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- Cartesian Space Schemes vs. Joint Space Schemes
- Cubic Polynomial
- Quintic Polynomial
- Linear Function with Parabolic Blends
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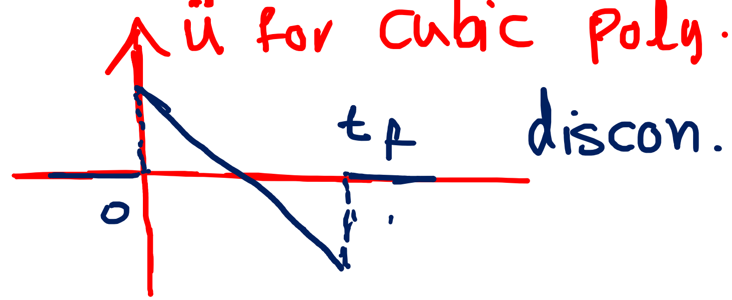
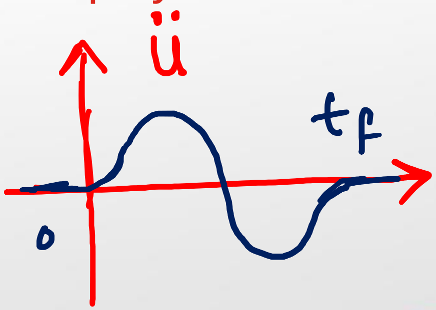
Quintic Polynomial → smooth acceleration

- Using the cubic polynomial, we can only specify 4 constraints, because the cubic polynomial only has 4 parameters.
  - Start and end positions
  - Start and end velocities
- We had no control over the accelerations.
  - From the numerical example, we saw that the acceleration started and ended at 40 and -40 respectively.



- If we want to be able to specify the start and end accelerations as well (i.e. now altogether 6 constraints), then we will need to use a polynomial with 6 parameters – The Quintic Polynomial.

$$\Rightarrow \quad u(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3 + a_4 t^4 + a_5 t^5$$



$$\begin{cases} u(0) = u_0 \\ u(t_f) = u_f \end{cases} \quad \begin{cases} \dot{u}(0) = 0 \\ \dot{u}(t_f) = 0 \end{cases} \quad \begin{cases} \ddot{u}(0) = 0 \\ \ddot{u}(t_f) = 0 \end{cases}$$

Quintic Polynomial

- The constraints are:

$$u(0) = a_0 + a_1 0 + a_2 0^2 + a_3 0^3 + a_4 0^4 + a_5 0^5 = a_0$$

$$u(t_f) = a_0 + a_1 t_f + a_2 t_f^2 + a_3 t_f^3 + a_4 t_f^4 + a_5 t_f^5$$

$$\dot{u}(0) = a_1 + 2a_2 0 + 3a_3 0^2 + 4a_4 0^3 + 5a_5 0^4 = a_1$$

$$\dot{u}(t_f) = a_1 + 2a_2 t_f + 3a_3 t_f^2 + 4a_4 t_f^3 + 5a_5 t_f^4$$

By differentiation of

$$u(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3$$

$$\ddot{u}(0) = 2a_2 + 6a_3 0 + 12a_4 0^2 + 20a_5 0^3 = 2a_2$$

$$\ddot{u}(t_f) = 2a_2 + 6a_3 t_f + 12a_4 t_f^2 + 20a_5 t_f^3$$

By differentiation of

$$\dot{u}(t) = a_1 + 2a_2 t + 3a_3 t^2 + 4a_4 t^3 + 5a_5 t^4$$

Quintic Polynomial

- Solving the simultaneous equations, the parameters are:

$$a_0 = u_0 \qquad a_1 = \dot{u}_0 = 0 \qquad a_2 = \frac{\ddot{u}_0}{2} = 0$$

$$a_3 = \frac{20u_f - 20u_0 - (8\dot{u}_f + 12\dot{u}_0)t_f - (3\ddot{u}_0 - \ddot{u}_f)t_f^2}{2t_f^3} = \frac{10}{t_f^3}(u_f - u_0)$$

$$a_4 = \frac{30u_0 - 30u_f + (14\dot{u}_f + 16\dot{u}_0)t_f + (3\ddot{u}_0 - 2\ddot{u}_f)t_f^2}{2t_f^4} = -\frac{15}{t_f^4}(u_f - u_0)$$

$$a_5 = \frac{12u_f - 12u_0 - (6\dot{u}_f + 6\dot{u}_0)t_f - (\ddot{u}_0 - \ddot{u}_f)t_f^2}{2t_f^5} = \frac{6}{t_f^5}(u_f - u_0)$$

Note!

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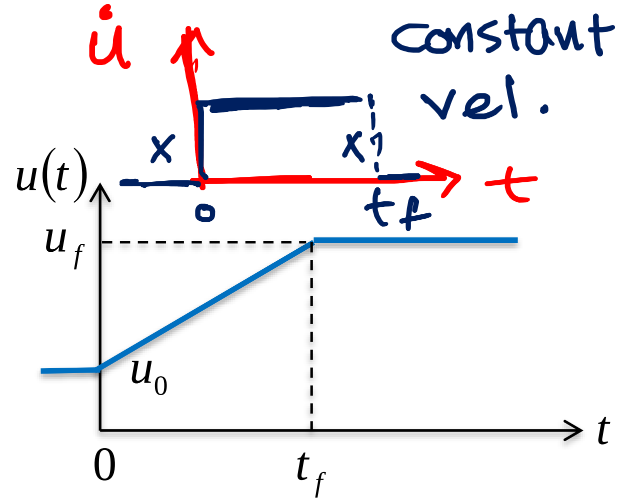
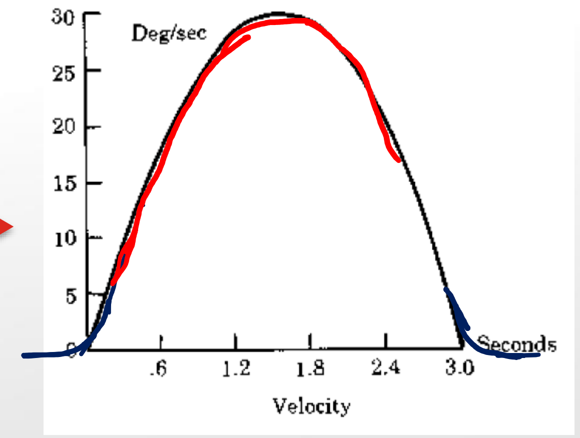
Linear Function w. Parabolic Blends

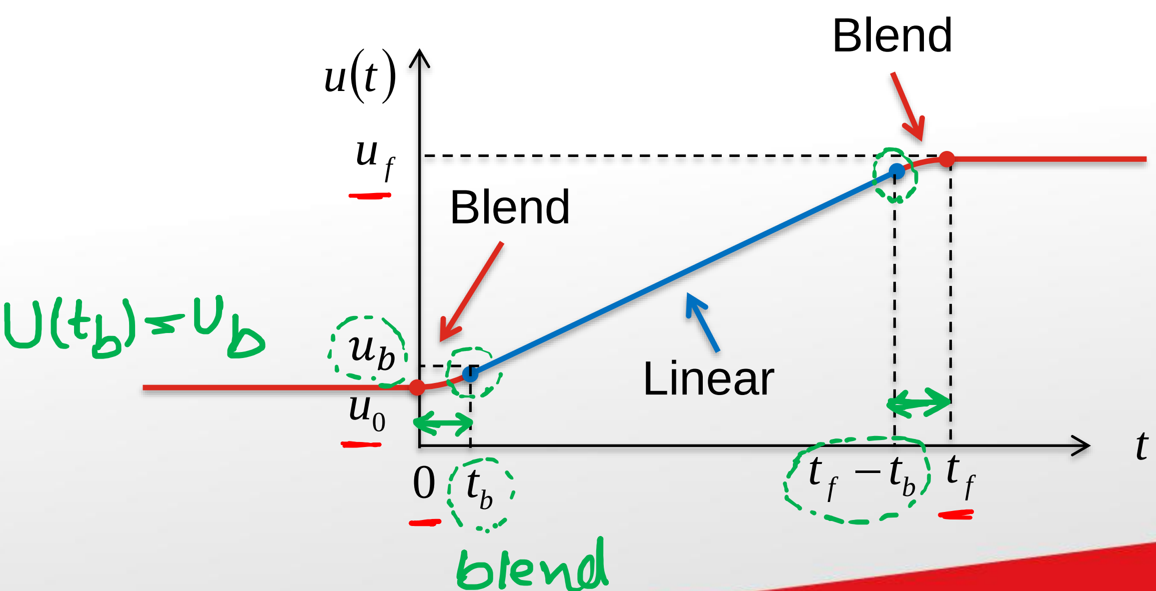
- The following summarizes the trajectories we have learnt so far:
  - Straight line:
    - Advantage: Constant velocity during motion.
    - Disadvantage: Discontinuous velocity at start and end points.
  - Polynomials:
    - Advantage: Smooth motion
    - Disadvantage: Velocity is not constant during motion.

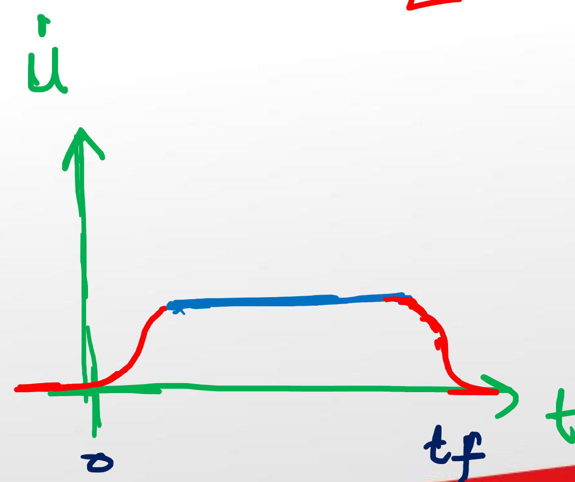




Linear Function w. Parabolic Blends

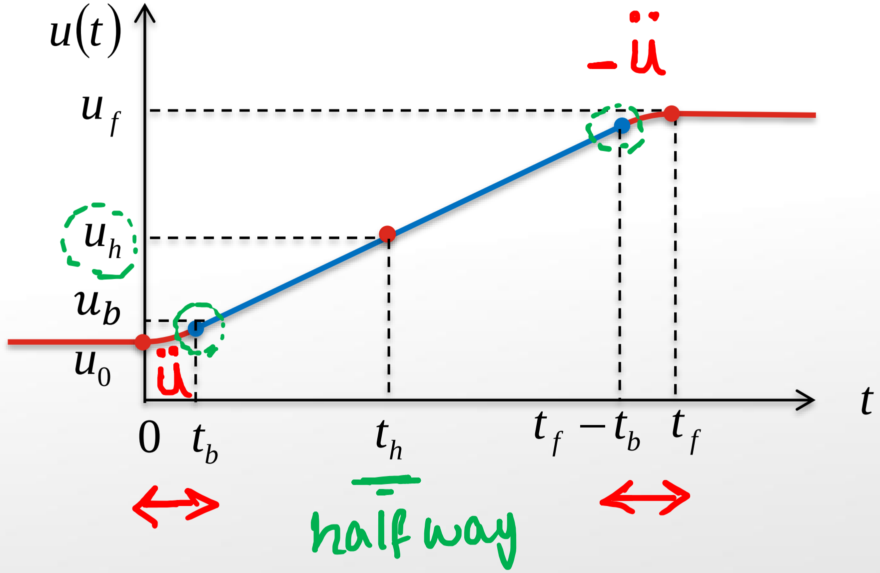
- Can we achieve:
  - Constant velocity during motion, AND
  - Smooth and continuous motion at the start and end points?
- Yes! We combine the ideas from the straight line and from the polynomial curves:
  - Linear Function with Parabolic Blends





Linear Function w. Parabolic Blends

- Assumptions / requirements:
  - Both the parabolic blends have the same time duration.
    - Therefore the same acceleration (apart from the sign) for both blends.
  - The solution is symmetric about the halfway point in time (t_h) and position (u_h).
  - The velocity at the end of blend region same as that of linear region.



- Question: how to get u_b and t_b?

Linear Function w. Parabolic Blends

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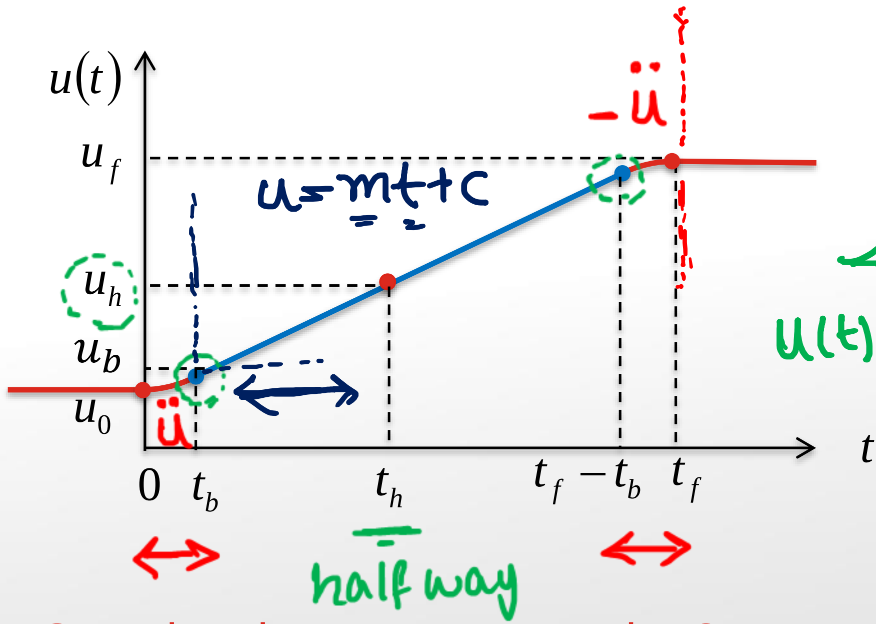
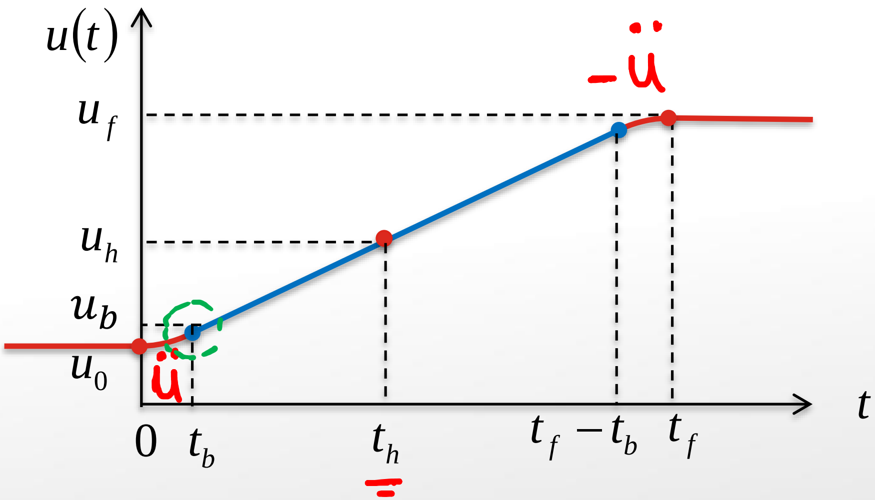


$$u(t) = \begin{cases} \frac{1}{2}\ddot{u}t^2 + u_0 & t < t_b \\ \left(\frac{u_h - u_b}{t_h - t_b}\right)(t - t_b) + u_b & t_b < t < (t_f - t_b) \\ -\frac{1}{2}\ddot{u}(t - t_f) + u_f & t > (t_f - t_b) \end{cases}$$

- Question: how to get u_b and t_b?

Linear Function w. Parabolic Blends

- Assumptions / requirements:
  - Both the parabolic blends have the same time duration.
    1 - Therefore the same acceleration (apart from the sign) for both blends.
  2 - The solution is symmetric about the halfway point in time (t_h) and position (u_h).
  3 - The velocity at the end of blend region same as that of linear region.



① & ②

$$t_h = 1/2\, t_f$$

$$u_h = 1/2\, (u_0 + u_f)$$

③

$$\ddot{u}\, t_b = \frac{u_h - u_b}{t_h - t_b}$$

- Question: how to get u_b and t_b?

$t_b \rightarrow u_b$

Linear Function w. Parabolic Blends

- The last requirement translates to the following equations:
  - $\ddot{u}\cdot t_b = \dfrac{u_h - u_b}{t_h - t_b}$ where $\ddot{u}$ is the constant acceleration during blend region.
- Next, u_b is given by: $u_b = u_0 + \frac{1}{2}\ddot{u}\cdot t_b^2$
- At the desired end point, the position is u_f and the time is t_f.
- Note that: $u_h = \frac{1}{2}(u_0 + u_f)$ and $t_h = \frac{1}{2}t_f$
- Combining all above equations and eliminating u_b, we have:

$$\ddot{u}\cdot t_b^2 - \ddot{u}\cdot t_f\cdot t_b + (u_f - u_0) = 0$$

- Thus we can solve the above quadratic equation to get t_b.
- And then calculate u_b using $u_b = u_0 + \frac{1}{2}\ddot{u}\cdot t_b^2$

$\ddot{u}$ ✓ t_b & u_b ? ?

$a = -\ddot{u}$

$b = t_f$

$c = u_f - u_0$

$ax^2 + bx + c = 0$

$\underset{\sim}{t_b}\; x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Linear Function w. Parabolic Blends

- Summary:
- The steps in obtaining the linear function with parabolic blends are:
  - Given u_0, u_f and t_f.
  - Choose desired acceleration $\ddot{u}$.
  - Calculate t_b based on: $\ddot{u} \cdot t_b^2 - \ddot{u} \cdot t_f \cdot t_b + (u_f - u_0) = 0$ → t_b
    - i.e. $$t_b = \frac{\ddot{u}t_f \pm \sqrt{\ddot{u}^2 t_f^2 - 4\ddot{u}(u_f - u_0)}}{2\ddot{u}}$$

Choose "minus" only since t_b should be less than $\frac{1}{2}t_f$

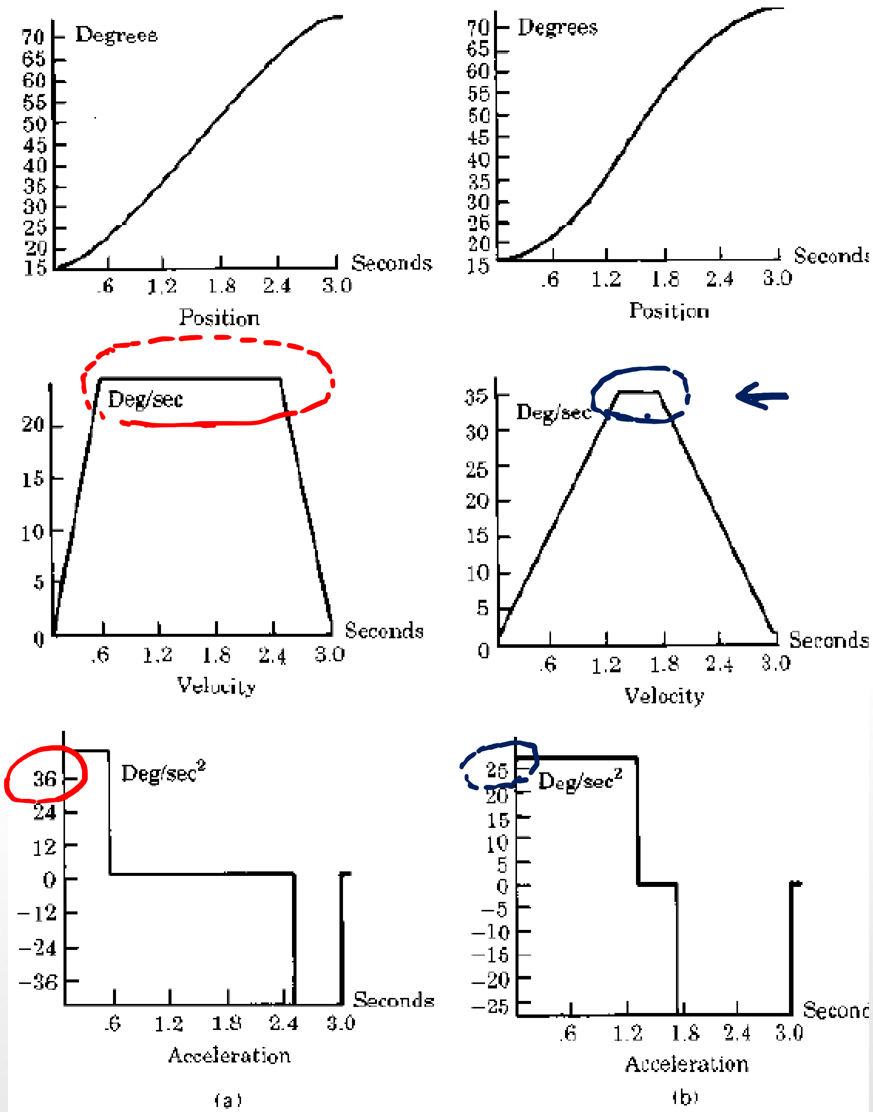
$t_b < t_h$

$t_b < \frac{1}{2} t_f$

  - Finally, calculate u_b based on:

$$t_b \longrightarrow u_b = u_0 + \frac{1}{2}\ddot{u} \cdot t_b^2$$

Linear Function w. Parabolic Blends



- Notes: Acceleration must be chosen to be high enough. Otherwise solution to t_b will not exist.
- E.g. if acceleration is small, the linear region shrinks.
- If acceleration is too small, there may be no more linear region.

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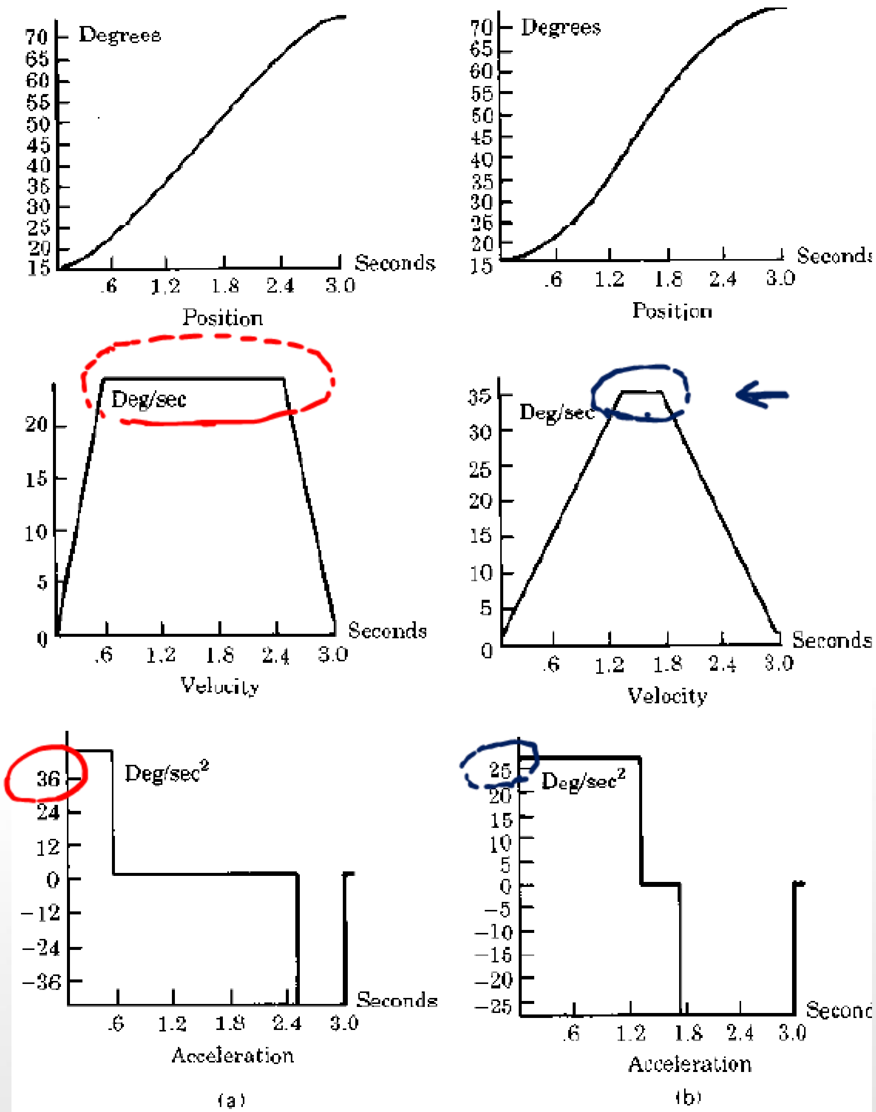
Linear Function w. Parabolic Blends



- Notes: Acceleration must be chosen to be high enough. Otherwise solution to t_b will not exist.
- E.g. if acceleration is small, the linear region shrinks.
- If acceleration is too small, there may be no more linear region.

$\ddot{u}$ guidance

$$\ddot{u} \geq \frac{4(u_f - u_o)}{t_f^2}$$

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Quintic Polynomial

- We will use Quintic Polynomial for the simulation:

$$u(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3 + a_4 t^4 + a_5 t^5$$

- The parameters are:

$$a_0 = u_0 \qquad a_1 = \dot{u}_0 \qquad a_2 = \frac{\ddot{u}_0}{2}$$

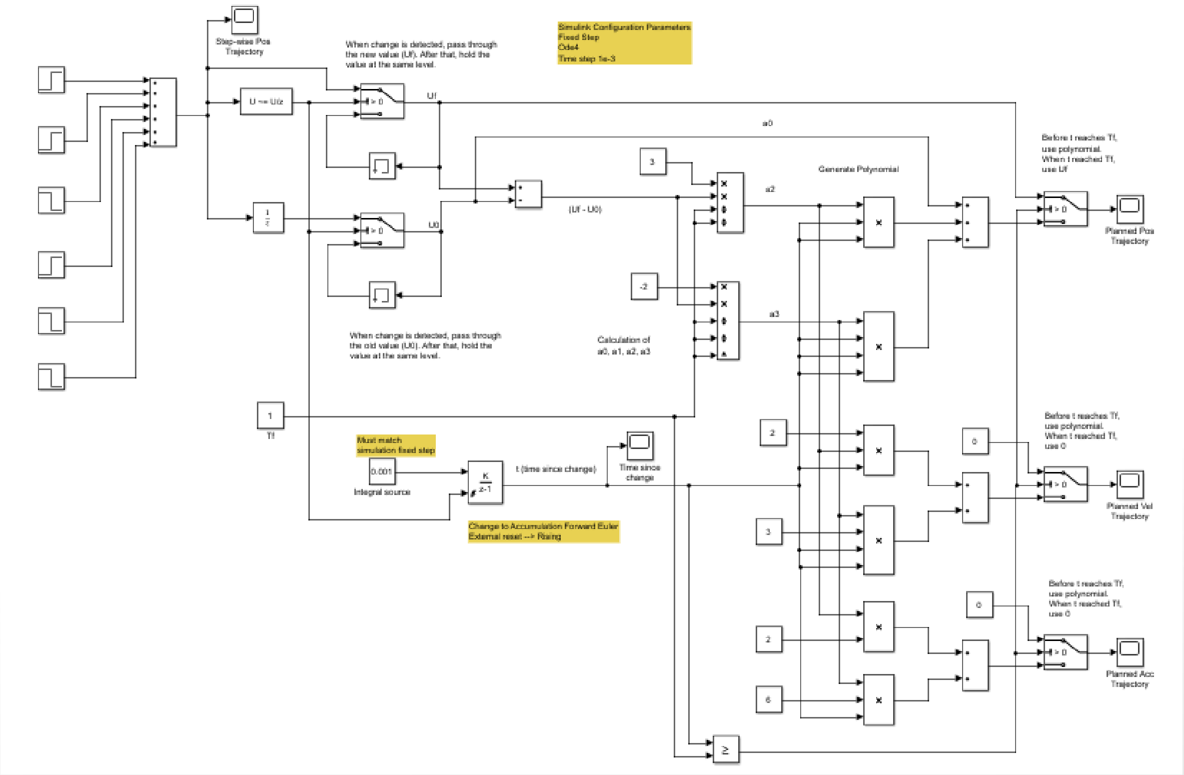
$$a_3 = \frac{20u_f - 20u_0 - (8\dot{u}_f + 12\dot{u}_0)t_f - (3\ddot{u}_0 - \ddot{u}_f)t_f^2}{2t_f^3}$$

$$a_4 = \frac{30u_0 - 30u_f + (14\dot{u}_f + 16\dot{u}_0)t_f + (3\ddot{u}_0 - 2\ddot{u}_f)t_f^2}{2t_f^4}$$

$$a_5 = \frac{12u_f - 12u_0 - (6\dot{u}_f + 6\dot{u}_0)t_f - (\ddot{u}_0 - \ddot{u}_f)t_f^2}{2t_f^5}$$

Quintic Polynomial

- This can be done in the following way:



- Please see attached Matlab Simulink file in Canvas.

Thank you!

Have a good evening.