QS. 1

Tuesday, 6 October 2020 4:38 PM

- **Question 1:**
  - A single-link robot with a rotary joint is motionless at θ = -5°. It is desired to move the joint in a smooth manner to θ = 80° in 4 seconds.
  - Find the coefficients of a cubic which accomplishes this motion and brings the arm to rest at the goal.
  - Sketch the position, velocity and acceleration of the joint as a function of time.

Cubic Poly. $u(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3$

$$\begin{cases} u(0) = -5^\circ \\ \dot{u}(0) = 0 \\ u(4) = 80^\circ \\ \dot{u}(4) = 0 \end{cases}$$

Question: Find a_0, a_1, a_2 & a_3

From Lecture :

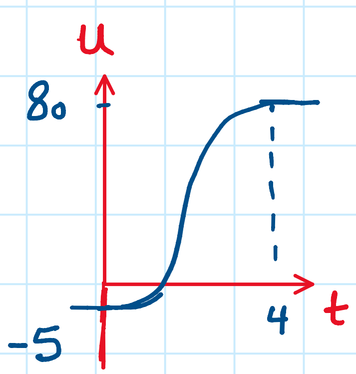
$$a_0 = u(0) = u_0 = -5$$

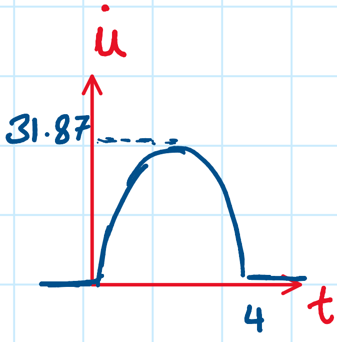
$$a_1 = 0$$

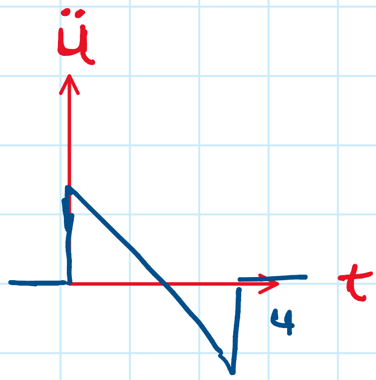
$$a_2 = \frac{3}{t_f^2}(u_f - u_0) = \frac{3}{4^2}(80-(-5)) = 15.9375$$

$$a_3 = -\frac{2}{t_f^3}(u_f - u_0) = -\frac{2}{4^3}(80-(-5)) = -2.6562$$

$$u(t) = -5 + 15.9375\,t^2 - 2.6562\,t^3$$







- **Question 2:**
  - A single-link robot with a rotary joint is motionless at θ = -5°. It is desired to move the joint in a smooth manner to θ = 80° in 4 seconds and also stop smoothly.
  - Find the corresponding parameters of a linear trajectory with parabolic blends.
  - Sketch the position, velocity and acceleration of the joint as a function of time.

1 _ Given $u_0 = -5°$, $u_f = 80°$ & $t_f = 4$ sec

2 _ Specify $\ddot{u}$:

Recall / Guidance : $\ddot{u} \geq \frac{4(u_f - u_0)}{t_f^2}$

So, $\ddot{u}_{min} = \frac{4(80-(-5))}{4^2} = \frac{85}{4}$

in this example, Let's choose $\ddot{u} = \underline{\underline{85}}$

3 _ calculate t_b : duration of Bend

Recall : $\ddot{u}\, t_b^2 - \ddot{u}\, t_f\, t_b + (u_f - u_0) = 0$ (Quadratic Equation)

$$85\, t_b^2 - (85)(4)\, t_b + (80 - (-5)) = 0$$

$$85\, t_b^2 - 340\, t_b + 85 = 0$$

$$t_b = \frac{-b \ominus \sqrt{b^2 - 4ac}}{2a}$$

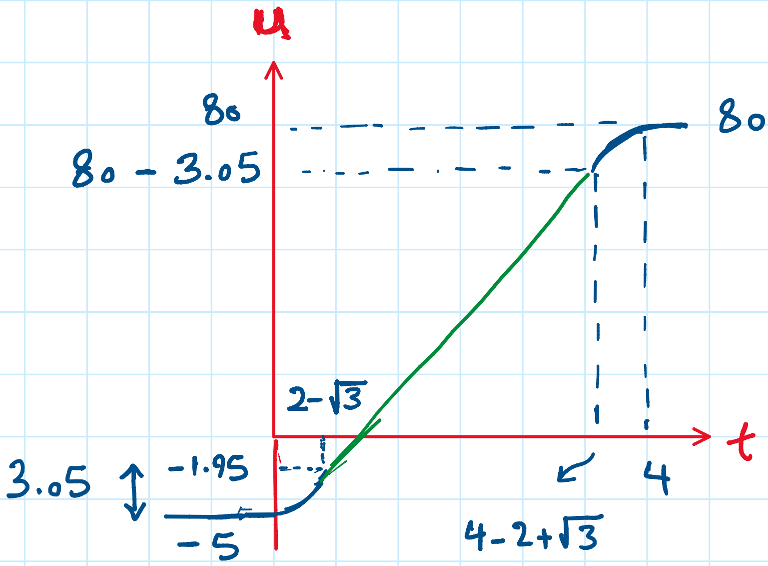
$$t_b = \frac{340 - \sqrt{340^2 - 4(85)(85)}}{2(85)} = 2 - \sqrt{3}$$

$$= 0.2679$$

4- calculate u_b: $\quad u(t_b)$

Recall: $u_b = \frac{1}{2}\ddot{u}\, t_b^2 + u_0$

$$u_b = \frac{1}{2}(85)(0.2679)^2 - 5 = -1.95$$



QS. 3

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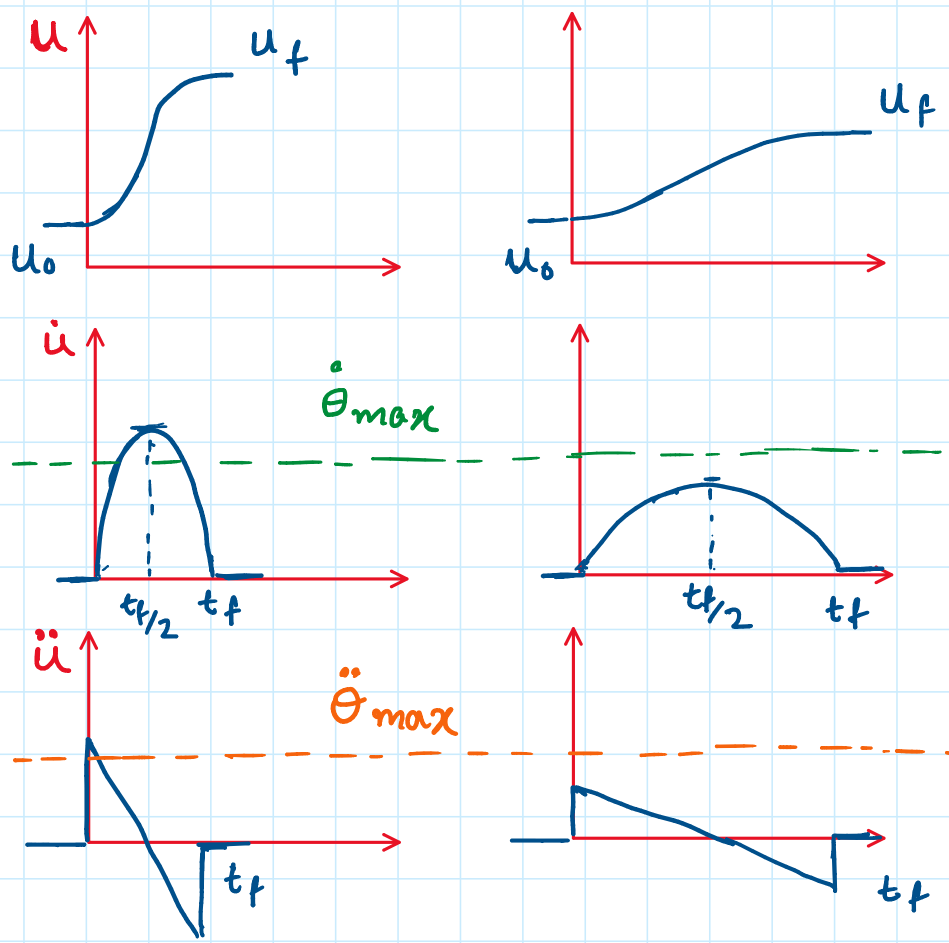
- **Question 3:**
  - We wish to move a single joint from θ_0 to θ_f starting from rest, ending at rest, in time t_f.
  - The values of θ_0 to θ_f are given, but we wish to compute t_f so that the velocity never exceeds a maximum value (θ-dot max), and the * acceleration never exceeds a maximum value (θ-double-dot max). *
  - Use a single cubic segment, and give an expression for t_f ? and for the cubic's coefficients.

Recall: effect of velocity on Traj. Profiles



$$u(t) = u_0 + \frac{3}{t_f^2}(u_f - u_0)\,t^2 - \frac{2}{t_f^3}(u_f - u_0)\,t^3$$

① $$\dot{u}(t) = \frac{6}{t_f^2}(u_f - u_0)\,t - \frac{6}{t_f^3}(u_f - u_0)\,t^2 \leq \dot{u}_{max}$$

$$t_f^2 \qquad\qquad t_f^3$$

② $$\ddot{u}(t) = \frac{6}{t_f^2}(u_f - u_0) - \frac{12}{t_f^3}(u_f - u_0)\,t \leqslant \ddot{u}_{max}$$

Note: Max $\dot{\theta}$ happens at $t = \frac{1}{2} t_f$ ①

$$\frac{6}{t_f^2}(\theta_f - \theta_0)\frac{t_f}{2} - \frac{6}{t_f^3}(\theta_f - \theta_0)\frac{t_f^2}{4} \leqslant \dot{\theta}_{max}$$

$$\frac{3}{2t_f}(\theta_f - \theta_0) \leqslant \dot{\theta}_{max}$$

$$t_f \geqslant \frac{3\,|\theta_f - \theta_0|}{2\,\dot{\theta}_{max}} \rightarrow A$$

Note: Max $\ddot{\theta}$ happens at $t = 0$ or $t = t_f$ ②

$$\frac{6}{t_f^2}(\theta_f - \theta_0) \leqslant \ddot{\theta}_{max} \qquad t_f \geqslant \sqrt{\frac{6}{\ddot{\theta}_{max}}\,|\theta_f - \theta_0|} \rightarrow B$$

Finally, choose the largest of A & B

e.g $A \rightarrow t_f = 3\,sec$ & $B \rightarrow t_f = 5\,sec$

Then $t_f = 5\,sec$ to satisfy Both A & B