Numer.AI Assignment 1

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1 Problem 1

Given the resonating property of the Bazooka gun, it will try to transmit x signals per second in the next state, if it had been transmitting x signals per second currently. But there might be fluctuations in this behavious.

1.1 Modelling as a Markov Process

So to model the above problem as a Markov chain with infinite states, we need to define the set of states and a transition matrix. The set of states are labeled as $0, 1, 2, \ldots, \infty$ where each state represents the number of signals transmitted per second.

Now to get the transition matrix, we need to model the transition probabilities. Given that the Bazooka is in state x, it will try to remain in state x but could transmit more or lesser signals later. So it is reasonable to assume a poisson PMF, with the average number of signals transmitted per second being x.

So the transition matrix will have each term as follows

$$\begin{bmatrix} 1 & 0 & 0 & \cdots \\ \frac{1^0 e^{-1}}{0!} & \frac{1^1 e^{-1}}{1!} & \frac{1^2 e^{-1}}{2!} & \cdots \\ \frac{2^0 e^{-2}}{0!} & \frac{2^1 e^{-2}}{1!} & \frac{2^2 e^{-2}}{2!} & \cdots \\ \frac{3^0 e^{-3}}{0!} & \frac{3^1 e^{-3}}{1!} & \frac{3^2 e^{-3}}{2!} & \cdots \\ \vdots & \vdots & \vdots & \vdots \end{bmatrix}$$

In general, except for i = 0;

$$P_{ij} = \frac{i^j e^{-i}}{j!}$$

where P_{ij} represents the probability of transition from the i_{th} state to the j_{th} state.

1.2 Martingale in this infinite random walk

Assume that the state of the system is defined by the random variable X_n after n steps or n seconds. Suppose $X_n = i$, then from the transition probability,

$$P[X_{n+1} = j | X_n = i] = \frac{i^j e^{-i}}{j!}$$

Note that because of the markov property, X_{n+1} depends only on X_n and not on the previous history. Thus we can establish the Martingale property as follows :

$$E[|X_n|] \leq \infty \quad \forall n \in \mathbb{N}$$

$$E[X_{n+1}|X_n] = \sum_{j=0}^{\infty} j \times \frac{i^j e^{-i}}{j!}$$

$$E[X_{n+1}|X_n] = \sum_{j=0}^{\infty} ie^{-i} \times \frac{i^{j-1}}{(j-1)!}$$

$$E[X_{n+1}|X_n] = i$$

Thus,

$$E[X_{n+1}|X_n] = X_n$$

1.3 Combined behaviour of Bazooka 5000 and Bazooka 5001

We know that Bazooka 5000 shows Martingale behaviour, and because Bazooka 5001 also has the resonating property, it will continue to show Martingale behaviour.

The combined machinery will be the product of the two numbers generated. So the question reduces to show that the product of two independent Martingales is a Martingale or not.

I claim that the product of two independent martingales is indeed a martingale. Proof follows from the fact that $E[XY] = E[X]E[Y]$ for two independent random variables X and Y.