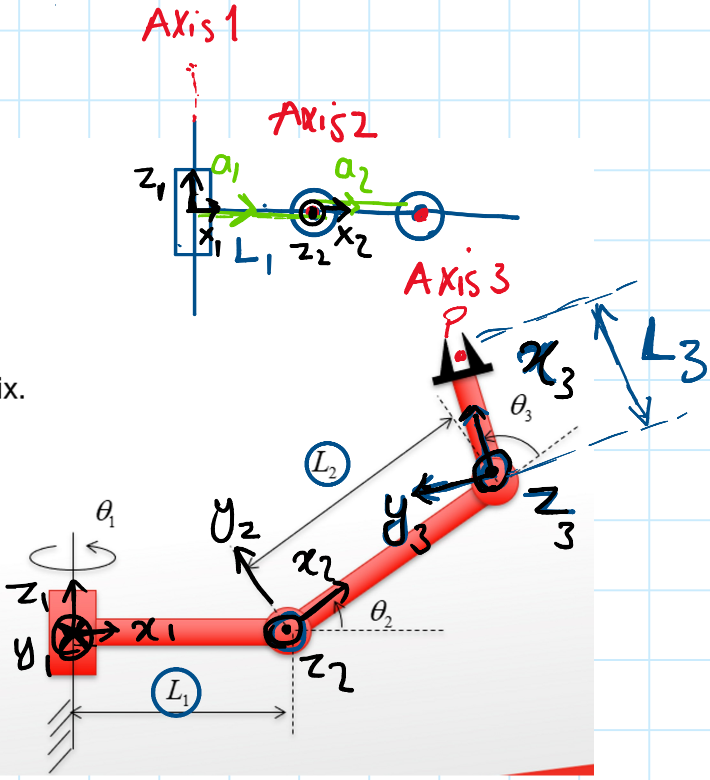
QS. 1

Thursday, 13 August 2020 5:13 PM

- **Question 1:**
  - For the robot shown on the right:
  - Derive the DH parameters,
  - Sketch the frames,
  - And calculate the Transform matrix.



1- Draw Axes

2- Draw Mutual ⊥

3- Link Length (do it from 1 to $n-1=2$)

$\boxed{a_1}$ = dist. from Ax1 to Ax2 along mutual ⊥ = $\boxed{L_1}$

$\boxed{a_2}$ = dist. from Ax2 to Ax3 along mutual ⊥ = $\boxed{L_2}$

$\boxed{a_0 = 0}$ by convention

4. Link Twist (do it from 1 to $n-1=2$)

$\boxed{\alpha_1}$ = Angle ∡ Between Ax.1 & Ax.2 about a_1 = $\boxed{90^\circ}$

$\boxed{\alpha_2}$ = Angle ∡ Between Ax.2 & Ax.3 about a_2 = $\boxed{0^\circ}$

$\boxed{\alpha_0 = 0}$ by Convention

5. Link offset (do it from 2 to $n-1=2$)

$\boxed{d_2}$ = dist. from a_1 to a_2 along Ax.2 = $\boxed{0}$

$\boxed{d_1 \& d_3 = 0}$ by Convention for Rotary Joints

6 - Join Angles (do it from 2 to $n-1=2$)

$\boxed{\theta_2}$ = Angle Between a_1 & a_2 about Ax.2 → $\boxed{\theta_2}$ var

$\boxed{\theta_2 =}$ Angle Between a_1 & a_2 about Ax.2 — $\boxed{\theta_2}$ var

$\boxed{\theta_1 \,\&\, \theta_3 = \theta_1 \,\&\, \theta_3}$ var by convention for Rot. Joints

7- Transfer into DH Table

i	α_{i-1}		a_{i-1}		d_i		θ_i	
1	α_0	0	a_0	0	d_1	0	θ_1	θ_1
2	α_1	90	a_1	L_1	d_2	0	θ_2	θ_2
3	α_2	0	a_2	L_2	d_3	0	θ_3	θ_3

8- Draw the frames in above Picture
$\left\{ \begin{array}{l} \text{Z along axis} \\ \text{origin is where } a \text{ intersects axis} \\ \text{X along } a \end{array} \right.$

9- Calculate all the transformations

$$ {}^{i-1}_{i}T = \begin{bmatrix} c\theta_i & -s\theta_i & 0 & a_{i-1} \\ c\alpha_{i-1}s\theta_i & c\alpha_{i-1}c\theta_i & -s\alpha_{i-1} & -s\alpha_{i-1}d_i \\ s\alpha_{i-1}s\theta_i & s\alpha_{i-1}c\theta_i & c\alpha_{i-1} & c\alpha_{i-1}d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^{0}_{1}T = \begin{bmatrix} c\theta_1 & -s\theta_1 & 0 & 0 \\ s\theta_1 & c\theta_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

look at the first row at DH Table
$\alpha_0 = a_0 = d_1 = 0$
θ_1 var

$$ {}^{1}_{2}T = \begin{bmatrix} c\theta_2 & -s\theta_2 & 0 & L_1 \\ 0 & 0 & -1 & 0 \\ s\theta_2 & c\theta_2 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

look at the 2nd row at DH Table
$\alpha_1 = 90^\circ$
$a_1 = L_1$ & θ_2 var

$[\; c\theta_2 \;\; -s\theta_3 \;\; 0 \;\; L_2 \;]$ look at the 2rd

$$ {}^{2}_{3}T = \begin{bmatrix} C\theta_3 & -S\theta_3 & 0 & L_2 \\ S\theta_3 & C\theta_3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

look at the 3rd row of DH Table

$d_2 = 0$

$a_2 = L_2$ & θ_3 var

$$ {}^{0}_{3}T = {}^{0}_{1}T \cdot {}^{1}_{2}T \cdot {}^{2}_{3}T = \begin{bmatrix} C_1 C_{23} & -C_1 S_{23} & S_1 & L_1 C_1 + L_2 C_1 C_2 \\ S_1 C_{23} & -S_1 S_{23} & -C_1 & L_1 S_1 + L_2 S_1 S_2 \\ S_{23} & C_{23} & 0 & L_2 S_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$S_{23} = \sin(\theta_2 + \theta_3)$ $\qquad$ $C_{23} = \cos(\theta_2 + \theta_1)$

1o) where is the endeffector? ${}^{0}P$?

First: in frame {3} $\rightarrow$ ${}^{3}P = [L_3 \ \ 0 \ \ 0]^T$

L_3 in x direction of {3}

Finally $\begin{bmatrix} {}^{0}P \\ \hline 1 \end{bmatrix} = {}^{0}_{3}T \begin{bmatrix} {}^{3}P \\ \hline 1 \end{bmatrix}$

QS. 2

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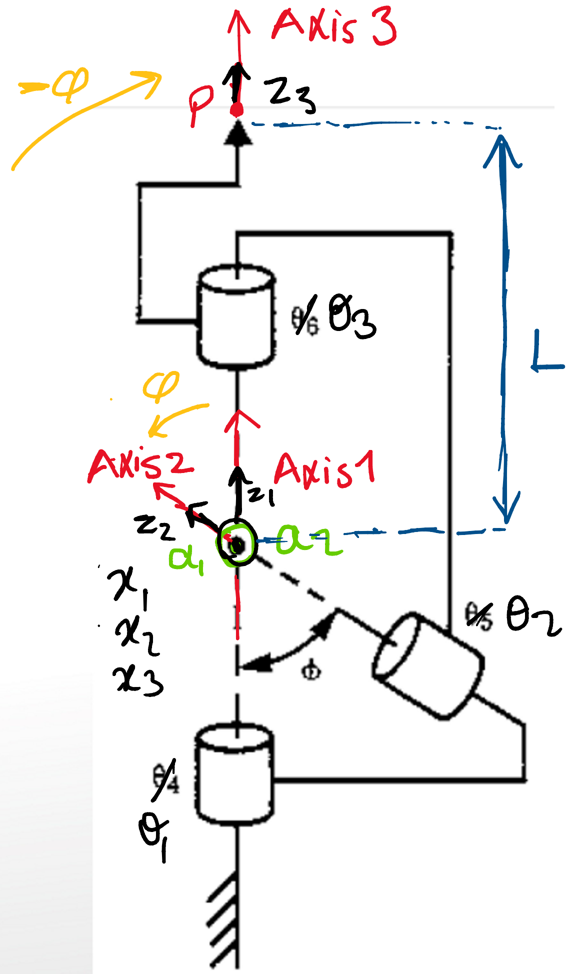
- **Question 2:**
  - For the robot shown on the right:
  - Derive the DH parameters (you may add in any constants i.e. link lengths or offset as required),
  - Sketch the frames,
  - And calculate the Transform matrix.



1- Draw Axes

2- Draw Mutual Prependiculars a_1 & a_2

3- Link Length (do it from 1 to n-1=2)

$\boxed{a_1}$ = dist. from Ax1 to Ax2 along mutual ⊥ = $\boxed{0}$

$\boxed{a_2}$ = dist. from Ax2 to Ax3 along mutual ⊥ = $\boxed{0}$ intersecting

$\boxed{a_0 = 0}$ by convention

4. Link Twist (do it from 1 to n-1=2)

$\boxed{\alpha_1}$ = Angle ∡ Between Ax.1 & Ax.2 about a_1 = $\boxed{\varphi}$

$\boxed{\alpha_2}$ = Angle ∡ Between Ax.2 & Ax.3 about a_2 = $\boxed{-\varphi}$

$\boxed{\alpha_0 = 0}$ by convention

5. Link offset (do it from 2 to n-1=2)

5. Link offset (do it from 2 to n-1=2)

$\boxed{d_2}$ = dist. from a_1 to a_2 along Ax. 2 = $\boxed{0}$

$\boxed{d_1 \,\&\, d_3 = 0}$ by Convention for Rotary Joints

6 - Join Angles (do it from 2 to n-1=2)

$\boxed{\theta_2 =}$ Angle Between a_1 & a_2 about Ax.2 — $\boxed{\theta_2}$ var

$\boxed{\theta_1 \,\&\, \theta_3 = \theta_1 \,\&\, \theta_3}$ var by convention for Rot. Joints

7- Transfer into DH Table

i	α_{i-1}	a_{i-1}	d_i	θ_i
1	α_0 0	a_0 0	d_1 0	θ_1 θ_1
2	α_1 φ	a_1 0	d_2 0	θ_2 θ_2
3	α_2 $-\varphi$	a_2 0	d_3 0	θ_3 θ_3

8- Draw the frames in above Picture $\left\{\begin{array}{l} \underline{Z} \text{ along } \underline{\text{axis}} \\ \text{origin is where } \underline{a} \text{ intersects } \underline{\text{axis}} \\ \underline{X} \text{ along } \underline{a} \end{array}\right.$

9- Calculate all the transformations

$$ {}^{i-1}_{\;\;i}T = \begin{bmatrix} C\theta_i & -S\theta_i & 0 & a_{i-1} \\ C\alpha_{i-1} S\theta_i & C\alpha_{i-1} C\theta_i & -S\alpha_{i-1} & -S\alpha_{i-1}\, d_i \\ S\alpha_{i-1} S\theta_i & S\alpha_{i-1} C\theta_i & C\alpha_{i-1} & C\alpha_{i-1}\, d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^0_1T = \begin{bmatrix} C\theta_1 & -S\theta_1 & 0 & 0 \\ S\theta_1 & C\theta_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

look at the first row of DH Table

$\alpha_0 = a_0 = d_1 = 0$

θ_1 var

$$ {}^1_2T = \begin{bmatrix} C\theta_2 & -S\theta_2 & 0 & 0 \\ \cdots \end{bmatrix} $$

look at the 2nd

$$ {}^{1}_{2}T = \begin{bmatrix} C\theta_2 & -S\theta_2 & 0 & 0 \\ C\varphi S\theta_2 & C\varphi C\theta_2 & -S\varphi & 0 \\ S\varphi S\theta_2 & S\varphi C\theta_2 & C\varphi & 0 \\ 0 & 0 & & 1 \end{bmatrix} $$

look at the 2nd row of DH Table

$\alpha_1 = \varphi$

$d_2, a_1 = 0$ & θ_2 var

$$ {}^{2}_{3}T = \begin{bmatrix} C\theta_3 & -S\theta_3 & 0 & 0 \\ C\varphi S\theta_3 & C\varphi C\theta_3 & S\varphi & 0 \\ -S\varphi S\theta_3 & -S\varphi C\theta_3 & C\varphi & 0 \\ 0 & 0 & & 1 \end{bmatrix} $$

look at the 3rd row of DH Table

$\alpha_2 = -\varphi$

$d_3, a_2 = 0$ & θ_3 var

Finally $\quad {}^{0}_{3}T = {}^{0}_{1}T \cdot {}^{1}_{2}T \cdot {}^{2}_{3}T$

10) where is the endeffector? ${}^{0}P$?

First: in frame $\{3\} \rightarrow {}^{3}P = \begin{bmatrix} 0 & 0 & L \end{bmatrix}^T$

Finally $\begin{bmatrix} {}^{0}P \\ \hline 1 \end{bmatrix} = {}^{0}_{3}T \begin{bmatrix} {}^{3}P \\ \hline 1 \end{bmatrix}$

QS. 3

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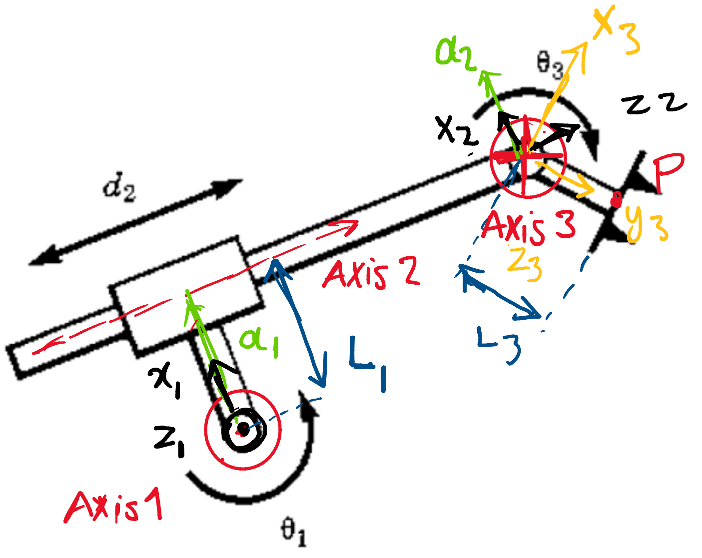
- **Question 3:**
  - For the robot shown below,
  - Derive the DH parameters (you may add in any constants i.e. link lengths or offset as required), sketch the frames, and calculate the Transform matrix.



Axis 3 is into the Plane

1- Draw Axes

2- Draw Mutual Prependiculars a_1 & a_2

3- Link Length (do it from 1 to n-1=2)

$\boxed{a_1}$ = dist. from Ax1 to AX2 along mutual ⊥ = $\boxed{L_1}$

$\boxed{a_2}$ = dist. from Ax2 to AX3 along mutual ⊥ = $\boxed{0}$ → intersecting

$\boxed{a_0 = 0}$ by convention

4. Link Twist (do it from 1 to n-1=2)

$\boxed{\alpha_1}$ = Angle ∡ Between Ax.1 & Ax.2 about a_1 = $\boxed{90}$

$\boxed{\alpha_2}$ = Angle ∡ Between Ax.2 & Ax.3 about a_2 = $\boxed{90}$

$\boxed{\alpha_0 = 0}$ by Convention

5. Link offset (do it from 2 to n-1=2)

$\boxed{d_2}$ = dist. from a_1 to a_2 along Ax. 2 = $\boxed{d_2}$ → Variable

$\boxed{d_1 \& d_3 = 0}$ by Convention for Rotary Joints

6 - Join Angles (do it from 2 to $n-1=2$)

$\boxed{\theta_2 =}$ Angle Between a_1 & a_2 about Ax.2 $-$ $\boxed{0}$

$\boxed{\theta_1 \& \theta_3 = \theta_1 \& \theta_3}$ var by convention for Rot. Joints

7- Transfer into DH Table

i	α_{i-1}	a_{i-1}	d_i	θ_i
1	α_0 0	a_0 0	d_1 0	θ_1 θ_1
2	α_1 90	a_1 L_1	d_2 d_2	θ_2 0
3	α_2 90	a_2 0	d_3 0	θ_3 θ_3

8- Draw the frames in above Picture

$\left\{ \begin{array}{l} \underline{Z} \text{ along } \underline{\text{axis}} \\ \text{origin is where } \underline{a} \text{ intersects } \underline{\text{axis}} \\ \underline{X} \text{ along } \underline{a} \end{array} \right.$

9- Calculate all the transformations

$$ {}^{i-1}_{\;\;i}T = \begin{bmatrix} c\theta_i & -s\theta_i & 0 & a_{i-1} \\ c\alpha_{i-1} s\theta_i & c\alpha_{i-1} c\theta_i & -s\alpha_{i-1} & -s\alpha_{i-1} d_i \\ s\alpha_{i-1} s\theta_i & s\alpha_{i-1} c\theta_i & c\alpha_{i-1} & c\alpha_{i-1} d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^{0}_{1}T = \begin{bmatrix} c\theta_1 & -s\theta_1 & 0 & 0 \\ s\theta_1 & c\theta_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

look at the first row of DH Table

$\alpha_0 = a_0 = d_1 = 0$

θ_1 var

$$ {}^{1}_{2}T = \begin{bmatrix} 1 & 0 & 0 & L_1 \\ 0 & 0 & -1 & -d_2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

look at the 2nd row of DH Table

$\alpha_1 = 90$

$a_1 = L_1$, $\theta_2 = 0$, d_2 var

$$ {}^{2}T = \begin{bmatrix} c\theta_3 & -s\theta_3 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} $$

look at the 3rd

$$ {}^{2}_{3}T = \begin{bmatrix} C\theta_3 & -S\theta_3 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ S\theta_3 & C\theta_3 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

look at the 3rd row at DH Table

$\alpha_2 = 90$

$d_3, a_2 = 0$ & θ_3 var

$$ {}^{0}_{3}T = {}^{0}_{1}T \cdot {}^{1}_{2}T \cdot {}^{2}_{3}T = \left[\begin{array}{c|c|c|c} C_1C_3 + S_1S_3 & -C_1S_3 + S_1C_3 & 0 & L_1C_1 + d_2S_1 \\ S_1C_3 - C_1S_3 & -S_1S_3 - C_1C_3 & 0 & L_1S_1 - d_2C_1 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right] $$

10) where is the endeffector? ${}^{0}P$?

First: in frame {3} $\rightarrow$ ${}^{3}P = [0 \;\; L_3 \;\; 0]^T$

Finally $\begin{bmatrix} {}^{0}P \\ \hline 1 \end{bmatrix} = {}^{0}_{3}T \begin{bmatrix} {}^{3}P \\ \hline 1 \end{bmatrix}$

QS. 4

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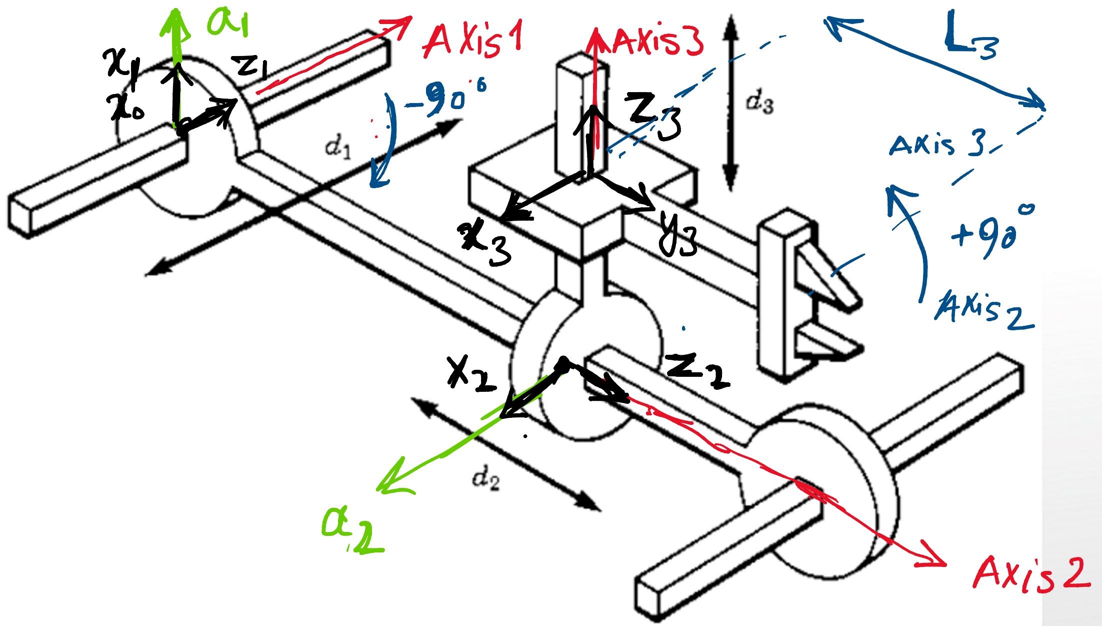
- **Question 4:**
  - For the robot shown below,
  - Derive the DH parameters (you may add in any constants i.e. link lengths or offset as required), sketch the frames, and calculate the Transform matrix.



1- Draw Axes

2- Draw Mutual Prependiculars a_1 & a_2

3- Link Length (do it from 1 to n-1=2)

$\boxed{a_1}$ = dist. from Ax1 to Ax2 along mutual ⊥ = $\boxed{0}$ intersecting

$\boxed{a_2}$ = dist. from Ax2 to Ax3 along mutual ⊥ = $\boxed{0}$

$\boxed{a_0 = 0}$ by convention

4. Link Twist (do it from 1 to n-1=2)

$\boxed{\alpha_1}$ = Angle ∡ Between Ax.1 & Ax.2 about a_1 = $\boxed{-90}$

$\boxed{\alpha_2}$ = Angle ∡ Between Ax.2 & Ax.3 about a_2 = $\boxed{90}$

$\boxed{\alpha_0 = 0}$ by convention

5. Link offset (do it from 2 to $n-1=2$)

$\boxed{d_2}$ = dist. from a_1 to a_2 along Ax. 2 = $\boxed{d_2}$ → Variable

$\boxed{d_1 \& d_3 = d_1 \& d_3}$ Variable by Convention for Prismatic Joints

6 - Join Angles (do it from 2 to $n-1=2$)

$\boxed{\theta_2 =}$ Angle Between $a_1 \& a_2$ about Ax. 2 → $\boxed{90}$

$\boxed{\theta_1 \& \theta_3 = 0}$ by Convention for Prismatic Joints

7- Transfer into DH Table

i	α_{i-1}	a_{i-1}	d_i	θ_i
1	α_0 0	a_0 0	d_1 d_1	θ_1 0
2	α_1 -90	a_1 0	d_2 d_2	θ_2 90
3	α_2 90	a_2 0	d_3 d_3	θ_3 0

8- Draw the frames in above Picture

{ Z along axis
origin is where a intersects axis
X along a

9- Calculate all the transformations

$$
{}^{i-1}_{\ \ i}T = \begin{bmatrix} c\theta_i & -s\theta_i & 0 & a_{i-1} \\ c\alpha_{i-1}\, s\theta_i & c\alpha_{i-1}\, c\theta_i & -s\alpha_{i-1} & -s\alpha_{i-1}\, d_i \\ s\alpha_{i-1}\, s\theta_i & s\alpha_{i-1}\, c\theta_i & c\alpha_{i-1} & c\alpha_{i-1}\, d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^{0}_{1}T = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

look at the first row of DH Table

$\alpha_0 = a_0 = 0$, $\theta_1 = 0$

d_1 var

$$
{}^{1}_{2}T = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & d_2 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

look at the 2nd row of DH Table

$\alpha_1 = -90$

$a_1 = 0$, $\theta_2 = 90$, d_2 var

$$\begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$\alpha_1 = -90$

$a_1 = 0$, $\theta_2 = 90$, d_2 var

$$^{2}_{3}T = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & -d_3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

look at the 3rd row at DH Table

$\alpha_2 = 90$

θ_3, $a_2 = 0$ & d_3 var

$$^{0}_{3}T = {}^{0}_{1}T \cdot {}^{1}_{2}T \cdot {}^{2}_{3}T = \begin{bmatrix} 0 & 0 & 1 & d_3 \\ 0 & 1 & 0 & d_2 \\ -1 & 0 & 0 & d_1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

10) where is the endeffector? ^{0}P ?

First: in frame {3} $\rightarrow$ $^{3}P = [0 \;\; L_3 \;\; 0]^T$

Finally $\begin{bmatrix} ^{0}P \\ \hline 1 \end{bmatrix} = {}^{0}_{3}T \begin{bmatrix} ^{3}P \\ \hline 1 \end{bmatrix}$