Content

- Adaptive Control of Manipulators (not included in exam)

Adaptive Control

- In the previous section, we saw that if the model parameters are correct, then we achieve zero tracking error:

$$\ddot{E} + K_v\dot{E} + K_pE = 0$$

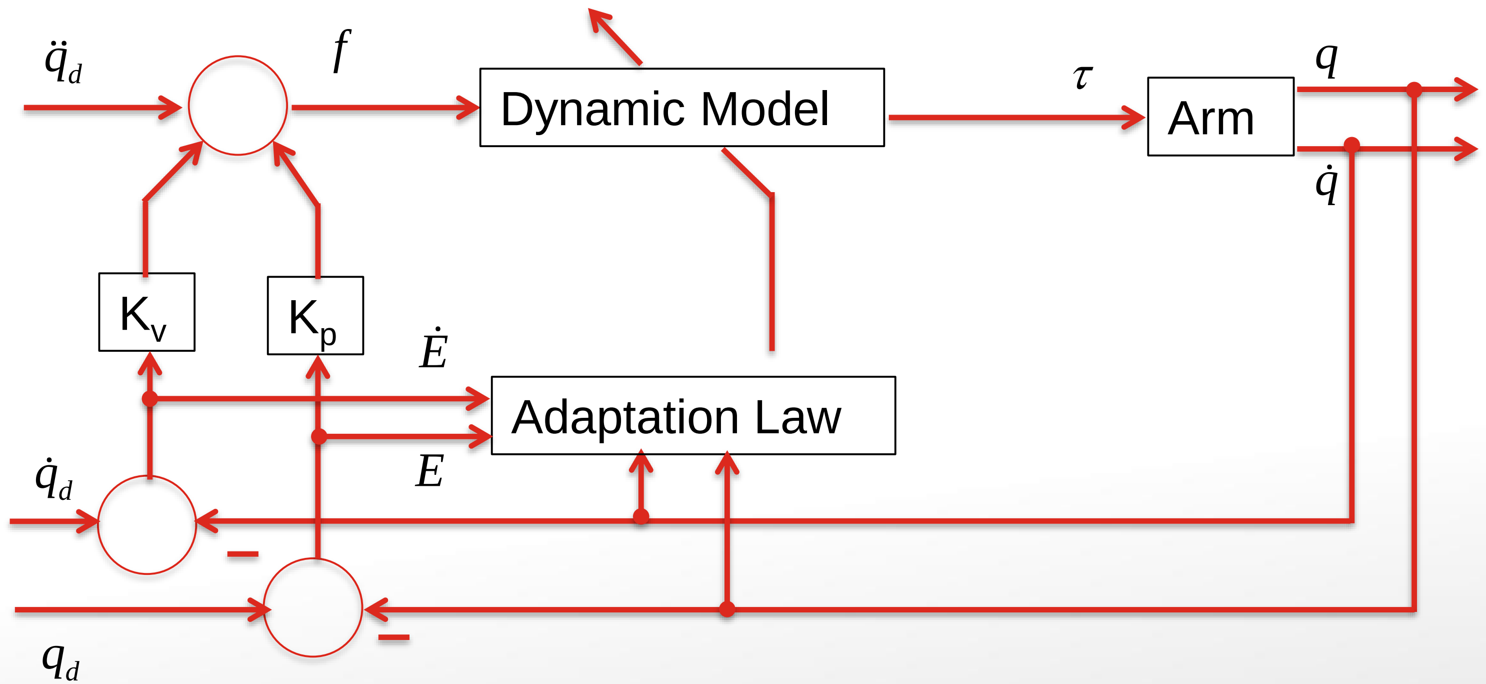
- However, if the model parameters do not match the real parameters, this will result in servo errors:

$$\ddot{E} + K_v\dot{E} + K_pE = \hat{M}^{-1}\left[\left(M - \hat{M}\right)\ddot{q} + \left(V - \hat{V}\right) + \left(G - \hat{G}\right) + \left(F - \hat{F}\right)\right]$$

- The adaptive control is based on the idea that the model parameters could be continuously updated until the servo error diminishes.

Adaptive Control

- One adaptive control scheme is as follows:



- If there is servo error (E and E-dot), the adaptation law will adjust the parameters in the dynamic model, until the error disappears.
- The system learns its own dynamic properties.

Indirect Adaptive Control

- Let’s derive one adaptive control scheme here. (There are many other algorithms and this is just one of them).
- The manipulator’s dynamic equation is:

$$M(q)\ddot{q}+V(q,\dot{q})+G(q)=\tau$$

- The model structure is known.
- However, the parameters are not accurately known (or they may change).
- Now, the land hand side of the equation can be written in a Linear-in-the-Parameters form:

$$M(q)\ddot{q}+V(q,\dot{q})+G(q)=\varphi^{T}\theta=\tau$$

  - where θ is the unknown parameters, and
  - φ is the known regressor.

Indirect Adaptive Control

- Aside: What does linear-in-parameters mean?
- It means the parameters can be separated into a vector.
- E.g. 2R robot:

$$\tau_1 = m_2 L_2^2\left(\ddot{\theta}_1+\ddot{\theta}_2\right)+m_2 L_1 L_2 c_2\left(2\ddot{\theta}_1+\ddot{\theta}_2\right)+\left(m_1+m_2\right)L_1^2\ddot{\theta}_1 - m_2 L_1 L_2 s_2 \dot{\theta}_2^2$$
$$-2m_2 L_1 L_2 s_2 \dot{\theta}_1\dot{\theta}_2+\left(m_1+m_2\right)gL_1c_1+m_2 g L_2 c_{12}$$
$$\tau_2 = m_2 L_1 L_2 c_2 \ddot{\theta}_1 + m_2 L_1 L_2 s_2 \dot{\theta}_1^2 + m_2 g L_2 c_{12} + m_2 L_2^2\left(\ddot{\theta}_1+\ddot{\theta}_2\right)$$

$$\begin{bmatrix}\tau_1\\ \tau_2\end{bmatrix} = \underbrace{\begin{bmatrix}\left(\ddot{\theta}_1+\ddot{\theta}_2\right) & c_2\left(2\ddot{\theta}_1+\ddot{\theta}_2\right)-s_2\dot{\theta}_2^2-2s_2\dot{\theta}_1\dot{\theta}_2 & \ddot{\theta}_1 & gc_1 & gc_{12}\\ \left(\ddot{\theta}_1+\ddot{\theta}_2\right) & c_2\ddot{\theta}_1+s_2\dot{\theta}_1^2 & 0 & 0 & gc_{12}\end{bmatrix}}_{\varphi^T}\underbrace{\begin{bmatrix}m_2L_2^2\\ m_2L_1L_2\\ \left(m_1+m_2\right)L_1^2\\ \left(m_1+m_2\right)L_1\\ m_2L_2\end{bmatrix}}_{\theta}$$

Indirect Adaptive Control

- The initially-inaccurate model can be written as:

$$\hat{M}(q)\ddot{q}+\hat{V}(q,\dot{q})+\hat{G}(q)=\varphi^T\hat{\theta}$$

  - where the “hat” symbol means estimates.

- We define:
  - $\tilde{\theta}=\theta-\hat{\theta}$ as the parameter estimation error.
  - $e=\varphi^T\theta-\varphi^T\hat{\theta}=\varphi^T\tilde{\theta}$ as the model error.
    - This model error is available to us, because:
    - $M(q)\ddot{q}+V(q,\dot{q})+G(q)=\varphi^T\theta=\tau$ is the torque values at the joints, which can be measured.
    - $\hat{M}(q)\ddot{q}+\hat{V}(q,\dot{q})+\hat{G}(q)=\varphi^T\hat{\theta}$ is calculated by multiplying the measured regressor (joint acceleration, velocity, angles) with the parameter estimates.

Indirect Adaptive Control

- How can we update the parameter estimates $\hat{\theta}$ such that it gets closer and closer to θ ?
- We will use the so-called Lyapunov method as a design tool.
- What is it?

Lyapunov Stability Analysis

- In the 19th century, a Russian mathematician, Lyapunov, introduced the method of deducing the stability of a system, by analyzing its energy and the rate of change of the energy.

- The idea is intuitive:
  - Assume the energy of a system is always non-negative (positive or zero).
  - If the rate of change of energy is negative, it means the system is losing energy.
    - System will slow down and eventually stops.
    - Asymptotically stable!
  - If the rate of change of energy is positive, then the is gaining energy.
    - System will speed up and eventually grow unbounded.
    - Unstable!

Lyapunov Stability Analysis

- Example: mass-spring-damper system: $m\ddot{x}+b\dot{x}+kx=0$
- The total energy of the system, V, is given by:

$$V=\frac{1}{2}m\dot{x}^2+\frac{1}{2}kx^2$$

  - Note that this energy is always non-negative.
- The rate of change of the energy is obtained by differentiating V wrt. time:

$$\dot{V}=m\dot{x}\ddot{x}+kx\dot{x}=\dot{x}(m\ddot{x}+kx)$$

- Substituting the first equation into V-dot gives:

$$\dot{V}=\dot{x}(m\ddot{x}+kx)=\dot{x}(-b\dot{x})=-b\dot{x}^2$$

- The rate of change of energy is always negative (for positive b), except that it becomes zero when x-dot becomes zero.
- This implies that the system will lose energy and come to a stop.

Indirect Adaptive Control

- Coming back to our system:
- Let: $$V = \frac{1}{2}\tilde{\theta}^T \Gamma^{-1} \tilde{\theta}$$
  - where $\Gamma = \Gamma^T > 0$ is a positive definite matrix, which determines the learning rate of the parameters.
    - Larger entries would make learning faster, but the parameter updates may become oscillatory.
    - Need some trade-offs.

Indirect Adaptive Control

- The derivatives of V is: $\dot{V} = \tilde{\theta}^T \Gamma^{-1} \dot{\tilde{\theta}}$
- Our aim is to design the updates of $\hat{\theta}$ such that $\dot{V}$ becomes non-positive.
- If we design: $\dot{\tilde{\theta}} = -\dot{\hat{\theta}} = -\Gamma \varphi e$

When model error is big, the parameters will be updated fast.

- Then: $$\begin{aligned} \dot{V} &= \tilde{\theta}^T \Gamma^{-1} (-\Gamma \varphi e) \\ &= -\underbrace{\tilde{\theta}^T \varphi}_{e^T} e \\ &= -2 e^T e \end{aligned}$$
- V-dot is non-positive, thus: $\frac{1}{2} \tilde{\theta}(t)^T \Gamma^{-1} \tilde{\theta}(t) \leq \frac{1}{2} \tilde{\theta}(0)^T \Gamma^{-1} \tilde{\theta}(0)$
  - The estimates are likely to become better!

Indirect Adaptive Control

- Finally, as the parameter estimates are being updated, we use the values into any control techniques, as if the parameters are correct.
  - This is called the Certainty Equivalence Principle.
- Summary:
  - Manipulator dynamics: $M(q)\ddot{q} + V(q,\dot{q}) + G(q) = \varphi^T \theta = \tau$
  - Model with estimated parameters: $\hat{M}(q)\ddot{q} + \hat{V}(q,\dot{q}) + \hat{G}(q) = \varphi^T \hat{\theta}$
  - Update the parameters as: $\dot{\hat{\theta}} = \Gamma \varphi e$
    - where $e = \varphi^T \theta - \varphi^T \hat{\theta} = \varphi^T \tilde{\theta}$
  - Use the estimated parameters in the control law, for e.g.

$$\tau = \hat{M}(q) f + \hat{V}(q,\dot{q}) + \hat{G}(q) \qquad f = \ddot{q}_d + K_v \dot{E} + K_p E$$

Thank you!

Have a good evening.

RMIT UNIVERSITY