Pseudorandomness IV and Encryption I

601.442/642 Modern Cryptography

19th February 2026

Logistics

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  - See updated schedule on Canvas

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  - In practice, PRPs and PRFs can be used interchangeably. We will prove this if we have time.

PRF Game

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$$k \xleftarrow{\$} \{0,1\}^\lambda$$
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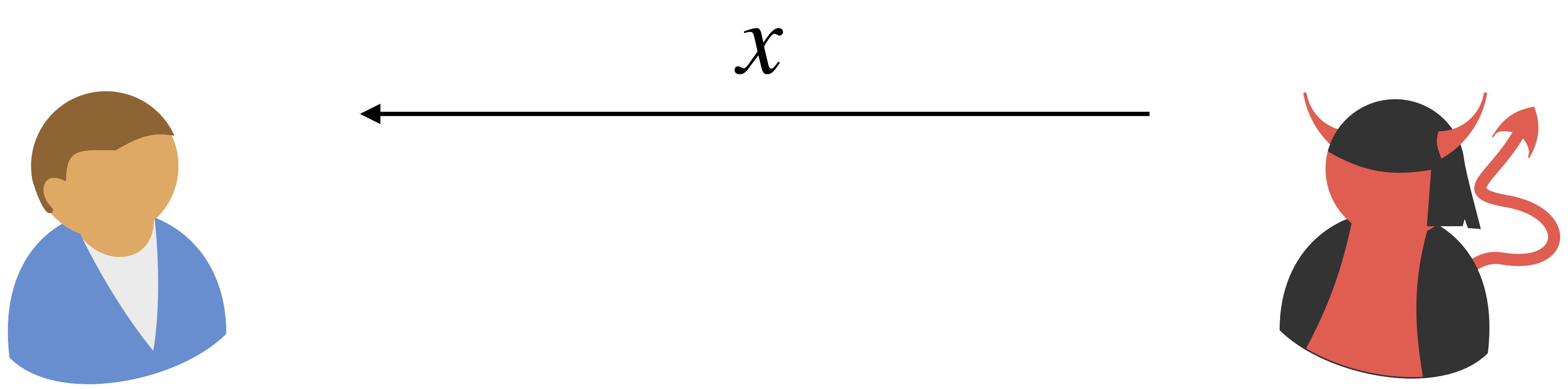
if $b = 0$

$y = F_k(x)$



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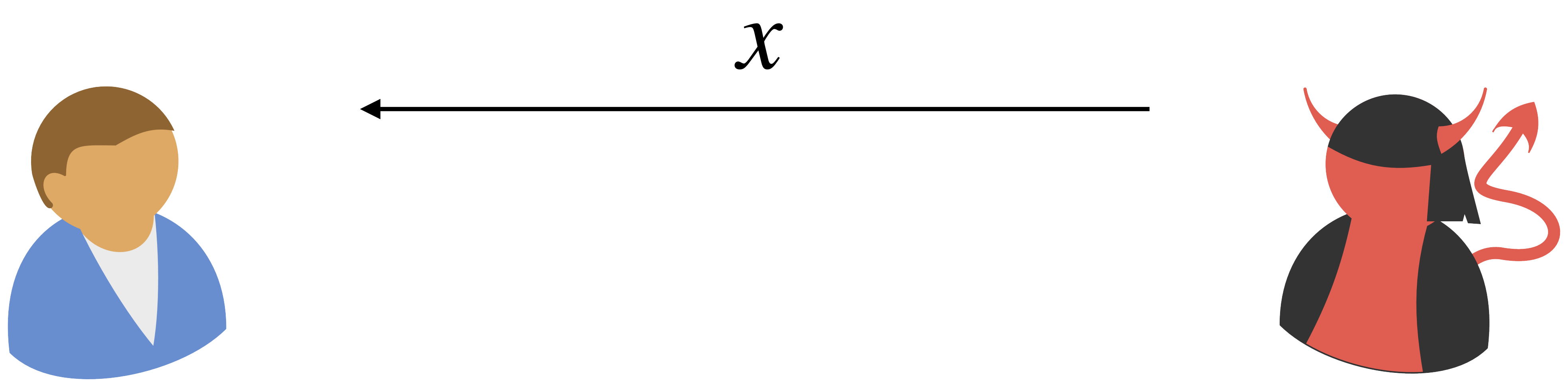

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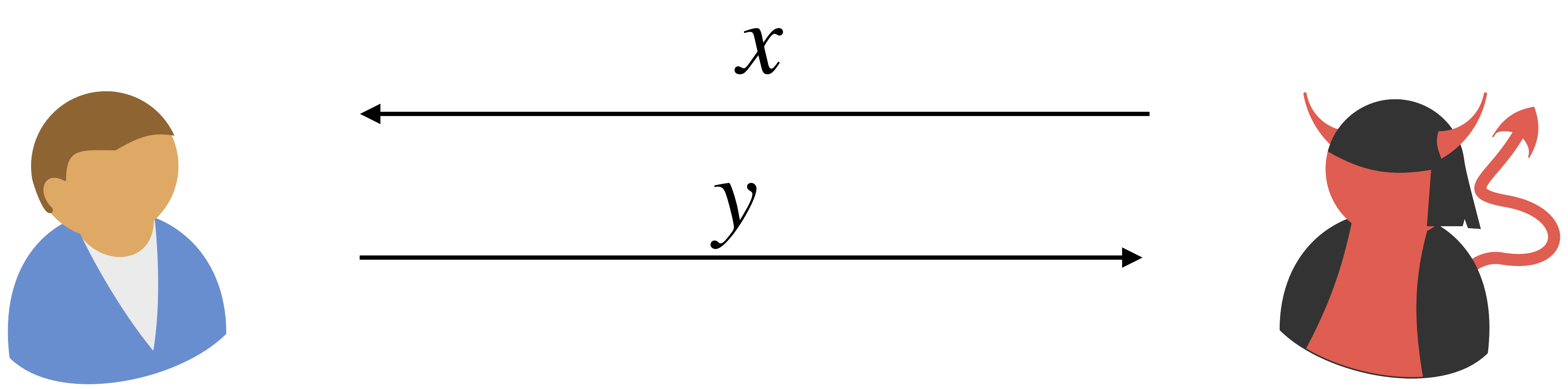

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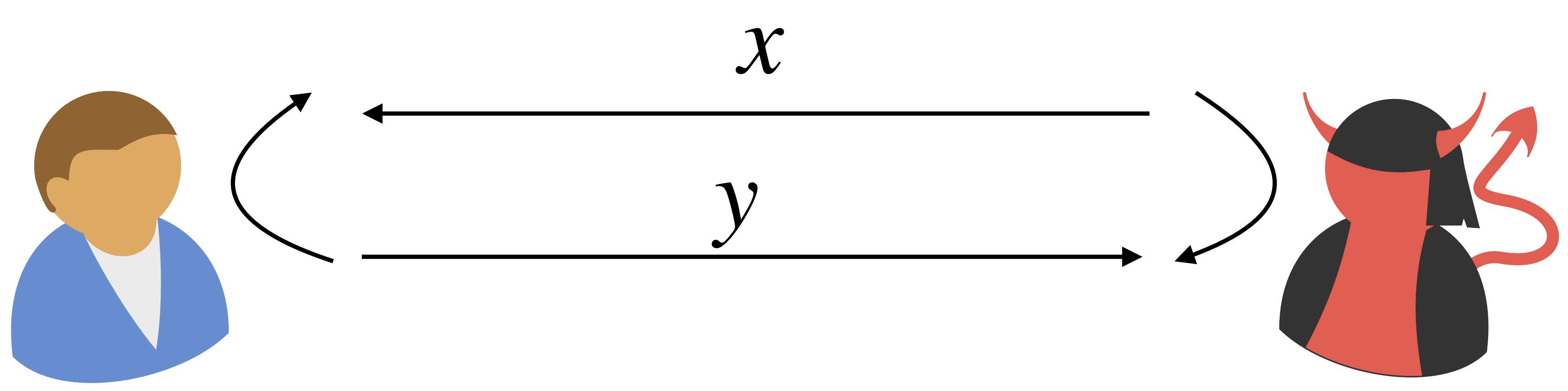

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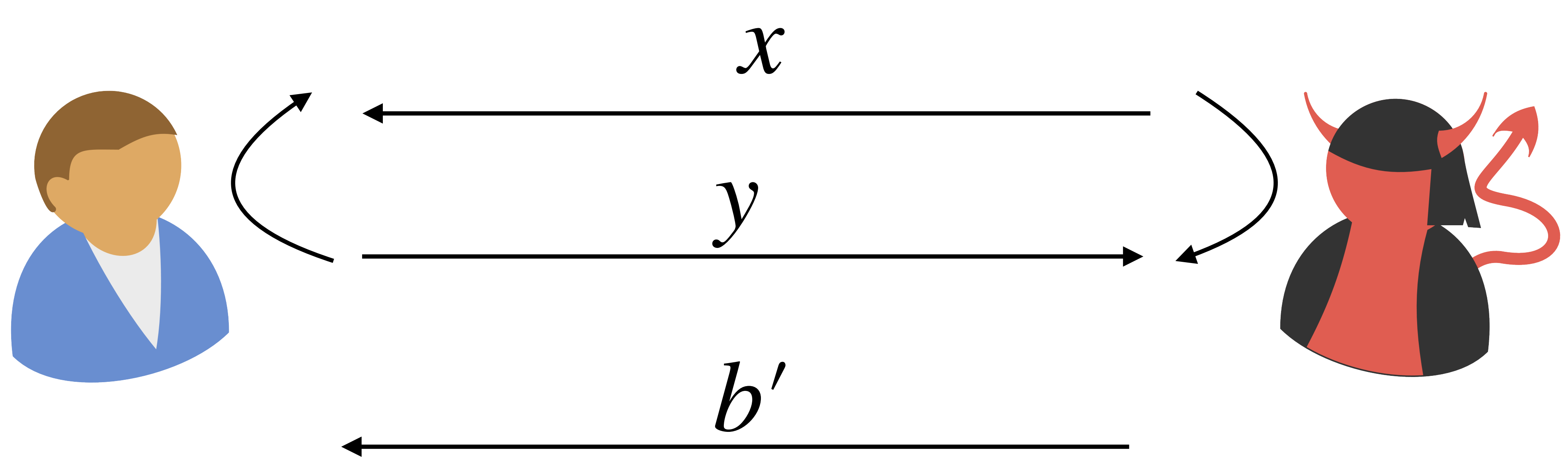

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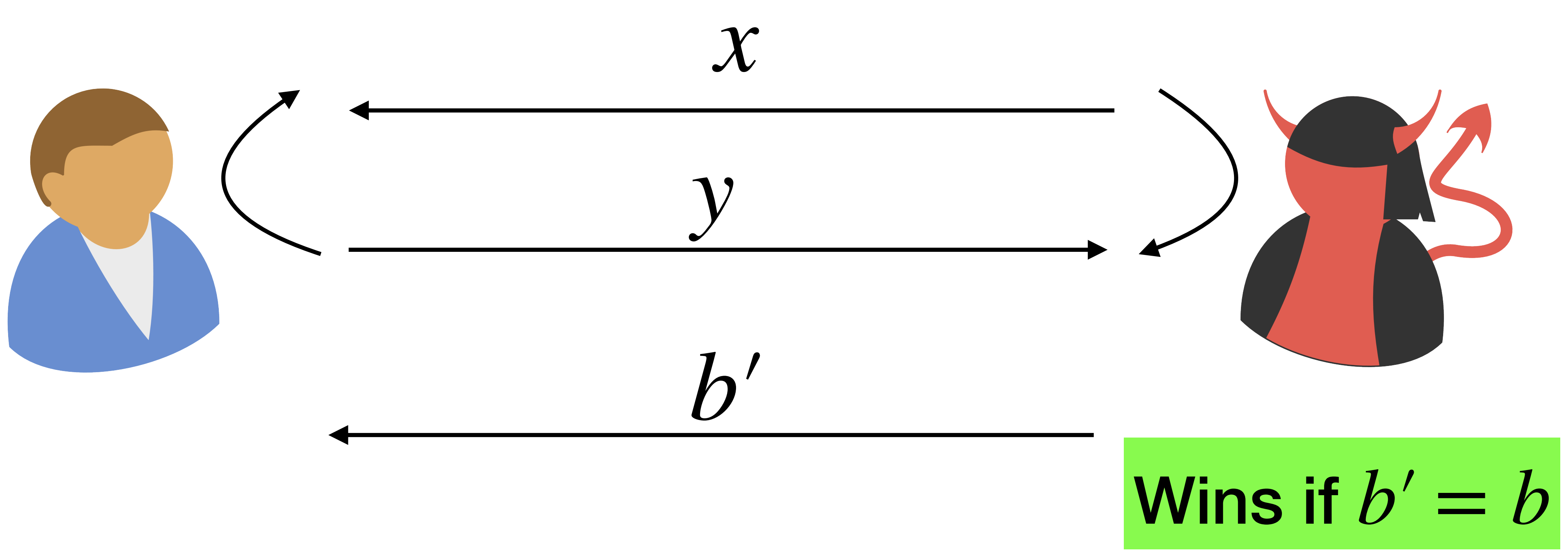

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$\mathcal{A}$ can be *adaptive*

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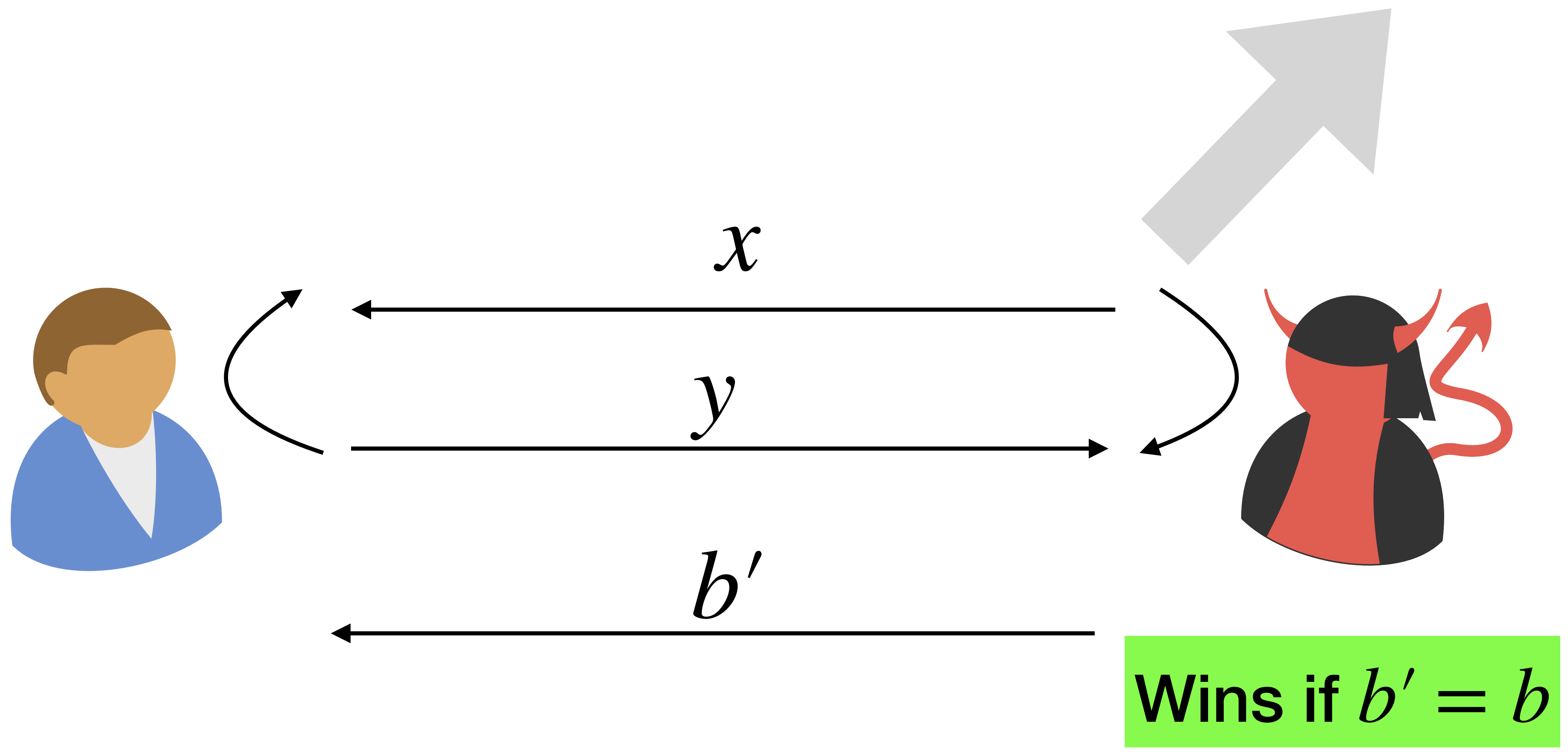

Multi-Message Security

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An encryption scheme with message length $\ell := \ell(\lambda)$ is multi-message secure if $\forall \{(m_0^i, m_1^i)\}_{i=1}^{q(\lambda)}$ where $q(\lambda)$ is a polynomial

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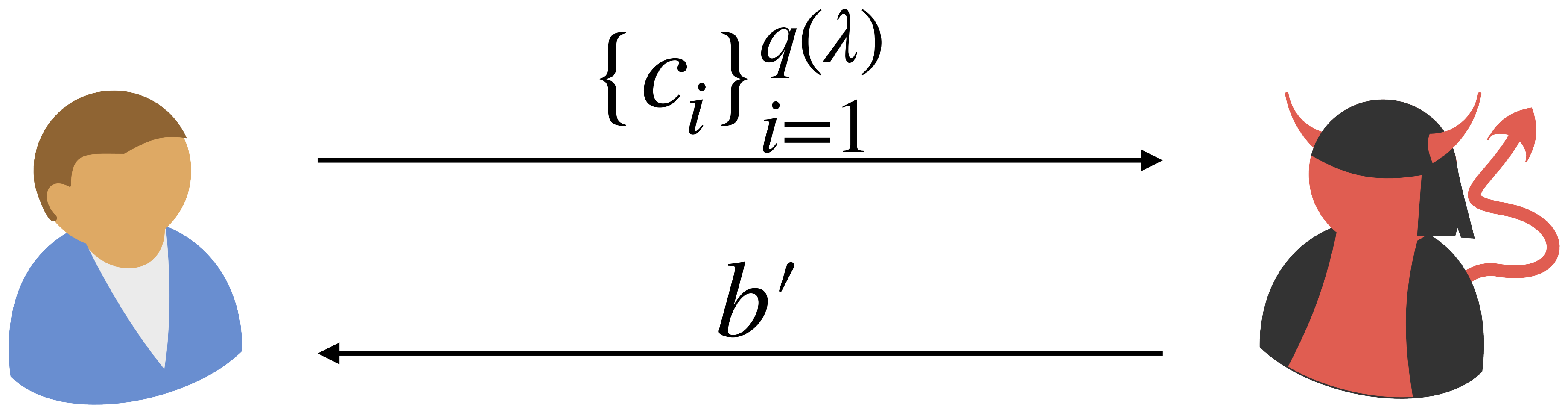

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No adaptivity!
All messages must be "chosen" upfront.

Adaptive Multi-Message Security

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$\mathsf{Enc}(k, m) \to c$

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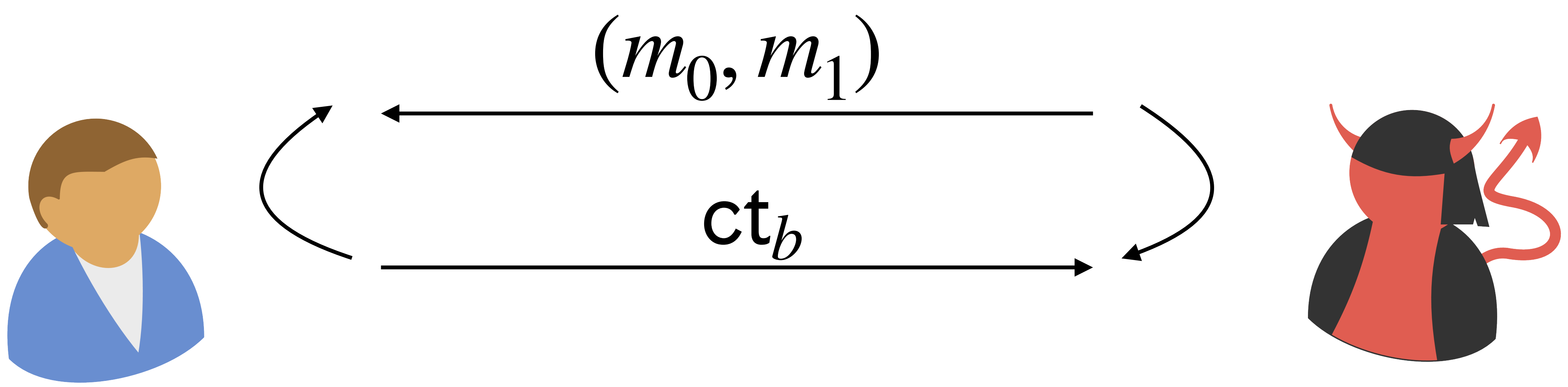

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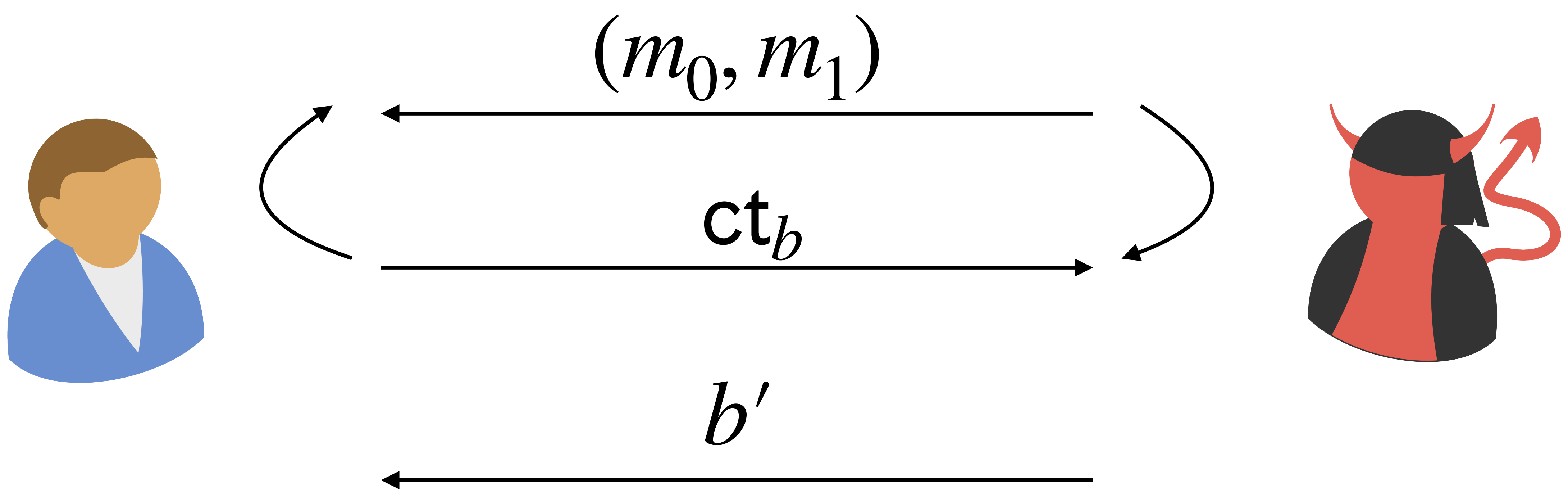

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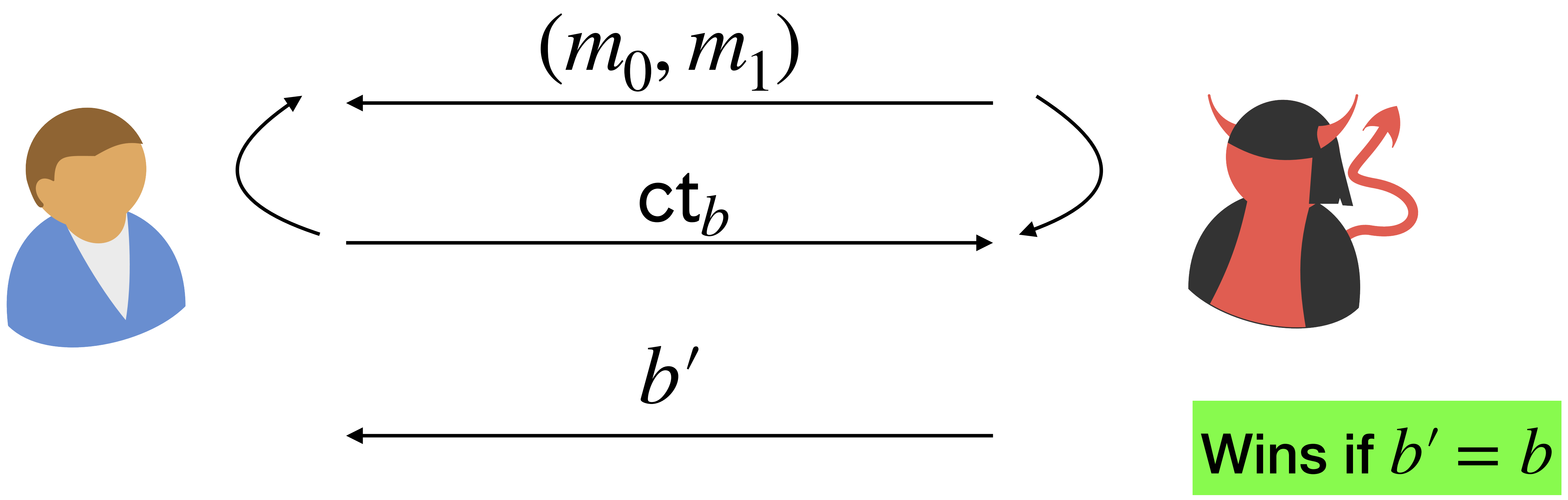

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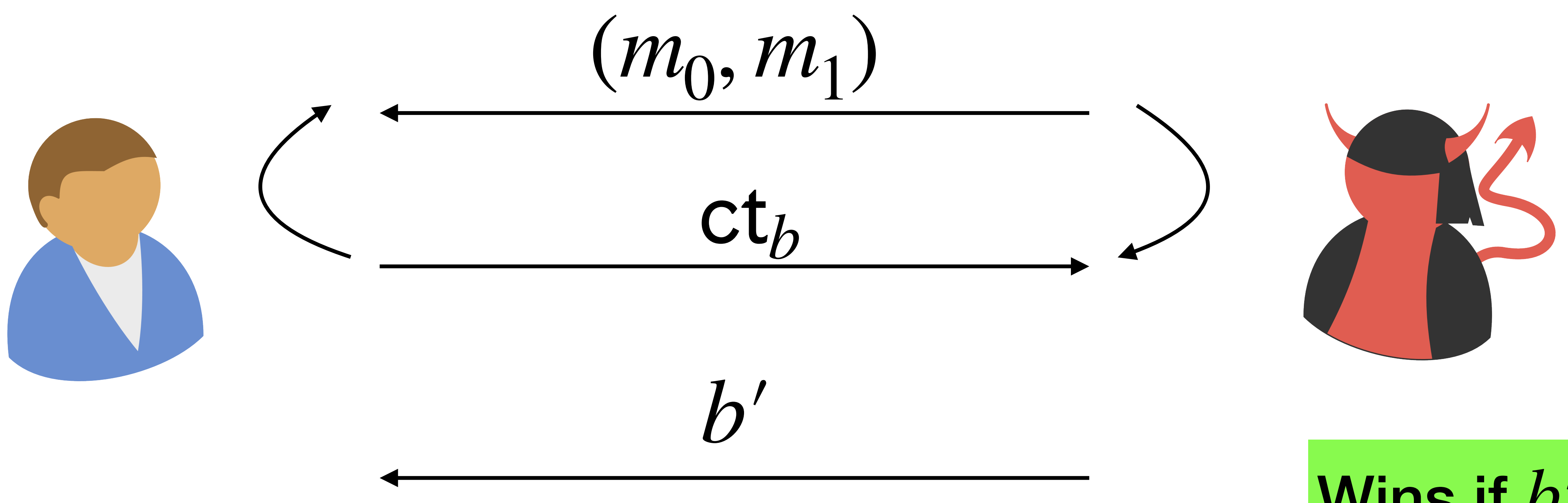

Wins if $b' = b$

Ciphertexts should be **indistinguishable**, even when the adversary gets to **choose** the plaintext

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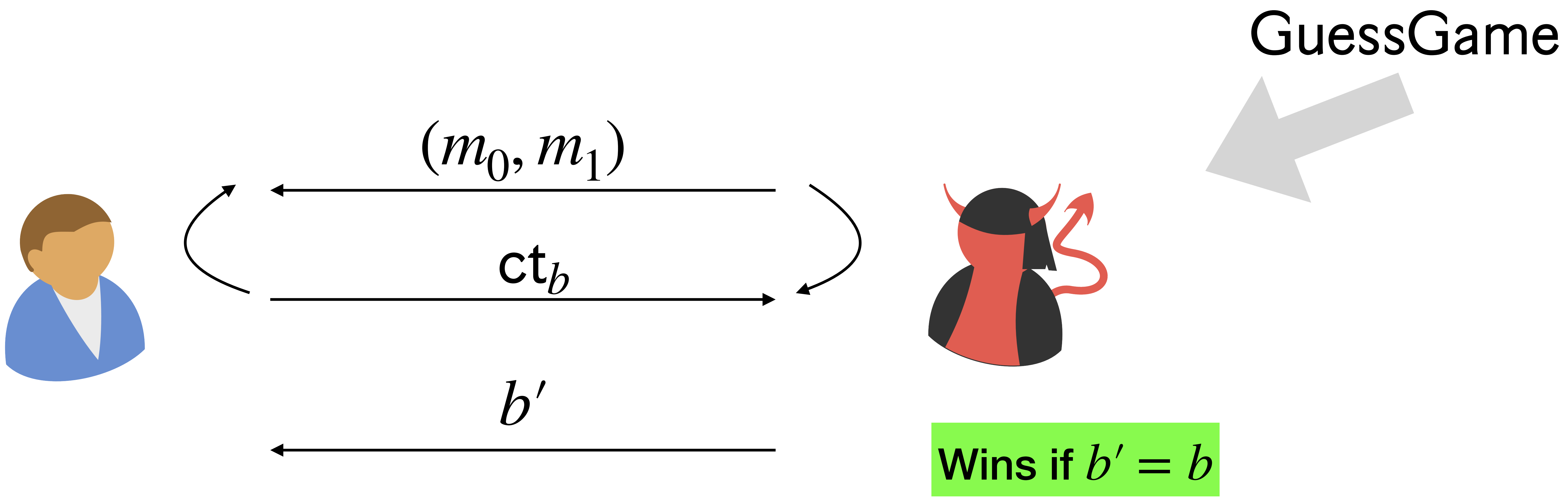

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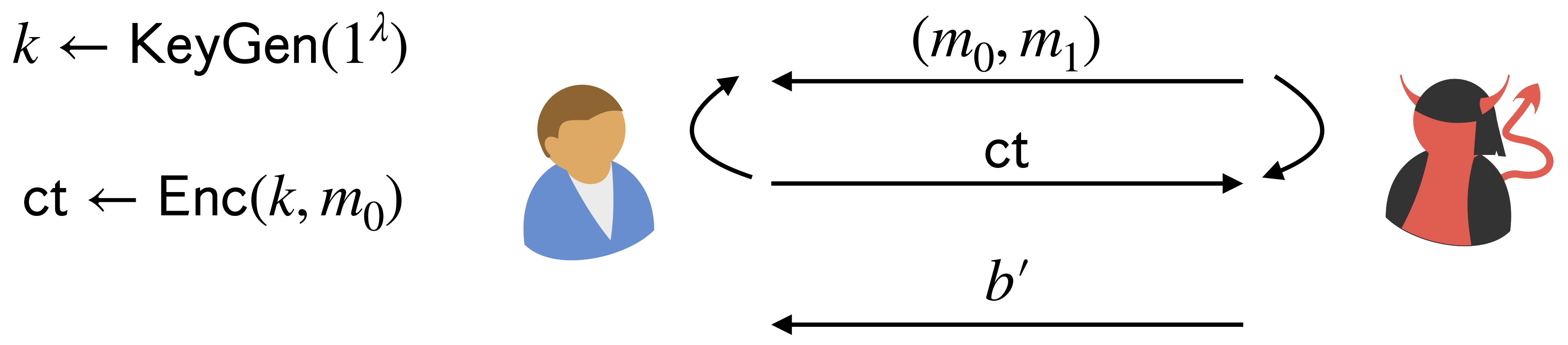
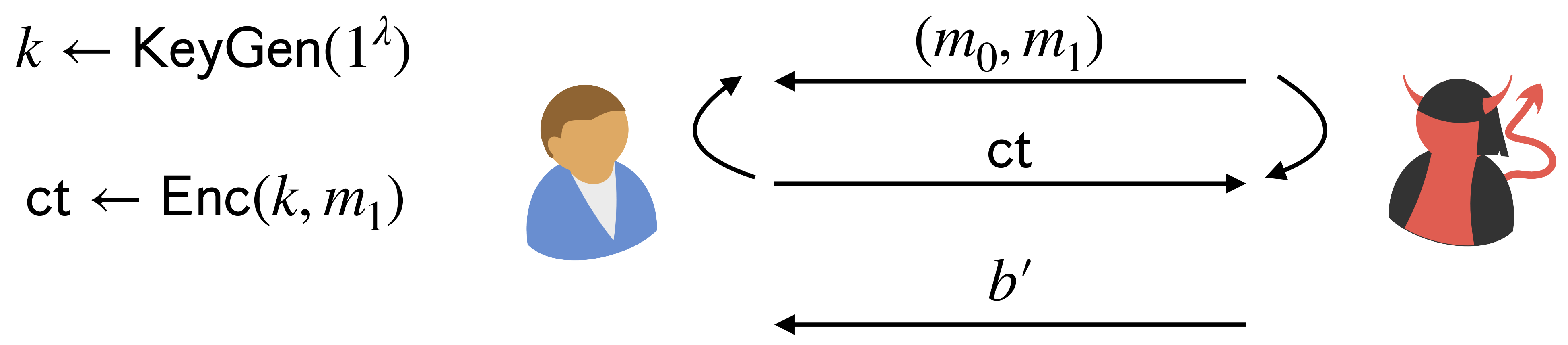
$$\left| \Pr[\mathscr{A} \text{ outputs } 1 \text{ in } \mathsf{Game}_0] - \Pr[\mathscr{A} \text{ outputs } 1 \text{ in } \mathsf{Game}_1] \right| \leq \nu(\lambda)$$

Game_0



Game_1



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Only applies to messages of fixed length $\ell(\lambda)$

Proof of Security

$\mathsf{KeyGen}(1^\lambda) : k \xleftarrow{\$} \{0,1\}^\lambda$

$\mathsf{Enc}(\mathsf{k}, \mathsf{m}) : \begin{array}{c} x \xleftarrow{\$} \{0,1\}^\lambda \\ \mathsf{ct} := (F_k(x) \oplus m, x) \end{array}$

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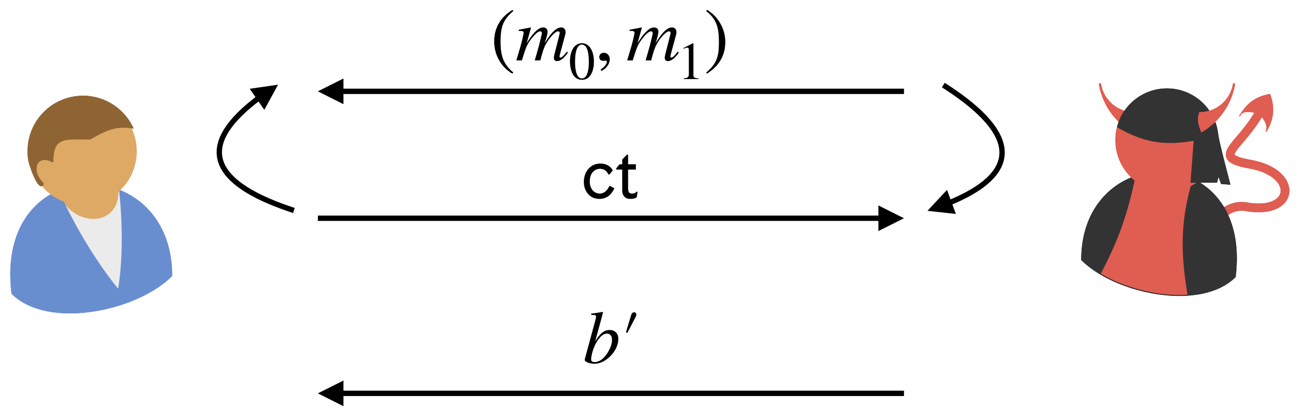
Proof of Security

H_0

$k \leftarrow \mathsf{KeyGen}(1^\lambda)$

$\mathsf{ct} \leftarrow \mathsf{Enc}(k, m_0)$



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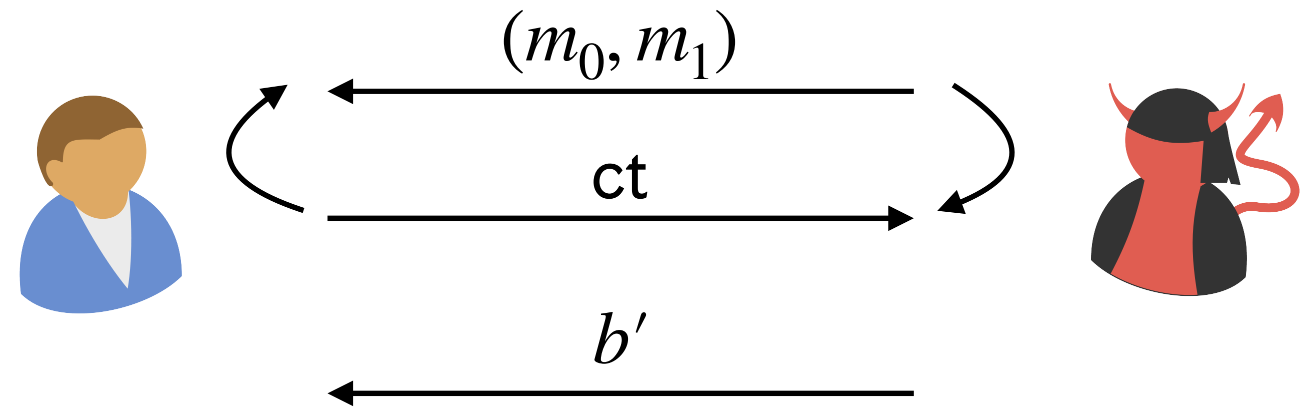

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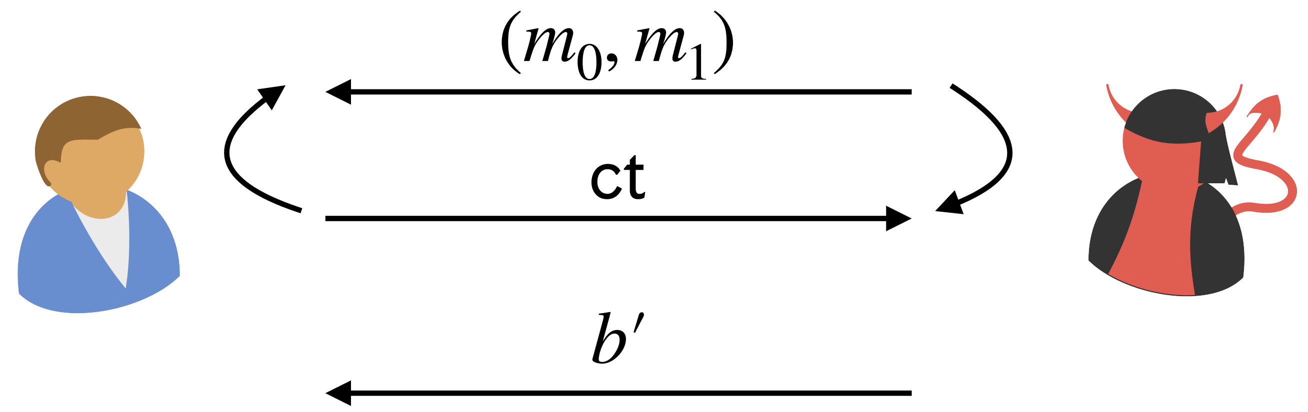

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Let W_i be the probability that $\mathscr{A}$ outputs 1 in H_i

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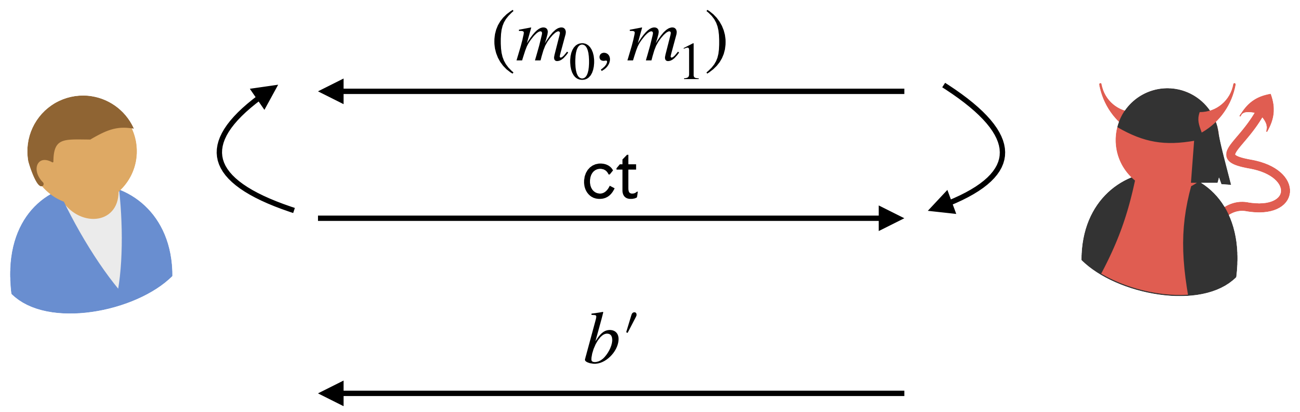

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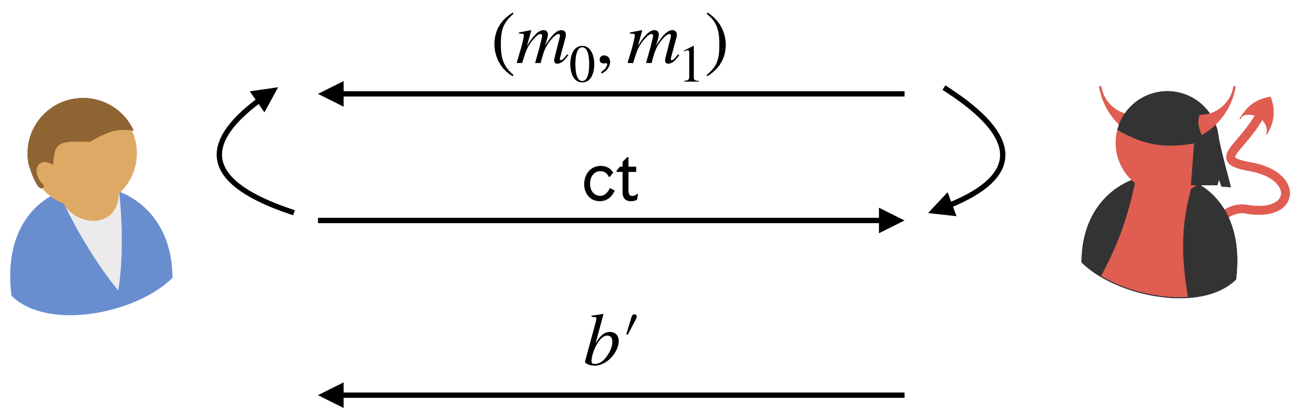

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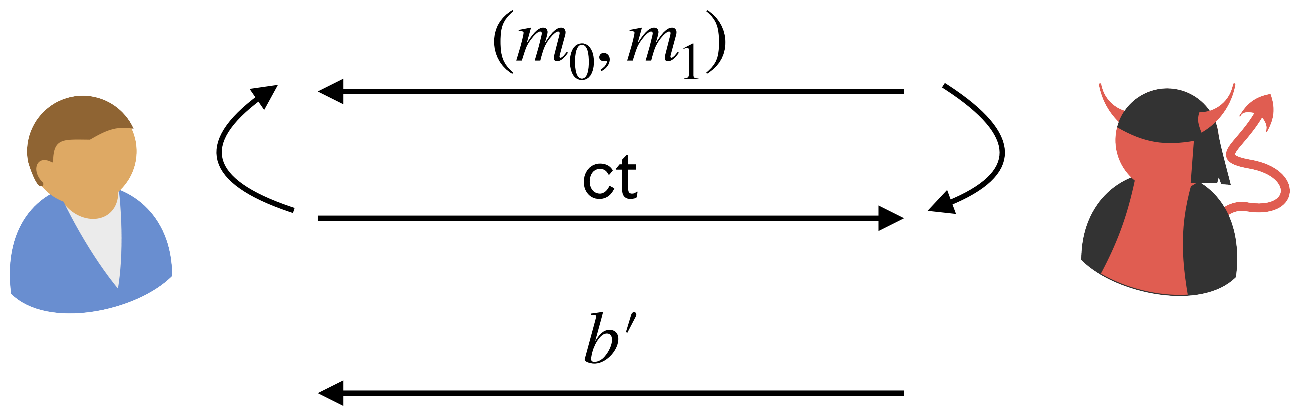

Goal: prove that $H_0 \overset{c}{\approx} H_?$, i.e. that

$$\left| \Pr\left[W_0\right] - \Pr\left[W_?\right] \right| \leq \mathsf{negl}(\lambda)$$

$H_?$ $k \leftarrow \mathsf{KeyGen}(1^\lambda)$
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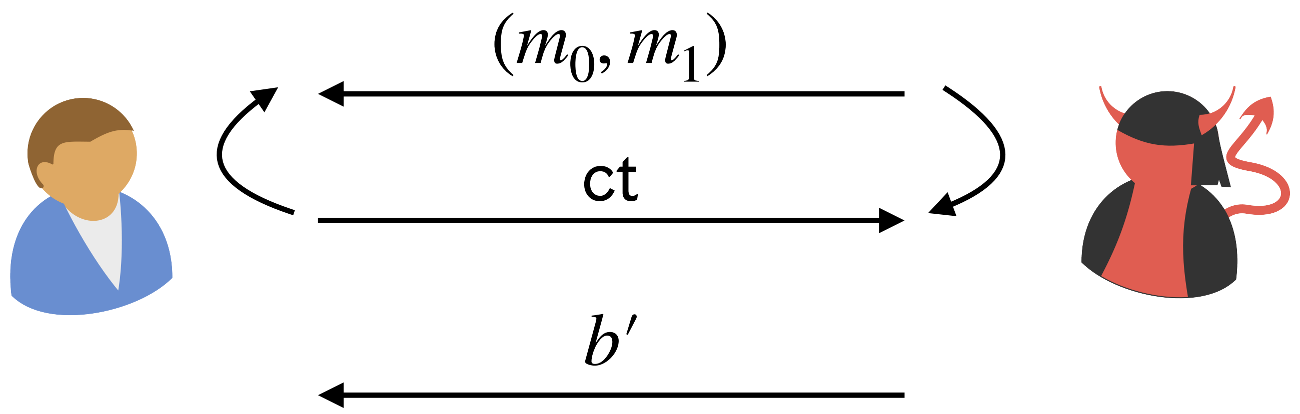

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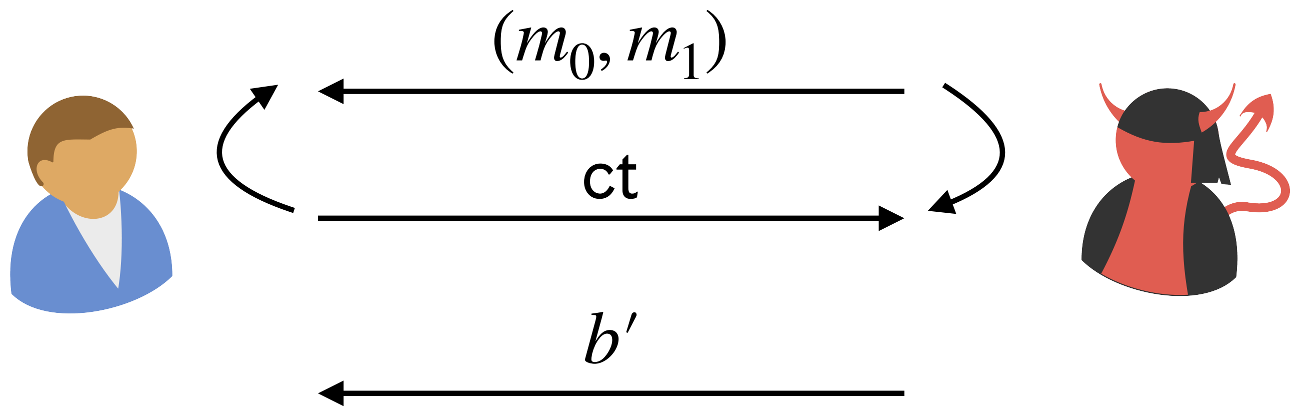

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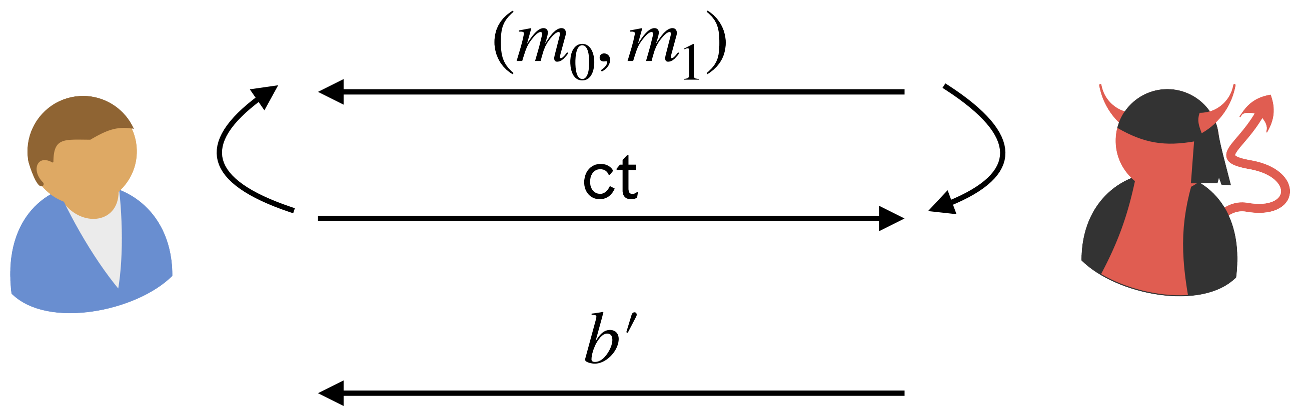

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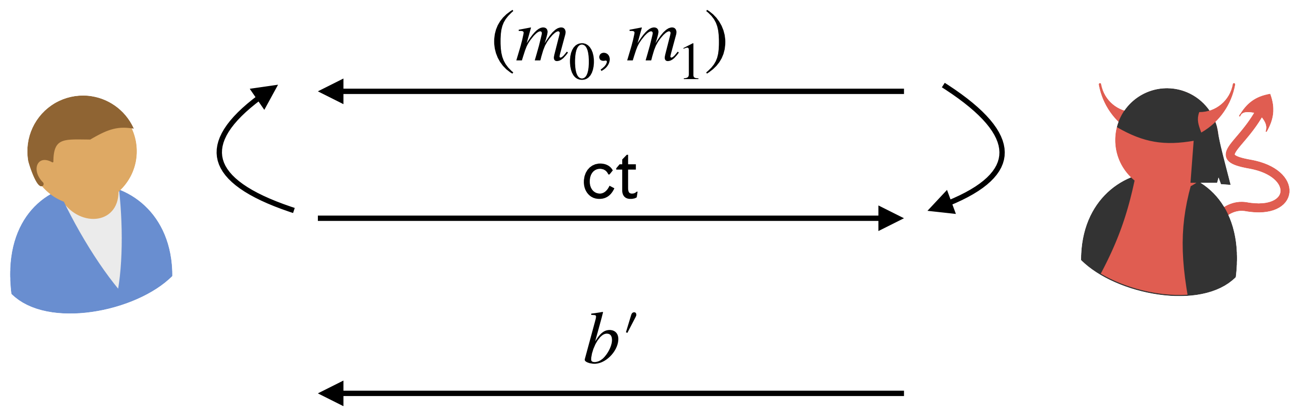
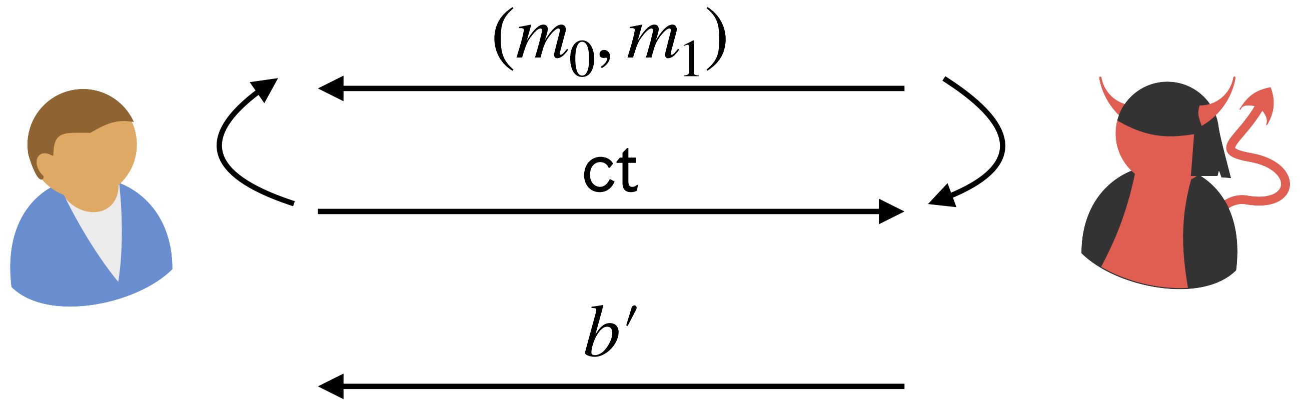


H_1 Some small change



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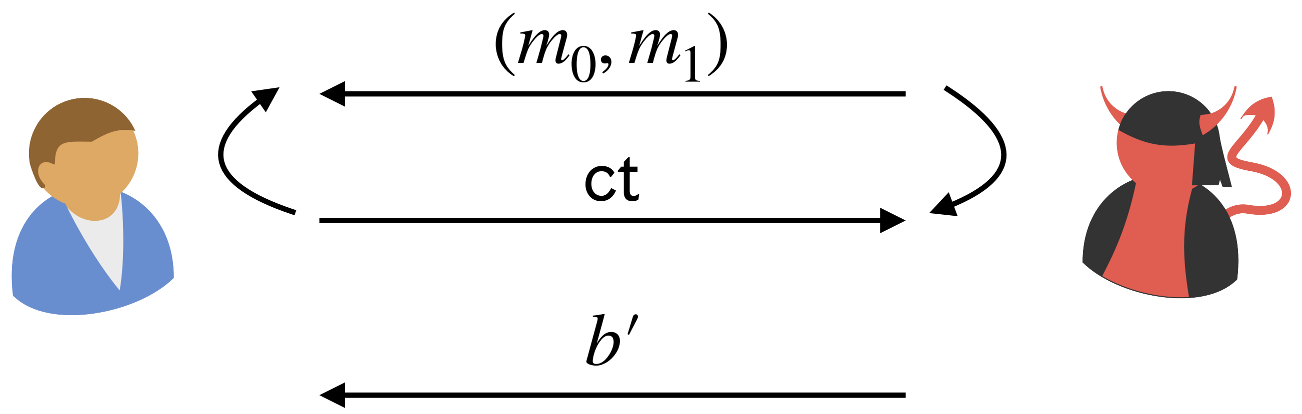

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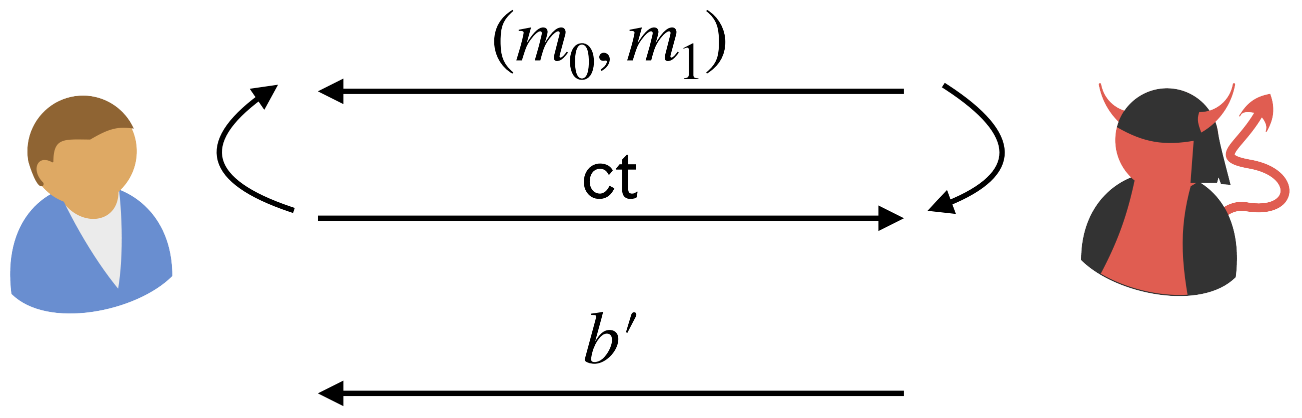

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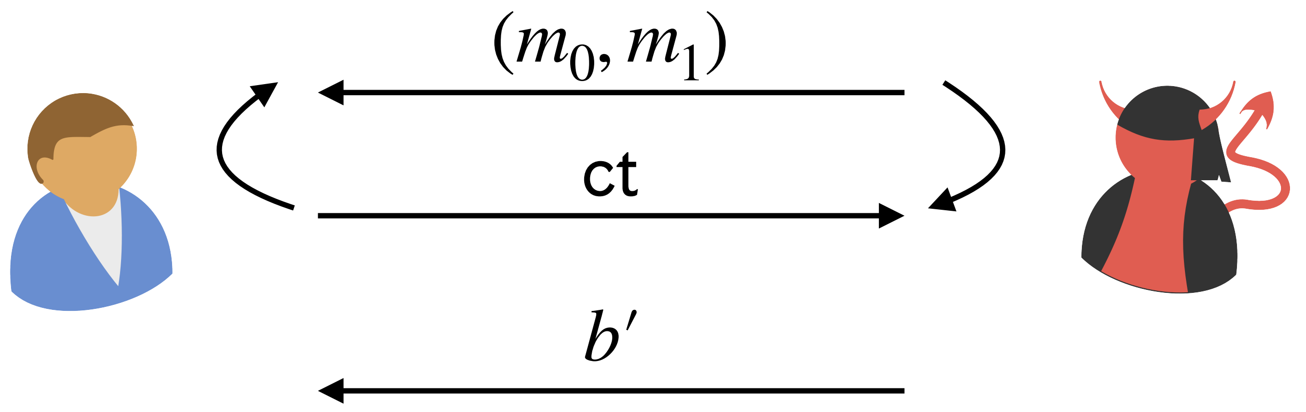

Prove that $H_0 \overset{c}{\approx} H_1$

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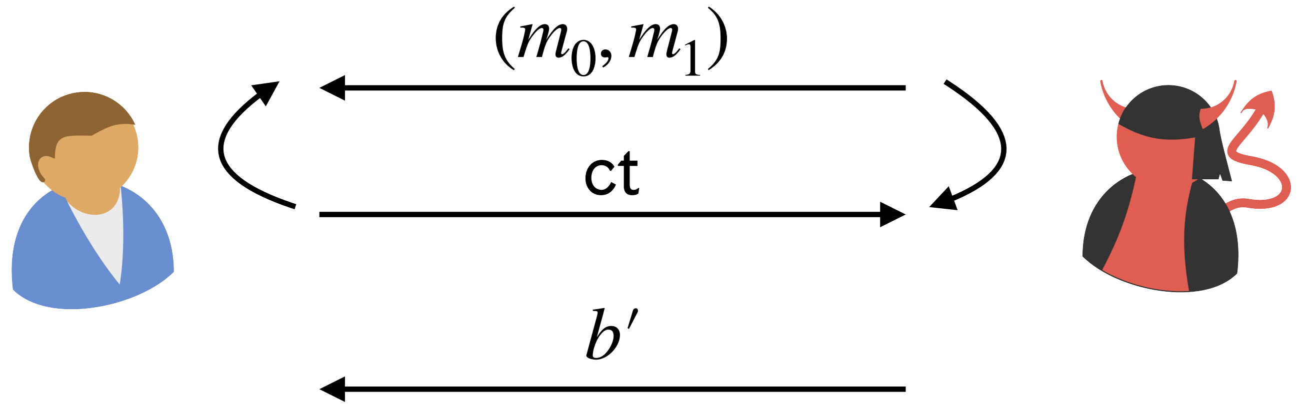

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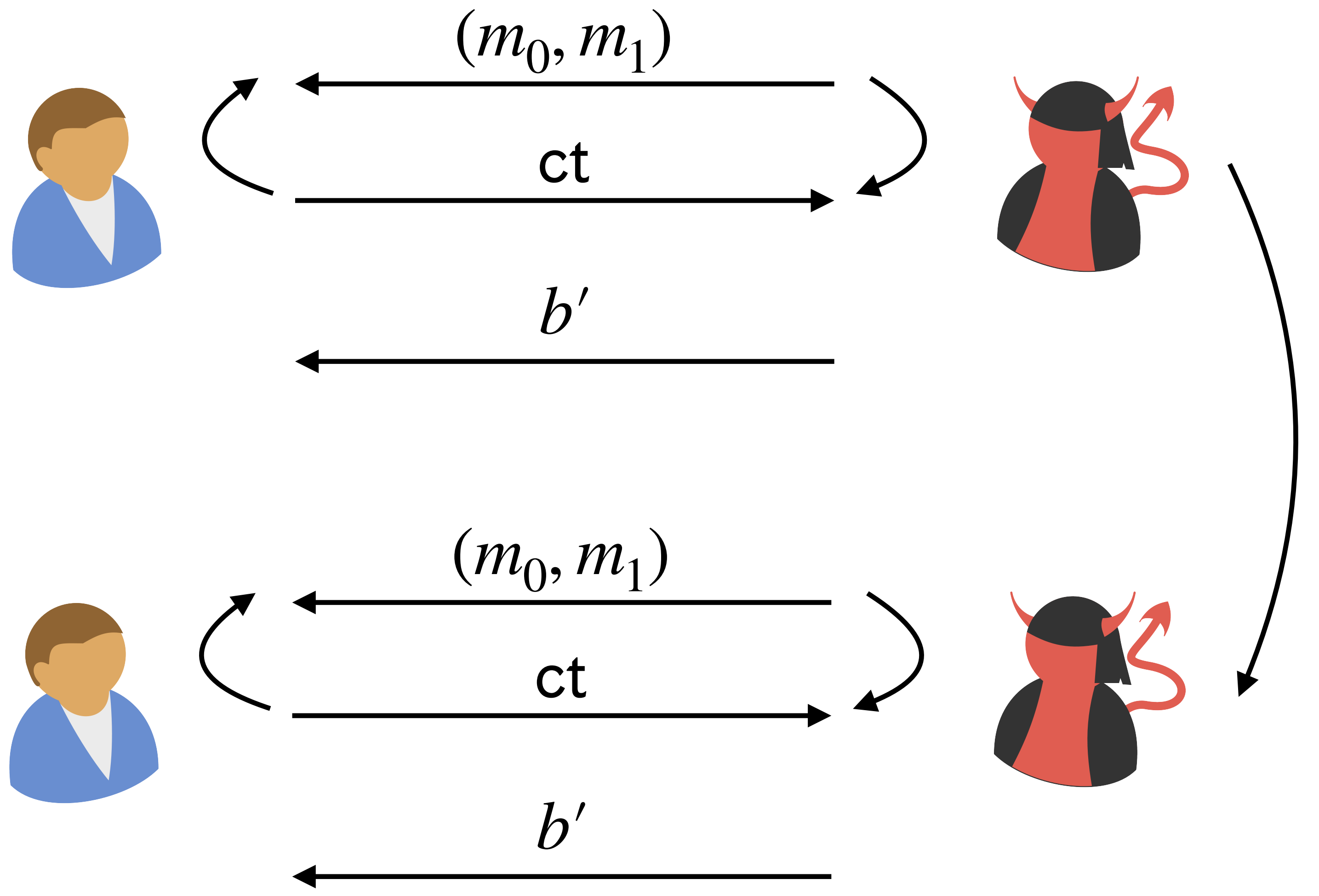

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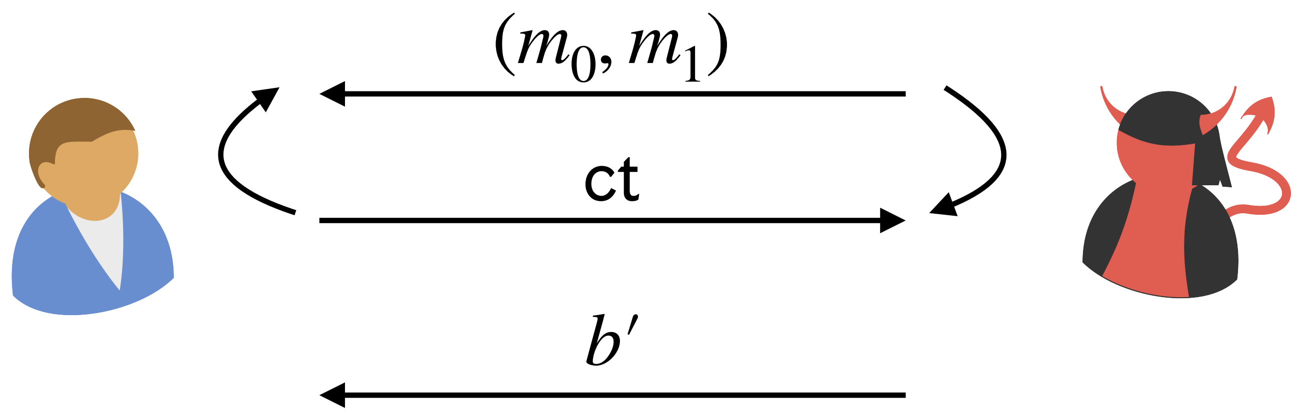

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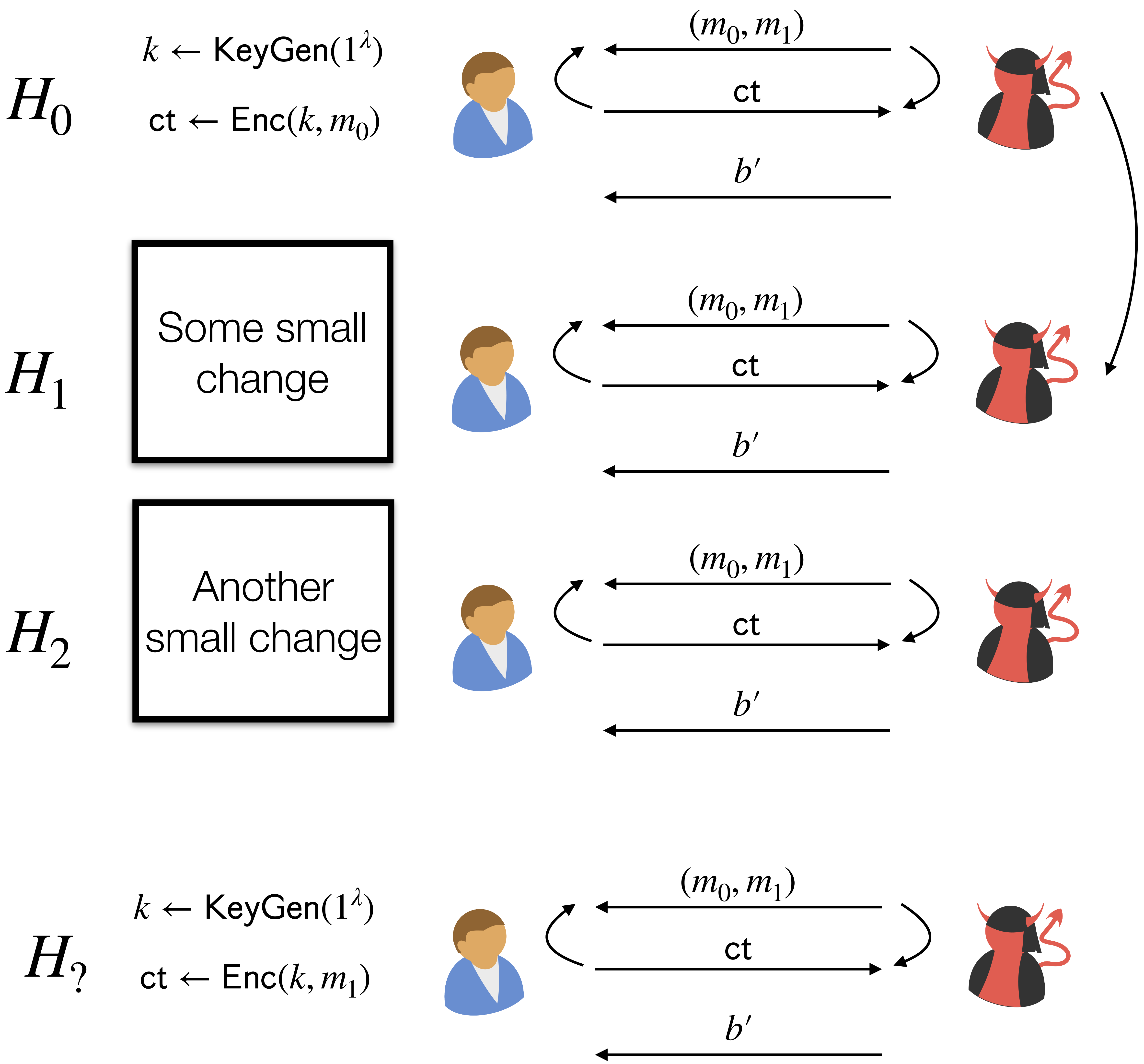

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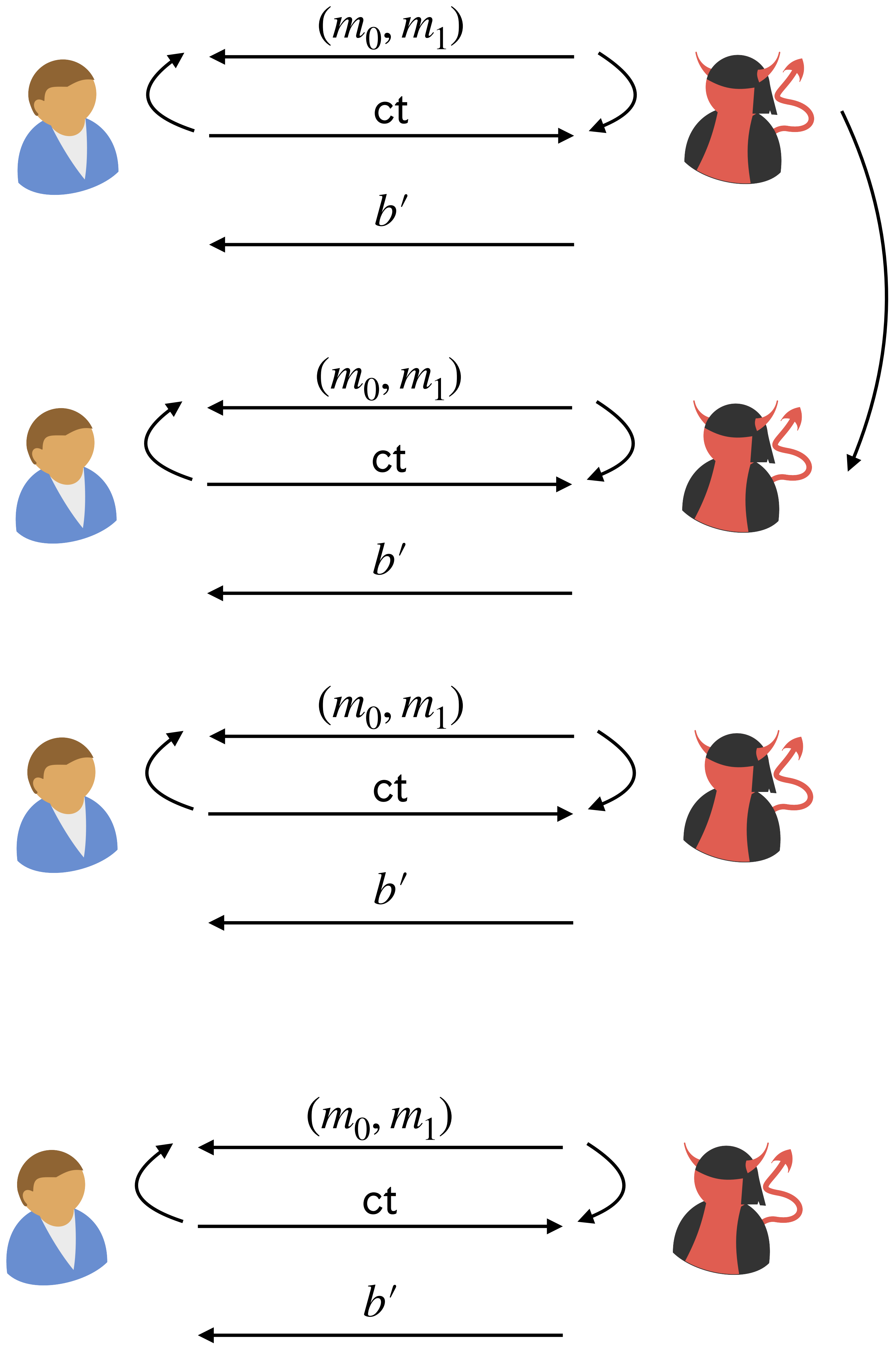

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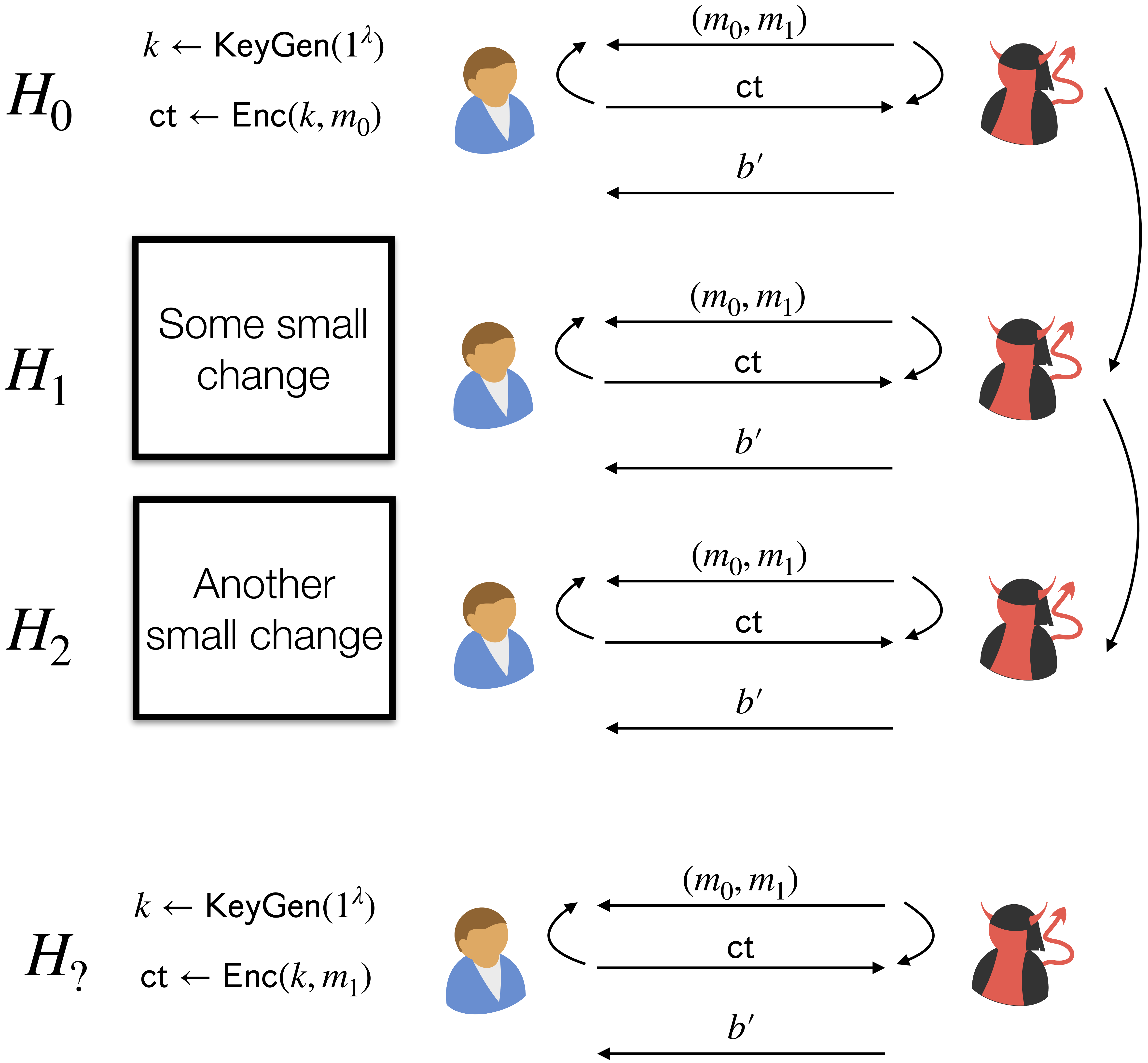

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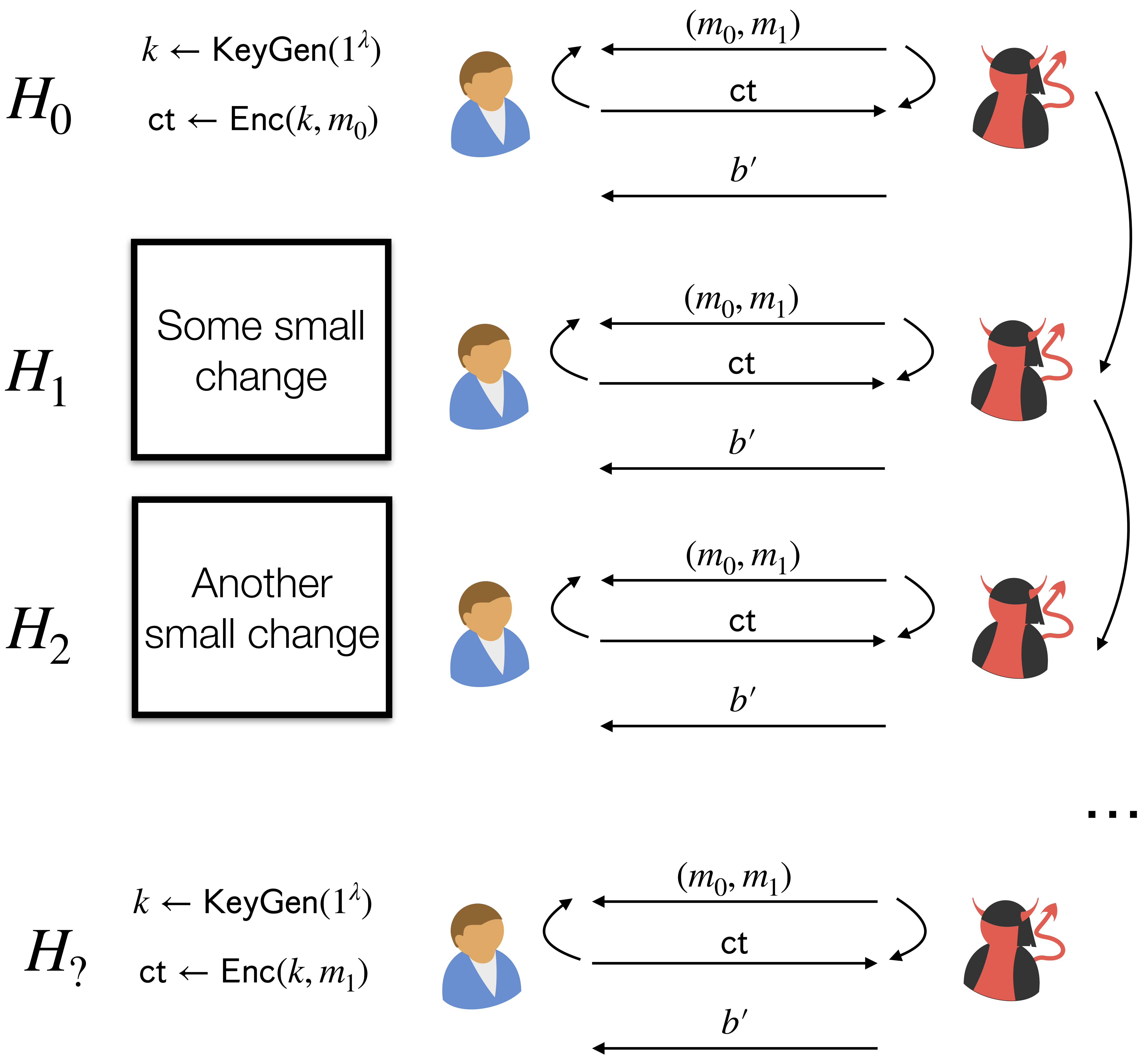

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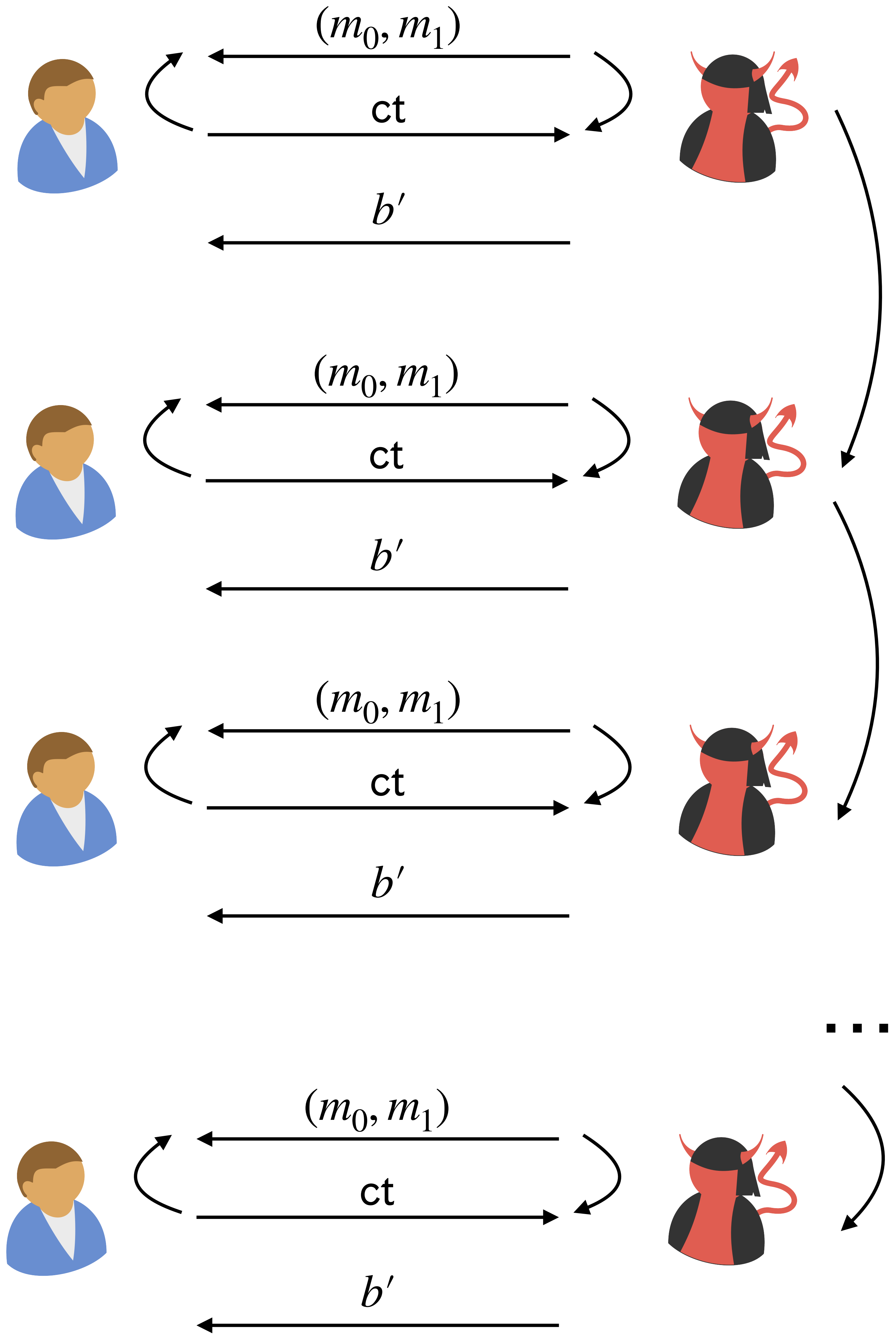
H_1 — Some small change

H_2 — Another small change

...

$H_?$ — $k \leftarrow \mathsf{KeyGen}(1^\lambda)$, $\mathsf{ct} \leftarrow \mathsf{Enc}(k, m_1)$



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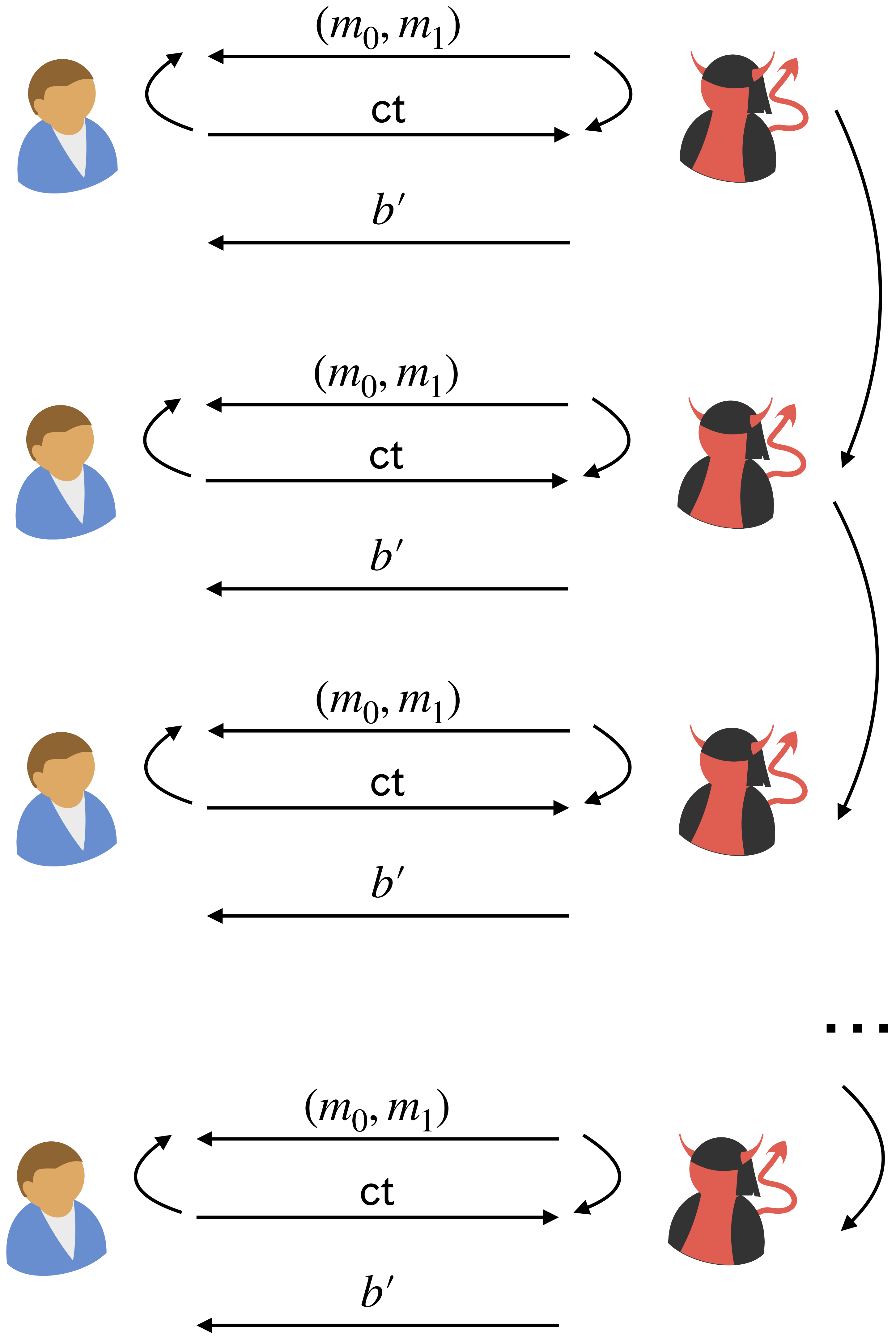

Prove that $H_0 \overset{c}{\approx} H_1$

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By hybrid lemma: $H_0 \overset{c}{\approx} H_?$

Proof of Security

H_0

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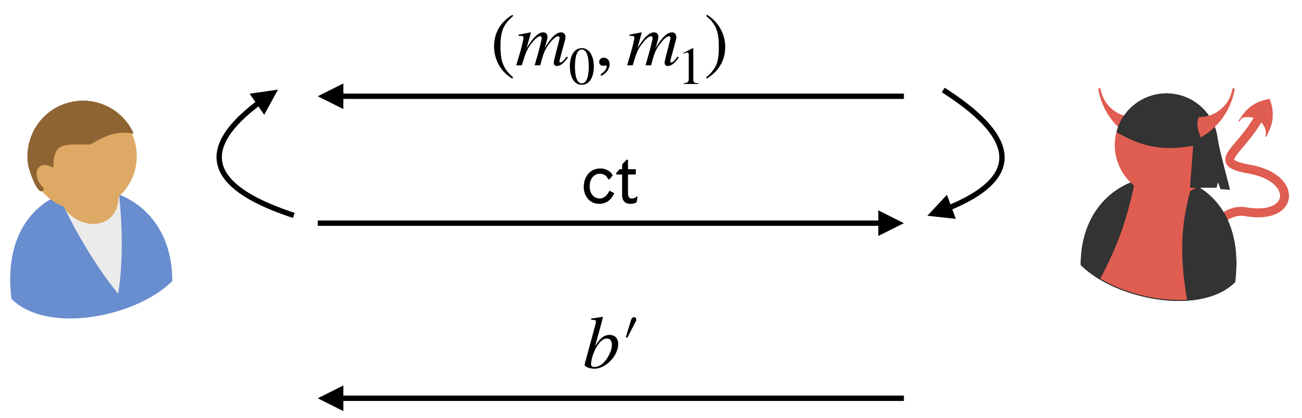

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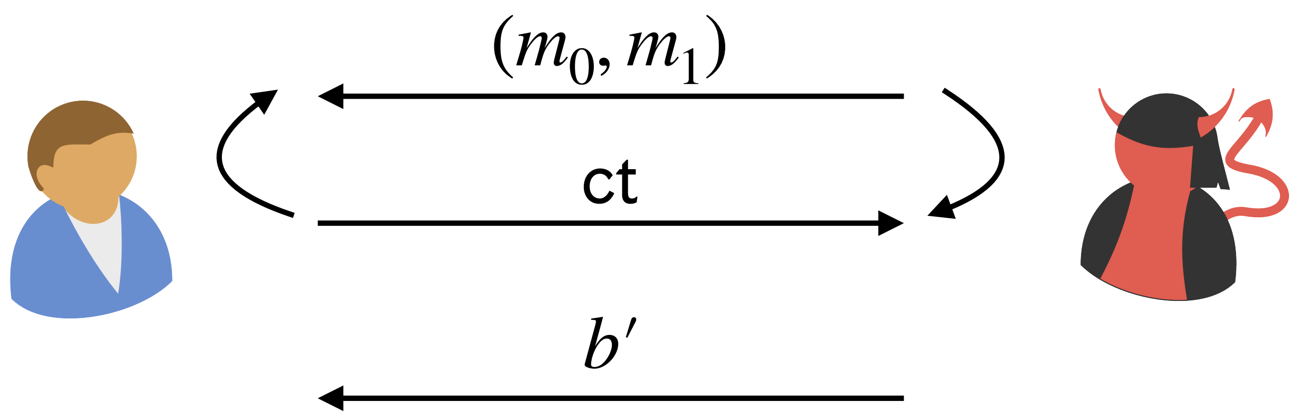

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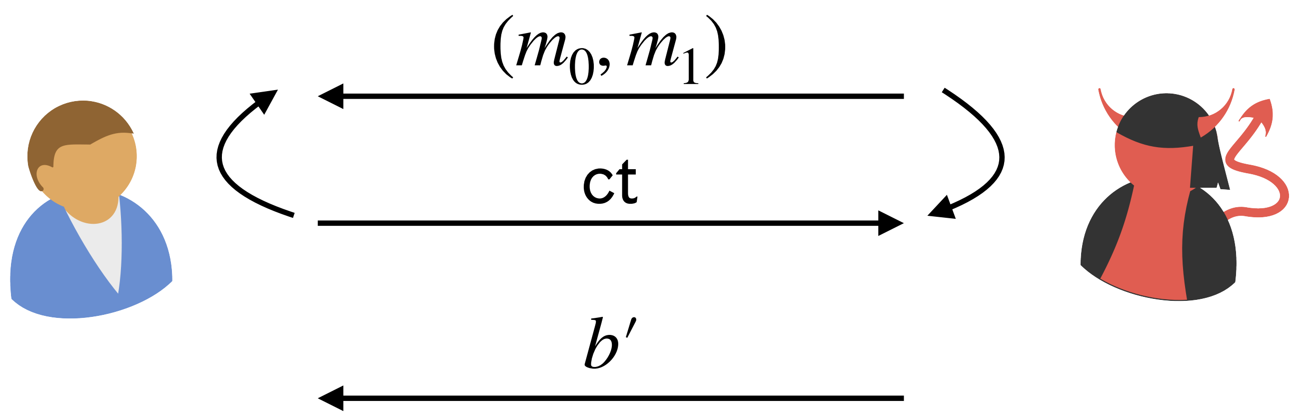

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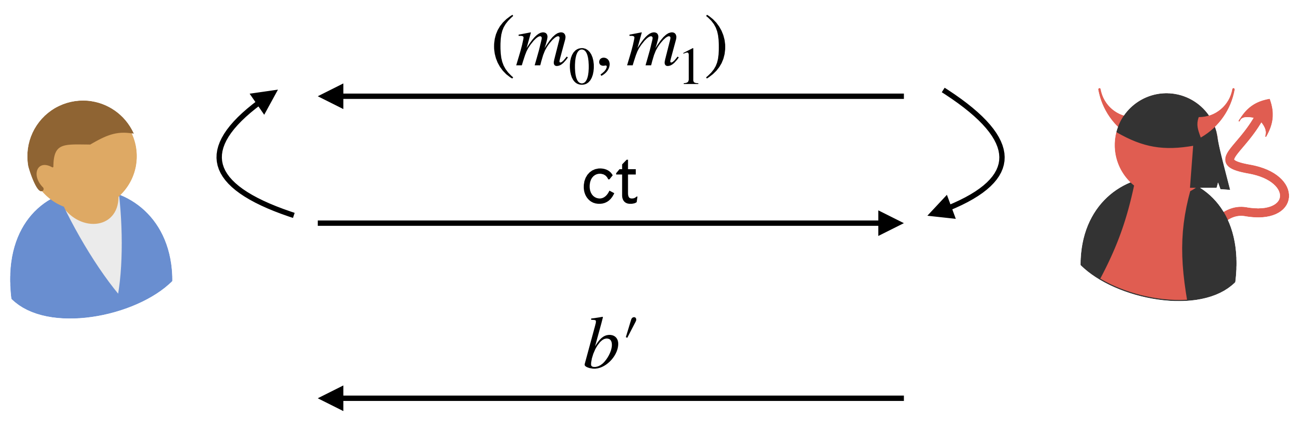

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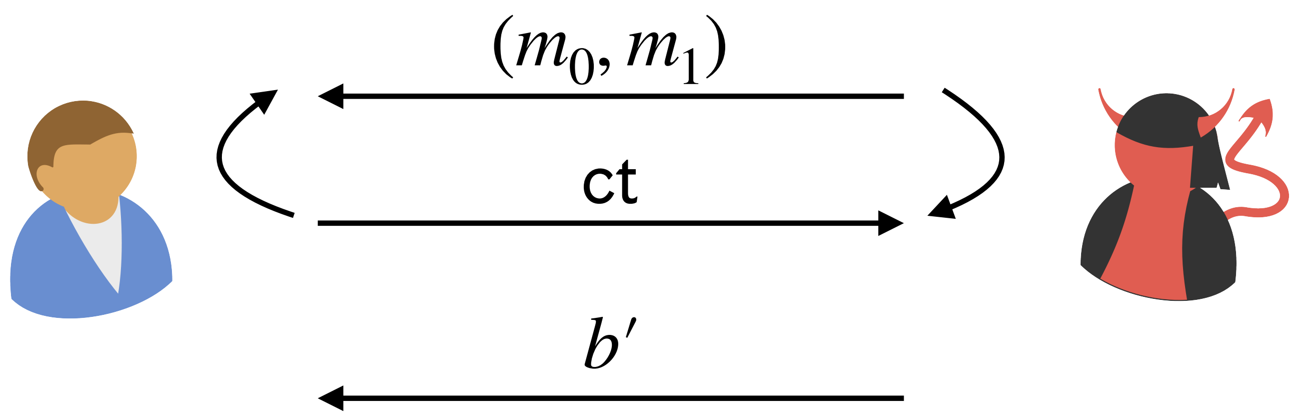
$\mathsf{ct} := (F_k(x) \oplus m_0, x)$



H_1

$T = \{\}$

$x \xleftarrow{\$} \{0,1\}^\lambda$

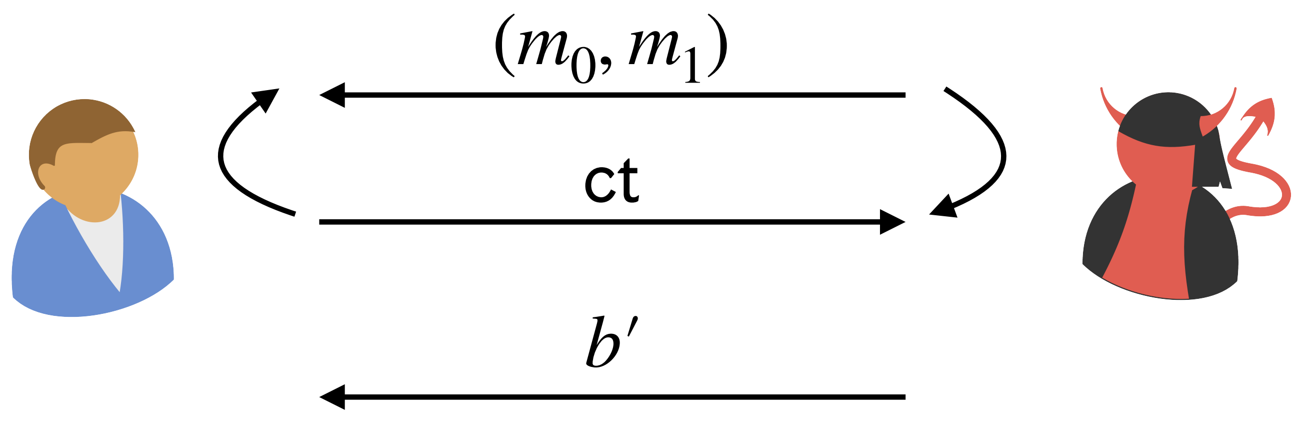
if $x \notin T$

$\quad r \xleftarrow{\$} \{0,1\}^\lambda$

$\quad T[x] = r$

$y = T[x]$

$\mathsf{ct} := (y \oplus m_0, x)$



Proof of Security

H_0

$k \xleftarrow{\$} \{0,1\}^\lambda$

$x \xleftarrow{\$} \{0,1\}^\lambda$

ct $:= (F_k(x) \oplus m_0, x)$

H_1

$T = \{\}$

$x \xleftarrow{\$} \{0,1\}^\lambda$

if $x \notin T$

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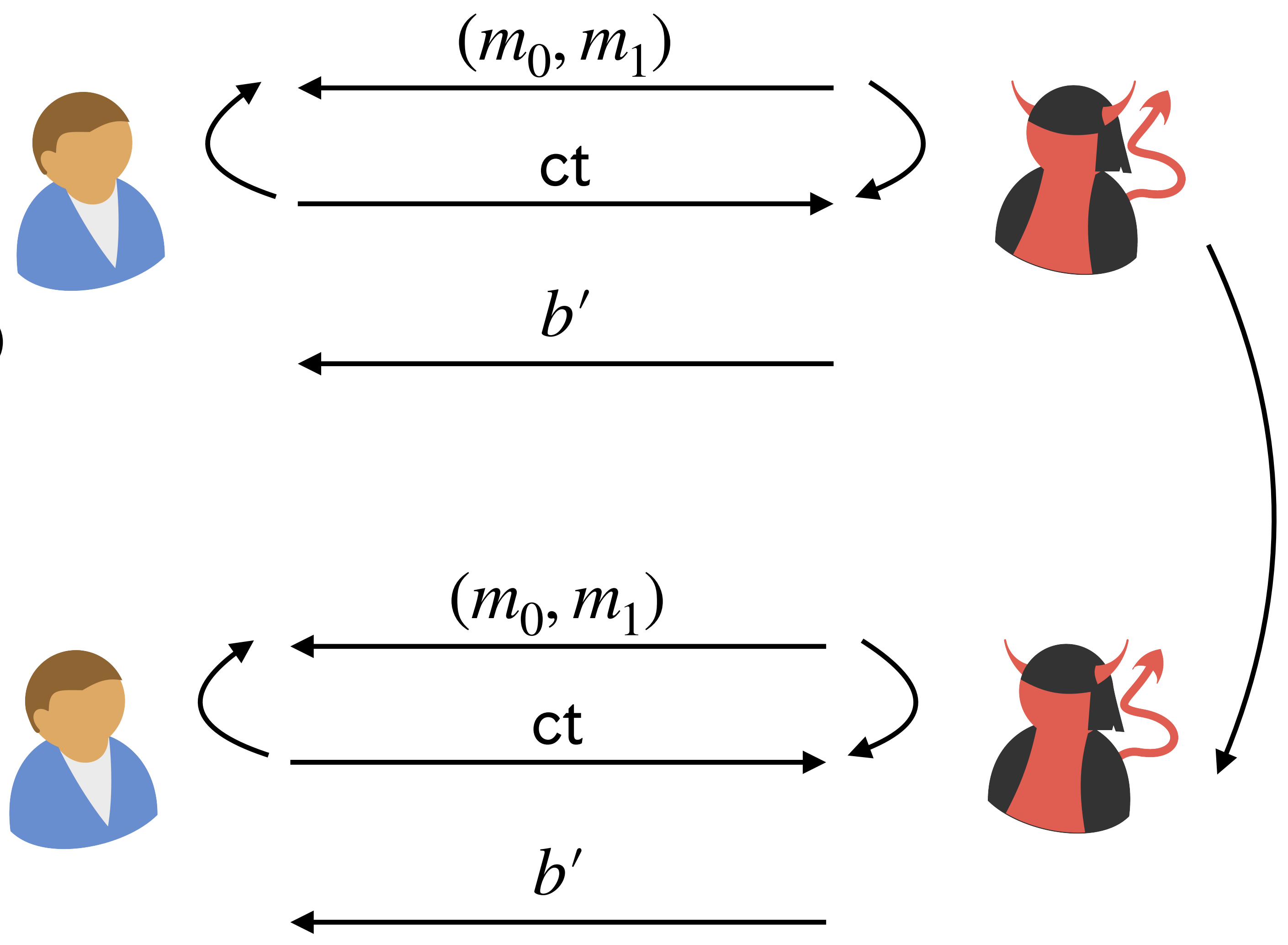
ct $:= (y \oplus m_0, x)$



Prove $H_0 \overset{c}{\approx} H_1$ via a reduction to the security of F

KeyGen(1^λ) : $k \xleftarrow{\$} \{0,1\}^\lambda$

Enc(k, m) : $x \xleftarrow{\$} \{0,1\}^\lambda$
ct $:= (F_k(x) \oplus m, x)$

Dec$(k, (c, x))$: $m' := F_k(x) \oplus c$

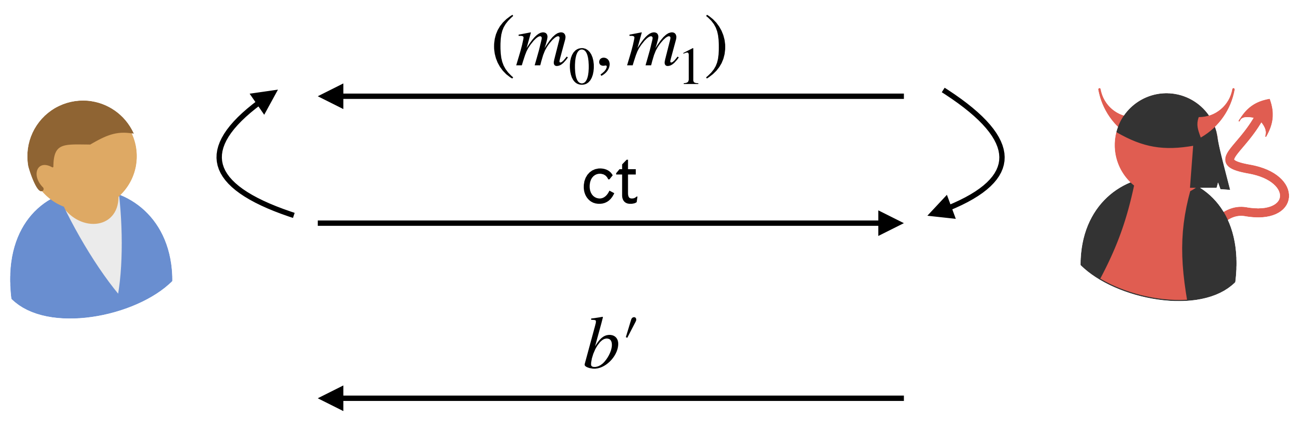
Proof of Security

H_0

$k \xleftarrow{\$} \{0,1\}^\lambda$

$x \xleftarrow{\$} \{0,1\}^\lambda$

ct $:= (F_k(x) \oplus m_0, x)$



H_1

$T = \{\}$

$x \xleftarrow{\$} \{0,1\}^\lambda$

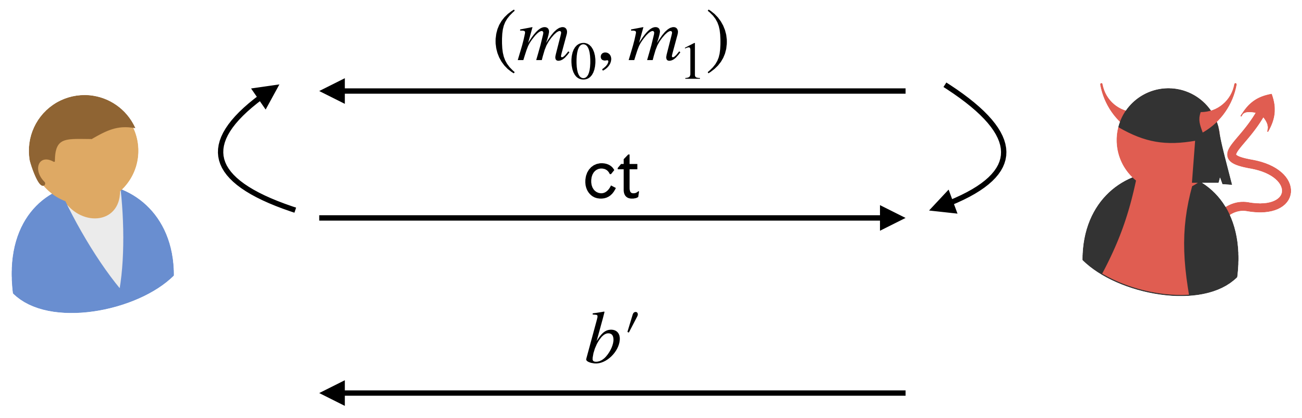
if $x \notin T$

$r \xleftarrow{\$} \{0,1\}^\lambda$

$T[x] = r$

$y = T[x]$

ct $:= (y \oplus m_0, x)$



Prove $H_0 \overset{c}{\approx} H_1$ via a reduction to the security of F

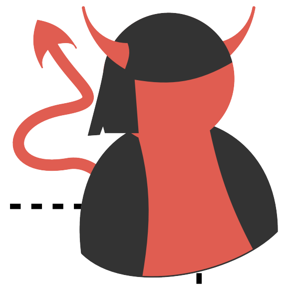
Ch_F

KeyGen(1^λ) : $k \xleftarrow{\$} \{0,1\}^\lambda$

Enc(k, m) : $x \xleftarrow{\$} \{0,1\}^\lambda$
ct $:= (F_k(x) \oplus m, x)$

Dec$(k, (c, x))$: $m' := F_k(x) \oplus c$

A_F



A_{H_0,H_1}

Proof of Security

$\mathsf{KeyGen}(1^\lambda) : k \xleftarrow{\$} \{0,1\}^\lambda$

$\mathsf{Enc}(k, m) :$ $x \xleftarrow{\$} \{0,1\}^\lambda$, $\mathsf{ct} := (F_k(x) \oplus m, x)$

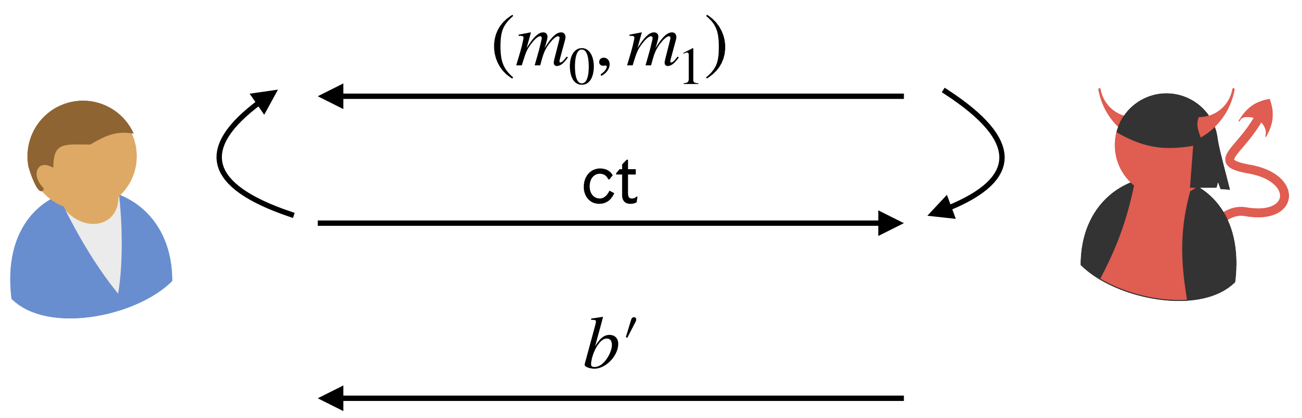
$\mathsf{Dec}(k, (c, x)) : m' := F_k(x) \oplus c$

H_0

$k \xleftarrow{\$} \{0,1\}^\lambda$

$x \xleftarrow{\$} \{0,1\}^\lambda$

$\mathsf{ct} := (F_k(x) \oplus m_0, x)$



Prove $H_0 \overset{c}{\approx} H_1$ via a reduction to the security of F

H_1

$T = \{\}$

$x \xleftarrow{\$} \{0,1\}^\lambda$

if $x \notin T$

$r \xleftarrow{\$} \{0,1\}^\lambda$

$T[x] = r$

$y = T[x]$

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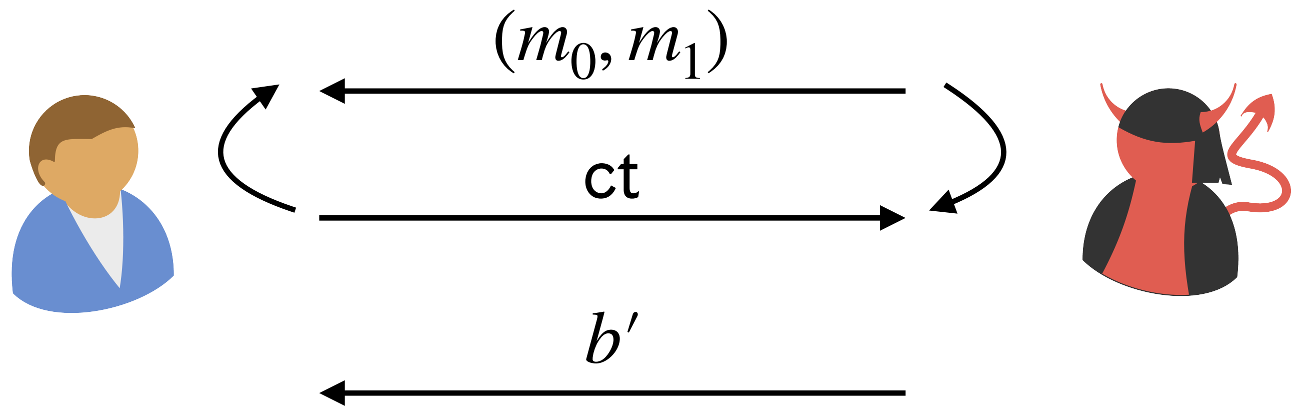
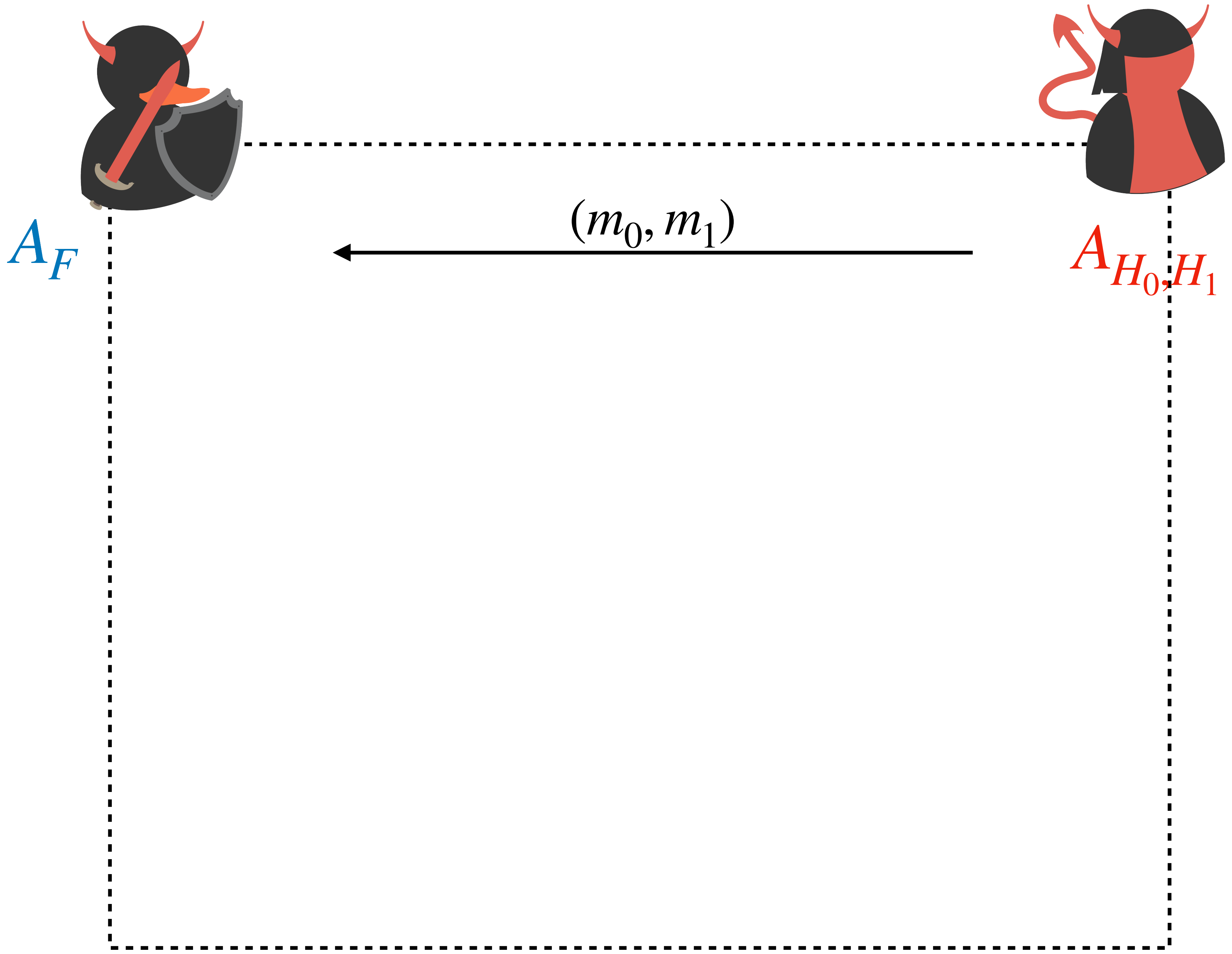


Ch_F



Proof of Security

H_0

$k \xleftarrow{\$} \{0,1\}^\lambda$

$x \xleftarrow{\$} \{0,1\}^\lambda$

ct $:= (F_k(x) \oplus m_0, x)$

(m_0, m_1)

ct

b'

H_1

$T = \{\}$

$x \xleftarrow{\$} \{0,1\}^\lambda$

if $x \notin T$

$r \xleftarrow{\$} \{0,1\}^\lambda$

$T[x] = r$

$y = T[x]$

ct $:= (y \oplus m_0, x)$

(m_0, m_1)

ct

b'

Prove $H_0 \overset{c}{\approx} H_1$ via a reduction to the security of F

Ch_F

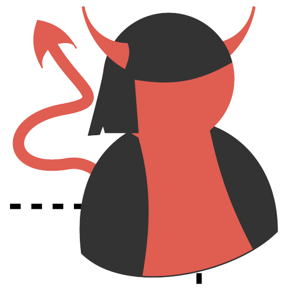
KeyGen(1^λ) : $k \xleftarrow{\$} \{0,1\}^\lambda$

Enc(k, m) : $x \xleftarrow{\$} \{0,1\}^\lambda$

ct $:= (F_k(x) \oplus m, x)$

Dec$(k, (c, x))$: $m' := F_k(x) \oplus c$

A_F



A_{H_0,H_1}

(m_0, m_1)

$x \xleftarrow{\$} \{0,1\}^\lambda$

Proof of Security

$\mathsf{KeyGen}(1^\lambda) : k \xleftarrow{\$} \{0,1\}^\lambda$

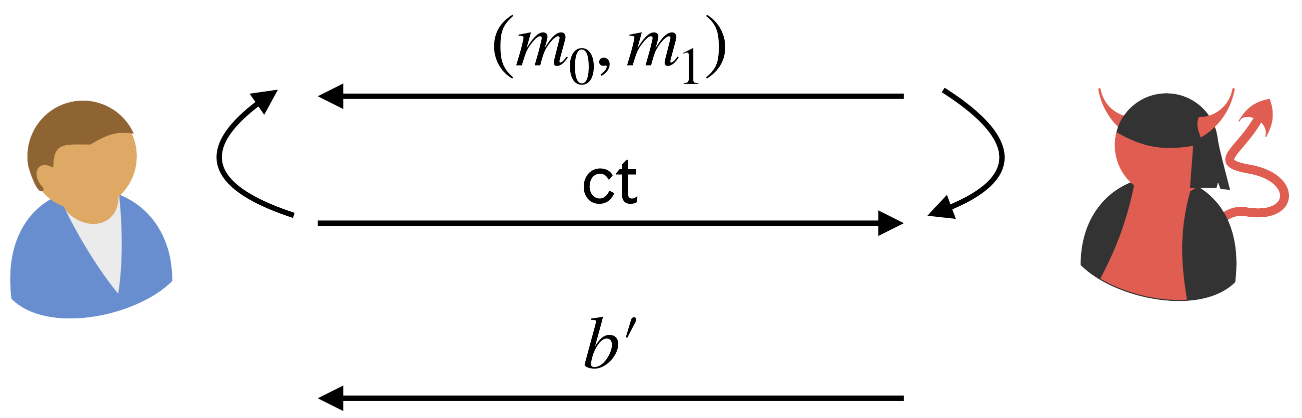
$\mathsf{Enc}(\mathsf{k}, \mathsf{m})$: $x \xleftarrow{\$} \{0,1\}^\lambda$
$\mathsf{ct} := (F_k(x) \oplus m, x)$

$\mathsf{Dec}(k, (c, x)) : m' := F_k(x) \oplus c$

H_0

$k \xleftarrow{\$} \{0,1\}^\lambda$

$x \xleftarrow{\$} \{0,1\}^\lambda$
$\mathsf{ct} := (F_k(x) \oplus m_0, x)$



H_1

$T = \{\}$
$x \xleftarrow{\$} \{0,1\}^\lambda$

if $x \notin T$
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$y = T[x]$

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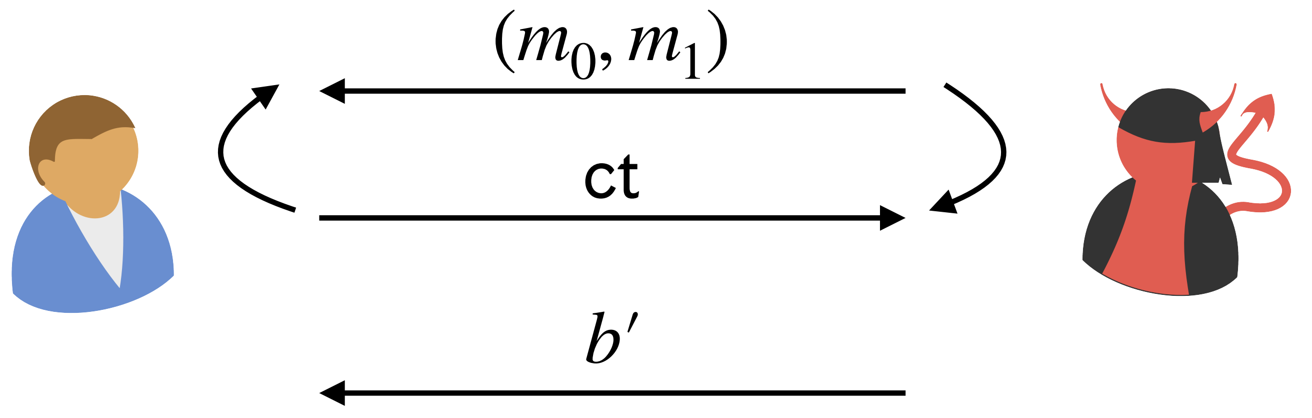
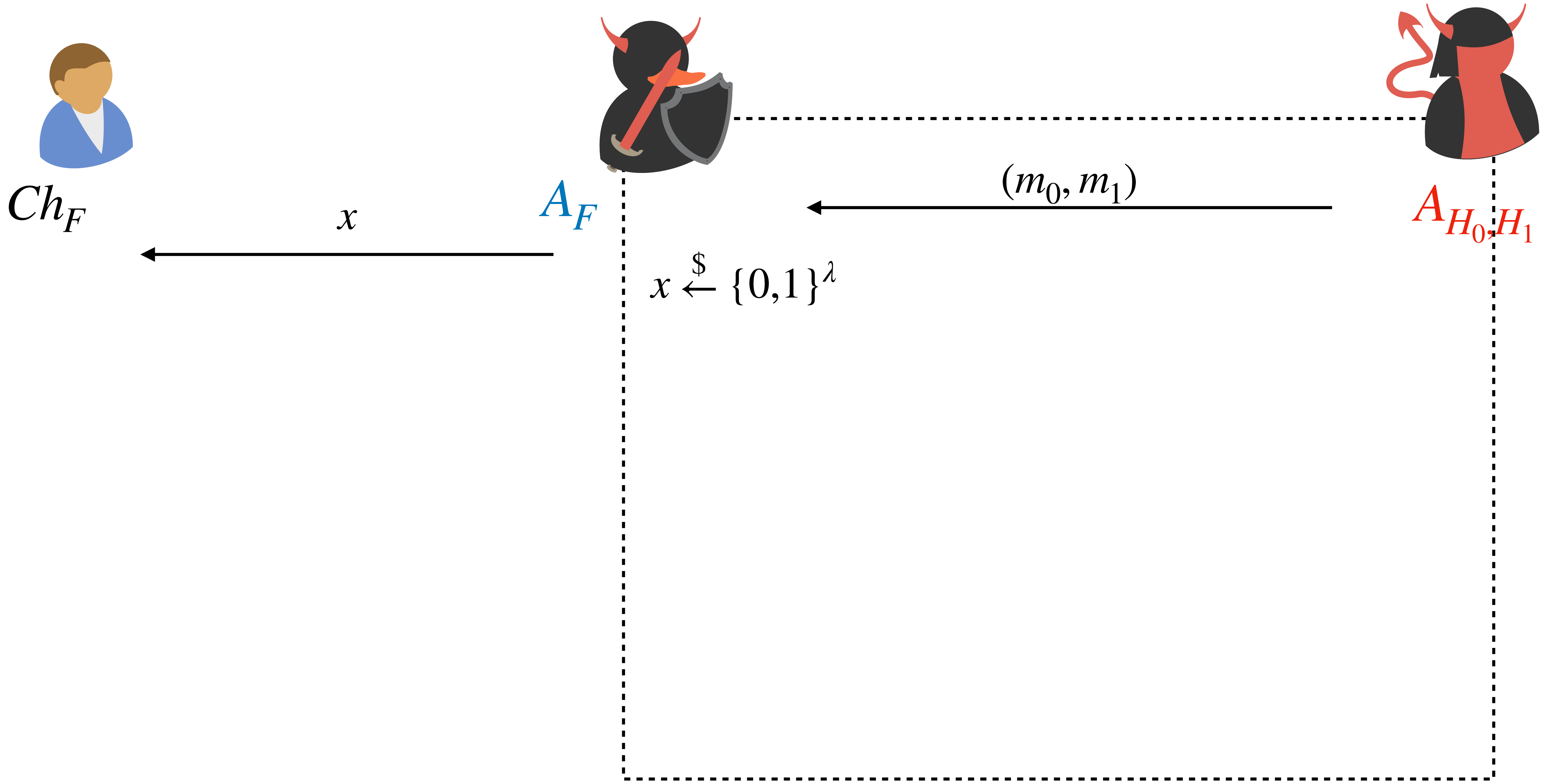


Prove $H_0 \overset{c}{\approx} H_1$ via a reduction to the security of F



Proof of Security

KeyGen(1^λ) : $k \xleftarrow{\$} \{0,1\}^\lambda$

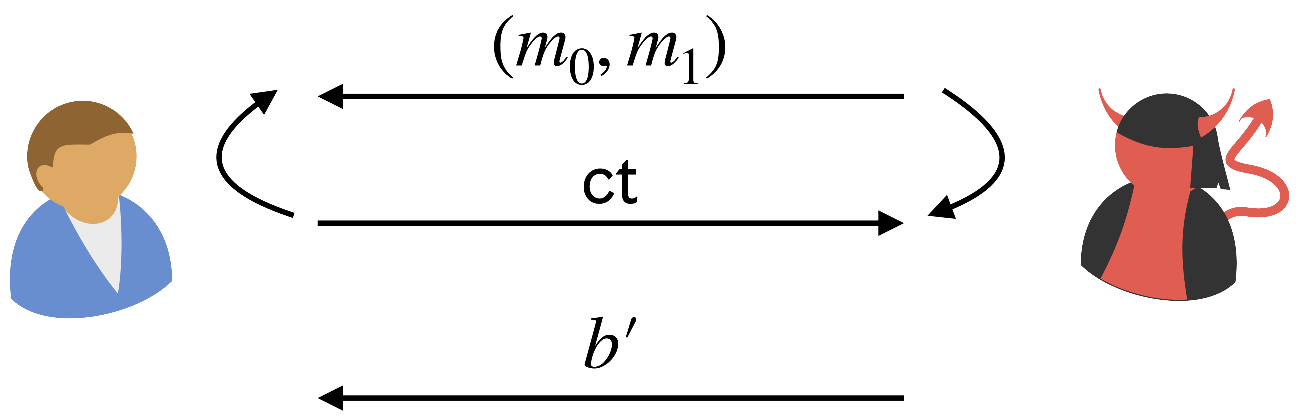
Enc(k, m) : $x \xleftarrow{\$} \{0,1\}^\lambda$
ct $:= (F_k(x) \oplus m, x)$

Dec$(k, (c, x))$: $m' := F_k(x) \oplus c$

H_0

$k \xleftarrow{\$} \{0,1\}^\lambda$

$x \xleftarrow{\$} \{0,1\}^\lambda$
ct $:= (F_k(x) \oplus m_0, x)$



Prove $H_0 \overset{c}{\approx} H_1$ via a reduction to the security of F

H_1

$T = \{\}$
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if $x \notin T$
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$T[x] = r$
$y = T[x]$

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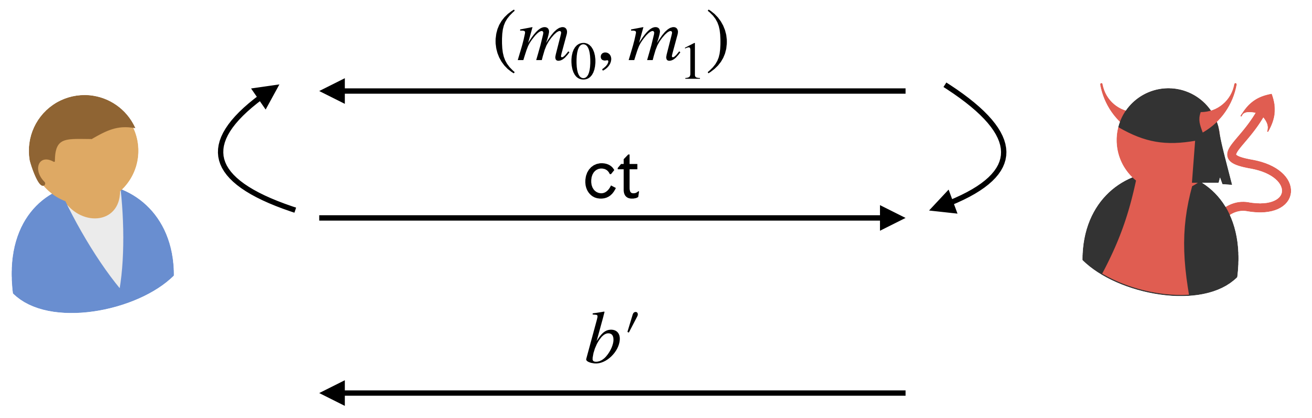
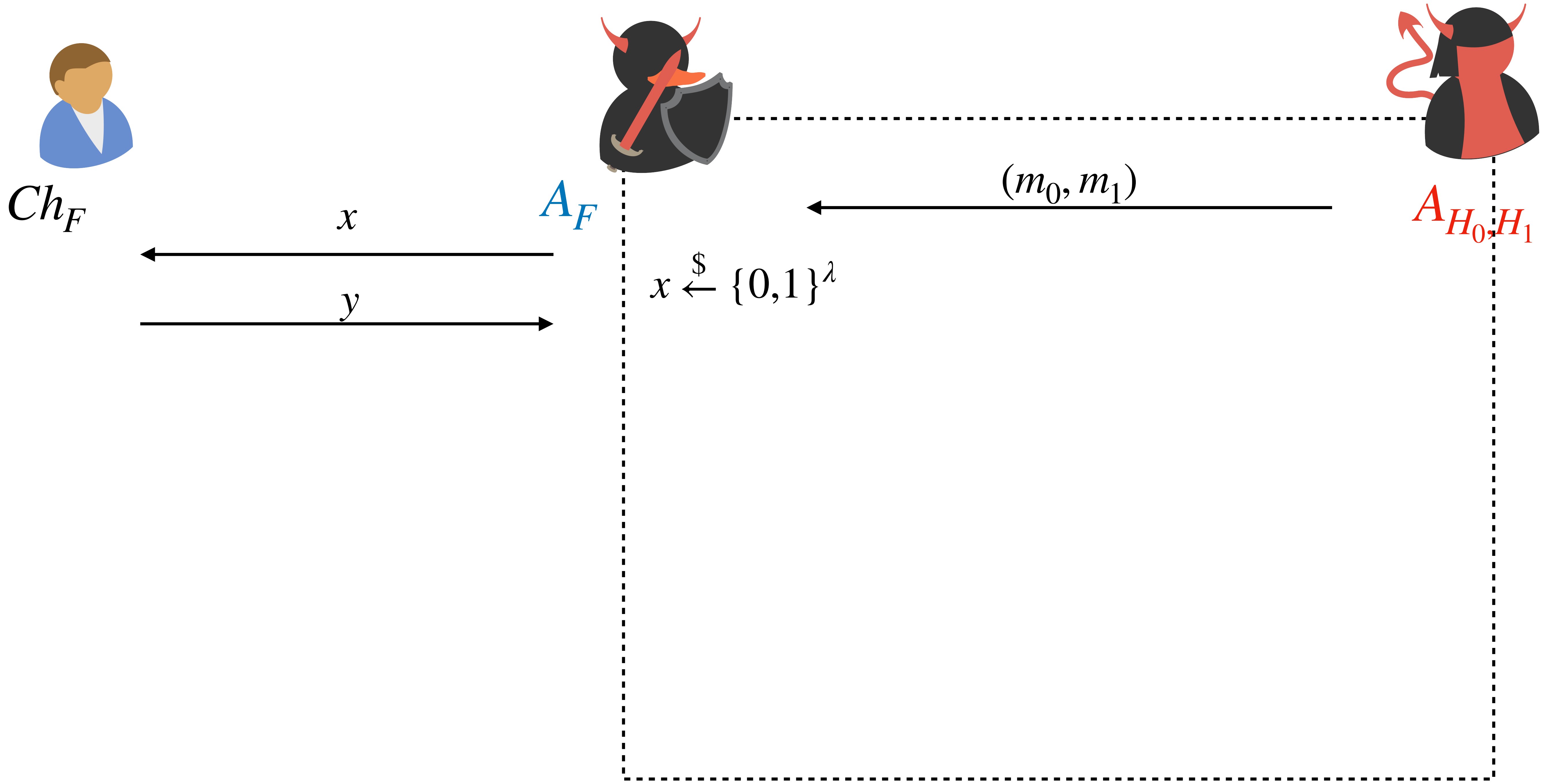




Proof of Security

$\mathsf{KeyGen}(1^\lambda) : k \xleftarrow{\$} \{0,1\}^\lambda$

$\mathsf{Enc}(\mathsf{k}, \mathsf{m}) :$ $x \xleftarrow{\$} \{0,1\}^\lambda$
$\mathsf{ct} := (F_k(x) \oplus m, x)$

$\mathsf{Dec}(k, (c, x)) : m' := F_k(x) \oplus c$

H_0

$k \xleftarrow{\$} \{0,1\}^\lambda$

$x \xleftarrow{\$} \{0,1\}^\lambda$
$\mathsf{ct} := (F_k(x) \oplus m_0, x)$

(m_0, m_1)

ct

b'

H_1

$T = \{\}$
$x \xleftarrow{\$} \{0,1\}^\lambda$

if $x \notin T$
$r \xleftarrow{\$} \{0,1\}^\lambda$
$T[x] = r$
$y = T[x]$

$\mathsf{ct} := (y \oplus m_0, x)$

(m_0, m_1)

ct

b'

Prove $H_0 \overset{c}{\approx} H_1$ via a reduction to the security of F

Ch_F

x

y

A_F

A_{H_0,H_1}

(m_0, m_1)

$x \xleftarrow{\$} \{0,1\}^\lambda$

$(y \oplus m_0, x)$

Proof of Security

$\mathsf{KeyGen}(1^\lambda) : k \xleftarrow{\$} \{0,1\}^\lambda$

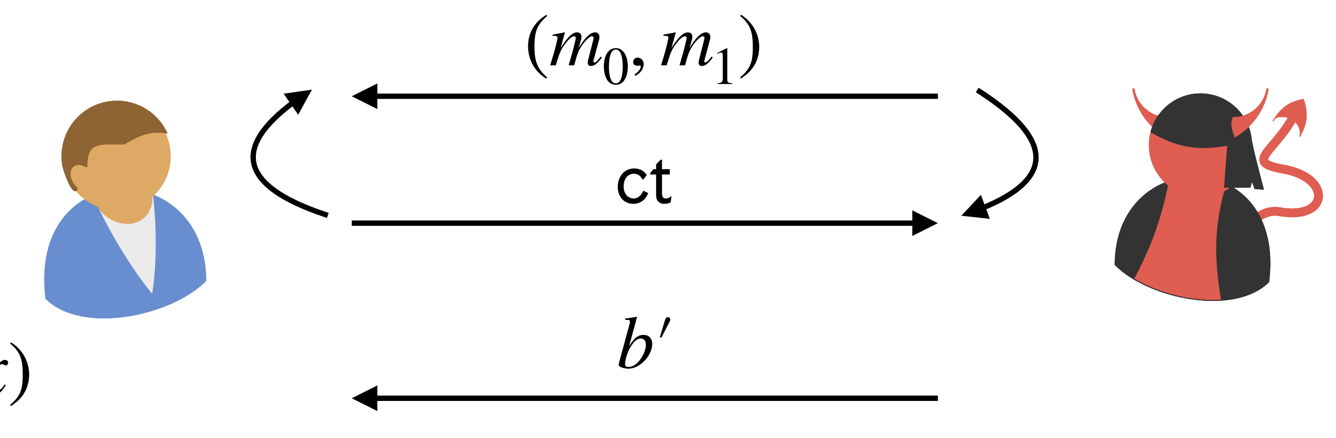
$\mathsf{Enc}(\mathsf{k}, \mathsf{m}) :$ $x \xleftarrow{\$} \{0,1\}^\lambda$
$\mathsf{ct} := (F_k(x) \oplus m, x)$

$\mathsf{Dec}(k, (c, x)) : m' := F_k(x) \oplus c$

H_0

$k \xleftarrow{\$} \{0,1\}^\lambda$

$x \xleftarrow{\$} \{0,1\}^\lambda$
$\mathsf{ct} := (F_k(x) \oplus m_0, x)$



H_1

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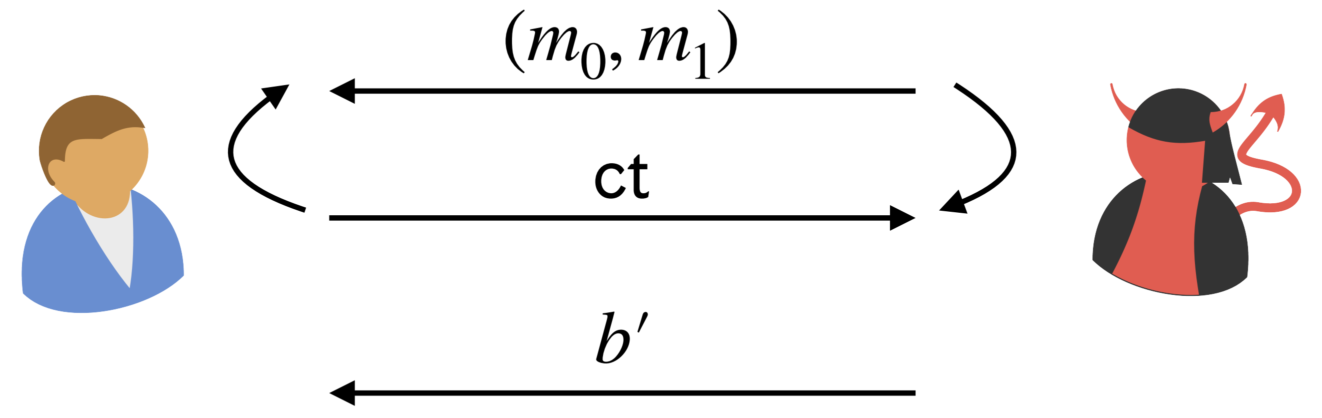
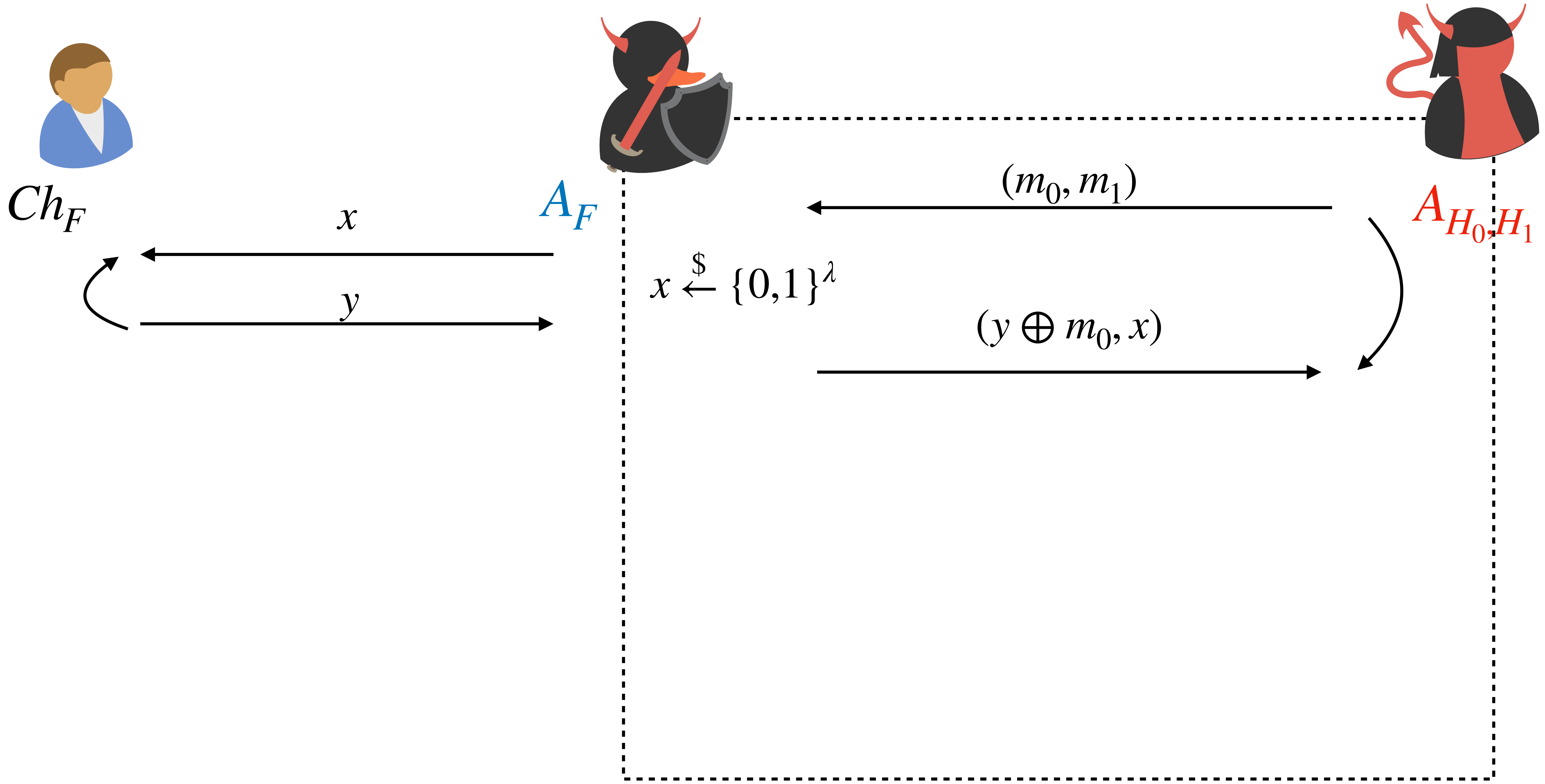


Prove $H_0 \overset{c}{\approx} H_1$ via a reduction to the security of F



Proof of Security

$$\mathsf{KeyGen}(1^\lambda) : k \xleftarrow{\$} \{0,1\}^\lambda$$

$$\mathsf{Enc}(k, m) : \begin{array}{l} x \xleftarrow{\$} \{0,1\}^\lambda \\ \mathsf{ct} := (F_k(x) \oplus m, x) \end{array}$$

$$\mathsf{Dec}(k, (c, x)) : m' := F_k(x) \oplus c$$

H_0

$k \xleftarrow{\$} \{0,1\}^\lambda$

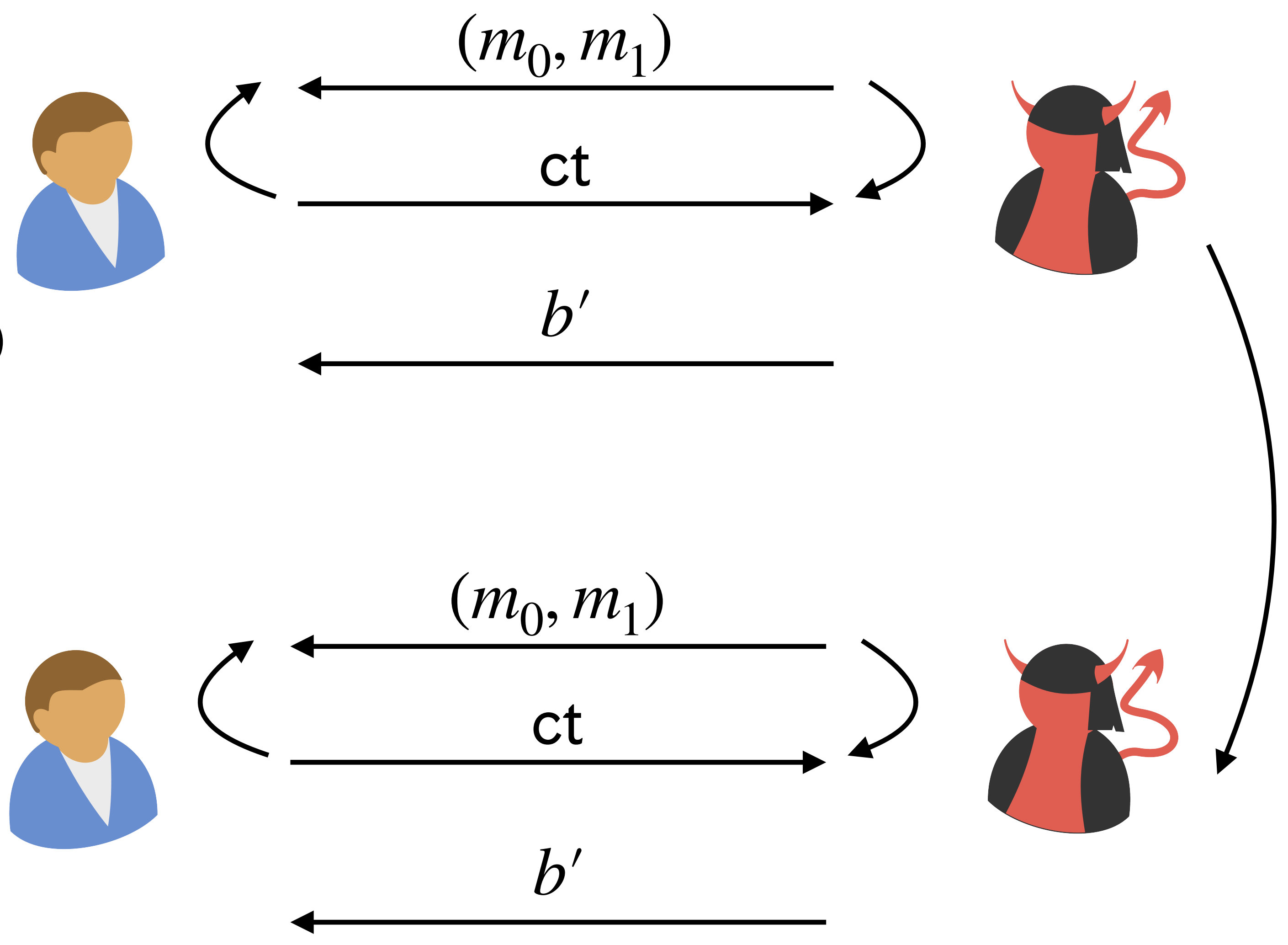
$x \xleftarrow{\$} \{0,1\}^\lambda$
ct $:= (F_k(x) \oplus m_0, x)$

H_1

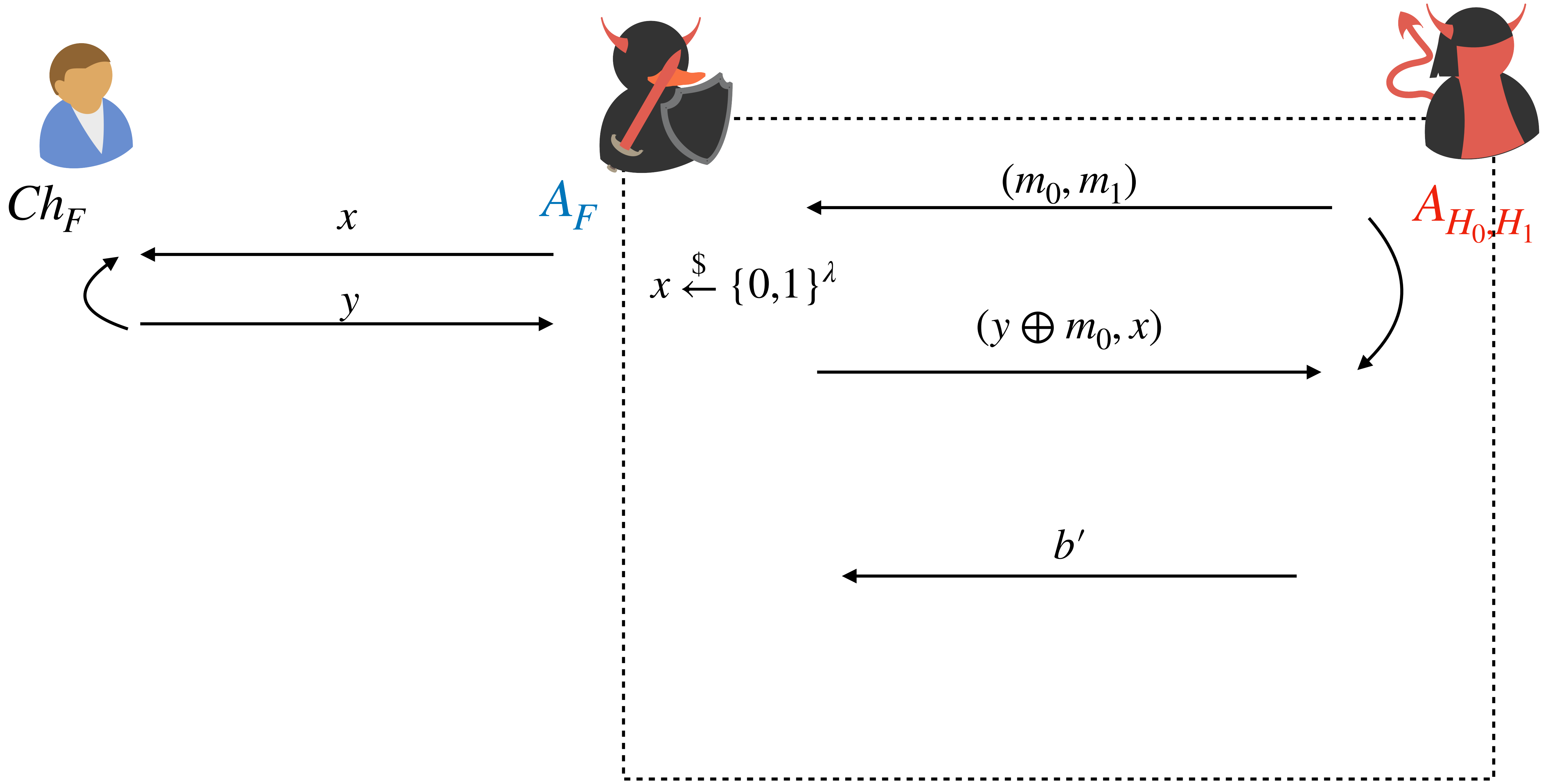
$T = \{\}$
$x \xleftarrow{\$} \{0,1\}^\lambda$

if $x \notin T$
  $r \xleftarrow{\$} \{0,1\}^\lambda$
  $T[x] = r$
$y = T[x]$

ct $:= (y \oplus m_0, x)$



Prove $H_0 \overset{c}{\approx} H_1$ via a reduction to the security of F



Proof of Security

$\mathsf{KeyGen}(1^\lambda) : k \xleftarrow{\$} \{0,1\}^\lambda$

$\mathsf{Enc}(k, m) :$ $x \xleftarrow{\$} \{0,1\}^\lambda$, $\mathsf{ct} := (F_k(x) \oplus m, x)$

$\mathsf{Dec}(k, (c, x)) : m' := F_k(x) \oplus c$

H_0

$k \xleftarrow{\$} \{0,1\}^\lambda$

$x \xleftarrow{\$} \{0,1\}^\lambda$

$\mathsf{ct} := (F_k(x) \oplus m_0, x)$

(m_0, m_1)

ct

b'

H_1

$T = \{\}$

$x \xleftarrow{\$} \{0,1\}^\lambda$

if $x \notin T$

$r \xleftarrow{\$} \{0,1\}^\lambda$

$T[x] = r$

$y = T[x]$

$\mathsf{ct} := (y \oplus m_0, x)$

(m_0, m_1)

ct

b'

Prove $H_0 \overset{c}{\approx} H_1$ via a reduction to the security of F

Ch_F

A_F

A_{H_0,H_1}

x

y

(m_0, m_1)

$x \xleftarrow{\$} \{0,1\}^\lambda$

$(y \oplus m_0, x)$

b'

b'

Proof of Security

KeyGen(1^λ) : $k \xleftarrow{\$} \{0,1\}^\lambda$

Enc(k, m) : $x \xleftarrow{\$} \{0,1\}^\lambda$
ct $:= (F_k(x) \oplus m, x)$

Dec$(k, (c, x))$: $m' := F_k(x) \oplus c$

H_0

$k \xleftarrow{\$} \{0,1\}^\lambda$

$x \xleftarrow{\$} \{0,1\}^\lambda$

ct $:= (F_k(x) \oplus m_0, x)$

(m_0, m_1)

ct

b'

H_1

$T = \{\}$

$x \xleftarrow{\$} \{0,1\}^\lambda$

if $x \notin T$

$r \xleftarrow{\$} \{0,1\}^\lambda$

$T[x] = r$

$y = T[x]$

ct $:= (y \oplus m_0, x)$

(m_0, m_1)

ct

b'

Prove $H_0 \overset{c}{\approx} H_1$ via a reduction to the security of F

Ch_F

A_F

A_{H_0,H_1}

x

y

b'

(m_0, m_1)

$x \xleftarrow{\$} \{0,1\}^\lambda$

$(y \oplus m_0, x)$

b'

<u>Input Mapping</u>

Proof of Security

$\mathsf{KeyGen}(1^\lambda) : k \xleftarrow{\$} \{0,1\}^\lambda$

$\mathsf{Enc}(\mathsf{k}, \mathsf{m}) :$ $x \xleftarrow{\$} \{0,1\}^\lambda$, $\mathsf{ct} := (F_k(x) \oplus m, x)$

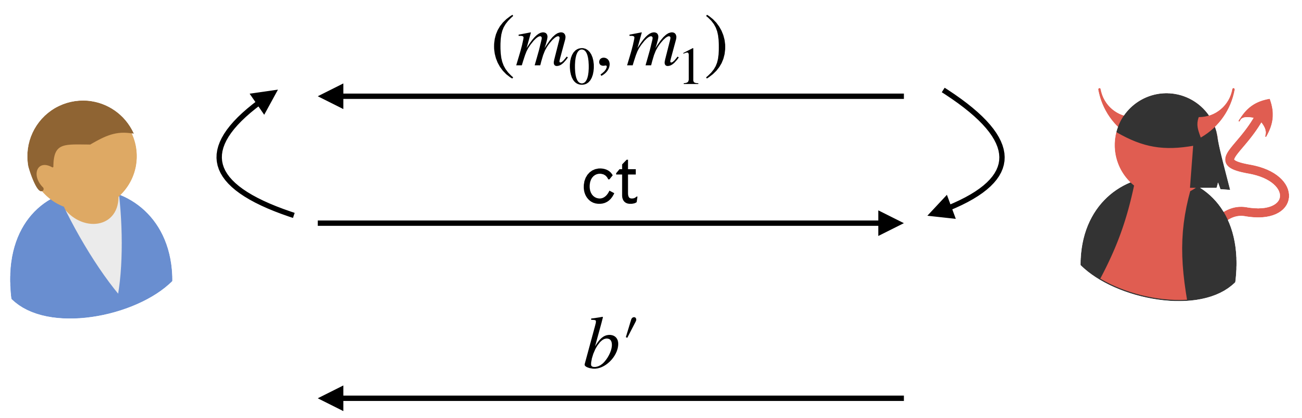
$\mathsf{Dec}(k, (c, x)) : m' := F_k(x) \oplus c$

H_0

$k \xleftarrow{\$} \{0,1\}^\lambda$

$x \xleftarrow{\$} \{0,1\}^\lambda$

$\mathsf{ct} := (F_k(x) \oplus m_0, x)$



H_1

$T = \{\}$

$x \xleftarrow{\$} \{0,1\}^\lambda$

if $x \notin T$

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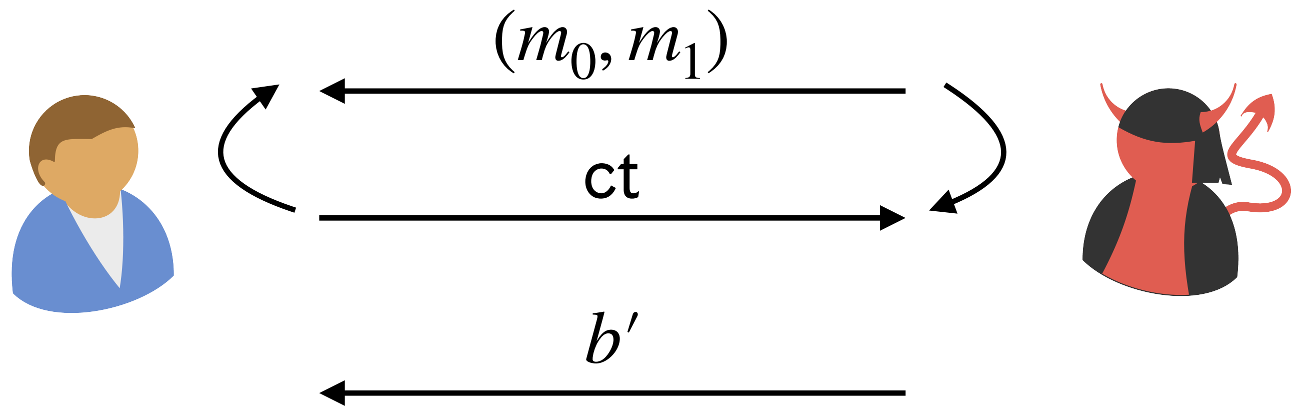
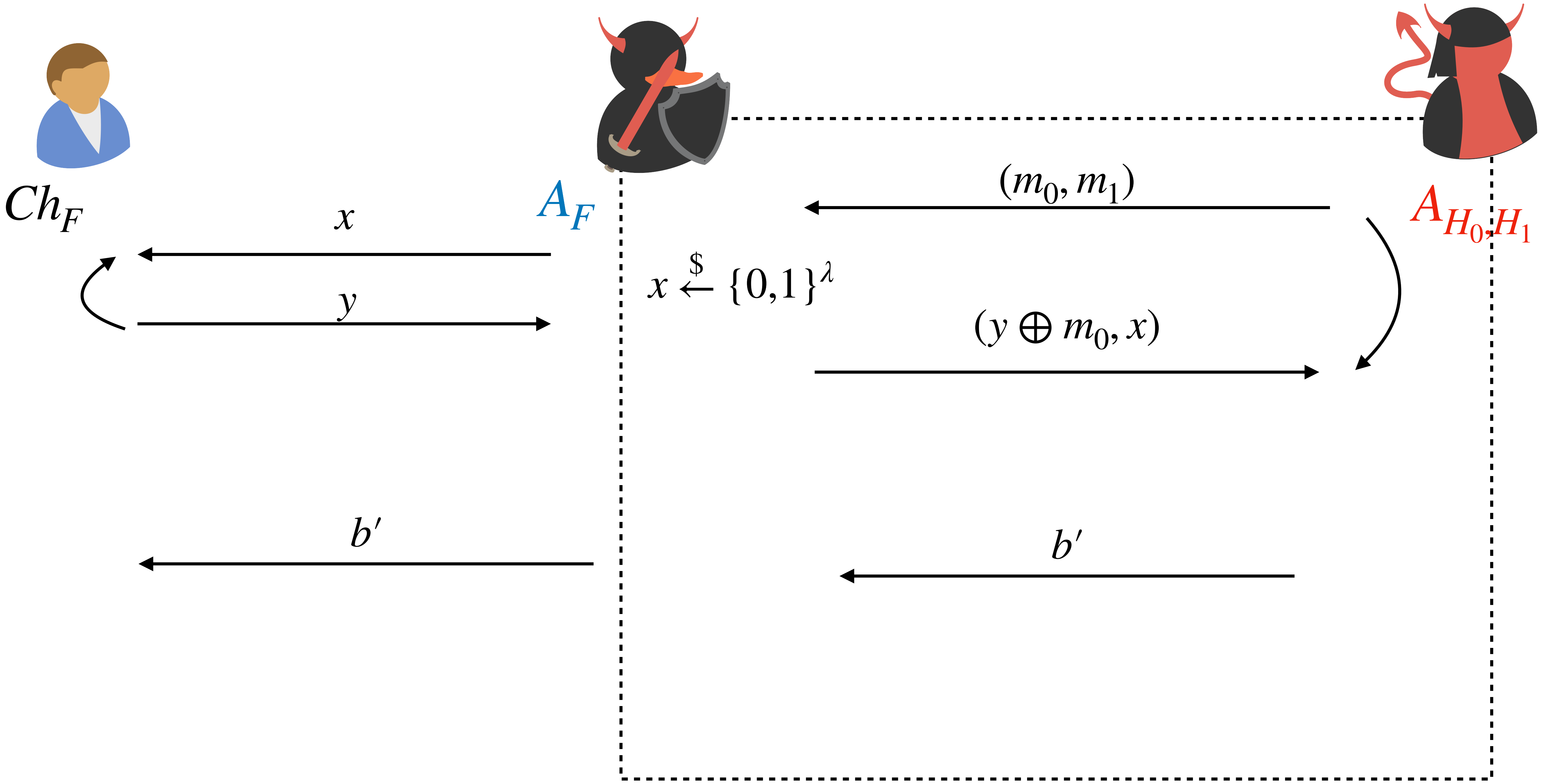
$\quad T[x] = r$

$y = T[x]$

$\mathsf{ct} := (y \oplus m_0, x)$



Prove $H_0 \overset{c}{\approx} H_1$ via a reduction to the security of F



<u>Input Mapping</u>

$b = 0$: $(y \oplus m_0, x) = (F_k(x) \oplus m_0, x)$

Proof of Security

$\mathsf{KeyGen}(1^\lambda) : k \xleftarrow{\$} \{0,1\}^\lambda$

$\mathsf{Enc}(k, m) :$ $x \xleftarrow{\$} \{0,1\}^\lambda$; $\mathsf{ct} := (F_k(x) \oplus m, x)$

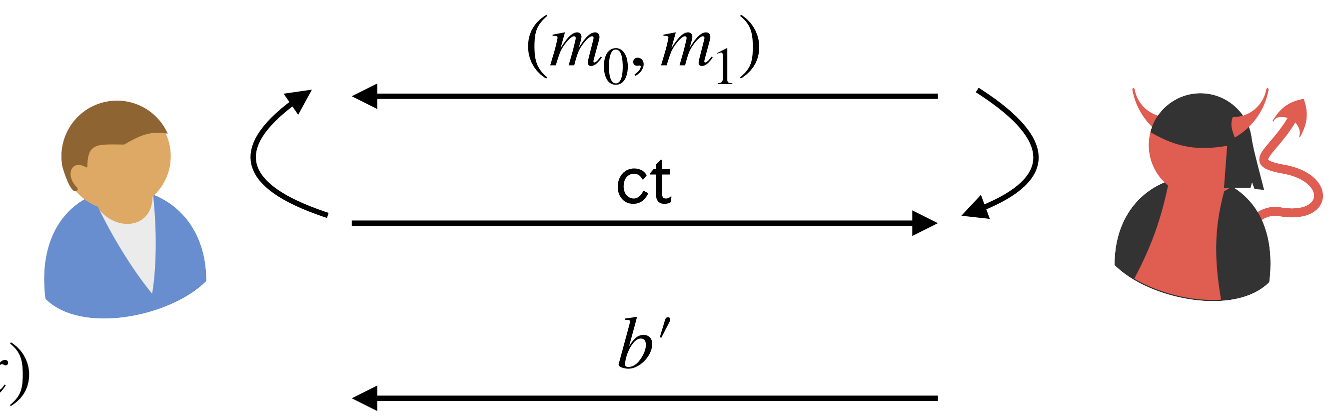
$\mathsf{Dec}(k, (c, x)) : m' := F_k(x) \oplus c$

H_0

$k \xleftarrow{\$} \{0,1\}^\lambda$

$x \xleftarrow{\$} \{0,1\}^\lambda$

$\mathsf{ct} := (F_k(x) \oplus m_0, x)$



H_1

$T = \{\}$

$x \xleftarrow{\$} \{0,1\}^\lambda$

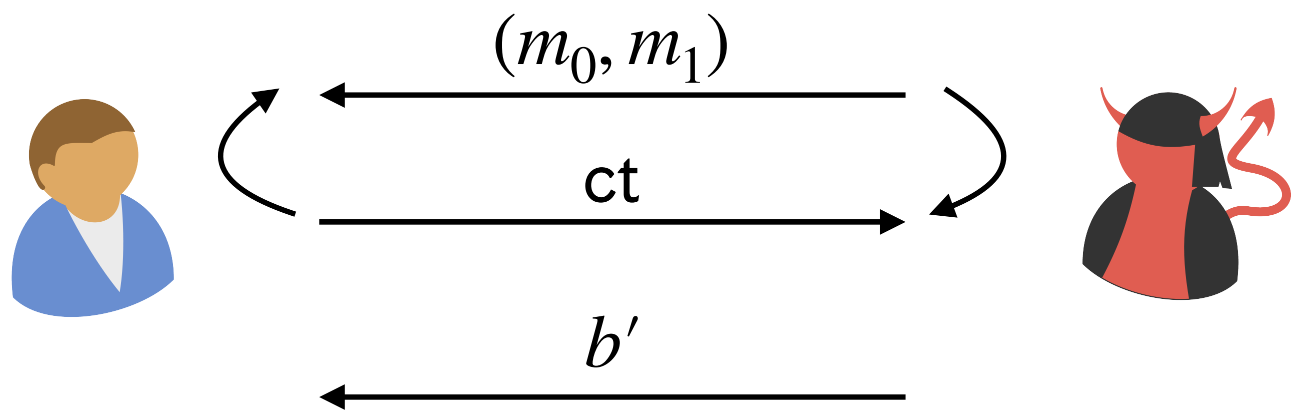
if $x \notin T$

$r \xleftarrow{\$} \{0,1\}^\lambda$

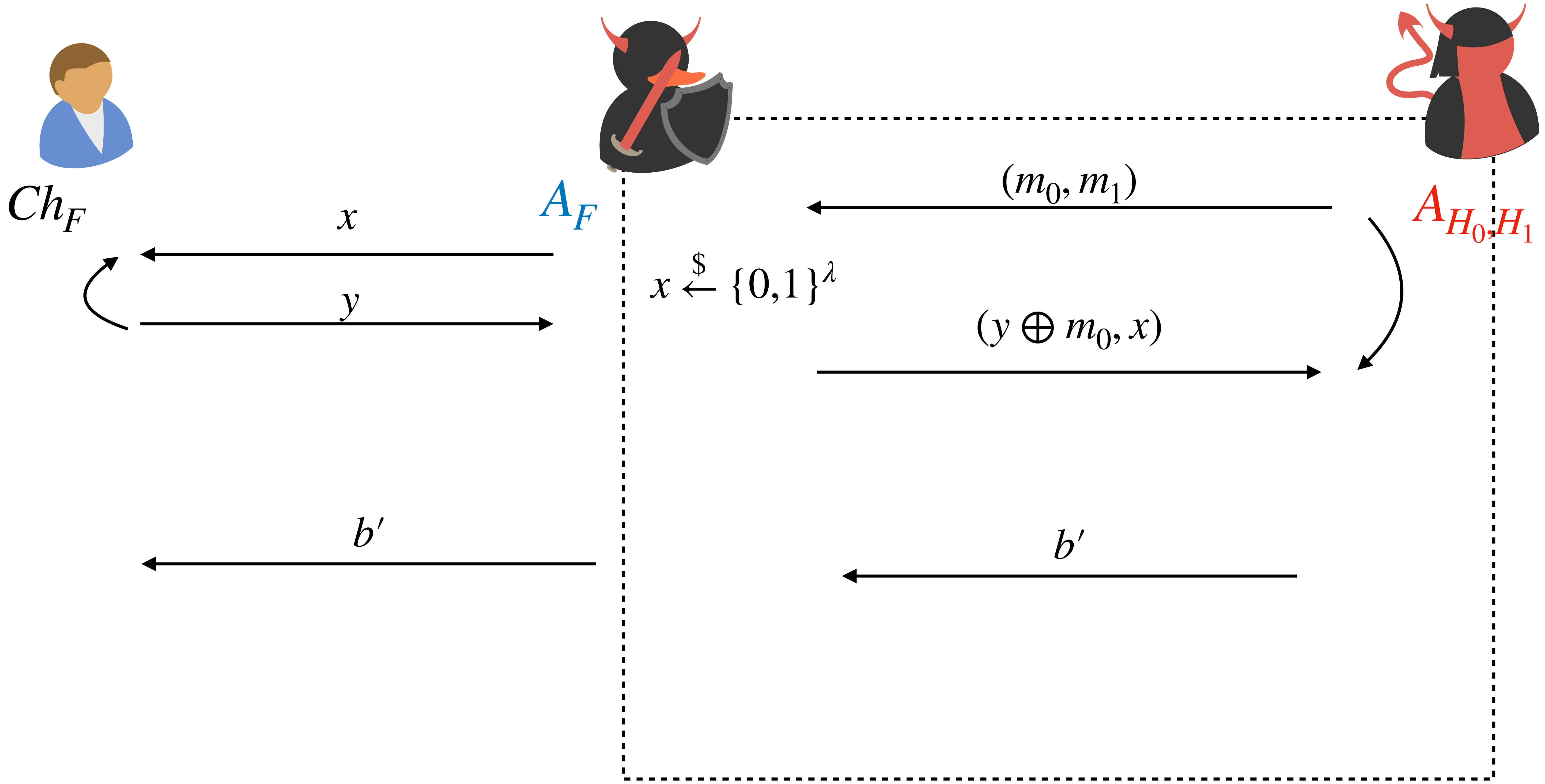
$T[x] = r$

$y = T[x]$

$\mathsf{ct} := (y \oplus m_0, x)$



Prove $H_0 \overset{c}{\approx} H_1$ via a reduction to the security of F



<u>Input Mapping</u>

$b = 0$: $(y \oplus m_0, x) = (F_k(x) \oplus m_0, x)$

$b = 1$: $(y \oplus m_0, x) = (T[x] \oplus m_0, x)$

Proof of Security

$\mathsf{KeyGen}(1^\lambda) : k \xleftarrow{\$} \{0,1\}^\lambda$

$\mathsf{Enc}(k, m) :$ $x \xleftarrow{\$} \{0,1\}^\lambda$, $\mathsf{ct} := (F_k(x) \oplus m, x)$

$\mathsf{Dec}(k, (c, x)) : m' := F_k(x) \oplus c$

H_0

$k \xleftarrow{\$} \{0,1\}^\lambda$

$x \xleftarrow{\$} \{0,1\}^\lambda$

$\mathsf{ct} := (F_k(x) \oplus m_0, x)$

(m_0, m_1)

ct

b'

H_1

$T = \{\}$

$x \xleftarrow{\$} \{0,1\}^\lambda$

if $x \notin T$

$r \xleftarrow{\$} \{0,1\}^\lambda$

$T[x] = r$

$y = T[x]$

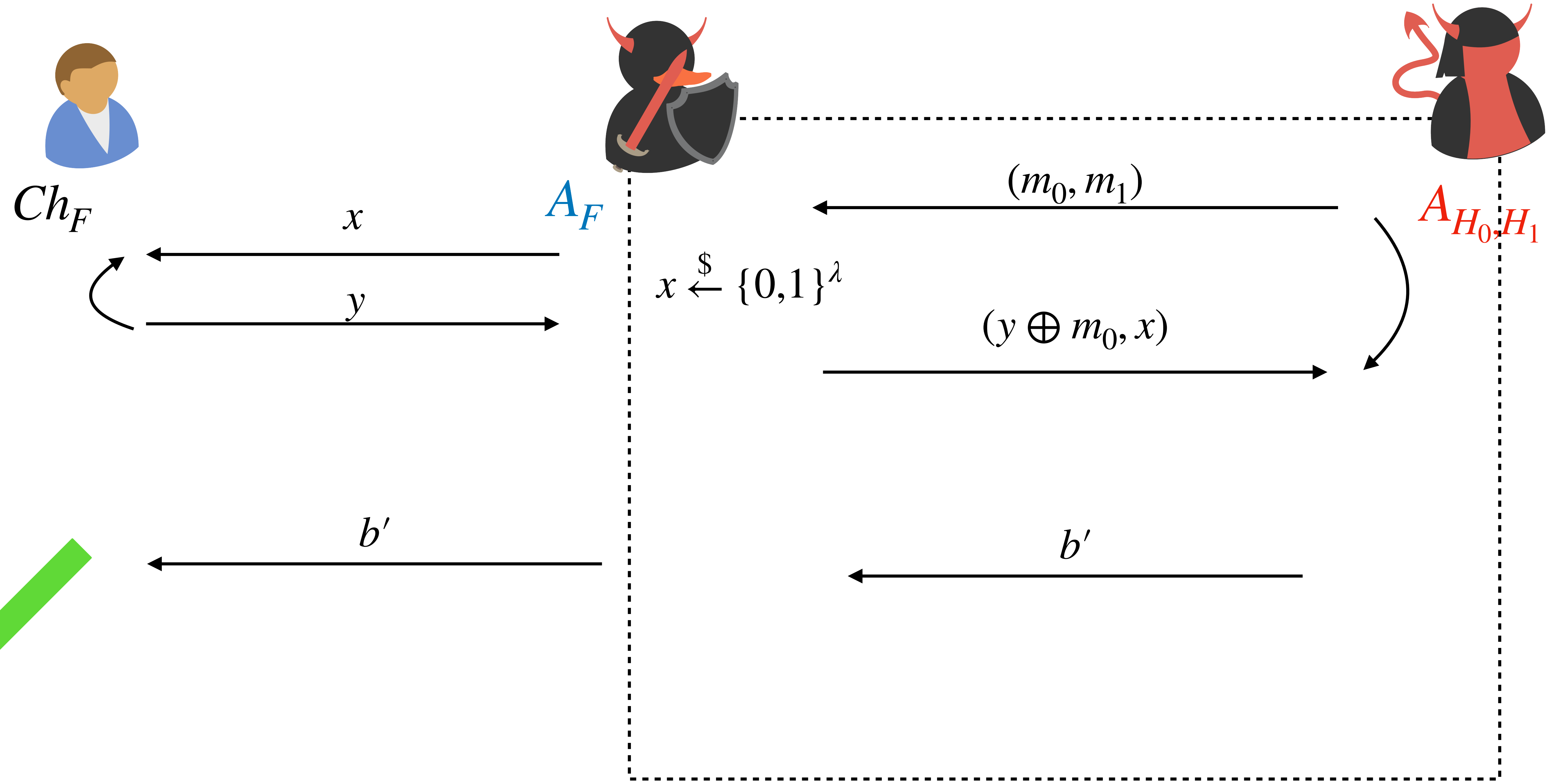
$\mathsf{ct} := (y \oplus m_0, x)$

(m_0, m_1)

ct

b'

Prove $H_0 \overset{c}{\approx} H_1$ via a reduction to the security of F



Input Mapping

$b = 0$: $(y \oplus m_0, x) = (F_k(x) \oplus m_0, x)$ ✓

$b = 1$: $(y \oplus m_0, x) = (T[x] \oplus m_0, x)$

Proof of Security

H_1

$T = \{\}$

$x \xleftarrow{\$} \{0,1\}^\lambda$

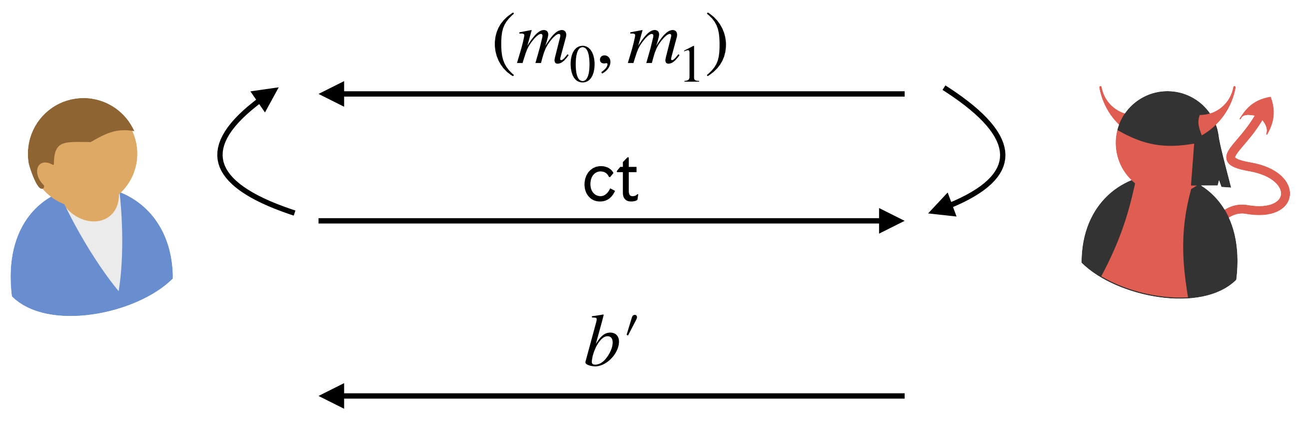
if $x \notin T$

$\quad r \xleftarrow{\$} \{0,1\}^\lambda$

$\quad T[x] = r$

$y = T[x]$

ct $:= (y \oplus m_0, x)$



KeyGen(1^λ) : $k \xleftarrow{\$} \{0,1\}^\lambda$

Enc(k, m) : $x \xleftarrow{\$} \{0,1\}^\lambda$

ct $:= (F_k(x) \oplus m, x)$

Dec$(k, (c, x))$: $m' := F_k(x) \oplus c$

Proof of Security

H_1

$T = \{\}$

$x \xleftarrow{\$} \{0,1\}^\lambda$

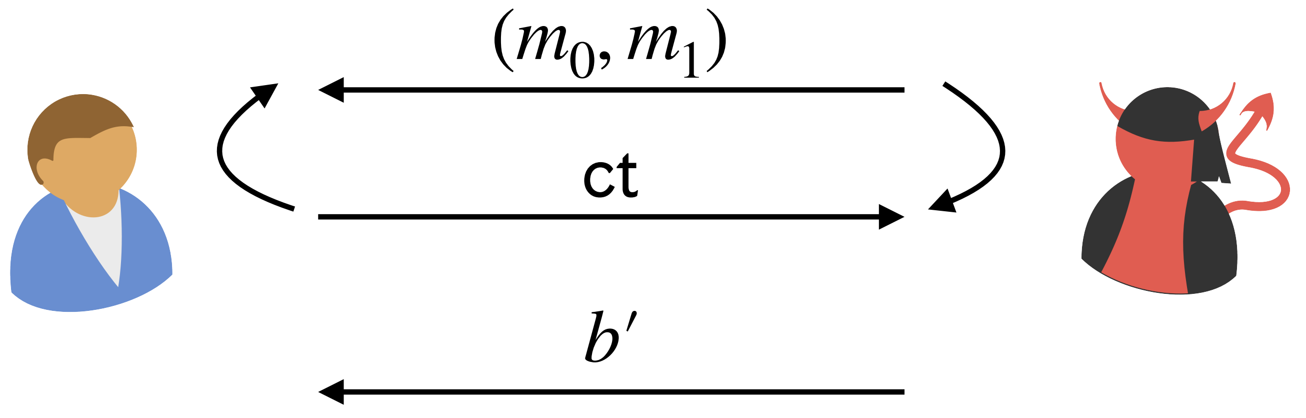
if $x \notin T$

$\quad r \xleftarrow{\$} \{0,1\}^\lambda$

$\quad T[x] = r$

$y = T[x]$

ct $:= (y \oplus m_0, x)$



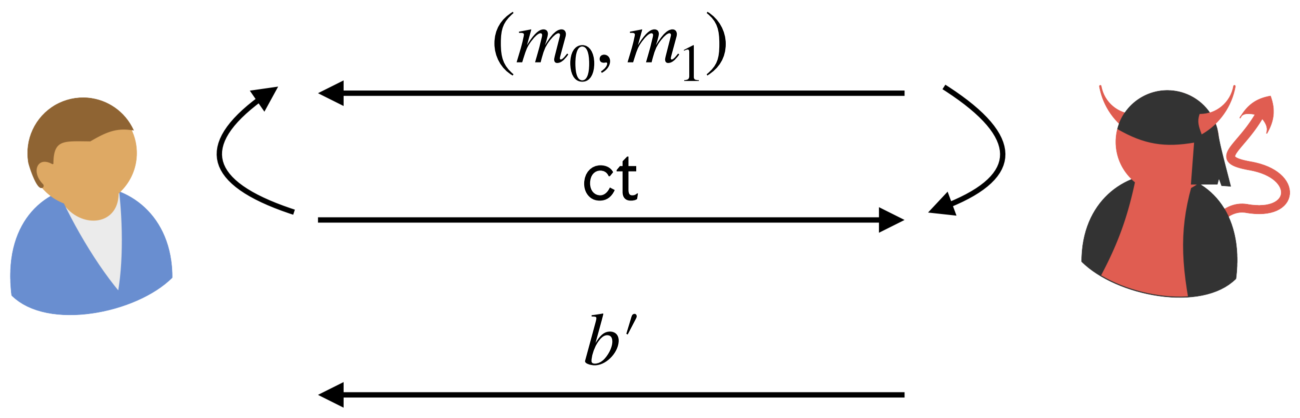
H_2

$T = \{\}$

$x \xleftarrow{\$} \{0,1\}^\lambda$

$y \xleftarrow{\$} \{0,1\}^\lambda$

ct $:= (y \oplus m_0, x)$



KeyGen(1^λ) : $k \xleftarrow{\$} \{0,1\}^\lambda$

Enc(k, m) : $x \xleftarrow{\$} \{0,1\}^\lambda$, ct $:= (F_k(x) \oplus m, x)$

Dec$(k, (c, x))$: $m' := F_k(x) \oplus c$

Proof of Security

H_1

$T = \{\}$

$x \xleftarrow{\$} \{0,1\}^\lambda$

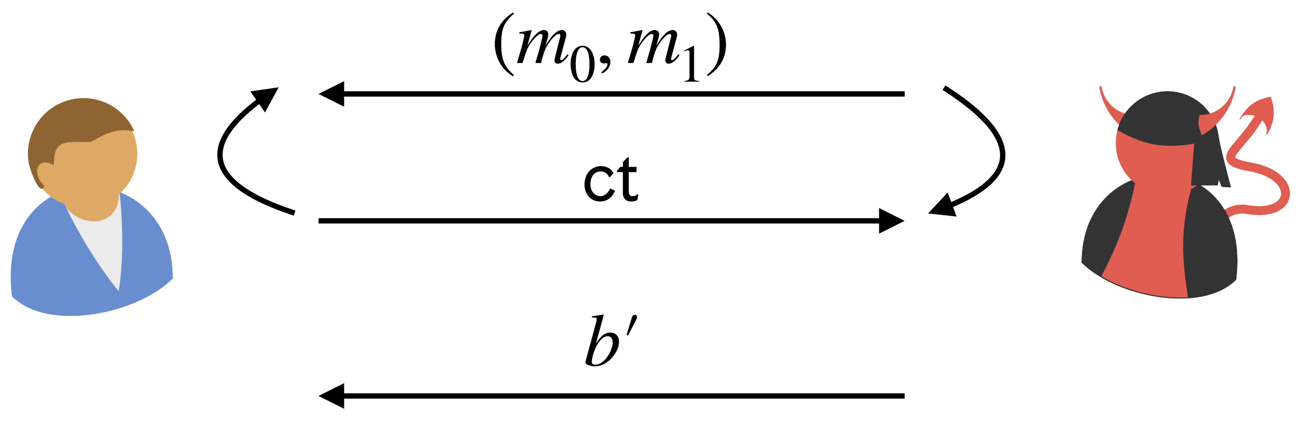
if $x \notin T$

$\quad r \xleftarrow{\$} \{0,1\}^\lambda$

$\quad T[x] = r$

$y = T[x]$

ct $:= (y \oplus m_0, x)$



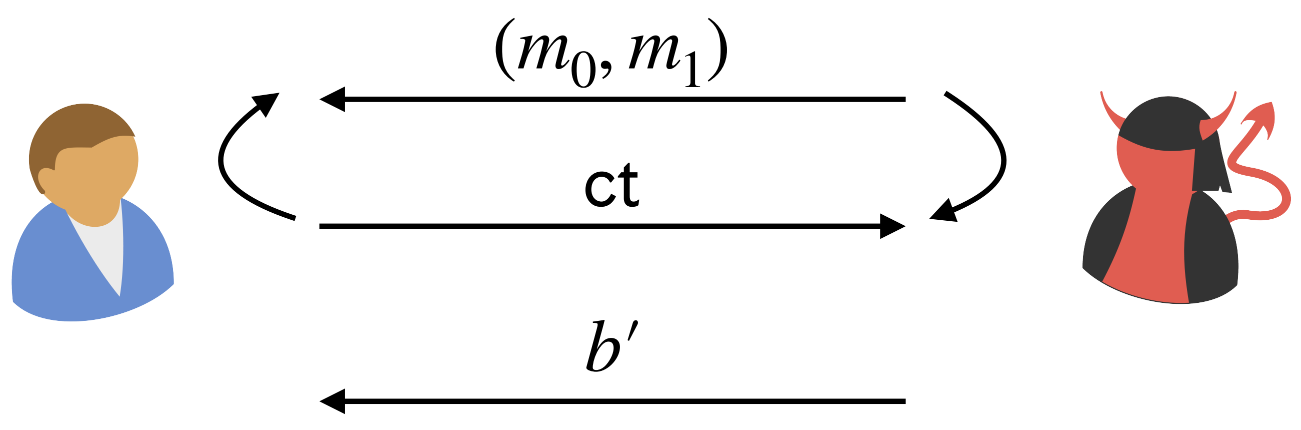
H_2

$T = \{\}$

$x \xleftarrow{\$} \{0,1\}^\lambda$

$y \xleftarrow{\$} \{0,1\}^\lambda$

ct $:= (y \oplus m_0, x)$



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Enc(k, m) : $x \xleftarrow{\$} \{0,1\}^\lambda$

ct $:= (F_k(x) \oplus m, x)$

Dec$(k, (c, x))$: $m' := F_k(x) \oplus c$

When does H_2 behave differently from H_1?

Proof of Security

H_1

$T = \{\}$

$x \xleftarrow{\$} \{0,1\}^\lambda$

if $x \notin T$

$\quad r \xleftarrow{\$} \{0,1\}^\lambda$

$\quad T[x] = r$

$y = T[x]$

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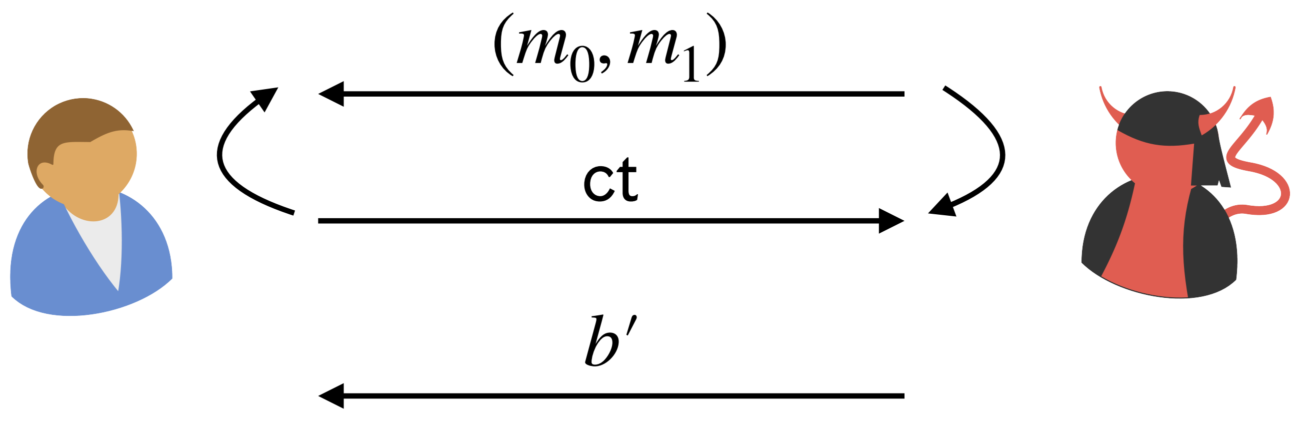


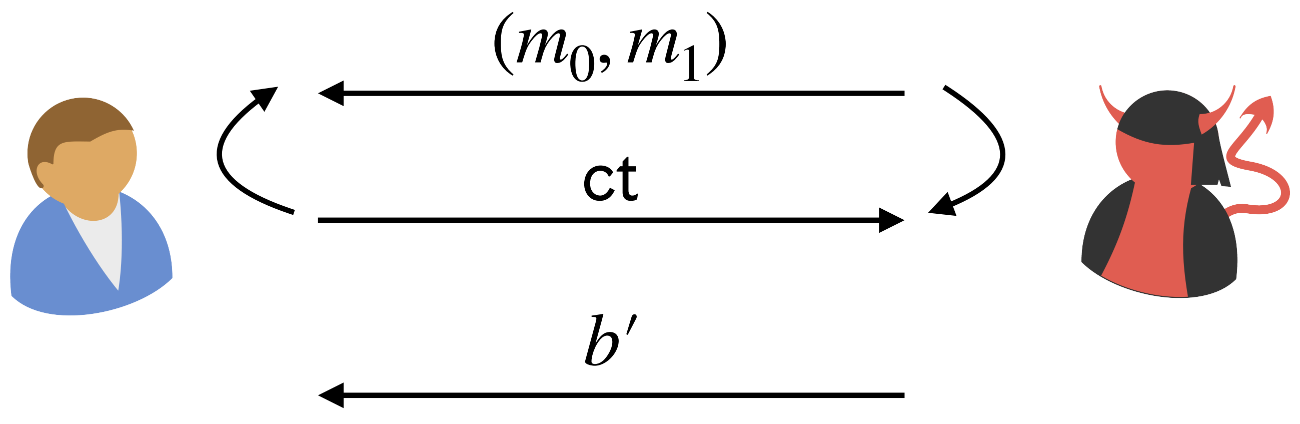
H_2

$T = \{\}$

$x \xleftarrow{\$} \{0,1\}^\lambda$

$y \xleftarrow{\$} \{0,1\}^\lambda$

ct $:= (y \oplus m_0, x)$



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Dec$(k, (c, x))$: $m' := F_k(x) \oplus c$

When does H_2 behave differently from H_1?

When the challenger samples the same x value twice!

Proof of Security

H_1

$T = \{\}$

$x \xleftarrow{\$} \{0,1\}^\lambda$

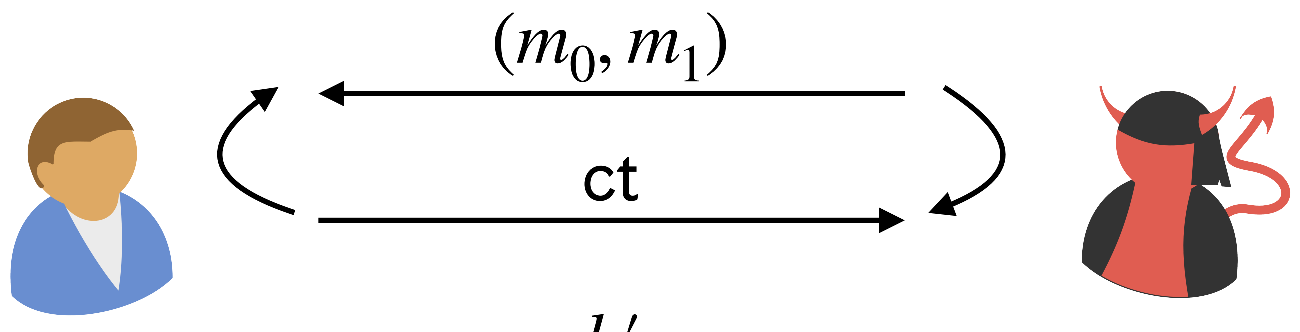
if $x \notin T$

$\quad r \xleftarrow{\$} \{0,1\}^\lambda$

$\quad T[x] = r$

$y = T[x]$

ct $:= (y \oplus m_0, x)$



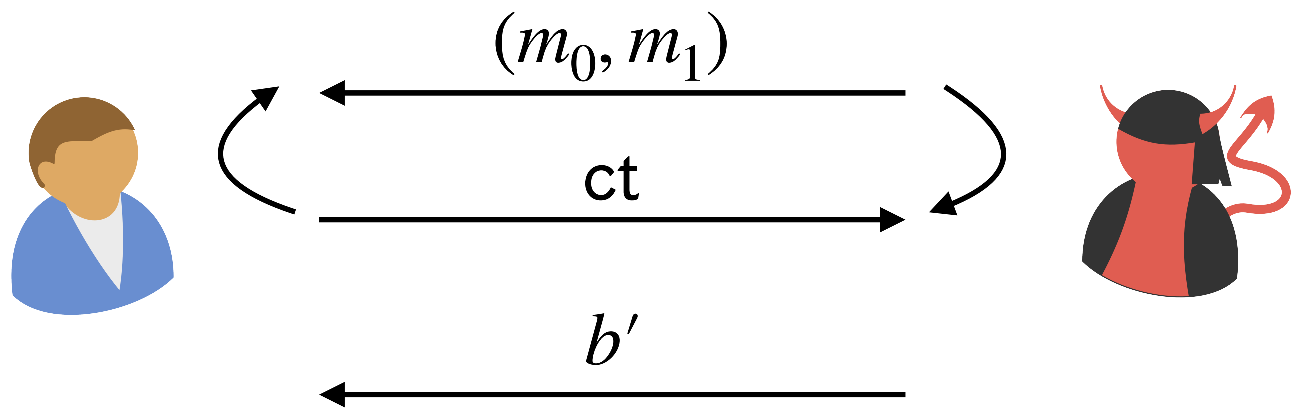
H_2

$T = \{\}$

$x \xleftarrow{\$} \{0,1\}^\lambda$

$y \xleftarrow{\$} \{0,1\}^\lambda$

ct $:= (y \oplus m_0, x)$



<u>Birthday Bound</u>: when sampling q elements from a space of size 2^λ, the probability of sampling some element twice is $P(q) \approx 1 - e^{\frac{-q^2}{2^{\lambda+1}}}$

KeyGen(1^λ) : $k \xleftarrow{\$} \{0,1\}^\lambda$

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When does H_2 behave differently from H_1?

When the challenger samples the same x value twice!

Proof of Security

H_1

$T = \{\}$

$x \xleftarrow{\$} \{0,1\}^\lambda$

if $x \notin T$

$\quad r \xleftarrow{\$} \{0,1\}^\lambda$

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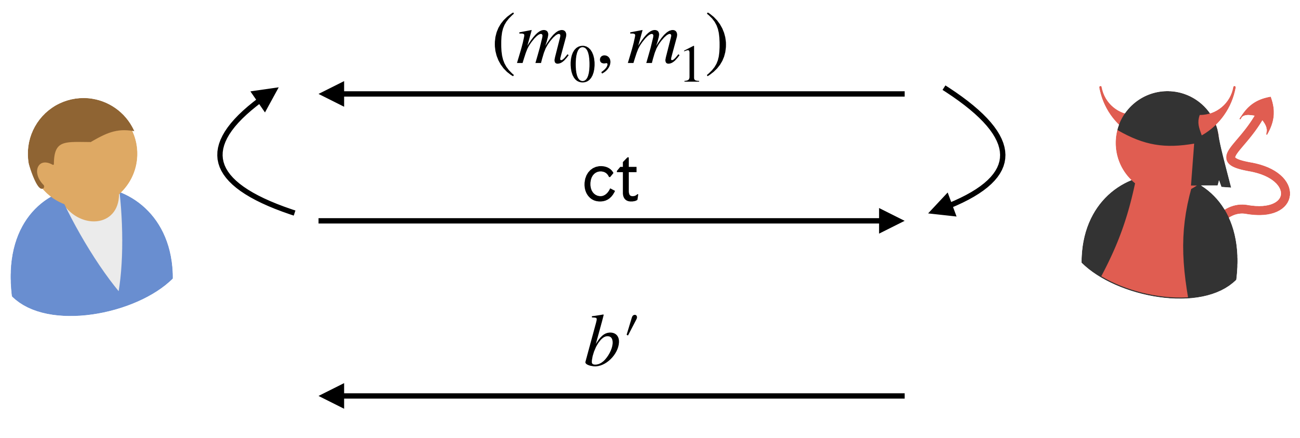
ct $:= (y \oplus m_0, x)$



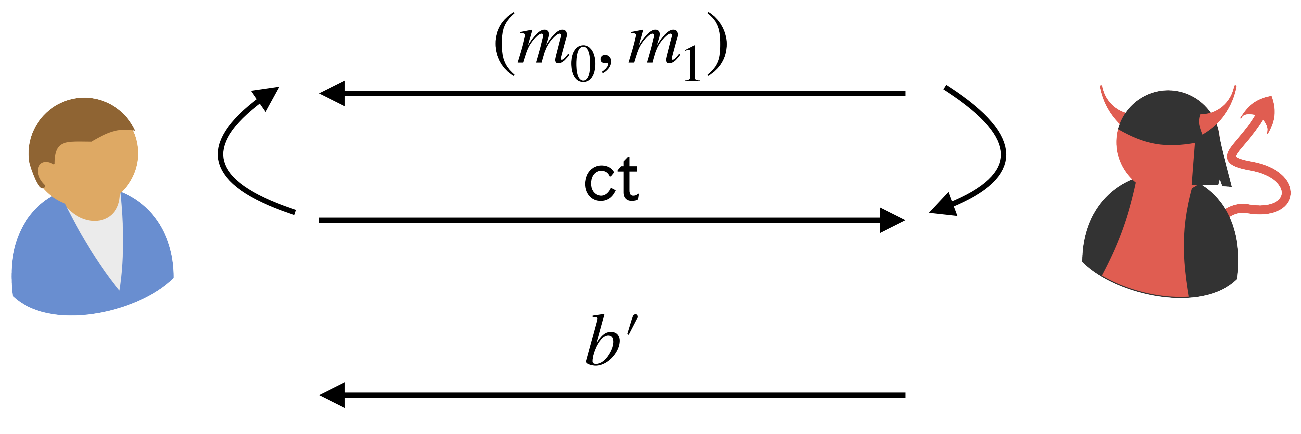
H_2

$T = \{\}$

$x \xleftarrow{\$} \{0,1\}^\lambda$

$y \xleftarrow{\$} \{0,1\}^\lambda$

ct $:= (y \oplus m_0, x)$



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When does H_2 behave differently from H_1?

When the challenger samples the same x value twice!

Birthday Bound: when sampling q elements from a space of size 2^λ, the probability of sampling some element twice is $P(q) \approx 1 - e^{\frac{-q^2}{2^{\lambda+1}}}$

q is polynomial in λ, so
$P(q) \approx 1 - e^{-\mathsf{negl}(\lambda)} \approx 1 - e^0 = 1 - 1 = 0$

Proof of Security

H_1

$T = \{\}$

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if $x \notin T$

$\quad r \xleftarrow{\$} \{0,1\}^\lambda$

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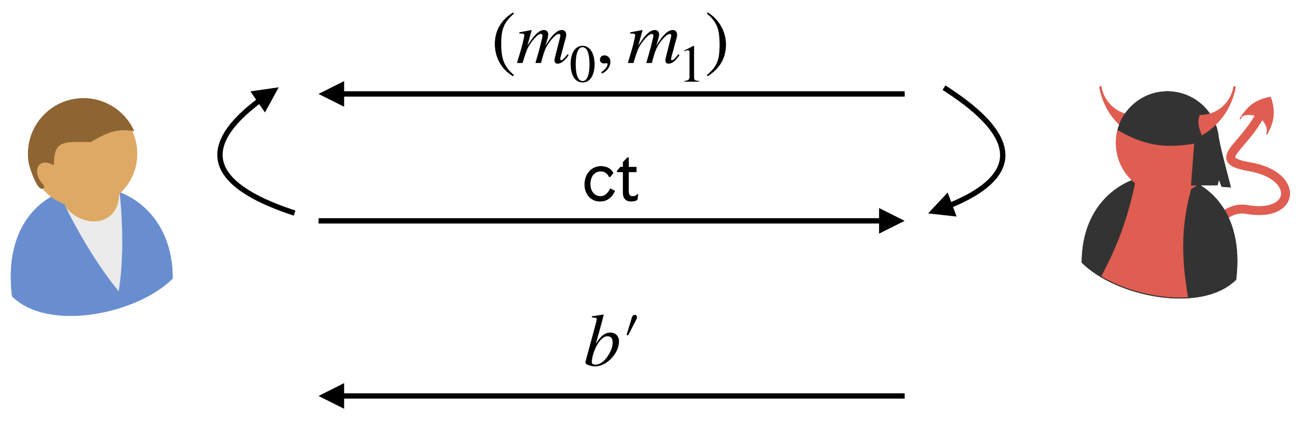

H_2

$T = \{\}$

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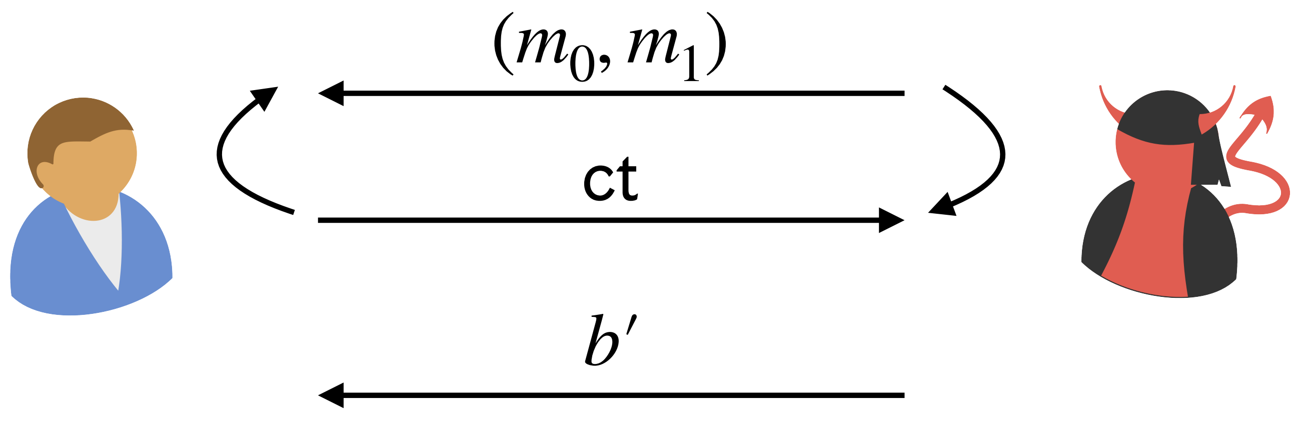

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Proof of Security

<u>Difference Lemma</u>: for events A, B, F, if $\Pr[A \mid F] = \Pr[B \mid F]$, then $\left|\Pr[A] - \Pr[B]\right| \leq \Pr[F]$

H_1

$T = \{\}$

$x \xleftarrow{\$} \{0,1\}^\lambda$

if $x \notin T$

$\quad r \xleftarrow{\$} \{0,1\}^\lambda$

$\quad T[x] = r$

$y = T[x]$

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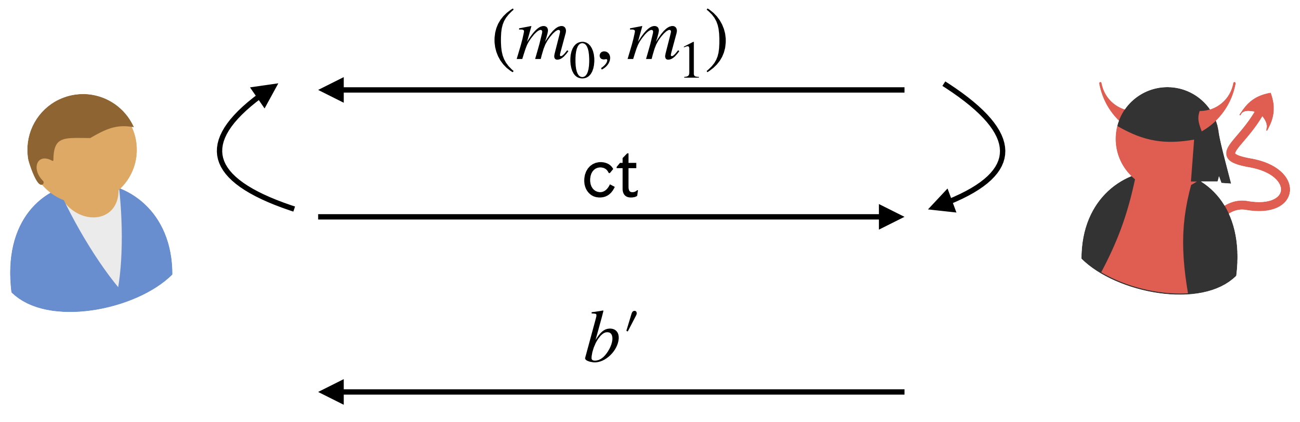


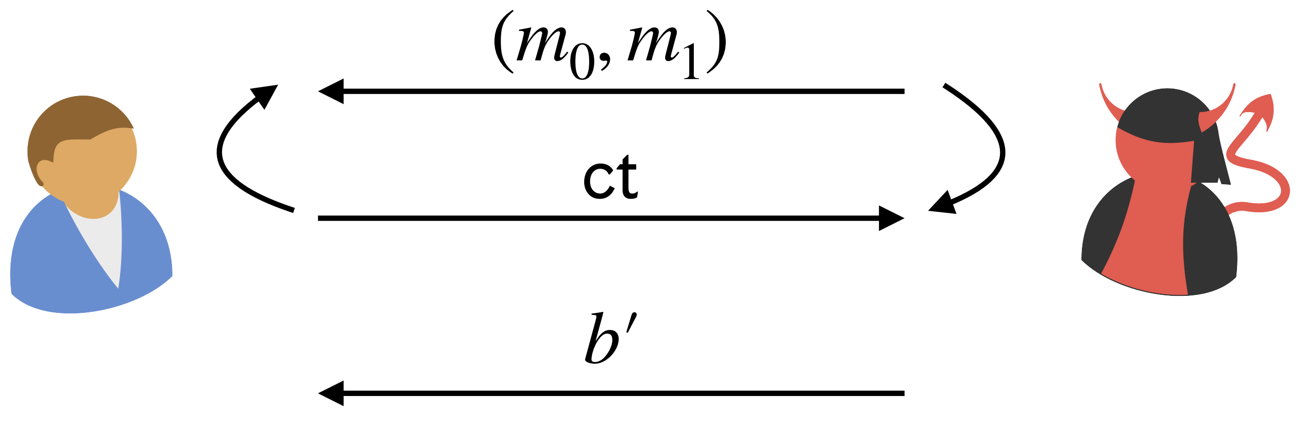
H_2

$T = \{\}$

$x \xleftarrow{\$} \{0,1\}^\lambda$

$y \xleftarrow{\$} \{0,1\}^\lambda$

ct $:= (y \oplus m_0, r)$



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Proof of Security

Difference Lemma: for events A, B, F, if $\Pr[A \mid F] = \Pr[B \mid F]$, then $\left|\Pr[A] - \Pr[B]\right| \le \Pr[F]$

H_1

$T = \{\}$

$x \xleftarrow{\$} \{0,1\}^\lambda$

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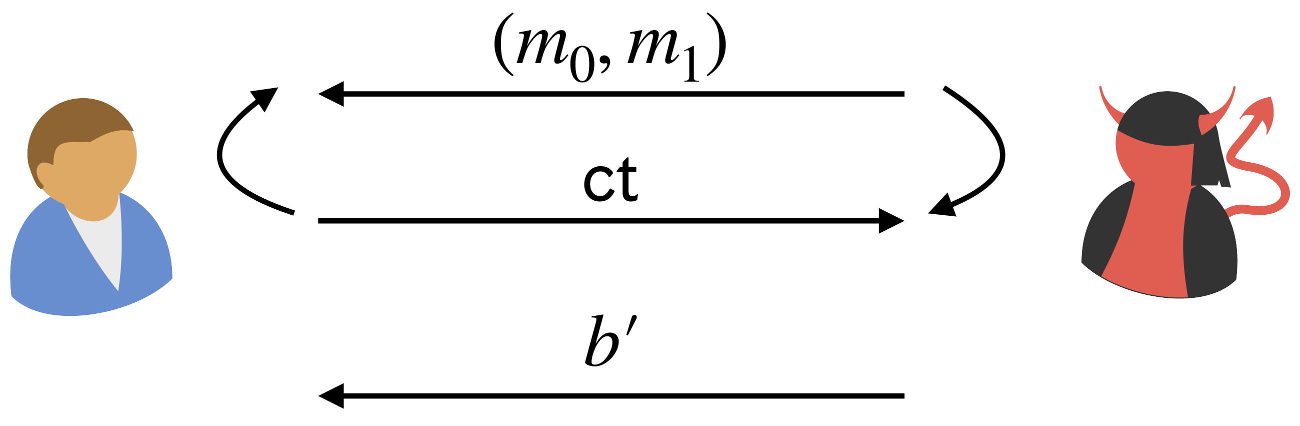

H_2

$T = \{\}$

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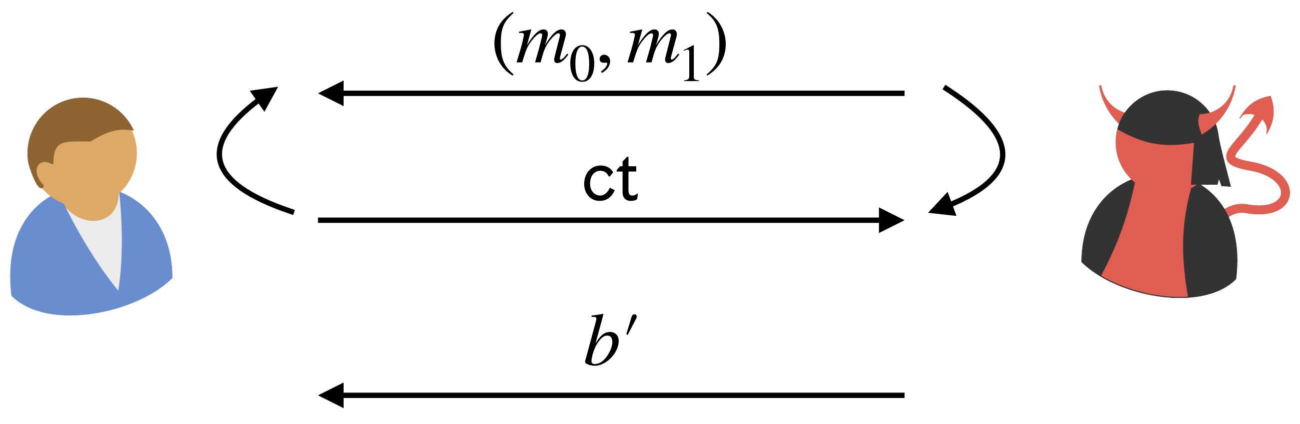

$$\left|\Pr\left[W_1\right] - \Pr\left[W_2\right]\right|$$

$$\le \Pr\left[C_{H_2} \text{ samples the same } x \text{ value twice}\right]$$

$$= P(q) = \mathsf{negl}(\lambda)$$

Birthday Bound: when sampling q elements from a space of size 2^λ, the probability of sampling some element twice is $P(q) \approx 1 - e^{\frac{-q^2}{2^{2\lambda+1}}}$

q is polynomial in λ, so

$P(q) \approx 1 - e^{-\mathsf{negl}(\lambda)} \approx 1 - e^0 = 1 - 1 = 0$

Proof of Security

H_2

$T = \{\}$

$x \xleftarrow{\$} \{0,1\}^\lambda$

$y \xleftarrow{\$} \{0,1\}^\lambda$

ct $:= (y \oplus m_0, r)$

(m_0, m_1)

ct

b'

KeyGen(1^λ) : $k \xleftarrow{\$} \{0,1\}^\lambda$

Enc(k, m) : $x \xleftarrow{\$} \{0,1\}^\lambda$
ct $:= (F_k(x) \oplus m, x)$

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Proof of Security

$$\mathsf{KeyGen}(1^\lambda) : k \xleftarrow{\$} \{0,1\}^\lambda$$

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H_2

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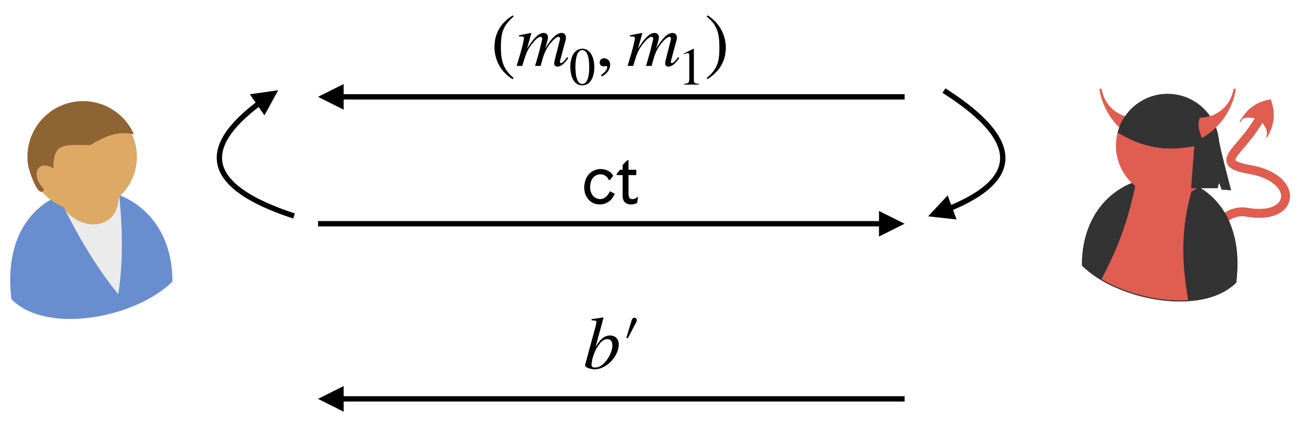


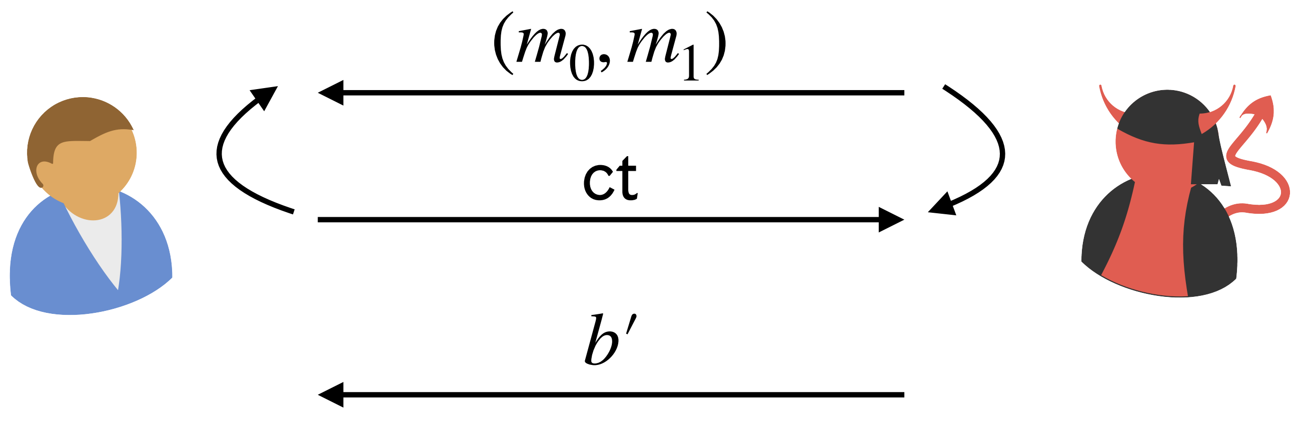
H_3

$T = \{\}$

$x \xleftarrow{\$} \{0,1\}^\lambda$

$y \xleftarrow{\$} \{0,1\}^\lambda$

$\mathsf{ct} := (y \oplus m_1, r)$



Proof of Security

$$\mathsf{KeyGen}(1^\lambda) : k \xleftarrow{\$} \{0,1\}^\lambda$$

$$\mathsf{Enc}(\mathsf{k}, \mathsf{m}) : \begin{array}{l} x \xleftarrow{\$} \{0,1\}^\lambda \\ \mathsf{ct} := (F_k(x) \oplus m, x) \end{array}$$

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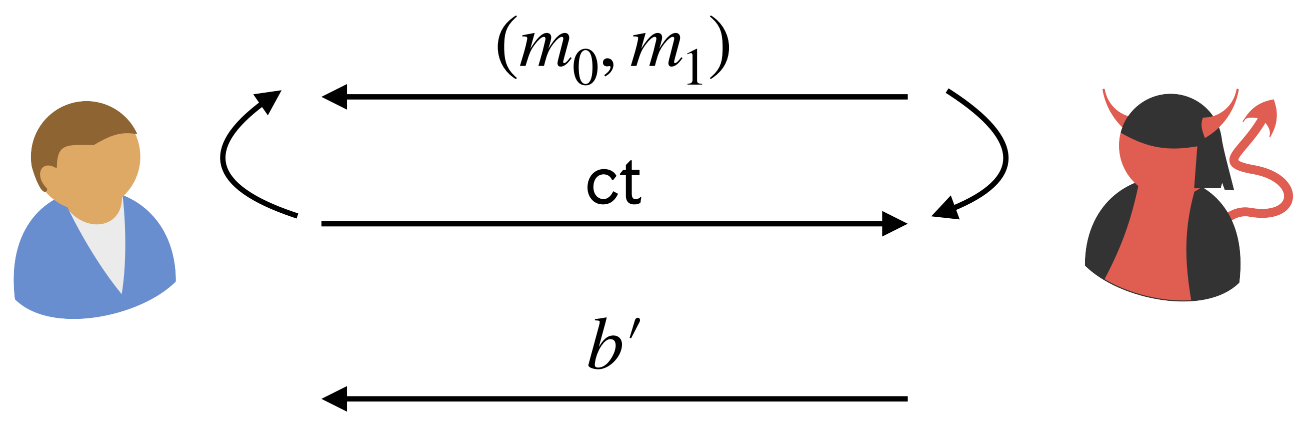
H_2

$T = \{\}$

$x \xleftarrow{\$} \{0,1\}^\lambda$

$y \xleftarrow{\$} \{0,1\}^\lambda$

$\mathsf{ct} := (y \oplus m_0, r)$



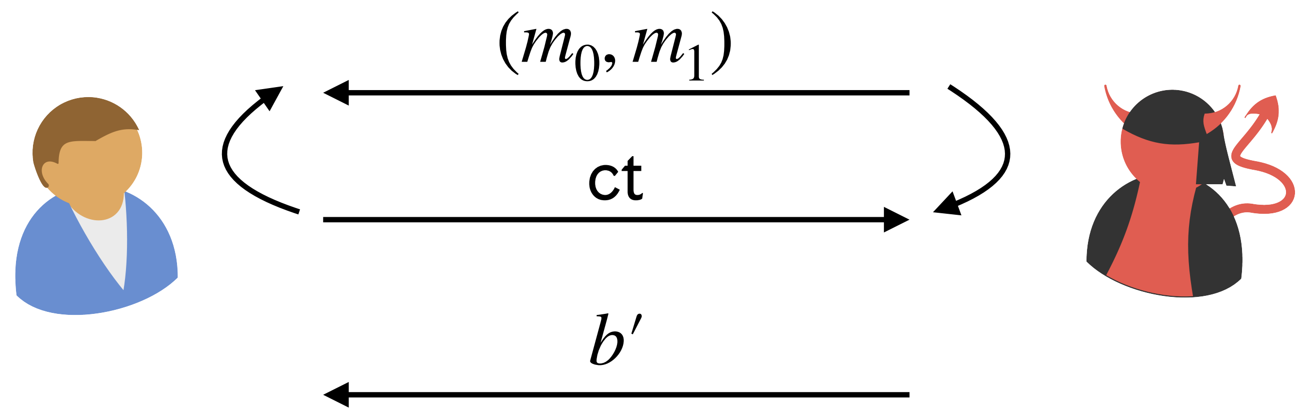
H_3

$T = \{\}$

$x \xleftarrow{\$} \{0,1\}^\lambda$

$y \xleftarrow{\$} \{0,1\}^\lambda$

$\mathsf{ct} := (y \oplus m_1, r)$



Messages are all one-time-padded, can just swap them!

Proof of Security

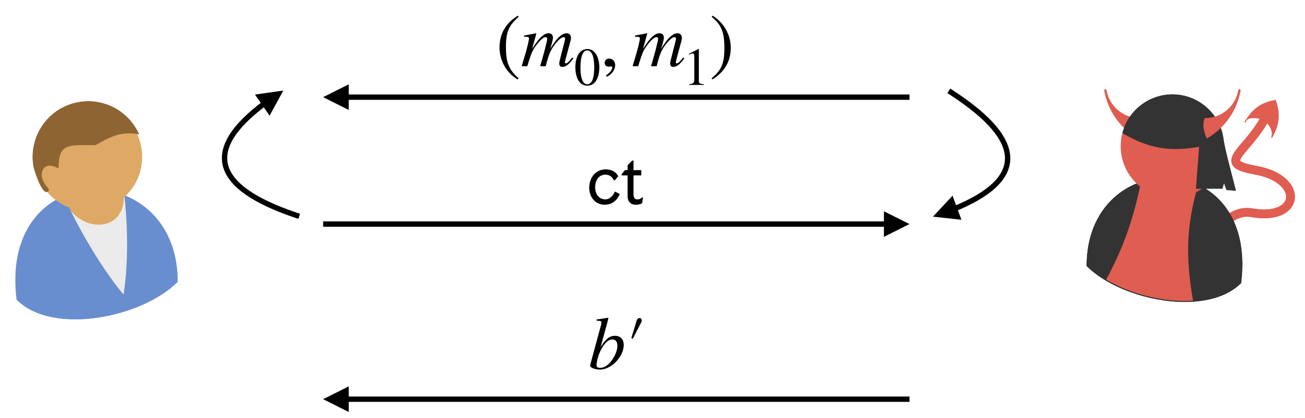
H_3

$T = \{\}$

$x \xleftarrow{\$} \{0,1\}^\lambda$

$y \xleftarrow{\$} \{0,1\}^\lambda$

ct $:= (y \oplus m_1, r)$



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Proof of Security

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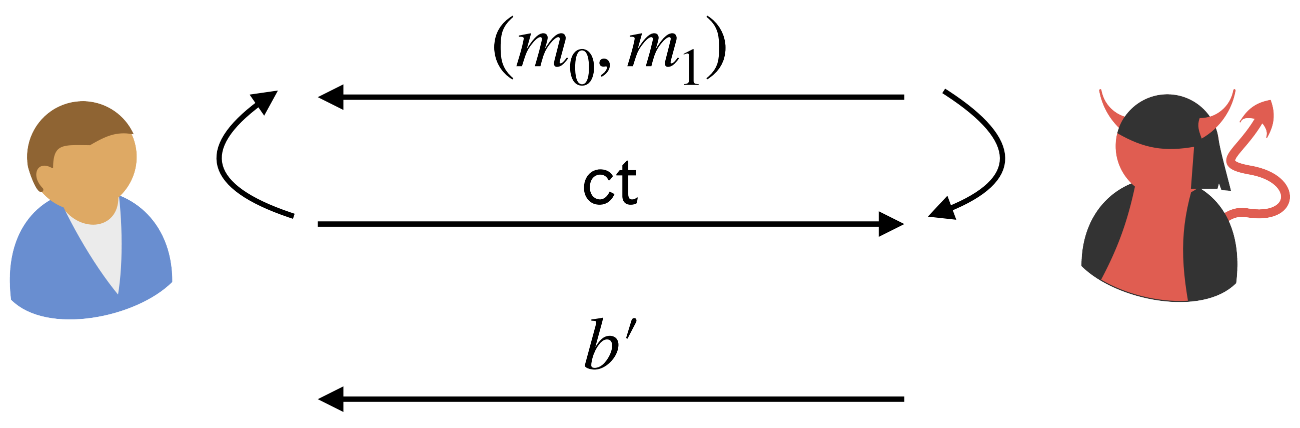
H_3

$T = \{\}$

$x \xleftarrow{\$} \{0,1\}^\lambda$

$y \xleftarrow{\$} \{0,1\}^\lambda$

$\mathsf{ct} := (y \oplus m_1, r)$



H_4

$T = \{\}$

$x \xleftarrow{\$} \{0,1\}^\lambda$

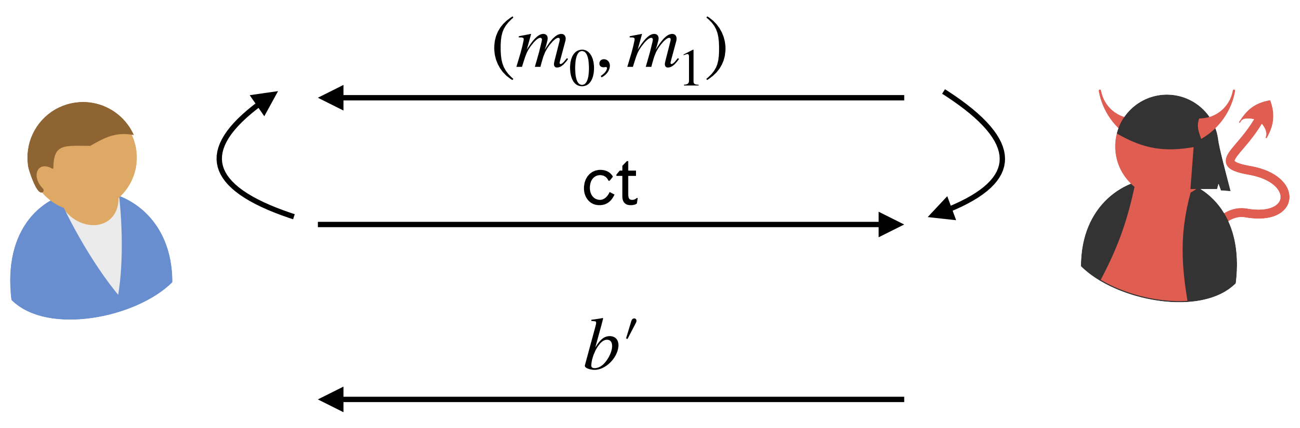
if $x \notin T$

$\quad r \xleftarrow{\$} \{0,1\}^\lambda$

$\quad T[x] = r$

$y = T[x]$

$\mathsf{ct} := (y \oplus m_1, r)$



Proof of Security

$$\mathsf{KeyGen}(1^\lambda) : k \xleftarrow{\$} \{0,1\}^\lambda$$

$$\mathsf{Enc(k, m)} : \begin{array}{l} x \xleftarrow{\$} \{0,1\}^\lambda \\ \mathsf{ct} := (F_k(x) \oplus m, x) \end{array}$$

$$\mathsf{Dec}(k, (c, x)) : m' := F_k(x) \oplus c$$

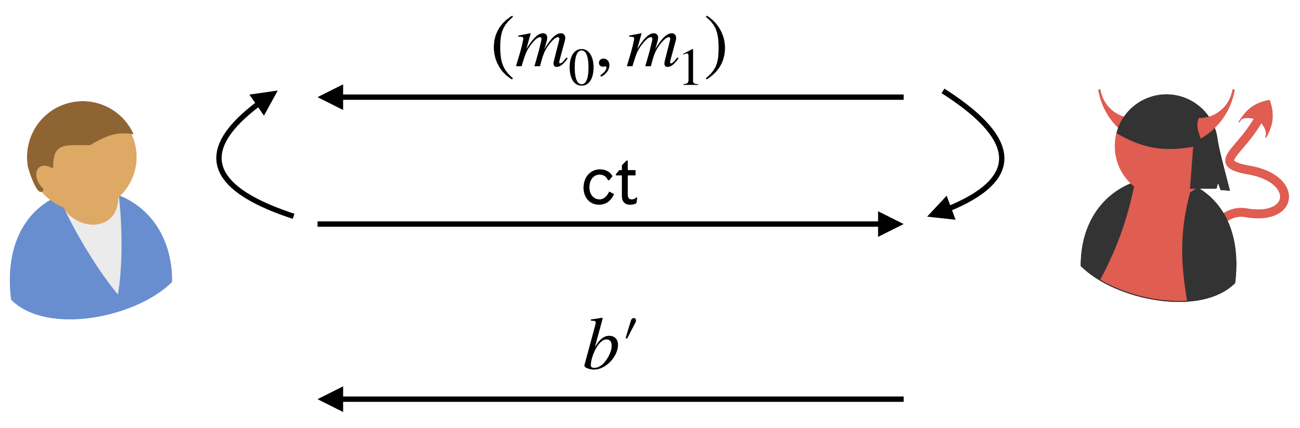
H_3

$T = \{\}$

$x \xleftarrow{\$} \{0,1\}^\lambda$

$y \xleftarrow{\$} \{0,1\}^\lambda$

ct $:= (y \oplus m_1, r)$



H_4

$T = \{\}$

$x \xleftarrow{\$} \{0,1\}^\lambda$

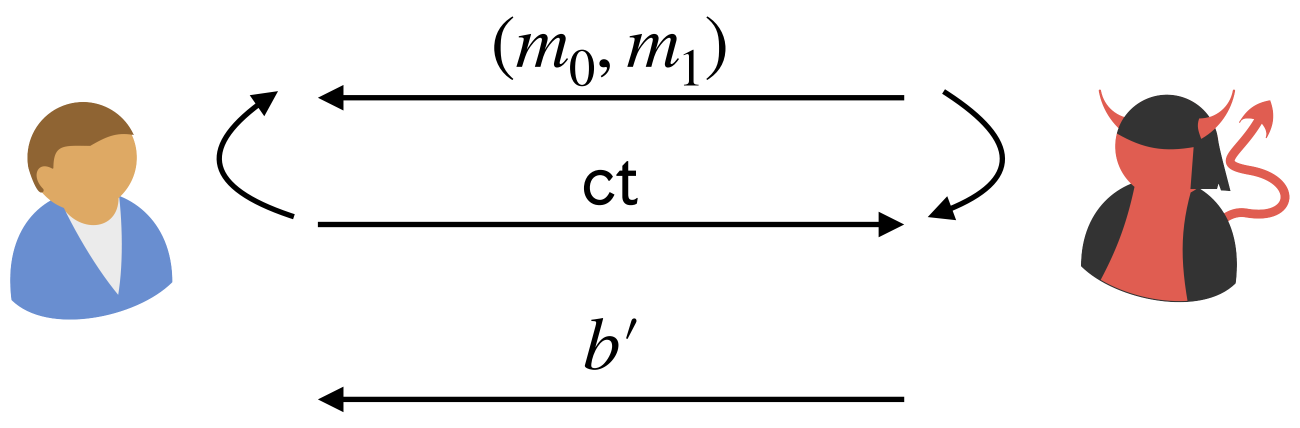
if $x \notin T$

$r \xleftarrow{\$} \{0,1\}^\lambda$

$T[x] = r$

$y = T[x]$

ct $:= (y \oplus m_1, r)$



Exactly the same transition as H_1 to H_2!

Proof of Security

KeyGen(1^λ) : $k \xleftarrow{\$} \{0,1\}^\lambda$

Enc(k, m) : $x \xleftarrow{\$} \{0,1\}^\lambda$
ct $:= (F_k(x) \oplus m, x)$

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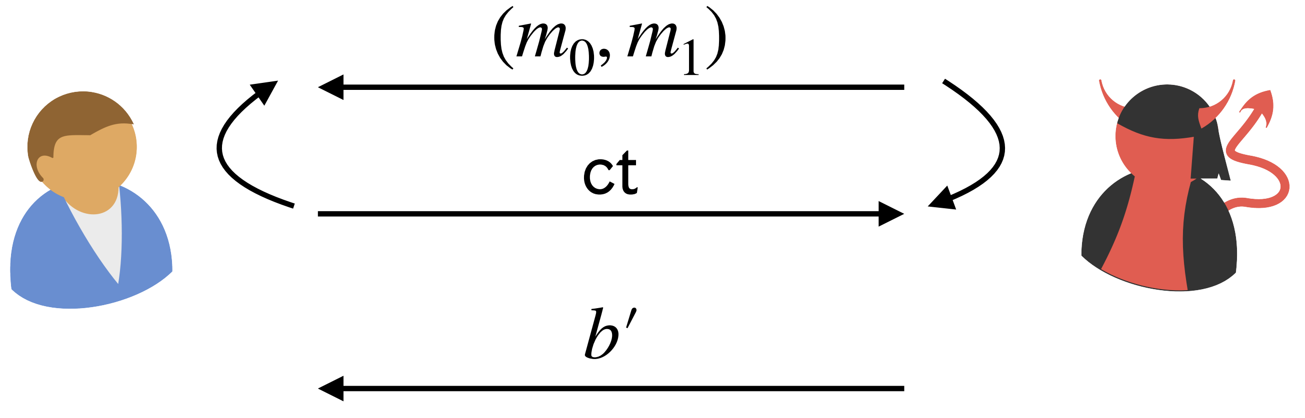
H_4

$T = \{\}$

$x \xleftarrow{\$} \{0,1\}^\lambda$



ct $:= (y \oplus m_1, r)$

Proof of Security

H_4

$T = \{\}$

$x \xleftarrow{\$} \{0,1\}^\lambda$

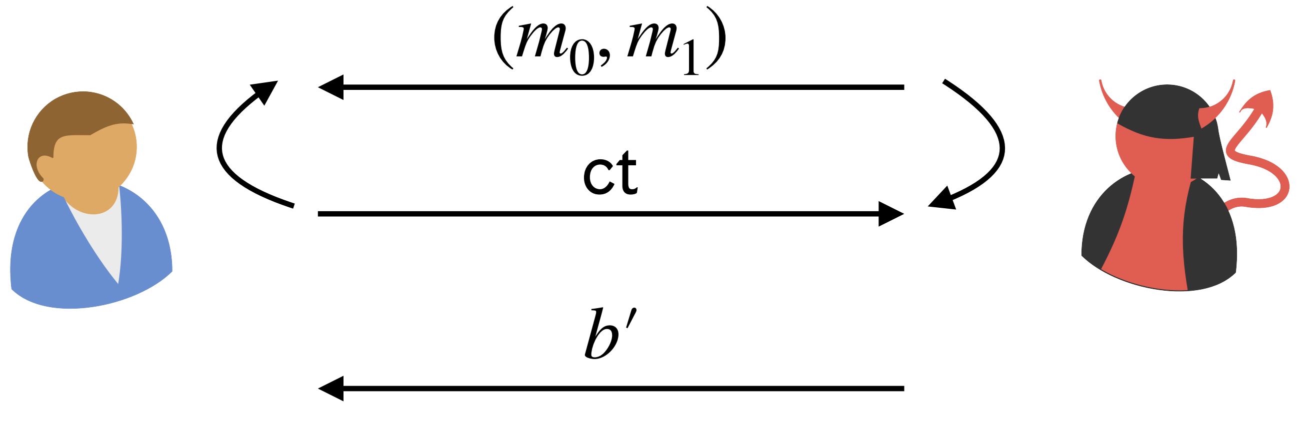
if $x \notin T$

$\quad r \xleftarrow{\$} \{0,1\}^\lambda$

$\quad T[x] = r$

$y = T[x]$

$\mathsf{ct} := (y \oplus m_1, r)$



$\mathsf{KeyGen}(1^\lambda) : k \xleftarrow{\$} \{0,1\}^\lambda$

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Proof of Security

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$$\mathsf{Dec}(k, (c, x)) : m' := F_k(x) \oplus c$$

H_4

$T = \{\}$

$x \xleftarrow{\$} \{0,1\}^\lambda$

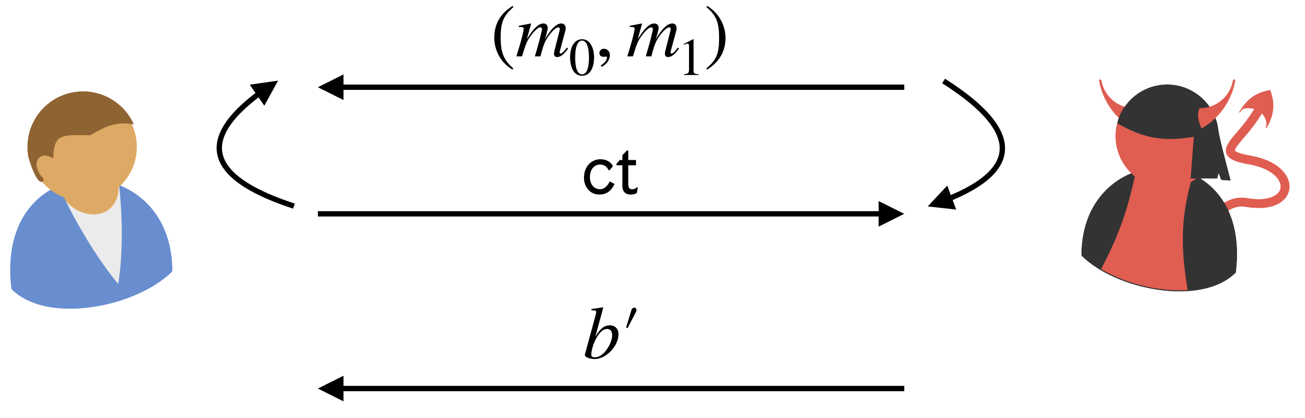
if $x \notin T$

$\quad r \xleftarrow{\$} \{0,1\}^\lambda$

$\quad T[x] = r$

$y = T[x]$

$\mathsf{ct} := (y \oplus m_1, r)$

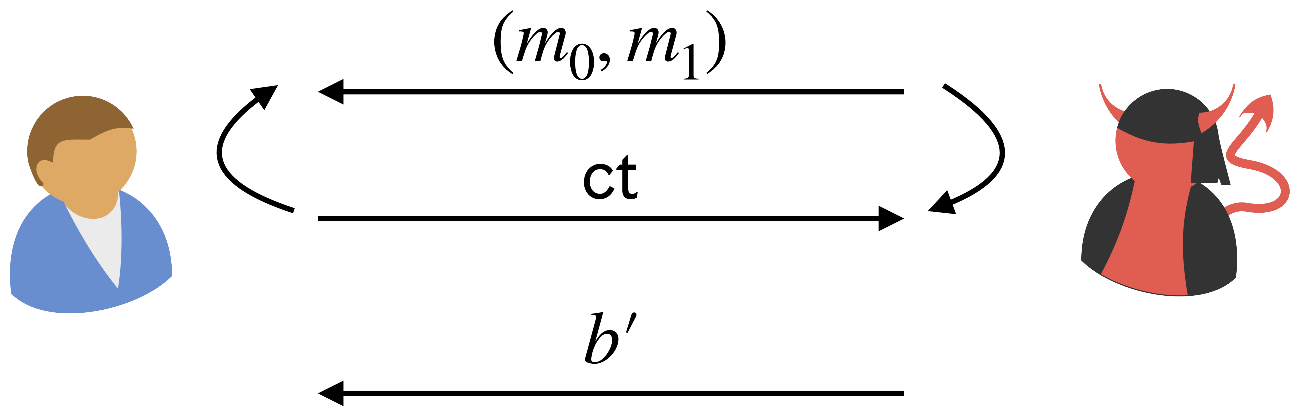


H_5

$k \xleftarrow{\$} \{0,1\}^\lambda$

$x \xleftarrow{\$} \{0,1\}^\lambda$

$\mathsf{ct} := (F_k(x) \oplus m_1, x)$



Proof of Security

$$\mathsf{KeyGen}(1^\lambda) : k \xleftarrow{\$} \{0,1\}^\lambda$$

$$\mathsf{Enc}(\mathsf{k}, \mathsf{m}) : \begin{array}{l} x \xleftarrow{\$} \{0,1\}^\lambda \\ \mathsf{ct} := (F_k(x) \oplus m, x) \end{array}$$

$$\mathsf{Dec}(k, (c, x)) : m' := F_k(x) \oplus c$$

H_4

$T = \{\}$

$x \xleftarrow{\$} \{0,1\}^\lambda$

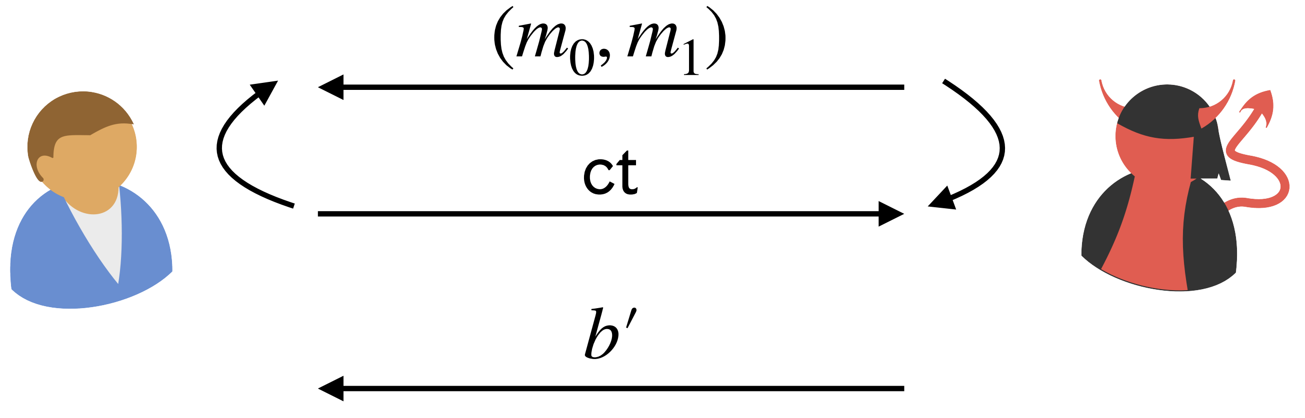
if $x \notin T$

$r \xleftarrow{\$} \{0,1\}^\lambda$

$T[x] = r$

$y = T[x]$

$\mathsf{ct} := (y \oplus m_1, r)$

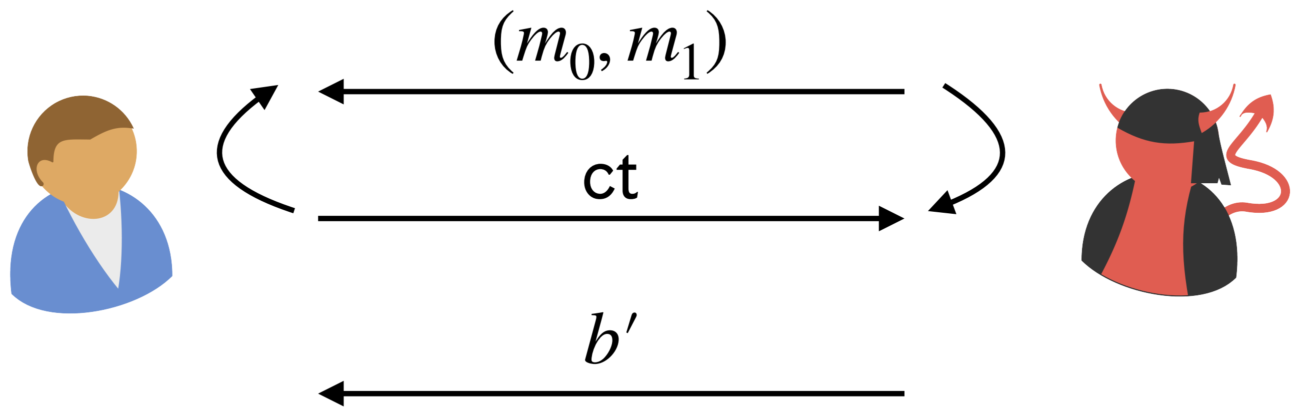


H_5

$k \xleftarrow{\$} \{0,1\}^\lambda$

$x \xleftarrow{\$} \{0,1\}^\lambda$

$\mathsf{ct} := (F_k(x) \oplus m_1, x)$



Exactly the same transition as H_0 to H_1!

Proof of Security

$\mathsf{KeyGen}(1^\lambda) : k \xleftarrow{\$} \{0,1\}^\lambda$

$\mathsf{Enc}(\mathsf{k}, \mathsf{m}) :$ $x \xleftarrow{\$} \{0,1\}^\lambda$
$\mathsf{ct} := (F_k(x) \oplus m, x)$

$\mathsf{Dec}(k, (c, x)) : m' := F_k(x) \oplus c$

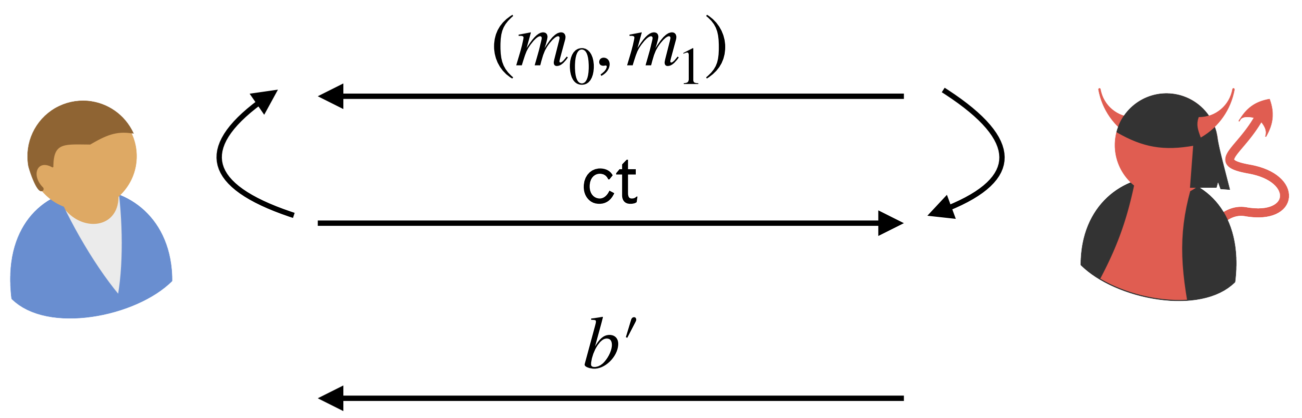
Proof of Security

H_0

$k \xleftarrow{\$} \{0,1\}^\lambda$

$x \xleftarrow{\$} \{0,1\}^\lambda$

ct $:= (F_k(x) \oplus m_0, x)$



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Proof of Security

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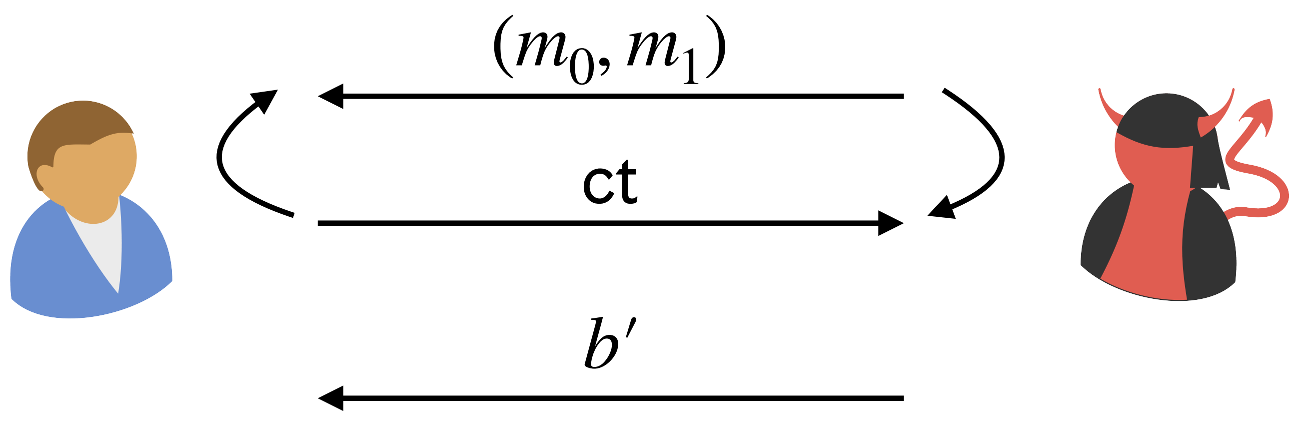

H_5

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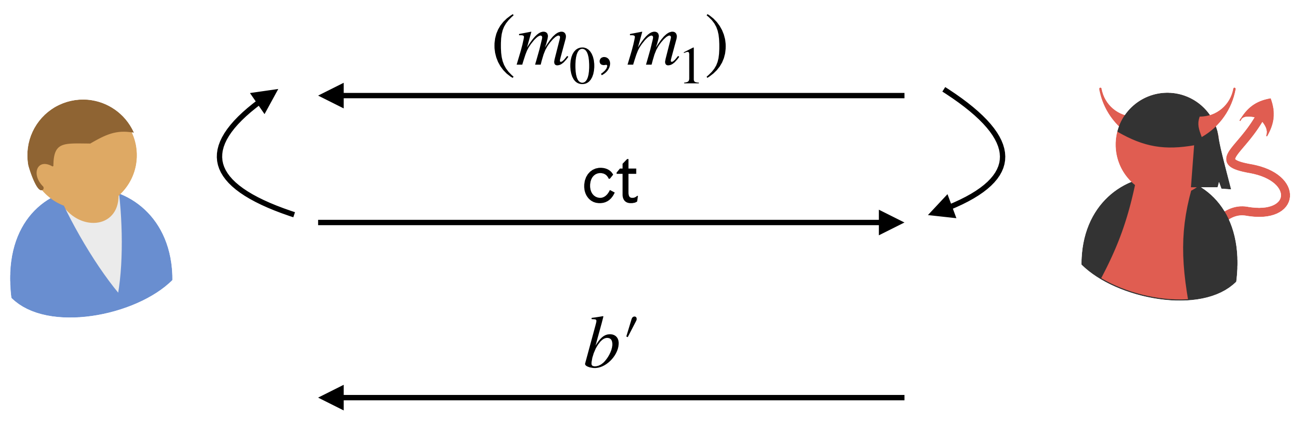

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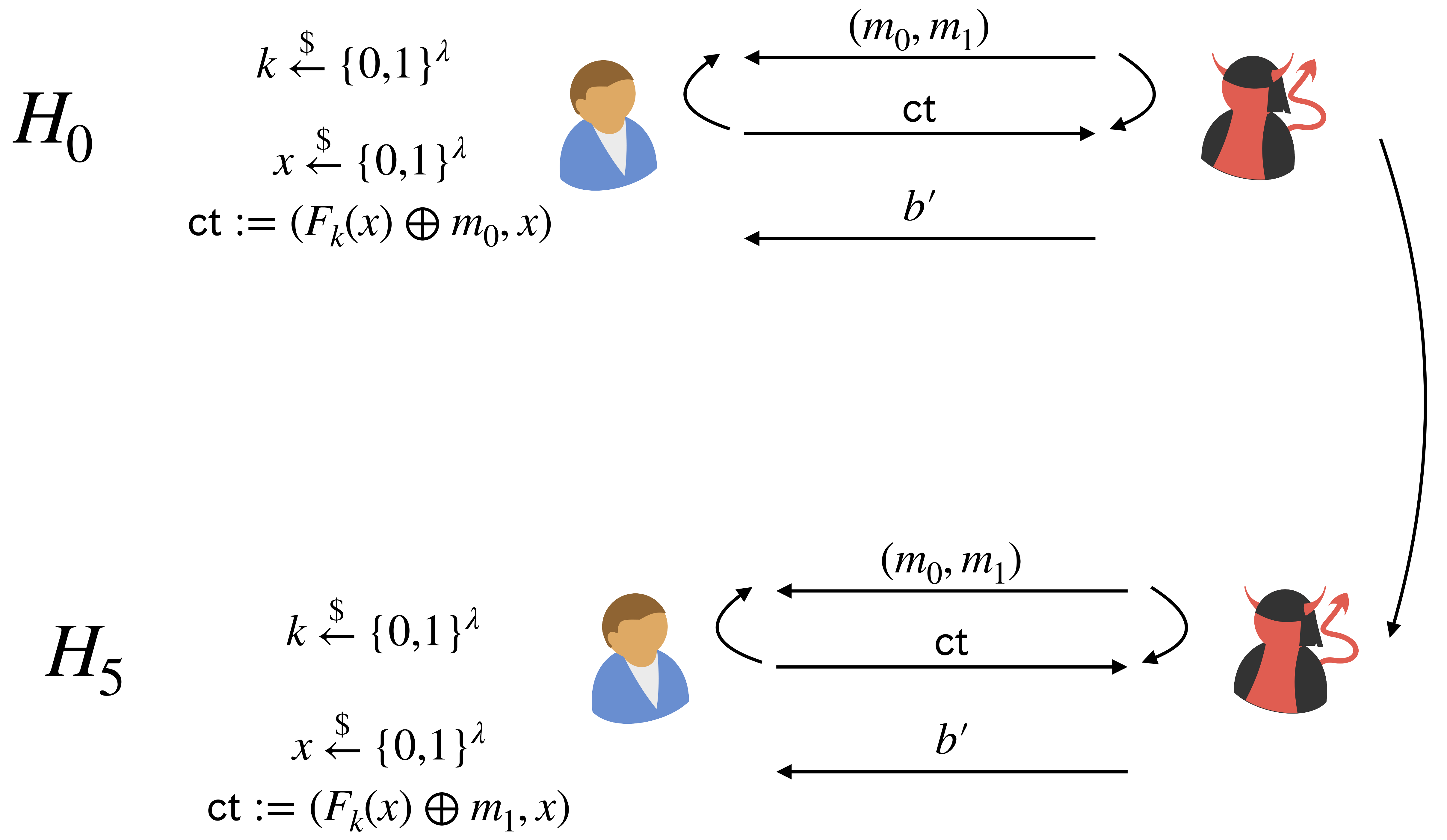

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By the hybrid lemma $H_0 \overset{c}{\approx} H_5$, and so our encryption scheme satisfies IND-CPA security

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- We may also refer to this as the *asymmetric* setting, and private key encryption as the *symmetric* setting

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Public Key Encryption Scheme Syntax

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- Dec$(sk,$ ct$) \rightarrow m$ takes a secret key sk and a ciphertext ct and outputs a message m

PKE Security

KeyGen $\rightarrow (pk, sk)$

Enc$(pk, m) \rightarrow$ ct

Dec$(sk, \text{ct}) \rightarrow m$

Wins if $b' = b$

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In the public key setting, security for a single message implies multi-message security

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$$\Pr[\mathscr{A} \text{ wins GuessGame}] \leq \frac{1}{2} + \nu(\lambda)$$

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<u>IND-CPA Security</u>

$$\Big|\Pr[\mathscr{A} \text{ outputs } 1 \text{ in } \mathsf{Game}_0] - \Pr[\mathscr{A} \text{ outputs } 1 \text{ in } \mathsf{Game}_1]\Big| \leq \nu(\lambda)$$

Game_0

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Game_1

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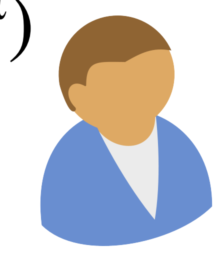

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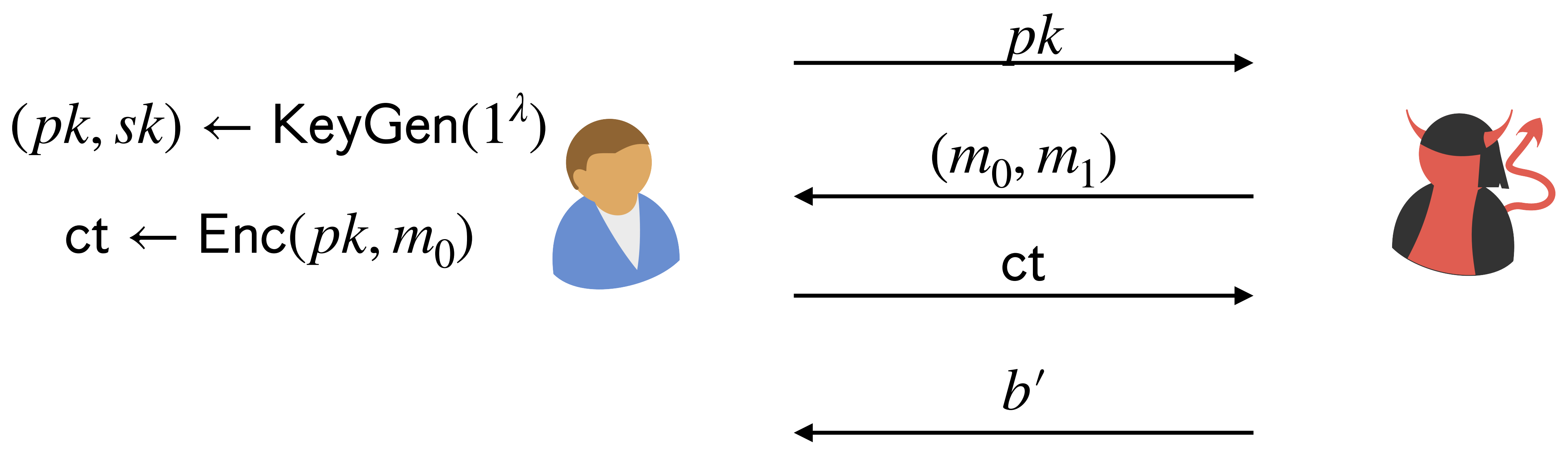
$$\Big| \Pr[\mathscr{A} \text{ outputs } 1 \text{ in Game}_0] - \Pr[\mathscr{A} \text{ outputs } 1 \text{ in Game}_1] \Big| \leq \nu(\lambda)$$

Game$_0$



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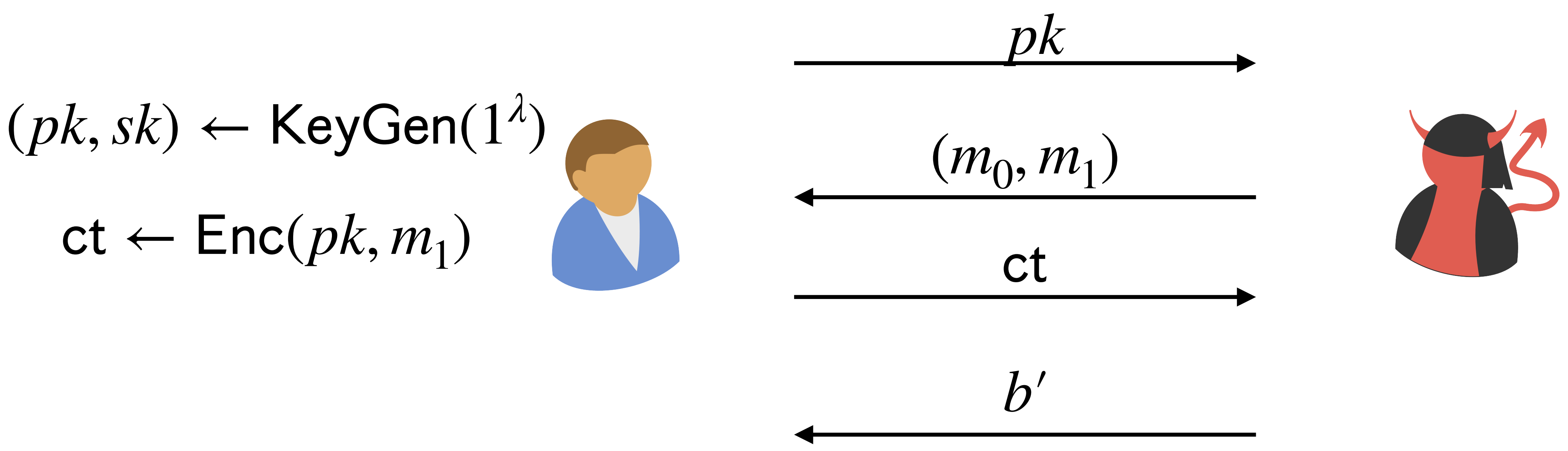

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- Building PKE seems to require some sort of *new* assumption.
    - RSA, Diffie-Hellman, LWE, etc.

Groups

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- Our assumption: given g^a for some randomly sampled a, it should be hard to find a

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Wins if $a' = a$

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Let $(G, *)$ be a cyclic group of order p (where p is a safe prime) with generator g, then for every NUPPT adversary $\mathscr{A}$, there exists a negligible function ν such that

$$\Pr\left[\mathscr{A} \text{ wins DLGame}\right] \leq \nu(\lambda)$$

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Note: This is a *search problem*. $\mathscr{A}$ needs to actually *find* the discrete logarithm