Sample for 1st Online Test: (Lectures week 1 - week 7)

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EXAM COVER SHEET

NOTE: DO NOT REMOVE this exam paper from the exam venue

EXAM DETAILS			
Course Code:			
Course Description:			
Date of exam: 17/08/2016	**Start time of exam:**		**Duration of exam:**
Total number of pages (incl. this cover sheet)			

ALLOWABLE MATERIALS AND INSTRUCTIONS TO CANDIDATES

1. Write your full name and student number on each exam booklet together with the number of exam books used.
2. Students must not write, mark in any way any exam materials, read any other text other than the exam paper or do any calculations during reading time.
3. All mobile phones must be switched off and placed under your desk. You are in breach of exam conditions if it is on your person (ie. pocket).
4. This is a **LIMITED TEXT** Exam.

   One double-sided A4 sheet of hand-written or typed material is allowed. If typed, font size 10 or above.
5. Non text storing calculators are allowed.
6. Bi-lingual dictionaries are not allowed.

Advanced Robotics

Final Exam (Semester 2, Year 2016)

Question 1 (3 Marks) (25min)

Frame {B} is initially coincident with frame {A}. First, {B} is rotated about X_B by 30 degrees, and is subsequently rotated about Y_B by 45 degrees, followed by a rotation about Z_B by 15 degrees.

(a) Give the rotation matrix which accomplishes these 3 rotations in the given order. (1 Marks) . Please give all values in the calculations for 4 decimal points.

(b) Calculate the Euler parameters for the above rotation matrix. **Note**: Do **NOT** calculate part (c) first and use the values to calculate answers for part (b). (1 Marks)

(c) Calculate the unit vector and the single angle of rotation, using the equivalent angle-axis representation, for the above rotation matrix. (Marks)

Space for answer:

(a) Euler angle : $X_B(30) \rightarrow Y_B(45) \rightarrow Z_B(15)$

$$^A_B R_{x'y'z'} = R_X \cdot R_Y \cdot R_Z$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & c30 & -s30 \\ 0 & s30 & c30 \end{bmatrix} \cdot \begin{bmatrix} c45 & 0 & s45 \\ 0 & 1 & 0 \\ -s45 & 0 & c45 \end{bmatrix} \cdot \begin{bmatrix} c15 & -s15 & 0 \\ s15 & c15 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0.866 & -0.5 \\ 0 & 0.5 & 0.866 \end{bmatrix} \begin{bmatrix} 0.7071 & 0 & 0.7071 \\ 0 & 1 & 0 \\ -0.7071 & 0 & 0.7071 \end{bmatrix} \begin{bmatrix} 0.9659 & -0.2588 & 0 \\ 0.2588 & 0.9659 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0.7071 & 0 & 0.7071 \\ 0.3536 & 0.866 & -0.3536 \\ -0.6123 & 0.5 & 0.6123 \end{bmatrix} \begin{bmatrix} 0.9659 & -0.2588 & 0 \\ 0.2588 & 0.9659 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \underline{\underline{\begin{bmatrix} 0.6830 & -0.1830 & 0.7071 \\ 0.5657 & 0.7450 & -0.3536 \\ -0.4620 & 0.6414 & 0.6123 \end{bmatrix}}}$$

(15 mins)

(b) Euler parameters:

$$\varepsilon_4 = \frac{1}{2}\sqrt{1 + r_{11} + r_{22} + r_{33}}$$

$$= \frac{1}{2}\sqrt{3.0403} = \underline{\underline{0.8718}}$$

$$\varepsilon_1 = \frac{r_{32} - r_{23}}{4\varepsilon_4} = \underline{\underline{0.2853}}$$

$$\varepsilon_2 = \frac{r_{13} - r_{31}}{4\varepsilon_4} = \underline{\underline{0.3353}}$$

$$\varepsilon_3 = \frac{r_{21} - r_{12}}{4\varepsilon_4} = \underline{\underline{0.2147}}$$

(5 mins)

(c)

$$\theta = \arccos\left(\frac{r_{11} + r_{22} + r_{33} - 1}{2}\right) = \arccos(0.52015) = \underline{\underline{58.6577^\circ}}$$

$$\hat{k} = \frac{1}{2\sin\theta}\begin{bmatrix} r_{32} - r_{23} \\ r_{13} - r_{31} \\ r_{21} - r_{12} \end{bmatrix} = \frac{1}{1.7081}\begin{bmatrix} 0.995 \\ 1.1691 \\ 0.7487 \end{bmatrix} = \underline{\underline{\begin{bmatrix} 0.5825 \\ 0.6844 \\ 0.4383 \end{bmatrix}}}$$

(5 mins)

Question 2 (2 Marks) (10 min) // 35 min

The rotation matrix from A to B is:

$$ {}^{A}_{B}R = \begin{bmatrix} 0.933 & 0.067 & 0.354 \\ 0.067 & 0.933 & -0.354 \\ -0.354 & 0.354 & 0.866 \end{bmatrix} $$

Also, the origin of frame {B} has the coordinate of $[5, 10, 2]^T$ when specified in frame {A}.

(a) Write down the homogenous transformation matrix ${}^{A}_{B}T$ (1 Mark).

(b) Calculate the inverse of the above homogenous transformation matrix (1 Marks).

Space for answer:

(a) $$ {}^{A}_{B}T = \begin{bmatrix} 0.933 & 0.067 & 0.354 & 5 \\ 0.067 & 0.933 & -0.354 & 10 \\ -0.354 & 0.354 & 0.866 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

(1 Min)

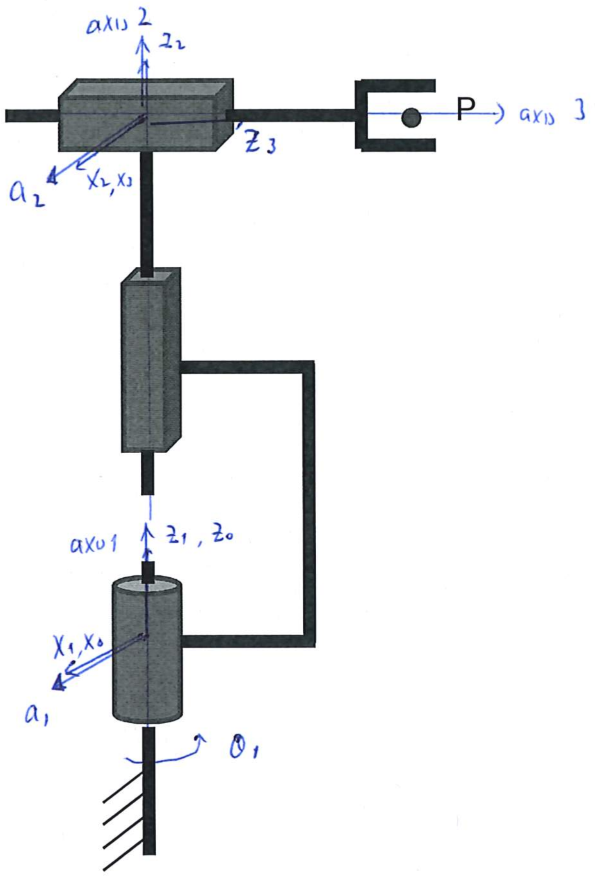
(b) $$ \left({}^{A}_{B}T\right)^{-1} = \left[\begin{array}{c|c} {}^{A}_{B}R^T & -{}^{A}_{B}R^T \cdot {}^{A}P_{BORG} \\ \hline 0\ 0\ 0 & 1 \end{array}\right] $$

$$ = \left[\begin{array}{ccc|c} 0.933 & 0.067 & -0.354 & -4.627 \\ 0.067 & 0.933 & 0.354 & -10.172 \\ 0.354 & -0.354 & 0.866 & +0.038 \\ \hline 0 & 0 & 0 & 1 \end{array}\right] $$

(9 mins)

Question 3 (5 Marks) (35 mins) // 70 min

The kinematic structure of a 3-link RPP robot is shown in the following diagram.



(a) Derive the DH-Parameters of the manipulator, and tabulate them. You should provide some simple explanations (e.g. convention, definition) of how you get those parameters instead of just writing the final answer (4 Marks)

(b) Sketch the frames {0}, {1}, {2} and {3} directly in the above figure. (2 Mark)

(c) Calculate the transformation matrices ${}^0_1T$, ${}^1_2T$, ${}^2_3T$ and ${}^0_3T$ (Marks).

(d) Calculate the linear Jacobian matrix for the manipulator. (1 Marks).

(e) Calculate the rotational Jacobian matrix for the manipulator. (1 Mark).

Space for answer:

(a) 1. Draw axis

2. Mutual $\perp$: axis 1 & 2 : arbitrary. Put at joint 1.

axis 2 & 3 : as shown.

3. Link lengths a_{i-1}

length of mutual ⊥ from axis i-1 to i.

a_1 from axis 1 to 2 : 0

a_2 from axis 2 to 3 : 0.

4 Link twist α_{i-1}

∡ btween axis i-1 & i, about a_{i-1}

α_1 (∡ axis 1 & 2) : 0°

α_2 (∡ axis 2 & 3 abt a_2) : -90°

5. θ_1 variable

θ_2 (∡ btween a_1 and a_2 abt joint 1) : 0°

θ_3 (∡ betwen a_2 and a_3 abt joint 2) = 0° by convention

6. d_1 (distance from a_0 to a_1) = 0°

d_2 variable

d_3 variable.

i	α_{i-1}	a_{i-1}	d_i	θ_i
1	0	0	0	θ_1
2	0	0	d_2	0
3	-90	0	d_3	0

(15 mins)

(b) (1 min)

(c)
$$ {}^{i-1}_{i}T = \begin{bmatrix} c\theta_i & -s\theta_i & 0 & a_{i-1} \\ c\alpha_{i-1}s\theta_i & c\alpha_{i-1}c\theta_i & -s\alpha_{i-1} & -s\alpha_{i-1}d_i \\ s\alpha_{i-1}s\theta_i & s\alpha_{i-1}c\theta_i & c\alpha_{i-1} & c\alpha_{i-1}d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^{0}_{1}T = \begin{bmatrix} c\theta_1 & -s\theta_1 & 0 & 0 \\ s\theta_1 & c\theta_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad {}^{1}_{2}T = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_2 \\ 0 & 0 & 0 & 0 \end{bmatrix} $$

$$ {}^{2}_{3}T = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

(14 min)

$$ {}^{0}_{3}T = {}^{0}_{1}T \cdot {}^{1}_{2}T \cdot {}^{2}_{3}T $$

$$ = \begin{bmatrix} c_1 & -s_1 & 0 & 0 \\ s_1 & c_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_2 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ = \begin{bmatrix} c_1 & -s_1 & 0 & 0 \\ s_1 & c_1 & 0 & 0 \\ 0 & 0 & 1 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ = \begin{bmatrix} c_1 & 0 & -s_1 & -s_1 d_3 \\ s_1 & 0 & c_1 & c_1 d_3 \\ 0 & -1 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

(d)

$$ x = f_1 = -s_1 d_3 $$
$$ y = f_2 = c_1 d_3 $$
$$ z = f_3 = d_2 $$

$$ \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix} = \begin{bmatrix} \frac{\partial f_1}{\partial \theta_1} & \frac{\partial f_1}{\partial d_2} & \frac{\partial f_1}{\partial d_3} \\ \frac{\partial f_2}{\partial \theta_1} & \frac{\partial f_1}{\partial d_2} & \frac{\partial f_2}{\partial d_3} \\ \frac{\partial f_3}{\partial \theta_1} & \frac{\partial f_3}{\partial d_2} & \frac{\partial f_3}{\partial d_3} \end{bmatrix} \begin{bmatrix} \dot{\theta}_1 \\ \dot{d}_2 \\ \dot{d}_3 \end{bmatrix} = \underbrace{\begin{bmatrix} -c_1 d_3 & 0 & -s_1 \\ -s_1 d_3 & 0 & c_1 \\ 0 & 1 & 0 \end{bmatrix}}_{J_v} \begin{bmatrix} \dot{\theta}_1 \\ \dot{d}_2 \\ \dot{d}_3 \end{bmatrix} $$

(5 min)

(e)

$$ J_w = \begin{bmatrix} \text{3rd column of } {}^{0}_{1}R & \begin{matrix} 0 \\ \text{prismatic} \end{matrix} & \begin{matrix} 0 \\ \text{prismatic} \end{matrix} \end{bmatrix} = \begin{bmatrix} \cancel{0} & 0 & 0 \\ \cancel{0} & 0 & 0 \\ \cancel{0} & 0 & 0 \end{bmatrix} $$

(1 min)

$$ \begin{matrix} 0 \\ 0 \\ 1 \end{matrix} \quad ! $$

$$ \begin{bmatrix} \dot{\alpha} \\ \dot{\beta} \\ \dot{\gamma} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{\theta}_1 \\ \dot{d}_2 \\ \dot{d}_3 \end{bmatrix} \Rightarrow \begin{matrix} \dot{\alpha} = 0 \\ \dot{\beta} = 0 \\ \dot{\gamma} = \dot{\theta}_1 \end{matrix} $$

Question 4 (10 Marks) (30 min) // 100 min

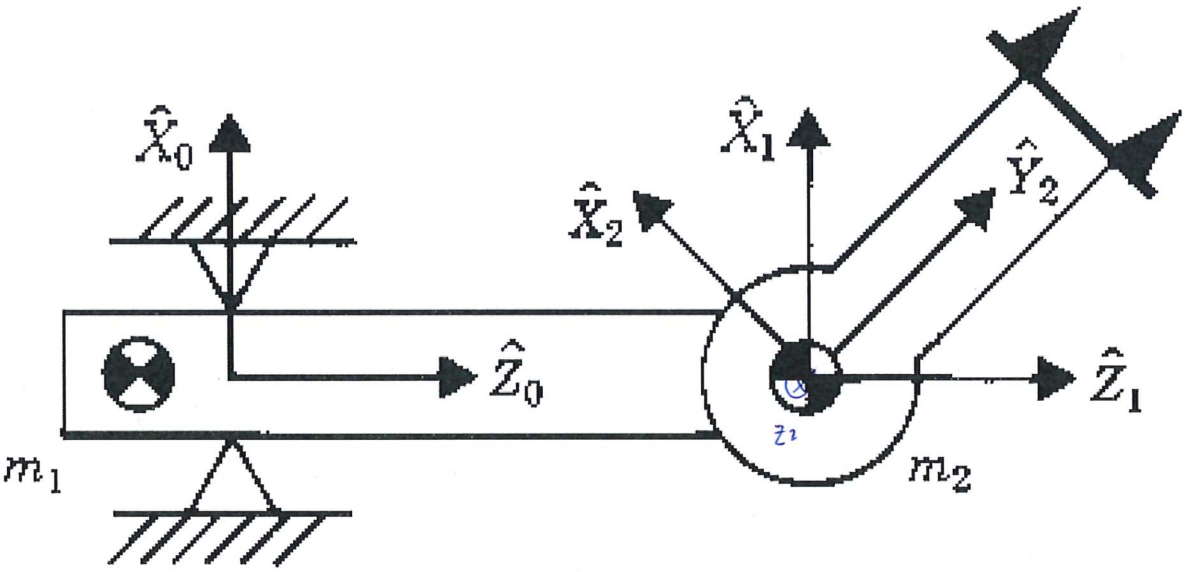
Consider the following PR Manipulator.

Friction is negligible, and gravity points in the negative X_0 direction.

The inertia tensor of the links are diagonal with moments Ixx1, Iyy1, Izz1 and Ixx2, Iyy2, Izz2.

The centers of mass for each link are given by:

$$
{}^{1}P_{C_1} = \begin{bmatrix} 0 \\ 0 \\ -l_1 \end{bmatrix} \quad {}^{2}P_{C_2} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}
$$



(a) Tabulate the DH-parameters of the manipulator. (Hint: The axes are already labelled, and thus the parameters can be read-off easily). (1 Marks)

(b) Calculate all the required quantities (velocities, accelerations, forces / moments) for the outward iterations of the iterative Newton-Euler dynamic algorithm. (Marks)

(c) Calculate all the required quantities (forces, torques) for the inward iterations of the iterative Newton-Euler dynamic algorithm. (3 Marks)

(d) Write down the dynamic equations for the manipulator. The answer should be given in the "general structure" involving mass matrix, Coriolis and centrifugal vector, as well as gravity vector. (2 Marks)

Space for answer: (~~30 MIN~~) //

(a)

i	α_{i-1}	a_{i-1}	d_i	θ_i
1	0	0	d_1	0
2	90°	0	0	θ_2

(5 mins)

(b) $${}^0_1R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad {}^1_2R = \begin{bmatrix} c_2 & -s_2 & 0 \\ s_2 & c_2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$${}^0\omega_0 = 0, \quad {}^0\dot{\omega}_0 = 0, \quad {}^0\dot{v}_0 = g\hat{X}_0$$

$${}^1\omega_1 = ({}^1_0R \cdot \underbrace{{}^0\omega_0}_{=0}) + (\underbrace{\dot{\theta}_1}_{=0} \cdot {}^1\hat{Z}_1)$$

$$= \underline{\underline{0}}$$

$${}^1\dot{\omega}_1 = ({}^1_0R \cdot \underbrace{{}^0\dot{\omega}_0}_{=0}) + ({}^1_0R \cdot {}^1\omega_1 \times \underbrace{\dot{\theta}_1}_{=0}\, {}^1\hat{Z}_1) + (\underbrace{\ddot{\theta}_1}_{=0}\, {}^1\hat{Z}_1)$$

$$= \underline{\underline{0}}$$

$${}^1\dot{v}_1 = (\underbrace{{}^1_0R}_{I} \cdot {}^0\dot{v}_0) + (\underbrace{2\,{}^1\omega_1}_{=0} \times \dot{d}_1\, {}^1\hat{Z}_1) + (\ddot{d}_1\, {}^1\hat{Z}_1) + {}^1_0R \cdot (\underbrace{{}^0\dot{\omega}_0}_{=0} \times {}^0P_1 + \underbrace{{}^0\omega_0}_{=0} \times \underbrace{{}^0\omega_0}_{=0} \times {}^0P_1)$$

$$= \begin{bmatrix} g \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \ddot{d}_1 \end{bmatrix} = \underline{\underline{\begin{bmatrix} g \\ 0 \\ \ddot{d}_1 \end{bmatrix}}}$$

$${}^1\dot{v}_{c1} = ({}^1\dot{v}_1) + (\underbrace{{}^1\dot{\omega}_1}_{=0} \times {}^1P_{c1}) + (\underbrace{{}^1\omega_1}_{=0} \times {}^1\omega_1 \times {}^1P_{c1})$$

$$= \underline{\underline{\begin{bmatrix} g \\ 0 \\ \ddot{d}_1 \end{bmatrix}}}$$

$${}^1F_1 = m_1 \cdot {}^1\dot{v}_{c1} = \underline{\underline{\begin{bmatrix} m_1 g \\ 0 \\ m_1 \ddot{d}_1 \end{bmatrix}}}$$

$${}^1N_1 = {}^{C_1}I_1 \cdot \underbrace{{}^1\dot{\omega}_1}_{=0} + \underbrace{{}^1\omega_1}_{=0} \times {}^{C_1}I_1 \cdot \underbrace{{}^1\omega_1}_{=0} = \underline{\underline{0}}$$

$$ {}^2\omega_2 = ({}^2_1R \cdot \underbrace{{}^1\omega_1}_{=0}) + (\dot\theta_2 \cdot {}^2\hat{Z}_2) = \begin{bmatrix} 0 \\ 0 \\ \dot\theta_2 \end{bmatrix} $$

$$ {}^2\dot\omega_2 = ({}^2_1R \cdot \underbrace{{}^1\dot\omega_1}_{=0}) + ({}^2_1R \cdot \underbrace{{}^1\omega_1}_{=0} \times \dot\theta_2 \, {}^2\hat{Z}_2) + (\ddot\theta_2 \, {}^2\hat{Z}_2) = \begin{bmatrix} 0 \\ 0 \\ \ddot\theta_2 \end{bmatrix} $$

$$ {}^2\dot V_2 = ({}^2_1R \cdot {}^1\dot V_1) + (2 \cdot {}^2\omega_2 \times \underbrace{\dot d_2}_{=0} \cdot {}^2\hat{Z}_2) + (\underbrace{\ddot d_2}_{=0} \cdot {}^2\hat{Z}_2) + {}^2_1R(\underbrace{{}^1\dot\omega_1}_{=0} \times {}^1P_2 + \underbrace{{}^1\omega_1}_{=0} \times \underbrace{{}^1\omega_1}_{=0} \times {}^1P_2) $$

$$ = \begin{bmatrix} c_2 & s_2 & 0 \\ -s_2 & c_2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} g \\ 0 \\ \ddot d_1 \end{bmatrix} = \begin{bmatrix} gc_2 \\ -gs_2 \\ \ddot d_1 \end{bmatrix} $$

$$ {}^2\dot V_{C2} = ({}^2\dot V_2) + ({}^2\dot\omega_2 \times \underbrace{{}^2P_{C2}}_{=0}) + ({}^2\omega_2 \times {}^2\omega_2 \times \underbrace{{}^2P_{C2}}_{=0}) = \begin{bmatrix} gc_2 \\ -gs_2 \\ \ddot d_1 \end{bmatrix} $$

$$ {}^2F_2 = m_2 \, {}^2\dot V_{C2} = \begin{bmatrix} m_2 g c_2 \\ -m_2 g s_2 \\ m_2 \ddot d_1 \end{bmatrix} $$

$$ {}^2N_2 = ({}^{C2}I_2 \cdot {}^2\dot\omega_2) + ({}^2\omega_2 \times {}^{C2}I_2 \cdot {}^2\omega_2) $$

$$ = \begin{bmatrix} I_{xx2} & & \\ & I_{yy2} & \\ & & I_{zz2} \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ \ddot\theta_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \dot\theta_2 \end{bmatrix} \times \begin{bmatrix} I_{xx2} & & \\ & I_{yy2} & \\ & & I_{zz2} \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ \dot\theta_2 \end{bmatrix} $$

$$ = \begin{bmatrix} 0 \\ 0 \\ I_{zz2}\ddot\theta_2 \end{bmatrix} + \begin{vmatrix} i & j & k \\ 0 & 0 & \dot\theta_2 \\ 0 & 0 & I_{zz2}\dot\theta_2 \end{vmatrix} = \begin{bmatrix} 0 \\ 0 \\ I_{zz2}\ddot\theta_2 \end{bmatrix} $$

(15 min)

(c) let ${}^3f_3 = 0 \quad {}^3n_3 = 0$

then: $${}^2f_2 = {}^2_3R \cdot \underbrace{{}^3f_3}_{=0} + {}^2F_2 = \begin{bmatrix} m_2 g c_2 \\ -m_2 g s_2 \\ m_2 \ddot{d}_1 \end{bmatrix}$$

$${}^2n_2 = \left({}^2_3R \cdot \underbrace{{}^3n_3}_{=0}\right) + \left(\underbrace{{}^2P_{c_2}}_{=0} \times {}^2F_2\right) + \left({}^2P_3 \times {}^2_3R \cdot \underbrace{{}^3f_3}_{=0}\right) + \left({}^2N_2\right) = \begin{bmatrix} 0 \\ 0 \\ I_{zz2}\ddot{\theta}_2 \end{bmatrix}$$

$${}^1f_1 = {}^1_2R \cdot {}^2f_2 + {}^1F_1$$

$$= \begin{bmatrix} c_2 & -s_2 & 0 \\ s_2 & c_2 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} m_2 g c_2 \\ -m_2 g s_2 \\ m_2 \ddot{d}_1 \end{bmatrix} + \begin{bmatrix} m_1 g \\ 0 \\ m_1 \ddot{d}_1 \end{bmatrix} = \begin{bmatrix} m_2 g + m_1 g \\ 0 \\ (m_1 + m_2)\ddot{d}_1 \end{bmatrix}$$

$${}^1n_1 = \left({}^1_2R \cdot {}^2n_2\right) + \left({}^1P_{c_1} \times {}^1F_1\right) + \left(\underbrace{{}^1P_2}_{=0} \times {}^1_2R \cdot {}^2f_2\right) + \left({}^1N_1\right)$$

$$= \begin{bmatrix} c_2 & -s_2 & 0 \\ s_2 & c_2 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 0 \\ 0 \\ I_{zz2}\ddot{\theta}_2 \end{bmatrix} + \begin{vmatrix} i & j & k \\ 0 & 0 & -l_1 \\ m_1 g & 0 & m_1\ddot{d}_1 \end{vmatrix} + 0$$

$$= \begin{bmatrix} 0 \\ 0 \\ I_{zz2}\ddot{\theta}_2 \end{bmatrix} + \begin{bmatrix} 0 \\ m_1 g l_1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ m_1 g l_1 \\ I_{zz2}\ddot{\theta}_2 \end{bmatrix}$$

(5 min)

(d) τ_1 = prismatic = ${}^1f_1^T \cdot {}^1\hat{Z}_1 = (m_1 + m_2)\ddot{d}_1$

τ_2 = revolute = ${}^2n_2^T \cdot {}^2\hat{Z}_2 = I_{zz2}\ddot{\theta}_2$

$$\begin{bmatrix} m_1 + m_2 & 0 \\ 0 & I_{zz2} \end{bmatrix}\begin{bmatrix} \ddot{d}_1 \\ \ddot{\theta}_2 \end{bmatrix} = \begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix}$$

(5 min)