Robustness and Equity in Power Grid Expansion Planning

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Why Should We Care About Power Grid "Robustness"

- Each year, weather related outages in the US incur an average cost of $18-$33 billion [1]
- The grid is also vulnerable to a variety of threats other than weather (cyber, terror, natural disasters, etc.)
- The grid is a **critical infrastructure**
- Disruptions cannot be totally avoided, but they can be mitigated
  - Infrastructure **design** (e.g., capacity upgrades, micro-grids, DG)
  - **Realtime** mitigation (e.g., islanding, restoration)
- How can we design more robust power grids?

[1] President's Council of Economic Advisers and the U.S. Department of Energy, Economic benefits of increasing electric grid resilience to weather outages, 2013.

What is Robustness?

Robustness [2]

- Robustness is related to *how much* damage occurs as a consequence of an unexpected perturbation.

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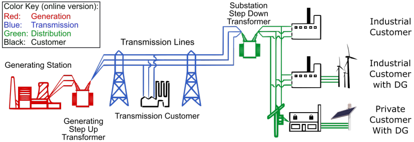
- Robustness is related to *how much* damage occurs as a consequence of an unexpected perturbation.
- The greater the damage, the system is less robust. For example:
  - Expected loss of load (electricity)
  - Probability for loss of load (of a certain amount)

[2] Cuadra *et al.*, A critical review of robustness in power grids using complex networks concepts. *Energies*, 2015.

Layers of the Grid

Generation, transmission, and distribution



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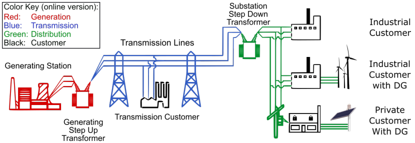
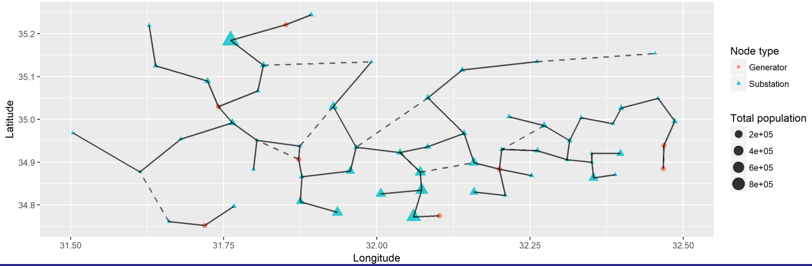


Eventually the layers are arranged into a network. The following illustrates the generation and transmission layers:



The Flow Equations

The DC load flow equations:

$$\sum_j f_{ij} = d_i \quad \forall i \in \mathcal{N}, \{i,j\} \in \mathcal{L}$$

$$\theta_i - \theta_j - x_{ij} f_{ij} = 0 \quad \forall \{i,j\} \in \mathcal{L}$$

Flow preservation + Active power (phase angle constraints), where:

- Decision variables: f_{ij} = flow in $\{i,j\}$; θ_i = phase angle at i
- Parameters: x_{ij} = reactance of $\{i,j\}$; d_i = demand (supply) at i

Lemma 1. Flow solution uniqueness

If the flow preservation and active power constraints have a solution, then this solution is unique in the f_{ij} variables.

Proof sketch. For trees, the flow can be determined incrementally, by traversing the nodes. For a graph with cycles it requires a bit more work but still holds (can be proved by contradiction).

What is a “Cascading Failure”?

The DC load flow model does not include line capacities, however, a transmission line cannot conduct infinite power. Hence, the power grid is vulnerable to a failure process, as follows:

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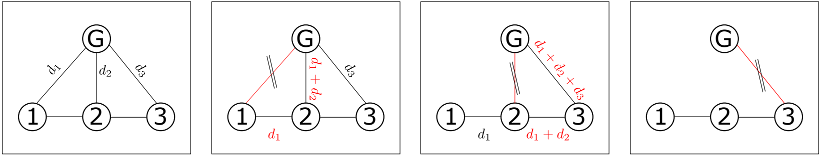
- The cascade process is usually described via a simulation model [3,4]

[3] Bienstock, Daniel and Mattia, Sara, Using mixed-integer programming to solve power grid blackout problems, *Discrete Optimization*, 2007.
[4] Soltan *et. al.*, Cascading failures in power grids: analysis and algorithms, *Proceedings of the 5th international conference on Future energy systems*, 2014.

Example of a Cascade

- Node i with demand d_i
- Edge $\{G, i\}$ with capacity $d_i + \epsilon$
- Edges $\{1, 2\}, \{2, 3\}$ with "infinite" (very large) capacity



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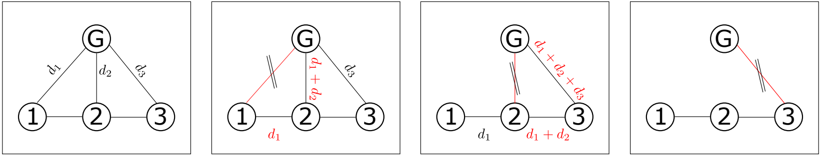

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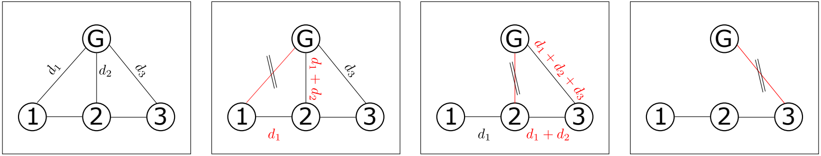

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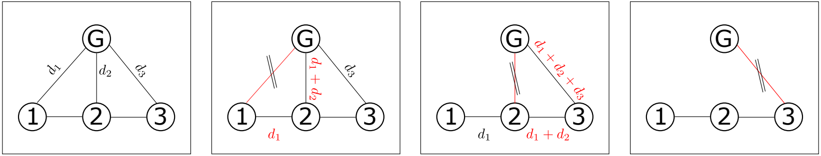

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"**Braess's paradox** in power grids"...

Design for Robustness

Design a power grid with **minimum loss of load** subject to:

- Failure scenarios (scenarios which describe failures due to external factors)
- Considering cascade outcomes due to initial failures
- Using the DC load flow model
- Limited budget

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- Establish new transmission lines
- Establish units with backup capacity at consumer nodes (DG or battery)
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Start simple – assume up to a single cascade:

0. Initial failure;
1. Flow after failure, which causes a cascade;
2. After cascade flow rearranges again and no further cascades

Design for Robustness – 1-cascade depth

What happens if only a single cascade may occur?

Minimum expected loss of load:

$$\sum_{s\in\mathcal{S}} p_s \sum_{i\in\mathcal{N}\setminus\{G\}} \left(d_i - w_i^{2,s}\right)$$

Variable domain:

$$\begin{aligned}
\forall i,j \in \mathcal{N}, t \in \{1,2\}, s \in \mathcal{S} :& \\
g_i^s, w_i^{t,s}, c_{ij}^l, c_i^g &\geq 0 \\
F_{ij}^s, Z_i &\in \{0,1\} \\
\forall (i,j) \in \mathcal{L}^e :& \\
X_{ij} &\in \{0,1\}
\end{aligned}$$

Investment cost constraint (budget):

$$\sum_{(i,j)\in\mathcal{L}} h_{ij} c_{ij}^l + \sum_{i\in\mathcal{N}} \left(h_i c_i^g + H_i Z_i\right) + \sum_{(i,j)\in\mathcal{L}^e} H_{ij} X_{ij} \leq C$$

Design for Robustness – 1-cascade depth – cont.

Conservation of flow (the DC load flow model, for $t = 1$):

$$\sum_{j\in\mathcal{N}|(j,i)\in\mathcal{L}\setminus\mathcal{F}^s} f_{ji}^{1,s} - \sum_{j\in\mathcal{N}|(i,j)\in\mathcal{L}\setminus\mathcal{F}^s} f_{ij}^{1,s} = d_i \quad \forall i \in \mathcal{N}, s \in \mathcal{S}$$

$$\theta_i^{1,s} - \theta_j^{1,s} - x_{ij} f_{ij}^{1,s} = 0 \quad \forall (i,j) \in \mathcal{L}^0, (i,j) \notin \mathcal{F}^s, s \in \mathcal{S}$$

$$\theta_i^{1,s} - \theta_j^{1,s} - x_{ij} f_{ij}^{1,s} \leq M(1 - X_{ij}) \quad \forall (i,j) \in \mathcal{L}^e \setminus \mathcal{F}^s, s \in \mathcal{S}$$

$$\theta_i^{1,s} - \theta_j^{1,s} - x_{ij} f_{ij}^{1,s} \geq -M(1 - X_{ij}) \quad \forall (i,j) \in \mathcal{L}^e \setminus \mathcal{F}^s, s \in \mathcal{S}$$

Cascade effects which occur at $t = 1$:

$$f_{ij}^{1,s} \leq c_{ij}^0 + c_{ij}^l + M \cdot F_{ij}^s \qquad \forall (i,j) \in \mathcal{L}\setminus\mathcal{F}^s, s \in \mathcal{S}$$

$$-f_{ij}^{1,s} \leq c_{ij}^0 + c_{ij}^l + M \cdot F_{ij}^s \qquad \forall (i,j) \in \mathcal{L}\setminus\mathcal{F}^s, s \in \mathcal{S}$$

Design for Robustness – 1-cascade depth – cont.

Transmission capacity:

$$f_{ij}^{t,s} \leq M \cdot X_{ij} \qquad \forall t \in \mathcal{T}, s \in \mathcal{S}, (i,j) \in \mathcal{L}^e$$
$$f_{ij}^{t,s} \geq -M \cdot X_{ij} \qquad \forall t \in \mathcal{T}, s \in \mathcal{S}, (i,j) \in \mathcal{L}^e$$

$$f_{ij}^{2,s} \leq M \cdot (1 - F_{ij}^s) \qquad \forall (i,j) \in \mathcal{L} \setminus \mathcal{F}^s, s \in \mathcal{S}$$
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Limit supply (do not exceed demand at $t = 2$):

$$\sum_{i \in \mathcal{N}} w_i^s = \sum_{i \in \mathcal{N}} g_i^s \qquad \forall s \in \mathcal{S}$$
$$w_i^s \leq d_i \qquad \forall i \in \mathcal{N} \setminus \{G\}$$

Design for Robustness – 1-cascade depth – cont.

Generation capacity:

$$\begin{aligned} g_i^s &\le c_i^g \qquad && \forall i \in \mathcal{N}, s \in \mathcal{S} \\ g_i^s &\le M \cdot Z_i \qquad && \forall i \in \mathcal{N} \end{aligned}$$

Conservation of flow after cascade ($t = 2$):

$$\begin{aligned} \sum_{(j,i)\in\mathcal{L}\setminus\mathcal{F}^s} f_{ji}^{2,s} - \sum_{(i,j)\in\mathcal{L}\setminus\mathcal{F}^s} f_{ij}^{2,s} + g_i^s - w_i^s &= 0 && \forall i \in \mathcal{N}, s \in \mathcal{S} \\ \theta_i^{2,s} - \theta_j^{2,s} - x_{ij} f_{ij}^{2,s} &\le MF_{ij}^s && \forall (i,j) \in \mathcal{L}^0\setminus\mathcal{F}^s, s \in \mathcal{S} \\ \theta_i^{2,s} - \theta_j^{2,s} - x_{ij} f_{ij}^{2,s} &\ge -MF_{ij}^s && \forall (i,j) \in \mathcal{L}^0\setminus\mathcal{F}^s, s \in \mathcal{S} \\ \theta_i^{2,s} - \theta_j^{2,s} - x_{ij} f_{ij}^{2,s} &\le M(1 - X_{ij}) + MF_{ij}^s && \forall (i,j) \in \mathcal{L}^e\setminus\mathcal{F}^s, s \in \mathcal{S} \\ \theta_i^{2,s} - \theta_j^{2,s} - x_{ij} f_{ij}^{2,s} &\ge -M(1 - X_{ij}) - MF_{ij}^s && \forall (i,j) \in \mathcal{L}^e\setminus\mathcal{F}^s, s \in \mathcal{S} \end{aligned}$$

Consistency and Limitations

Theorem 1. Consistency

The optimal solution is "physically consistent", i.e.:

1. The flow at $t = 1$ ($f_{ij}^{1,s}$) is equal to the DC load flow solution for a power grid at step $t = 1$;
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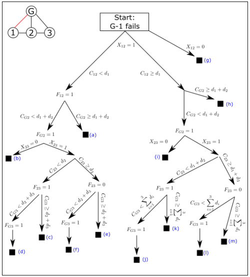
What happens when we cannot guarantee a single cascade depth?

- Approach fails when trying a "straightforward" generalization
- The additional degrees of freedom (of generation variables) gives a solution which is not "physically consistent"
- Can be used with transmission line capacities as an approximation – but how good would that approximation be?

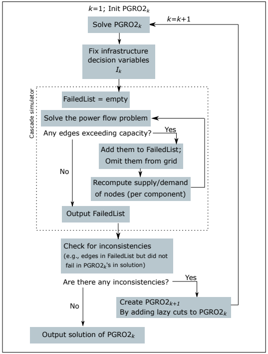
Second Approach "Cascade Trees"

If we could "preprocess" the solution tree and resulting cascades at each node, then we can essentially enumerate all possible failure combinations.



Unfortunately, such preprocessing is too complex (too many options), but this can be generated **"on-the-fly"** using a **cutting plane** "lazy" constraints framework.

Algorithm for an n-Cascade Depth Design Problem



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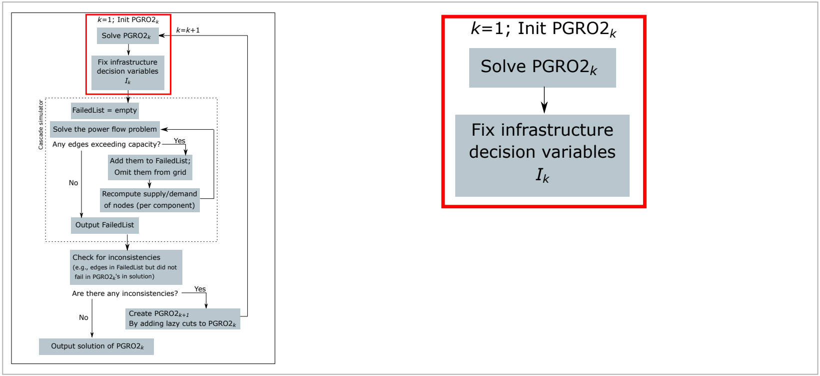

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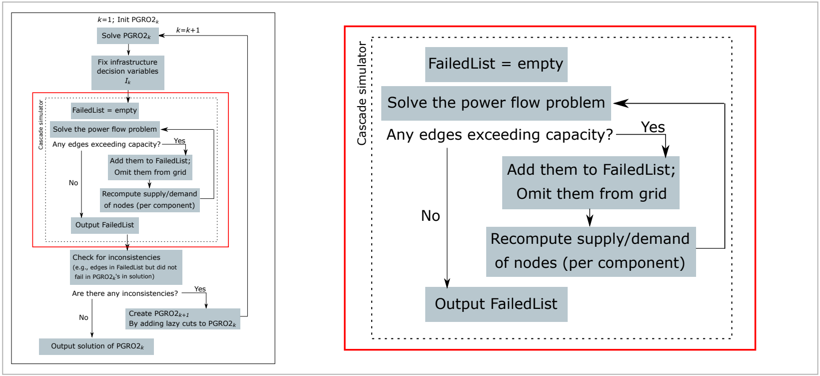

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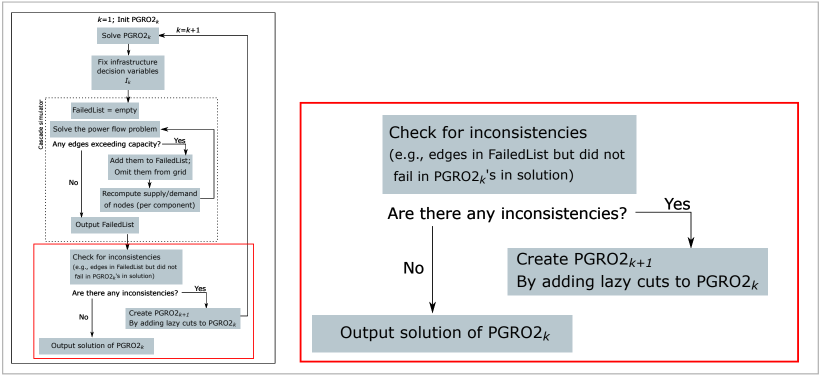

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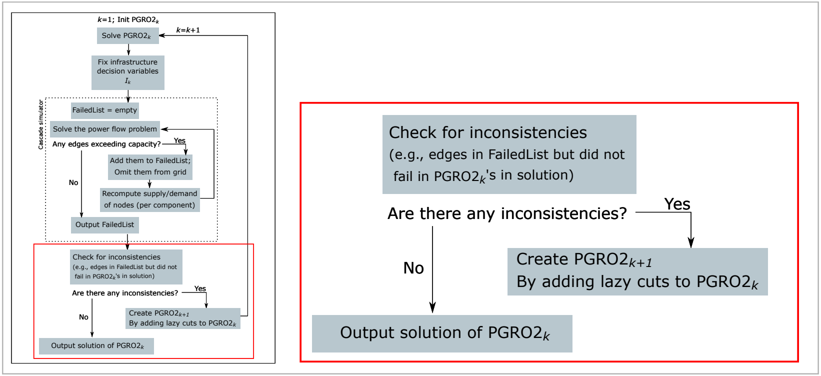

- Add-on: “branching priorities” – save the failure variables (F) for last!

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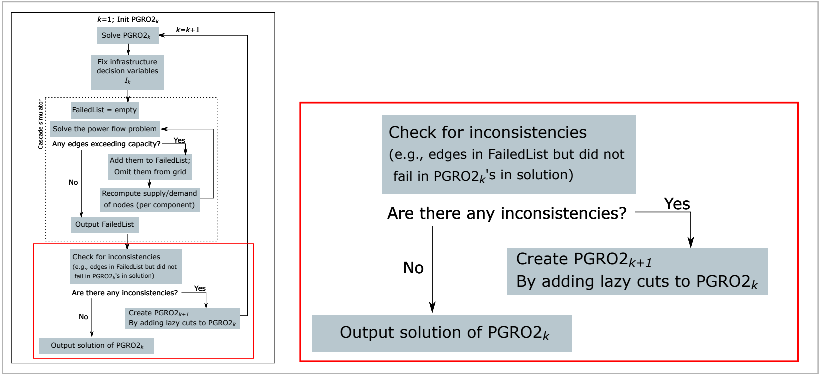


- Add-on: "branching priorities" – save the failure variables (F) for last!
- Heuristic callback: update incumbent solution during simulation step

Third Approach: a Large Neighborhood Search Heuristic (LNS)

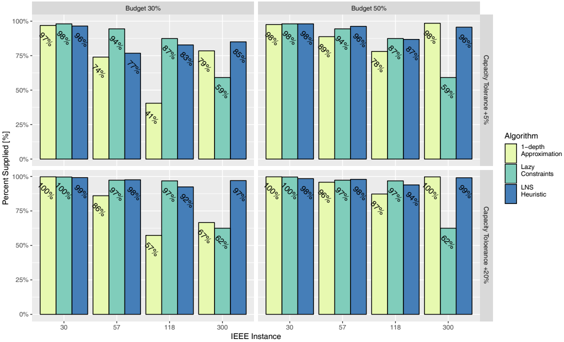
This meta-heuristic is based on "destroying" and "rebuilding" solutions. How does it work? (Pisinger, 2010)

- start with a feasible solution x.
- **repeat**
  - "Destroy" the solution x (perturb the solution, until it is infeasible).
  - "Rebuild" the infeasible solution back to a feasible solution x'.
  - Check if x' is better than x. If it is, update $x = x'$
- **until** stop criterion is met
- **return** x

In our case, destroying the solution is upgrading the grid slightly above budget, and repairing the solution is giving up on some upgrades to converge back to budget.

Results

Experiment based on four IEEE grids (30, 57, 118, and 300 nodes).



Results (2)

Algorithm	Percent Supplied Average [%] IEEE30	IEEE57	IEEE118	IEEE300	Overall Average [%]
Lazy Constraints	98.8	95.9	92.1	60.8	86.9
LNS Heuristic	98.0	92.1	89.0	94.2	93.3
1-depth Approximation	98.5	86.1	65.8	85.8	84.1

Table 1: Optimization results. Three algorithms are compared: the lazy constraint algorithm, the large neighborhood search heuristic, and the 1-depth approximation algorithm. Results are reported as average percent supplied of total demand.

Conclusions

This method still needs some "pondering"

- A new method to combine investment decisions with the cascade phenomena
- It takes long to prove/close the optimality gap
- Needs an upper bound generation strategy
- Failure variables (F) introduce a challenge (more scenarios increase the complexity significantly)
- Customize the branching process further

Thank you

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