

MLE for Bernoulli parameter p

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2021-03-10

The likelihood function of a Bernoulli random variable is the probability of the results, i.e. a probability of a binomial random variable with v success out of n trials, with a probability p for success:

$$L(p) = p^v(1-p)^{(n-v)}$$

Taking the $\log(L)$ we get:

$$\log(L(p)) = v \log(p) + (n-v) \log(1-p)$$

Then

$$\frac{d \log(L(p))}{dp} = \frac{v}{p} - \frac{n-v}{1-p}$$

Require that this derivative is 0 (to find the maximum):

$$\frac{v}{p} = \frac{n-v}{1-p}$$

If and only if:

$$v(1-p) = p(n-v)$$

If and only if:

$$np = v$$

Hence

$$\hat{p} = \frac{v}{n}$$

Using the second derivative (to make sure that this is indeed a **maximum** of the likelihood):

$$\frac{d^2 \log(L(p))}{d^2 p} = -\frac{v}{p^2} - \frac{n-v}{(1-p)^2} < 0$$

□