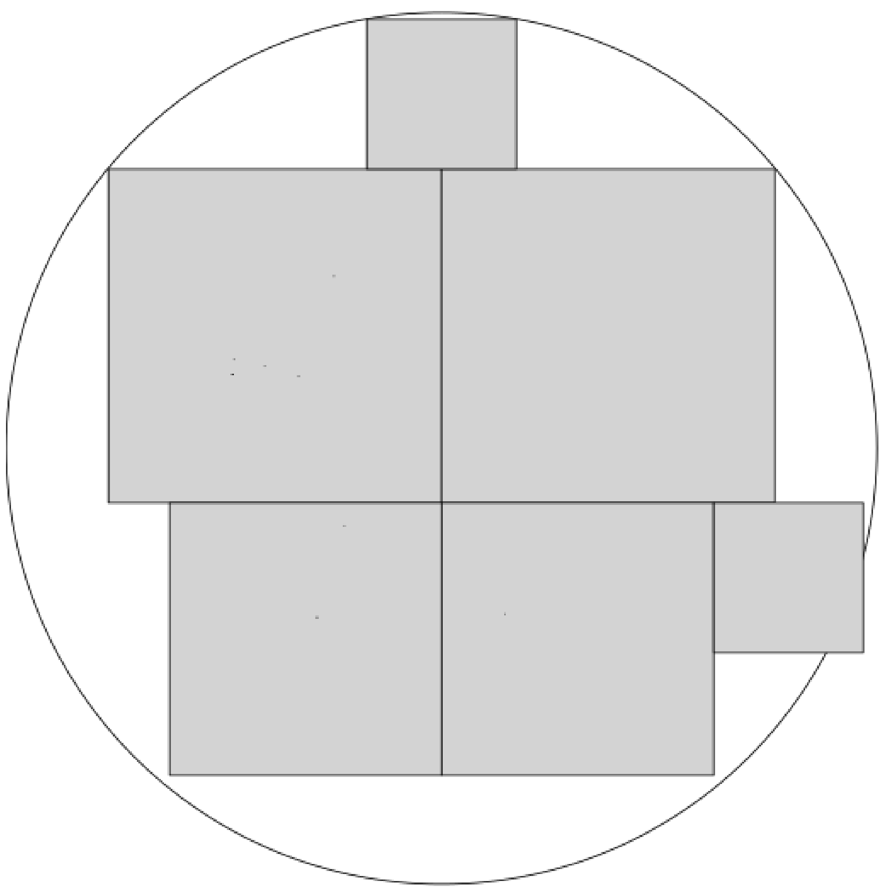
Framing Squares

To Prove: There is a unique arrangement of the framing squares in the disk.

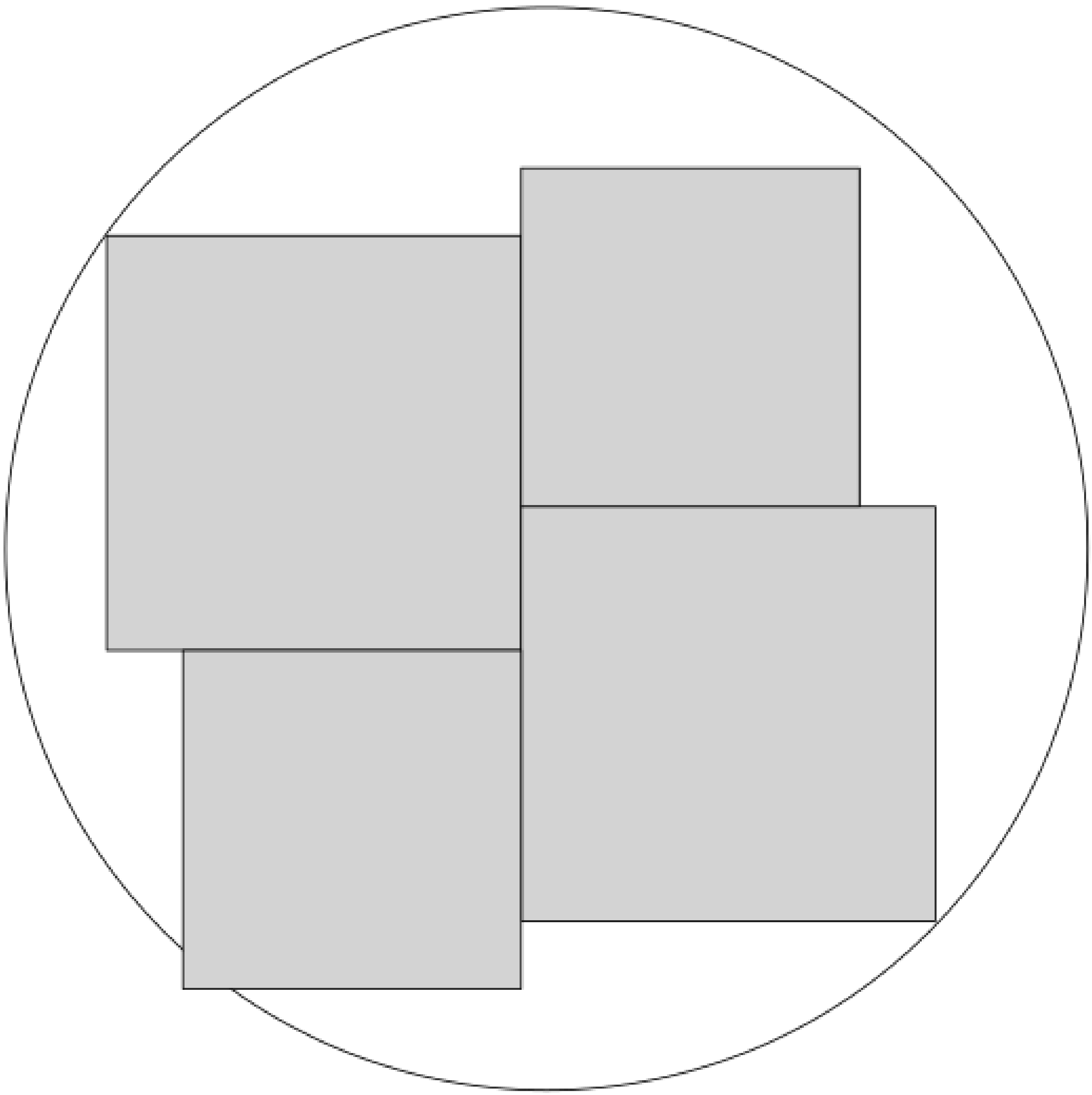
There are a total of 8 framing squares having four different side lengths such that there are exactly two of each type. Let their side lengths be a,b,c,d where a > b > c > d. The squares are packed in the disk in decreasing order of side lengths.

First, the squares with side length 'a' are packed adjacent to each other. Then the squares with side length 'b' are packed. We observe that with the remaining space left it is not possible to pack all the remaining squares.

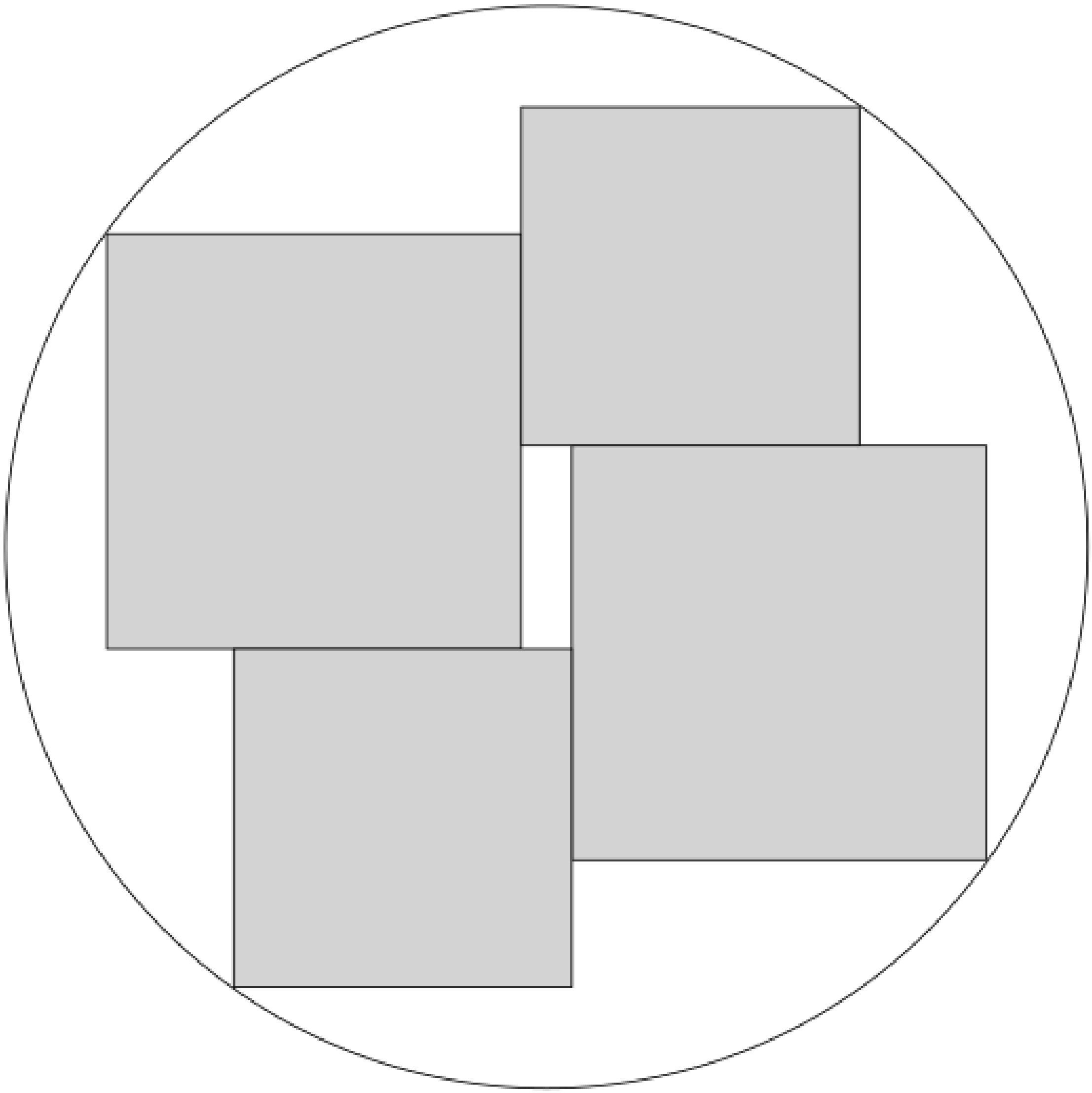


We can conclude that the squares with side length 'a' cannot be packed adjacent to each other in the way shown in the figure above.

Now consider that the squares the length 'a' are packed not exactly adjacent but slightly diagonally opposite to each other. Even the squares with side length 'b' are packed similarly.

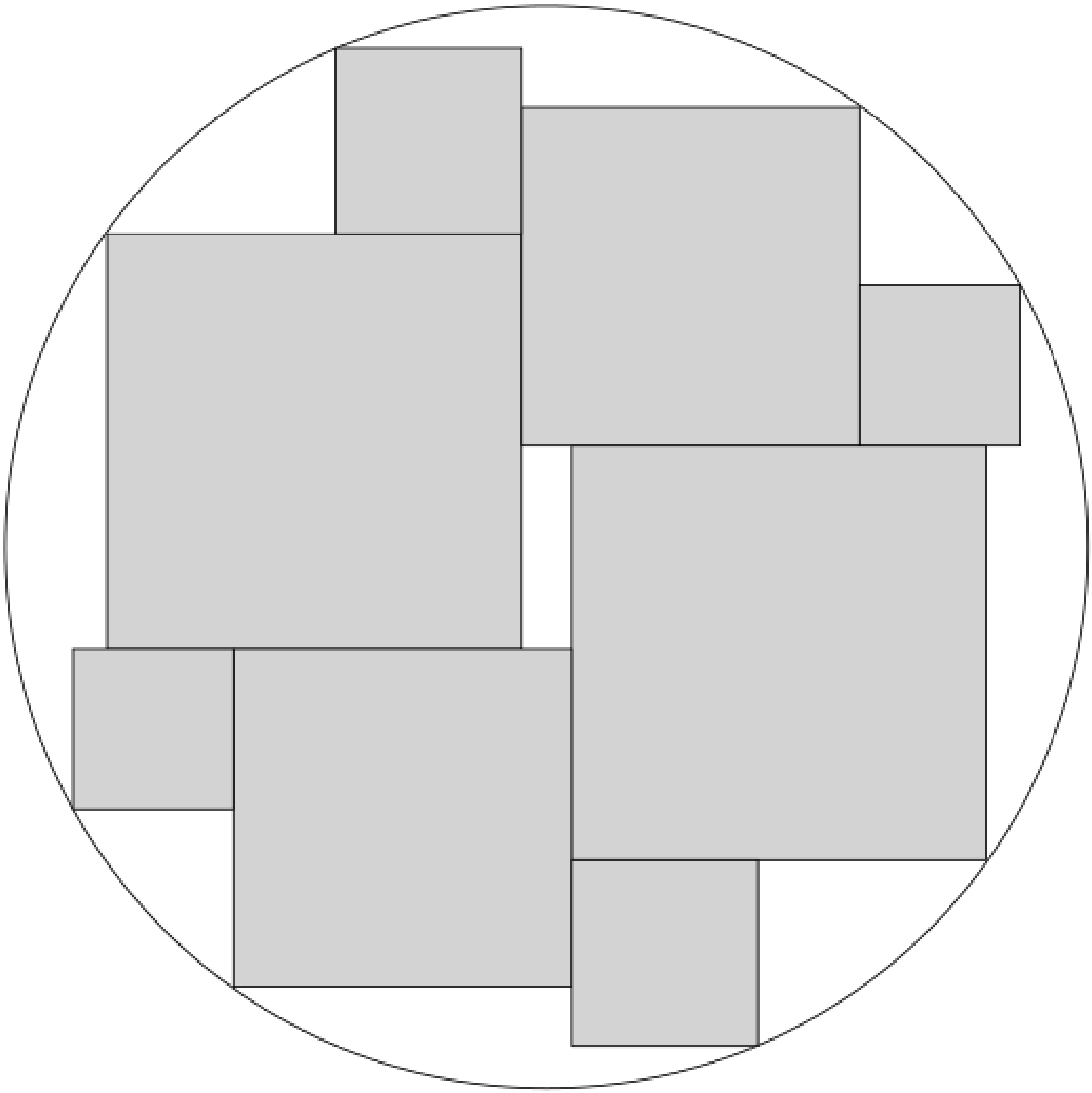


The bottom-left square is outside the disk. It needs to be shifted towards the right which pushes the bottom-right square towards the top in order to ensure that it remains within the disk. As a result of which the top right square is also pushed upwards. This process induces a central rectangular pocket. This is illustrated in the following image.



The arrangement above shows the biggest four squares packed in the disk. Now we have to pack the remaining four squares in the disk. The remaining squares will simply be packed in the spaces left in the disk.

In the figure below, the squares with side length ’c’ are packed to the top and bottom of the disk. The squares with side length ’d’ are packed to the left and right of the disk.



There can be no other arrangement possible which successfully packs all the squares in the disk. This arrangement also induces a central rectangular pocket. This proves that the arrangement of the framing squares in the disk is unique and induces a central rectangular pocket.