

Wave energy Part 1

Wave energy resource

OCEN4007 Ocean Renewable Energy

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1 Main questions

The main question we would like to answer through these series of lectures on wave energy is this: *Given a site with a supposedly good wave power resource, how do we harness this potential?* To answer this question, we may ask the following sub questions, which we hope to answer by the end of these lectures:

- What makes a good site?
- How do we calculate the wave energy resource at this site?
- What makes a good wave energy converter (WEC)?
- How do we calculate its power output?
- What practical aspects do we need to consider?

2 What makes wave energy attractive

What makes wave energy attractive? Here are some reasons:

- Wave energy is more *persistent* (less intermittent) than wind or solar energy. Waves in deep water and open ocean can travel over long distances almost without energy loss. There may be days when the wind is not blowing, but even on those days, you will find swells at the beach.
- Wave has a higher energy intensity than solar or wind (see Section 4 for how much denser wave energy is compared to wind and solar).
- Wave has a temporal variability that is complementary to solar and wind. Wave is more energetic in winter, where days are shorter and there is less solar energy. In most of WA coast, it is more windy in summer than in winter.
- The potential is enormous. Globally, it is estimated that there is enough wave energy to provide the electricity needs of about 1 billion homes.

- WECs can act as coastal protection, because they absorb energy from the waves that pass through them. Hence, waves in the lee of WECs carry less energy than the waves before them.
- Minimal visual intrusion. Most WECs do not extend far above the sea surface. Some of them are completely submerged.

3 Brief history of wave energy

The history of wave energy is relatively short compared to that of wind energy. The first known wave energy patent dates back to 1799, a time when windmills, which are not very different in principle from modern wind turbines we see today, were already widespread.

Around 1900s, devices to harness wave energy used to be called wave motors. Some of these were actually built and deployed, but little is known about their fate.

In 1947, the Japanese inventor Yoshio Masuda began testing wave energy devices in Japan. He proposed a number of concepts, some of which still persist in different variants today. At about the same time, Walton Bott began working on wave energy in Mauritius. The scheme he proposed was the precursor of overtopping devices.

In 1965, Ryokuseisha, a Japanese company, began manufacturing wave-powered navigation buoys. This company is still in operation today.

Wave energy received a greater attention in the 1970s. The 1973 oil crisis motivated serious research and development into alternative energy sources, including wave energy. At the University of Edinburgh, Stephen Salter experimented with WECs of various shapes in a wave flume and ended up with a shape looking like a nodding duck. He was able to demonstrate, for the first time, that near-complete wave absorption was possible.

As a response to the oil crisis, the UK government initiated the UK Wave Power Programme, and a number of devices were deemed promising. These include the Salter Duck, the NEL oscillating water column (OWC), the Lancaster flexible bag, the Bristol cylinder, the Vickers device, and the Clam (see Fig. 1). Some other notable devices were proposed around this time, including the Cockerell raft (see Fig. 2), which was invented by Sir Christopher Cockerell, the inventor of the hovercraft, the Russell rectifier (see Fig. 2), the Kaimei, and the Triplate. Meanwhile, in Norway, Kjell Budal and Johannes Falnes were experimenting with *point absorbers*, devices of much smaller extent than typical wavelengths.

As the oil price declined, however, the UK Wave Power Programme was terminated. Devices that were so close to being put to sea were abandoned. Similar situation happened in other countries. Funding for wave energy research was drastically cut. Nevertheless, this period saw the construction of two full-scale plants in Norway: the tapered channel and the Kvaerner OWC (see Fig. 3). Both operated for a few years before being damaged during a severe winter storm.

Wave energy entered a period of relative inactivity which lasted for about two decades, from early 1980s to late 1990s. At the turn of the millennium, research activities in wave energy picked up again, along with increasing awareness of the negative consequences of fossil fuels. A number of devices were tested at sea during this period, including the PowerBuoy, Pelamis, Wavebob, Limpet, Wavestar, Wave Dragon, Oyster, and Seabased, to name a few.

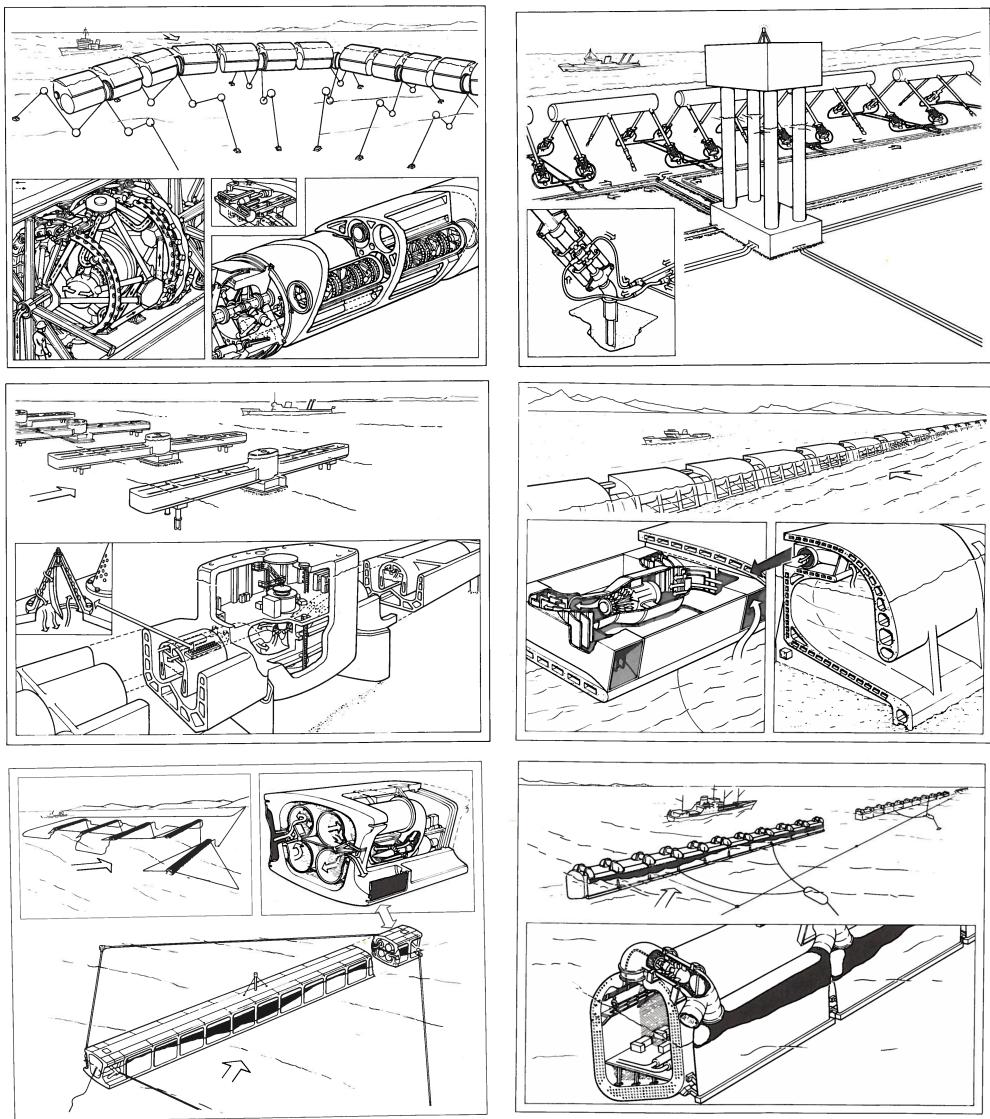


FIGURE 1: Wave energy converters from the UK Wave Energy Programme, reproduced from [5]. Clockwise from top left: Salter Duck, Bristol Cylinder, NEL oscillating water column, Clam, Lancaster flexible bag, Vickers device.

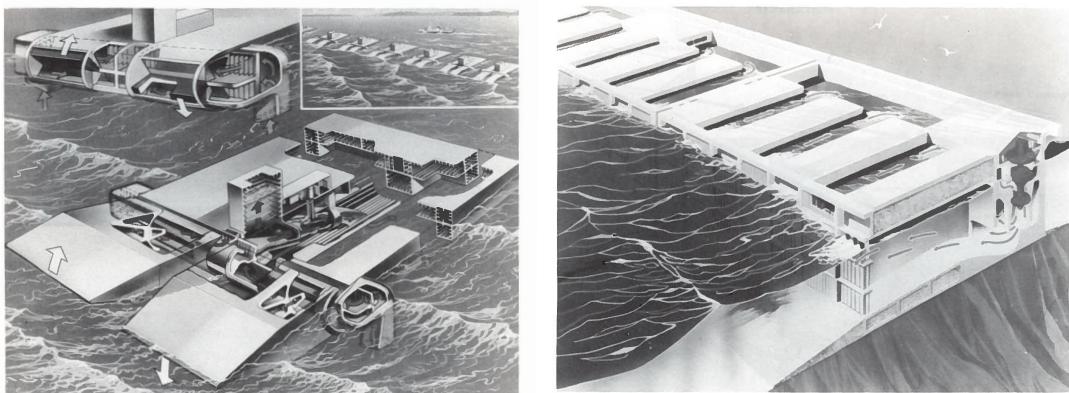


FIGURE 2: The Cockerell raft (left) and the Russell rectifier (right), reproduced from [5].

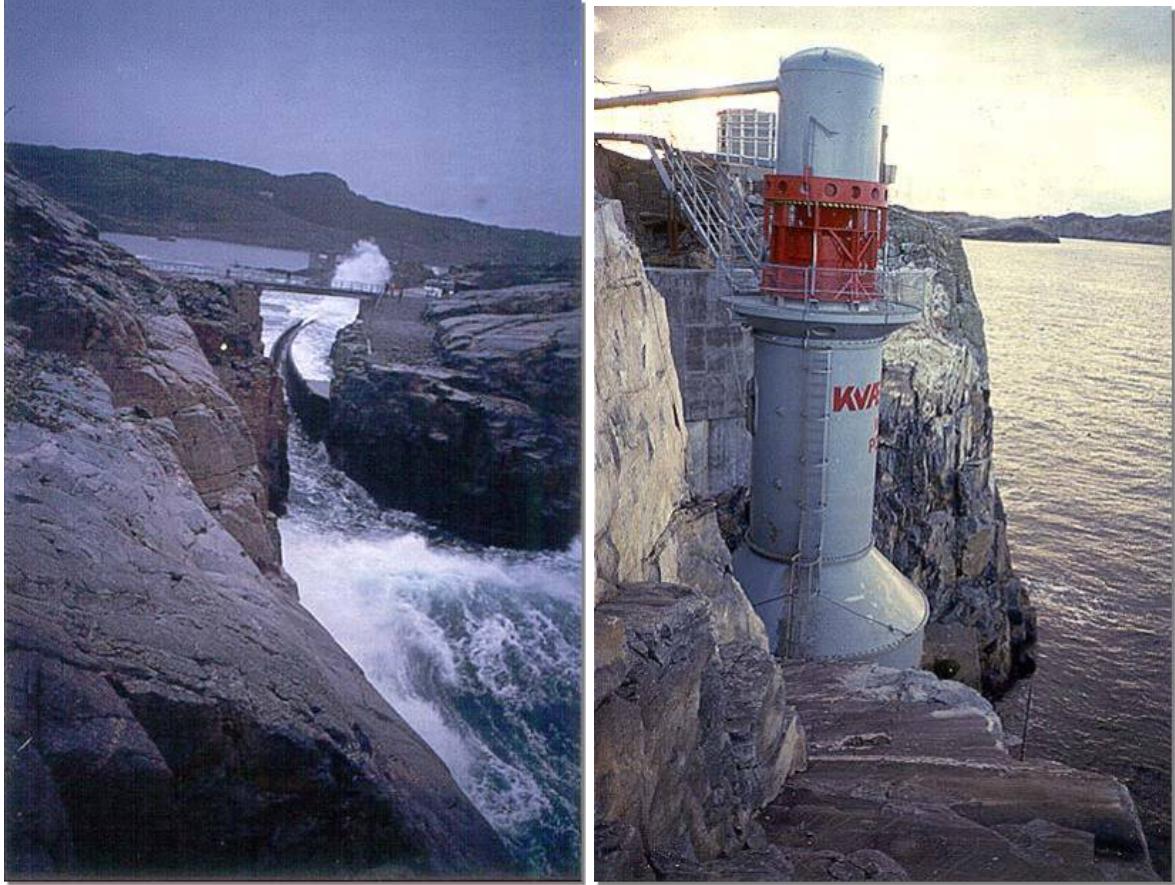


FIGURE 3: The tapered channel (left) and the Kvaerner oscillating water column (right), reproduced from [2].

More recent developments include the establishment of [Wave Energy Scotland](#) to reformulate the approach to wave energy development using a stage-gated model and a competitive Pre-Commercial Procurement (PCP) program. This ensures that only the best of the competing concepts get funded and progress to the next stage of development. This model is now followed by the [EuropeWave](#) program.

A summary of important events in the history of wave energy is presented in Fig. 4. As of today, there is yet no convergence in wave energy technology.

4 How waves are generated

Before discussing how to calculate the energy available in the waves, let us briefly consider how ocean waves are generated.

You might have seen ripples generated as a result of wind blowing over a puddle. You would have noticed that as the wind blew over the water surface, the ripples gathered energy and grew in size as they traveled from one side of the puddle to the other side. Ocean waves are formed in much the same way, at a larger scale. Wind generates ripples, which grow into waves. When the waves stop growing, we call this state a fully-developed sea. Waves that have travelled away from their generation area are called *swells* (see Fig. 5).

Swells can travel thousands of kilometres in the open sea with little loss of energy. Swells

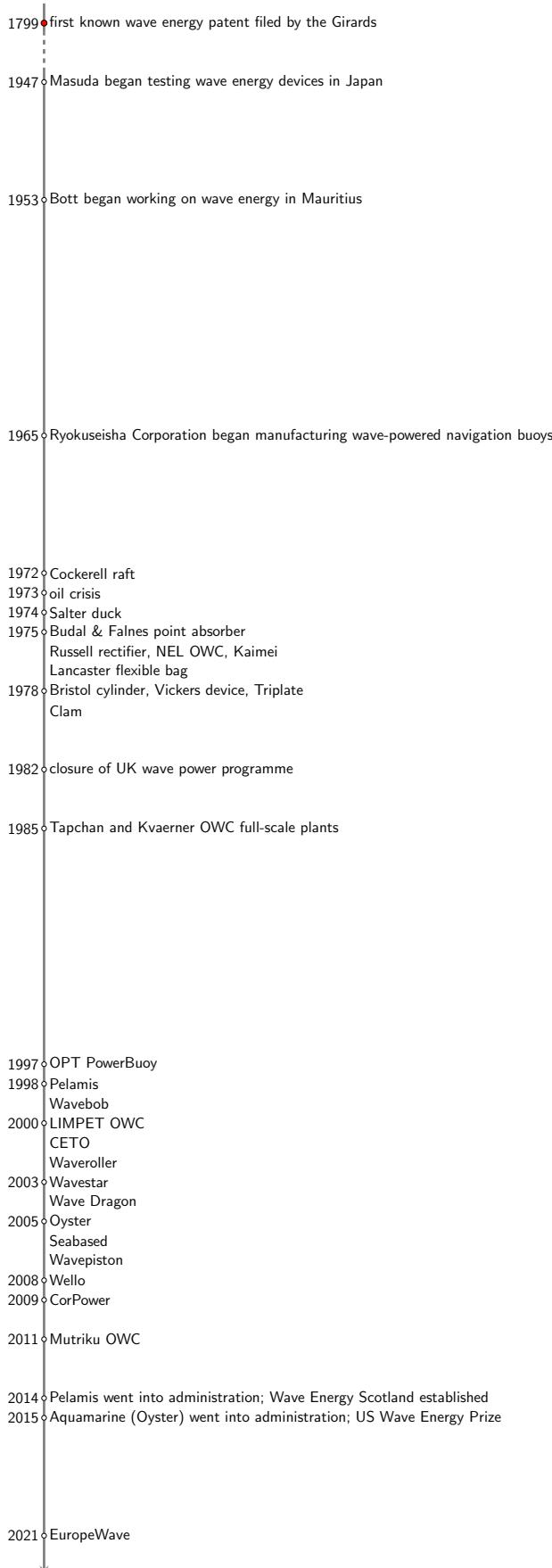


FIGURE 4: Wave energy timeline. Except for the period between 1799 and 1947, the length of the timeline is proportional to time.

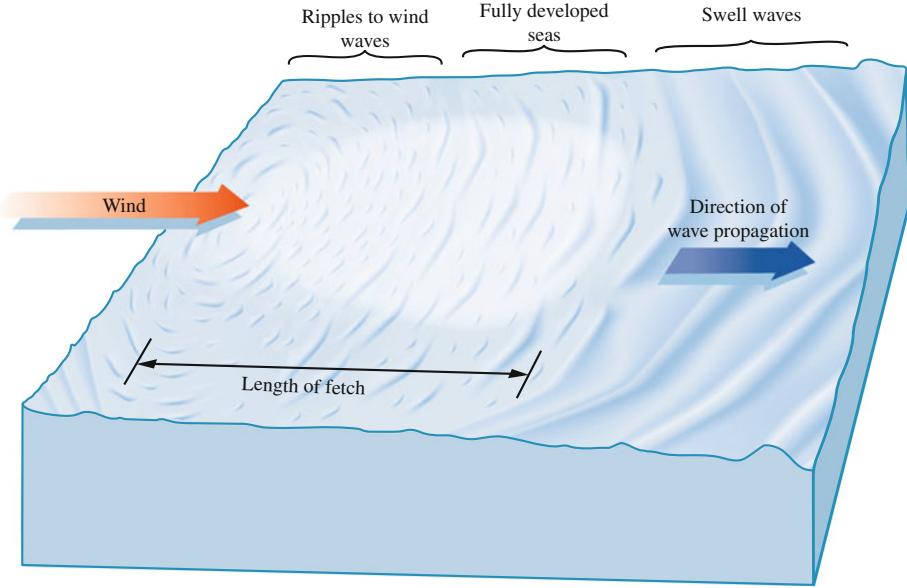


FIGURE 5: Generation of ocean surface waves [4].

Google Maps Swells arriving at South Western Australia might have originated from South Africa.

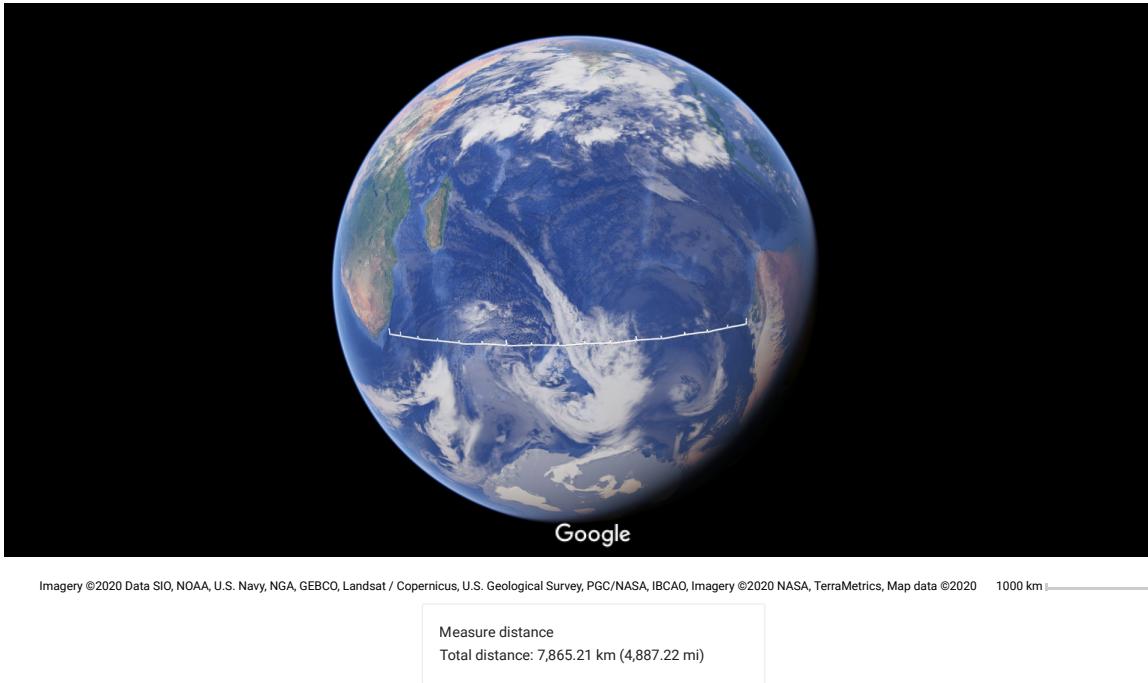


FIGURE 6: Swells arriving at southwestern coast of Australia might have originated from south of Africa. That is nearly 8000 km away!

arriving at the southwestern coast of Australia, for example, may have originated from a storm thousands of kilometres away (see Fig. 6). As there is little land mass in the Southern Ocean, swells arriving at the southern coast of Australia travel almost uninterrupted. That is why you can find waves at the beach even when it is not particularly windy.

Wind itself is in turn caused by the difference in atmospheric pressure on the earth surface. This difference is mainly caused by uneven heating of the earth surface by the sun. As energy

is transformed from solar energy into wind energy and then into wave energy, the flow becomes intensified. The amount of solar radiation that the earth receives on average is about 0.1-0.3 kW/m² (of horizontal area), whereas wind transports about 0.5 kW/m² (of area perpendicular to the wind direction). Water wave, on the other hand, transports about 2-3 kW/m² (of vertical area) just below the water surface, where wave energy is most concentrated. Compared to wind and solar, wave energy is a much denser form of energy (see Fig. 7).

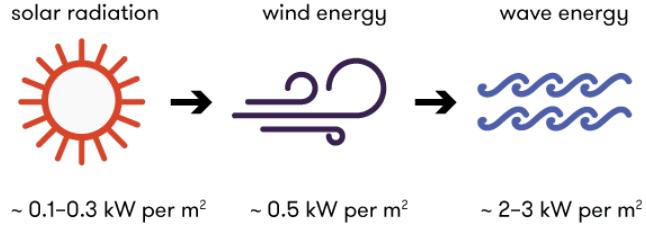


FIGURE 7: Wave energy is a denser form of energy than solar or wind. (Image from [7]. Numbers from [3]).

5 How much energy do the waves carry?

As wave propagates, energy is propagated with it, while the water particles only move locally (in elliptical orbits). See <https://www.acs.psu.edu/drussell/demos/waves/wavemotion.html> for some nice animations of waves (scroll down to water waves). The amount of energy that the waves carry is normally expressed in terms of the *wave-power level* (a.k.a. the *wave energy transport*). It is defined as the wave energy transported per unit time through a vertical strip of unit width (extending from the free surface down to the sea bed), perpendicular to the wave direction (or parallel to the wave front). It is normally expressed in kW/m.

5.1 Some useful approximate formulas

There exists two useful approximate formulas for the wave-power level of waves in deep water: one for *regular waves*, and the other for *irregular waves*. In the context of water waves, we say that the water is deep if the water depth h is greater than half the wavelength λ , that is, if $h > \lambda/2$, or $kh \gg 1$, where $k = 2\pi/\lambda$ is the so-called *wavenumber*. We will first show these approximations, before deriving the exact expressions in section 5.2.

5.1.1 Regular waves in deep water

For regular waves (i.e. waves of uniform height and period) of period T seconds and height H metres in deep water, the wave-power level, which we denote by J , can be approximated as

$$J \approx TH^2 \text{ kWm}^{-1}. \quad (1)$$

For example, a deep-water regular wave of period $T = 10$ s and height $H = 2$ m has a wave-power level of approximately 40 kW/m.

5.1.2 Irregular waves in deep water

For irregular waves (i.e. waves of varying heights and periods) of *energy period* T_J (to be defined later) seconds and *significant wave height* H_s metres in deep water,

$$J \approx \frac{1}{2} T_J H_s^2 \text{ kWm}^{-1}. \quad (2)$$

The significant wave height is defined as four times the standard deviation of the free-surface elevation, that is, $H_s = 4\sigma_\eta$.

For example, an irregular wave of energy period $T_J = 10$ s and significant wave height $H_s = 2$ m has a wave-power level of approximately $J \approx 20$ kW/m.

Equations (1) and (2) tell us that the wave-power level varies linearly with the wave period, but quadratically with the wave height. Thus, if the wave height increases by two times, the power increases by four times.

5.2 Derivations

To derive the above expressions (1)–(2), we start with the *intensity*, which is the time-average energy transport per unit time and per unit area. In other words, it is the power per surface area. For waves propagating in the x -direction, the intensity is

$$I = \overline{pv_x}, \quad (3)$$

where p is the dynamic pressure of the fluid (water), v_x is the water particle velocity in the x -direction, and the overbar denotes time average. Remember that power is force times velocity, and so intensity (which is power per surface area) is pressure times velocity.

The wave-power level is defined earlier as the wave energy transported per unit time through a vertical strip of unit width extending from the free surface to the sea bed. So, by definition, the wave-power level is the intensity integrated over the water depth:

$$J = \int_{-h}^0 I dz. \quad (4)$$

The integration is taken from the sea bed $z = -h$ up to the mean free surface $z = 0$ instead of the instantaneous free surface $z = \eta$ because the contribution of the integral from the mean free surface to the free surface is of a higher order and may therefore be neglected.

5.2.1 Derivation: Regular waves

For regular waves of amplitude A propagating in the x -direction, inserting expressions for p and v_x from linear wave theory, we obtain

$$I = \frac{1}{2} \frac{\rho g^2 k}{\omega} A^2 e^2(kz), \quad (5)$$

where ρ is the mass density of the water, g is the acceleration due to gravity, $\omega = 2\pi/T$ is the angular frequency of the wave, and the function $e(kz)$ is defined as

$$e(kz) = \frac{\cosh(kz + kh)}{\cosh(kh)}. \quad (6)$$

Derivation of (5) is left as an exercise. Inserting (5) into (4), we have

$$J = \int_{-h}^0 I dz = \frac{\rho g^2}{4\omega} A^2 2k \int_{-h}^0 e^2(kz) dz. \quad (7)$$

Defining the *depth function* $D(kh) \equiv 2k \int_{-h}^0 e^2(kz) dz$, we can write (7) as

$$J = \frac{\rho g^2 D(kh)}{4\omega} A^2. \quad (8)$$

Making use of a known relationship between the depth function $D(kh)$ and the *group velocity* v_g :

$$D(kh) = \frac{2\omega}{g} v_g, \quad (9)$$

we can alternatively write (8) as

$$J = \frac{\rho g}{2} v_g A^2, \quad (10)$$

where the group velocity is

$$v_g = \frac{\omega}{2k} \left[1 + \frac{2kh}{\sinh(2kh)} \right]. \quad (11)$$

Equation (8) or (10) gives the general expression for the wave-power level of regular waves of amplitude A in an arbitrary water depth h .

To obtain the approximate expression (1) for regular waves in deep water, recall that wavelength λ , wave period T , and water depth h are related through the *dispersion equation*:

$$\omega^2 = gk \tanh(kh). \quad (12)$$

For deep water, $kh \gg 1$, the dispersion equation becomes $\omega^2 \rightarrow gk$, the group velocity (11) becomes $v_g \rightarrow \frac{\omega}{2k} = \frac{g}{2\omega}$, and $D(kh) \rightarrow 1$ (how?). So, from (8), and using $\omega = 2\pi/T$ and $A = H/2$, we have

$$J = \frac{\rho g^2}{32\pi} TH^2 = 0.976 TH^2 \text{ kW/m} \approx TH^2 \text{ kW/m} \quad \square \quad (13)$$

if T is expressed in seconds and H in metres, which proves the approximate formula (1).

5.2.2 Derivation: Irregular waves

For irregular waves, the result follows from the regular wave case by recalling that an irregular wave may be expressed as a superposition of regular waves of different amplitudes and

frequencies. Thus, from the regular wave result (10), the irregular wave result is

$$\begin{aligned} J &= \rho g \sum_n \frac{1}{2} A^2(\omega_n) v_g(\omega_n) \\ &= \rho g \sum_n \Delta\omega S(\omega_n) v_g(\omega_n). \end{aligned} \quad (14)$$

The last equality follows from the definition of the *wave spectrum*, $\Delta\omega S(\omega_n) = \frac{1}{2} A^2(\omega_n)$. Replacing the sum by an integral as $\Delta\omega \rightarrow 0$, we have

$$J = \rho g \int_0^\infty S(\omega) v_g d\omega, \quad (15)$$

where $S(\omega)$ is the wave spectrum. This is the general expression for the wave-power level of irregular waves in an arbitrary water depth h .

To obtain the approximate formula (2) for deep water, we again use $v_g \rightarrow \frac{g}{2\omega}$, valid for deep water. Substituting this into (15), we have

$$J = \frac{\rho g^2}{2} \int_0^\infty S(\omega) \omega^{-1} d\omega. \quad (16)$$

To proceed further, we define the energy period as

$$T_J \equiv \frac{2\pi \int_0^\infty S(\omega) \omega^{-1} d\omega}{\int_0^\infty S(\omega) d\omega}. \quad (17)$$

Using this and the relationship between the significant wave height H_s and the zeroth-moment of the wave spectrum,

$$\int_0^\infty S(\omega) d\omega = \sigma_\eta^2 \equiv H_s^2/16, \quad (18)$$

we have

$$J = \frac{\rho g^2}{2} \frac{T_J}{2\pi} \frac{H_s^2}{16} = \frac{\rho g^2}{64\pi} T_J H_s^2 = 0.488 T_J H_s^2 \text{ kW/m} \approx \frac{1}{2} T_J H_s^2 \text{ kW/m}, \quad \square \quad (19)$$

if T_J is expressed in seconds and H_s in metres. This proves the approximate formula (2).

Hence, provided we know the wave spectrum $S(\omega)$ and the water depth h at a given site, we can calculate the wave-power level at that particular location with confidence using (15). This formula is exact, not approximate. However, if the wave spectrum is not given but we know the significant wave height H_s and the energy period T_J , we may use the deep-water approximation (2) to estimate the wave-power level at that location.

6 Group velocity and wave energy

Let us say a few words about the group velocity v_g , which plays an important role in wave energy propagation. Consider a superposition of two waves of equal amplitudes but slightly different frequencies:

$$\eta = \eta_1 + \eta_2, \quad (20)$$

where

$$\eta_1 = A \cos(\omega_1 t - k_1 x), \quad \eta_2 = A \cos(\omega_2 t - k_2 x). \quad (21)$$

Using the trigonometric identity

$$\cos \alpha + \cos \beta = 2 \cos \frac{\alpha - \beta}{2} \cos \frac{\alpha + \beta}{2}, \quad (22)$$

we can write (20) as

$$\eta = 2A \cos\left(\frac{\omega_2 - \omega_1}{2}t - \frac{k_2 - k_1}{2}x\right) \cos\left(\frac{\omega_2 + \omega_1}{2}t - \frac{k_2 + k_1}{2}x\right). \quad (23)$$

Because the two waves have slightly different frequencies, we can write

$$\omega_1 = \omega - \Delta\omega \quad \omega_2 = \omega + \Delta\omega \quad (24)$$

$$k_1 = k - \Delta k \quad k_2 = k + \Delta k, \quad (25)$$

where $\Delta\omega \ll \omega$. Thus,

$$\eta = 2A \cos(\Delta\omega t - \Delta k x) \cos(\omega t - kx). \quad (26)$$

We can see that this is a wave with slowly-varying amplitudes. The wave, $\cos(\omega t - kx)$, which propagates with the *phase speed* $v_p = \omega/k$, has slowly-varying amplitudes, $2A \cos(\Delta\omega t - \Delta k x)$, whose envelope propagates slower, with the velocity

$$\frac{\Delta\omega}{\Delta k} \rightarrow \frac{d\omega}{dk} \text{ (as } \Delta\omega \rightarrow 0\text{).} \quad (27)$$

This is the group velocity v_g . See Fig. 8 for an illustration.

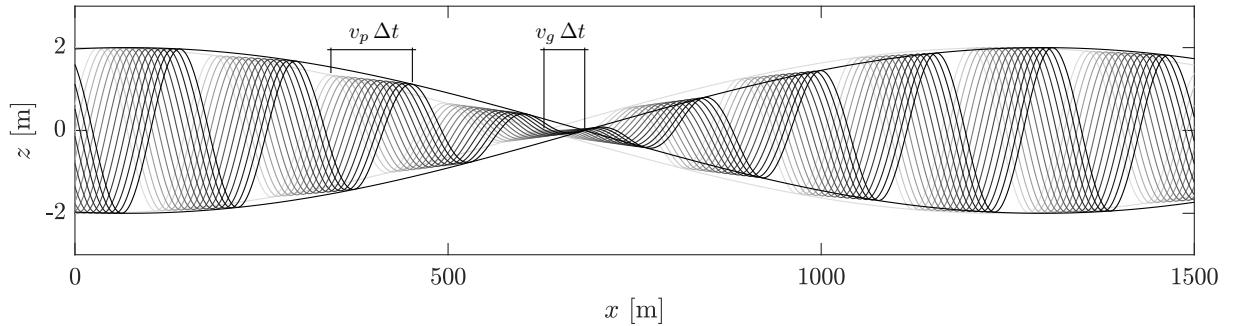


FIGURE 8: A train of waves with slowly-varying amplitudes, resulting from a superposition of two regular waves with equal amplitudes but slightly different frequencies. Individual waves move at the phase speed v_p , while the envelope moves with the group velocity $v_g \leq v_p$.

You might have seen circular waves generated by dropping a stone in a lake or any water body. You would have noticed that the individual waves moved faster than the front, such that they always catch up the front. The individual waves move at the phase speed, while the front moves with the group velocity, which is slower. To calculate the time taken for a swell to propagate from a storm centre, we use the group velocity because it is the velocity with which the wave front moves.

Example Assume that the sea is calm. Suddenly, a storm develops $l = 300$ km from shore. How long afterwards do we record a swell of period $T = 14$ s at the shore? Assume deep-water wave propagation.

Answer On deep water, $v_g = \frac{g}{2\omega} = \frac{gT}{4\pi}$. Time for swell to reach shore:

$$\begin{aligned}\Delta t &= \frac{l}{v_g} = \frac{4\pi l}{gT} \\ &= \frac{4\pi \times 3 \times 10^5}{9.81 \times 14} \\ &= 27 \times 10^3 \text{ s} = 7.6 \text{ hours.}\end{aligned}$$

6.1 Energy per unit horizontal area

In the following, we will show that wave energy propagates at the group velocity. We first introduce the concept of energy per unit horizontal area. Per unit (horizontal) area, the potential energy of the water relative to the sea bed equals the product of the water weight per unit area, $\rho g(h + \eta)$, and the height of the centre of the water mass above the sea bed, $(h + \eta)/2$:

$$\frac{\rho g}{2}(h + \eta)^2 = \frac{\rho g}{2}h^2 + \rho gh\eta + \frac{\rho g}{2}\eta^2. \quad (28)$$

Since the potential energy of calm water relative to the sea bed is $\rho gh \times \frac{h}{2} = \frac{\rho g}{2}h^2$, the increase in potential energy relative to calm water is

$$\rho gh\eta + \frac{\rho g}{2}\eta^2. \quad (29)$$

Because the free-surface elevation η is sinusoidal, the first term has a zero time average. Hence, the time-average potential energy per unit (horizontal) area is

$$E_p(x, y) = \frac{\rho g}{2}\overline{\eta^2(x, y, t)}, \quad (30)$$

where the overbar denotes time average. For a regular wave of amplitude A , that is, the free-surface elevation is of the form $\eta = A \cos(\omega t - kx)$, we have

$$E_p = \frac{\rho g}{4}A^2. \quad (31)$$

It can be shown that there is an equal amount of time-average kinetic energy. Thus, the total is

$$E = E_p + E_k = 2E_p = 2E_k = \frac{\rho g}{2}A^2. \quad (32)$$

Previously (see (10)), we have derived $J = \frac{\rho g}{2}v_gA^2$ for a regular wave. Comparing it with (32), we see that

$$J = v_g E, \quad (33)$$

which implies that energy propagates at the group velocity.

The scatter diagram is also useful for calculating the expected power output from a wave energy converter (WEC). More on this in Part 2.

8 Global wave-power resource and variability

Different locations around the world have different wave climates, and hence different wave-power levels. In general, the average wave-power level is higher in the extratropical regions than in the tropics (Fig. 10). If we integrate the wave-power level along the total length of the world's coast lines, then the estimated global potential is in the order of 1 billion kW. That is enough electricity for roughly 1 billion homes! Even if we only manage to harness 1% of this, it is still a significant resource.

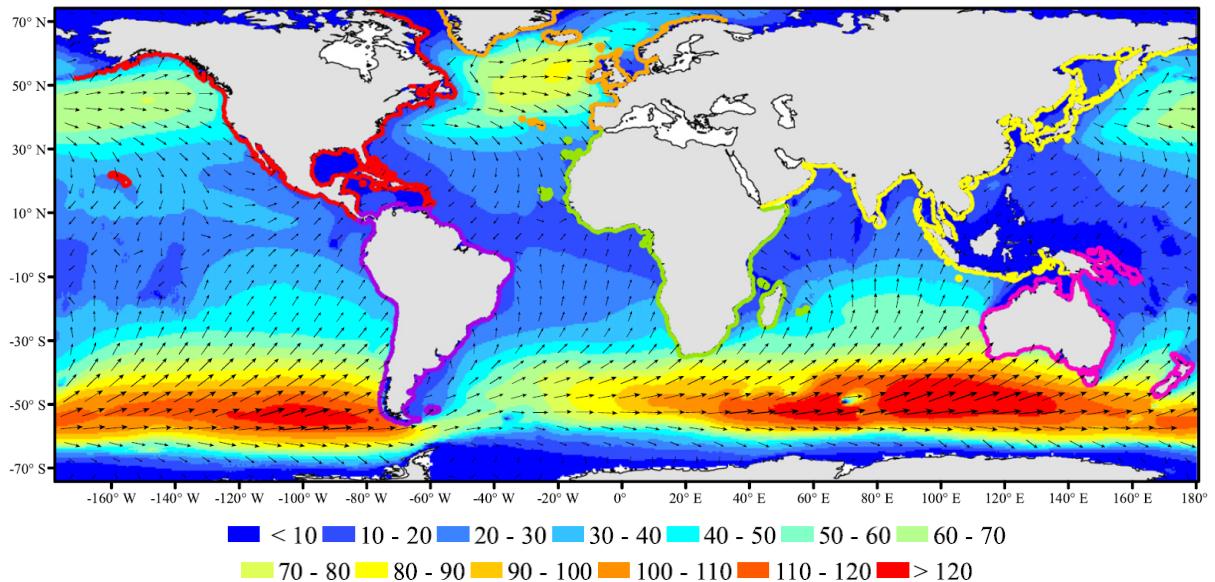


FIGURE 10: Global mean wave-power level (in kW/m) [6].

While mean wave-power levels give an indication about the potential of a given site in term of its resource, it is not a sufficient criterion for choosing a suitable location for deployment of WECs. Wave-power levels vary over time, on different time scales. Offshore wave-power levels may vary from a few kW/m during calm weather to a few hundreds kW/m during storms (remember, wave-power level is proportional to the wave height squared!). Fig. 11 shows the global distribution of wave-power level during different months of the year, demonstrating its seasonal variation.

It is clear then that a WEC will have to encounter variable wave conditions during its lifetime. Low to moderate waves occur most of the time and so they dictate the WEC's revenue, but extreme waves drive its cost since it has to survive these conditions (which do not occur as often, but do occur). In addition, wave direction also varies over time and real sea waves are multidirectional (purely unidirectional waves only exist in theory!).

For these reasons, the best locations to deploy WECs (in terms of resource) are not necessarily those with the highest mean wave-power levels. Variability must be taken into account, and sites with a low variability but not the highest mean may in fact be more economically attractive.

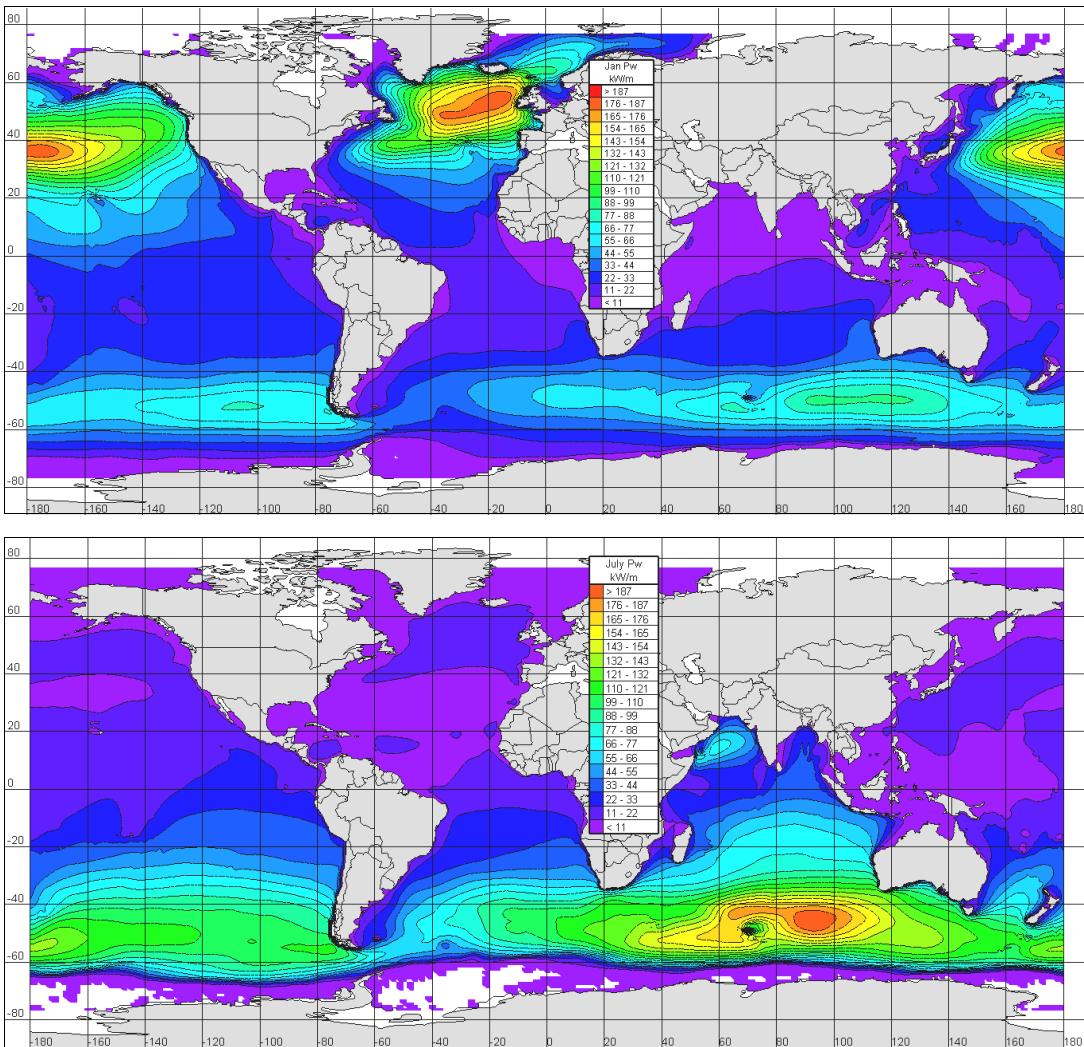


FIGURE 11: Global mean wave-power level in January (top) and July (bottom) [1].

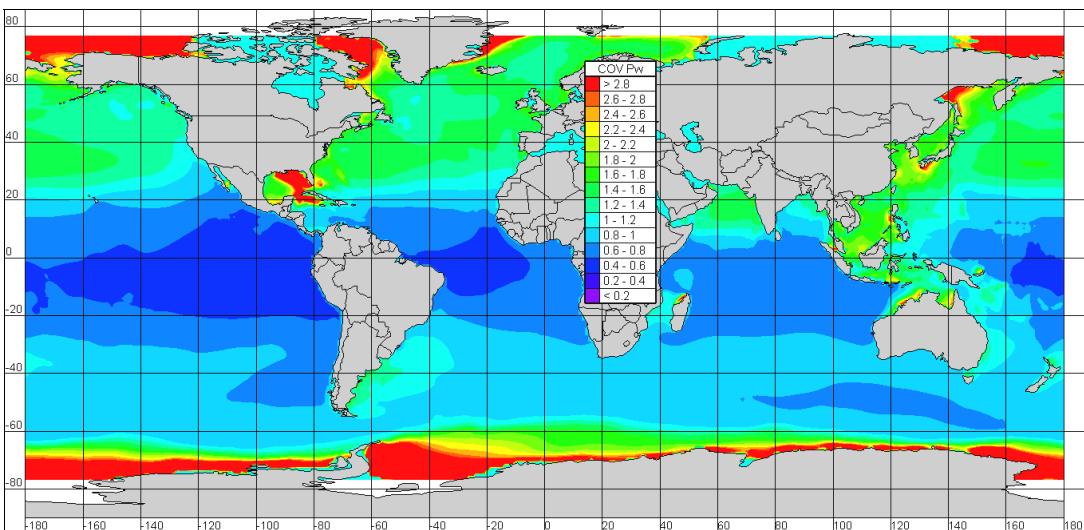


FIGURE 12: Coefficient of variation (COV) of wave-power level [1] around the world. Low COV means low variability.

Fig. 12 shows the coefficient of variation of the wave-power level globally, which when combined with Fig. 10, suggests that there are locations with relatively low variability and decent mean. Fig. 13 highlights the difference in variability of wave-power level one could get at two different locations in the world. Which location would you choose to deploy your WEC?

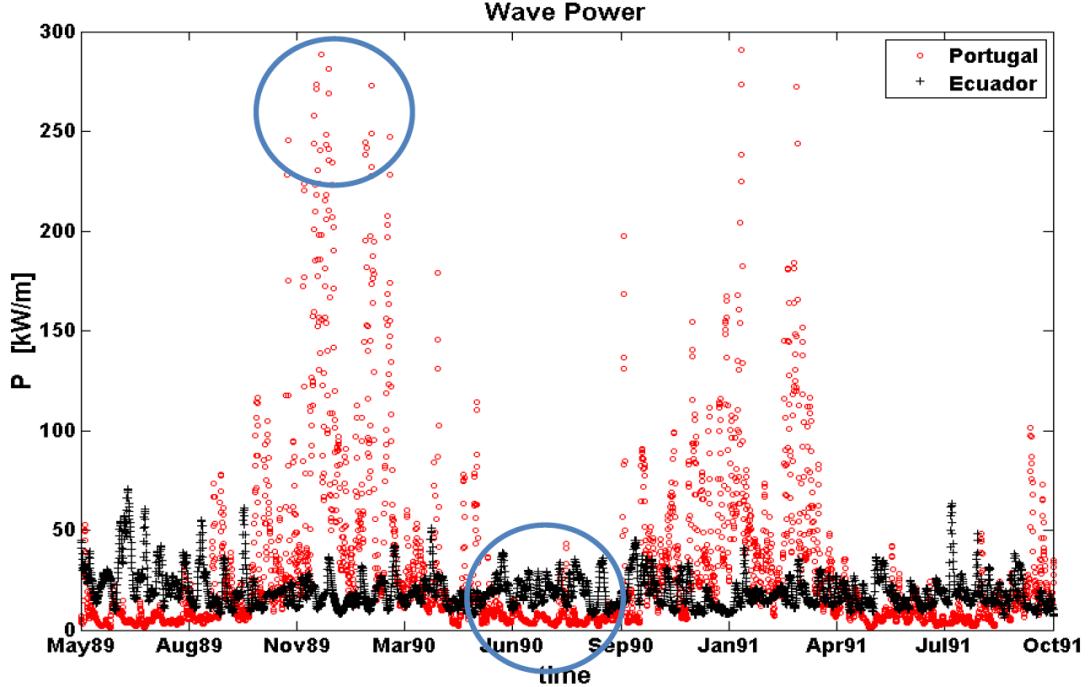


FIGURE 13: Comparison of wave-power level as a function of time, for two locations: Portugal in North Atlantic and Ecuador in Equatorial Pacific [8].

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