Dimensions of Framing Squares

Obtaining the dimensions of the framing squares

Consider the arrangement of the framing squares in Figure 1. We have to obtain the dimensions of all the squares in terms of the radius which is 1 and the dimensions of the central pocket which is taken as a 2a x 2b rectangle. There are four squares in total ,two of each type. Let the squares have sides x_1, x_2, x_3, x_4 in order of their decreasing magnitude.

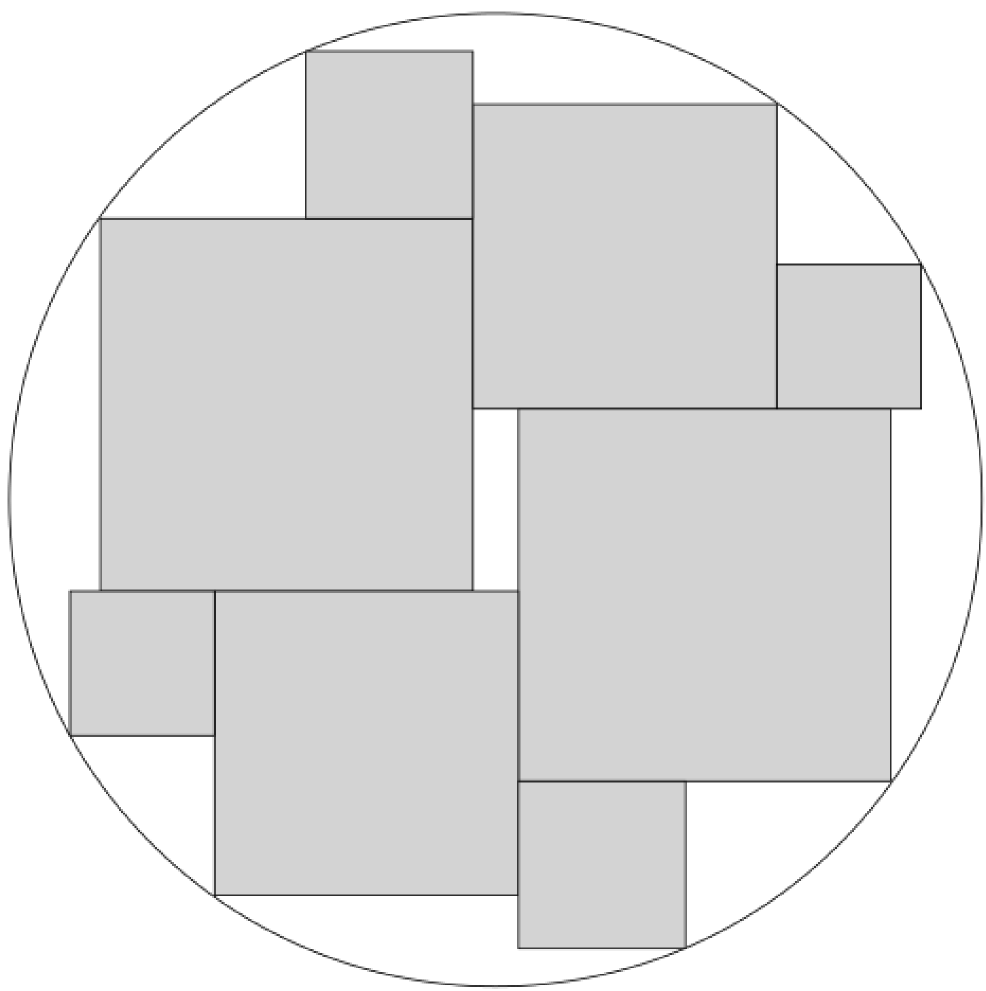

Figure 1

Now consider a portion of Figure 1 shown in Figure 2. We are going to find x_1 by Pythagoras' Theorem.

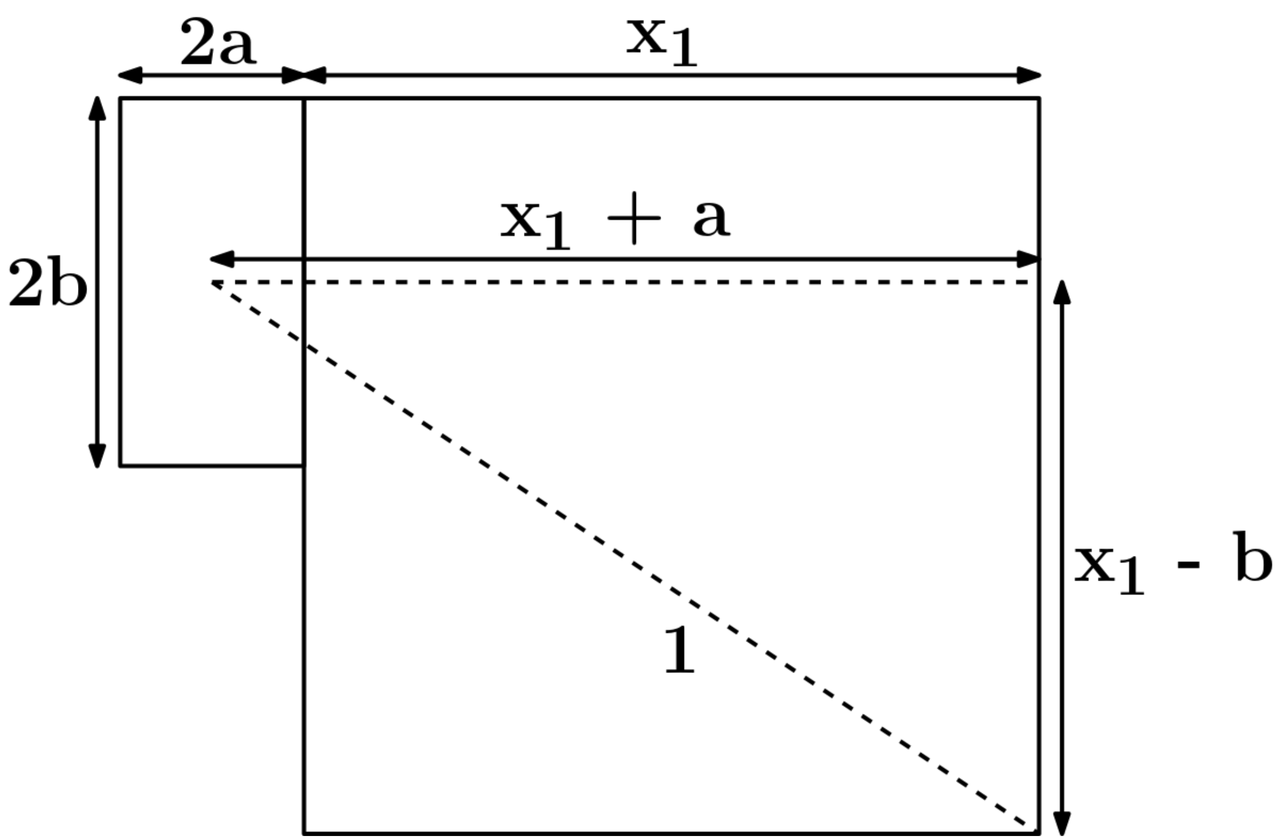

Figure 2

$$(x_1 + a)^2 + (x_1 - b)^2 = 1$$
$$x_1^2 + 2ax_1 + a^2 + x_1^2 - 2bx_1 + b^2 - 1 = 0$$

$$x_1 = \frac{-2(a-b) \pm \sqrt{4(a^2+b^2-2ab) - 8(a^2+b^2-1)}}{4}$$

$$x_1 = \frac{2(b-a) \pm \sqrt{8-4a^2-4b^2-8ab}}{4}$$

$$x_1 = \frac{(b-a) \pm \sqrt{2-a^2-b^2-2ab}}{2}$$

$$x_1 = \frac{(b-a) + \sqrt{2-(a+b)^2}}{2}$$

Considering +ve sign as x_1 has to be positive.

Now we are going to find x_2 from the following figure.

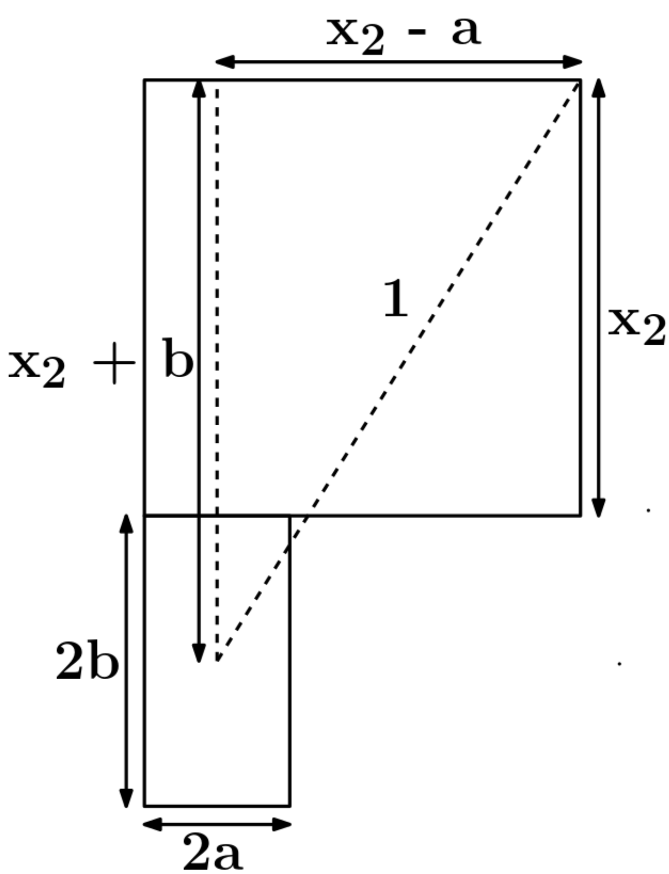

Figure 3

$(x_2 - a)^2 + (x_2 + b)^2 = 1 \ \ (Pythagoras' \ \ Theorem)$
$x_2^2 - 2ax_2 + a^2 + x_2^2 + 2bx_2 + b^2 - 1 = 0$
$2x_2^2 + (2b - 2a)x_2 + a^2 + b^2 - 1 = 0$

$x_2 = \frac{-(2b-2a) \pm \sqrt{4b^2 + 4a^2 - 8ab - 8(a^2+b^2-1)}}{4}$

$x_2 = \frac{-(2b-2a) \pm \sqrt{8 - 4a^2 - 4b^2 - 8ab}}{4}$

$x_2 = \frac{(a-b) + \sqrt{2-(a+b)^2}}{2}$

Considering +ve sign as x_2 has to be positive.

Now we have to find x_3 using the already found x_1 from the following figure.

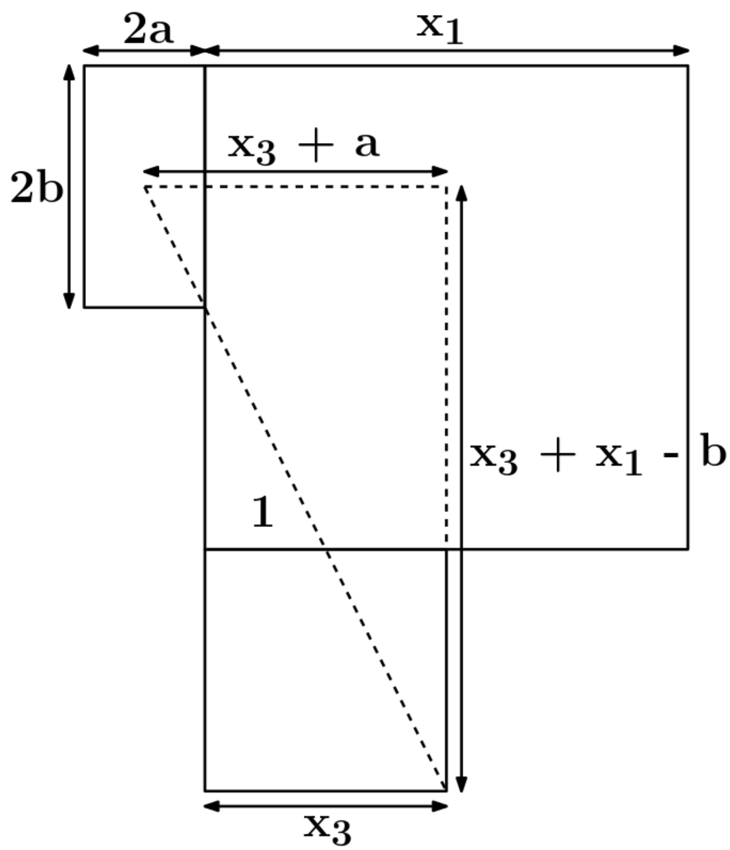

Figure 4

Let $c = x_1 - b$

$(x_3 + a)^2 + (x_3 + c)^2 = 1 \;\; (Pythagoras' \;\; Theorem)$

$2x_3^2 + 2(a + c)x_3 + a^2 + c^2 - 1 = 0$

$x_3 = \frac{-2(a+c)\pm\sqrt{4a^2+4c^2+8ac-8(a^2+c^2-1)}}{4}$

$x_3 = \frac{-2(a+c)\pm\sqrt{8-4a^2-4c^2+8ac}}{4}$

$x_3 = \frac{-(a+c)\pm\sqrt{2-a^2-c^2+2ac}}{2}$

$x_3 = \frac{-(a+c)+\sqrt{2-(c-a)^2}}{2}$

$x_3 = \frac{-(a+x_1-b)+\sqrt{2-(x_1-b-a)^2}}{2}$

Considering +ve sign as x_3 has to be positive.

Finally we have to find x_4 from the following figure.

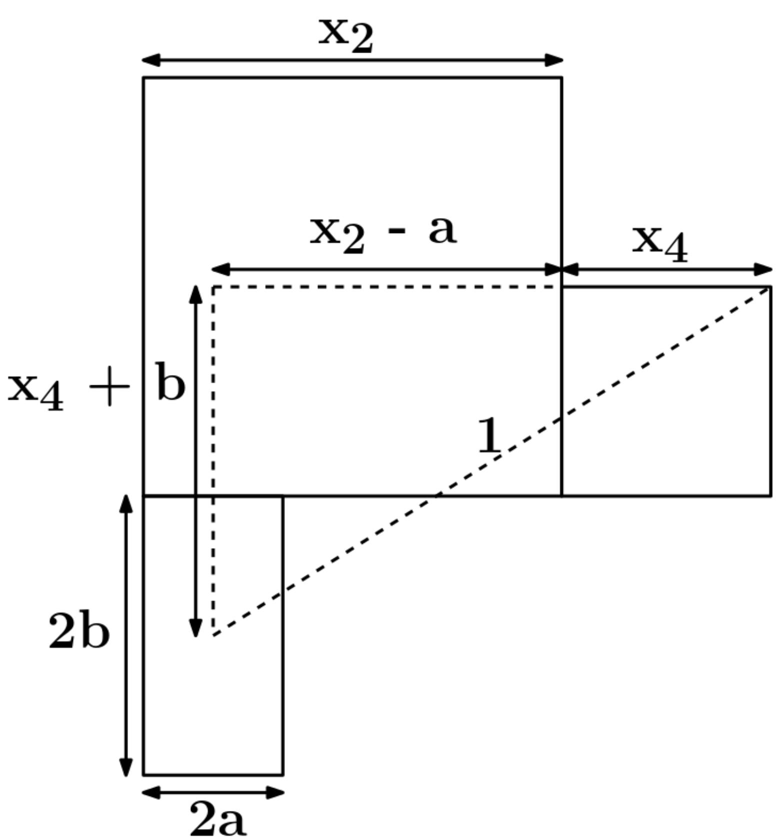

Figure 5

Let d = x_2 - a

$(x_4 + d)^2 + (x_4 + b)^2 = 1 \;\; (Pythagoras' \;\; Theorem)$

$2x_4^2 + 2(d + b)x_4 + b^2 + d^2 - 1 = 0$

$x_4 = \frac{-2(d+b)\pm\sqrt{4d^2+4b^2+8bd-8b^2-8d^2+8}}{4}$

$x_4 = \frac{-2(d+b)\pm\sqrt{8-4b^2-4d^2+8bd}}{4}$

$$x_4 = \frac{-(d+b) \pm \sqrt{2-b^2-d^2+2bd}}{2}$$

$$x_4 = \frac{-(d+b) + \sqrt{2-(d-b)^2}}{2}$$

$$x_4 = \frac{-(b+x_2-a) + \sqrt{2-(x_2-a-b)^2}}{2}$$

Considering +ve sign as x_4 has to be positive.

We have now calculated x_1 ,x_2 ,x_3 and x_4.