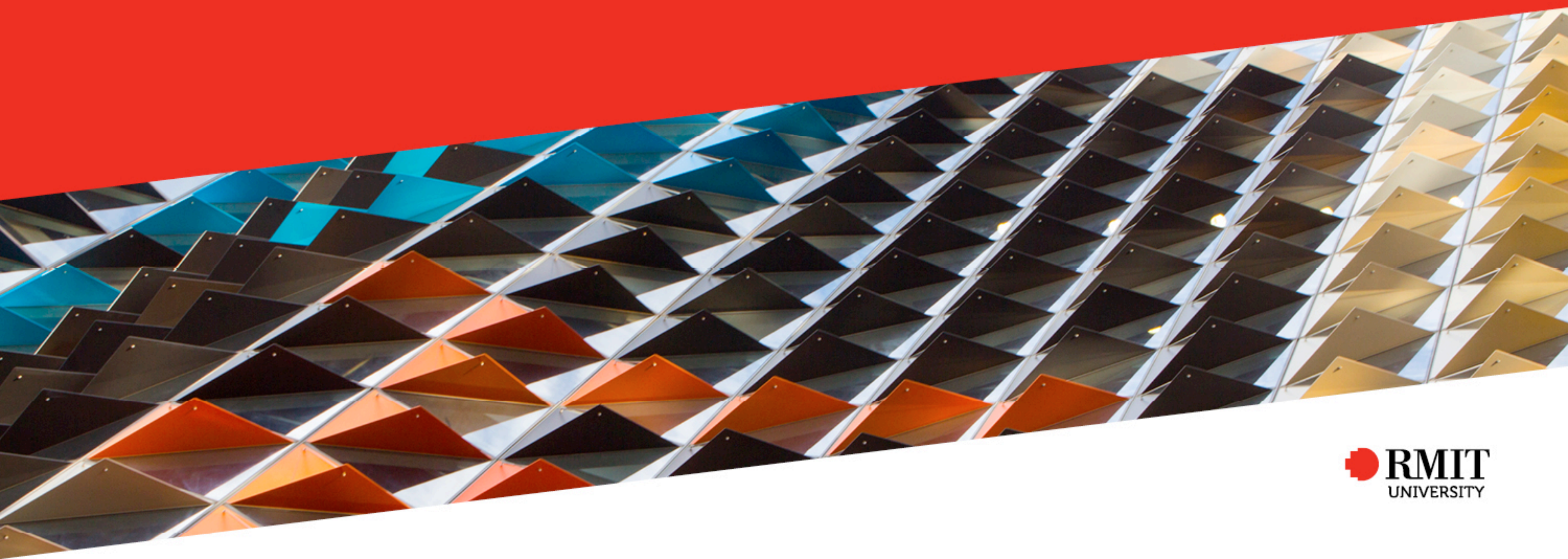
Monte Carlo Simulation using @Risk20

Risk and Project Management, OENG1117



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Background

- Lowrance (1976) defines **risk** as a measure of the **probability** and **severity** of adverse effects.
- Risk = Likelihood X Consequence
- Risk and uncertainty arise from measurement errors and from the **underlying variability** of complex, natural, social, and economic situations (P., S., & P. 1980).
- **Deterministic** vs. **probabilistic** approach
- Probability is used to model **variability** and **frequency** or to quantify the level of confidence we have in the information. Does probability exist as a physical entity?
- A **model** should be as **simple** as possible and **as complex as needed** to answer the expected **questions** (Haimes Y. 2016).
- Generating distributions
    - Historical data
    - Expert judgment
    - Interviews / questionnaire
    - Field / market experiments and investigations

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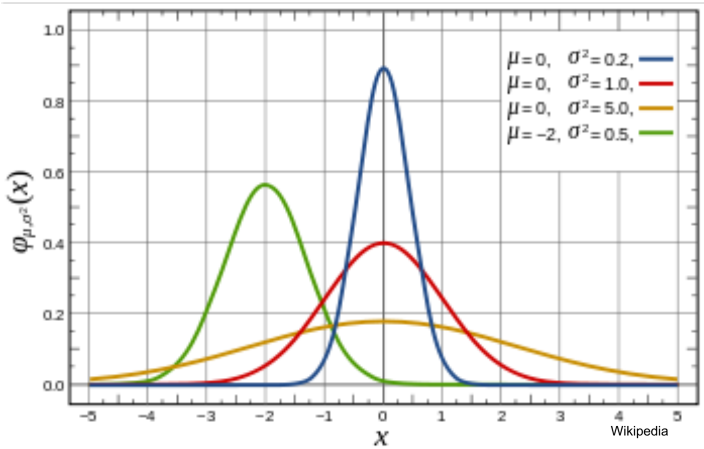
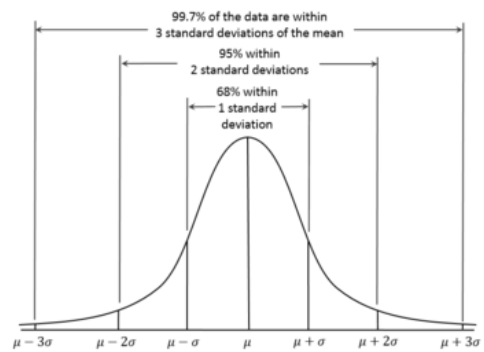
Probability Distribution

Normal distribution





$$f(x \mid \mu, \sigma^2) = \frac{1}{\sigma\sqrt{2\pi}}\, e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

Mean   Variance

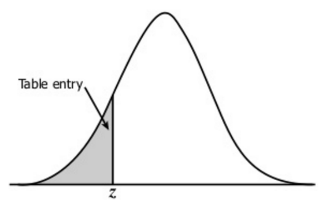


Table entry for z is the area under the standard normal curve to the left of z.

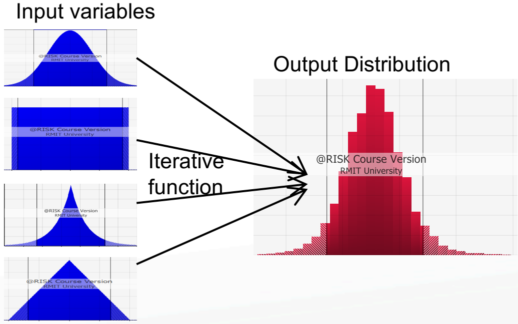
z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
−3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
−3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
−3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
−3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
−3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
−2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
−2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
−2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
−2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036

To calculate the area under the curve:

- Integral
- Z table
- Numerical analysis
  - Direct
  - Iterative

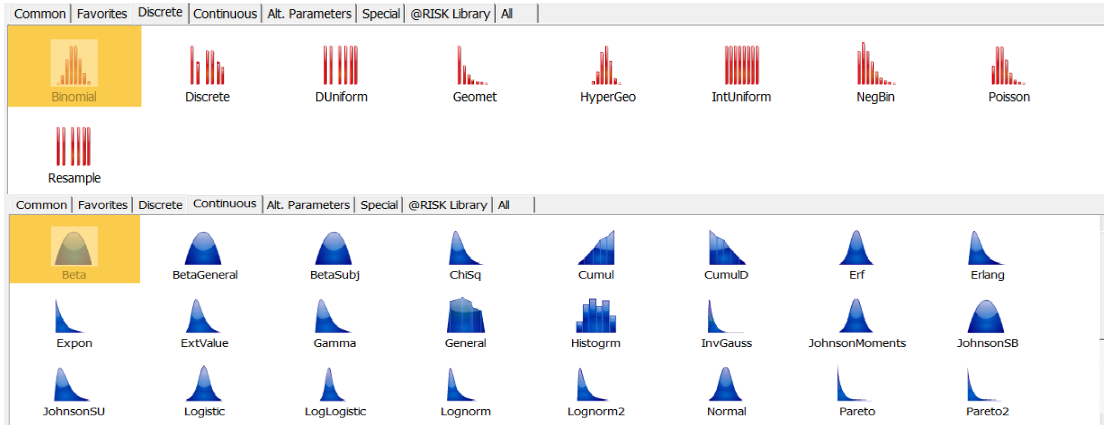
Monte Carlo Simulation

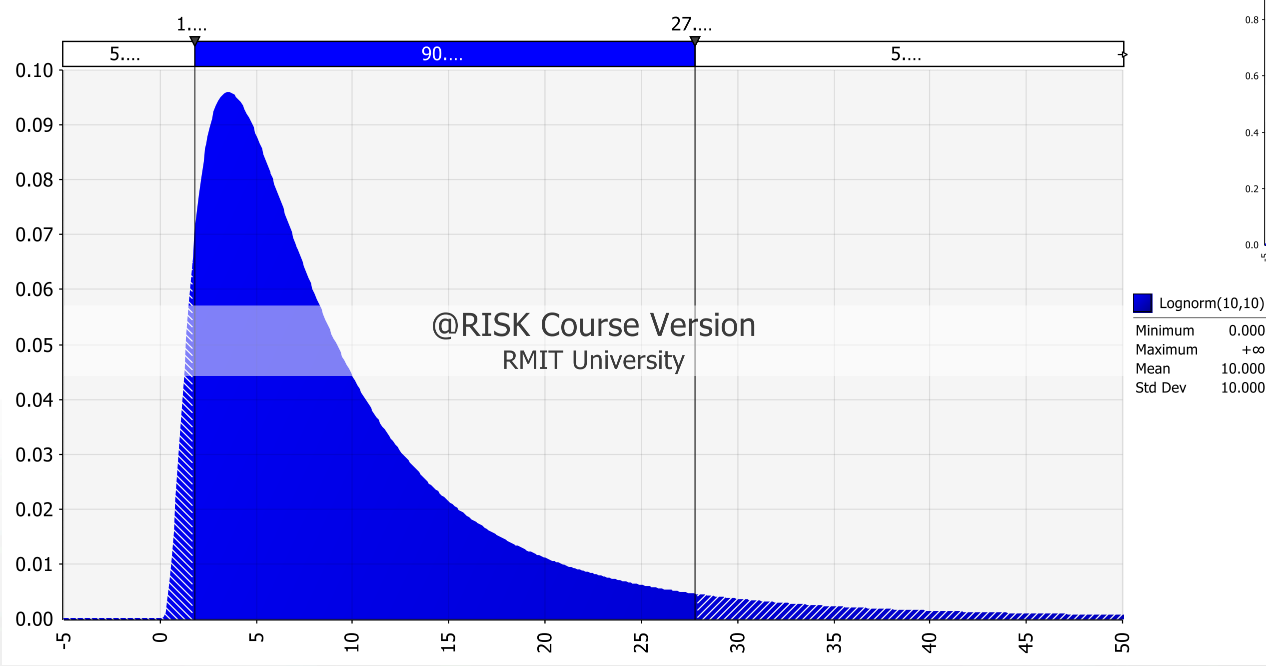
- Classic calculations and closed-form formula vs. random sampling using computer power
- Sampling random variable in a model to obtain numerical results
- Computational algorithm which can be done 1000’s + times to generate a reasonable representation of distribution of input variables
    - Not as prescriptive as deterministic models
    - Possible outcomes and their probabilities
    - More realistic results
    - Requires better understanding of the input variables
    - Used for decision making and understanding the risk involved in decisions
- Monte Carlo gives an information-rich outcome for gaining insight into the model (Prabhakar R., 2011)

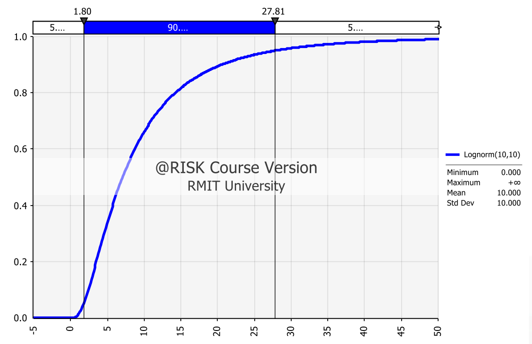


Probability Distribution

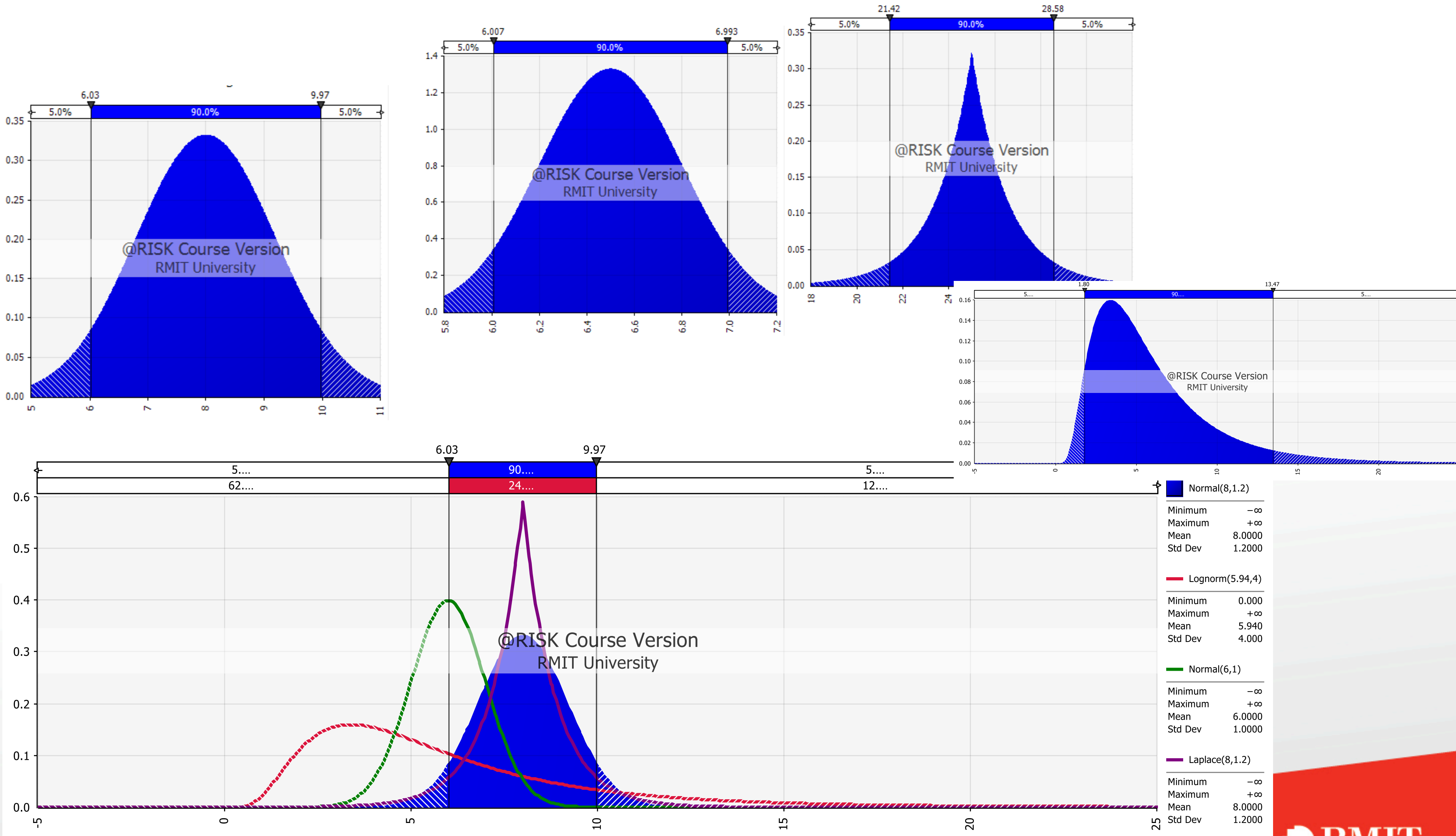
- Discrete vs. continuous distribution
- Probability density function (pdf)
    - Variables (x axis)
    - Relative likelihood (y axis)
- Cumulative distribution function (cdf)







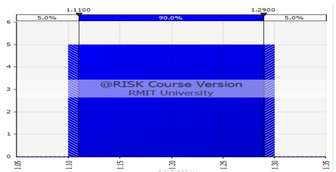
Combining Distributions

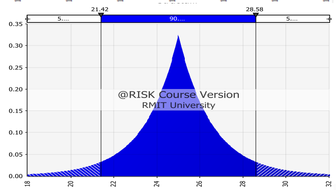


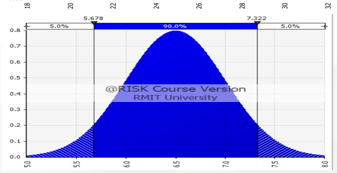
Creating Model – Input / Output Definition

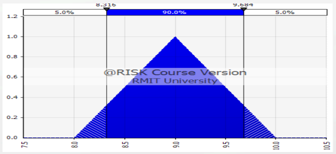
Let's assume a multiple building construction site requires the following:

	Cost (m$)	Cost range	Cost distribution
• Excavation	1.2	1.1 – 1.3	Uniform
• Foundation	3.7	3.7 - 4	Lognormal
• Structural	25	20 - 27	Laplace
• Electrical	4.5	4.3 - 4.7	Weibull
• Mechanical	6.5	6.3 - 6.6	Normal
• Finishing	8	5.5 - 9.5	Normal
• Labour	9.5	8 - 10	Triang
	58.4	48.9 – 63.1	?





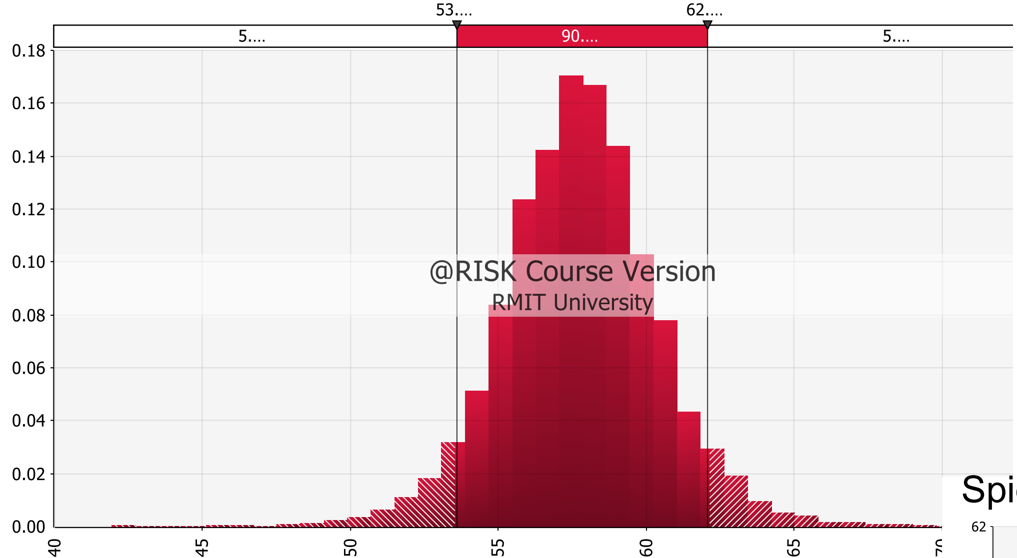




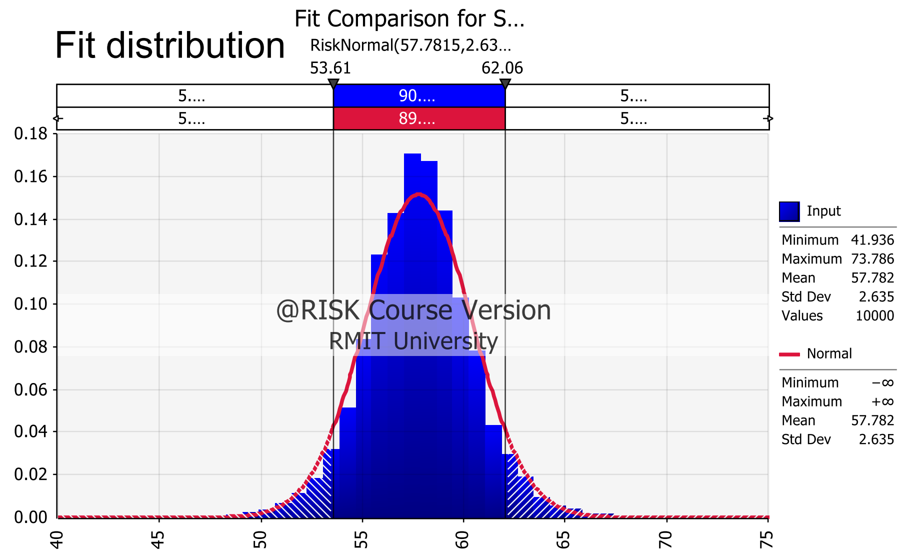
Total construction cost = EW + S + El + M + F + L

Creating Model – Generating Output

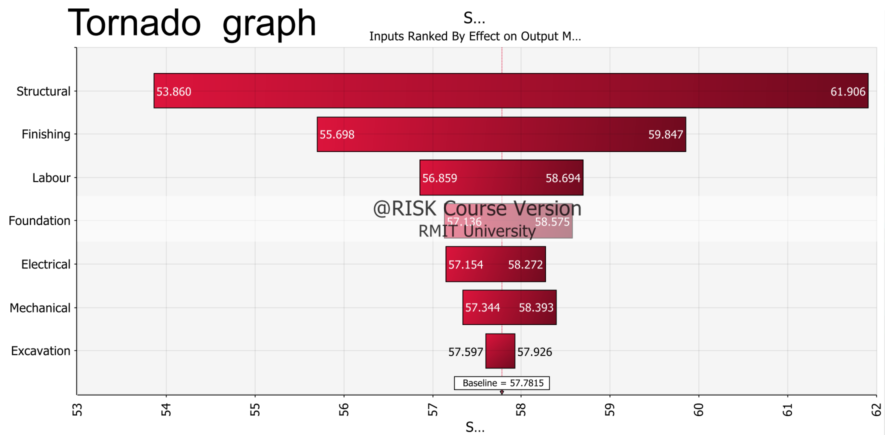
Total construction cost



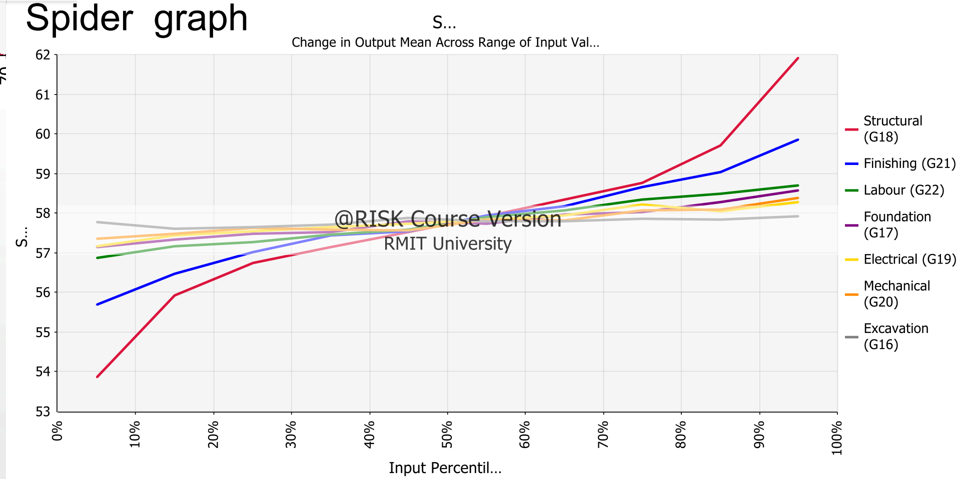
Fit distribution



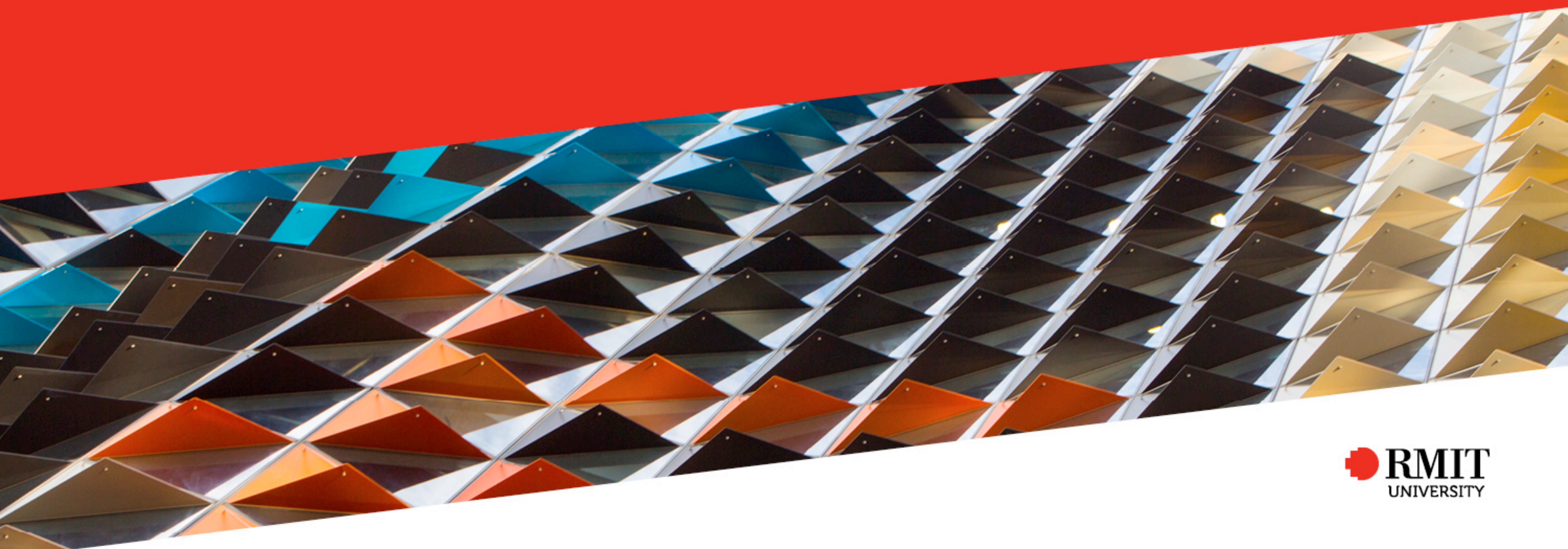
Tornado graph



Spider graph



@Risk - Example



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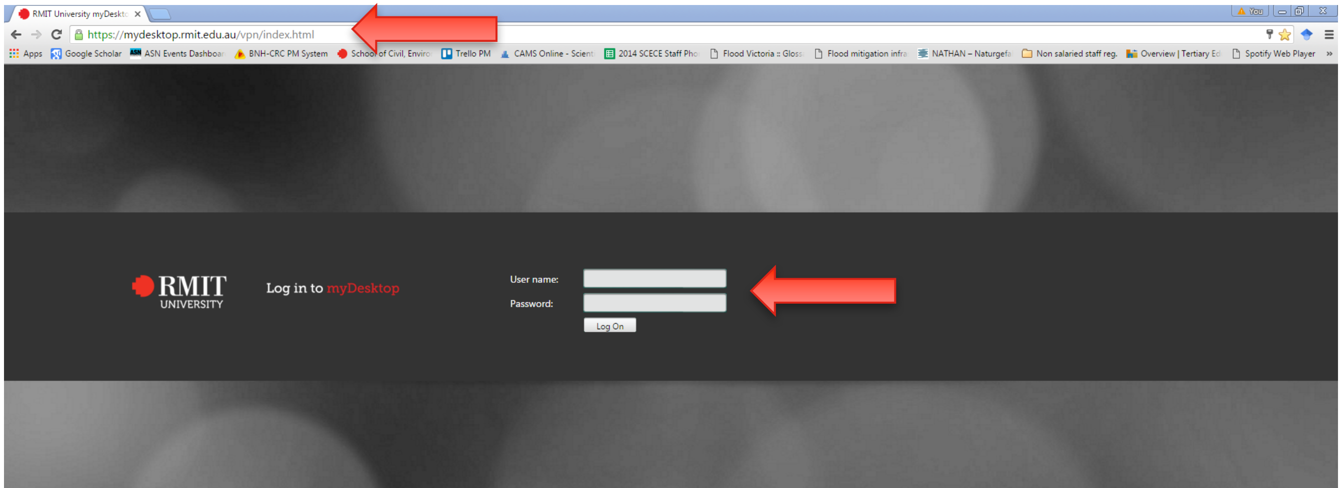
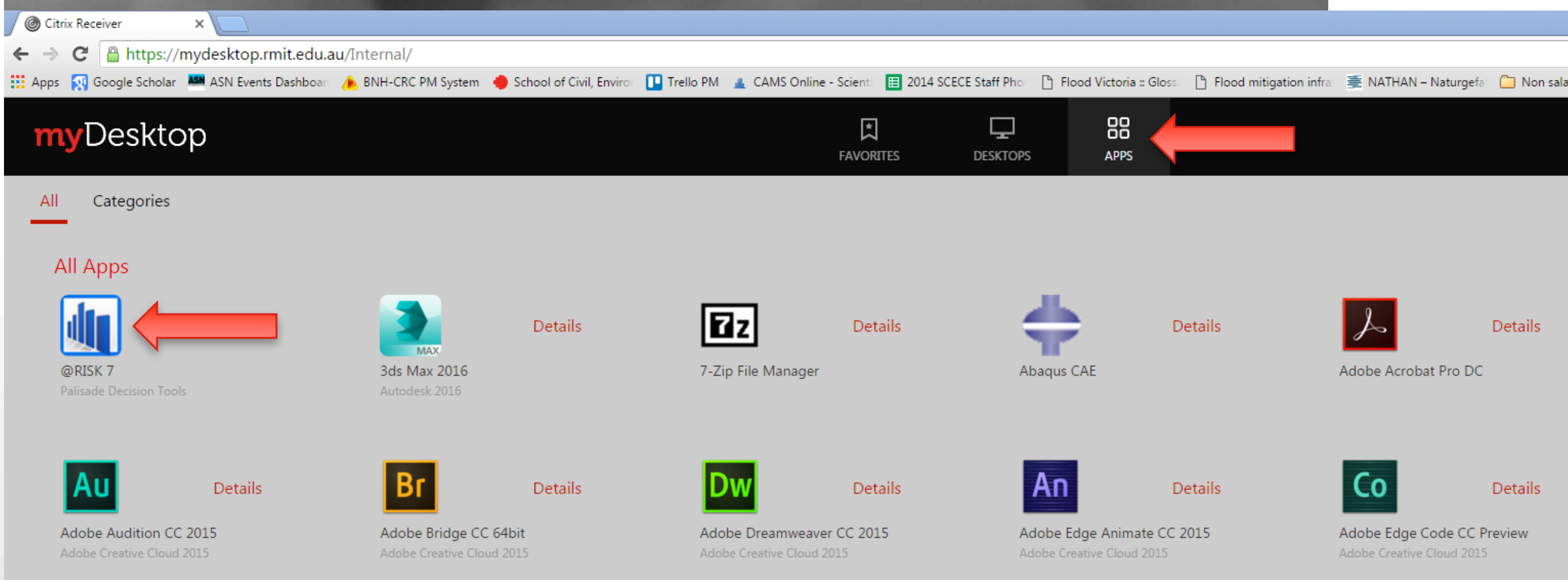
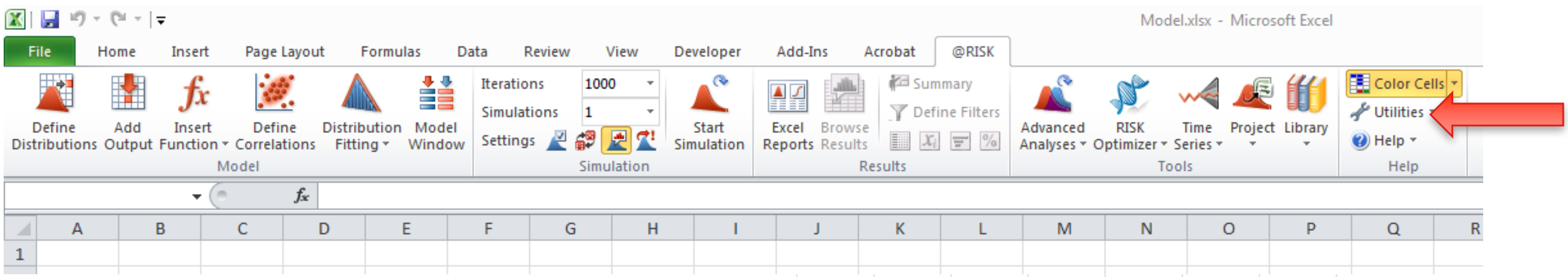
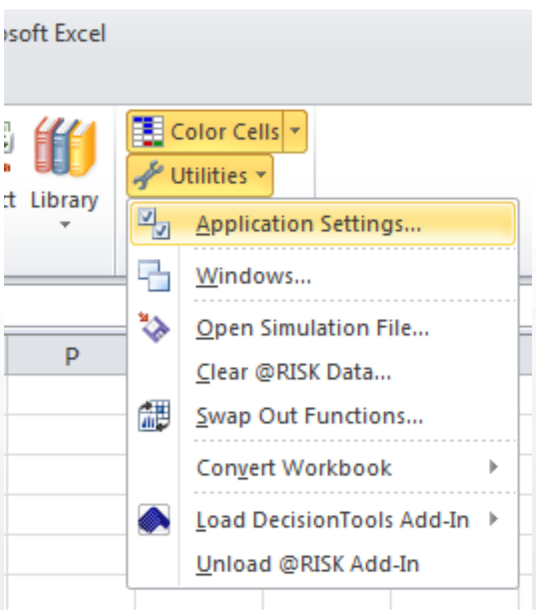


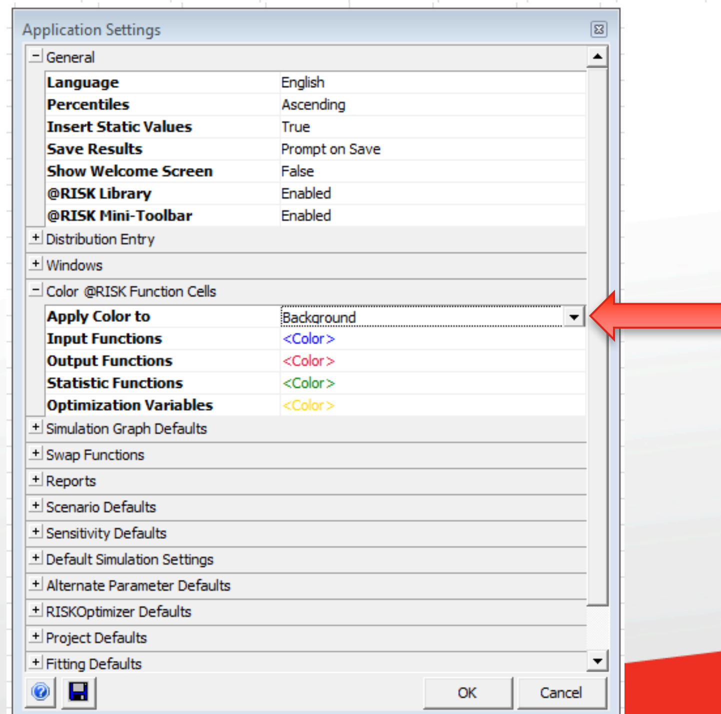


@Risk

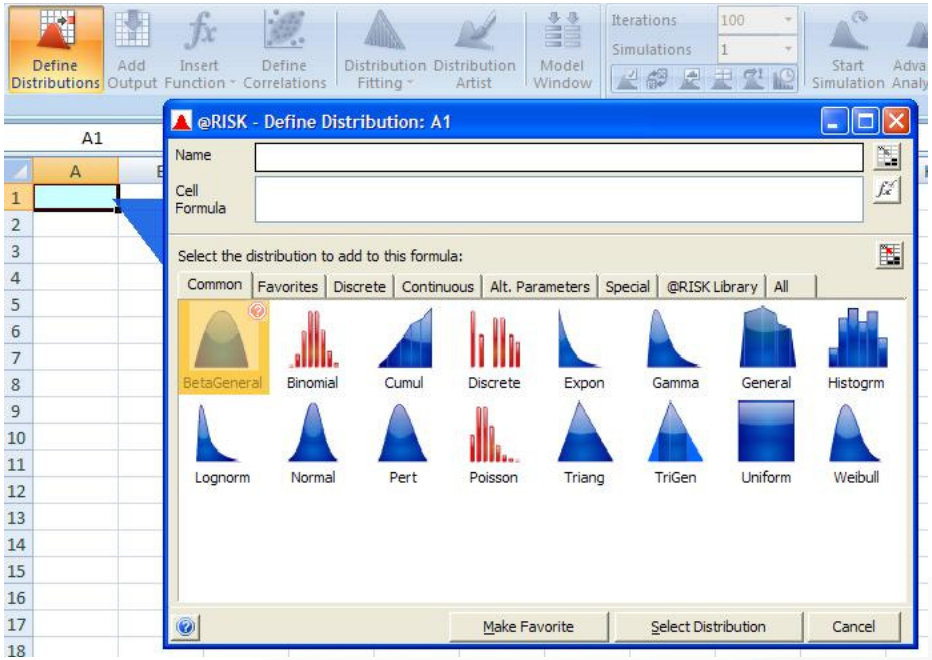
Program settings: to distinguish between inputs and outputs

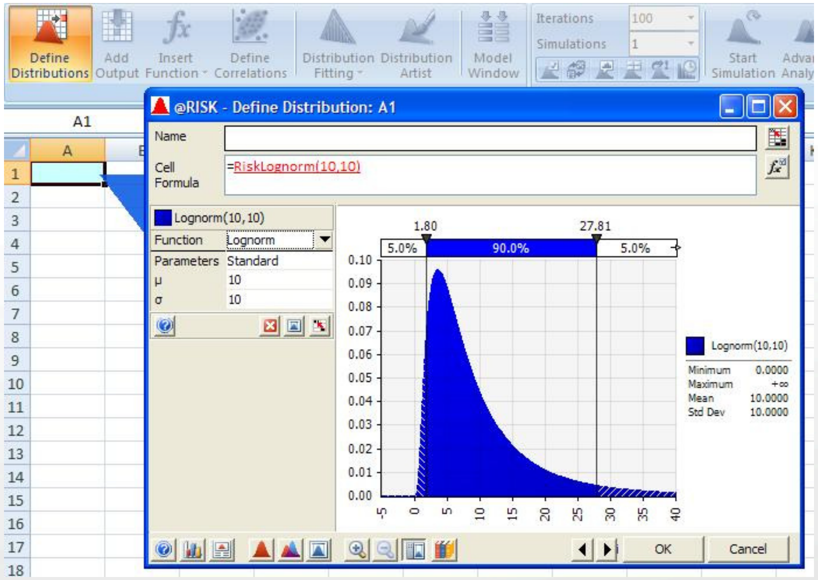






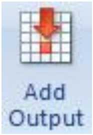
Selecting and defining a distribution

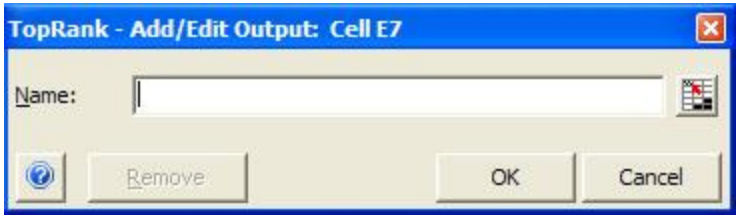




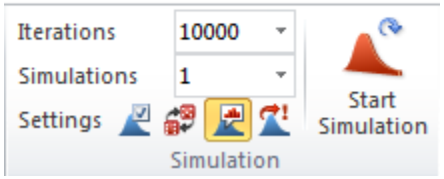
Defining an output

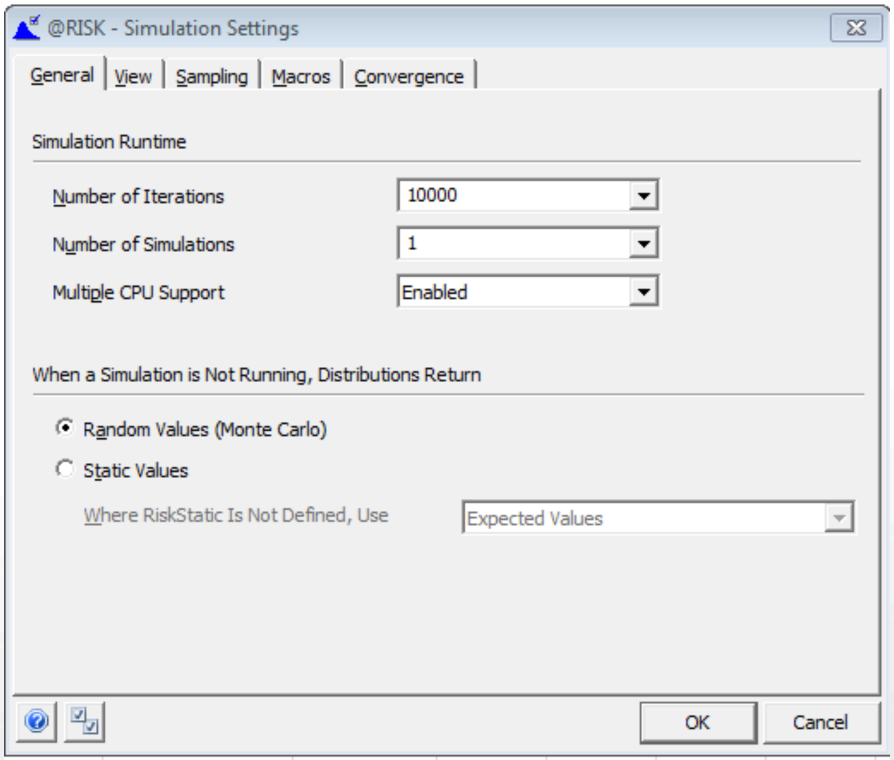
- Outputs are the target cells / cells of interest in the model
- They can contain Excel and/or @Risk functions
- Click on the cell and hit the “Add Output” button

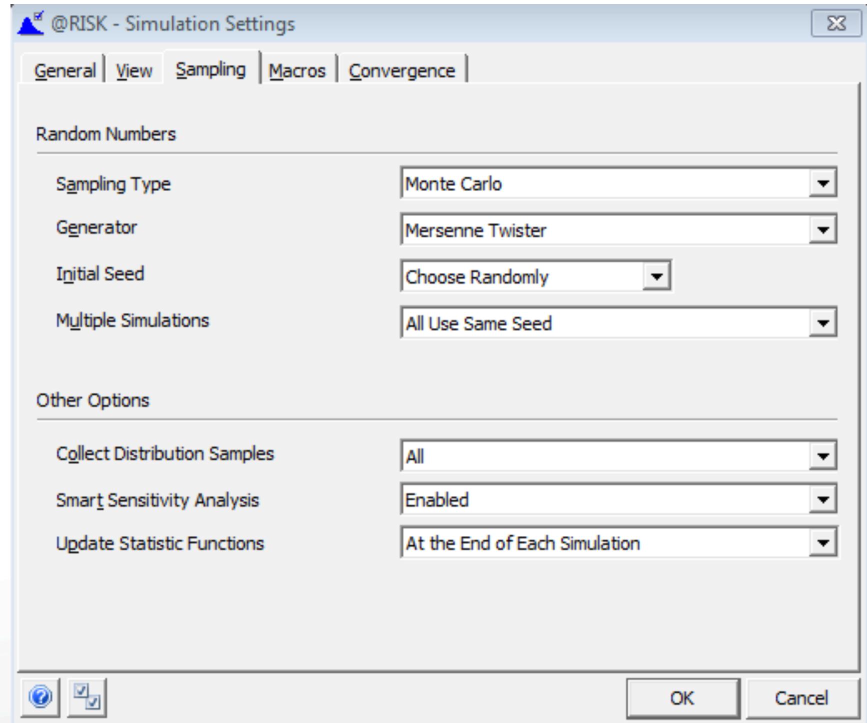




Simulation settings

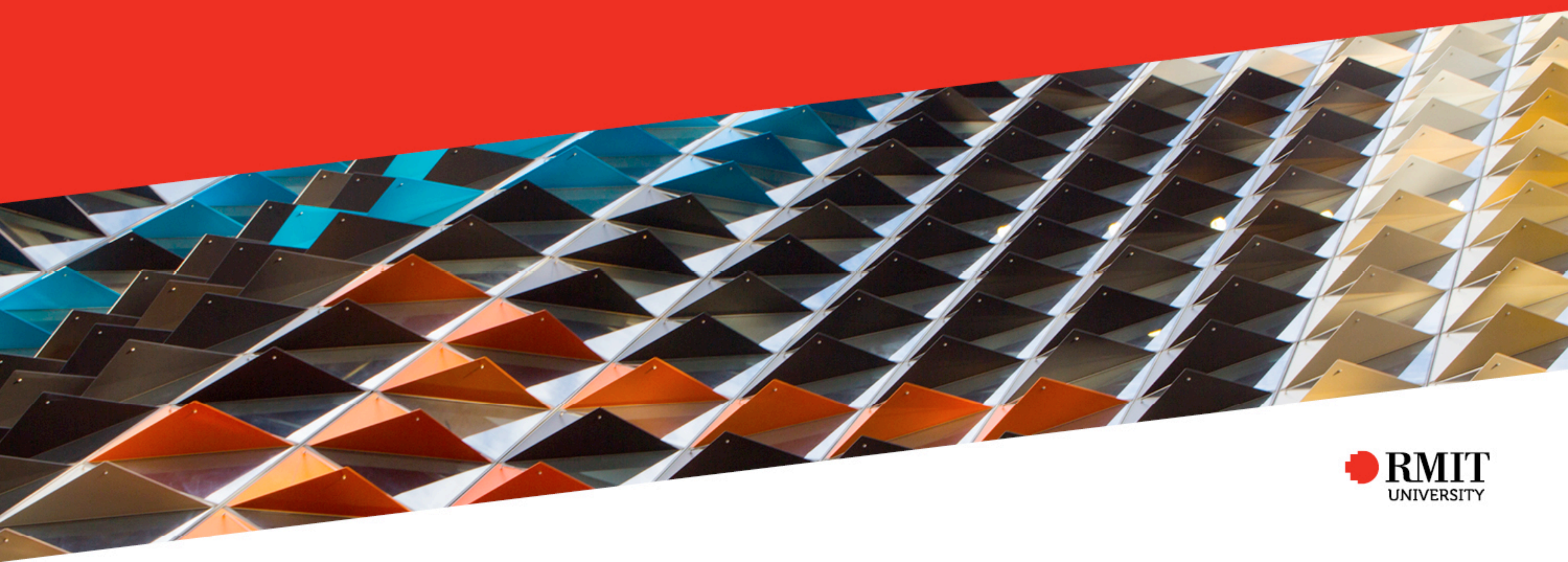






Exercise 1

Building Project



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Variable Distribution Input

Define the following input distributions

Task	Distribution Type	Parameters	
Excavation	Uniform	Min	Max
		1.1	1.3
Foundation	Lognormal	μ	σ
		3.7	0.5
Structural	Laplace	μ	σ
		25	2.2
Electrical	Weibull	α	β
		14.3	4.5
Mechanical	Normal	μ	σ
		6.5	0.3
Finishing	Normal	μ	σ
		8	1.2
Labour	Uniform	Min	Max
		8	10

Variable Distribution Input

Task	Distribution Type	Parameters		Input Parameters
Excavation	Uniform	**Min**	**Max**	
		1.1	1.3	1.2
Foundation	Lognormal	**μ**	**σ**	
		3.7	0.5	3.7
Structural	Laplace	**μ**	**σ**	
		25	2.2	25
Electrical	Weibull	**α**	**β**	
		14.3	4.5	4.34
Mechanical	Normal	**μ**	**σ**	
		6.5	0.3	6.5
Finishing	Normal	**μ**	**σ**	
		8	1.2	8
Labour	Uniform	**Min**	**Max**	
		8	10	9

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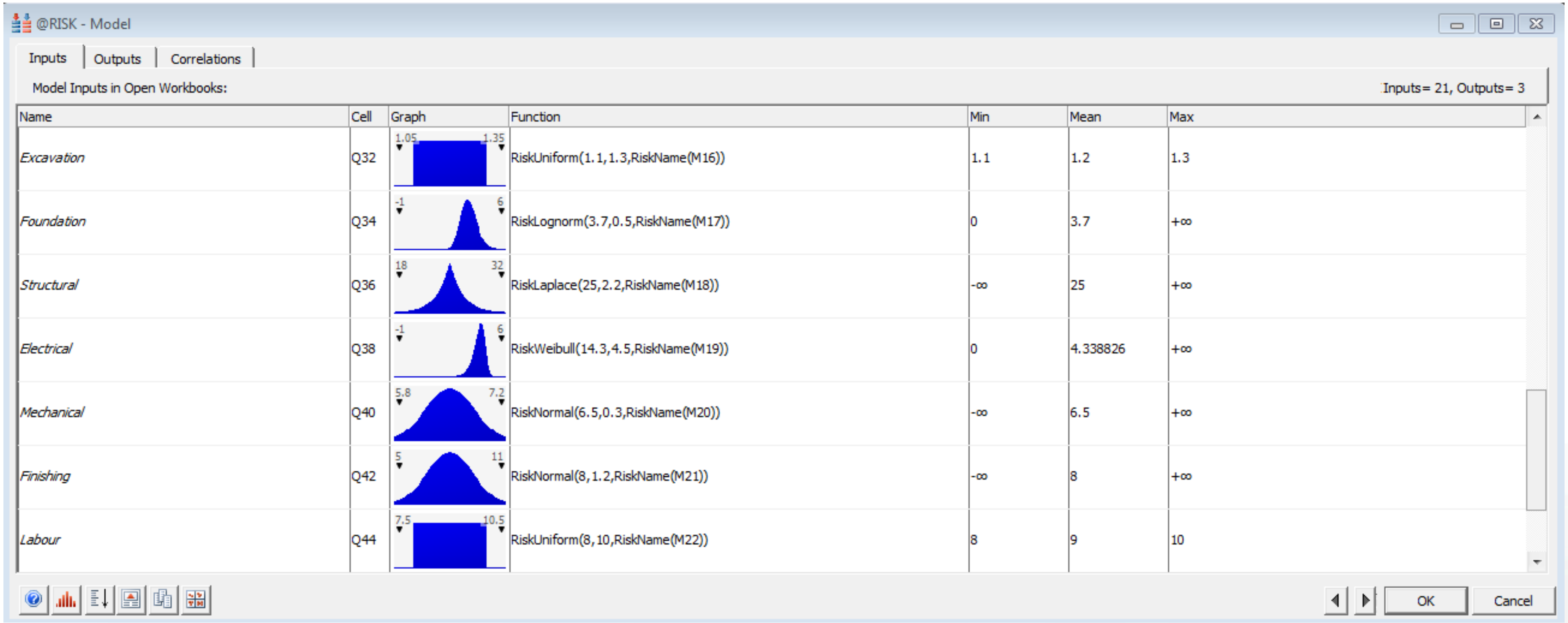
Output Definition

Task	Distribution Type	Parameters		Input Parameters
Excavation	Uniform	**Min**	**Max**	
		1.1	1.3	1.2
Foundation	Lognormal	**μ**	**σ**	
		3.7	0.5	3.7
Structural	Laplace	**μ**	**σ**	
		25	2.2	25
Electrical	Weibull	**α**	**β**	
		14.3	4.5	4.34
Mechanical	Normal	**μ**	**σ**	
		6.5	0.3	6.5
Finishing	Normal	**μ**	**σ**	
		8	1.2	8
Labour	Uniform	**Min**	**Max**	
		8	10	9
			Total Cost	57.74

Total construction cost = EW + S + El + M + F + L

Model Window



@RISK - Model

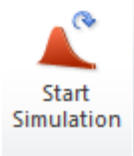
Inputs | Outputs | Correlations

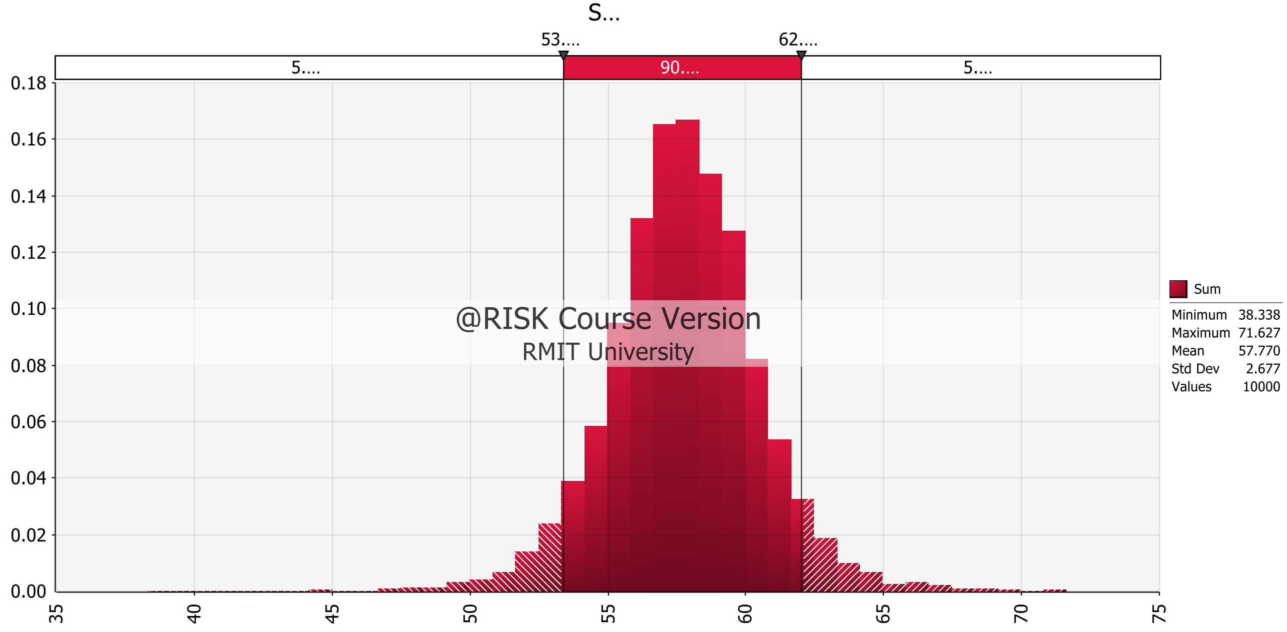
Model Inputs in Open Workbooks: Inputs= 21, Outputs= 3

Name	Cell	Graph	Function	Min	Mean	Max
Excavation	Q32	1.05 1.35	RiskUniform(1.1,1.3,RiskName(M16))	1.1	1.2	1.3
Foundation	Q34	-1 6	RiskLognorm(3.7,0.5,RiskName(M17))	0	3.7	+∞
Structural	Q36	18 32	RiskLaplace(25,2.2,RiskName(M18))	-∞	25	+∞
Electrical	Q38	-1 6	RiskWeibull(14.3,4.5,RiskName(M19))	0	4.338826	+∞
Mechanical	Q40	5.8 7.2	RiskNormal(6.5,0.3,RiskName(M20))	-∞	6.5	+∞
Finishing	Q42	5 11	RiskNormal(8,1.2,RiskName(M21))	-∞	8	+∞
Labour	Q44	7.5 10.5	RiskUniform(8,10,RiskName(M22))	8	9	10

OK Cancel

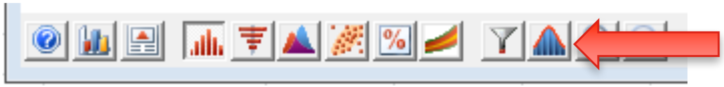
Run Simulation – Explore Tornado & Spider Graphs

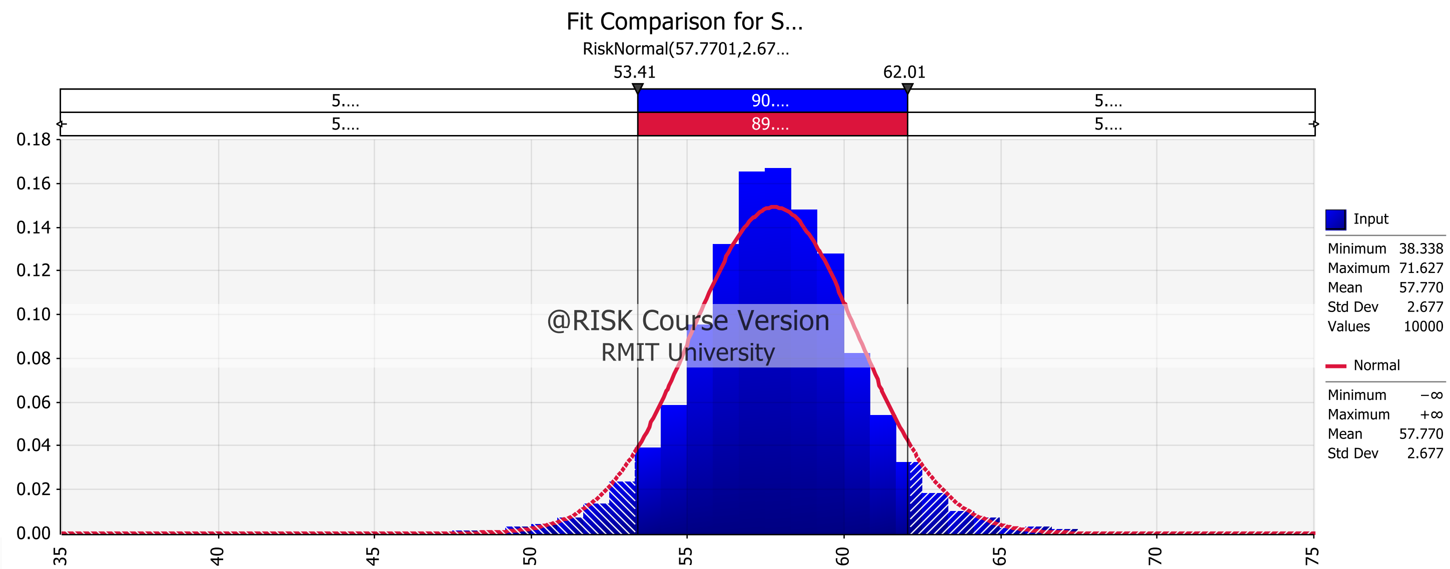




Fit the resultant distribution

- Fit to a Normal distribution
- Check the statistics





Statistics

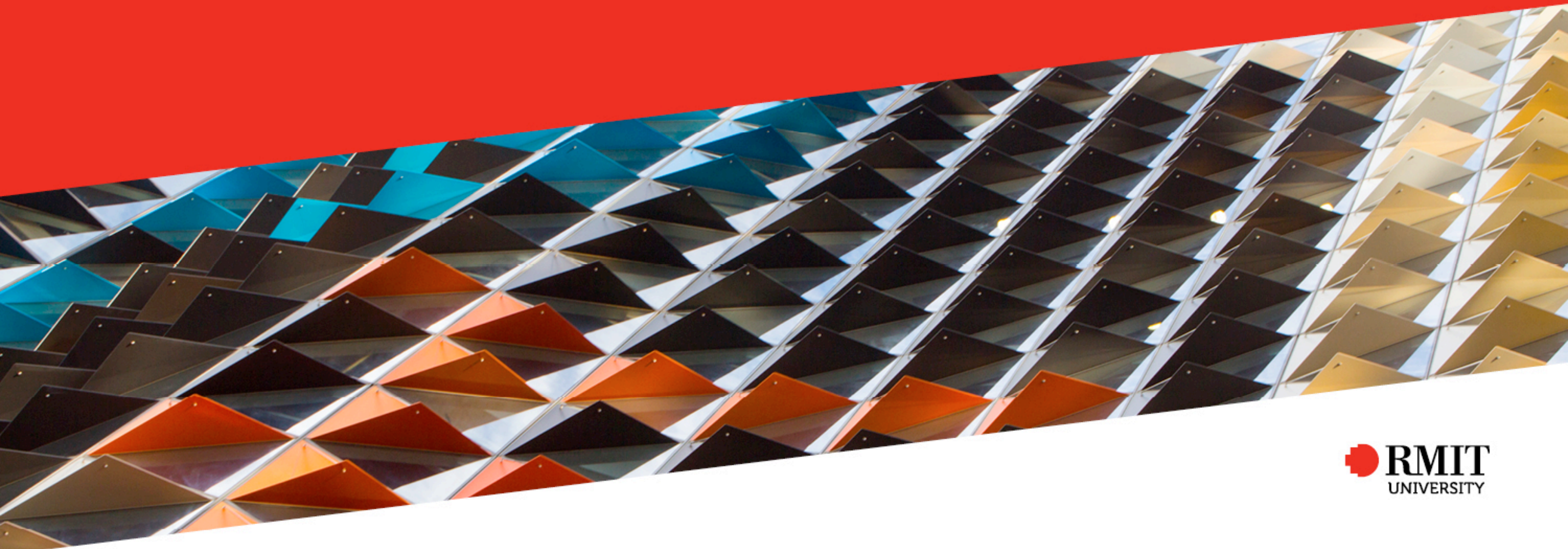
	Input	Normal
Minimum	38.338	−∞
Maximum	71.627	+∞
Mean	57.770	57.770
Mode	≈57.488	57.770
Median	57.754	57.770
Std Dev	2.677	2.677
Skewness	-0.0037	0.0000
Kurtosis	4.6792	3.0000
Left X	53.41	53.41
Left P	5.0%	5.2%
Right X	62.01	62.01
Right P	95.0%	94.4%
Dif. X	8.608	8.608
Dif. P	90.0%	89.2%
1%	50.984	51.542
5%	53.407	53.366
10%	54.581	54.339
15%	55.237	54.995
20%	55.757	55.517
25%	56.181	55.964
30%	56.530	56.366
35%	56.875	56.738
40%	57.183	57.092
45%	57.467	57.434
50%	57.754	57.770
55%	58.057	58.107
60%	58.354	58.448
65%	58.688	58.802
70%	59.024	59.174
75%	59.390	59.576
80%	59.772	60.023
85%	60.287	60.545
90%	60.971	61.201
95%	62.014	62.174
99%	64.737	63.998

Interpret the results

- What is the probability that the project is done with less than m$50?
- If we budget the project for m$58.4, what is the probability to go overbudget?
- Which variables should we do more research on to get better accuracy in the results?
- Considering the consequence of overbudgeting, what is the risk associated with the project?

Exercise 2

Construction cost model - Bridge



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Bridge Construction

Create the following model.

- Define the variables and generate the output
- Show which distribution fits best the output
- Generate a tornado graph

Task	Distribution Type	Parameters	
Electrical	Lognormal	μ	σ
		3.7	0.25
Structural	Laplace	μ	σ
		25	1.1
Labour	Weibull	α	β
		28.6	4.5
Roadworks	Normal	μ	σ
		6.5	0.15
Earthworks	Normal	μ	σ
		8	0.6