CS736 Assignment 2

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1 Problem 1

1.1 d) Initial Pointsets

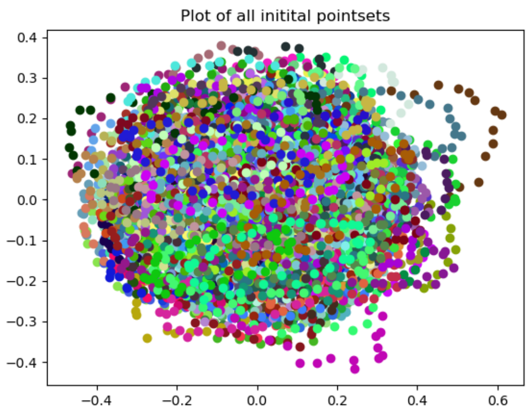

Figure 1: Scatter plot

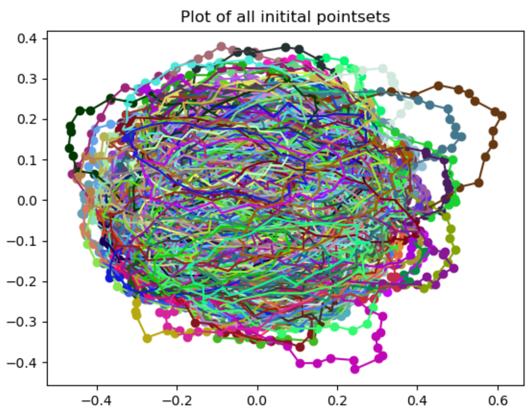

Figure 2: Each pointset represented by polylines

1.2 e)

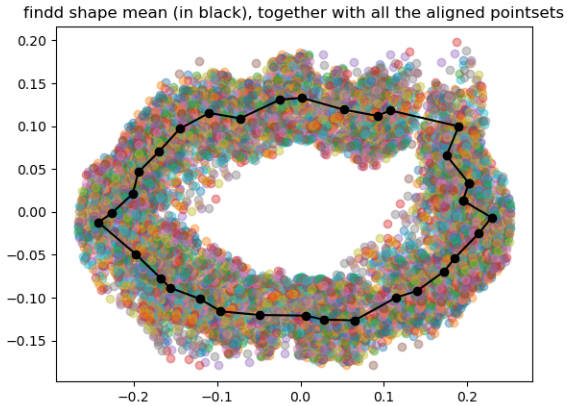

Figure 3: Result based on code11

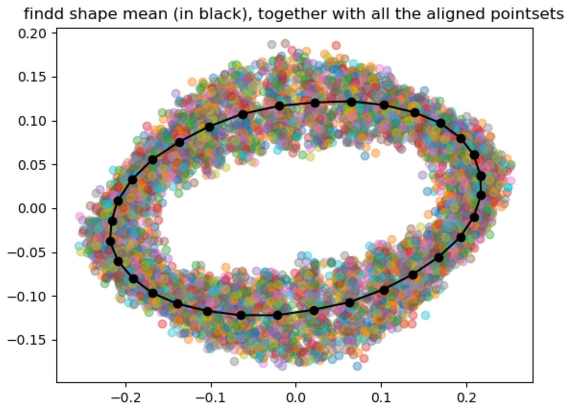

Figure 4: Result based on code22

1.3 f)

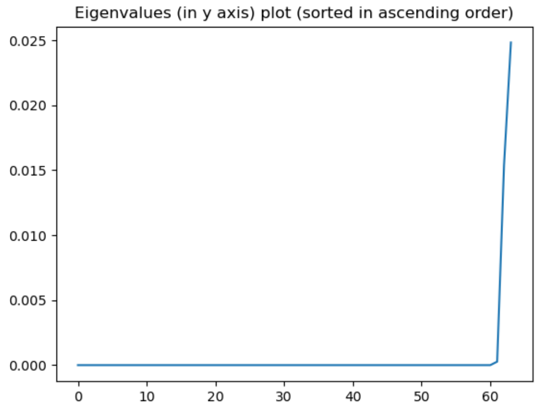

Figure 5: Result based on code11. First two eigenvalues are significantly greater than other eigenvalues which are close to 0

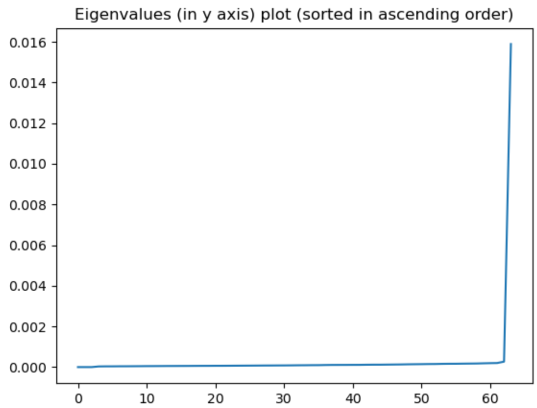

Figure 6: Result based on code22

1.4 g)

1.4.1 Results based on code11

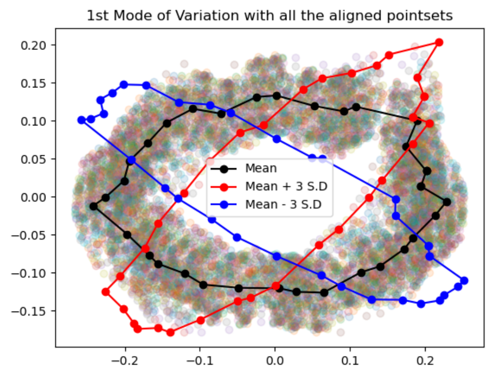

Figure 7: First Mode of variation

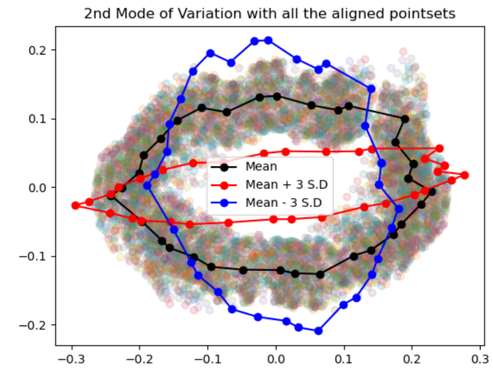

Figure 8: Second Mode of variation

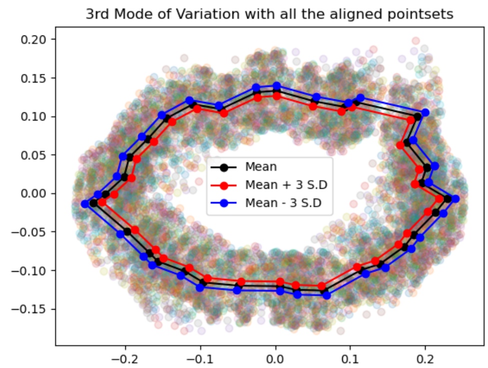

Figure 9: Third mode of variation

1.4.2 Results based on code22

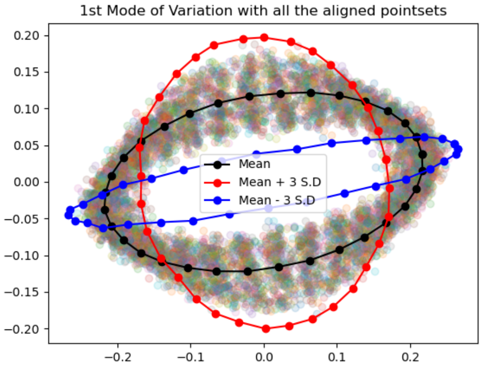

Figure 10: First Mode of variation

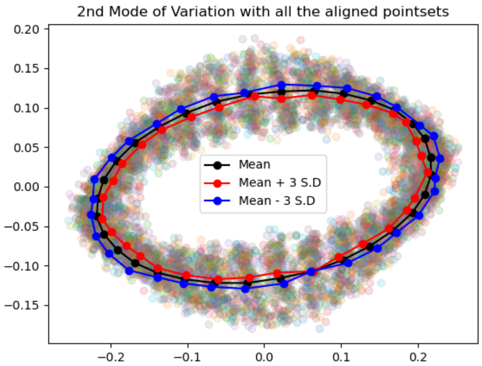

Figure 11: Second Mode of variation

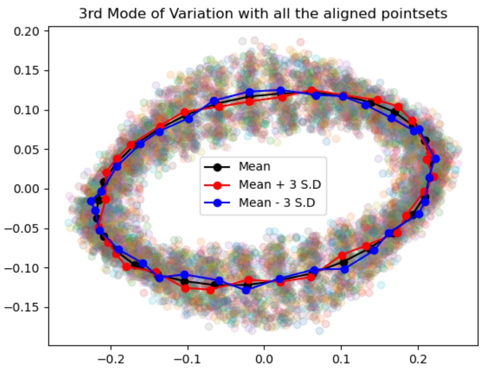

Figure 12: Third mode of variation

2 Problem 2

2.1 d) Initial Pointsets

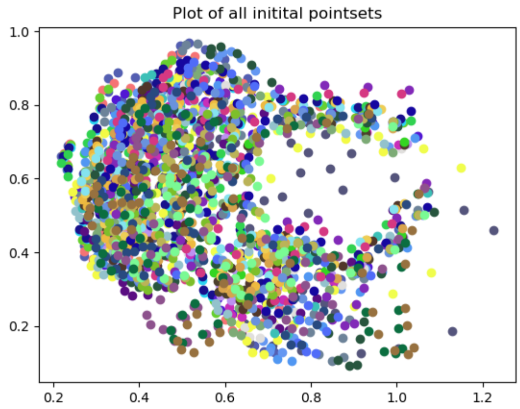

Figure 13: Scatter plot

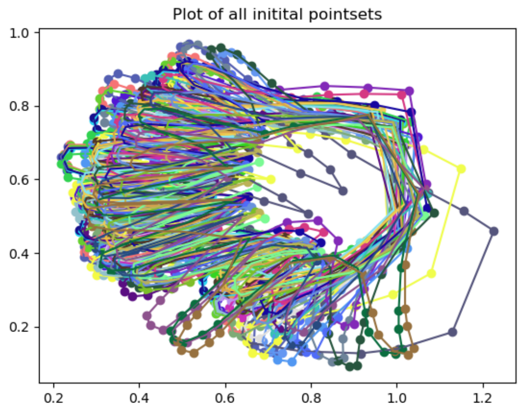

Figure 14: Each pointset represented by polylines

2.2 e)

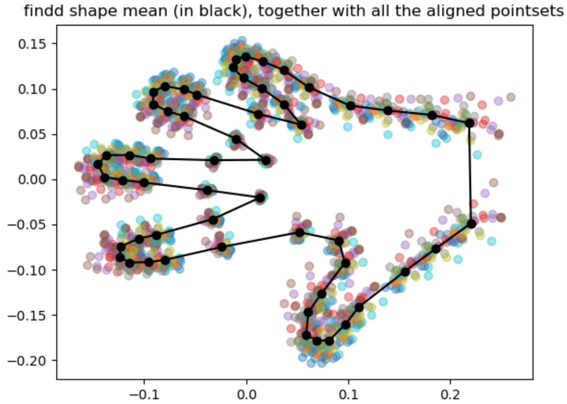

Figure 15: Result based on code11

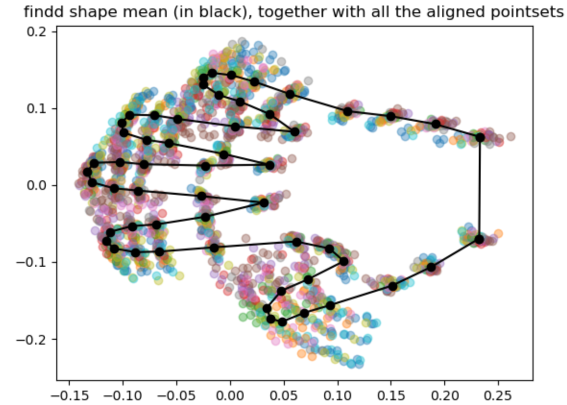

Figure 16: Result based on code22

2.3 f)

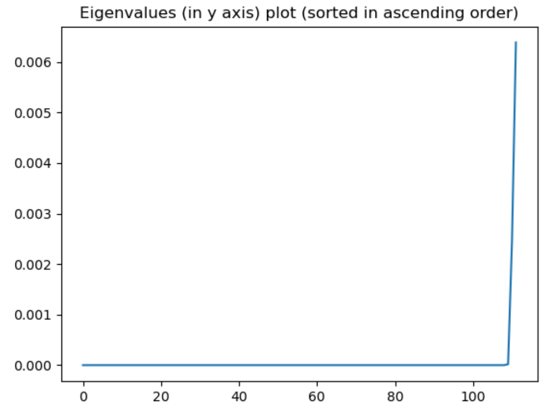

Figure 17: Result based on code11. First two eigenvalues are significantly greater than other eigenvalues which are close to 0

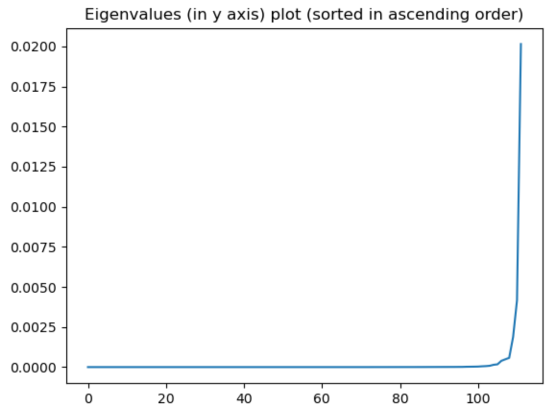

Figure 18: Result based on code22

2.4 g)

2.4.1 Results based on code11

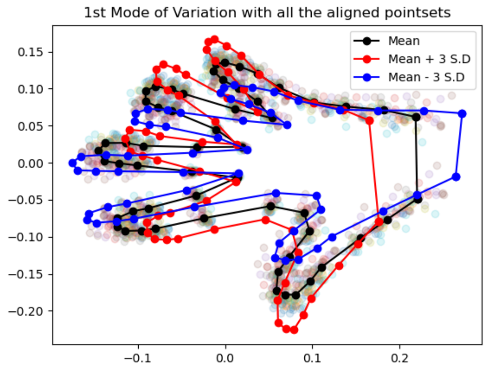

Figure 19: First Mode of variation captures

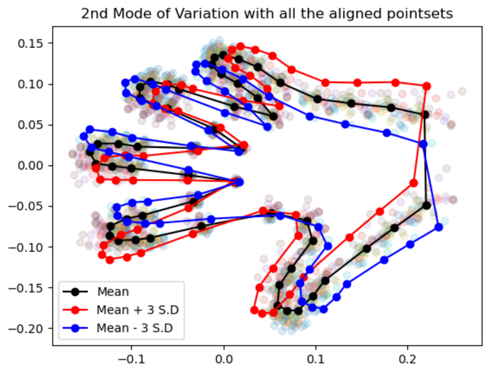

Figure 20: Second Mode of variation

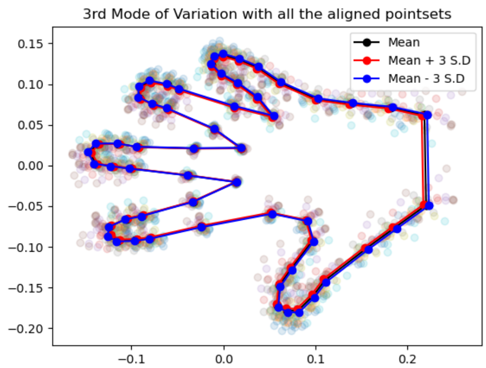

Figure 21: Third mode of variation

2.4.2 Results based on code22

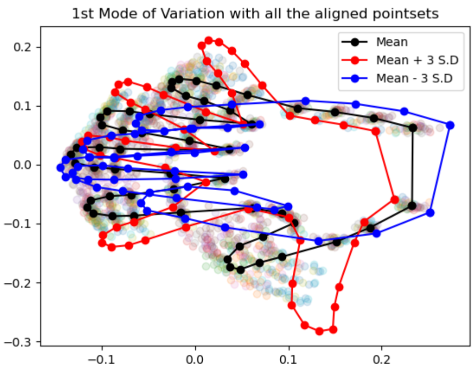

Figure 22: First Mode of variation

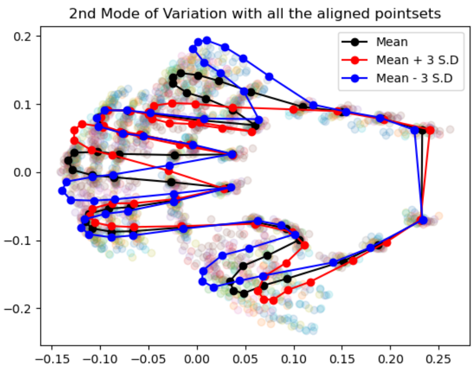

Figure 23: Second Mode of variation

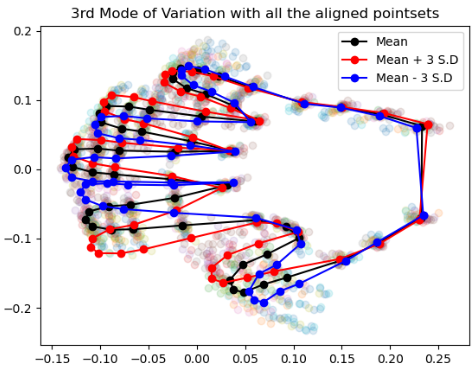

Figure 24: Third mode of variation

3 Problem 3

3.1 Procrustes Distance

The simplest way to define proximity is just the magnitude of the Euclidean norm of the difference between the two shape vectors in $\mathbb{R}^{dN}$ which is the square of the Procrustes distance between the shapes. Procrustes distance between z_1 and z_2 (resulting from aligning z_2 to z_1 by similarity transformations is

$$min_{\theta,T,s} \sum_{n=1,...,N} ||z_{1n} - sM_\theta z_{2n} - T||$$

where s is the scaling factor, M_θ is the rotation matrix of dimensions $d \times d$ and T represents a translation in $\mathbb{R}^d$.

3.2 K - means algorithm in the Shape space

We apply the k-means algorithm to the points $X_1, X_2, \ldots, X_n$ by using the Procrustes distance and Procrustes Mean.

3.3 Objective Function

$$W(\mathcal{C}, [Z_1], \ldots, [Z_k]) = \sum_{i=1}^{k} \sum_{l \in C_i} Procrustes\_distance([X_l], [Z_i])$$

3.4 Algorithm for clustering

1. **Initialisation condition** The starting centroid vector can be assigned in different ways: at random, using prior information, etc. In our application we will choose it at random.
2. Given a vector of shapes $Z = ([Z_1], \ldots, [Z_k])$ $\;[Z_i] \in \sum_3^n \;\; i = 1, ..., k$ we minimize with respect to $\mathcal{C} = (C_1, \ldots, C_k)$ assigning each shape $([X_1], \ldots, [X_n])$ to the class whose centroid has the minimum Procrustes distance to it.
3. Given $\mathcal{C}$, we minimize with respect to the vector shapes Z, taking $Z = ([\hat{\mu_1}], \ldots, [\hat{\mu_k}])$, and $[\hat{\mu_i}]$ $\; i = 1, ..., k$, the Procrustes mean of shapes in C_i. This is calculated using the algorithm mentioned below.
4. Repeat steps 2 and 3 till convergence.
5. **Stopping condition** When the value of the objective function doesn't decrease further, we stop.

3.5 Algorithm to find the Procrustes Means

1. Center the configurations to remove location. Let $X_i, i = 1, \ldots, n$ be the configurations. Initially let
$$X_i^P = X_i, i = 1, \ldots, n$$
2. Calculate $G = \frac{1}{n} \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} ||X_i^P - X_j^P||^2$ For the ith configuration let
$$\overline{X}_{(i)} = \frac{1}{n-1} \sum_{j \neq i} X_i^P$$
We can optimise $||\overline{X}_{(i)} - X_i^P M||^2$ over rotations. Set $X_i^P = X_i^P \hat{M}$, where $\hat{M}$ is the optimal rotation matrix. Repeat this for all i. Calculate the new G value. This process is repeated until G can't be reduced further.

3. For the ith configuration calculate $\hat{\beta}_i = \frac{\sum_{k=1}^{n} ||X_k^P||^2}{||X_i^P||^2}^{1/2} \phi_i$, where ϕ_i is the ith component of the eigenvector ϕ corresponding to the largest eigenvalue of the correlation matrix ϕ of the $vector(X_i^P)$. Set $X_i^P = \hat{\beta}_i X_i^P$. Repeat this for all i and calculate the new G value.
4. Repeat steps 2 and 3 until G cannot be reduced further.
5. $$[\hat{\mu}] = \frac{1}{n} \sum_{i}^{n} X_i^P$$