Week 11 – Linear & Nonlinear Control of Manipulators

Advanced Robotic Systems – MANU2453

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RMIT UNIVERSITY

Lectures

Wk	Date	Lecture (NOTE: video recording)	Maths Difficulty	Hands-on Activity	Related Assessment
1	24/7	• Introduction to the Course • Spatial Descriptions & Transformations			
2	31/7	• Spatial Descriptions & Transformations • Robot Cell Design			Robot Cell Design Assignment
3	7/8	• Forward Kinematics • Inverse Kinematics			
4	14/8	• ABB Robot Programming via Teaching Pendant • ABB RobotStudio Offline Programming		ABB RobotStudio Offline Programming	Offline Programming Assignment
5	21/8	• Jacobians: Velocities and Static Forces			
6	28/8	• Manipulator Dynamics			
7	11/9	• Manipulator Dynamics		MATLAB Simulink Simulation	
8	18/9	• Robotic Vision		MATLAB Simulation	Robotic Vision Assignment
9	25/9	• Robotic Vision II		MATLAB Simulation	
10	2/10	• Trajectory Generation			
11	9/10	• Linear & Nonlinear Control		MATLAB Simulink Simulation	
12	16/10	• Introduction to I4.0 • Revision			Final Exam

Content

- Introduction
- Control of Second Order Linear Systems
- Control-Law Partitioning
- Trajectory Following
- Nonlinear and Time-Varying Systems
- Nonlinear Control of Multi-Input-Multi-Output System
- Matlab Simulink Simulation

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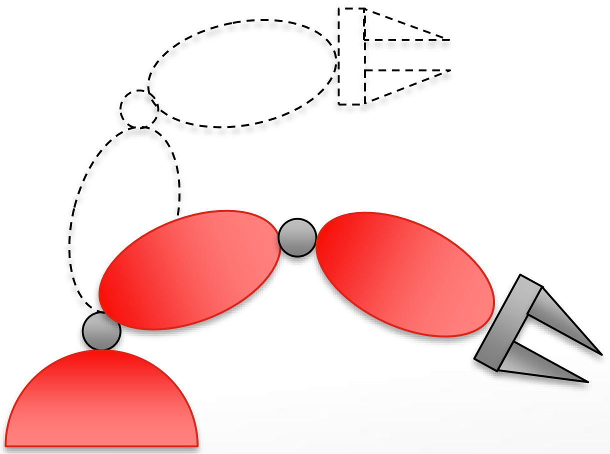
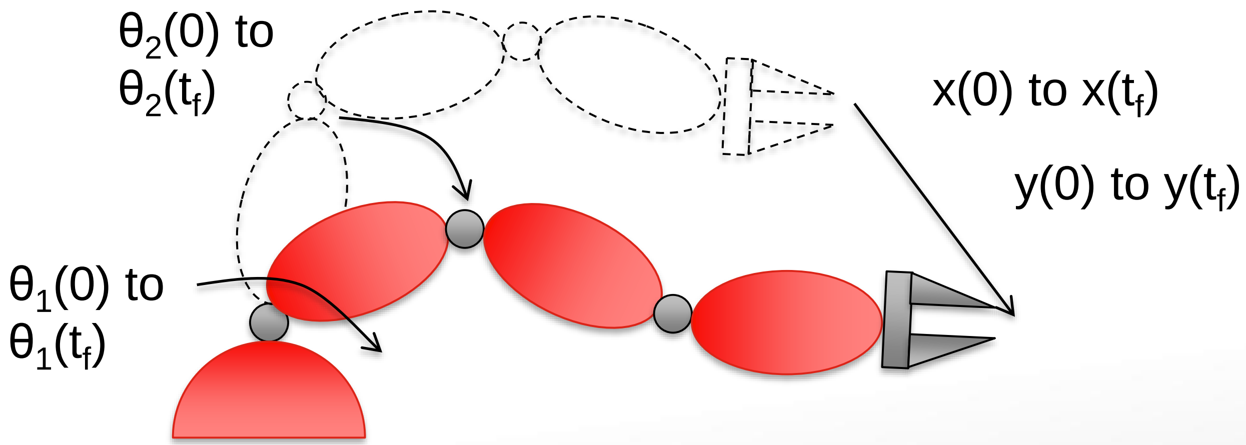
Introduction



Introduction

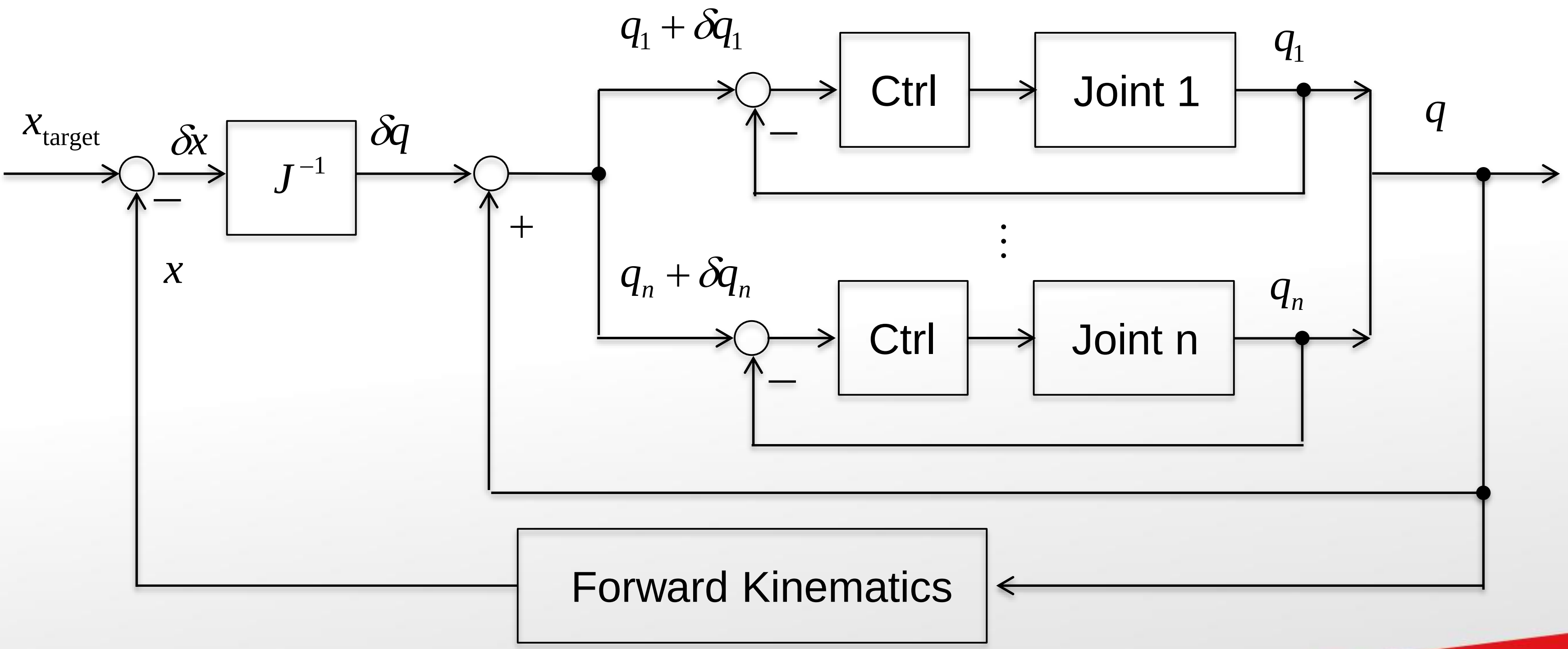
- Last week, we discussed about the trajectory which the robot is required to follow.



- In today's lecture, we will study how we can control the robot (or joints) so that they follow the desired trajectory.

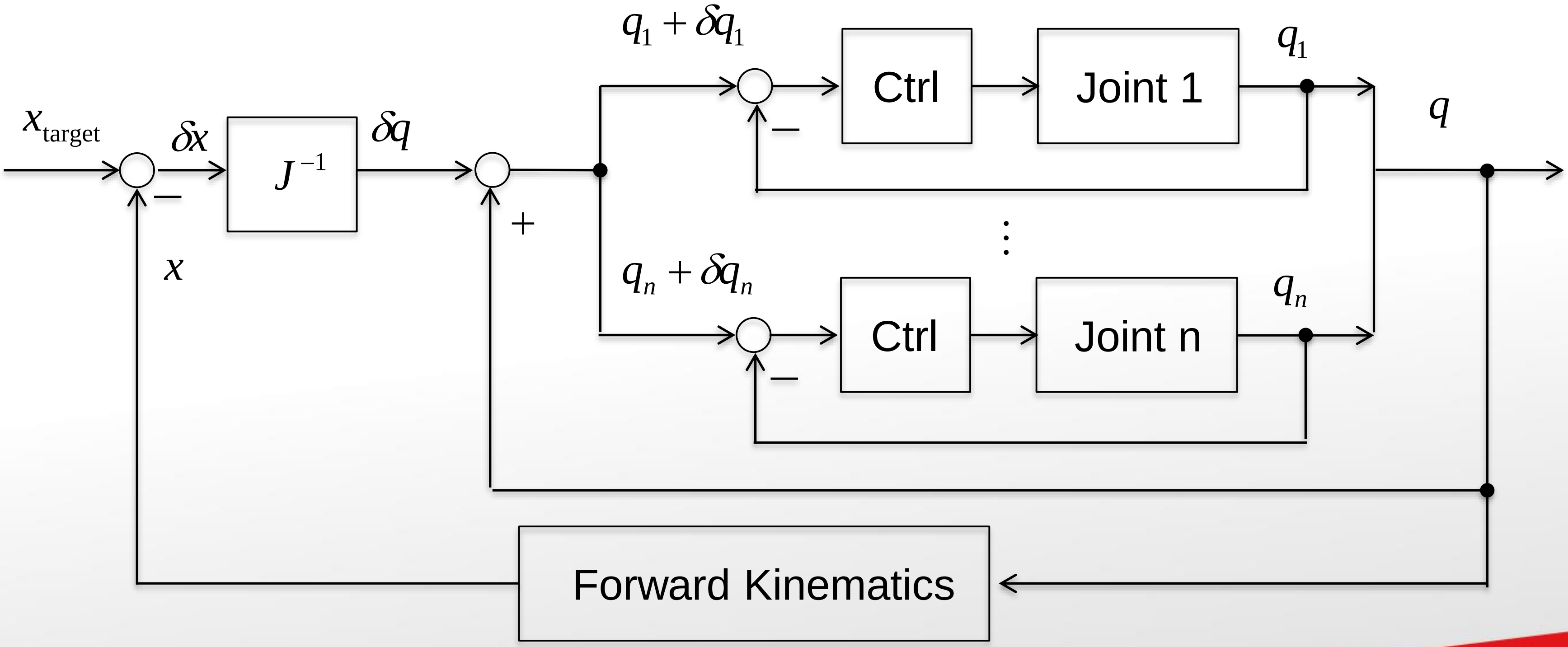
Introduction – Linear Control

- We will first explore linear control techniques.
- That is, we assume or approximate the nonlinear, coupled manipulator as a few linear and decoupled joints/links, and we control each joints individually.



Introduction – Linear Control

- The controller of each joint only cares about bringing that particular joint to reach a goal, or to track a trajectory,
- while ignoring coupling effects from all other links or just treat them as disturbances.



Introduction – Linear Control

- While this method seems crude, it is in fact quite widely used in industrial robotic manipulators.
- Advantage:
  - Simple
  - Acceptable performance.
- Disadvantage:
  - Performance not as good as using nonlinear control.
  - Performance may vary at different configurations.

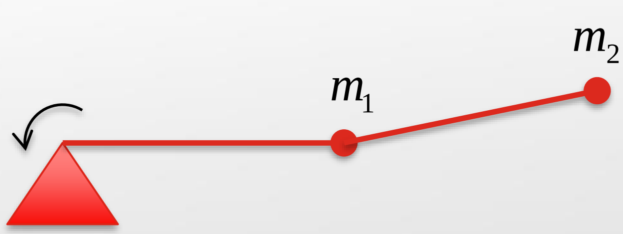
Introduction – Nonlinear Control

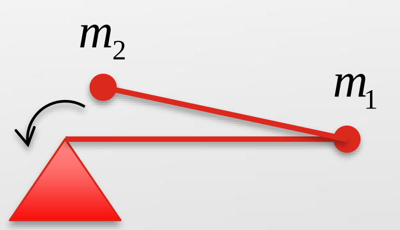
- The disadvantages of linear control method are due to the following reasons:
  - The joints or links are highly coupled.
  - The inertia (and other) matrices are NOT constant.

$$\underbrace{\begin{bmatrix} (m_1+m_2)L_1^2 + m_2L_2^2 + 2m_2L_1L_2c_2 & m_2L_2^2 + m_2L_1L_2c_2 \\ m_2L_2^2 + m_2L_1L_2c_2 & m_2L_2^2 \end{bmatrix}}_{M(q)} \begin{bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{bmatrix}$$

$$+ \underbrace{\underbrace{\begin{bmatrix} -m_2L_1L_2s_2\dot{\theta}_2^2 \\ m_2L_1L_2s_2\dot{\theta}_1^2 \end{bmatrix}}_{\text{Centrifugal}} + \underbrace{\begin{bmatrix} -2m_2L_1L_2s_1\dot{\theta}_1\dot{\theta}_2 \\ 0 \end{bmatrix}}_{\text{Coriolis}}}_{V(q,\dot{q})} + \underbrace{\begin{bmatrix} m_2gL_2c_{12} + (m_1+m_2)gL_1c_1 \\ m_2gL_2c_{12} \end{bmatrix}}_{G(q)} = \begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix}$$

large inertia seen from joint 1





small inertia seen from joint 1

Introduction – Nonlinear Control

- The use of linear control will therefore lead to undesirable results.
  - E.g. the damping will NOT be uniform throughout the workspace.
- Thus, we will also learn about some nonlinear control techniques to achieve better performance.
- Using nonlinear techniques, we will design the controller for the robot as a multi-input-multi-output system, instead of individual joints.

Introduction – Open Loop Control

- We have the dynamic equation of the robot:

$$M(q)\ddot{q}+V(q,\dot{q})+G(q)=\tau$$

- We also have the desired trajectories for position, speed and acceleration.

$$q_d, \dot{q}_d, \ddot{q}_d$$

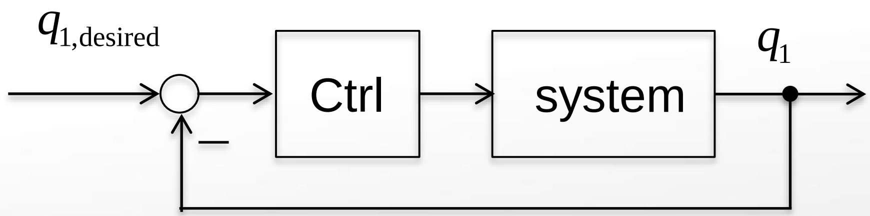
- In an ideal world where there is no modeling error or disturbance, then designing the joint torques as:

$$\tau = M(q_d)\ddot{q}_d + V(q_d, \dot{q}_d) + G(q_d)$$

  - could make the robot follow the desired trajectories!
- → Open Loop Control.
- Unfortunately, real world system definitely has modelling error and disturbances.
  - Therefore the robot will deviate from the desired trajectory.

Introduction – Feedback Control

- To make sure the robot joint actually follows the desired trajectory, we need feedback control.
  - Use sensors to measure joint angles and velocities.
  - If there are errors (difference between desired and actual trajectory), then provide corrective actions (increase or reduce torque) so that the actual trajectory moves back towards the desired trajectory.



  - We need to ensure the stability of such closed-loop systems.

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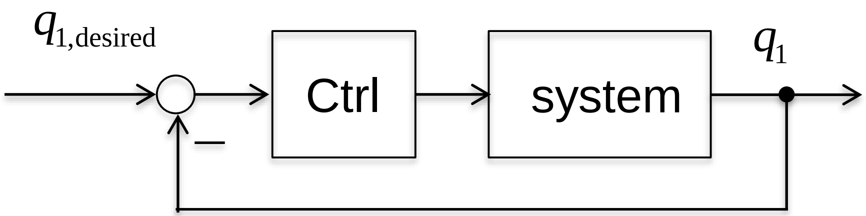
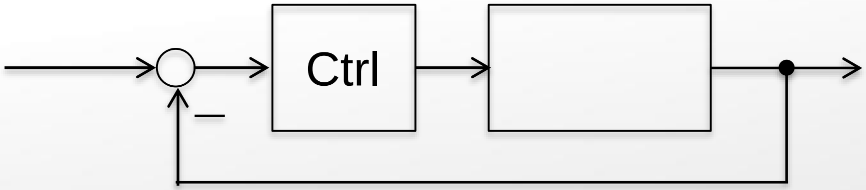
Control of only one joint/link

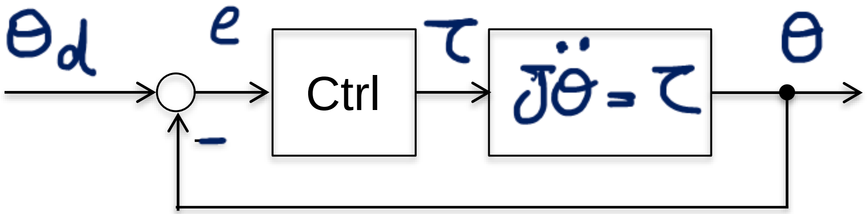
- To control a single joint with feedback control.





Control of only one joint/link

- P-Control, assuming desired Theta is zero .



- To analyse the closed loop system

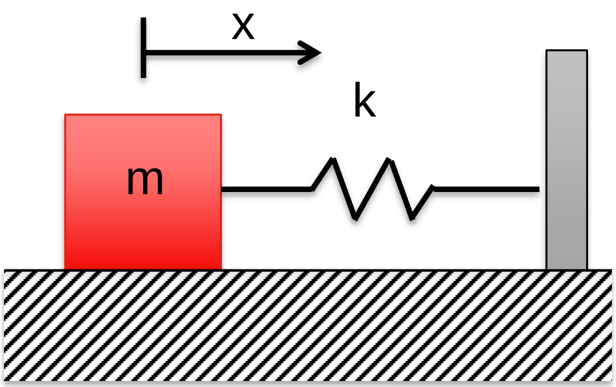
Natural Systems

- let's have an understanding of natural systems first.
- Imagine you have a mass-spring-system on a frictionless surface:

$m\ddot{x} = -kx$

or

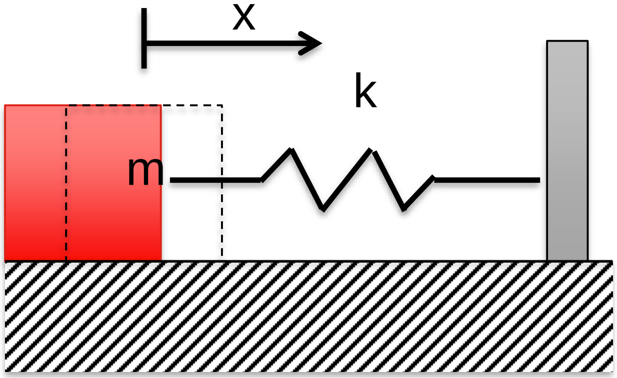
$m\ddot{x} + kx = 0$

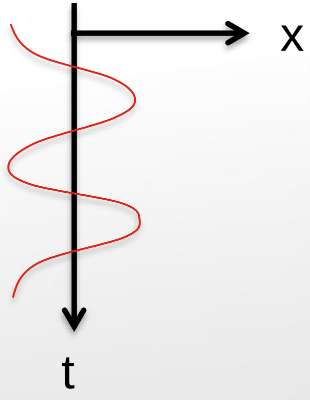


- Let the equilibrium position be x = 0.

Natural Systems

- If you perturb the mass from its equilibrium position, and then release it, the mass will swing back and forth continuously.

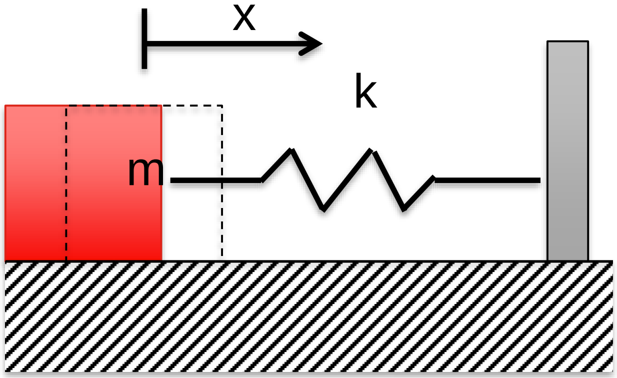




Natural Systems

- Mathematically, the differential equation (Newton's Law) for the mass-spring system on frictionless surface is:

$$m\ddot{x} = -kx \quad \text{or} \quad m\ddot{x} + kx = 0$$



Natural Frequency

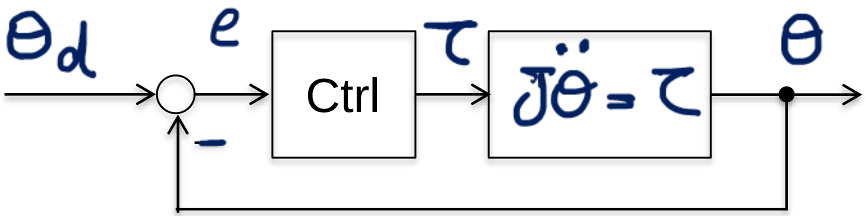
$$\omega_n = \sqrt{\frac{k}{m}}$$

- Solving the differential equation gives:

$$x = C_1 \cos\left(\sqrt{\frac{k}{m}}t\right) + C_2 \sin\left(\sqrt{\frac{k}{m}}t\right)$$

Control of only one joint/link

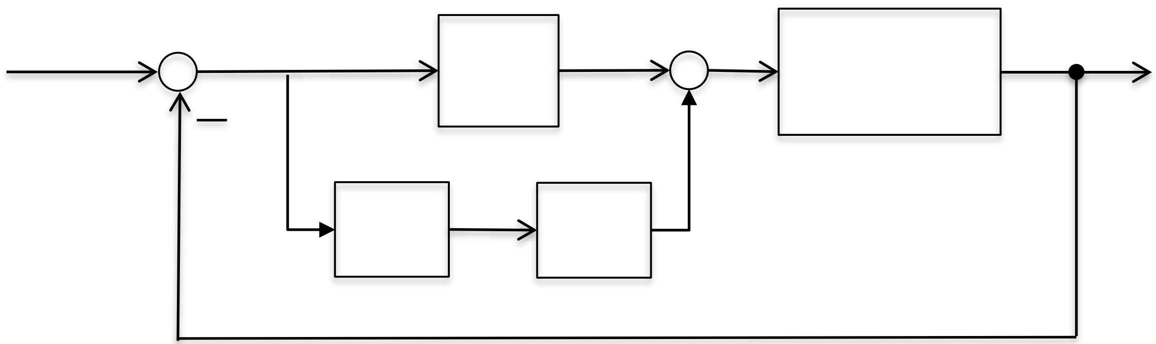
- P-Control, assuming desired Theta is zero .



- To analyse the closed loop system

Control of only one joint/link

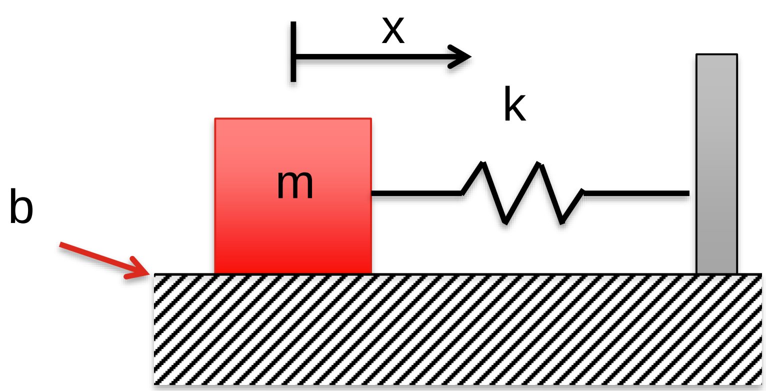
- PD-Control, assuming desired Theta is zero .



- To analyse the closed loop system

Natural Systems

- Now, imagine the surface is not frictionless anymore.
- Instead it has a viscous friction b.



- Let the equilibrium position be x = 0.

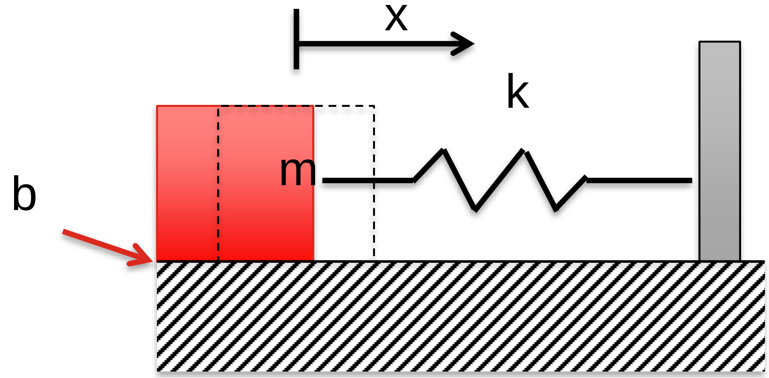
Natural Systems

- Mathematically, the differential equation (Newton's Law) for the mass-spring system on surface with viscous friction is:

$$m\ddot{x} = -b\dot{x} - kx$$

or

$$m\ddot{x} + b\dot{x} + kx = 0$$



- This is also equivalent to a mass-spring-damper system.
- The roots of the characteristic equation $ms^2 + bs + k = 0$ are:

$$s_1 = \frac{-b + \sqrt{b^2 - 4mk}}{2m} \qquad s_2 = \frac{-b - \sqrt{b^2 - 4mk}}{2m}$$

- Depending on the relationship between b and k, we have one of the three possible solutions:

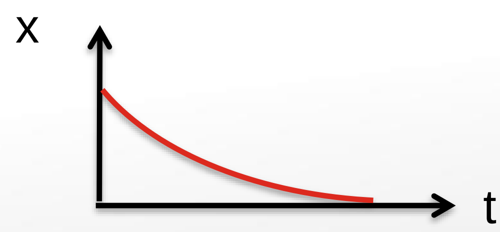
Natural Systems

- If $b^2 > 4mk$: Real and unequal roots

$$s_{1,2} = \frac{-b \pm \sqrt{b^2 - 4mk}}{2m} = \lambda \pm \mu i$$

- Overdamped response.
- Decreases to equilibrium position (0) very slowly.

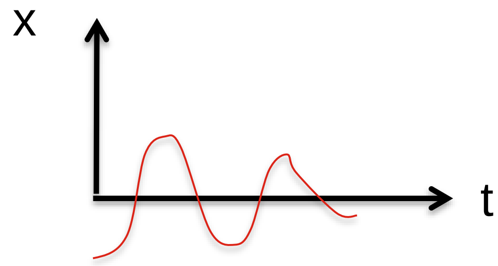
$$x(t) = c_1 e^{s_1 t} + c_2 e^{s_2 t}$$



Natural Systems

- If $b^2 < 4mk$: Complex roots $$s_{1,2} = \frac{-b \pm \sqrt{b^2 - 4mk}}{2m} = \lambda \pm \mu i$$
  - Oscillatory response, with reducing amplitude.

$$x(t) = c_1 e^{\lambda t} \cos(\mu t) + c_2 e^{\lambda t} \sin(\mu t)$$



- Another way to write 2nd order system is: $s^2 + 2\xi\omega_n s + \omega_n^2 = 0$
- where ξ is the damping ratio.
- Comparing all equations, we have: $\lambda = -\xi\omega_n$ $\mu = \omega_n\sqrt{1-\xi^2}$

$$\xi = \frac{b}{2\sqrt{km}} \qquad \omega_n = \sqrt{\frac{k}{m}}$$

Damped natural frequency

Natural Systems

- Finally, if b^2 = 4mk:

$$s_1 = s_2$$

$$x(t) = (c_1 + c_2 t)e^{s_1 t}$$

- Critically damped response.

- System reaches equilibrium position rapidly and without oscillation!
- Highly desirable!
- In this case:

$$\xi = \frac{b}{2\sqrt{km}} = \frac{b}{2\sqrt{b^2/4}} = 1$$

Damping ratio for critically damped system

Example (1)

- Find the response of the mass-spring-damper system for m =1, b = 5, k = 6, when the block is released from position x = -1.
- Answer:
  - b^2 (= 25) > 4mk (= 24).
  - Roots of characteristic equation: $$s_{1,2} = \frac{-b \pm \sqrt{b^2 - 4mk}}{2m} = \frac{-5 \pm 1}{2} = -3 \,\&\, -2$$
  - Thus:

$$x(t) = c_1 e^{-3t} + c_2 e^{-2t} \qquad \dot{x}(t) = -3c_1 e^{-3t} - 2c_2 e^{-2t}$$

  - To find c_1 and c_2, use initial conditions:

$$x(0) = c_1 + c_2 = -1 \qquad \dot{x}(0) = -3c_1 - 2c_2 = 0$$

  - This gives: $c_1 = 2$ $c_2 = -3$
  - Thus the complete solution is: $$x(t) = 2e^{-3t} - 3e^{-2t}$$

Example (2)

- Find the response of the mass-spring-damper system for m =1, b = 1, k = 1, when the block is released from position x = -1.
- Answer:
  - b^2 (= 1) < 4mk (= 4).
  - Roots of characteristic equation: $s_{1,2} = \frac{-b \pm \sqrt{b^2 - 4mk}}{2m} = \frac{-1 \pm \sqrt{1-4}}{2} = \frac{-1 \pm \sqrt{3}i}{2}$
  - Thus: $x(t) = c_1 e^{-\frac{1}{2}t} \cos\left(\frac{\sqrt{3}}{2}t\right) + c_2 e^{-\frac{1}{2}t} \sin\left(\frac{\sqrt{3}}{2}t\right) = e^{-\frac{1}{2}t}\left(c_1 \cos\left(\frac{\sqrt{3}}{2}t\right) + c_2 \sin\left(\frac{\sqrt{3}}{2}t\right)\right)$

$$\dot{x}(t) = -\frac{1}{2}e^{-\frac{1}{2}t}\left(c_1 \cos\left(\frac{\sqrt{3}}{2}t\right) + c_2 \sin\left(\frac{\sqrt{3}}{2}t\right)\right) + e^{-\frac{1}{2}t}\left(-\frac{\sqrt{3}}{2}c_1 \sin\left(\frac{\sqrt{3}}{2}t\right) + \frac{\sqrt{3}}{2}c_2 \cos\left(\frac{\sqrt{3}}{2}t\right)\right)$$

  - To find c_1 and c_2, use initial conditions:

$$x(0) = c_1 = -1 \qquad \dot{x}(0) = -\frac{1}{2}c_1 + \frac{\sqrt{3}}{2}c_2 = \frac{1}{2} + \frac{\sqrt{3}}{2}c_2 = 0$$

  - This gives: $x(t) = e^{-\frac{1}{2}t}\left(-\cos\left(\frac{\sqrt{3}}{2}t\right) - \frac{1}{\sqrt{3}}\sin\left(\frac{\sqrt{3}}{2}t\right)\right)$

Example (3)

- Find the response of the mass-spring-damper system for m =1, b = 4, k = 4, when the block is released from position x = -1.
- Answer:
  - b^2 (= 16) < 4mk (= 16).
  - Roots of characteristic equation: $s_{1,2} = \frac{-b \pm \sqrt{b^2 - 4mk}}{2m} = \frac{-4 \pm \sqrt{16-16}}{2} = -2$
  - Thus:

$$x(t) = (c_1 + c_2 t)e^{-2t} \qquad \dot{x}(t) = -2(c_1 + c_2 t)e^{-2t} + c_2 e^{-2t}$$

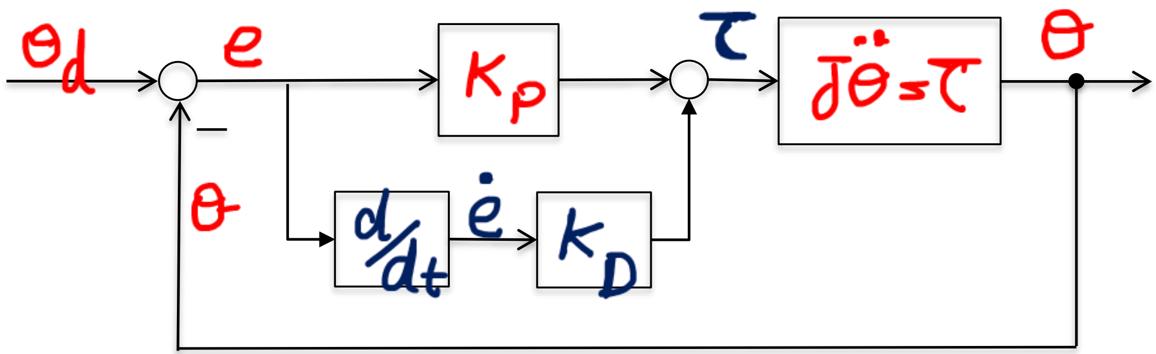
  - To find c_1 and c_2, use initial conditions:

$$x(0) = c_1 = -1 \qquad \dot{x}(0) = -2c_1 + c_2 = 0$$

  - This gives: $x(t) = (-1 - 2t)e^{2t}$

Control of only one joint/link

- PD-Control, assuming desired Theta is zero .

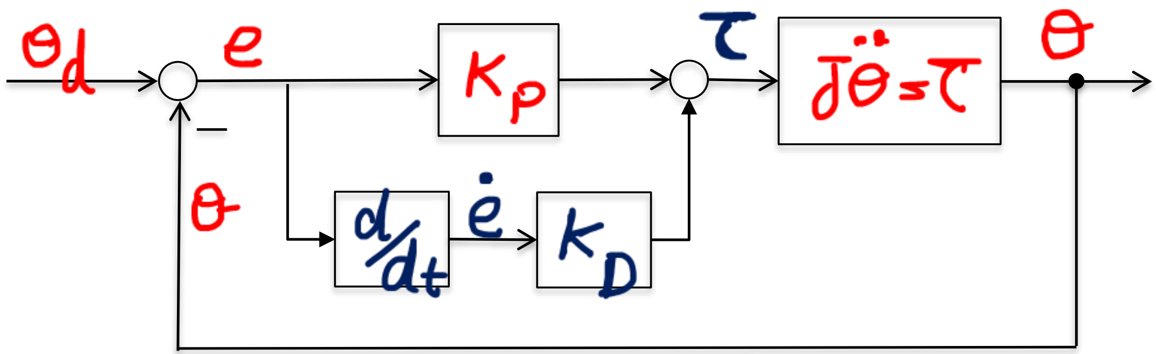


- We learnt that if the relationship between b, k and m is $b^2 = 4mk$, then we have the critically damped response which is fast and non-oscillatory.
- Therefore, we decide on a good k_p (this determines the stiffness of the mass), and then let:

$$k_D^2 = 4Jk_p \quad \Rightarrow \quad k_D = 2\sqrt{Jk_p}$$

Control of only one joint/link

- PD-Control, assuming desired Theta is zero .



- Problems of this method
- It relies on model $J\ddot{\theta} = \tau$; In real world, J may change : e.g. if robot pick an object
- If model is not $J\ddot{\theta} = \tau$, then the solution is not optimal. e.g. system has some friction.

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Control-Law Partitioning

- We know want a controller which is largely independent of the actual dynamics

- Control-Law Partitioning: The controller is partitioned into:
  - Model based portion – system parameters appear here
  - Servo portion – Independent of the system parameters

Control of only one joint/link

- For the system: $$J\ddot{\theta} + b\dot{\theta} + k\theta = \tau$$
- We design: $$\tau = J\alpha + b\dot{\theta} + k\theta$$

Model-based compensation

- And we can design $$\alpha = \ddot{\theta}_d + k_D(\dot{\theta}_d - \dot{\theta}) + k_P\,(\,\theta_d - \theta\,)$$
  - Setting a desired stiffness k_p which is now independent of J.
  - Then we calculate $$k_D = 2\sqrt{k_p}$$ which is also independent of J.

Control of only one joint/link

- To analyse the closed loop system

$$J\ddot{\theta} + b\dot{\theta} + k\theta = J\alpha \ + b\dot{\theta} + k\theta$$

$$J\ddot{\theta} + b\dot{\theta} + k\theta = J\ [\ddot{\theta}_d \ + \ k_D(\dot{\theta}_d - \dot{\theta}) + k_P\ (\ \theta_D - \theta)] + b\dot{\theta} + k\theta$$

- And we get

$$\ddot{\theta} = \ddot{\theta}_d + k_D(\dot{\theta}_d - \dot{\theta}) + k_P\ (\ \theta_D - \theta\ )$$

Trajectory Following

- Suppose now we not only want to regulate to a constant position, but to track a desired trajectory.
- Assume trajectory $\theta_d(t)$ is smooth, and thus $\dot{\theta}_d(t)$ and $\ddot{\theta}_d(t)$ are available.
- Define: $e = \theta_d - \theta$ thus we also have $\dot{e}$ and $\ddot{e}$
- We can then design the trajectory following controller as follows:

- For system: $J\ddot{\theta} + b\dot{\theta} + k\theta = \tau$
- Design: $\tau = J\alpha + b\dot{\theta} + k\theta$
  - with: $\alpha = \ddot{\theta}_d + k_D(\dot{\theta}_d - \dot{\theta}) + k_P\,(\theta_d - \theta)$
- Set a desired stiffness k_p
- Then calculate $k_D = 2\sqrt{k_p}$

Trajectory Following

- The closed loop system then becomes:

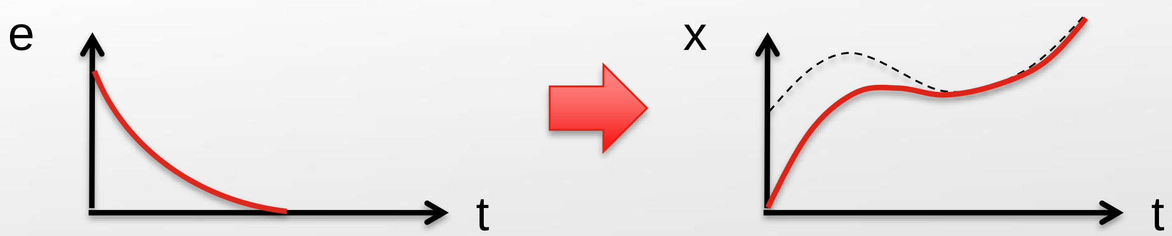
$$J\ddot{\theta} + b\dot{\theta} + k\theta = J\,[\ddot{\theta}_d \;+\; k_D(\dot{\theta}_d - \dot{\theta}) + k_P\,(\,\theta_d - \theta)] + b\dot{\theta} + k\theta$$

$$0 \;= (\ddot{\theta}_d - \ddot{\theta}) + k_D(\dot{\theta}_d - \dot{\theta}) + k_P\,(\,\theta_d - \theta\,)$$

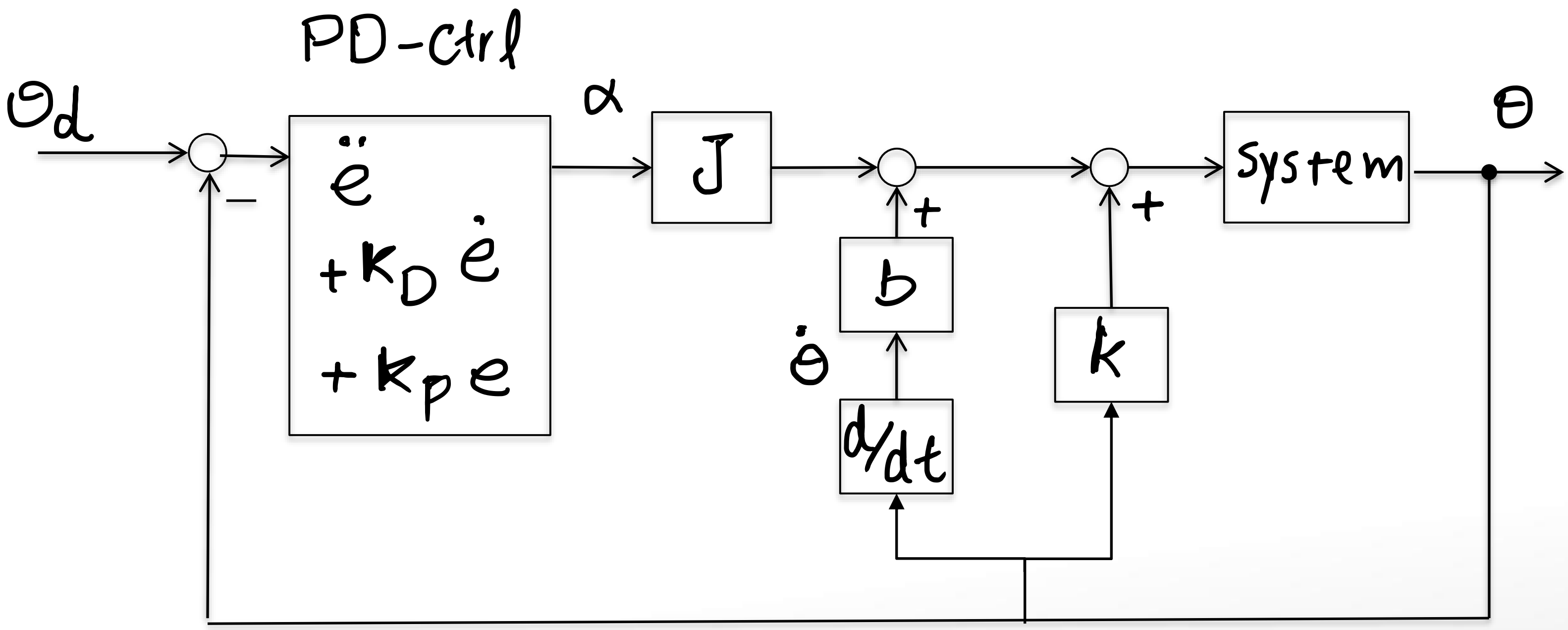
- from which we can obtain the error dynamics:

$$\ddot{e} + k_D\,\dot{e} + k_p\,e = 0$$

- The control parameters have been chosen to achieve critically damped response.
- Therefore, error (x_d – x) decays rapidly and we achieve trajectory following.



Control Law Partitioning



Control Law Partitioning

- Note:
  - In the linear case, the advantage of such control law partitioning might not be obvious.
  - m, b, k are mostly constants and thus it wouldn't be too difficult to calculate the controller gains directly from original equation.
  - However, the M, V, G matrices for a robot manipulator are nonlinear, and vary according to the robot configuration and speed.
  - By using the control law partitioning method, we will be able to calculate the controller parameters easily.
  - The model-based compensation for V and G, and the scaling of f by M will allow a constant stiffness and damping for the robot, regardless of the configuration and speed.

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Nonlinear and Time-Varying Systems

- Now that you have understood control law partitioning, we will use a very simple nonlinear control method to achieve constant performance (stiffness and damping) throughout the workspace:
  - Just cancel off the nonlinear or time-varying portion of the model!
  - This is called a linearizing control law.
- The control law partitioning method is particularly useful to achieve this.
- Let’s see a few examples to understand the concept.

Nonlinear and Time-Varying Systems

- E.g. 2nd order system with nonlinear spring: $m\ddot{x} + b\dot{x} + qx^3 = F$
  - We shall design the controller F as:

$$F = mf + b\dot{x} + qx^3 \quad \text{with} \quad f = \ddot{x}_d + k'_D(\dot{x}_d - \dot{x}) + k'_p(x_d - x) = \ddot{x}_d + k'_D\dot{e} + k'_p e$$

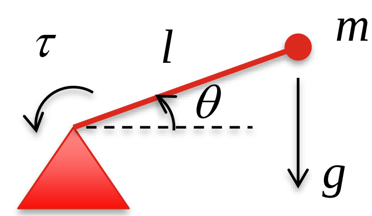
Model-based portion,
incorporating the
nonlinear term

  - The controller leads to the following closed-loop system:

$$\begin{aligned} m\ddot{x} + b\dot{x} + qx^3 &= F \\ &= mf + b\dot{x} + qx^3 \\ &= m\left(\ddot{x}_d + k'_D\dot{e} + k'_p e\right) \end{aligned} \Rightarrow \begin{aligned} \ddot{x} &= \ddot{x}_d + k'_D\dot{e} + k'_p e \\ 0 &= \ddot{e} + k'_D\dot{e} + k'_p e \end{aligned}$$

  - Set the desired stiffness kp', and let $k'_D = 2\sqrt{k'_p}$

Nonlinear and Time-Varying Systems



- E.g. Single-link Manipulator with Coulomb and viscous friction.
  - Its dynamic model is: $ml^2\ddot{\theta} + b\dot{\theta} + c\,\mathrm{sgn}(\dot{\theta}) + mgl\cos(\theta) = \tau$
  - We shall design the controller T as:

$$\tau = ml^2\alpha + b\dot{\theta} + c\,\mathrm{sgn}(\dot{\theta}) + mgl\cos(\theta)$$

with

$$\begin{aligned}\alpha &= \ddot{\theta}_d + k_D'(\dot{\theta}_d - \dot{\theta}) + k_p'(\theta_d - \theta)\\ &= \ddot{\theta}_d + k_D'\dot{e} + k_p'e\end{aligned}$$

Model-based portion, incorporating the nonlinear term

  - The controller leads to the following closed-loop system:

$$\begin{aligned}ml^2\ddot{\theta} + b\dot{\theta} + c\,\mathrm{sgn}(\dot{\theta}) + mgl\cos(\theta) &= \tau\\ &= ml^2\alpha + b\dot{\theta} + c\,\mathrm{sgn}(\dot{\theta}) + mgl\cos(\theta)\end{aligned}$$

$$\Rightarrow \quad \ddot{\theta} = \ddot{\theta}_d + k_D'\dot{e} + k_p'e \quad \Rightarrow \quad 0 = \ddot{e} + k_v'\dot{e} + k_p'e$$

Nonlinear and Time-Varying Systems

- As can be seen from the examples, by using the control law partitioning method, it is not difficult to design a nonlinear controller.
  - Make use of the model to design a model-based control law which “cancels” off the nonlinearities.
  - Then, design a linear servo law for unit mass to achieve desired stiffness and critical damping.

- NOTE: This method is also called the “Computed Torque Control”.
- IMPORTANT ASSUMPTION: The model and the parameters are exactly known.
  - In practice, this can be a problem.

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Multi-Input-Multi-Output System

- Apart from being nonlinear and time-varying, the robotic manipulator also has strong coupling amongst its many joints.
- To handle this issue, we will first look at solving a multi-input-multi-output (MIMO) control problem.
  - Instead of one single joint variable (x or θ), we now have a vector of joint positions: $X = [q_1 \quad q_2 \quad \cdots \quad q_n]^T$
  - along with its time derivatives (velocities and accelerations).
- Let the dynamic model of the MIMO system be: $f\ddot{X} + \beta = F$
- Design the control law as: $F = f\alpha + \beta$
- The closed loop system then becomes: $\ddot{X} = \alpha$ or $\begin{bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \\ \vdots \\ \ddot{q}_n \end{bmatrix} = \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \vdots \\ \alpha_n \end{bmatrix}$

Multi-Input-Multi-Output System

- We see that, again, by using the control law partitioning method, we are able to reduce the problem to that of n independent unit mass.
- Therefore, the model based portion of the control law is called "Linearizing and Decoupling" control law.
- Finally, we will design a servo control law for each of the joints:

$$\ddot{X} = \ddot{X}_d + K_D\dot{E} + K_pE$$

usually diagonal

usually diagonal

or

$$\begin{bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \\ \vdots \\ \ddot{q}_n \end{bmatrix} = \begin{bmatrix} \ddot{q}_{1d} + k_{D1}\dot{e}_1 + k_{p1}e_1 \\ \ddot{q}_{2d} + k_{D2}\dot{e}_2 + k_{p2}e_2 \\ \vdots \\ \ddot{q}_{nd} + k_{Dn}\dot{e}_n + k_{pn}e_n \end{bmatrix}$$

$$K_D = \begin{bmatrix} k_{D1} & 0 & 0 & 0 \\ 0 & k_{D2} & 0 & 0 \\ 0 & 0 & k_{D3} & 0 \\ 0 & 0 & 0 & k_{D4} \end{bmatrix}$$

$$K_p = \begin{bmatrix} k_{p1} & 0 & 0 & 0 \\ 0 & k_{p2} & 0 & 0 \\ 0 & 0 & k_{p3} & 0 \\ 0 & 0 & 0 & k_{p4} \end{bmatrix}$$

Manipulator Control

- The same idea of control law partitioning will be used for linearizing, decoupling and servoing of the manipulator.
- The dynamic model of manipulator is:

$$M(q)\ddot{q} + V(q,\dot{q}) + G(q) = \tau$$

- We could also include non-rigid body effects, e.g. friction into the model:

$$M(q)\ddot{q} + V(q,\dot{q}) + G(q) + F(q,\dot{q}) = \tau$$

- Now, design the model-based control law as:

$$\tau = M(q)\alpha + V(q,\dot{q}) + G(q) + F(q,\dot{q})$$

- The servo portion is then designed as:

$$\alpha = \ddot{q}_d + K_D\dot{E} + K_p E$$

Manipulator Control

- The control law leads to the following closed loop system:

$$
\begin{aligned}
M(q)\ddot{q} + V(q,\dot{q}) + G(q) + F(q,\dot{q}) = \tau \\
&= M(q)\alpha + V(q,\dot{q}) + G(q) + F(q,\dot{q}) \\
&= M(q)\left(\ddot{X}_d + K_D\dot{E} + K_pE\right) + V(q,\dot{q}) + G(q)
\end{aligned}
$$

- Or: $\ddot{q} = \ddot{q}_d + K_D\dot{E} + K_pE$ $\Rightarrow$ $\ddot{E} + K_D\dot{E} + K_pE = 0$

- Note that the system is decoupled. K_D and K_p are diagonal, thus we can write the closed loop equation for each joint: $\ddot{e}_i + k_{Di}\dot{e}_i + k_{pi}e_i = 0$

  - This is an asymptotically stable system, and the error will decay to zero, meaning that tracking of reference is achieved. $x_i \rightarrow x_{di}$

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MATLAB Simulink Simulation

- The Matlab Simulink files are uploaded on Canvas.
- Run the files in the order of A.. B.. C.

Tutorial Assignments

- **Question 1:**
  - Determine the motion of a mass-spring-damper system if parameter values are $m = 2$, $b = 6$ and $k = 4$, and the mass (initially at rest) is released from the position $x = 1$.

Tutorial Assignments

- **Question 2:**
  - Determine the motion of a mass-spring-damper system if parameter values are $m = 1$, $b = 2$ and $k = 1$, and the mass (initially at rest) is released from the position $x = 4$.

Tutorial Assignments

- **Question 3:**
  - Determine the motion of a mass-spring-damper system if parameter values are $m = 1$, $b = 4$ and $k = 5$, and the mass (initially at rest) is released from the position $x = 2$.

Tutorial Assignments

- **Question 4:**
  - Consider a mass-spring-damper system with parameter values m = 1, b = 4 and k = 5.
  - The system is known to possess an unmodeled resonance at $\omega_{res} = 6$ rad/sec.
  - Determine the gains k_v and k_p which will critically damp the system with as high a stiffness as reasonable.

Tutorial Assignments

- **Question 5:**
  - Give the nonlinear control equations for the system:

$$\left(2\sqrt{\theta}+1\right)\ddot{\theta}+3\dot{\theta}^2-\sin(\theta)=\tau$$

  - Choose gains so that this system is always critically damped with closed-loop stiffness K_{CL} of 10.

Tutorial Assignments

- **Question 6:**
  - Give the nonlinear control equations for the system:

$$2\ddot{\theta} + 5\theta\dot{\theta} - 13\dot{\theta}^3 + 5 = \tau$$

  - Choose gains so that this system is always critically damped with closed-loop stiffness K_{CL} of 10.

Tutorial Assignments

- **Question 7:**
  - Design a trajectory-following control system for a system with the following dynamic equations:

$$m_1 l_1^2 \ddot{\theta}_1 + m_1 l_1 l_2 \dot{\theta}_1 \dot{\theta}_2 = \tau_1$$
$$m_2 l^2 \left(\ddot{\theta}_1 + \ddot{\theta}_2\right) + v_2 \dot{\theta}_2 = \tau_2$$

Thank you!

Have a good evening.

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