Exp. (*b*)

Given, $A \cap B = A \cap C$ and $A \cup B = A \cup C$

$\therefore \quad B = C$

Exp. (b)

Given, graph is symmetrical about the line $x = 2$.

$\therefore$ $$f(2 + x) = f(2 - x)$$

Exp. (a)

Given that, $f(x)=\sqrt{\log_{10}\left(\frac{5x-x^2}{4}\right)}$

For domain of $f(x)$,

$$\log_{10}\left(\frac{5x-x^2}{4}\right)\geq 0$$

$$\Rightarrow \quad \frac{5x-x^2}{4}\geq 1$$

$$\Rightarrow \quad x^2-5x+4\leq 0$$

$$\Rightarrow \quad (x-1)(x-4)\leq 0$$

$$\Rightarrow \quad x\in[1,4]$$

Exp. (c)

Given, relation R is defined as

$R = \{(x, y) | x, y$ are real numbers and $x = wy$ for some rational number $w\}$

(i) **Reflexive** $xRx \Rightarrow x = wx$

$\therefore \quad w = 1 \in$ rational number

The relation R is reflexive.

(ii) **Symmetric** $xRy \nRightarrow yRx$ as $0R1$

But $1R0 \Rightarrow 1 = w \cdot (0)$

which is not true for any rational number.

The relation R is not symmetric.

Thus, R is not equivalence relation.

Now, for relation S which is defined as

$S = \left\{\left(\frac{m}{n}, \frac{p}{q}\right) \middle| m, n, p \text{ and } q \in \text{integers such that } n, q \neq 0 \text{ and } qm = pn\right\}$

(i) **Reflexive** $\frac{m}{n} R \frac{m}{n} \Rightarrow mn = mn$ [true]

The relation S is reflexive.

(ii) **Symmetric** $\frac{m}{n} R \frac{p}{q} \Rightarrow mq = np$

$$\Rightarrow \quad np = mq \Rightarrow \frac{p}{q} R \frac{m}{n}$$

The relation S is symmetric.

(iii) **Transitive** $\frac{m}{n} R \frac{p}{q}$

$$\text{and} \quad \frac{p}{q} R \frac{r}{s}$$

$$\Rightarrow \quad mq = np$$

$$\text{and} \quad ps = rq$$

$$\Rightarrow \quad mq \cdot ps = np \cdot rq$$

$$\Rightarrow \quad ms = nr$$

$$\Rightarrow \quad \frac{m}{n} = \frac{r}{s}$$

$$\Rightarrow \quad \frac{m}{n} R \frac{r}{s}$$

The relation S is transitive.

Hence, the relation S is equivalence relation.

Exp. (c)

Given, $R = \{(1, 3), (4, 2), (2, 4), (2, 3), (3, 1)\}$ is a relation on the set $A = \{1, 2, 3, 4\}$.

(a) Since, $(2, 4) \in R$ and $(2, 3) \in R$. So, R is not a function.

(b) Since, $(1, 3) \in R$ and $(3, 1) \in R$ but $(1, 1) \notin R$. So, R is not transitive.

(c) Since, $(2, 3) \in R$ but $(3, 2) \notin R$. So, R is not symmetric.

(d) Since, $(1, 1), (2, 2), (3, 3), (4, 4) \notin R$. So, R is not reflexive.

Exp. (b)

Given that, $f(x) = \sin^4 x + \cos^4 x$

$\therefore \quad f(x) = (\sin^2 x + \cos^2 x)^2 - 2\sin^2 x \cos^2 x$

$$= 1 - \frac{1}{2}(2\sin x \cos x)^2 = 1 - \frac{1}{2}(\sin 2x)^2$$

$$= 1 - \frac{1}{2}\left(\frac{1-\cos 4x}{2}\right) = \frac{3}{4} + \frac{1}{4}\cos 4x$$

$\therefore$ The period of $f(x) = \frac{2\pi}{4} = \frac{\pi}{2}$

[$\because \cos x$ is periodic with period 2π]

Exp. (b)

$$f(x) = (x-1)^2 + 1, \quad \text{as } x \geq 1$$

When $\quad f(x) = f^{-1}(x)$

$\Rightarrow \quad f(x) = x$

$\Rightarrow \quad (x-1)^2 + 1 = x$

$\Rightarrow \quad (x-1)^2 = x - 1$

$\Rightarrow \quad (x-1)^2 - (x-1) = 0$

$\Rightarrow \quad (x-1)\{x-1-1\} = 0$

$\Rightarrow \quad x = 1, 2$

$\therefore \quad \{x : f(x) = f^{-1}(x)\} = \{1, 2\}$

Also, let $\quad f(x) = y$

$\therefore \quad y = (x-1)^2 + 1$

$\Rightarrow \quad (x-1) = \pm\sqrt{y-1}$

$\therefore \quad x = 1 \pm \sqrt{y-1}$

$\because \quad x \geq 1$

Neglecting $1 - \sqrt{y-1}$

$\Rightarrow \quad x = 1 + \sqrt{y-1}$

or $\quad f^{-1}(y) = 1 + \sqrt{y-1}$

$\therefore \quad f^{-1}(x) = 1 + \sqrt{x-1}$

So, both statements are correct and Statement II is correct explanation of Statement I.

Exp. (d)

Given, $f(x-y)=f(x)f(y)-f(a-x)f(a+y)$

Let $x=0=y$

$\Rightarrow \quad f(0)=[f(0)]^2-[f(a)]^2$

$\Rightarrow \quad 1=1-[f(a)]^2 \quad$ [given, $f(0)=1$]

$\Rightarrow \quad f(a)=0$

$\therefore \quad f(2a-x)=f\{a-(x-a)\}$

$=f(a)f(x-a)-f(a-a)f(x)$

$=0-f(x)\cdot 1=-f(x)$

Exp. (a)

Since, domain of $\sin^{-1} x$ is $[-1, 1]$.

$$\therefore \quad -1 \le \log_3\left(\frac{x}{3}\right) \le 1$$

$$\Rightarrow \quad 3^{-1} \le \frac{x}{3} \le 3 \Rightarrow 1 \le x \le 9$$

Hence, domain of $\sin^{-1}\left[\log_3\left(\frac{x}{3}\right)\right]$ is $[1, 9]$.

Exp. (a)

Given that $p(x) = f(x) - g(x)$ has only one root -1.

$\Rightarrow p(x) = (a - a_1)x^2 + (b - b_1)x + (c - c_1)$ has one root, -1 only.

$\Rightarrow p'(x)$ will also has root as -1.

$\Rightarrow p'(x) = 0$ at $x = -1$

$\Rightarrow 2(a - a_1)x + (b - b_1) = 0$ at $x = -1$.

$\Rightarrow -2(a - a_1) + (b - b_1) = 0$

$$\Rightarrow \quad \frac{-(b - b_1)}{(a - a_1)} = -2 \quad \ldots(\text{i})$$

Now, $p(x) = (a - a_1)x^2 + (b - b_1)x + (c - c_1)$

$$\Rightarrow \quad \frac{p(x)}{a - a_1} = x^2 + \frac{b - b_1}{a - a_1}x + \frac{(c - c_1)}{a - a_1}$$

$\because \quad p(-1) = 0$

$$\therefore \quad 0 = (-1)^2 - \frac{(b - b_1)}{a - a_1} + \frac{(c - c_1)}{a - a_1}$$

$$\Rightarrow \quad 0 = 1 - 2 + \frac{(c - c_1)}{a - a_1} \quad [\text{using Eq. (i)}]$$

$$\Rightarrow \frac{c - c_1}{a - a_1} = 1 \quad \ldots(\text{ii})$$

Also, given that $p(-2) = 2$

$\Rightarrow 4(a - a_1) - 2(b - b_1) + (c - c_1) = 2 \quad \ldots(\text{iii})$

From Eqs. (i), (ii) and (iii), we have

$$4(a - a_1) - 4(a - a_1) + (a - a_1) = 2$$

$$\Rightarrow \quad a - a_1 = 2$$

On substituting $a - a_1 = 2$ in Eq. (ii), we get

$$c - c_1 = 2$$

On substituting $a - a_1 = 2$ in Eq. (i), we get

$$b - b_1 = 4$$

Now, $p(2) = 4(a - a_1) + 2(b - b_1) + (c - c_1)$

$$= 4 \times 2 + 2 \times 4 + 2$$

$$= 8 + 8 + 2 = 18$$

Exp. (a)

Given, $x \in (-1, 1)$

$\Rightarrow \quad \tan^{-1} x \in \left(-\frac{\pi}{4}, \frac{\pi}{4}\right)$

$\Rightarrow \quad 2\tan^{-1} x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Given that,

$$f(x) = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$$

$$= 2\tan^{-1} x \qquad [\because x^2 < 1]$$

So, $f(x) \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Hence, function is one-one onto.

Exp. (b)

We know that,

$$\sin^2 \theta = \frac{1 - \cos 2\theta}{2} = \frac{1}{2} - \frac{1}{2}\cos 2\theta$$

$\therefore$ Period of $\sin^2 \theta = \frac{2\pi}{2} = \pi$

Exp. (c)

Given,

$R = \{(A, B) : A = P^{-1} BP$ for some invertible matrix $P\}$

For Statement I

(i) **Reflexive**

$ARA \Rightarrow A = P^{-1}AP$, which is true only, if $P = I$.

Since, $A = P^{-1}BP$ for some invertible matrix P.

$\therefore$ We can assume $P = I$

$\Rightarrow \quad ARA \Rightarrow A = I^{-1}AI$

$\Rightarrow \quad A = A$

$\Rightarrow R$ is Reflexive

Note *Here, due to some invertible matrix, P is used (reflexive) but if for all invertible matrix is used, then R is not reflexive.*

(ii) **Symmetric**

$ARB \Rightarrow A = P^{-1}BP$

$\Rightarrow \quad PAP^{-1} = P(P^{-1}BP)P^{-1}$

$\Rightarrow \quad PAP^{-1} = (PP^{-1})B(PP^{-1})$

$\therefore \quad B = PAP^{-1}$

Since, for some invertible matrix P, we can let $Q = P^{-1}$

$\therefore \quad B = (P-1)^{-1} AP^{-1}$

$\Rightarrow \quad B = Q^{-1} AQ$

$\Rightarrow BRA$

$\Rightarrow R$ is symmetric.

(iii) **Transitive**

ARB and BRC

$\Rightarrow \quad A = P^{-1}BP$

and $\quad B = P^{-1}CP$

$\Rightarrow \quad A = P^{-1}(P^{-1}CP)P$

$= (P^{-1})^2 C (P)^2$

So, ARC, for some $P^2 = P$

$\Rightarrow R$ is transitive

So, R is an equivalence relation.

For Statement II It is always true that

$$(MN)^{-1} = N^{-1}M^{-1}$$

Hence, both statements are true but second is not the correct explanation of first.

The contrapositive of $p \rightarrow \sim q$ is

$\sim(\sim q) \rightarrow \sim p$ or $q \rightarrow \sim p$

Also, converse of $q \rightarrow \sim p$ is $\sim p \rightarrow q$

Exp. (d)

Since, for every elements of A, there exists elements $(3, 3), (6, 6), (9, 9), (12, 12) \in R \Rightarrow R$ is reflexive relation.

Now, $(6, 12) \in R$ but $(12, 6) \notin R$, so it is not a symmetric relation.

Also, $\quad (3, 6), (6, 12) \in R$

$\Rightarrow \quad (3, 12) \in R$

$\therefore$ R is transitive relation.

Exp. (b)

$$y = \frac{1}{\sqrt{|x| - x}}$$

For domain, $|x| - x > 0 \Rightarrow |x| > x$

i.e., only possible, if $x < 0$.

$\therefore$ $x \in (-\infty, 0)$

Exp. **(a)**

Let $W = \{CAT, TOY, YOU, \ldots\}$

Clearly, R is reflexive and symmetric but not transitive.

$$[\because {}_{CAT}R_{TOY},\ {}_{TOY}R_{YOU} \nRightarrow {}_{CAT}R_{YOU}]$$

Exp. **(b)**

Given that, $f(x) = \log(x + \sqrt{x^2 + 1})$

Now, $\quad f(-x) = \log(-x + \sqrt{x^2 + 1})$

$$\therefore \quad f(x) + f(-x) = \log(x + \sqrt{x^2 + 1}) + \log(-x + \sqrt{x^2 + 1})$$

$$= \log(1) = 0$$

Hence, $f(x)$ is an odd function.

Exp. (b)

Statement I

$A = \{(x, y) \in R \times R : y - x \text{ is an integer}\}$

(a) **Reflexive**

$xRx : (x - x)$ is an integer.

i.e., true

$\therefore$ Reflexive

(b) **Symmetric**

$xRy : (x - y)$ is an integer.

$\Rightarrow -(y - x)$ is an integer.

$\Rightarrow (y - x)$ is an integer.

$\Rightarrow yRx$

$\therefore$ Symmetric

(c) **Transitive**

xRy and yRz

$\Rightarrow (x - y)$ is an integer and $(y - z)$ is an integer.

$\Rightarrow (x - y) + (y - z)$ is an integer.

$\Rightarrow (x - z)$ is an integer.

$\Rightarrow xRz$

$\therefore$ Transitive

Hence, A is an equivalence relation.

Statement II

$B = \{(x, y) \in R \times R : x = \alpha y \text{ for some rational number } \alpha\}$

If $\alpha = \frac{1}{2}$, then for reflexive, we have

$xRx \Rightarrow x = \frac{1}{2}x$, which is not true, $\forall\ x \in R - \{0\}$.

$\therefore$ B is not reflexive on R.

Hence, B is not an equivalence relation on R.

Hence, statement I is true, statement II is false.

Exp. **(d)**

Given that,

$$f(x) = 4^{-x^2} + \cos^{-1}\left(\frac{x}{2} - 1\right) + \log(\cos x)$$

Here, 4^{-x^2} is defined for $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$,

$\cos^{-1}\left(\frac{x}{2} - 1\right)$ is defined, if $-1 \le \frac{x}{2} - 1 \le 1$

$$\Rightarrow \quad 0 \le \frac{x}{2} \le 2$$

$$\Rightarrow \quad 0 \le x \le 4$$

And $\log(\cos x)$ is defined, if $\cos x > 0$.

$$\Rightarrow \quad -\frac{\pi}{2} < x < \frac{\pi}{2}$$

Hence, $f(x) = 4^{-x^2} + \cos^{-1}\left(\frac{x}{2} - 1\right) + \log(\cos x)$

is defined, if $x \in \left[0, \frac{\pi}{2}\right)$.

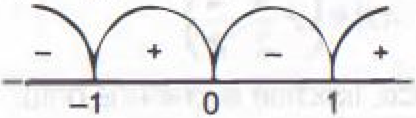
Exp. **(d)**

Given, $f(x) = \dfrac{3}{4 - x^2} + \log_{10}(x^3 - x)$

For domain of $f(x)$,

$$x^3 - x > 0$$

$\Rightarrow \quad x(x-1)(x+1) > 0$



$\Rightarrow \quad x \in (-1, 0) \cup (1, \infty)$ and $4 - x^2 \neq 0$

$\Rightarrow \quad x \neq \pm 2$

$\Rightarrow \quad x \in (-\infty, -2) \cup (-2, 2) \cup (2, \infty)$

So, common region is $(-1, 0) \cup (1, 2) \cup (2, \infty)$.

Exp. (b)

Given A set $X = \{1, 2, 3, 4, 5\}$

To find The number of different ordered pairs (Y, Z) such that $Y \subseteq X$, $Z \subseteq X$ and $Y \cap Z = \phi$

Since, $Y \subseteq X$, $Z \subseteq X$, hence we can only use the elements of X to construct sets Y and Z.

Method 1

$n(Y)$	Number of ways to make Y	Number of ways to make Z such that $Y \cap Z = \phi$
0	${}^5C_0$	2^5
1	${}^5C_1$	2^4
2	${}^5C_2$	2^3
3	${}^5C_3$	2^2
4	${}^5C_4$	2^1
5	${}^5C_5$	2^0

Let us explain anyone of the above 6 rows say third row. In third row,

Number of elements in $Y = 2$

$\therefore$ Number of ways to select $Y = {}^5C_2$ ways

Because any 2 elements of X can be part of Y.

Now, if Y contains any 2 elements, then these 2 elements cannot be used in any way to construct Z, because we want $Y \cap Z = \phi$. And from the remaining 3 elements which are not present in Y, 2^3 subsets can be made each of which can be equal to Z and still $Y \cap Z = \phi$ will be true.

Hence, total number of ways to construct sets Y and Z such that $Y \cap Z = \phi$

$$= {}^5C_0 \times 2^5 + {}^5C_1 \times 2^{5-1} + \dots + {}^5C_5 \times 2^{5-5}$$

$$= (2+1)^5 = 3^5$$

Exp. (a)

Since, $(1, 2) \in S$ but $(2, 1) \notin S$

So, S is not symmetric.

Hence, S is not an equivalence relation.

Given, $T = \{(x, y) : (x - y) \in I\}$

Now, $x - x = 0 \in I$, it is reflexive relation.

Again now, $(x - y) \in I$

$y - x \in I$, it is symmetric relation.

Let $\quad x - y = I_1$

and $\quad y - z = I_2$

Now, $\quad x - z = (x - y) + (y - z)$

$$= I_1 + I_2 \in I$$

So, T is also transitive.

Hence, T is an equivalence relation.

Exp. (d)

$$\sum_{r=1}^{n} f(r) = f(1) + f(2) + f(3) + \ldots + f(n)$$

$$= f(1) + 2f(1) + 3f(1) + \ldots + nf(1)$$

$$[\because f(x+y) = f(x) + f(y)]$$

$$= (1 + 2 + 3 + \ldots + n)f(1)$$

$$= f(1)\Sigma n$$

$$= \frac{7n(n+1)}{2} \qquad [\because f(1) = 7, \text{ given}]$$

Exp. (d)

Here, g is the inverse of $f(x)$.

$\Rightarrow \quad fog\ (x) = x$

On differentiating w.r.t. x, we get

$f'\{g(x)\} \times g'(x) = 1$

$$g'(x) = \frac{1}{f'\{g(x)\}} = 1 + \{g(x)\}^5 \left[\because f'(x) = \frac{1}{1+x^5}\right]$$

$\Rightarrow g'(x) = 1 + \{g(x)\}^5$

Exp. **(*d*)**

Given, $f(x) = (x+1)^2 - 1,\ x \geq -1$

$\Rightarrow \quad f'(x) = 2(x+1) \geq 0, \quad \text{for } x \geq -1$

$\Rightarrow$ $f(x)$ is one-one.

Since, codomain of the given function is not given, hence it can be considered as R, the set of real and consequently f is not onto.

Hence, f is not bijective. Statement II is false.

Also, $f(x) = (x+1)^2 - 1 \geq -1 \quad \text{for } x \geq -1$

$\Rightarrow \quad R_f = [-1, \infty)$

Clearly, $f(x) = f^{-1}(x)$ at $x = 0$ and $x = -1$

$\therefore$ Statement I is true.

Exp. (c)

Given that, $f(n) = \begin{cases} \frac{n-1}{2}, & \text{when } n \text{ is odd} \\ -\frac{n}{2}, & \text{when } n \text{ is even} \end{cases}$

and $f : N \to I$, where N is the set of natural numbers and I is the set of integers.

Let $x, y \in N$ and both are even.

Then, $$f(x) = f(y)$$

$$\Rightarrow \quad -\frac{x}{2} = -\frac{y}{2}$$

$$\Rightarrow \quad x = y$$

Again, $x, y \in N$ and both are odd.

Then, $$f(x) = f(y)$$

$$\Rightarrow \quad \frac{x-1}{2} = \frac{y-1}{2}$$

$$\Rightarrow \quad x = y$$

So, mapping is one-one.

Since, each negative integer is an image of even natural number and positive integer is an image of odd natural number. So, mapping is onto. Hence, mapping is one-one onto.

Exp. (d)

$\because \quad X = \{4^n - 3n - 1 : n \in N\}$

$X = \{0, 9, 54, 243, \ldots\}$ [put $n = 1, 2, 3, \ldots$]

$Y = \{9(n-1) : n \in N\}$

$Y = \{0, 1, 18, 27, \ldots\}$

It is clear that $X \subset Y$.

$\therefore \quad X \cup Y = Y$

Exp. **(c)**

Given $f(x) = x^3 + 5x + 1$

Now, $f'(x) = 3x^2 + 5 > 0, \ \forall \ x \in R$

Thus, $f(x)$ is strictly increasing function.

So, $f(x)$ is one-one function.

Clearly, $f(x)$ is a continuous function and also increasing on R.

$$\therefore \quad \lim_{x \to -\infty} f(x) = -\infty$$

$$\text{and} \quad \lim_{x \to \infty} = \infty$$

Hence, $f(x)$ takes every value between $-\infty$ and ∞.

Thus, $f(x)$ is onto function.

Exp. (b)

Given function $f(x) = \dfrac{\sin^{-1}(x-3)}{\sqrt{9-x^2}}$ will be defined, if

$$-1 \le (x-3) \le 1$$

$$\Rightarrow \quad 2 \le x \le 4 \quad \ldots(i)$$

and $\quad 9 - x^2 > 0$

$$\Rightarrow \quad -3 < x < 3 \quad \ldots(ii)$$

From Eqs. (i) and (ii), we get

$$2 \le x < 3$$

Hence, domain of the given function is [2, 3).

Exp. **(c)**

$$\Delta x \cdot \Delta v \geq \frac{h}{4\pi m}$$

$$\Delta x = \frac{6.63 \times 10^{-34}}{4 \times 3.14 \times 9.1 \times 10^{-31} \times 300 \times 0.001 \times 10^{-2}}$$

$$= 0.01933 = 1.93 \times 10^{-2} \text{ m}$$

(a) : $c = \upsilon\lambda$

$$\lambda = \frac{c}{\upsilon} = \frac{3\times10^{17}}{6\times10^{15}} = 50\text{ nm}$$

(c) : Elements (a), (b) and (d) belong to the same group since each one of them has two electrons in valence shell. In contrast, element (c) has seven electrons in the valence shell, and hence it lies in other group.

Exp. (b)

$$\lambda = \frac{h}{mV} = \frac{6.63 \times 10^{-34}}{1.67 \times 10^{-27} \times 1 \times 10^{3}}$$

$$= 3.97 \times 10^{-10}\ \text{m} \approx 0.40\ \text{nm}$$

(a) : No. of radial nodes in $3p$-orbital $= n - l - 1$

$= 3 - 1 - 1 = 1$

(d) : Abnormally high difference between 2^{nd} and 3^{rd} ionisation energy means that the element has two valence electrons, which is a case in configuration (d).

Exp. (d)

Energy required for 1 Cl_2 molecule

$$= \frac{242 \times 10^3}{N_A} \text{ J}$$

$$E = \frac{hc}{\lambda} \quad \text{or} \quad \lambda = \frac{hc}{E}$$

$$= \frac{6.626 \times 10^{-34} \times 3 \times 10^8 \times 6.02 \times 10^{23}}{242 \times 10^3}$$

$$= 494 \times 10^{-9} \text{ m} = 494 \text{ nm}$$

(b) : Both He and Li^{+} contain 2 electrons each.

(c) : Electronic configuration of an element is

$1s^2 2s^2 2p^6 3s^2 3p^6 3d^{10} 4s^2 4p^3$

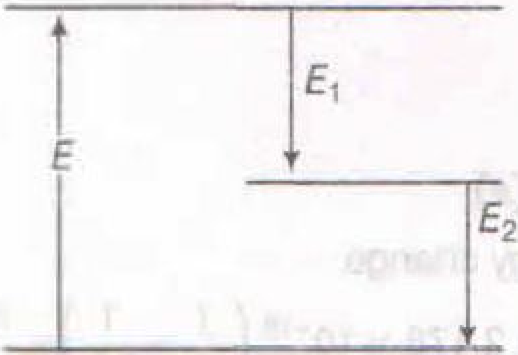
Hence it lies in fifth or 15^{th} group.

Exp. (c)

Energy values are always additive.

$$E_{total} = E_1 + E_2$$

$$\frac{hc}{\lambda} = \frac{hc}{\lambda_1} + \frac{hc}{\lambda_2}$$



$$\therefore \quad \frac{1}{\lambda} = \frac{1}{\lambda_1} + \frac{1}{\lambda_2}$$

$$\frac{1}{355} = \frac{1}{680} + \frac{1}{\lambda_2}$$

$$\lambda_2 = 742.77 \text{ nm} \approx 743 \text{ nm}$$

Exp. (a)

Heisenberg uncertainty principle

$$\Delta x \cdot m\Delta v \approx \frac{h}{4\pi}$$

Uncertainty in velocity,

$$\Delta v = \frac{h}{4 \times 3.14 \times \Delta x \times m}$$

$$= \frac{6.6 \times 10^{-34}}{4 \times 3.14 \times 0.025 \times 10^{-5}}$$

$$= 2.1 \times 10^{-28} \text{ m/s}$$

(a) : These are isoelectronic ions (ions with same number of electrons) and for isoelectronic ions, greater the positive nuclear charge, greater is the force of attraction on the electrons by the nucleus and the smaller is the size of the ion. Thus Al^{3+} has the smallest size.

Exp. (*b*)

	n	l	nl (suborbit)	$n + l$
(a)	4	1	$4p$	5
(b)	4	0	$4s$	4
(c)	3	2	$3d$	5
(d)	3	1	$3p$	4

Higher the value of $(n + l)$, higher is the energy. If $(n + l)$ are same, then suborbit with lower value of n has lower energy. Thus,

$$3p < 4s < 3d < 4p$$

$$(4) < (2) < (3) < (1)$$

Exp. (*b*)

$N^{3-} = 7 + 3 = 10$ electrons

$F^{-} = 9 + 1 = 10$ electrons

$Na^{+} = 11 - 1 = 10$ electrons

(c) : Among the halogens the electron affinity value of 'F' should be maximum. But due to small size the 7-electrons in its valence shell are much more crowded, so that it feels difficulty in entry of new electrons. Thus, the E.A. value is slightly lower than chlorine and the order is

$$I < Br < F < Cl.$$

Exp. (a)

Molar mass 108 g mol^{-1}

$$\text{Total part by weight} = 9 + 1 + 3.5 = 13.5$$

$$\text{Weight of carbon} = \frac{9}{13.5} \times 108 = 72 \text{ g}$$

$$\text{Number of carbon atoms} = \frac{72}{12} = 6$$

$$\text{Weight of hydrogen} = \frac{1}{13.5} \times 108 = 8 \text{ g}$$

$$\text{Number of hydrogen atoms} = \frac{8}{1} = 8$$

$$\text{Weight of nitrogen} = \frac{3.5}{13.5} \times 108 = 28 \text{ g}$$

$$\text{Number of nitrogen atom} = \frac{28}{14} = 2$$

Hence, molecular formula = $C_6H_8N_2$.

Exp. (*d*)

$$Fe_{26} = [Ar]\,3d^6, 4s^2$$

$$Fe^{2+}_{(24)} = [Ar]\,3d^6, 4s^0$$

(a) : Ionic radius in the n^{th} orbit is given by

$$r_n = \frac{n^2 a_0}{Z^*} \quad \text{or,} \quad r_n \propto \frac{1}{Z^*}$$

Where n is principal quantum number, a_0 the Bohr's radius of H-atom and Z^*, the effective nuclear charge.

Exp. (*c*)

560 g of Fe

$$\text{Number of moles} = \frac{560\text{ g}}{56\text{ g mol}^{-1}} = 10\text{ mol}$$

For 70 g of N

$$14\text{ g N} = 1\text{ mol of N-atom}$$
$$70\text{ g N} = 5\text{ mol of N-atom}$$

For 20 g of H

$$1\text{ g H} = 1\text{ mol of H-atom}$$
$$20\text{ g H} \equiv 20\text{ mol of H-atom}$$

Exp. (a)

Wavelength associated with the velocity of tennis ball.

$$\lambda = \frac{h}{mv} = 1105 \times 10^{-33} \text{ m}$$

(a) : $X - X$ bond F – F Cl – Cl Br – Br I – I

$X - X$ bond	F – F	Cl – Cl	Br – Br	I – I
Bond dissociation energy (kcal/mol)	38	57	45.5	35.6

The lower value of bond dissociation energy of fluorine is due to the high inter-electronic repulsion between non-bonding electrons in the 2*p*-orbitals of fluorine. As a result F – F bond is weaker in comparison to Cl – Cl and Br – Br bonds.

Exp. (*d*)

$$2Al(s) + 6HCl(aq) \longrightarrow 2Al^{3+}(aq) + 6Cl^-(aq) + 3H_2(g)$$

From the equation, it is clear that,

6 mol of HCl produces 3 mol of H_2

or 1 mole of HCl $= \frac{3 \times 22.4}{6}$ L of H_2 = 11.2 L of H_2

Exp. (c)

Any suborbit is represented as nl such that n is the principal quantum number (in the form of values) and l is the azimuthal quantum number (its name).

Value of $l < n$

$$l = 0 \quad 1 \quad 2 \quad 3 \quad 4$$
$$s \quad p \quad d \quad f \quad g$$

Value of $m: -l, -l+1 \ldots\ldots 0, \ldots\ldots +l$

Value of $s: +\frac{1}{2}$ or $-\frac{1}{2}$

Thus, for $4f$; $n = 4, l = 3, m =$ any value between -3 to $+3$ and $s =$ may be $\pm\frac{1}{2}$.

(b) : The atomic radii decrease on moving from left to right in a period, thus order of sizes for Cl, P and Mg is Cl < P < Mg. Down the group size increases. Thus overall order is : Cl < P < Mg < Ca.

Exp. (*d*)

Total mass of solution = 1000 g water + 120 g urea

$$= 1120 \text{ g}$$

$$\text{Density of solution} = 1.15 \text{ g / mL}$$

$$\text{Thus, volume of solution} = \frac{\text{mass}}{\text{density}} = \frac{1120 \text{ g}}{1.15 \text{ g / mL}}$$

$$= 973.91 \text{ mL} = 0.974 \text{ L}$$

$$\text{Moles of solute} = \frac{120}{60} = 2 \text{ mol}$$

$$\text{Molarity} = \frac{\text{moles of solute}}{\text{volume (L) of solution}}$$

$$= \frac{2 \text{ mol}}{0.974 \text{ L}} = 2.05 \text{ mol L}^{-1}$$

Exp. (a)

On applying Rydberg formula,

$$\frac{1}{\lambda} = \bar{\nu}_H = R_H \left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$$

$$= 1.097 \times 10^7 \left[\frac{1}{1^2} - \frac{1}{\infty^2}\right]$$

$$\therefore \quad \lambda = \frac{1}{1.097 \times 10^7} \text{ m}$$

$$= 9.11 \times 10^{-8} \text{m}$$

$$= 91.1 \times 10^{-9} \text{ m}$$

$$= 91.1 \text{ nm} \qquad [\because 1 \text{ nm} = 10^{-9} \text{ m}]$$

(c) : $S^{2-} > Cl^- > K^+ > Ca^{2+}$

Among isoelectronic species, ionic radii increases with increase in negative charge. This happens because effective nuclear charge (Z_{eff}) decreases. Similarly, ionic radii decreases with increase in positive charge as Z_{eff} increases.

Exp. (a)

The molarity of a resulting solution is given by

$$M = \frac{M_1V_1 + M_2V_2}{V_1 + V_2}$$

$$= \frac{750 \times 0.5 + 250 \times 2}{750 + 250} = \frac{875}{1000} = 0.875\,\text{M}$$

Exp. (a)

(A) $1s$ (B) $2s$

(C) $2p$ (D) $3d$

(E) $3d$

In the absence of any field, $3d$ in (D) and (E) will be of equal energy.

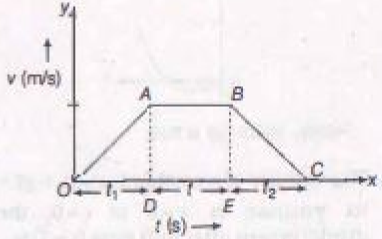
(a) : $Na \rightarrow Na^{+} + e^{-}$; $\Delta H = 5.1$ eV

$Na^{+} + e^{-} \rightarrow Na$; $\Delta H = -5.1$ eV

Exp. (*d*)

The velocity-time graph for the given situation can be drawn as below.

Magnitudes of slope of $OA = f$



and slope of $BC = \frac{f}{2}$

$$v = ft_1 = \frac{f}{2} t_2$$

$$\therefore \quad t_2 = 2t_1$$

In graph, area of ΔOAD gives distance,

$$s = \frac{1}{2} ft_1^2 \qquad \ldots\text{(i)}$$

Area of rectangle $ABED$ gives distance travelled in time t.

$$s_2 = (ft_1)t$$

Distance travelled in time t_2,

$$s_3 = \frac{1}{2}\frac{f}{2}(2t_1)^2$$

Thus, $\quad s_1 + s_2 + s_3 = 15s$

$\Rightarrow \quad s + (ft_1)t + ft_1^2 = 15s$

or $\quad s + (ft_1)t + 2s = 15s \qquad \left[\because s = \frac{1}{2} ft_1^2\right]$

or $\quad (ft_1)t = 12s \qquad \ldots\text{(ii)}$

From Eqs. (i) and (ii), we have

$$\frac{12s}{s} = \frac{(ft_1)t}{\frac{1}{2}(ft_1)t_1}$$

or $\quad t_1 = \frac{t}{6}$

From Eqs. (i) and (ii), we get

$$s = \frac{1}{2} f(t_1)^2$$

Hence, none of the given options is correct.

Exp. (b)

From Newton's equations, we have

$$v^2 = u^2 - 2as$$

Given, $v = 0$ [car is stopped]

As friction provide the retardation

$$a = \mu g, v = 100 \text{ ms}^{-1}$$

$$\therefore \quad (100)^2 = 2 \mu gs$$

$$\Rightarrow \quad s = \frac{100 \times 100}{2 \times 0.5 \times 10}$$

$$= \frac{100 \times 100}{5 \times 2} = 1000 \text{ m}$$

37. (b) : Impulse = Change in momentum

$$F \cdot \Delta t = m \cdot v\ ;\ F = \frac{m \cdot v}{\Delta t} = \frac{150 \times 10^{-3} \times 20}{0.1} = 30 \text{ N}.$$

Exp. (a)

Given, $v = \alpha\sqrt{x}$

or $\dfrac{dx}{dt} = \alpha\sqrt{x}$ $\left[\because v = \dfrac{dx}{dt}\right]$

or $\dfrac{dx}{\sqrt{x}} = \alpha\, dt$

On integrating, we get

$$\int_0^x \frac{dx}{\sqrt{x}} = \int_0^t \alpha\, dt$$

[$\because$ at $t = 0$, $x = 0$ and let at any time t, particle be at x]

$\Rightarrow$ $\left[\dfrac{x^{1/2}}{1/2}\right]_0^x = \alpha t$ or $x^{1/2} = \dfrac{\alpha}{2} t$

or $x = \dfrac{\alpha^2}{4} \times t^2$ or $x \propto t^2$

Exp. (c)

This is the question based on impulse-momentum theorem.

$$|F \cdot \Delta t| = |\text{Change in momentum}|$$

$$\Rightarrow \quad F \times 0.1 = |p_f - p_i|$$

As the ball will stop after catching

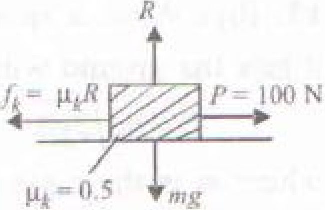
$$p_i = mv_i = 0.15 \times 20 = 3, \; p_f = 0$$

$$\Rightarrow \quad F \times 0.1 = 3$$

$$\Rightarrow \quad F = 30 \text{ N}$$

34. (b) : $m = 10$ kg,

$R = mg$



$\therefore$ Frictional force $= f_k$

$= \mu_k R = \mu_k mg$

$= 0.5 \times 10 \times 10$

$= 50$ N [$g = 10$ m/sec^2]

$\therefore$ Net force acting on the body $= F = P - f_k$

$= 100 - 50 = 50$ N.

$\therefore$ Acceleration of the block $= a = F/m$

$= 50/10 = 5$ m/sec^2.

Exp. (b)

Given, $v = v_0 + gt + ft^2$

After differentiating with respect to time, we get

$$\frac{dx}{dt} = v_0 + gt + ft^2$$

$$\Rightarrow \quad dx = (v_0 + gt + ft^2)\,dt$$

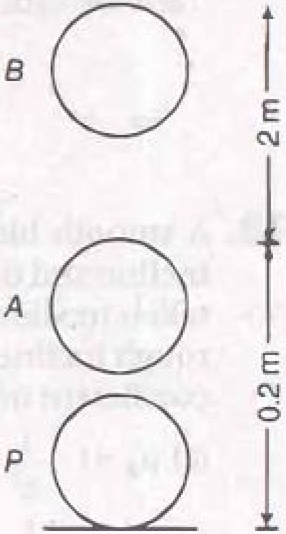
So, $$\int_0^x dx = \int_0^1 (v_0 + gt + ft^2)\,dt$$

$$\Rightarrow \quad x = v_0 + \frac{g}{2} + \frac{f}{3}$$

Since, vertical component of velocity is zero.

Exp. (*d*)

The situation is shown in figure. At initial time, the ball is at P, then under the action of a force (exerted by hand) from P to A and then from A to B let acceleration of ball during PA be a ms^{-2} (assumed to be constant) in upward direction and velocity of ball at be is v m/s.



Then, for PA,

$$v^2 = 0^2 + 2a \times 0.2$$

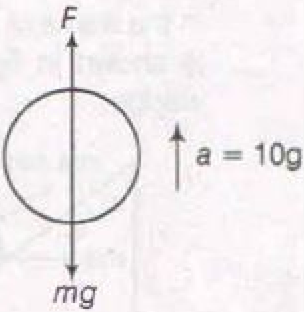
For AB,

$$0 = v^2 - 2 \times g \times 2$$

$$\Rightarrow \quad v^2 = 2g \times 2$$

From above equations,

$$a = 10g = 100 \text{ ms}^{-2}$$



Then, for PA, FBD of ball is

$$F - mg = ma$$

[F is the force exerted by hand on ball]

$$\Rightarrow \quad F = m(g + a)$$

$$= 0.2(11g) = 22 \text{ N}$$

Alternate Method Using work-energy theorem

$$W_{mg} + W_F = 0$$

$$\Rightarrow \quad -mg \times 2.2 + F \times 0.2 = 0$$

or $\quad F = 22$ N

31. (d) : When the lift is accelerating upwards with acceleration a, then reading on the scale

$$R = m(g + a) = 80(10 + 5)\ \text{N} = 1200\ \text{N}.$$

Exp. (c)

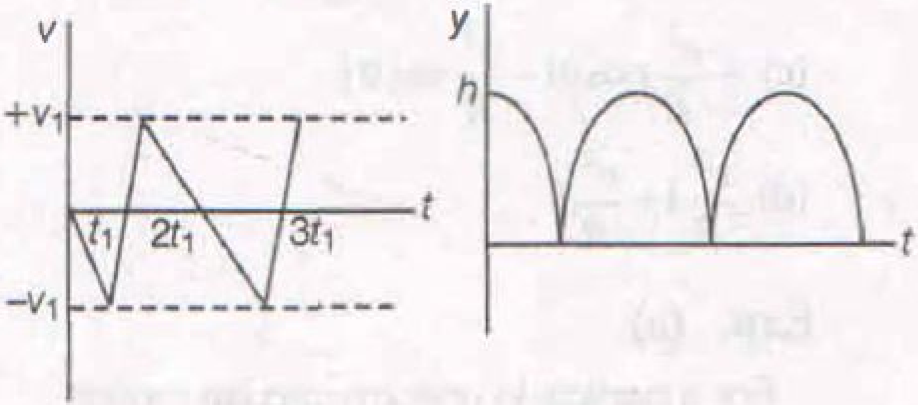
As we know that for vertical motion,

$$h = \frac{1}{2} g t^2 \qquad \text{[parabolic]}$$

$v = -gt$ and after the collision,

$v = gt$ (straight line).

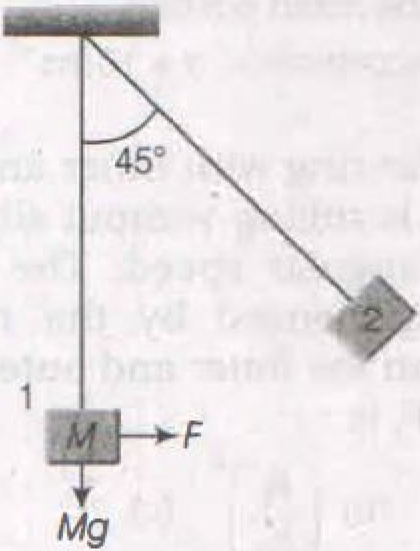
Collision is perfectly elastic, then ball reaches to same height again and again with same velocity.



Hence, option (c) is true.

Exp. (d)

Here, the constant horizontal force required to take the body from position 1 to position 2 can be calculated by using work-energy theorem. Let us assume that body be taken slowly, so that its speed does not change, then



$$\Delta K = 0 = W_F + W_{Mg} + W_{\text{tension}}$$

[symbols have their usual meanings]

$$W_F = F \times l \sin 45° = \frac{Fl}{\sqrt{2}}$$

$$W_{Mg} = Mg(l - l\cos 45°), W_{\text{tension}} = 0$$

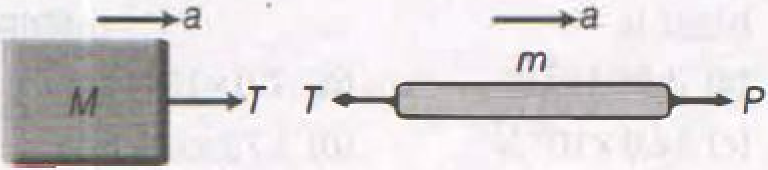
$$\therefore \quad F = Mg(\sqrt{2} - 1)$$

Exp. (d)

Let acceleration of system (rope + block) be a along the direction of applied force. Then,

$$a = \frac{P}{M + m}$$

Draw the FBD of block and rope as shown in figure.

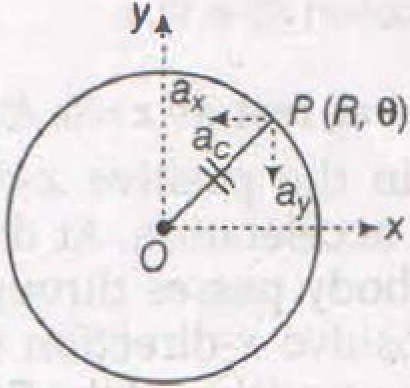


where, T is the required parameter.

For block, $T = Ma \quad \Rightarrow T = \frac{MP}{M + m}$

Exp. (c)

For a particle in uniform circular motion,



$$\mathbf{a} = \frac{v^2}{R} \text{ towards centre of circle}$$

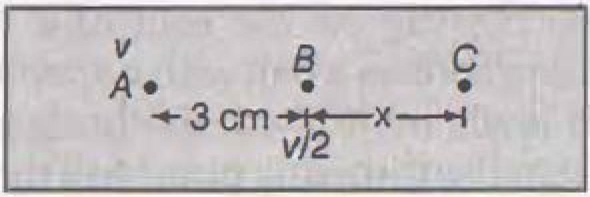
[centripetal acceleration]

$$\therefore \qquad \mathbf{a} = \frac{v^2}{R}(-\cos\theta\mathbf{i} - \sin\theta\,\mathbf{j})$$

or $$\mathbf{a} = -\frac{v^2}{R}\cos\theta\,\mathbf{i} - \frac{v^2}{R}\sin\theta\,\mathbf{j}$$

Exp. (*a*)

Let initial velocity of body at point A be v, AB is 3 cm.



From $$v^2 = u^2 - 2as$$

$\Rightarrow$ $$\left(\frac{v}{2}\right)^2 = v^2 - 2a \times 3$$

or $$a = \frac{v^2}{8}$$

Let on penetrating 3 cm in a wooden block, the body moves x distance from B to C.

So, for B to C, $u = \frac{v}{2}$, $v = 0$,

$$s = x, \quad a = \frac{v^2}{8} \quad \text{[deceleration]}$$

$\therefore$ $$(0)^2 = \left(\frac{v}{2}\right)^2 - 2 \cdot \frac{v^2}{8} \cdot x$$

or $$x = 1\,\text{cm}$$

Note Here, it is assumed that retardation is uniform.

Exp. (c)

Let coefficient of friction be μ, then retardation will be μg. From equation of motion, $v = u + at$

$$\Rightarrow \quad 0 = 6 - \mu g \times 10$$

$$\text{or} \quad \mu = \frac{6}{100} = 0.06$$

Exp. (c)

As the force is exponentially decreasing, so its acceleration, *i.e.*, rate of increase of velocity will decrease with time. Thus, the graph of velocity will be an increasing curl with decreasing slope with time.

$$\therefore \qquad a = \frac{F}{M} = \frac{F_0 e^{-bt}}{m} = \frac{dv}{dt}$$

$$\Rightarrow \qquad \int_0^v dv = \int_0^t \frac{F_0}{m} e^{-bt} dt$$

$$\Rightarrow \qquad v = \frac{F_0}{m}\left[\left(\frac{1}{-b}\right)e^{-bt}\right]_0^t$$

$$= \frac{F_0}{mb}[e^{-bt}]_t^0$$

$$= \frac{F_0}{mb}(e^0 - e^{-bt}) = \frac{F_0}{mb}(1 - e^{-bt})$$

$$v_{max} = \frac{F_0}{mb}$$

Exp. (c)

At the highest point of its flight, vertical component of velocity is zero and only horizontal component is left which is

$$u_x = u\cos\theta$$

Given, $\theta = 45°$

$$\therefore \quad u_x = u\cos 45° = \frac{u}{\sqrt{2}}$$

Hence, at the highest point, kinetic energy is

$$E' = \frac{1}{2}mu_x^2$$

$$= \frac{1}{2}m\left(\frac{u}{\sqrt{2}}\right)^2 = \frac{1}{2}m\left(\frac{u^2}{2}\right)$$

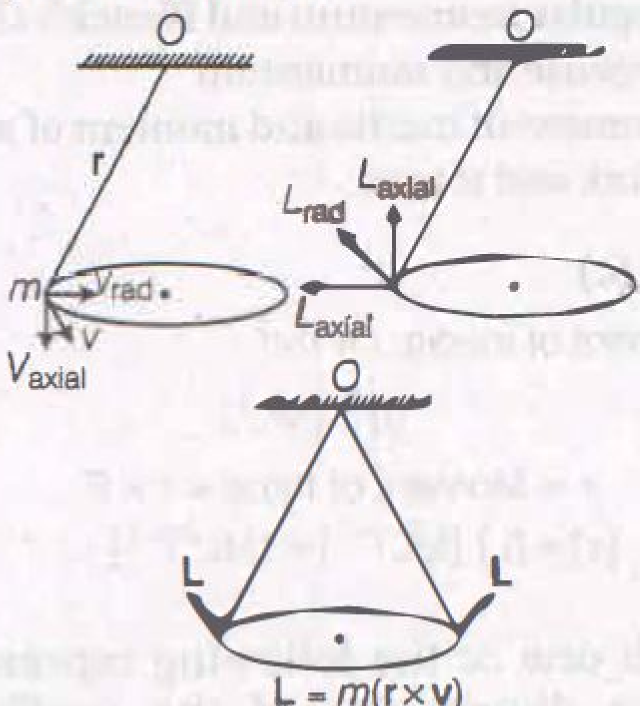
$$= \frac{E}{2} \qquad \left[\because \frac{1}{2}mu^2 = E\right]$$

Exp. (*d*)

Resultant force is zero, as three forces acting on the particle can be represented in magnitude and direction by three sides of a triangle in same order. Hence, by Newton's 2nd law $\left(\mathbf{F} = m\frac{d\mathbf{v}}{dt}\right)$, the velocity ($\mathbf{v}$) of particle will be same.

Exp. (c)

Angular momentum of the pendulum about the suspension point O is



Then, v can be resolved into two components, radial component v_{rad} and axial component v_{axial}. Due to v_{rad}, will be axial and due to v_{axial}, L will be radially outwards as shown.

Exp. (b)

The given coordinate

$$x = \alpha t^3, \quad y = \beta t^3$$

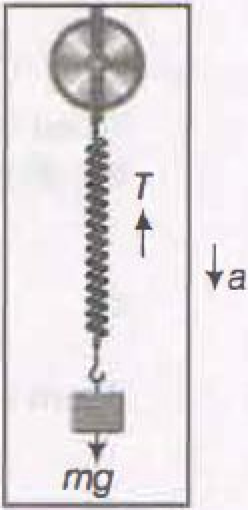
Then, $v_x = \dfrac{dx}{dt} = 3\alpha t^2$ and $v_y = \dfrac{dy}{dt} = 3\beta t^2$

$\therefore$ Resultant velocity, $v = \sqrt{v_x^2 + v_y^2}$

$$= \sqrt{9\alpha^2 t^4 + 9\beta^2 t^4}$$

$$= 3t^2\sqrt{\alpha^2 + \beta^2}$$

Exp. (a)



In stationary position, spring balance reading = $mg = 49$

or $$m = \frac{49}{9.8} = 5\,\text{kg}$$

When lift moves downward,

$$mg - T = ma$$

Reading of balance,

$$\begin{aligned} T &= mg - ma \\ &= 5(9.8 - 5) \\ &= 5 \times 4.8 \\ &= 24.0\,\text{N} \end{aligned}$$

Exp. (b)

Planck's constant (in terms of unit)

$$h = \text{J-s} = [ML^2T^{-2}][T] = [ML^2T^{-1}]$$

Momentum $(p) = \text{kg - ms}^{-1} = M][L][T^{-1}] = [MLT^{-1}]$

Exp. (a)

Man will catch the ball, if the horizontal component of velocity becomes equal to the constant speed of man, *i.e*,

$$v_0 \cos \theta = \frac{v_0}{2}$$

or $$\cos \theta = \frac{1}{2}$$

$\Rightarrow$ $$\cos \theta = \cos 60^\circ$$

or $$\theta = 60^\circ$$

Exp. (*d*)

Force exerted by machine gun on man's hand in firing a bullet = Change in momentum per second on a bullet or rate of change of momentum

$$= \left(\frac{40}{1000}\right) \times 1200 = 48 \text{ N}$$

Force exerted by man on machine gun

$$= \text{Force exerted on man by machine gun}$$

$$= 144 \text{ N}$$

Hence, number of bullets fired $= \frac{144}{48} = 3$.

Exp. (b)

Magnetic energy $= \frac{1}{2} L I^2 = \frac{Lq^2}{2t^2}$ $\left[\text{as } I = \frac{q}{t}\right]$

where L = inductance, I = current

Energy has the dimensions $= [ML^2T^{-2}]$

Equate the dimensions, we have

$$[ML^2T^{-2}] = [\text{henry}] \times \frac{[Q^2]}{[T^2]}$$

$$\Rightarrow \qquad [\text{henry}] = \frac{[ML^2]}{[Q^2]}$$

Exp. (*b*)

For a particle moving in a circle with constant angular speed, velocity vector is always along the tangent to the circle and the acceleration vector always points towards the centre of circle or is always along radius of the circle.

Since, tangential vector is perpendicular to radial vector, therefore velocity vector will be perpendicular to the acceleration vector. But in no case, acceleration vector is tangent to the circle.

Exp. (c)

According to work-energy theorem,

$$\text{Work done} = \text{Change in kinetic energy}$$

$$W = \Delta K = 0$$

$\Rightarrow$ Work done by friction + Work done by gravity $= 0$

$$\Rightarrow \quad -(\mu\, mg \cos\phi)\frac{l}{2} + mgl \sin\phi = 0$$

or $$\frac{\mu}{2}\cos\phi = \sin\phi$$

or $$\mu = 2\tan\phi$$

Exp. (a)

So, momentum,

$$p = mv = 17.565 \text{ kg-ms}^{-1}$$

where $m = 3.513$ kg and $v = 5.00 \text{ ms}^{-1}$

As the number of significant digits in m is 4 and in v is 3 so, p must have 3 (minimum) significant digits. Hence,

$$p = 17.6 \text{ kg-ms}^{-1}$$

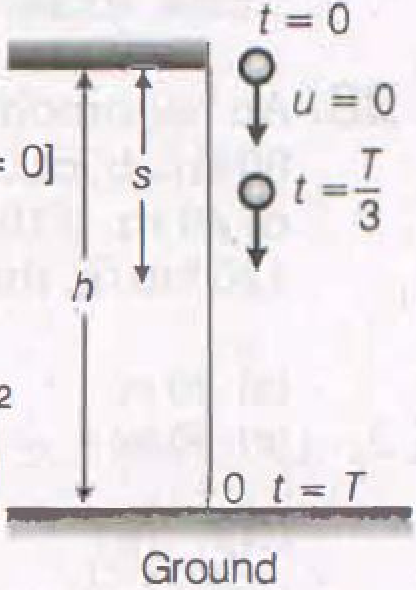
Exp. (c)

Second law of motion gives

$$s = ut + \frac{1}{2}gT^2$$

or $\quad h = 0 + \frac{1}{2}gT^2 \quad [\because u = 0]$

$\Rightarrow \quad T = \sqrt{\left(\frac{2h}{g}\right)}$



At $t = \frac{T}{3}$ s, $s = 0 + \frac{1}{2}g\left(\frac{T}{3}\right)^2$

or $\quad s = \frac{1}{2}g \cdot \frac{T^2}{9}$

$\Rightarrow \quad s = \frac{g}{18} \times \frac{2h}{g} \quad \left[\because T = \sqrt{\frac{2h}{g}}\right]$

or $\quad s = \frac{h}{9}$ m

Hence, the position of ball from the ground

$$= h - \frac{h}{9} = \frac{8h}{9} \text{ m}$$

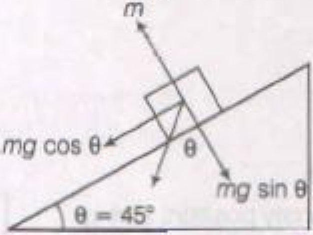
Exp. (*a*)

When friction is absent

$$ma_1 = mg\sin\theta$$

$$a = g\sin\theta$$

$$\therefore \quad s_1 = \frac{1}{2}a_1 t_1^2 \quad \ldots(\text{i})$$



When friction is present, friction is in opposite to the direction of motion

$$a_2 = g\sin\theta - \mu_k g\cos\theta$$

$$\therefore \quad s_2 = \frac{1}{2}a_2 t_2^2 \quad \ldots(\text{ii})$$

From Eqs. (i) and (ii), we get

$$\frac{1}{2}a_1 t_1^2 = \frac{1}{2}a_2 t_2^2$$

$$\Rightarrow \quad a_1 t_1^2 = a_2 (nt_1)^2 \quad [\because t_2 = nt_1]$$

$$\text{or} \quad a_1 = n^2 a_2$$

$$\Rightarrow \quad \frac{a_2}{a_1} = \frac{g\sin\theta - \mu_k g\cos\theta}{g\sin\theta} = \frac{1}{n^2}$$

$$\text{or} \quad \frac{g\sin 45^\circ - \mu_k g\cos 45^\circ}{g\sin 45^\circ} = \frac{1}{n^2}$$

$$\text{or} \quad 1 - \mu_k = \frac{1}{n^2}$$

$$\text{or} \quad \mu_k = 1 - \frac{1}{n^2}$$

JEE Mains Solutions

The solutions are lined up in sequential order from 1 to 90.

For the questions, the student can revisit the test link for taking reference.