Quantitative Verification of Thin Lense Equation

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Abstract. Multiple experiments were conducted to verify the focal lengths in various lenses.

INTRODUCTION

Optics and lesnes are a cornerstone of modern physics with their applications ranging from the study of the cosmos to the development of modern technology. However, the fundementals of such optical systems are often overlooked. In this experiment, the thin lens equation was quantiatively verified using a variety of lenses. The experiment was conducted in two parts. The first focused on verifying the thin lens equation, the second investigating if this still holds for thick lenses. Moreover, additional experimentation was conducted to investigate the diffraction patterns of light passing through a lens.

The Thin Lens Equation

The discovery of the thin lens equaton is of some debate. While the theory of lenses was well understood by the time of the ancient Greeks, some historians name Barrow (1669) and Huygens (1653) [2] as accounting for some of the first semi-verbal descriptions of the thin lens equation. Such debate over (1) is not suprising given its importance.

$$\frac{1}{f} = \frac{1}{p} + \frac{1}{q} \tag{1}$$

(1) describes the fundemental relationship between the focal length of a lens, the object distance, and the image distance. One may also derive the relationship with magnetization (2).

$$M = -\frac{q}{p} \tag{2}$$

However, there is a fundemental assumption in both (1) and (2) which is that the lens is thin. In effect, this means that we approximate the lens as a 2D plane instead of a 3D object. In reality, a lens more acts as two seperate planes, a front plane and a back plane as can be seen in Fig. 1.

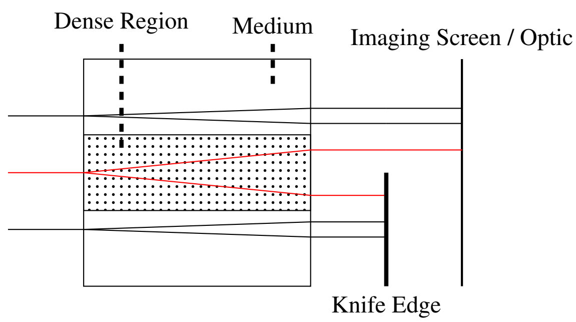

FIGURE 1: Illustration of primary planes in a 3-dimension lens as inspired from the laboratory manual [1].

The Thick Lens Equation

Chromatic Abberations

METHODOLOGY

The experiment was conducted using a 1.5 m optical bench equipped with a light source, diaphragms, lenses, and a screen. The apparatus was arranged in a classical object-lens-image configuration to investigate the thin lens equation. Three types of lenses—thin biconvex, spherical, and hemispherical—were studied.
Proper alignment of the optical components was critical to ensure accurate measurements:

1. The lamp was positioned behind a diaphragm with

a small aperture to produce a collimated beam.

2. A screen was placed at a short distance from the diaphragm to check for alignment by marking the central light spot.
3. The lens was positioned approximately 7 cm from the diaphragm, and its angle was adjusted to retro-reflect light back through the aperture.
4. The screen was moved about 1 m away to observe the far-field focus, ensuring the beam remained centered.
5. Fine adjustments were made iteratively to confirm proper alignment.

The distances between optical elements were recorded using the built-in ruler of the optical bench. To account for systematic errors, offsets between the mount edges and the optical centers of the elements were measured using calipers with a precision of 0.1 mm. Uncertainties in these measurements were estimated by taking multiple readings and calculating the standard deviation.

Focal Length Estimation

The focal length of each lens was estimated through two methods:

1. Measuring the image distance when the lens was far from the light source.
2. Using the lensmaker's equation:

$$f = \frac{R}{2(n-1)} \tag{3}$$

   where R is the radius of curvature and n is the refractive index of the lens material.

Data Collection

A transparent grid was used as the object, and a screen was placed at varying distances to obtain a sharp focus. The following measurements were recorded:

- Source, lens, and image positions along the optical bench.
- Image magnification by measuring the grid line spacing in the image.
- Multiple readings (3–5) for each configuration to determine uncertainty.

Data was collected for a range of object and image distances covering extreme cases ($q = f$ and $p = f$).

Data Analysis and Iterative Improvements

After an initial set of measurements, the data was plotted and analyzed to check consistency with the thin lens equation:

$$\frac{1}{f} = \frac{1}{p} + \frac{1}{q} \tag{4}$$

If inconsistencies were found, additional measurements were taken to refine the dataset. The analysis was conducted using MATLAB scripts (datacrunch.m and fitter.m), which performed least-squares fitting to determine the best estimates for lens parameters. The goodness-of-fit was evaluated using the chi-squared test.

Thick Lens Measurements

For thicker lenses, additional measurements were taken to determine whether the thin lens approximation remained valid. The locations of the front and back principal planes were estimated, and deviations from the ideal thin lens behavior were analyzed.

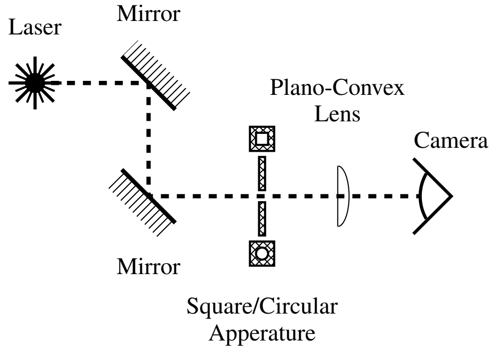

FIGURE 2: Figure

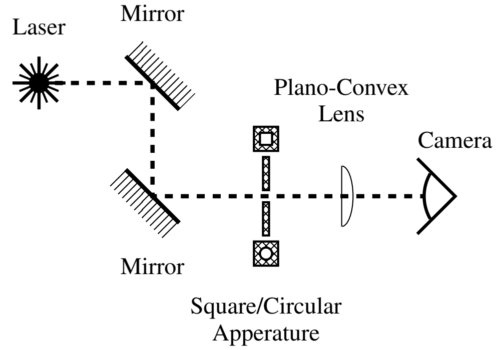

FIGURE 3: Figure

RESULTS & ANALYSIS

DISCUSSION

CONCLUSION

ACKNOWLEDGMENTS

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Appendix

Raw Data

Analysis Code