Introduction to Feedback Control of Underactuated VTOL Vehicles

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A REVIEW OF BASIC CONTROL DESIGN IDEAS AND PRINCIPLES

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This article is an introduction to feedback control design for a family of robotic aerial vehicles with vertical take-off and landing (VTOL) capabilities such as quadrotors, ducted-fan tail-sitters, and helicopters. Potential applications for such devices, like surveillance, monitoring, or mapping, are varied and numerous. For these applications to emerge, motion control algorithms that guarantee a good amount of robustness against state measurement/estimation errors and unmodeled dynamics like, for example, aerodynamic perturbations, are needed. The feedback control methods considered here range from basic linear control schemes to more elaborate nonlinear control solutions.

This article is an introduction to feedback control design for a family of robotic aerial vehicles with vertical take-off and landing capabilities such as quadrotors, ducted-fan tail-sitters, and helicopters.

The modeling of the dynamics of these systems is first recalled and discussed. Then several control algorithms are presented and commented upon in relation to implementation issues and various operating modes encountered in practice, from teleoperated to fully autonomous flight. Particular attention is paid to the incorporation of integral-like control actions, often overlooked in nonlinear control studies despite their practical importance to render the control performance more robust with respect to unmodeled or poorly estimated additive perturbations.

INTRODUCTION

The growing interest for the robotics research community in VTOL vehicles as exemplified by quadrotors [10], [19], [21], [23], [65], [82], ducted-fan tail-sitters [43], [58], [63], and helicopters [16], [32], [39] is partly due to the numerous applications that can be addressed with such systems like surveillance, inspection, or mapping. Recent technological advances in sensors, batteries, and processing cards that allow embarking on small vehicles all components necessary for autonomous flights at a reasonable cost also constitute a favorable factor. The development of these applications raises several issues. First, from a mechanical viewpoint, the vehicle must have good flying capabilities and provide enough payload for embarked sensors and energy. Then real-time pose estimation, necessary for feedback loops implementation, is often challenging. Since in a noninstrumented environment no single sensor can produce direct measurements of the pose, a combination of sensors has to be used. Data fusion algorithms must be designed to cope with sensor limitations associated with, for instance, unreliable magnetometers, low-precision global positioning satellite (GPS) devices, and low-frequency visual processing of camera images. Finally, feedback control laws must achieve fast response and robustness to aerodynamic perturbations (such as wind gusts). All these difficulties are amplified on small-scale vehicles due to the limited payload, the high sensitivity to aerodynamic perturbations, and the complexity of aerodynamic effects at this scale.

Many autonomous flight experiments have been reported in recent years. In particular, aggressive maneuvers have been achieved with quadrotors, like spins and flips [45], dance in the air [75], or high-speed flight through narrow environments [54]. The control algorithms used for these extreme maneuvers rely on feedforward action to account for and anticipate strong dynamic effects. This type of performance has been obtained in indoor environments by taking advantage of full state measurements acquired at high-frequency rates with an external three-dimensional (3D) tracking system. Reported results for aerial robotic vehicles using onboard sensors only, such as GPS, laser range finders, cameras, or acoustic sensors, address far less aggressive maneuvers [29], [9], [19], [23], [62], [77] because the system then suffers from a heavier payload (sensors and data processing cards) that reduces its maneuverability and also from low-frequency measurement rates and poor quality state estimates little compatible with the monitoring of aggressive maneuvers.

This article focuses essentially on feedback control aspects, knowing that mechanical design and state estimation aspects are equally important. The main objective is to provide the reader with an introduction to feedback control methods, linear and nonlinear, developed for translational velocity and/or position stabilization and trajectory tracking applications. Whereas linearization of the system's dynamics yields well-known control analysis and design tools that are local in scope, the nonlinear approach exploits the physics of the motion control problem at hand in a more natural way. This aspect reflects in algorithms that are not more complex and a computational effort not more important than that of linear controllers. Under certain conditions, nonlinear controllers can also be endowed with provable global stability and convergence properties. As for robustness, although no robustness analysis per say is reported, it is a permanent preoccupation whose related issues are discussed at various places in this article. For VTOL vehicles, these issues are all the more important that:

- precise aerodynamic models, valid in a large range of operating conditions, are extremely difficult to obtain
- wind gusts are unpredictable, and embarked power is not always sufficient to counterbalance the destabilizing effects of these perturbations
- pose measurement/estimation errors, in relation to the size and inertia of the vehicle, can be significant.

This article is organized as follows. Control models are reviewed, with complementary comments on the contribution of aerodynamic forces and torques. Then an overview on existing feedback control techniques developed for the class of underactuated VTOL vehicles is given. First, linear techniques based on linearized control models are reviewed.

The growing interest for the robotics research community in VTOL vehicles is partly due to the numerous applications that can be addressed with such systems like surveillance, inspection, or mapping.

The classical pole placement method invoked for the determination of the control gains can be complemented with an optimization policy (e.g., linear quadratic regulator (LQR), linear quadratic Guassian (LQG), H_2, and H_∞) and also extended to slowly time-varying operating conditions via gain-scheduling techniques. Then nonlinear techniques (exact input–output feedback linearization, backstepping, and hierarchical control) are presented, and a more detailed exposition of the control approach proposed in [26]. Finally, concluding remarks and perspectives are given. Suggestions of references for complementary reading are pointed out throughout the article.

SYSTEM MODELING

In the first approximation, VTOL vehicles can be modeled as rigid bodies immersed in a fluid and moving in 3D space. These vehicles are usually controlled via a thrust force $\vec{T}$ along a body-fixed direction $\vec{k}$ to create translational motion and a torque vector $\Gamma \in \mathbb{R}^3$ for attitude monitoring. In practice, the torque actuation is typically generated via propellers (for example, multirotors), rudders or flaps (for example, ducted-fan tail-sitters), or a swashplate mechanism (for example, helicopters).

We assume that the thrust $\vec{T} = -T\vec{k}$, with $T \in \mathbb{R}$, applies at a point that lies on, or close to, the axis $\{G;\vec{k}\}$, with G the vehicle's center of mass (CoM) so that it does not create an important torque at G. All external forces acting on the vehicle (gravity and buoyancy forces, added-mass forces, and dissipative aerodynamic reaction forces, etc.) are summed up in a vector $\vec{F}_e$. Due to aerodynamic reaction forces, this vector generally depends on the airflow's velocity and direction relatively to the vehicle, as well as on its translational and angular accelerations (via added-mass effects). Applying the Newton-Euler formalism, one obtains the following equations of motion of the vehicle (see, for example, [17, Ch. 2], [78, Ch. 1], [26]):

$$m\ddot{\xi} = -TRe_3 + F_e(\dot{\xi},\ddot{\xi},R,\omega,\dot{\omega},d(t)) + R\Sigma_R\Gamma \tag{1}$$

$$\dot{R} = RS(\omega) \tag{2}$$

$$\mathbf{I}\dot{\omega} = -S(\omega)\mathbf{I}\omega + \Gamma + \Gamma_e(\dot{\xi},\ddot{\xi},R,\omega,\dot{\omega},d(t)) + \tau_g + \Sigma_T T e_3 \tag{3}$$

where the following notation is used:

- m is the vehicle's mass, $\mathbf{I} \in \mathbb{R}^{3\times3}$ is its inertia matrix.
- $\mathcal{I} = \{O;\vec{\imath}_o,\vec{\jmath}_o,\vec{k}_o\}$ is a fixed (inertial or Galilean) frame with respect to which the vehicle's absolute pose (position + orientation) is measured. This frame is typically chosen as the north-east-down (NED) frame with $\vec{\imath}_o$ pointing to the north, $\vec{\jmath}_o$ pointing to the east, and $\vec{k}_o$ pointing to the center of the Earth. $\mathcal{B} = \{G;\vec{\imath},\vec{\jmath},\vec{k}\}$ is a frame attached to the body. The vector $\vec{k}$ is parallel to the thrust force axis. This leaves two possible and opposite directions for this vector. The direction here chosen ($\vec{k}$ pointing downward nominally) is consistent with the convention used for VTOL vehicles (see Figure 1).
- $\xi = (\xi_1,\xi_2,\xi_3)^\top \in \mathbb{R}^3$ is the vector of coordinates of the vehicle's CoM position expressed in the inertial frame $\mathcal{I}$.
- $R \in SO(3)$ is the rotation matrix representing the orientation of the body-fixed frame $\mathcal{B}$ with respect to the inertial frame $\mathcal{I}$. The column vectors of R correspond to the vectors of coordinates of $\vec{\imath},\vec{\jmath},\vec{k}$ expressed in the basis of $\mathcal{I}$.
- $\omega = (\omega_1,\omega_2,\omega_3)^\top \in \mathbb{R}^3$ is the angular velocity vector of the body-fixed frame $\mathcal{B}$ relative to the inertial frame $\mathcal{I}$ and expressed in $\mathcal{B}$.
- Γ_e is the external torque vector induced by all external forces.
- $S(\cdot)$ is the skew-symmetric matrix associated with the cross product (i.e., $S(u)v = u \times v, \forall u,v \in \mathbb{R}^3$).
- $d(t)$ represents external disturbances, including wind, that do not depend on the vehicle's position and motion.
- τ_g is the gyroscopic torque associated with rotor crafts.

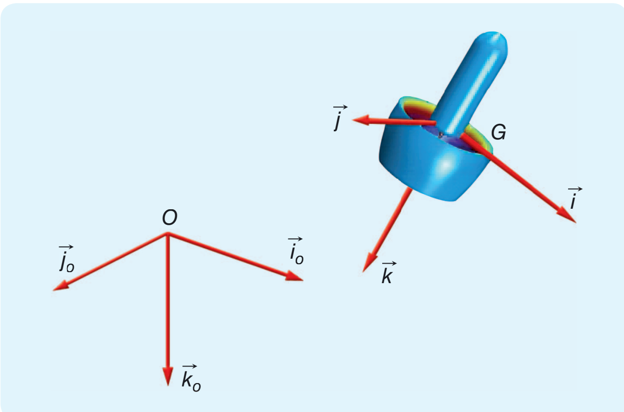

FIGURE 1 Inertial frame $\mathcal{I}$ and body frame $\mathcal{B}$.

FIGURE 2 X4-flyer: quadrotor developed by CEA-List.

- » $e_3 = (0,0,1)^\top$ is the third vector of the canonical basis of $\mathbb{R}^3$ and also the vector of coordinates in $\mathcal{B}$ of the thrust direction vector $\vec{k}$.
- » Σ_T and Σ_R denote 3×3 (approximately) constant coupling matrices.

To be complete, the dynamical model should also include a modeling of the generation of force and torque control inputs, since these inputs are typically produced via actuators that have their own dynamics. Experience shows that the actuators' dynamics of a well-designed VTOL system are sufficiently fast with respect to the vehicle's dynamics so that they can be neglected, at least in the first approximation. Another reason for considering this approximation is that it allows for the decoupling of the generic aspects of the control problem from the specifics attached to each particular vehicle. Nevertheless, the value of this decoupling assumption needs in practice to be reassessed and, eventually, reconsidered to bring adequate modifications and refinements to the control algorithms. For instance, in the case of vehicles with propellers and deflecting surfaces, the way actuation thrust force and torques depend on the airflow needs to be looked at closely. Advanced specialized issues like this one, addressed in [29] for instance, call for complementary studies which are beyond the scope of this article.

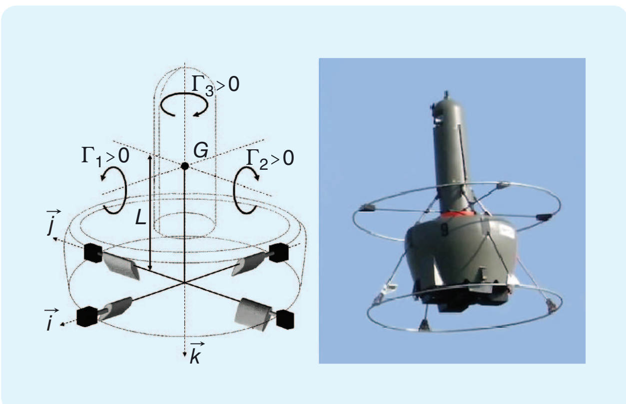

FIGURE 3 Ducted-fan tail-sitter HoverEye of Bertin Technologies.

About the Coupling Between Thrust and Actuation Torques

The constant matrices Σ_T and Σ_R in (1) and (3) represent the coupling between the thrust force and the actuated torques. For instance, the term $\Sigma_T T e_3$ in (3) accounts for the fact that the thrust force vector $\vec{T}$ may not apply exactly at the vehicle's CoM. For most VTOL vehicles, this term is relatively small so that it can be compensated for by control torque actions. By contrast, the influence of the torque control inputs on the translational dynamics via the coupling matrix Σ_R in (1), whose expression mainly depends on the vehicle's torque generation mechanism, is more involved. The coupling term $R\Sigma_R\Gamma$ is often referred to as small body forces in the literature. This coupling is negligible (i.e., $\Sigma_R \approx 0$) for quadrotors [21], [65], [10] such as the CEA-List X4-flyer (see Figure 2), but it can be significant for helicopters, due to the swashplate mechanism [24, Ch. 1], [15], [39], [46], [59, Ch. 5], and for ducted-fan tail-sitters, due to the rudder system [60, Ch. 3], [63]. For instance, the coupling matrix Σ_R of a conventional helicopter is approximately given by (see, for example, [24, Ch. 1], [59, Ch. 5], and [39])

$$\Sigma_R = \begin{bmatrix} 0 & \varepsilon_1 & 0 \\ \varepsilon_2 & \varepsilon_3 & \varepsilon_4 \\ 0 & 0 & 0 \end{bmatrix}$$

where the ε_is depend on the position of the main and tail rotor disks. Referring to the model of ducted-fan tail-sitters, such as the HoverEye (see Figure 3), the coupling matrix Σ_R is given by (see, e.g., [63], [60, Ch. 3])

$$\Sigma_R = -\frac{1}{L} S(e_3), \tag{4}$$

with L the distance between the plane of controlled fins and the vehicle's CoM. More details on the difficulties associated with this coupling term and control solutions proposed in the literature will be provided further on.

About External Forces and Torques

The modeling of the external force F_e and torque Γ_e acting on the vehicle is a major problem due to the complexity of fluid dynamics and of the interactions between the vehicle, the actuators, and the surrounding fluid (for more details about this issue, see, e.g., [65], [29], and [10] for quadrotors; [35], [38], [60, Ch. 3], and [61] for ducted-fan tail-sitters; and [56], [67], and [83] for helicopters). In particular, when the vehicle is designed so as to take advantage of strong aerodynamic lift forces, as in the case of airplanes, modeling the dependence of lift and drag forces upon the vehicle's orientation relatively to the airflow direction (i.e., the so-called angle of attack and side-slip angle) is not a simple matter. To our knowledge, no analytical representation of these forces over the vehicle's entire operating envelop has ever been worked out. Existing models are often based on a superposition principle that consists of i) decomposing the vehicle in distinct rigid parts (propellers, rudders, and duct), ii) computing a force/torque acting on each of these parts by neglecting the aerodynamic effects created by the other parts, and iii) summing up all these forces/torques to obtain the resultant force/torque. For ducted-fan tail-sitters, for example, the vehicle is decomposed into the duct structure, the propeller, and rudders. Aerodynamics forces include lift and drag forces (and respectively, momentum drag force) generated by the airflow circulating outside (respectively, through) the duct structure, as well as lift and drag forces on the propeller and rudders generated by the airflow inside the duct. Evaluating these forces separately is hardly justified since, rigorously, the vehicle moves in a fluid that exerts, by the rotation of the propeller and by airflow deflection from the rudders, a force on the fluid that in turn applies a force on the duct structure.

A good understanding of aerodynamic effects and fine modeling of these effects are crucial to design the vehicle's geometrical and mechanical characteristics to optimize the system's maneuverability and energetic efficiency. They are also useful for simulation purposes to evaluate the performance, robustness, and also the limitations of a controller. For control design, however, the knowledge of a precise and well-tuned model is not as important. A classical reason is that a well-designed feedback control is expected to grant robustness in the sense of performance insensitivity with respect to model inaccuracies. In this respect, energy dissipation due to friction also tends to be a favorable factor. In practice, it is possible to use an approximate model or a real-time estimation of the terms F_e and Γ_e. The latter possibility is preferred when adequate and accurate on-board sensors allow for it. Moreover, experiments show that in most cases an explicit model of Γ_e is not necessary. The possibility of obtaining a good online estimation of Γ_e, via dynamic inversion or high-gain observer, mainly depends on the quality and frequency acquisition of the vehicle's attitude estimation and of the measurement of its angular velocity.

A Simplified Control Model

The control model (1)–(3) is very general, but it involves terms—aerodynamic forces and torques in particular—that are very complex. For many VTOL vehicles, simplified models of these terms can be sufficiently accurate for feedback control design purposes. To keep the exposition simple without impairing its generality too much, feedback control will be discussed on the basis of the following simplified control model:

$$m\ddot{\xi} = -TRe_3 + F_e(\dot{\xi}, d(t)) + R\Sigma_R\Gamma \tag{5}$$

$$\dot{R} = RS(\omega) \tag{6}$$

$$\mathbf{I}\dot{\omega} = -S(\omega)\mathbf{I}\omega + \Gamma. \tag{7}$$

Assumptions under which the reduction of system (1)–(3) to the above form is justified are the following:

- » Added mass effects are negligible (i.e., F_e and Γ_e do not depend on $\ddot{\xi}$ and $\dot{\omega}$).
- » Aerodynamic forces do not depend on the vehicle's orientation, and they apply close to the vehicle's CoM (i.e., F_e and Γ_e depend neither on R nor on ω). This assumption corresponds ideally to the case of a spherical vehicle with a perfectly smooth surface, for which the resultant of drag forces applies at the CoM.
- » Both τ_g and Σ_T are either negligible or known so that they can be directly compensated for by the control torque action.

These assumptions, although restrictive, hold with a reasonably good degree of accuracy for many VTOL vehicles for which the resultant aerodynamic reaction force is, in nominal conditions, dominated by the drag component, as in the case of quadrotors, ducted fans, and helicopters.

System (5)–(7) shows full actuation of the rotational dynamics, via the 3D torque control vector Γ, and underactuation of the translational dynamics, via the monodimensional thrust intensity T. At the translational level, in the case where $\Sigma_R = 0$, controllability results from the nonlinear coupling term TRe_3 between the control input T and the vehicle's orientation characterized by the rotation matrix R. Feedback control thus has to exploit this coupling to stabilize the vehicle's translational dynamics. The next sections gives an overview of feedback methods, starting with linear control schemes.

BASIC CONTROL DESIGN FOR VTOL VEHICLES: LINEAR CONTROL SCHEMES

Linear control techniques are based on linear approximations of the system's dynamics about desired feasible state trajectories. They are widely used in airplanes autopilots. For these vehicles, the angle of attack over the nominal flight envelop is small, and the linearization of aerodynamics models, which are precisely tuned by using extensive wind tunnel measurements over a restricted set of nominal cruising velocities, then yields accurate linearized models. For VTOL vehicles, the variations of the vehicle's attitude with respect to the airflow direction can be much more important. Nevertheless, a similar control design approach can be adopted. This approach basically consists of considering a set of reference trajectories associated with constant translational velocities, in developing control laws for each of them, and in combining these laws via interpolation to obtain a practical control algorithm. To simplify the exposition, the focus is hereafter put on the case of quasi-stationary flight in the absence of wind, which can be seen as an extension to the problem of stabilizing a fixed position in space with a reference velocity equal to zero. The reviewed approaches can be extended to other trajectories and flight conditions, such as constant velocity cruising with constant wind, provided that aerodynamic forces are either measured or estimated online so that the equilibrium about which linearization is done can be calculated.

In the case of hovering and in the absence of wind, aerodynamic forces arising from relative vehicle/wind velocity can be neglected in front of the gravity force since their intensities are proportional to the square of this velocity. As a consequence, the external forces are essentially reduced to $F_e = mge_3$ (with g being the gravity acceleration). The equilibrium orientation matrix R_e associated with hovering at a constant position ξ and zero yaw angle (with $\dot{\xi} = 0$ and $\omega = 0$) is the identity matrix. A first-order approximation of the matrix R about this equilibrium is $R \approx I_3 + S(\Theta)$, with $\Theta \in \mathbb{R}^3$ any minimal parameterization of $SO(3)$ around the identity matrix (e.g., the vector of Euler angles ϕ: roll, θ: pitch, ψ: yaw). This yields the following linear approximation of system (5)–(7) at the equilibrium $(\xi = 0,\ \dot{\xi} = 0,\ \Theta = 0,\ \omega = 0,\ T = mg,\ \Gamma = 0)$:

$$m\ddot{\xi} = -mgS(\Theta)e_3 - \tilde{T}e_3 + \Sigma_R\Gamma \tag{8}$$

$$\dot{\Theta} = \omega \tag{9}$$

$$I\dot{\omega} = \Gamma \tag{10}$$

with $\tilde{T} = T - mg$. Without loss of generality, let us assume that the vehicle's inertia matrix is diagonal (i.e., $\mathbf{I} = \mathrm{diag}(\mathbf{I}_1, \mathbf{I}_2, \mathbf{I}_3)$). For most VTOL vehicles and especially for tail-sitters and helicopters, the coupling matrix Σ_R requires close attention. This coupling, which induces marginally stable or unstable zero dynamics, depending on the vehicle's actuation configuration, has motivated various research studies [24, App. A], [39], [61], [65]. As a case study, let us consider the class of ducted-fan tail-sitters for which the coupling matrix Σ_R is given by (4). In this case, system (8)–(10) can be decomposed into four independent single-input, single-output (SISO) linear systems. The first two systems concern the vehicle's altitude and Euler yaw angle dynamics. Their equations are

$$m\ddot{\xi}_3 = -\tilde{T}$$

$$\mathbf{I}_3\ddot{\psi} = \Gamma_3.$$

The origins $(\xi_3, \dot{\xi}_3) = (0,0)$ and $(\psi, \omega_3) = (0,0)$ of these double integrators are exponentially stabilized by classical proportional derivative (PD) controllers that can be complemented with an integral action [yielding proportional-integral-derivative (PID) controllers]. One remarks that the third torque control variable Γ_3 is only used for the control of the vehicle's yaw angle but is of no use for the control of the vehicle's position ξ. The other two independent SISO linear systems are

$$\begin{cases} m\ddot{\xi}_1 &= -mg\phi + \Gamma_2/L \\ \mathbf{I}_2\ddot{\phi} &= \Gamma_2 \end{cases} \tag{11}$$

$$\begin{cases} m\ddot{\xi}_2 &= mg\phi - \Gamma_1/L \\ \mathbf{I}_1\ddot{\phi} &= \Gamma_1. \end{cases} \tag{12}$$

By an adequate change of coordinates, each of these two systems can be rewritten as

$$\begin{cases} \dot{x}_1 = x_2 & \text{(13a)} \\ \dot{x}_2 = x_3 + \varepsilon u & \text{(13b)} \\ \dot{x}_3 = x_4 & \text{(13c)} \\ \dot{x}_4 = u & \text{(13d)} \end{cases}$$

with $(x_1, x_2, x_3, x_4) = (\xi_1, \dot{\xi}_1, -g\theta, -g\dot{\theta})$, $u = -(g/\mathbf{I}_2)\Gamma_2$, and $\varepsilon = -\mathbf{I}_2/(mgL)$ for system (11); and (x_1, x_2, x_3, x_4) $(\xi_2, \dot{\xi}_2, g\phi, g\dot{\phi})$, $u = (g/\mathbf{I}_1)\Gamma_1$, and $\varepsilon = -\mathbf{I}_1/(mgL)$ for system (12). The linear system (13a)–(13d) is controllable but the stabilization of its origin $x = 0$, although obvious, deserves close attention. A certain number of contributions on the nonlinear control of VTOL vehicles turn out to have a close kinship with the linear control approaches summarized below:

The first approach focuses on the exponential stabilization of $(x_1, x_2) = (0,0)$ by using the control input εu when $\varepsilon \neq 0$. Setting $v := x_3 + \varepsilon u$, results in the double integrator

$$\begin{cases} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= v \end{cases} \tag{14}$$

whose origin is exponentially stabilized by the PD controller

$$v = -k_0x_1 - k_1x_2, \text{ with } k_{0,1} > 0.$$

The closed-loop exponential convergence of v to zero also follows. Once this convergence is achieved (i.e., when $x_1 = x_2 = 0$), the dynamics of the variables x_3 and x_4 are given by

$$\begin{cases} \dot{x}_3 = x_4 \\ \dot{x}_4 = -x_3/\varepsilon \end{cases}. \tag{15}$$

It appears from these equations that the origin of system (15) is at best marginally stable. It is so when $\varepsilon > 0$, but it is unstable when $\varepsilon < 0$. Besides, even when it is marginally stable as in the case of a helicopter, for instance, it is oscillatory at a high frequency, since ε is often very small and thus highly sensitive to the slightest perturbation. For these reasons this approach, which looks simple and attractive in the first place because it exploits the coupling terms associated with a nonzero matrix Σ_R,, is not recommended.

The second approach consists of designing a full-state feedback controller

$$u = -k_0x_1 - k_1x_2 - k_2x_3 - k_3x_4, \text{ with } k_{0,1,2,3} > 0,$$

yielding the exponential stabilization of the origin of system (13a)–(13d). In this case, when ε is small the term εu affects the control performance marginally only. In fact, it suffices to choose the control gains k_i $(i = 0, \ldots, 4)$ so that the characteristic polynomial of the closed-loop system, given by

$$p^4 + (k_3 + \varepsilon k_1)p^3 + (k_2 + \varepsilon k_0)p^2 + k_1p + k_0,$$

is Hurwitz. For example, by choosing k_2 and k_3 large enough with respect to εk_0, the precise knowledge of ε does not matter. Although this approach looks simple enough, it is seldom used in practice. A probable reason is that practitioners prefer to decompose the fourth-order system (13a)–(13d) into a cascade of subsystems of order two at most so as to facilitate control implementation (with on-the-shelf PID controllers) and failure diagnosis. In particular, a separation principle between the control of x_1 (in a so-called guidance loop) and the control of x_3 (in a so-called control loop) is often adopted. It is partly justified by the different control and measurement rates associated with the two levels: low frequency for the guidance loop and high frequency for the control loop. This yields the hierarchical control described next.

Classical hierarchical control essentially neglects the coupling term εu in the translational dynamics (i.e., by setting $\varepsilon = 0$ or equivalently $\Sigma_R = 0$) and consists of considering x_3 as a control input for subsystem (13a)–(13b). Then the desired value of x_3, here denoted as x_3^d, is used as a reference value for subsystem (13c)–(13d). Either backstepping or a high-gain technique can be applied. For instance, backstepping leads to rewriting the system as

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = x_3^d + \tilde{x}_3 \\ \dot{\tilde{x}}_3 = \tilde{x}_4 \\ \dot{\tilde{x}}_4 = -\ddot{x}_3^d + u \end{cases}$$

with $\tilde{x}_3 = x_3 - x_3^d$ and $\tilde{x}_4 = x_4 - \dot{x}_3^d$. With this approach x_3^d represents a desired control input for the asymptotic stabilization of the origin of subsystem (13a)–(13b). With a PD controller of the form $x_3^d = -k_0 x_1 - k_1 x_2$, with $k_{0,1} > 0$, closing the loop yields $\dot{x}_3^d = -k_0 x_2 - k_1 x_3$ and $\ddot{x}_3^d = -k_0 x_3 - k_1 x_4$. It then remains to design a control input u which asymptotically stabilizes the "errors" $\tilde{x}_3$ and $\tilde{x}_4$ at zero. One option is to take

$$u = \ddot{x}_3^d - k_2 \tilde{x}_3 - k_3 \tilde{x}_4, \text{ with } k_{2,3} > 0. \tag{16}$$

Since the characteristic polynomial of the closed-loop system is

$$(p^2 + k_1 p + k_0)(p^2 + k_3 p + k_2),$$

the positivity of all gains k_i $(i = 0, \ldots, 3)$ indeed ensures, by a straightforward application of the Routh-Hurwitz criterion, the exponential stability of the origin of the initial fourth-order system. A variation of the backstepping technique consists in using a high-gain controller for the second subsystem [i.e., subsystem (13c)–(13d)] while neglecting the dynamics of the reference variable x_3^d (i.e., by setting $\dot{x}_3^d = \ddot{x}_3^d = 0$ in expression (16) of the control input u). This yields the simpler controller $u = -k_2 \tilde{x}_3 - k_3 x_4$ and the closed-loop characteristic polynomial

$$p^4 + k_3 p^3 + k_2 p^2 + k_1 k_2 p + k_0 k_2$$

which is Hurwitz if and only if $k_3 > k_1$ and $k_2 > k_3^2 k_0 / (k_1(k_3 - k_1)) > 0$, by application of the Routh-Hurwitz criterion. The choice of high gains k_2 and k_3 is consistent with these inequalities and is legitimate in practice when the attitude dynamics can be rendered fast compared to the translational dynamics, and high-frequency measurements of the vehicle's attitude and angular velocities are available. With recent advances on MEMS gyrometers, precise high-frequency angular velocity measurements from an embedded inertial measurement unit (IMU) are now available at a modest cost. However, estimating the vehicle's orientation is much more difficult. The use of accelerometers to recover roll and pitch angles is often invoked, but the subject remains controversial [53]. Alternative approaches have been proposed lately based on complementary measurements (issued from a GPS, for instance). This still constitutes an active research topic [13], [25], [47], [52], [72]. A separation of the system's dynamics that better takes the above-mentioned measurement rates and limitations into account consists of moving the orientation variables to the slow guidance loops, together with the position and translational velocity variables. This yields the hierarchical control scheme described next.

Another hierarchical control strategy consists of using x_4 (i.e., the variable associated with roll and pitch angular velocities) as a virtual control input for the third-order subsystem (13a)–(13c), with the coupling term εu being neglected. The desired value of x_4, denoted as x_4^d, is then used as a reference for the second subsystem that, in this case, is reduced to the first-order equation (13d). For instance, applying a backstepping technique yields the system:

$$\begin{cases} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= x_3 \\ \dot{x}_3 &= x_4^d + \tilde{x}_4 \\ \dot{\tilde{x}}_4 &= -\dot{x}_4^d + u \end{cases}$$

with $\tilde{x}_4 = x_4 - x_4^d$ and x_4^d typically chosen of the form $x_4^d = -k_0 x_1 - k_1 x_2 - k_2 x_3$ with $k_{0,1,2} > 0$. Then, the control input u is designed to exponentially stabilize $\tilde{x}_4 = 0$. For instance, the feedback controller

$$u = \dot{x}_4^d - k_3 \tilde{x}_4, \text{ with } k_3 > 0, \tag{17}$$

yields the following closed-loop characteristic polynomial

$$(p^3 + k_2 p^2 + k_1 p + k_0)(p + k_3)$$

which is Hurwitz provided that all gains $k_i (i = 0, \ldots, 3)$ are positive and $k_1 k_2 > k_0$. A simplified solution consists of using a high-gain controller for the second subsystem [i.e., subsystem (13d)] while neglecting the dynamics of x_4^d (i.e., by setting $\dot{x}_4^d = 0$ in the expression (17) of u). Then, the closed-loop characteristic polynomial is

$$p^4 + k_3(p^3 + k_2 p^2 + k_1 p + k_0)$$

and it is Hurwitz if and only if $k_1 k_2 > k_0$ and $k_3 > k_1^2/(k_1 k_2 - k_0)$. The latter inequality is consistent with the choice of a high gain k_3 for the control of the second subsystem.

Beyond classical pole placement techniques, modern linear control theory is concerned with the determination of optimal control gains associated with optimal control problems. Typical underlying preoccupations are energy efficiency and robustness with respect to model uncertainties and/or aerodynamic disturbances. Examples of published studies along this direction are

- [7], where an LQR controller is proposed for the control of the rotational dynamics of a quadrotor helicopter and compared to a PID controller; the LQR control technique has also been applied to the stabilization of the lateral position and Euler roll angle of a quadrotor helicopter [11] and to velocity stabilization of a ducted-fan tail-sitter [79]
- [3], where an LQG controller, incorporating a quadratic criterion for weighting the model sensibility with respect to parametric variations, is applied to a model of quadrotor helicopter
- [12], [44], [48], [66], and [80], where robust H_2 and H_∞ control techniques are applied to helicopter models.

The main limitation of linear control techniques is the local nature of the control design and analysis. More precisely, the following issues should be kept in mind when applying these techniques, especially for cruising flight (with nonzero reference velocity):

- In the case of reference trajectories with time-varying velocities, or in the case of wind gusts, the equilibrium orientation varies with time. The linearized system associated with the error dynamics is then also time-varying, and ensuring the stability of the origin of this system becomes much more involved. Such is the case, for instance, of the "gain-scheduling" approach used for helicopters (e.g., [36]) and airplanes autopilots (e.g., [30]), which is based on interpolating local linear controllers calculated for a set of nominal operating conditions. By essence, this approach does not account for rapidly changing operating conditions that render the stability analysis untractable even in the ideal case where all forces are measured exactly.
- Due to aerodynamic disturbances, the vehicle may at some point depart from its nominal operating domain and even get far from it. In this case, the linearized dynamics are no longer representative of the real dynamics.
- A more purely technical issue, here mentioned essentially to point out useless complications, is that linearization involves a minimal parameterization of the vehicle's attitude. This yields representation singularities that artificially limit the domain of application of the controller, unless a switching procedure between several local minimal representations, specifically tailored to solve this problem, is added to the controller. In this respect, some parameterizations (for example, Rodrigues parameterization) are known to be better than others because their domain of validity is larger. For instance, despite their historical importance and frequent use, Euler angles are not best appropriate for the control of small VTOL vehicles.

NONLINEAR CONTROL

Compared to control design methods based on linearization, nonlinear methods can yield controllers with a significantly enlarged domain of stability and enhanced robustness, especially in the case of highly nonlinear dynamics, input saturations, or fast time-varying perturbations. To simplify the exposition, nonlinear feedback control laws for underactuated VTOL vehicles are here classified into two families, depending on the design approach considered.

Controllers Based on Dynamic Extensions

The approach essentially consists of considering TRe_3 as a system state variable and in differentiating it until three independent control variables allowing for the exact linearization of the translational dynamics are obtained. Early work [22], based on exact input–output linearization [31], has focused on the control of a planar VTOL (PVTOL) aircraft. Extensions to the 3D case for a nonlinear model of a conventional helicopter are reported in [39]. To provide the reader with a more precise idea of the approach, without complicating the exposition too much, let us again assume that the external forces acting on the vehicle are reduced to the vehicle's weight (i.e., $F_e = mge_3$) and that the coupling term $\Sigma_R \Gamma$ in the translational dynamics can be neglected. Then, the following relations can be derived from the equations of system (5)–(7):

$$\begin{cases} \dot{X}_1 &= X_2/m \\ \dot{X}_2 &= X_3 \\ \dot{X}_3 &= X_4 \\ \dot{X}_4 &= U \end{cases} \tag{18}$$

with

$$\begin{aligned} X &= (X_1, X_2, X_3, X_4)^\top \\ &= (\xi, m\dot{\xi}, -TRe_3 + mge_3, -R\delta)^\top, \\ \delta &= (T\omega_2, -T\omega_1, \dot{T})^\top, \\ U &= R(-\ddot{T}e_3 + TS(e_3)\mathbf{I}^{-1}\Gamma + 2\dot{T}S(e_3)\omega - TS(\omega)^2 e_3 \\ &\quad - TS(e_3)\mathbf{I}^{-1}S(\omega)\mathbf{I}(\omega). \end{aligned}$$

Provided that $T \neq 0$, the application $(\ddot{T}, \Gamma) \to U$ is surjective, thus allowing U to be viewed as a new control vector. Since system (18) is linear and controllable, the asymptotic stabilization of a given reference trajectory for $\xi = X_1$ is then a simple matter. Although this approach looks attractive at first glance, its implementation raises a few issues:

» For rotary-wing VTOL vehicles, such as ducted-fans and quadrotors, the thrust force is generated by propeller(s) and T is a function of the associated motor's angular velocity, a quantity commonly used as a reference input by motor manufacturers. If, instead of this velocity, $\ddot{T}$ is used as a control variable this means that the motor's jerk becomes a control input that has to be monitored. This, in turn, induces serious complications along the necessity to incorporate the motor's dynamics into the control design.

» In view of the expression of U, the calculation of the control inputs $\ddot{T}$ and Γ involves the knowledge of T and $\dot{T}$. For most platforms, these two variables are not available to measurement. Theoretically, these variables could be estimated online from measurements of the vehicle's velocity. In practice, however, the low frequency rate of these measurements (approximately 10 Hz with a GPS) does not allow for accurate estimations.

» The control is well defined as long as $T \neq 0$. But the noncrossing of zero by T, as a state variable, is not guaranteed.

Hierarchical Controllers

Many nonlinear control laws of this type for underactuated VTOL vehicles have been proposed. Problems addressed with this class of methods include:

» way-point navigation [62]

» visual servoing [40]

» control with partial state measurement [1], [6]

» robustness with respect to unmodeled dynamics like, e.g., variations of the vehicle's mass or gravitational field [21], [41]; aerodynamic disturbances [26], [62], [63], [73]; and parametric uncertainties [33], [49], [50]

» robustness with respect to measurement errors [14]

» actuators saturation [20], [50], [51], [74], [86].

Most design methods are based on "classical" Lyapunov-type approaches, but other techniques have also been used such as sliding-mode control [8], [42], [85] or predictive control [5], [34], [37].

A common denominator of hierarchical nonlinear controllers is their focus on the determination of the thrust vector TRe_3 to control the vehicle's translational dynamics. They are reminiscent of the hierarchical linear controllers described in earlier by their two-stage architecture composed of a "low-level" fast inner-loop and a "high-level" slow outer-loop. The principles of hierarchical control for the stabilization of reference translational velocities are presented next. Indications of how these principles can be extended to other control objectives, such as the stabilization of reference position trajectories and the compensation of unmodeled dynamics and perturbations via the use of an integral action, are then given.

Let us first further simplify the control model (5)–(7) by neglecting the coupling term $\Sigma_R \Gamma$ in (5) and by using ω as a control variable. Both simplifications are often made. While the validity of the former should be assessed on a case-by-case basis, the latter corresponds to the linear hierarchical strategy considered last in the previous section, with x_4 taken as a virtual control input. It is justified by the full actuation of the vehicle's orientation and by the high-frequency rate measurement of the angular velocity vector ω from an embedded set of gyrometers, which allows for the fast convergence of ω to a desired reference value ω_d. The control model is then reduced to the following equations:

$$\begin{cases} m\ddot{\xi} = -TRe_3 + F_e(\dot{\xi}, d(t)) \\ \dot{R} \;=\; RS(\omega) \end{cases} \tag{19}$$

with T and ω as control inputs.

Velocity Control

Let $\dot{\xi}_r$ denote a reference translational velocity expressed in the inertial frame I, $\ddot{\xi}_r$ its time-derivative, and $\dot{\tilde{\xi}} := \dot{\xi} - \dot{\xi}_r$ the velocity error expressed in the same frame. The error model can be obtained

$$\begin{cases} m\ddot{\tilde{\xi}} = -TRe_3 + F(\dot{\xi}, t) & \text{(20a)} \\ \dot{R} = RS(\omega) & \text{(20b)} \end{cases}$$

with

$$F(\dot{\xi}, t) := F_e(\dot{\xi}, d(t)) - m\ddot{\xi}_r(t) \tag{21}$$

and with either the integral of the velocity error $\tilde{\xi} := \int_0^t \dot{\tilde{\xi}}(s)\,ds$ or the position tracking error $\tilde{\xi} := \xi - \xi_r$ when a reference trajectory ξ_r is specified.

Equilibrium Analysis

To asymptotically stabilize zero for the velocity error $\dot{\tilde{\xi}} := \dot{\xi} - \dot{\xi}_r$, $\dot{\tilde{\xi}} \equiv 0$ must be an equilibrium of (20a). This in turn implies that $\ddot{\tilde{\xi}} \equiv 0$ must be satisfied and thus, in view of (20a), that

$$T\eta = F(\dot{\xi}_r, t), \tag{22}$$

with $\eta := Re_3$ the unit vector characterizing the thrust force direction. The underlying geometrical interpretation is clear: at the equilibrium the thrust direction is aligned with the direction of the external force (corrected by the reference acceleration) and the thrust intensity counterbalances the external force intensity. The above basic relation calls for simple but important remarks:

» As long as $F(\dot{\xi}_r, t)$ is different from zero, there exist only two solutions (T_r, η_r) to (22), given by

$$(T_r, \eta_r) = \left(\pm |F(\dot{\xi}_r, t)|, \pm \frac{F(\dot{\xi}_r, t)}{|F(\dot{\xi}_r, t)|} \right). \tag{23}$$

In practice, the chosen solution is most of the time imposed by physical considerations, such as the positivity of the thrust intensity.

- As already mentioned in the linear control section, monitoring the translational motion of a flying vehicle does not impose by itself any constraint upon the yaw angle (i.e., the modification of which does not change the thrust direction). This angle can thus be controlled independently to achieve a complementary objective. This also means, as one could logically expect, that only three independent actuation inputs corresponding to three degrees of freedom are needed to control the three translational velocity (or position) coordinates of a vehicle.
- When $F(\dot{\xi}_r, t) = 0$, any unit vector η satisfies (22) with $T = 0$. This singular case corresponds to specific operating conditions that are not commonly encountered in practice with VTOL vehicles. This is due to the gravity term involved in $F(\xi_r, t)$ that keeps this force away from zero when aerodynamic forces and translational accelerations are not too strong. However, intense wind gusts or very aggressive reference trajectories may provoke this situation whose severity, in terms of control, can be appreciated by the fact that the linearization of the error system (20) at any equilibrium point $(\dot{\tilde{\xi}}, R) = (0, R^*)$ satisfying the equilibrium condition $F(\dot{\xi}_r, \cdot) \equiv 0$ is not controllable, even though the system itself remains theoretically (mathematically) controllable. Stabilization of such equilibria is a challenging problem whose solution involves very specific nonlinear control techniques (see, e.g., [55]), the exposition of which is beyond the scope of this article.
- When $F(\dot{\xi}_r, t) \neq 0$, the well-posedness of the solutions (23) to (22) is much related to the dependence of F_e upon the vehicle's orientation (i.e., upon the rotation matrix R). More precisely, when F_e and, thus, F do not depend on R, as in the hovering case for which F_e is essentially reduced to the vehicle's weight, it is clear that the equation has a unique solution η_r once the sign of T_r has been chosen. Another favorable case is when the aerodynamic force F_e is dominated by its drag (versus lift) component whose direction is (by definition) given by the airflow and is thus essentially independent of the vehicle's orientation. Otherwise, the equation becomes implicit in η_r and existence and uniqueness of the solution (given the sign of T_r) are no longer systematic. Several subcases can then be considered to attempt determining sufficient conditions under which these properties are satisfied. We will come back to this issue.

Orientation Control

In a first stage, consider the problem of stabilizing a given desired direction characterized by a unit vector η_d, independently of other possible control objectives. This desired direction may equally be specified by the orientation of a user's joystick or be the (locally) unique solution η_r previously evoked for velocity control purposes. For instance, in the case of hovering, one can take $\eta_d = e_3$ and nonlinear solutions to the control problem can be compared to the linear ones presented in the previous section. Defining $\omega_{1,2} := (\omega_1, \omega_2)^\top$, a possible nonlinear control solution, among many other possibilities, is [26]

$$\omega_{1,2} = \left(R^\top \left(k_0 \frac{\eta \times \eta_d}{(1 + \eta^\top \eta_d)^2} - S(\eta)^2 (\eta_d \times \dot{\eta}_d) \right) \right)_{1,2} \tag{24}$$

with $k_0 > 0$. This control ensures the exponential stability of the equilibrium point $\eta = \eta_d$ provided that the initial conditions $\eta(0)$ and $\eta_d(0)$ are not opposite to each other. It is obtained by considering the storage function $\nu := 1 - \eta^\top \eta_d$ whose nonpositive time derivative, along any solution to the controlled system, satisfies

$$\dot{\nu} = -k_0 \frac{|\eta \times \eta_d|^2}{(1 + \eta^\top \eta_d)^2}.$$

Two remarks are in order at this point. The first one concerns the time-derivative $\dot{\eta}_d$, which is present in the right-hand side of (24). This term can be problematic when the desired direction η_d depends on the vehicle's orientation R because its time derivative then involves ω. Relation (24) then becomes an implicit equation in ω that may not have a (bounded) solution. Note that, in the case where η_d is taken equal to η_r, this difficulty may arise even when the well-posedness of η_r is established. In this case, another control solution must be sought. The second remark concerns the calculation of η_r as a reference direction. In practice, and in general, the expression of aerodynamic forces along arbitrary reference trajectories is not known. As a consequence, neither $F(\dot{\xi}_r, t)$ nor η_r is known. On the other hand, it is often possible to obtain an online estimation of F at the current point (i.e., $F(\dot{\xi}, t)$) via, for example, accelerometer measurements or high-gain observers (see [27] and [24, Ch. 2] for more details). For this reason, control approaches based on this latter information are often preferable. The velocity control schemes discussed next take this route, knowing that stabilization of the reference thrust direction is not by itself sufficient to ensure velocity stabilization and that complementary terms involving the velocity error $\dot{\tilde{\xi}}$ must also be introduced in the control law.

Classical Control Design

This approach (see, e.g., [62] and [63]) is an extension of the previous orientation control design to translational velocity control. First, the velocity error dynamics is rewritten as [compare with (20a)–(21)]

$$m\ddot{\tilde{\xi}} = -\beta\left(\dot{\tilde{\xi}}\right) - T\eta + \bar{F}(\dot{\xi}, t), \tag{25}$$

with

$$\bar{F}(\dot{\xi}, t) := \beta\left(\dot{\tilde{\xi}}\right) + F\left(\dot{\tilde{\xi}}, t\right) \tag{26}$$

and $\beta(\cdot)$ a function chosen so as to make $\dot{\tilde{\xi}} = 0$ a globally asymptotically stable equilibrium of the equation

$m\ddot{\tilde{\xi}} = -\beta(\dot{\tilde{\xi}})$. A simple example is the linear function defined by $\beta(x) := k_\beta x$, with $k_\beta > 0$. It then only remains to ensure the asymptotic stability of $\bar{F} - T\eta = 0$ via an adequate choice of the thrust intensity T and of the control of the thrust direction η. Assuming that $\bar{F}$ does not vanish, a possibility consists of defining T and a desired thrust direction η_d according to [compare with (23)]

$$(T, \eta_d) = \left(\pm |\bar{F}(\dot{\xi}, t)|, \pm \frac{\bar{F}(\dot{\xi}, t)}{|\bar{F}(\dot{\xi}, t)|} \right)$$

and in making η converge to η_d. To this purpose, if η_d does not (or little) depend(s) on the vehicle's orientation, the orientation control $\omega_{1,2} := (\omega_1, \omega_2)^\top$ defined by (24) can be used.

Let us briefly comment upon the choice of the feedback term $\beta(\dot{\tilde{\xi}})$. Beside the infinite number of possibilities for the choice of the function β, it matters to limit the risk of a vanishing $\bar{F}$. To this end a function uniformly smaller in norm than the vehicle's weight (corresponding to the norm of F_e when hovering in the absence of wind) seems preferable to the linear function used in linear controllers. One can consider, for instance,

$$\beta(\dot{\tilde{\xi}}) := \mathrm{sat}_\Delta(K_\beta \dot{\tilde{\xi}}),$$

with K_β diagonal positive gain matrix, $\Delta < mg$, and $\mathrm{sat}_\Delta(\cdot)$ either the classical saturation function defined by $\mathrm{sat}_\Delta(x) := x\min(1, \Delta/|x|)$ or a differentiable approximation such as $\mathrm{sat}_\Delta(x) := \Delta x/(1 + |x|)$.

Other Control Design

A nonlinear control design, recently proposed by the authors [26] and with a structure slightly different from the classical design presented previously, involves the "current" force $F(\dot{\xi}, t)$ of relation (21) whose intensity T_c and direction η_c are given by

$$(T_c, \eta_c) = \left(\pm |F(\dot{\xi}, t)|, \pm \frac{F(\dot{\xi}, t)}{|F(\dot{\xi}, t)|} \right).$$

The proposed controller, whose angular velocity components are reminiscent of these defined by (24) with η_d replaced by η_c, is

$$\begin{cases} T = T_c + T_c\left(\eta^\top \eta_c - 1 + k_2 \eta^\top \dot{\tilde{\xi}}\right) \\ \quad = T_c \eta^\top \left(\eta_c + k_2 \dot{\tilde{\xi}}\right) & \text{(27a)} \\ \omega_{1,2} = R^\top \left(k_0 \dfrac{\eta \times \eta_c}{(1 + \eta^\top \eta_c)^2} - S(\eta)^2 (\eta_c \times \dot{\eta}_c) \right)_{1,2} \\ \quad + \left(k_1 T_c R^\top \left(\eta \times \dot{\tilde{\xi}}\right)\right)_{1,2}. & \text{(27b)} \end{cases}$$

It is derived by considering the storage function

$$\mathcal{L} := \frac{mk_1}{2} \left|\dot{\tilde{\xi}}\right|^2 + (1 - \eta^\top \eta_c)$$

whose time derivative

$$\dot{\mathcal{L}} = -k_0 \frac{|\eta \times \eta_d|^2}{(1+\eta^\top \eta_d)^2} - k_1 k_2 T_c (\eta^\top \dot{\tilde{\xi}})^2$$

along any solution to the controlled system is nonpositive. It ensures the asymptotic stability of the equilibrium $(\dot{\tilde{\xi}}, \eta) = (0, \eta_c)$ and the convergence to this equilibrium of all system's solutions with initial conditions such that $\eta(0)$ and $\eta_c(0)$ are not opposite to each other.

The main difference with the classical control design is the use of F rather than $\bar{F}$ in the definition of the "desired" thrust direction η_c, with the advantage of not adding the contribution of the feedback term $\beta(\dot{\tilde{\xi}})$) to the risk of encountering the singular case associated with the vanishing of $\bar{F}$. Being less prone to ill-posedness, this control solution potentially applies to a larger operating domain.

Remarks

- Several studies (see, for example, [18], [50], [51], and [59, Ch. 5]) propose to define a desired vehicle's orientation $R_d \in SO(3)$ instead of a desired thrust direction η_d $(= R_d e_3)$. For example, R_d may be computed from η_d and another noncollinear direction that defines a desired value for the yaw angle. Stabilization of R to R_d can then be achieved with any of the numerous control solutions proposed in the literature (see, e.g., [50], [51], [33], [73], [81], [82], [84], [18], [59, Ch. 5], and [64]). However, the approach presented here shows that there is no necessity in the first place to determine the vehicle's orientation completely when the objective is just to control its translational velocity (or its position). Controlling the yaw angle corresponds to a complementary decoupled objective that is achieved via the determination of the remaining angular velocity component ω_3, provided, of course, that this degree of freedom is itself associated with an independent means of actuation. For instance, in the case of helicopters and airplanes in cruising mode, it usually matters to keep the side-slip angle, i.e., the lateral angle between the airflow and the vehicle's longitudinal axis, small. This is achieved via the monitoring of the yaw angle (called the roll angle in the case of airplanes, because the longitudinal axis and thrust direction are quasi-horizontal) either by relying on passive aerodynamic forces associated with the geometry and shape of the vehicle (helicopter's tail, airplane's dihedral angle) or by using complementary actuators (helicopter's tail rotor, airplanes ailerons) that allow for a more precise control of ω_3 and the yaw (respectively, roll) angle in a large operating range.
- As pointed out before, well-posedness of the nonlinear velocity control solutions considered here partly relies on the assumption of a force $F(\dot{\xi}, t)$ or $\bar{F}(\dot{\xi}, t)$ always different from zero. Particularly harsh unpredictable flying conditions capable of inducing large tracking errors can clearly invalidate this property, no matter how good the control system is. However, the fact that these forces essentially reduce to the resultant external force $F_e(\xi_r, d(t))$ at the equilibrium (when the reference acceleration is small) suggests to select reference

velocities for which the magnitude of this force remains larger than a certain "security" threshold. Furthermore, using the dissipativity (passivity) of aerodynamic forces, it has been shown (see [26] for details) that properties of quasi-global stability can be obtained under the sole condition that $F(\dot{\xi}_r, d(t)) \neq 0$ for all t.

» Another potential cause, also pointed out before, for ill-posedness of the controllers is the strong dependence of the external force F_e upon the vehicle's orientation R. In particular, a detailed analysis of equilibrium thrust directions in the case where the lift component is important has revealed that, given a reference translational velocity, there may exist several such equilibrium directions and, furthermore, that their number may vary with the reference velocity [68]. This in turn implies that some of these equilibria may disappear when the reference velocity changes, in which case the asymptotic stabilization of such an equilibrium is clearly meaningless. This difficulty, intimately related to the appearance of the so-called "stall" phenomenon, calls for complementary investigations.

» The control expressions (24) and (27b) involve the time derivative of the desired (respectively, current) thrust direction η_d (respectively, η_c). Since these directions themselves depend on the vector of external forces F_e, an estimation of the time derivative of this vector is needed to calculate the control inputs. Obtaining a good estimation of this derivative for arbitrary flight conditions is not easy. However, for favorable flight conditions, such as hovering or constant velocity cruising with constant wind, this derivative can be taken equal to zero without much affecting the control performance. More details on the issue can be found in [4] in the particular hovering case. This simplification is also commonly made in practice for way-point navigation and visual servo-control.

» In relation to the previous remark, not only can F_e vary quickly in some cases, because of either rapidly changing environmental conditions or important reference accelerations required by the execution of extreme maneuvers, the possible dependence of this force upon the vehicle's orientation makes the time derivative of this force a function depending linearly on the angular velocity ω, thus rendering the control expressions (24) and (27b) implicit in the control inputs $\omega_{1,2}$. As long as the corresponding "gains" are small, this dependence can be neglected in the calculation of the control inputs. Otherwise, it has to be evaluated via a model of F_e and eventually used in a modified expression of the control. This issue particularly concerns vehicles with large wing surfaces that produce strong lift forces depending on their orientation with respect to the airflow. Exploring it further is of utmost importance to extend and generalize the nonlinear control methods presented here.

Basic extensions of the previous nonlinear velocity controllers to the more demanding control objectives of velocity control with integral correction and of position control are presented next. For the sake of brevity, these extensions are only specified for the classical design version, but they apply similarly to the other design (see [26] for details).

Velocity Control with Integral Correction

Robustness against model uncertainties is discussed next. The control laws presented so far involve the knowledge of the external force vector F_e. In practice, whatever the method used for estimating F_e (for example, functional modeling and online estimation from embarked sensors data), the knowledge of this term is never totally accurate. To counterbalance the negative effects resulting from imprecise knowledge of this force and from other unmodeled terms involved in the system's dynamics, an integral action can be incorporated into the control law. For example, as suggested in [62], [63], and [73], one can modify the feedback term β in (25) so as to introduce an integral correction term, i.e.,

$$\beta\Big(\int \dot{\tilde{\xi}}, \dot{\tilde{\xi}}\Big) = K_{\beta_2} \int \dot{\tilde{\xi}} + \mathrm{sat}_\Delta(K_{\beta_1}\dot{\tilde{\xi}}), \tag{28}$$

with Δ a positive number and $(K_{\beta_1}, K_{\beta_2})$ a pair of positive diagonal gain matrices. Note, in view of (26), that the expression of $\bar{F}$ is then also modified.

It is common knowledge that integral correction action (i.e., the term $\int \dot{\tilde{\xi}}$) constitutes an effective means to compensate for model uncertainties. But it is also well known that this type of correction may generate instability and wind-up problems (like, for example, large overshoot, slow desaturation, and actuator saturation). For instance, even when $\dot{\tilde{\xi}}$ stays close to zero, its integral can grow arbitrarily large. As a consequence, there is an increased risk for $\bar{F}$ to cross zero and for the control expression to become singular. On the other hand, the use of a saturation function $\mathrm{sat}_{\Delta_I}(\int \dot{\tilde{\xi}})$ in place of $\int \dot{\tilde{\xi}}$ in the expression of $\beta(\cdot)$ does not prevent this integral term from growing arbitrarily large, leading to slow desaturation and sluggish dynamics. A possible remedy consists of using an antiwindup integrator technique (see, e.g., [26], [28], and [76] for antiwindup solutions). For instance, the term $\int \dot{\tilde{\xi}}$ in (28) may be replaced by the "bounded" integral term z calculated from (see, e.g., [76])

$$\dot{z} = k_z(-z + \mathrm{sat}_{\Delta_z}(z + \dot{\tilde{\xi}})), \quad |z(0)| < \Delta_z,$$

with $k_z > 0$ and $\Delta_z > 0$. Using the classical saturation function, this relation yields a pure integrator $\dot{z} = k_z \dot{\tilde{\xi}}$ when $|z + \dot{\tilde{\xi}}| \leq \Delta_z$, and also $|z(t)| \leq \Delta_z$ and $|\dot{z}(t)| \leq 2k_z\Delta_z$, $\forall t \geq 0$.

Position Control

As in the case of velocity control with integral correction, position trajectory tracking can be achieved via a suitable modification of the feedback term β. A first solution is

$$\beta(\tilde{\xi},\dot{\tilde{\xi}}) = \mathrm{sat}_{\Delta_2}(K_{\beta_2}\tilde{\xi}) + \mathrm{sat}_{\Delta_1}(K_{\beta_1}\dot{\tilde{\xi}}),$$

with (Δ_1, Δ_2) a pair of positive numbers and $(K_{\beta_1}, K_{\beta_2})$ a pair of positive diagonal gain matrices. The position feedback term $\mathrm{sat}_{\Delta_2}(K_{\beta_2}\tilde{\xi})$ provides the correction necessary to the convergence of $\tilde{\xi}$ to zero, while saturation functions are again used to reduce the risk of a vanishing $\bar{F}$.

To further add robustness with respect to unmodeled dynamics, a bounded nonlinear integrator with anti-windup properties can be also incorporated into the controller. The solution proposed in [26] involves new tracking error variables defined by $\bar{\tilde{\xi}} := \tilde{\xi} + z, \dot{\bar{\tilde{\xi}}} := \dot{\tilde{\xi}} + \dot{z}$, with z a "bounded integral" of $\tilde{\xi}$, twice differentiable, and with a second time derivative uniformly bounded in norm by an arbitrary number specified by the user. Examples of calculation of such a term are given in [26] and [28]. In the latter reference z is calculated as follows

$$\begin{aligned} \ddot{z} &= -k_{vz}\dot{z} + \mathrm{sat}_{\ddot{z}_{\max}/2}(k_{pz}(-z + \mathrm{sat}_{\Delta_z}(z + \tilde{\xi}))), \\ & z(0) = 0, \dot{z}(0) = 0, \end{aligned} \tag{29}$$

with the classical saturation function, and k_{vz}, k_{pz}, Δ_z, $\ddot{z}_{\max}$ denoting positive numbers. This relation yields the following upper bounds: $|z| < \Delta_z + \ddot{z}_{\max}/(2k_{vz}^2)$, $|\dot{z}| < \ddot{z}_{\max}/(2k_{vz})$, $|\ddot{z}| < \ddot{z}_{\max}$. Using the definitions of $\bar{\tilde{\xi}}$ and $\dot{\bar{\tilde{\xi}}}$ in the equations of system (20) yields

$$\begin{cases} m\ddot{\bar{\tilde{\xi}}} = -TRe_3 + F_z(\dot{\xi}, \ddot{z}, t) & \text{(30a)} \\ \dot{R} = RS(\omega) & \text{(30b)} \end{cases}$$

with $F_z(\dot{\xi}, \ddot{z}, t) := F(\dot{\xi}, t) + m\ddot{z}$. Since (30a)–(30b) are of the same form as those of the original system (20), one can use the position controller evoked previously, modulo the change of $(\tilde{\xi}, \dot{\tilde{\xi}}, F)$ to $(\bar{\tilde{\xi}}, \dot{\bar{\tilde{\xi}}}, F_z)$, to obtain the asymptotic stability of $(\bar{\tilde{\xi}}, \dot{\tilde{\xi}}) = (0,0)$. If z were the exact (unbounded) integral of $\tilde{\xi}$, the convergence of $\bar{\tilde{\xi}}$ to zero would clearly imply the one of $\tilde{\xi}$ to zero. It is not difficult to show that the same conclusion holds when z is calculated according to (29).

CONCLUSION

Basic control design ideas and principles for VTOL vehicles have been reviewed. One of the messages that we have tried to pass on is that the nonlinear approach to the control problems has definite assets with respect to the well-established and universally used linearization approach. It is founded on the physics of flight, it is respectful of the geometry of motion in space, and it allows for a large operating domain. The control schemes derived from it are not more complex and may in fact be seen as natural extensions of locally approximating linear control schemes. The interest of this latter point of view is that it reconciles the two approaches by also pointing out that linear control gain optimization techniques are useful for the tuning of a certain number of parameters involved in nonlinear control schemes. This article is an introduction to control principles on which it is possible to elaborate fully operational automatic control flight systems. As such it does not pretend to cover all aspects of flight control. Nor does it address all VTOL vehicles. In this respect, the assumption of independence of aerodynamic forces with respect to the vehicle's orientation is a severe limitation. Many problems, a certain number of which have been pointed out in this article, are still unsolved and directions for further exploration are numerous (sensory fusion for state estimation, coupling of state estimation with control, and vision-based control, to name a few). Let us just again mention here the importance of taking aerodynamic forces into account in the control design if the objective is to fly small devices in adverse wind conditions and/or when lift becomes the dominant aerodynamic force component. Convertible aerial vehicles capable of hovering and of economy cruising by using lift are particularly concerned by this issue and the adjacent problem of transition between these two modes [69], [70], [57]. The spectrum of vehicles of this type, from blimps to flapping-wing devices [2], [71], is very large. Beyond the case of flying vehicles, many other "thrust-propelled" vehicles, such as marine ships, hovercrafts, and submarines, can also be addressed with similar ideas and techniques by properly taking the specific nature and properties of external forces into account. All works in this direction participate in the development of a unified control theory for the ever growing family of underactuated vehicles.

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