Discussion 01

Number Representation

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About Me

- Grade: Junior
- # Teaching Sem: 4
  - 61a: 3
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- Major: Computer Science
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- Hobbies:
  - Reading, Gymming, Bouldering (?)

Announcements 📣

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Agenda

- Intro
- Unsigned
- Sign-Magnitude
- 2's Complement
- Q & A

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Number Representation

- Everything is bits!
- Numbers
  - 19 -> `0b00001011`
- ASCII Characters
  - Even letters and punctuation can be represented with bits
  - `a` -> `0b1100001`
- Computer Instructions
  - `3 + 0` -> `0b00000000001100000000000010010011`

Conversions between Representations

- Base a to decimal
  - __ __ __ __ __ __ __ __
    a^7 a^6 a^5 a^4 a^3 a^2 a^1 a^0
  - Example: 61
  - Binary: `0b0111101`
  - Octal (Base 8): `75`
  - Hex (Base 16): `3D`
- Binary to Hex
  - Convert groups of 4 bits at a time
  - Example: `0b00111100`
  - What if length is not multiple of `4`?

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Speedy Conversions

- `2^n -> n+1` bit is `1`, rest are `0`
  - `64 = 2^6 = 0b01000000`
- `2^n - 1 -> n-1` bits that are all 1
  - `63 = 2^6 - 1 = 0b00111111`

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Unsigned

- Range: `[0, 2^n - 1]`
- `__ __ __ __ __ __ __ __`
  `2^7 2^6 2^5 2^4 2^3 2^2 2^1 2^0`

What about negative numbers?

Q: How to identify if a number is representable or not in a certain binary notation?

A:

- Check if the number is in the range of the representable numbers
- As a clarifying check, try to convert it to binary, then look at the binary and convert it back - is it the same number?

Sign-Magnitude Notation

- Range: `[-(2^n¯1 - 1), 2^n¯1-1]`
- `____ __ __ __ __ __ __ __`
  `sign 2⁶ 2⁵ 2⁴ 2³ 2² 2¹ 2⁰`
- `-1^sign * magnitude` (convert normally)
- Pros
- Cons
  - `+0` and `-0`
  - addition?

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Two's Complement

- Range: $[-2^{n-1}, 2^{n-1}-1]$
- ____ __ __ __ __ __ __ __
  $-2^7\ 2^6\ 2^5\ 2^4\ 2^3\ 2^2\ 2^1\ 2^0$
- `-1^sign * magnitude` (convert normally)
- Positive: same as unsigned
- Negative: `~(positive) + 1`

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Bias Notation

- Range: `[bias, 255 + bias]`
- Applying a shift to the unsigned interpretation of number
- Bias is typically negative
  - Standard bias: $-(2^{n-1}-1)$
- Converting to bias notation
  - `x + b = n`
  - `x` = binary in bias notation
  - `b` = chosen bias
  - `n` = number want to represent
- When is bias notation useful?

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Thank you!

Feedback

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