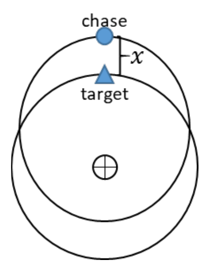
Astro Engr 423 – Homework

Name:

Due: Lesson 7

You may receive help from any person; however, you are required to turn in your own homework. This assignment must be documented in accordance with the policies explained in the DFAS Policy Letter and course handbook. Show all work – just writing the answer is not sufficient. If you use Matlab or some other software, provide a copy of your code. Submit your homework with this coversheet and per the homework format guidance in the course handbook.

Documentation:

Exercise 1 (40 pts)
In class, we discussed that the condition for periodic motion in the HCW equations ($\dot{y}_0 + 2n_T x_0 = 0$) will only yield periodic relative motion to the first-order, i.e., only for the HCW model. Additionally, there exist initial condition sets which satisfy the energy matching condition and therefore yield periodic relative motion but violate the HCW condition. Demonstrate this by doing the following: a) Construct a scenario where the chase satellite is in an elliptical orbit with $a_C = a_T = 7000\ km$, and the target satellite orbit is circular ($e_T = 0$). Assume the chase satellite begins at the apogee of its orbit at a distance x km from the target satellite in the $\hat{x}$ direction (see the diagram below).    b) (20 pts) Develop expressions for the following quantities as functions of x: • $\vec{\rho}\|_R$: Initial relative position of the chase with respect to the target, resolved (written) in the RIC frame. • ${}^{I}\dot{\vec{\rho}}\|_R$: Initial relative velocity between the chase and target with respect to the inertial frame, resolved in the RIC frame. ***Hint: to calculate relative inertial velocity, use the two-body problem velocity equations from A310 to calculate inertial velocities for the target and chase, then subtract these two velocities to obtain relative inertial velocity.*** • Use the prior two results and transport theorem to develop an expression for ${}^{R}\dot{\vec{\rho}}\|_R$: the initial relative velocity between the chase and target with respect to the RIC frame as a function of x, resolved in the RIC frame.

c) (10 pts) Using Matlab and the expressions you developed above, vary the value of x from 0 km to 500 km in steps of 5 km and calculate the value of the HCW condition for periodic motion and the energy matching condition for each x-value.
Plot both of these conditions vs x. You should see that while the value of the energy matching condition remains constant and ~0, the value of the HCW condition diverges from 0 as x increases. ***Explain this result.***

d) (10 pts) Simulate the relative motion for one period of the target's orbit using the two relative motion models we have learned so far:
- Numerically integrate the "General non-linear EOM for ProxOps" for the two cases where x=5 km and x=500 km. For your numerical integration, assume that $\theta_T(t = 0) = 0^o$.
- Calculate the trajectory for these same two cases over one period using the solution of the HCW equations.

Plot the trajectory resulting from each method in the x-y plane. You should see that while the numerically integrated trajectory remains periodic for both cases, the HCW trajectory with x=500 km is not periodic.

Hint: To complete parts c and d, use the provide matlab script A423_HW03.m and fill in the missing portions of code, to include writing the function HCW.m, or resusing your HCW function from Astro 321.