Sample for Control

Question (Marks) (20 min)

Consider a robotic system described by:

$$M(q)\ddot{q} + V(q,\dot{q}) + G(q) = \tau$$

(a) Design a nonlinear controller such that the manipulator has constant stiffness and damping throughout its workspace. (Marks).

(b) State one advantage and one disadvantage of your design (Marks).

(c) Assume you want to control the robot such that its joints follow certain trajectories. For joint 1, the constraints are:

$\theta_{10} = 10^0$

$\theta_{1f} = 50^0$

All start and ending velocities and accelerations are zero.
The end time is 2 seconds.

Calculate the parameters of a quintic polynomial which can satisfy all the given constraints (Marks).

(a) $M(q)\ddot{q} + V(q,\dot{q}) + G(q) = \tau$

Design $\tau = M(q) f + V(q,\dot{q}) + G(q) + F(q,\dot{q})$

where $f = \ddot{q}_d + K_v \dot{E} + K_p E$

K_v & K_p shall be diagonal. (2 Mins)

(b) Advantage: asymptotically stable, constant stiffness

Disadvantage: Need accurate model. (1 min)

(c) $u(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3 + a_4 t^4 + a_5 t^5$

where $a_0 = u_0 = 10^\circ$

$a_1 = \dot{u}_0 = 0$

$a_2 = \frac{\ddot{u}_0}{2} = 0$

$$a_3 = \frac{20u_f - 20u_0 - (8\dot{u}_f + 12\dot{u}_0)t_f - 3(\ddot{u}_0 - \ddot{u}_f)t_f^2}{2t_f^3}$$

$$= \frac{20(50^\circ) - 20(10^\circ) - 0}{2(2)^3} = \underline{\underline{50}}$$

$$a_4 = \frac{30u_0 - 30u_f + (14\dot{u}_f + 16\dot{u}_0)t_f + (3\ddot{u}_0 - 2\ddot{u}_f)t_f^2}{2t_f^4}$$

$$= \frac{30(10) - 30(50) + 0}{2(2)^4} = \underline{\underline{-37.5}}$$

$$a_5 = \frac{12u_f - 12u_0 - (6\dot{u}_f + 6\dot{u}_0)t_f - (\ddot{u}_0 - \ddot{u}_f)t_f^2}{2t_f^5}$$

$$= \frac{12(50) - 12(10) - 0}{2(2)^5} = \underline{\underline{7.5}}$$

hence: $u(t) = \underline{\underline{10 + 50t^3 + 37.5t^4 + 7.5t^5}}$ (7 min)

Sample for Trajectory Planning

Question (5 Marks)

A robot joint is required to move from q1 = 30° to q2 = 80° in 5 seconds. Additional requirements include zero velocity and zero acceleration at the start and at the end of the motion.

(a) Calculate the parameters of a quintic polynomial, which would be able to achieve all the requirements. Show your work out, not just the final answer. (2 marks)

(b) If the robot is required to pass through a via point q_{via} = 100 at t = 3 seconds during the motion from q1 to q2, with the velocity at the via point as 2 degrees/second, and accelerations as 0 degrees/second^2, what are the two quintic polynomials for the portion q1→q_{via}, and q_{via}→q2? (3 marks)

4) a) Quintic polynomial :

$u(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3 + a_4 t^4 + a_5 t^5$

$a_0 = u_0 = 30$

$a_1 = 0$

$a_2 = 0$

$a_3 = \frac{20(80) - 20(30)}{2(5^3)} = 4$

$a_4 = \frac{30(30) - 30(80)}{2(5^4)} = -1.2$

$a_5 = \frac{12(80) - 12(30)}{2(5^5)} = 0.096.$

Then: $\underline{\underline{u(t) = 30 + 4t^3 - 1.2t^4 + 0.096t^5}}$

~~(max 3 mins)~~

b) 1st potion :

$u_0 = 30$

$u_f = u_{via} = 100$

$\dot{u}_0 = 0$

$\dot{u}_{via} = 2$

$\ddot{u}_0 = 0$

$\ddot{u}_{via} = 0$ $\quad\quad t_f = 3$

Then $a_0 = u_0 = 30$

$a_1 = \dot{u}_0 = 0$ $\quad\quad$ (3 mins)

$a_2 = \ddot{u}_0/2 = 0$

$a_3 = \frac{20(100) - 20(30) - (8\times2 + 12\times0)3 - (3\times0 - 0)3^2}{2(3)^3} =$

$= \frac{1400 - 48}{54} = 25.04$

$$a_4 = \frac{30(70) - 30(100) + 14(2)3 + 0}{2 \cdot 3^4} = -12.44$$

$$a_5 = \frac{12(100) - 12(30) \cancel{+} - 6(2)3 - 0}{2 \cdot 3^5} = \cancel{1.80} \; 1.654$$

$\therefore$ First portion: $\underline{u(t) = 30 + 25.04t^3 - 12.44t^4 + 1.8t^5}$

second portion:

$u_0 = 100$

$u_f = 80$

$\dot{u}_0 = 2$

$\dot{u}_f = 0$

$\ddot{u}_0 = 0$

$\ddot{u}_f = 0$ $\quad$ $t_f = 2$

$\therefore a_0 = u_0 = 100$

$a_1 = \dot{u}_0 = 2$

$a_2 = \ddot{u}_0/2 = 0$

$$a_3 = \frac{20(80) - 20(100) - (12 \cdot 2)2}{2 \cdot 2^3} = -28$$

$$a_4 = \frac{30(100) - 30(80) + 16 \cdot 2 \cdot 2}{2 \cdot 2^4} = 20.75$$

$$a_5 = \frac{12(80) - 12(100) - (6 \cdot 2)2}{2 \cdot 2^5} = -4.125$$

$\therefore \underline{u(t) = 100 + 2t - 28t^3 + 20.75t^4 - 4.125t^5}$

(max 15 mins)