2013 21st Mediterranean Conference on Control & Automation (MED)
Platanias-Chania, Crete, Greece, June 25-28, 2013

Trajectory Control of Multirotor Helicopters with Thrust Vector Constraints

Davi Antônio dos Santos[1], Osamu Saotome[2], and Arben Cela[3]

***Abstract*— The multirotor control system structure that has mostly been adopted is constituted by an inner and an outer control loop. In this scheme, the inner loop carries out attitude control while the outer loop is responsible for trajectory control. The present work addresses the problem of safely controlling the trajectory of a multirotor helicopter by taking into account specified constraints on both the total thrust magnitude and the inclination of the rotor plane. The proposed solution partitions the whole problem into an altitude and an horizontal position control. The control laws of the two parts combine the feedback linearization principle with saturated proportional-derivative controllers. The proposed method is evaluated by computational simulations, which show its effectiveness to control the vehicle along a spiral trajectory as well as respond to abrupt position commands.**

I. Introduction

Multirotor unmanned aerial vehicles (UAV) have been attracting most attention of the academia and the industry in the last five years. Such interest can be explained by the relative low cost and construction simplicity of this type of drone compared with other aerial vehicles (e.g., airplanes, conventional helicopters, and blimps). Thanks to their hovering capability, the multirotors are quite appropriate for applications such as traffic monitoring, power line inspection, agricultural monitoring, search and rescue. Nevertheless, this technology has a good deal of aspects to be improved so as to enable autonomous safe flights both indoors and outdoors. The recent reference [1] provides a nice overview about the main multirotor control issues.

Recently, a plenty of works on flight control of multirotor helicopters have been appearing in the control and automation literature. In general, the control system structure that has mostly been adopted is constituted by an inner and an outer loop, as illustrated in Figure 1. In a wide fashion, the function of such a system is to make the true vehicle position $\mathbf{r} \in \mathbb{R}^3$ track some desired position command $\mathbf{r}_{\mathrm{d}} \in \mathbb{R}^3$. For a sufficiently complete explanation, the scheme also depicts a navigation system, which at large has the role of producing attitude estimates, $\hat{\mathbf{D}} \in SO(3)$, angular velocity estimates $\hat{\boldsymbol{\omega}} \in \mathbb{R}^3$, position estimates $\hat{\mathbf{r}} \in \mathbb{R}^3$, and linear velocity estimates $\hat{\dot{\mathbf{r}}} \in \mathbb{R}^3$. The inner loop carries out attitude control while the outer loop is responsible for trajectory control. Since the translation of the vehicle depends on its orientation with respect to the horizontal plane, the trajectory control law is primarily supposed to compute a desired attitude command $\mathbf{D}_{\mathrm{d}} \in SO(3)$ that would cause desired accelerations on the local horizontal plane. Moreover, this law also calculates the total thrust magnitude $f \in \mathbb{R}$ that would allow the desired local-vertical acceleration. On the other hand, the attitude control law aims at providing a suitable torque $\boldsymbol{\tau} \in \mathbb{R}^3$ that makes the vehicle follow the attitude command $\mathbf{D}_{\mathrm{d}}$ requested by the trajectory control law.

Various papers have already addressed the trajectory control problem [2], [3], [4], [5]. Lee *et al.* [2] used feedback linearization for designing a LQR position control. Bouabdallah and Siegwart [3] used the backstepping method to derive a simple PD law to control both the attitude and the translation motion. The merit of the work reported in [4] resides in the complete system design and flight tests, but the adopted control strategy assumes that the three degrees of freedom (DOF) of the rotational dynamics are decoupled and consists of simple PID controllers for each DOF separately. Lee *et al.* [5] used the integrator backstepping method to obtain an output feedback law to control the vehicle translation and heading. None of the aforementioned papers take into account any constraints on the total thrust control vector.

In fact, few works on multirotor control have faced constraint issues [6], [7]. In [6], the authors divided the system into smaller subsystems with two DOFs. For each subsystem, they applied the nested saturated controller to attain global stability while respecting a maximum constraint on the total thrust magnitude. However, the subsystems were assumed to have uncoupled dynamics, which is not true in general. In [7],

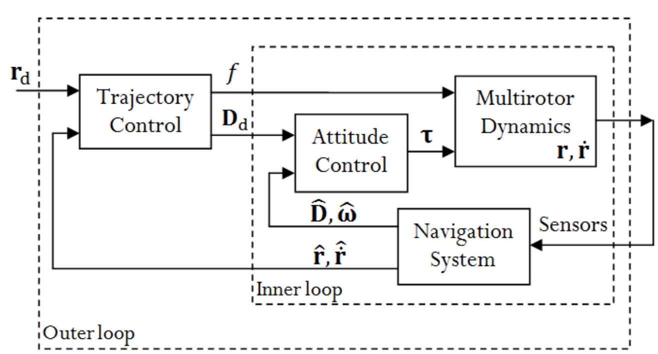

Fig. 1. A typical structure of multirotor control systems.

[1] D. A. dos Santos is with the Mechanical Division of the Instituto Tecnológico de Aeronáutica, 12228-900, São José dos Campos, Brazil and is a visiting researcher at Université Paris-Est, ESIEE Paris, 93162 Noisy-le-Grand, France. davists@ita.br.

[2] O. Saotome is with the Electronic Division of the Instituto Tecnológico de Aeronáutica, 12228-900, São José dos Campos, Brazil and is a visiting researcher at Université Paris-Est, ESIEE Paris, 93162 Noisy-le-Grand, France. osaotome@ita.br.

[3] A. Cela is with the Department of Embedded Systems of ESIEE Paris, Université Paris-Est, 93162 Noisy-le-Grand, France. celaa@esiee.fr.

to avoid an unbounded growth of the actuation, the authors presented an asymptotically stable controller that saturates the position errors.

The present paper is concerned with the safe trajectory control of multirotor helicopter UAVs considering three constraints on the total thrust control vector: its maximum angle with respect to the local vertical and its maximum and minimum magnitudes. The true position and velocity are assumed to be available for feedback. The proposed method is able to follow smooth, abrupt, ascending, and descending desired trajectories. Moreover, it is simple to design and implement, making it a great choice for low-cost UAV systems. The remaining text is organized in the following manner. Section II formally define the safe trajectory control problem for multirotor helicopter UAVs. Section III provides a solution to the main problem of the paper. This solution consists of two saturated PD controllers with feedback linearization. Section IV evaluates the proposed method using computational simulations. Finally, Section V presents the conclusions of the paper.

II. Problem Statement

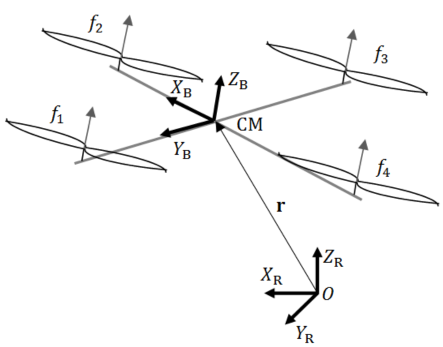

Fig. 2. Cartesian coordinate systems.

Consider the multirotor vehicle and the two Cartesian coordinate systems (CCS) illustrated in Figure 2. The body CCS is denoted by $S_{\mathrm{B}} = \{X_{\mathrm{B}}, Y_{\mathrm{B}}, Z_{\mathrm{B}}\}$. It is assumed to be attached to the vehicle structure and centered at its center of mass (CM). The reference CCS is denoted by $S_{\mathrm{R}} = \{X_{\mathrm{R}}, Y_{\mathrm{R}}, Z_{\mathrm{R}}\}$. It is assumed to be fixed on the ground at point O. For simplicity, the time dependence of the vectors will not be explicitly denoted hereafter.

Assume that S_{R} is an inertial frame and neglect any disturbance force. The direct application of the second Newton's law gives the following translational dynamic model represented in S_{R}:

$$\ddot{\mathbf{r}} = \frac{1}{m} f\mathbf{n} + \begin{bmatrix} 0 \\ 0 \\ -g \end{bmatrix}, \tag{1}$$

where $\mathbf{r} \triangleq [r_1\ r_2\ r_3]^{\mathrm{T}} \in \mathbb{R}^3$ denotes the position of CM, $\mathbf{n} \in \mathbb{R}^3$ is the unit vector normal to the rotor plane, g is the gravitational acceleration, m is the mass of the vehicle, and $f \triangleq f_1 + f_2 + f_3 + f_4$ is the magnitude of the total thrust. Note that if instead of a quadrotor, we considered an hexa-rotor or an octo-rotor, the model given by equation (1) would not change in any aspect, except the meaning of f as the sum of either six or eight individual thrust magnitudes.

Consider the following notations. Let $\|\mathbf{a}\|$ denote the Euclidian norm of the vector $\mathbf{a} \in \mathbb{R}^n$, with $n \in \mathbb{Z}_+$. Define $\boldsymbol{\varepsilon}_i \in \mathbb{R}^3$ as the unit vector with the i-th component equal to one and the remaining ones equal to zero. Denote the components of $\mathbf{n}$ by $n_i \triangleq \mathbf{n}^{\mathrm{T}}\boldsymbol{\varepsilon}_i$, for $i = 1, 2, 3$. Denote the S_{R}-representation of the total thrust vector by $\mathbf{f} \triangleq f\mathbf{n}$. Define the inclination angle ϕ of $\mathbf{n}$ to be

$$\phi \triangleq \cos^{-1} n_3. \tag{2}$$

Note that ϕ consists of the angle between $\mathbf{n}$ and Z_{R}.

Let $\mathbf{r}_{\mathrm{d}} \triangleq [r_{\mathrm{d},1}\ r_{\mathrm{d},2}\ r_{\mathrm{d},3}]^{\mathrm{T}} \in \mathbb{R}^3$ denote the position command, and define the tracking error $\tilde{\mathbf{r}} \in \mathbb{R}^3$ by

$$\tilde{\mathbf{r}} \triangleq \mathbf{r} - \mathbf{r}_{\mathrm{d}}. \tag{3}$$

Problem 1. Let $\phi_{\max} \in \mathbb{R}$ be the maximum allowable value of the inclination angle ϕ and $f_{\min} \in \mathbb{R}$ and $f_{\max} \in \mathbb{R}$ be, respectively, the minimum and the maximum allowable values of the total thrust magnitude f. The safe trajectory control problem is to find a feedback control law for $\mathbf{f}$ that makes $\tilde{\mathbf{r}}$ converge somehow to a neighborhood of the origin of $\mathbb{R}^3$ while respecting the constraints $f_{\min} \leq f \leq f_{\max}$ and $\phi \leq \phi_{\max}$. $\square$

III. Problem Solution

In order to solve Problem 1, the present paper partitions the whole trajectory control into an altitude and a horizontal position control law.

A. Altitude Control

From equation (1), the dynamic model of the altitude r_3 is given by

$$\ddot{r}_3 = \frac{n_3}{m} f - g. \tag{4}$$

In principle, the altitude r_3 could be controlled by varying n_3. However, that would inevitably cause variations of the horizontal acceleration components. Therefore, the most convenient form of controlling system (4) is by means of f. The following proposition presents a control law for f that consists of a saturated proportional-derivative controller with feedback linearization.

Proposition 1. Consider the system modeled by equation (4) and the minimum $f_{\min} \geq 0$ and maximum $f_{\max} > f_{\min}$ allowable values of the thrust magnitude f. Assume that m, g, and $n_3 \neq 0$ are exactly known. The feedback control law

$$f = \begin{cases} f_{\min}, & \alpha_3 < f_{\min} \\ \alpha_3, & \alpha_3 \in [f_{\min}, f_{\max}] \\ f_{\max}, & \alpha_3 > f_{\max} \end{cases} \tag{5}$$

with

$$\alpha_3 \triangleq \frac{m}{n_3}\left(g - k_1\left(r_3 - r_{\mathrm{d},3}\right) - k_2\dot{r}_3\right) \tag{6}$$

1) respects the total thrust magnitude constraint $f_{\min} \leq f \leq f_{\max}$ and
2) is such that if $f_{\min} \leq \alpha_3 \leq f_{\max}$, then the vehicle altitude r_3 responds to the altitude command $r_{\mathrm{d},3}$ as if it were governed by the following second order linear time-invariant (LTI) system:

$$\ddot{r}_3 + k_2\dot{r}_3 + k_1 r_3 = k_1 r_{\mathrm{d},3}, \tag{7}$$

where $k_1 > 0 \in \mathbb{R}$ and $k_2 > 0 \in \mathbb{R}$ are controller coefficients.

Proof. Item 1 can be verified by a simple inspection of equation (5). Now, assume that $f_{\min} \leq \alpha_3 \leq f_{\max}$, substitute (6) into (5) and then replace the resulting expression for f into (4). This procedure results in equation (7), thus proving item 2. □

It is worth mentioning that if one chose $f_{\min} = -f_{\max}$, a global stability proof would be provided by Theorem 2.1 of reference [8]. However, for the present application, $f_{\min}$ has to be positive since the propellers can only produce upward thrusts. A global stability study for system (4) with control law (5)-(6) is currently being pursued. For the moment, since the closed-loop dynamics (7) is a second order LTI system, on can compute k_1 and k_2 so as to make the unsaturated system reach some specified performance parameters, like overshoot and peak instant. Note that the condition $n_3 \neq 0$ is very reasonable since, on the contrary, axis Z_{B} would belong to the horizontal plane and, consequently, the total thrust would have null vertical projection, which is a pretty undesirable scenario for the scope of the present work.

B. Horizontal Position Control

Let $\mathbf{r}_{12} \triangleq [r_1\ r_2]^{\mathrm{T}} \in \mathbb{R}^2$ and $\mathbf{n}_{12} \triangleq [n_1\ n_2]^{\mathrm{T}} \in \mathbb{R}^2$ denote the horizontal projections of $\mathbf{r}$ and $\mathbf{n}$, respectively. From equation (1), one can write the horizon position dynamic model as

$$\ddot{\mathbf{r}}_{12} = \frac{f}{m}\mathbf{n}_{12}. \tag{8}$$

Different from the altitude control, here $\mathbf{n}_{12}$ is chosen as the control variable. Note that if f were chosen instead of $\mathbf{n}_{12}$, the controller would uniquely change the horizontal acceleration magnitude $\|\ddot{\mathbf{r}}_{12}\|$, nothing doing with its direction. Thus, in order to be capable of controlling both the magnitude and the direction of the horizontal position $\mathbf{r}_{12}$, the most convenient strategy is to vary $\mathbf{n}_{12}$. Physically, one can see that, in order to produce horizontal acceleration in a desired direction, it suffices to tilt the unit vector $\mathbf{n}$ such as its horizontal projection $\mathbf{n}_{12}$ be aligned with the same direction of the desired acceleration (see Figure 3). Moreover, in order to avoid large horizontal accelerations as well as prevent from loss of lift, when tilting $\mathbf{n}$, one can argue that it is necessary to respect some given maximum inclination angle, as established in Problem 1.

In light of the above explanation, the following proposition gives an horizontal position controller.

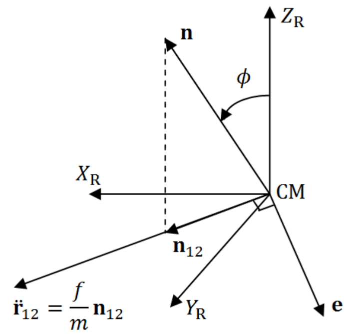

Fig. 3. Relationship between the normal unit vector $\mathbf{n}$ and the horizontal acceleration $\ddot{\mathbf{r}}_{12}$.

Proposition 2. Consider the system modeled by equation (8) and the maximum allowable value $\phi_{\max} > 0$ of the inclination angle ϕ. Assume that m and $f \neq 0$ are exactly known. The feedback control law

$$\mathbf{n}_{12} = \begin{cases} \boldsymbol{\alpha}_{12}, & \|\boldsymbol{\alpha}_{12}\| \leq \sin\phi_{\max} \\ \sin\phi_{\max}\frac{\boldsymbol{\alpha}_{12}}{\|\boldsymbol{\alpha}_{12}\|}, & \|\boldsymbol{\alpha}_{12}\| > \sin\phi_{\max} \end{cases} \tag{9}$$

with

$$\boldsymbol{\alpha}_{12} \triangleq -\frac{m}{f}\left[k_3\left(\mathbf{r}_{12} - \mathbf{r}_{\mathrm{d},12}\right) + k_4\dot{\mathbf{r}}_{12}\right] \tag{10}$$

1) respects the inclination angle constraint $\phi \leq \phi_{\max}$ and
2) is such that if $\|\boldsymbol{\alpha}_{12}\| \leq \sin\phi_{\max}$, then $\mathbf{r}_{12}$ responds to the horizontal position command $\mathbf{r}_{\mathrm{d},12} \triangleq [r_{\mathrm{d},1}\ r_{\mathrm{d},2}]^{\mathrm{T}}$ as if it were governed by the following pair of second order LTI systems:

$$\ddot{\mathbf{r}}_{12} + k_4\dot{\mathbf{r}}_{12} + k_3\mathbf{r}_{12} = k_3\mathbf{r}_{\mathrm{d},12}, \tag{11}$$

where $k_3 > 0 \in \mathbb{R}$ and $k_4 > 0 \in \mathbb{R}$ are controller coefficients.

Proof. From Figure 3, one can conclude that $\sin\phi = \|\mathbf{n}_{12}\|$, which allows to convert the inclination angle constraint $\phi \leq \phi_{\max}$ into $\|\mathbf{n}_{12}\| \leq \sin\phi_{\max}$. The condition $\|\mathbf{n}_{12}\| \leq \sin\phi_{\max}$ can be verified by simple inspection of (9), which thus proves item 1. Now, from (9), if $\|\boldsymbol{\alpha}_{12}\| \leq \sin\phi_{\max}$, then $\mathbf{n}_{12} = \boldsymbol{\alpha}_{12}$. In this case, substituting (9) into (8) one can immediately obtain equation (11), which thus proves item 2. □

Remark 1. Proposition 2 provides the horizontal projection $\mathbf{n}_{12}$ of the normal vector $\mathbf{n}$. Since $\mathbf{n}$ is a unit vector, one can thus write $\mathbf{n} = [\mathbf{n}_{12}^{\mathrm{T}}\ \sqrt{1 - \|\mathbf{n}_{12}\|^2}]^{\mathrm{T}}$. Therefore, Propositions 1-2 give together a solution $\mathbf{f} = f\mathbf{n}$ to Problem 1.

Remark 2. In practice, the attitude commands for the internal attitude control loop (see Figure 1) need to be computed from the normal unit $\mathbf{n}$. Note that there are infinite attitudes of S_{B} with respect to S_{R} for which the Z_{B} axis coincides with $\mathbf{n}$. In order to specify a unique attitude, it is necessary to select a heading angle. For example, one can choose a zero heading angle just by taking into consideration the attitude represented by the principal Euler angle/axis $(\phi, \mathbf{e})$, where the unit vector $\mathbf{e} \in \mathbb{R}^3$ (see Figure 3) is given by

$$\mathbf{e} = \frac{\mathbf{n} \times \mathbf{n}_{12}}{\|\mathbf{n} \times \mathbf{n}_{12}\|}. \tag{12}$$

From $(\phi, \mathbf{e})$ one can thus represent the attitude of S_{B} with respect to S_{R} using any other parameterization (e.g., quaternions, modified Rodrigues parameters, Euler angles, and direction cosine matrix) [9].

IV. Computational Simulation

The system is simulated by integrating equation (1) with the total thrust magnitude f given by Proposition 1 and the normal unit vector $\mathbf{n}$ given by both Proposition 2 and Remark 1. For this end, the Euler method with an integration step of $T = 0.01$ s is used. The multirotor vehicle has a mass of $m = 0.5$ kg and the gravitational acceleration is $g = 9.8$ m/s^2. The controller parameters k_1, k_2, k_3, and k_4 are chosen on the basis of item 2 of Propositions 1-2 (i.e., the system is assumed to behave as equations (7) and (11)) and thus computed to reach an overshoot of $M_{\mathrm{p}}^{\mathrm{ref}} = 0.01$ m and a peak instant of $t_{\mathrm{p}}^{\mathrm{ref}} = 2$ s. The obtained values are $k_1 = k_3 = 7.77$ and $k_2 = k_4 = 4.61$.

In order to evaluate the proposed control method, two simulation tests were carried out. In the first one, the system was subject to abrupt (step) position commands with initial point $\mathbf{r}_i = [0\ 1\ 2]^{\mathrm{T}}$ m and final point $\mathbf{r}_f = [1\ 2\ 3]^{\mathrm{T}}$ m. This test was repeated for some values of $\phi_{\max}$ and $f_{\max}$, while $f_{\min} = 2$ N was maintained constant. Two indices, I_f and I_ϕ, were adopted to assess the frequency of saturation of the control laws of equation (5) and (9), respectively, as the two constraints $\phi_{\max}$ and $f_{\max}$ are varied. These indices are defined as the percentage of discrete-time steps in which the saturations are active in a simulation with duration of 20 s. Table 1 shows the results and Figure 4 depicts a typical response taken for $\phi_{\max} = 10$ deg and $f_{\max} = 5$ N. One can note that I_f (and I_ϕ) decreases as $f_{\max}$ (and $\phi_{\max}$) are increased. Furthermore, the smaller the indices I_f and I_ϕ the better the overshoots and peak instants approach their reference values $M_{\mathrm{p}}^{\mathrm{ref}} = 0.01$ m and $t_{\mathrm{p}}^{\mathrm{ref}} = 2$ s.

In the second test, the system was commanded to follow the spiral trajectory $\mathbf{r}_{\mathrm{d}}(t) = [r_{\mathrm{d},1}(t)\ r_{\mathrm{d},2}(t)\ r_{\mathrm{d},3}(t)]^{\mathrm{T}}$, with $r_1(t) = 2\cos 0.1t$, $r_2(t) = -2\sin 0.1t$, and

$$r_{\mathrm{d},3}(t) = \begin{cases} 0.3t + 1 & 0 \le t < 100 \\ -0.3t + 61 & 100 \le t \le 200 \end{cases},$$

where t denotes time in second. The system starts from position $\mathbf{r}_i = [2\ 0\ 1]^T$ m and the controller constraints are fixed in $\phi_{\max} = 10$ deg, $f_{\max} = 5$ N, and $f_{\min} = 2$ N. An additive Gaussian disturbance force with zero mean and covariance $Q_{\mathrm{d}} = 0.25 I_3$ N^2 is taken into account in this test. The system response is shown in Figure 5. The saturation indices are evaluated as $I_\phi = 0$ and $I_f = 23.8\%$. One can note that even with a great deal of saturations on the thrust magnitude, the vehicle is able to track well the desired trajectory.

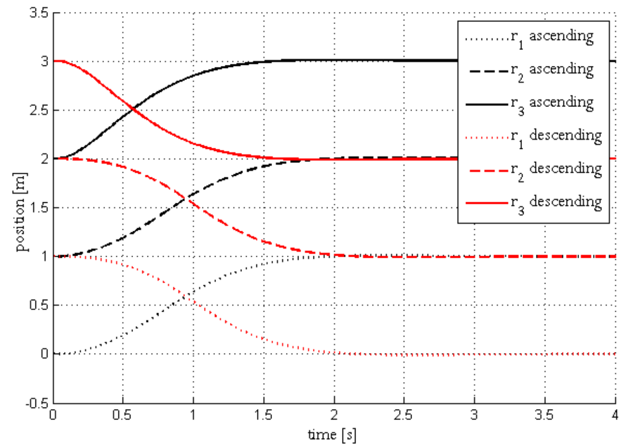

Fig. 4. A typical system response to ascending and descending abrupt position commands.

V. Conclusions

This paper presented a new multirotor trajectory control method that takes into account three actuation bounds: the maximum inclination of the rotor plane, the maximum and the minimum values of the total thrust magnitude. By respecting these bounds, the method is able to produce bounded control forces in all directions. The resulting control laws, which consist of saturated PD controllers with feedback linearization, are very simple to implement. We argue that this method is quite appropriate to attain safe indoor flights even with low-cost onboard computers, since it requires a small computational burden. For a future work, a global asymptotic stability proof for the controller given in Propositions 1-2 is being pursued. Moreover, we are preparing experimental tests, which may reveal the necessity of dealing with practical aspects like delays, disturbances, and parameter coupling.

Acknowledgment

The authors would like to thank CNPq/Brazil by the post-doctoral grants 237570/2012-2 and 201505/2011-8 and the Université Paris-Est/ESIEE Paris by the necessary support to realize the present work.

References

[1] R. Mahony, V. Kumar, and P. Corke, Multirotor Aerial Vehicles - Modeling, Estimation, and Control of Quadrotor, *IEEE Robotics and Automation Magazine*, 2012.

[2] S. H. Lee, S. H. Kang, and Y. Kim, Trajectory Tracking Control of Quadrotor UAV, *11th International Conference on Control, Automation, Systems*, Gyeonggi-do, Korea, 2011.

[3] S. Bouabdallah and R. Siegwart, Backstepping and Sliding-mode Techniques Applied to an Indoor Micro Quadrotor, *Proceedings of the 2005 IEEE International Conference on Robotics and Automation*, Barcelona, Spain, April, 2005.

[4] G. M. hoffmann, S. L. Waslander, and C. J. Tomlin, Quadrotor Helicopter Trajectory Tracking Control, *Proceedings of the AIAA Guidance, Navigation, and Control Conference*, Honolulu, Hawaii, 2008.

[5] D. Lee, T. C. Burg, B. Xian, and D. M. Dawson, Output Feedback Tracking Control of an Underactuated Quadrotor UAV, *Proceedings of the American Control Conference*, New York, NY, 2007.

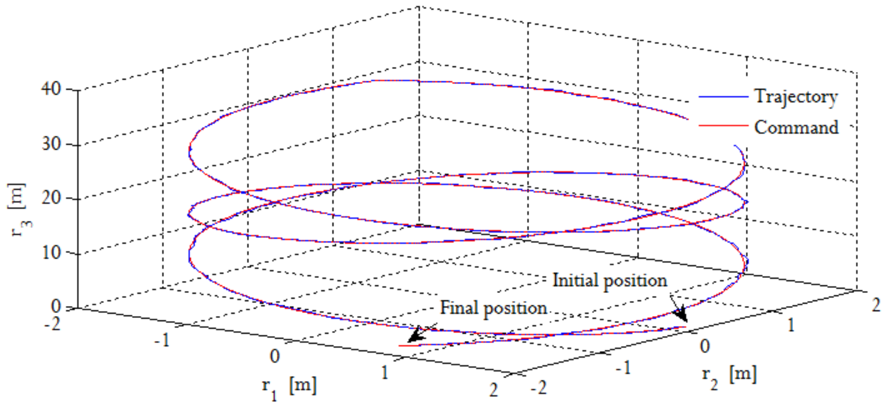

Fig. 5. System response to a spiral trajectory command with ascending and descending phases.

TABLE I

RESULTS OF EVALUATION TESTS USING ABRUPT POSITION COMMANDS.

$\phi_{\max}$ [deg]	$f_{\max}$ [N]	I_f [%]	I_ϕ [%]	r_1		r_2		r_3	
				M_p [cm]	t_p [s]	M_p [cm]	t_p [s]	M_p [cm]	t_p [s]
5	5	14.6	12.4	9.2	2.69	9.2	2.69	0.4	4.56
	10	0	12.3	9.2	2.64	9.2	2.64	1.1	1.99
	15	0	12.3	9.2	2.64	9.2	2.64	1.1	1.99
10	5	17.4	3.7	0.9	2.46	0.9	2.46	0.5	5.13
	10	0	4.2	0.9	2.36	0.9	2.36	1.1	1.98
	15	0	4.2	0.9	2.36	0.9	2.36	1.1	1.98
15	5	20.3	2.5	1.0	2.26	1.0	2.26	0.5	5.57
	10	0	2.0	1.0	2.15	1.0	2.15	1.1	1.98
	15	0	2.0	1.0	2.15	1.0	2.15	1.1	1.98

[6] P. Castillo, R. Lozano, and A. Dzul, Stabilization of a Mini Rotorcraft with Four Rotors, *IEEE Control System Magazine*, 2005.

[7] R. Cunha, D. Cabecinhas, C. Silvestre, Nonlinear Trajectory Tracking Control of a Quadrotor Vehicle, *Proceedings of the American Control Conference*, New York, NY, 2009.

[8] F. Tyan and D. S. Bernstein, Global Stabilization of Systems Containing a Double Integrator Using a Saturated Linear Controller, *International Journal of Robust and Nonlinear Control*, 9, 1999, 1143-1156.

[9] M. Shuster, A Survey of Attitude Representations, *The Journal of the Astronautical Sciences*, 41(4), 1993, 439-517.