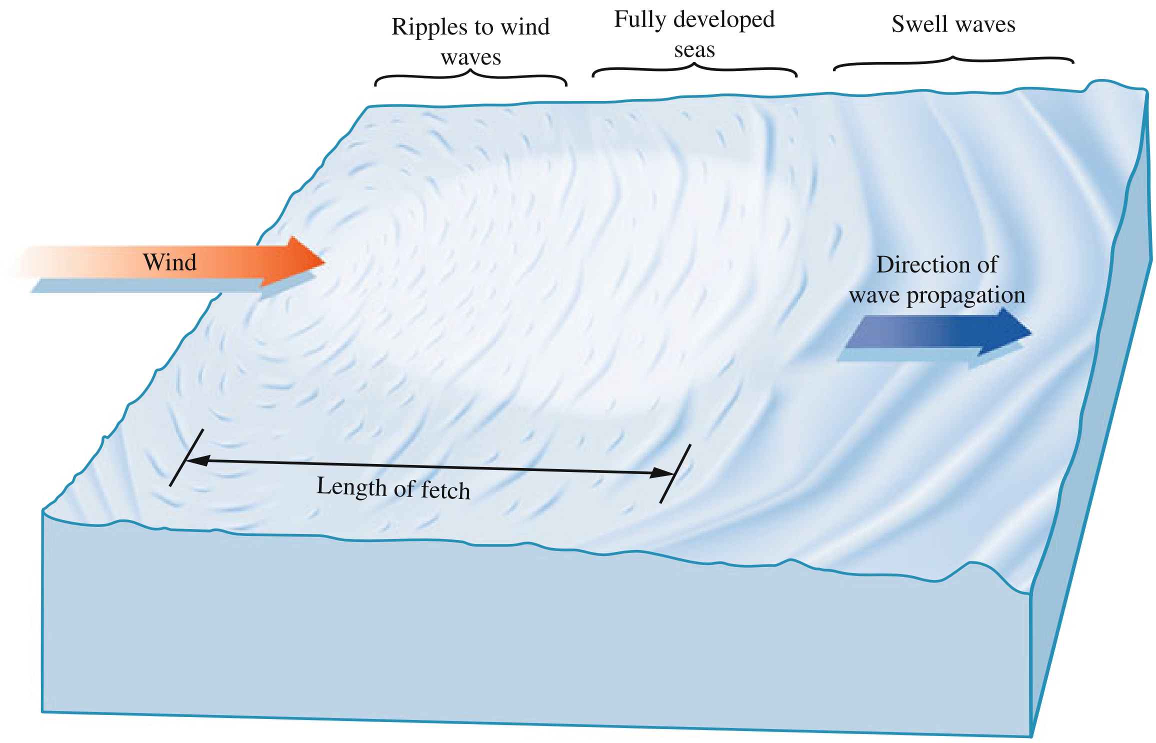

Figure 1: Generation of ocean surface waves [3].

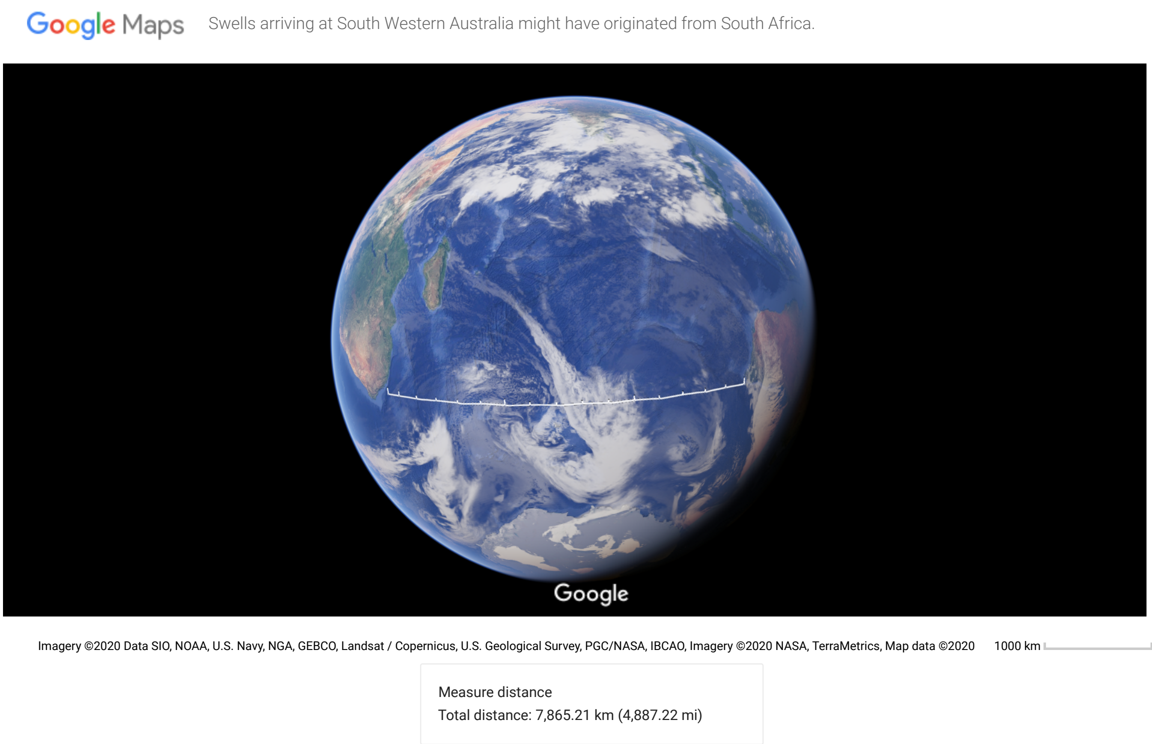

Figure 2: Swells arriving at southwestern coast of Australia might have originated from south of Africa. That is nearly 8000 km away!

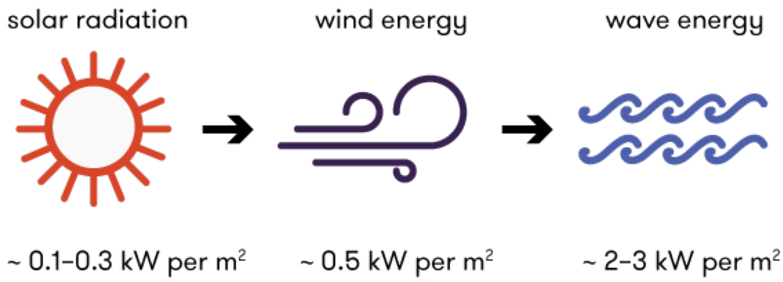

FIGURE 3: Wave energy is a denser form of energy than solar or wind. (Image from [5]. Numbers from [2]).

transformed from solar energy into wind energy and then into wave energy, energy flow becomes intensified. The amount of solar radiation that the earth receives on average is about 0.1-0.3 kW per square metre (of horizontal area), whereas wind transports about 0.5 kW of power per square metre of area perpendicular to the wind direction. Water wave, on the other hand, transports about 2-3 kW per square metre (of vertical area) just below the water surface, where wave energy is most concentrated. Compared to wind and solar, wave energy is a much denser form of energy (see Fig. 3).

3 How much energy do the waves carry?

As wave propagates, energy is propagated with it, while the water particles only move locally (in elliptical orbits). See `https://www.acs.psu.edu/drussell/demos/waves/wavemotion.html` for some nice animations of waves (scroll down to water waves). The amount of energy that the waves carry is normally expressed in terms of the wave-power level (a.k.a. the wave energy transport). It is defined as the wave energy transported per unit time through a vertical strip (extending from the free surface down to the sea bed) of unit width, perpendicular to the wave direction (or parallel to the wave front). It is normally expressed in kW/m.

3.1 Some useful approximate formulas

There exists two useful approximate formulas for the wave-power level of waves in deep water: one for regular waves, and the other for irregular waves. In the context of water waves, we say that the water is deep if the water depth h is greater than half the wavelength λ, that is, $h > \lambda/2$, or $kh \gg 1$, where $k = 2\pi/\lambda$ is the so-called wavenumber.

3.1.1 Regular waves in deep water

For regular waves (i.e. waves of uniform height and period) of period T seconds and height H metres in deep water, the wave-power level, which we denote by J, can be approximated as

$$J \approx TH^2 \text{ kWm}^{-1}. \tag{1}$$

As an example, for a regular wave of period $T = 10$ s and height $H = 2$ m, the wave-power level is approximately $J \approx 40$ kW/m.

3.1.2 Irregular waves in deep water

For irregular waves (i.e. waves of varying heights and periods) of energy period T_J (to be defined later) seconds and significant wave height H_s metres in deep water,

$$J \approx \frac{1}{2} T_J H_s^2 \text{ kWm}^{-1}. \tag{2}$$

The siginificant wave height is defined as four times the standard deviation of the free-surface elevation, that is, $H_s = 4\,\sigma_\eta$.

As an example, for an irregular wave of energy period $T_J = 10$ s and significant wave height $H_s = 2$ m, the wave-power level is approximately $J \approx 20$ kW/m.

Equations (1) and (2) tell us that the wave-power level varies linearly with the wave period, but quadratically with the wave height. Thus, if the wave height increases by two times, the power increases by four times.

3.2 Derivations

To arrive at the above expressions (1)–(2), we start with the *intensity*, which is defined as the time-average energy transport per unit time and per unit area in the direction of propagation. In other words, it is the power per surface area. For water waves propagating in the x-direction, the intensity is

$$I = \overline{p v_x}, \tag{3}$$

where p is the dynamic pressure of the fluid (water), v_x is the water particle velocity in the x-direction, and overbar denotes time average. Remember that power is force times velocity, and so intensity (which is power per surface area) is pressure times velocity.

The wave-power level (wave energy transport) is defined earlier as the wave energy transported per unit time through a vertical strip of unit width extending from the free surface to the sea bed. Therefore, by definition, the wave-power level is just the intensity integrated over the water depth, or

$$J = \int_{-h}^{0} I \,\mathrm{d}z. \tag{4}$$

The integration is taken up to the mean free surface $z = 0$ instead of the instantaneous free surface $z = \eta$ because the contribution of the integral from the mean free surface to the free surface is of a higher order and may therefore be neglected.

3.2.1 Derivation: Regular waves

For regular waves of amplitude A propagating in the x-direction, inserting expressions for p and v_x from linear wave theory, we obtain

$$I = \frac{1}{2}\frac{\rho g^2 k}{\omega} A^2 e^2(kz), \tag{5}$$

where the function $e(kz)$ is defined as

$$e(kz) = \frac{\cosh(kz + kh)}{\cosh(kh)}. \tag{6}$$

Derivation of (5) is left as an exercise.

Thus, inserting (5) into (4), we have

$$J = \int_{-h}^{0} I \, \mathrm{d}z = \frac{\rho g^2}{4\omega} A^2 \, 2k \int_{-h}^{0} e^2(kz) \mathrm{d}z. \tag{7}$$

Defining the depth function $D(kh) \equiv 2k \int_{-h}^{0} e^2(kz)\mathrm{d}z$, we have

$$J = \frac{\rho g^2 D(kh)}{4\omega} A^2. \tag{8}$$

Making use of a known relationship between the depth function $D(kh)$ and the group velocity v_g:

$$D(kh) = \frac{2\omega}{g} v_g, \tag{9}$$

we can alternatively write (8) as

$$J = \frac{\rho g}{2} v_g A^2, \tag{10}$$

where the group velocity is

$$v_g = \frac{\omega}{2k} \left[1 + \frac{2kh}{\sinh(2kh)} \right]. \tag{11}$$

Equation (8) or (10) gives the general expression for the wave-power level of regular waves of amplitude A in an arbitrary water depth h.

For deep water, $kh \gg 1$. In this case, the dispersion equation becomes $\omega^2 \to gk$. Recall that wavelength λ, wave period T, and water depth h are related through the dispersion equation:

$$\omega^2 = gk \tanh(kh). \tag{12}$$

Here, $k = 2\pi/\lambda$ is the wavenumber (where λ is the wavelength) and $\omega = 2\pi/T$ is the wave angular frequency (where T is the wave period). The group velocity (11) becomes $v_g \to \frac{\omega}{2k} = \frac{g}{2\omega}$ and $D(kh) \to 1$. So, from (8), and using $\omega = 2\pi/T$ and $A = H/2$, we have

$$J = \frac{\rho g^2}{32\pi} T H^2 = 0.976 \, T H^2 \text{ kW/m} \approx T H^2 \text{ kW/m} \quad \square \tag{13}$$

if T is expressed in seconds and H in metres, which proves (1).

3.2.2 Derivation: Irregular waves

For irregular waves, the result follows from the regular wave case by recalling that an irregular wave is a superposition of regular waves of different periods or frequencies. Thus, from the regular wave result (10), the irregular wave result is

$$\begin{aligned} J &= \rho g \sum_n \frac{1}{2} A^2(\omega_n) v_g(\omega_n) \\ &= \rho g \sum_n \Delta\omega\, S(\omega_n) v_g(\omega_n). \end{aligned} \tag{14}$$

The last equality follows from the definition of the wave spectrum, $\Delta\omega\, S(\omega_n) = \frac{1}{2}A^2(\omega_n)$. Replacing the sum by an integral as $\Delta\omega \to 0$, we have

$$J = \rho g \int_0^\infty S(\omega) v_g \, \mathrm{d}\omega, \tag{15}$$

where ρ is the mass density of the water, g is the acceleration due to gravity, and $S(\omega)$ is the wave spectrum. This is the general expression for the wave-power level of irregular waves in an arbitrary water depth h.

To obtain the approximate formula (2) for deep water, we again use $v_g \to \frac{g}{2\omega}$, valid for deep water. Substituting this into (15), we have

$$J = \frac{\rho g^2}{2} \int_0^\infty S(\omega) \omega^{-1} \mathrm{d}\omega. \tag{16}$$

We define the energy period T_J as

$$T_J \equiv \frac{2\pi \int_0^\infty S(\omega)\omega^{-1}\mathrm{d}\omega}{\int_0^\infty S(\omega)\mathrm{d}\omega}. \tag{17}$$

Using this definition and the relationship between the significant wave height H_s and the zeroth-moment of the wave spectrum,

$$\int_0^\infty S(\omega)\mathrm{d}\omega = \sigma_\eta^2 \equiv H_s^2/16, \tag{18}$$

we have

$$J = \frac{\rho g^2}{2} \frac{T_J}{2\pi} \frac{H_s^2}{16} = \frac{\rho g^2}{64\pi} T_J H_s^2 = 0.488 T_J H_s^2 \text{ kW/m} \approx \frac{1}{2} T_J H_s^2 \text{ kW/m}, \quad \square \tag{19}$$

if T_J is expressed in seconds and H_s in metres. Check if you get the same by substituting (17) and (18) into (16). This proves the approximate formula (2).

Hence, provided we know the wave spectrum $S(\omega)$ and the water depth h at a given site, we can calculate the wave-power level at that particular location with confidence using (15). Alternatively, if we know the significant wave height H_s and the energy period T_J, we can use the deep-water approximation (2) to estimate the wave-power level at that location.

4 Group velocity and wave energy

Let us say a few words about the group velocity v_g, which plays an important role in wave energy propagation. Consider a superposition of two waves of equal amplitudes but slightly different

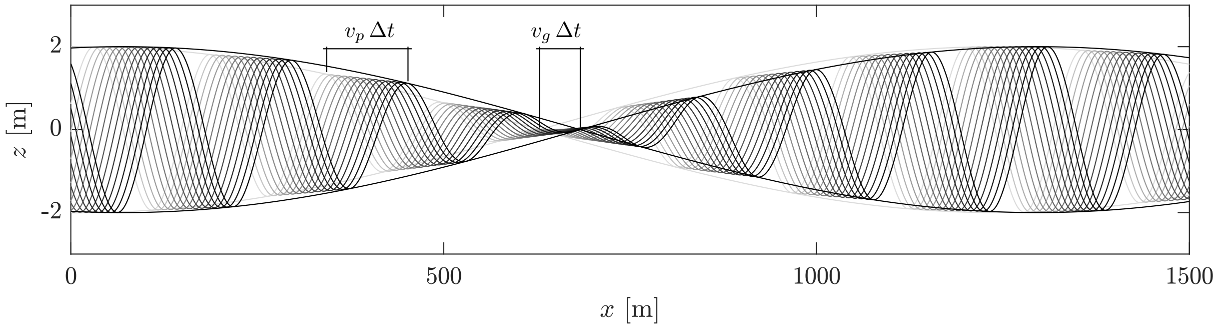

FIGURE 4: A train of waves with slowly-varying amplitudes, resulting from a superposition of two regular waves with equal amplitudes but slightly different frequencies. Individual waves move at the phase speed v_p, while the envelope moves with the group velocity v_g.

frequencies:

$$\eta = \eta_1 + \eta_2, \tag{20}$$

where

$$\eta_1 = A\cos(\omega_1 t - k_1 x), \tag{21}$$

$$\eta_2 = A\cos(\omega_2 t - k_2 x). \tag{22}$$

Using the trigonometric identity

$$\cos\alpha + \cos\beta = 2\cos\frac{\alpha-\beta}{2}\cos\frac{\alpha+\beta}{2}, \tag{23}$$

we can write (20) as

$$\eta = 2A\cos\left(\frac{\omega_2-\omega_1}{2}t - \frac{k_2-k_1}{2}x\right)\cos\left(\frac{\omega_2+\omega_1}{2}t - \frac{k_2+k_1}{2}x\right). \tag{24}$$

Because the two waves have slightly different frequencies, we can write

$$\omega_1 = \omega - \Delta\omega \qquad \omega_2 = \omega + \Delta\omega \tag{25}$$

$$k_1 = k - \Delta k \qquad k_2 = k + \Delta k, \tag{26}$$

where $\Delta\omega \ll \omega$. Thus,

$$\eta = 2A\cos(\Delta\omega\, t - \Delta k\, x)\cos(\omega t - kx). \tag{27}$$

We can see that this is a wave with slowly-varying amplitudes (see Fig. 4). The wave, $\cos(\omega t - kx)$, which propagates with the phase speed $v_p = \omega/k$, has slowly-varying amplitudes, $2A\cos(\Delta\omega\, t - \Delta k\, x)$, whose envelope propagates slower, with the velocity

$$\frac{\Delta\omega}{\Delta k} \to \frac{\mathrm{d}\omega}{\mathrm{d}k} \text{ (as } \Delta\omega \to 0). \tag{28}$$

This is the group velocity v_g.

You might have seen circular waves generated by dropping a stone in a lake or any body of water. You would have noticed that the individual waves moved faster than the front, such that they always catch up the front. The individual waves move at the phase speed, while the front moves with the group velocity, which is slower. To calculate the time taken for a swell to propagate from a storm centre, we use the group velocity because it is the velocity with which the wave front moves.

Example Assume that the sea is calm. Suddenly, a storm develops $l = 300$ km from shore. How long afterwards do we record a swell of period $T = 14$ s at the shore? Assume deep water wave propagation.

Answer On deep water, $v_g = \frac{g}{2\omega} = \frac{gT}{4\pi}$.
Time for swell to reach shore:

$$\begin{aligned}\Delta t &= \frac{l}{v_g} = \frac{4\pi l}{gT} \\ &= \frac{4\pi \times 3 \times 10^5}{9.81 \times 14} \\ &= 27 \times 10^3 \text{ s} = 7.6 \text{ hours.}\end{aligned}$$

4.1 Energy per unit horizontal area

In the following, we will show that wave energy propagates at the group velocity. We first introduce the concept of energy per unit horizontal area. Per unit (horizontal) area, the potential energy of the water relative to the sea bed equals the product of the water weight per unit area, $\rho g(h+\eta)$, and the height of the centre of the water mass above the sea bed, $(h+\eta)/2$:

$$\frac{\rho g}{2}(h+\eta)^2 = \frac{\rho g}{2}h^2 + \rho g h\eta + \frac{\rho g}{2}\eta^2. \tag{29}$$

Since the potential energy of calm water relative to the sea bed is $\rho g h \times \frac{h}{2} = \frac{\rho g}{2}h^2$, the increase in potential energy relative to calm water is

$$\rho g h\eta + \frac{\rho g}{2}\eta^2. \tag{30}$$

Because the free-surface elevation η is sinusoidal, the first term has a zero time average. Hence, the time-average potential energy per unit (horizontal) area is

$$E_p(x,y) = \frac{\rho g}{2}\overline{\eta^2(x,y,t)}, \tag{31}$$

where the overbar denotes time average. For a regular wave of amplitude A, that is, the free-surface elevation is of the form $\eta = A\cos(\omega t - kx)$,

$$E_p = \frac{\rho g}{4}A^2. \tag{32}$$

It can be shown that there is an equal amount of time-average kinetic energy. Thus, the total is

$$E = E_p + E_k = 2E_p = 2E_k = \frac{\rho g}{2}A^2. \tag{33}$$

Scatter diagram

Hs \ Tz	3.5	4.5	5.5	6.5	7.5	8.5	9.5	10.5	11.5	12.5	13.5	14.5
0.25	0.0066	0.0056	0.0030	0.0023	0.0011	0.0007	0.0003	0.00005				
1	0.0453	0.1650	0.0906	0.0347	0.0131	0.0047	0.0019	0.00069	0.0001	0.00004	0.00007	0.00005
2	0.0018	0.0368	0.1604	0.0650	0.0229	0.0099	0.0032	0.00121	0.00009	0.00005	0.00005	
3		0.0003	0.0187	0.1084	0.0335	0.0071	0.0033	0.00171	0.0004	0.00007		0.00002
4			0	0.01021	0.05565	0.01163	0.00209	0.00052	0.00034	0.00021	0.00005	
5				0.00002	0.00729	0.02391	0.00301	0.00069	0.00031	0.00014	0.00005	0.00005
6					0.00012	0.00603	0.00691	0.00052	0.00007			
7				0.00002	0.00009	0.00026	0.00352	0.00152	0.00016	0.00005		
8							0.00062	0.00288	0.00017			
9								0.00086	0.00073	0.00002		
10								0.00002	0.00043	0.00016		
11									0.00011	0.00014		
12										0.00004		

FIGURE 5: An example of a scatter diagram (Folley, 2017).

Previously (see (10)), we have derived $J = \frac{\rho g}{2} v_g A^2$ for a regular wave. Comparing it with (33), we see that

$$J = v_g E, \tag{34}$$

which implies that energy propagates at the group velocity.

5 Scatter diagram and mean wave-power level

We have looked at how to calculate the wave-power level of regular and irregular waves. To calculate the mean wave-power level at a given site, we need the joint probability of wave heights and periods (a.k.a. scatter diagram) at the site. This is basically a 2D (bivariate) histogram of the wave heights and periods. The scatter diagram is obtained from wave measurements at the site over a long duration. A variety of wave measuring instruments exists, such as wave measuring buoys, acoustic doppler current profilers (ADCPs), pressure sensors, lasers, and radars. These operate on different principles and are suited to different environments and needs. Some are more suited to shallow water, some require fixed attachment points, etc. In the absence of field measurements (either due to difficulties of maintaining instruments over a long duration or the need for information over a longer duration than what is available from the instruments), long-term numerical wave modelling, which relies on the meteorological archive, can be used.

To arrive at a scatter diagram, measurements are organised into short-term sea states, each sufficiently long (between 0.5 to 3 hours) for the sea state to be considered stationary, i.e., its statistics do not change within the sea state. Each sea state is described by a pair of statistical wave height (usually the significant wave height H_s is used) and period (either the spectral peak period T_p, the mean period T_z, or the energy period T_J may be used). A histogram can be constructed by collecting sea states having similar characteristics into bins and counting their number of occurrences over the long duration. An example of a scatter diagram is shown in Fig. 5.

The mean wave-power level $\bar{J}$ at the site can then be obtained simply as the sum of the wave-power level of every bin in the scatter diagram, weighted by its probability of occurrence:

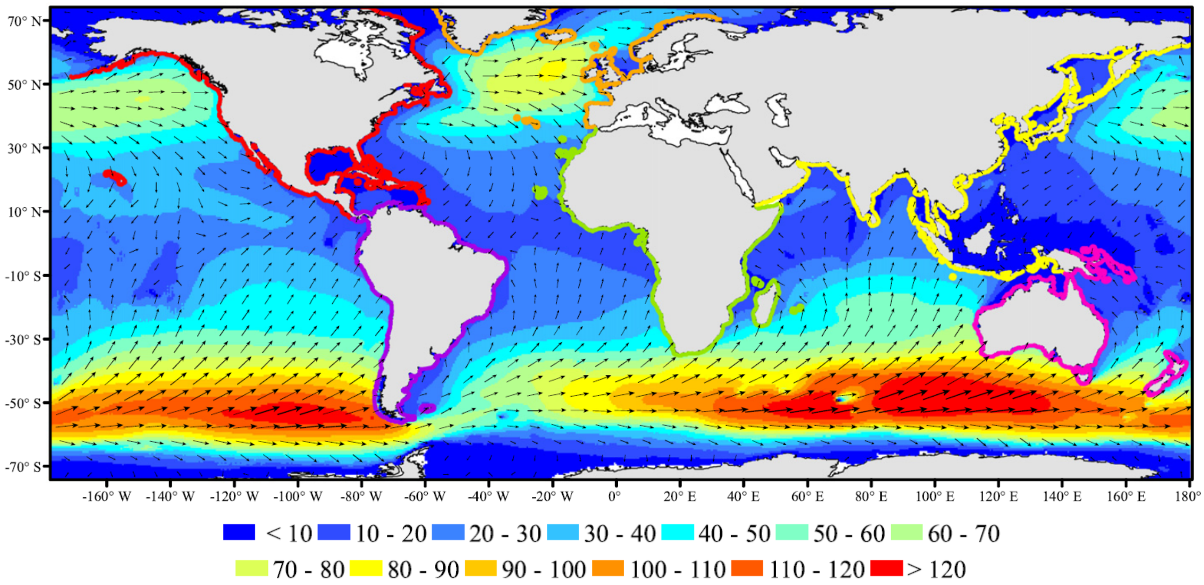

FIGURE 6: Global mean wave-power level (in kW/m) [4].

$$\bar{J} = \sum_{H_s} \sum_{T_p} J(H_s, T_p) C(H_s, T_p), \tag{35}$$

where

$$\sum_{H_s} \sum_{T_p} C(H_s, T_p) = 1. \tag{36}$$

The scatter diagram is useful also for calculating the expected power output from a wave energy converter (WEC). More on this in Part 2.

6 Global wave-power resource and variability

Different locations around the world have different wave climates, and hence different wave-power levels. In general, the average wave-power level is higher in the extratropical regions than in the tropics (Fig. 6). If we integrate the wave-power level along the total length of the world's coast lines, then the estimated global potential is in the order of 1 billion kW. That is enough electricity for roughly 1 billion homes! Even if we only manage to harness 1% of this, it is still a significant resource.

While mean wave-power levels give an indication about the potential of a given site in term of its resource, it is not a sufficient criterion for choosing a suitable location for deployment of WECs. Wave-power levels vary over time, on different time scales. Offshore wave-power levels may vary from a few kW/m during calm weather to several MW/m during storms. Fig. 7 shows the global distribution of wave-power level during different months of the year, demonstrating its seasonal variation.

It is clear that a WEC will have to encounter variable wave conditions during its lifetime. Low to moderate waves occur most of the time and so they dictate the WEC's revenue, but its cost is driven by extreme waves since it has to survive these conditions (which do not occur as

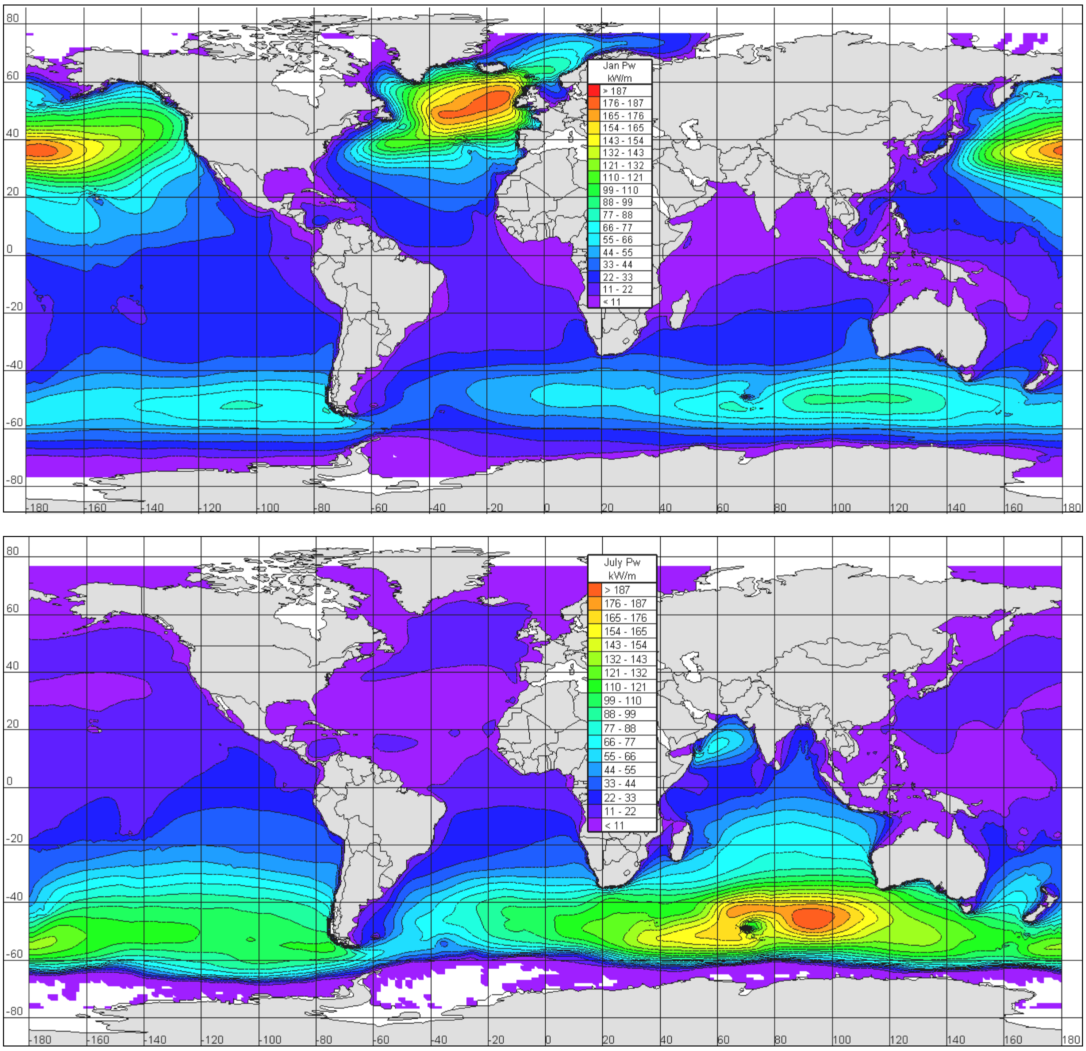

FIGURE 7: Global mean wave-power level in January (top) and July (bottom) [1].

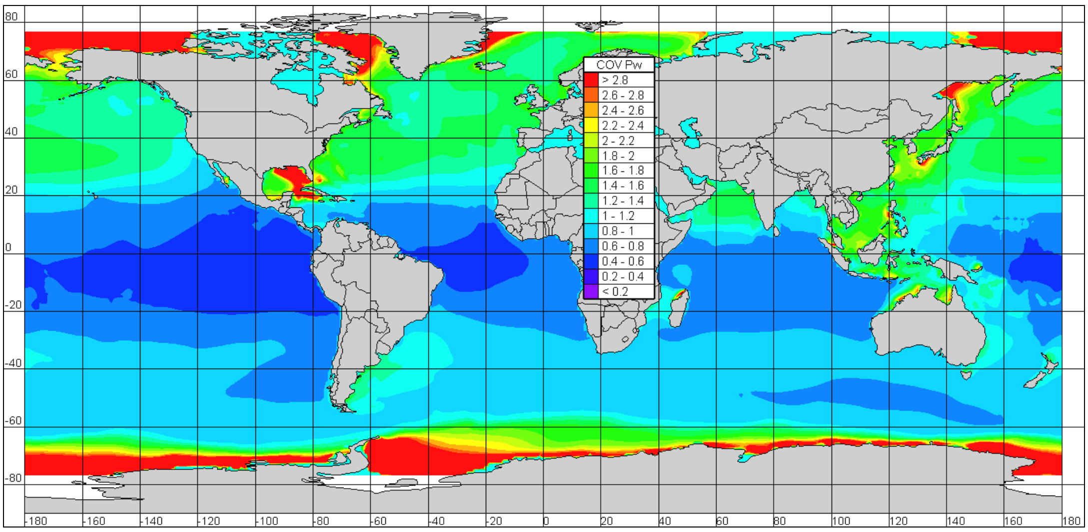

FIGURE 8: Coefficient of variation (COV) of wave-power level [1] arond the world. Low COV means low variability.

often, but do occur). In addition, wave direction also varies over time and real sea waves have some degree of spreading (purely unidirectional waves only exist in theory!).

For these reasons, the best locations to deploy WECs (in terms of resource) are probably not those with the highest mean wave-power levels. Variability must be taken into account, and sites with a low variability but not the highest mean may in fact be more economically attractive. Fig. 8 shows the coefficient of variation of the wave-power level globally, which when combined with Fig. 6, suggests that there are locations with relatively low variability and decent mean. Fig. 9 highlights the difference in variability of wave-power level one could get at two different locations in the world. Which location would you choose to deploy your WEC?

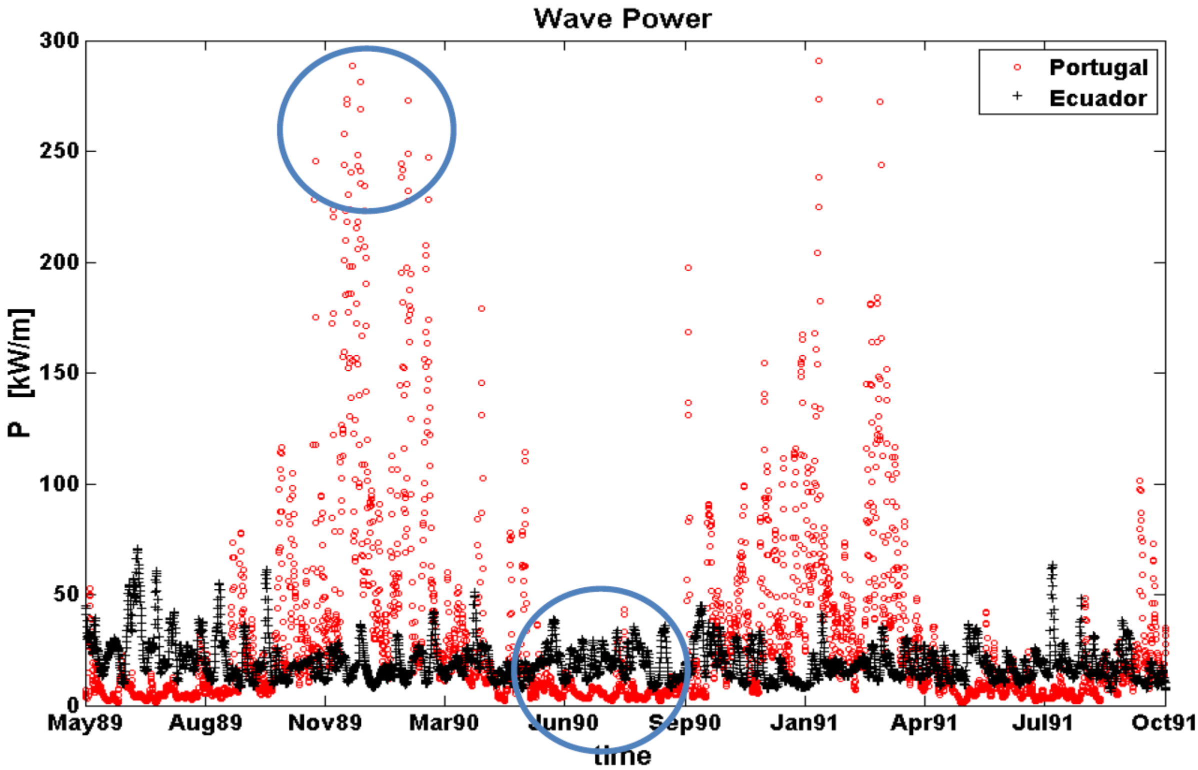

FIGURE 9: Comparison of wave-power level as a function of time, for two locations: Portugal in North Atlantic and Ecuador in Equatorial Pacific [6].

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