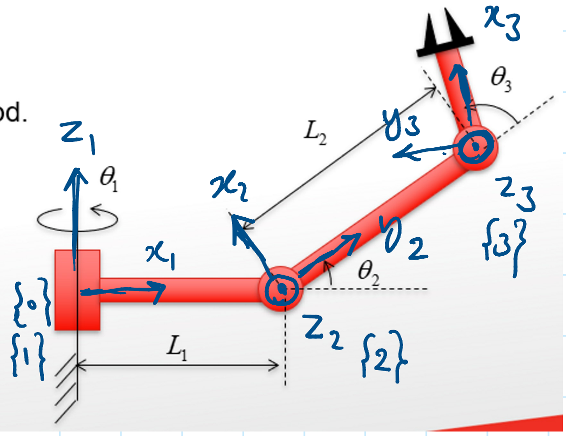
QS. 1

Monday, 24 August 2020 4:08 PM

- **Question 1:**
  - Find the Jacobian of the manipulator shown on the right. (You should already have some information about this robot in earlier tutorials).
  - Write it in terms of frame {3} at the wrist of robot.
    - A • Velocity propagation method.
    - B • Differentiation of kinematic equations.
  - C • Write also in terms of frame {4} at the tip of hand, having same orientation as {3}.



$$
{}^0_1T = \underbrace{\begin{bmatrix} c\theta_1 & -s\theta_1 & 0 \\ s\theta_1 & c\theta_1 & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{{}^0_1R}\ \begin{matrix}0\\0\\0\end{matrix} \quad \text{i.e.} \quad {}^0_1T = \begin{bmatrix} c\theta_1 & -s\theta_1 & 0 & 0 \\ s\theta_1 & c\theta_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^1_2T = \begin{bmatrix} c\theta_2 & -s\theta_2 & 0 & L_1 \\ 0 & 0 & -1 & 0 \\ s\theta_2 & c\theta_2 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad ({}^1_2R \text{ upper-left } 3\times3)
$$

$$
{}^2_3T = \begin{bmatrix} c\theta_3 & -s\theta_3 & 0 & L_2 \\ s\theta_3 & c\theta_3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad ({}^2_3R \text{ upper-left } 3\times3)
$$

$$
{}^0_3T = {}^0_1T \cdot {}^1_2T \cdot {}^2_3T = \begin{bmatrix} c_1c_{23} & -c_1s_{23} & s_1 & L_1c_1 + L_2c_1c_2 \\ s_1c_{23} & -s_1s_{23} & -c_1 & L_1s_1 + L_2s_1c_2 \\ s_{23} & c_{23} & 0 & L_2s_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

${}^0_3R$ ${}^0P_{\{3\}ORG}$

A) Velocity Propagation

A.1 ${}^0\omega_0 = 0$ ${}^0v_0 = 0$ {0} is fixed

A.2

${}^1\Omega_1$

Joint 1 : ${}^1\omega_1 = {}^1_0R \cdot \cancel{{}^0\omega_0} + \dot\theta_1\, {}^1\hat{Z}_1 = [0 \;\; 0 \;\; \dot\theta_1]^T$ ✓

Revolute $i=0$

${}^1v_1 = {}^1_0R\,(\cancel{{}^0v_0} + \cancel{{}^0\omega_0} \times {}^0P_1) = 0$ ✓

Joint 2 : ${}^2\omega_2 = {}^2_1R \cdot {}^1\omega_1 + \dot\theta_2\, {}^2\hat{Z}_2$

Revolute $i=1$

$= {}^1_2R^T \cdot {}^1\omega_1 + \dot\theta_2\, {}^2\hat{Z}_2$

$$\begin{bmatrix} C\theta_2 & 0 & S\theta_2 \\ -S\theta_2 & 0 & C\theta_2 \\ 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ \dot\theta_1 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \dot\theta_2 \end{bmatrix} = \begin{bmatrix} S_2\dot\theta_1 \\ C_2\dot\theta_1 \\ \dot\theta_2 \end{bmatrix} \quad \checkmark$$

$$ {}^2v_2 = {}^2_1R\,(\underbrace{\cancel{{}^1v_1}}_{0} + {}^1\omega_1 \times {}^1P_2) \qquad {}^1P_2 = L_1\hat X_1 = \begin{bmatrix} L_1 \\ 0 \\ 0 \end{bmatrix}$$

$$ = {}^1_2R^T(\underbrace{\cancel{{}^1v_1}}_{0} + {}^1\omega_1 \times {}^1P_2)$$

$$ = \begin{bmatrix} C\theta_2 & 0 & S\theta_2 \\ -S\theta_2 & 0 & C\theta_2 \\ 0 & -1 & 0 \end{bmatrix} \left\{ \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} + \begin{vmatrix} i & j & K \\ 0 & 0 & \dot\theta_1 \\ L_1 & 0 & 0 \end{vmatrix} \right\}$$

$$ = \begin{bmatrix} C\theta_2 & 0 & S\theta_2 \\ -S\theta_2 & 0 & C\theta_2 \\ 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ L_1\dot\theta_1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -L_1\dot\theta_1 \end{bmatrix} \quad \checkmark$$

Joint 3 : Revolute $i=2$

$$ {}^3\omega_3 = {}^3_2R \cdot {}^2\omega_2 + \dot\theta_3\, {}^3\hat Z_3$$

$$ = {}^2_3R^T \cdot {}^2\omega_2 + \dot\theta_3\, {}^3\hat Z_3$$

$$\begin{bmatrix} C\theta_3 & +S\theta_3 & 0 \\ -S\theta_3 & C\theta_3 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} S_2\dot\theta_1 \\ C_2\dot\theta_1 \\ \dot\theta_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \dot\theta_3 \end{bmatrix} =$$

$$\begin{bmatrix} S_2C_3\dot\theta_1 + C_2S_3\dot\theta_1 \\ -S_2S_3\dot\theta_1 + C_2C_3\dot\theta_1 \\ \dot\theta_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \dot\theta_3 \end{bmatrix} = \begin{bmatrix} S_{23}\,\dot\theta_1 \\ C_{23}\,\dot\theta_1 \\ \dot\theta_2 + \dot\theta_3 \end{bmatrix}$$

$$ {}^3v_3 = {}^3_2R\,({}^2v_2 + {}^2\omega_2 \times {}^2P_3) \qquad {}^2P_3 = L_2\hat X_2 = \begin{bmatrix} L_2 \\ 0 \\ 0 \end{bmatrix}$$

$$ = {}^2_3R^T({}^2v_2 + {}^2\omega_2 \times {}^2P_3)$$

$$ = \begin{bmatrix} C\theta_3 & +S\theta_3 & 0 \\ -S\theta_3 & C\theta_3 & 0 \\ 0 & 0 & 1 \end{bmatrix} \left\{ \begin{bmatrix} 0 \\ 0 \\ -L_1\dot\theta_1 \end{bmatrix} + \begin{vmatrix} i & j & K \\ S_2\dot\theta_1 & C_2\dot\theta_1 & \dot\theta_2 \\ L_2 & 0 & 0 \end{vmatrix} \right\}$$

$$= \begin{bmatrix} -S\theta_3 & C\theta_3 & 0 \\ 0 & 0 & 1 \end{bmatrix} \left\{ \begin{bmatrix} 0 \\ -L_1\dot\theta_1 \end{bmatrix} + \begin{bmatrix} \cdots & \cdots & \cdots \\ L_2 & 0 & 0 \end{bmatrix} \right\}$$

$$= \begin{bmatrix} C\theta_3 & +S\theta_3 & 0 \\ -S\theta_3 & C\theta_3 & 0 \\ 0 & 0 & 1 \end{bmatrix} \left\{ \begin{bmatrix} 0 \\ 0 \\ -L_1\dot\theta_1 \end{bmatrix} + \begin{bmatrix} 0 \\ L_2\dot\theta_2 \\ -L_2C_2\dot\theta_1 \end{bmatrix} \right\}$$

$$= \begin{bmatrix} L_2S_3\dot\theta_2 \\ L_2C_3\dot\theta_2 \\ -L_1\dot\theta_1 - L_2C_2\dot\theta_1 \end{bmatrix}$$

$n=3$ $\quad {}^3v_3$

A.3 we have already Propogated vel. to the wrist ${}^3\omega_3$

Last step is to transform back to frame{0}

$${}^0\omega_3 = {}^0_3R \cdot {}^3\omega_3$$

$$= \begin{bmatrix} C_1C_{23} & -C_1S_{23} & S_1 \\ S_1C_{23} & -S_1S_{23} & -C_1 \\ S_{23} & C_{23} & 0 \end{bmatrix} \begin{bmatrix} S_{23}\dot\theta_1 \\ C_{23}\dot\theta_1 \\ \dot\theta_2+\dot\theta_3 \end{bmatrix}$$

$$= \begin{bmatrix} S_1(\dot\theta_2+\dot\theta_3) \\ -C_1(\dot\theta_2+\dot\theta_3) \\ S_{23}^2\dot\theta_1 + C_{23}^2\dot\theta_1 \end{bmatrix} = \begin{bmatrix} S_1(\dot\theta_2+\dot\theta_3) \\ -C_1(\dot\theta_2+\dot\theta_3) \\ \dot\theta_1 \end{bmatrix} \quad \checkmark$$

$${}^0v_3 = {}^0_3R \cdot {}^3v_3$$

$$= \begin{bmatrix} C_1C_{23} & -C_1S_{23} & S_1 \\ S_1C_{23} & -S_1S_{23} & -C_1 \\ S_{23} & C_{23} & 0 \end{bmatrix} \begin{bmatrix} L_2S_3\dot\theta_2 \\ L_2C_3\dot\theta_2 \\ -L_1\dot\theta_1 - L_2C_2\dot\theta_1 \end{bmatrix}$$

$$= \begin{bmatrix} L_2C_1C_{23}S_3\dot\theta_2 - L_2C_1S_{23}C_3\dot\theta_2 - L_1S_1\dot\theta_1 - L_2S_1C_2\dot\theta_1 \\ L_2S_1C_{23}S_3\dot\theta_2 - L_2S_1S_{23}C_3\dot\theta_2 + L_1C_1\dot\theta_1 + L_2C_1C_2\dot\theta_1 \\ L_2S_{23}S_3\dot\theta_2 + L_2C_{23}C_3\dot\theta_2 \end{bmatrix} \quad \checkmark$$

Write the Jacobian Matrix ?

$$\begin{bmatrix} {}^0v_x \\ {}^0v_y \\ {}^0v_z \end{bmatrix} = \left[\begin{array}{c|c|c} -L_1S_1 - L_2S_1C_2 & L_2C_1C_{23}S_3 - L_2C_1S_{23}C_3 & 0 \\ +L_1C_1 + L_2C_1C_2 & L_2S_1C_{23}S_3 - L_2S_1S_{23}C_3 & 0 \\ 0 & L_2S_{23}S_3 + L_2C_{23}C_3 & 0 \end{array}\right] \begin{bmatrix} \dot\theta_1 \\ \dot\theta_2 \\ \dot\theta_3 \end{bmatrix}$$

$${}^0J_v \quad \left[\begin{array}{c|c|c} -L_1S_1 - L_2S_1C_2 & -L_2C_1S_2 & 0 \\ +L_1C_1 + L_2C_1C_2 & -L_2S_1S_2 & 0 \\ 0 & L_2C_2 & 0 \end{array}\right] \begin{bmatrix} \dot\theta_1 \\ \dot\theta_2 \\ \dot\theta_3 \end{bmatrix}$$

$$\begin{bmatrix} {}^0\omega_x \\ {}^0\omega_y \\ {}^0\omega_z \end{bmatrix} = \left[\begin{array}{c|c|c} 0 & S_1 & S_1 \\ 0 & -C_1 & -C_1 \\ 1 & 0 & 0 \end{array}\right] \begin{bmatrix} \dot\theta_1 \\ \dot\theta_2 \\ \dot\theta_3 \end{bmatrix}$$

${}^0J_\omega$

B) Differentiation (we need position vector of wrist)

Position of wrist can be obtained from the 4^{th} column of ${}^0_3T$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} f_x(q) \\ f_y(q) \\ f_z(q) \end{bmatrix} = \begin{bmatrix} L_1C_1 + L_2C_1C_2 \\ L_1S_1 + L_2S_1C_2 \\ L_2S_2 \end{bmatrix}$$

$$\begin{bmatrix} \dot x \\ \dot y \\ \dot z \end{bmatrix} = \begin{bmatrix} \partial f_x/\partial q_1 & \partial f_x/\partial q_2 & \partial f_x/\partial q_3 \\ \partial f_y/\partial q_1 & \partial f_y/\partial q_2 & \partial f_y/\partial q_3 \\ \partial f_z/\partial_1 & \partial f_z/\partial_2 & \partial f_z/\partial_3 \end{bmatrix} \begin{bmatrix} \dot q_1 \\ \dot q_2 \\ \dot q_3 \end{bmatrix} = \left[\begin{array}{c|c|c} -L_1S_1 - L_2S_1C_2 & -L_2C_1S_2 & 0 \\ L_1C_1 + L_2C_1C_2 & -L_2S_1S_2 & 0 \\ 0 & L_2C_2 & 0 \end{array}\right] \begin{bmatrix} \dot\theta_1 \\ \dot\theta_2 \\ \dot\theta_3 \end{bmatrix}$$

C). Speed at the Tip of hand

end of hand $\omega_e = \omega_n =$ last frame end of hand $\rightarrow \omega_e = \omega_n \leftarrow$ last frame

$$^n v_e = {}^n v_n + {}^n \omega_n \times {}^n P_e$$

Express everything in frame {3}

$$^3 v_e = {}^3 v_3 + {}^3 \omega_3 \times {}^3 P_e$$

$$= \begin{bmatrix} L_2 S_3 \dot\theta_2 \\ L_2 C_3 \dot\theta_2 \\ -L_1\dot\theta_1 - L_2 C_2 \dot\theta_1 \end{bmatrix} + \begin{vmatrix} i & j & K \\ S_{23}\dot\theta_1 & C_{23}\dot\theta_1 & \dot\theta_2 + \dot\theta_3 \\ L_3 & 0 & 0 \end{vmatrix}$$

$$\begin{bmatrix} L_2 S_3 \dot\theta_2 \\ L_2 C_3 \dot\theta_2 + L_3(\dot\theta_2 + \dot\theta_3) \\ -L_1\dot\theta_1 - L_2 C_2 \dot\theta_1 - L_3 C_{23}\dot\theta_1 \end{bmatrix}$$

change to {0}

$$^0 v_e = {}^0_3 R \cdot {}^3 v_e$$

$$= \begin{bmatrix} C_1 C_{23} & -C_1 S_{23} & S_1 \\ S_1 C_{23} & -S_1 S_{23} & -C_1 \\ S_{23} & C_{23} & 0 \end{bmatrix} \begin{bmatrix} L_2 S_3 \dot\theta_2 \\ L_2 C_3 \dot\theta_2 + L_3(\dot\theta_2 + \dot\theta_3) \\ -L_1\dot\theta_1 - L_2 C_2 \dot\theta_1 - L_2 C_{23}\dot\theta_1 \end{bmatrix}$$

$$= \left[\begin{array}{c|c|c} -L_1 S_1 - L_2 S_1 C_2 - L_3 S_1 C_{23} & -L_2 C_1 S_2 - L_3 C_1 S_{23} & -L_3 C_1 C_{23} \\ L_1 C_1 + L_2 C_1 C_2 + L_3 C_1 C_{23} & -L_2 S_1 S_2 - L_3 S_1 S_{23} & -L_3 S_1 S_{23} \\ 0 & L_2 C_2 + L_3 C_{23} & L_3 C_{23} \end{array}\right] \begin{bmatrix} \dot\theta_1 \\ \dot\theta_2 \\ \dot\theta_3 \end{bmatrix}$$

QS. 2

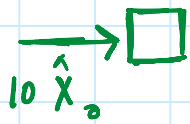
Wednesday, 26 August 2020 9:07 AM

- **Question 2:**
  - A 2-ink manipulator has the following Jacobian:

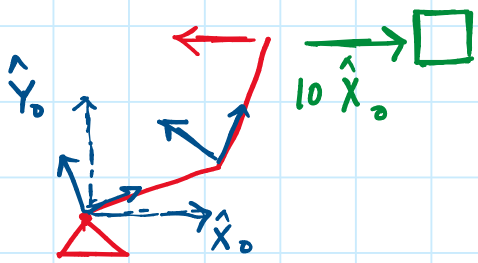
$$ {}^{0}J(\theta) = \begin{bmatrix} -l_1 s_1 - l_2 s_{12} & -l_2 s_{12} \\ l_1 c_1 + l_2 c_{12} & l_2 c_{12} \end{bmatrix} $$

  - If we ignore gravity, what are the joint torques required so that the manipulator can apply a static force of ${}^{0}F = 10\hat{X}_0$?

The Force applied by robot is in $\rightarrow$ direction



External Force applied to the Robot $\rightarrow$ ${}^{0}F_{ext} = -10\hat{X}_0$



$$ \tau = J^T F = \begin{bmatrix} -l_1 S_1 - l_2 S_{12} & l_1 C_1 + l_2 C_{12} \\ -l_2 S_{12} & l_2 C_{12} \end{bmatrix} \begin{bmatrix} -10 \\ 0 \end{bmatrix} $$

$$ \tau = \begin{bmatrix} 10\, l_1 S_1 + 10\, l_2 S_{12} \\ 10\, l_2 S_{12} \end{bmatrix} $$

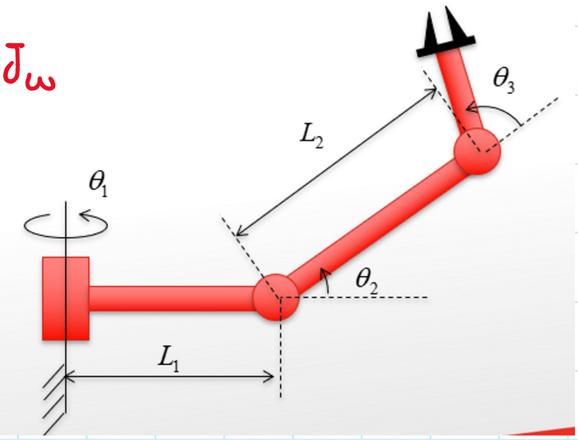
$\rightarrow$ will change depends on θ_1 & θ_2

QS. 3

Wednesday, 26 August 2020 9:07 AM

- **Question 3:**
  - Find the Jacobian of the manipulator shown on the right. (You should already have some information about this robot in earlier tutorials).
  - Write it in terms of frame {3} at the wrist of robot.
    - Explicit form for Jacobian.
  - Write also in terms of frame {4} at the tip of hand, having same orientation as {3}.

${}^{0}J_{\omega}$



To use this method, you need ${}^{0}_{1}T$, ${}^{0}_{2}T$, ${}^{0}_{3}T$

3rd col.

$$ {}^{0}_{1}T = \begin{bmatrix} C_1 & -S_1 & 0 & 0 \\ S_1 & C_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

3rd col.

$$ {}^{0}_{2}T = \underbrace{{}^{0}_{1}T \cdot {}^{1}_{2}T}_{\text{Please See question 1}} = \begin{bmatrix} C_1C_2 & -C_1S_2 & S_1 & L_1C_1 \\ S_1C_2 & -S_1S_2 & -C_1 & L_1S_1 \\ S_2 & C_2 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^{0}_{3}T = \begin{bmatrix} C_1C_{23} & -C_1S_{23} & S_1 & L_1C_1 + L_2C_1C_2 \\ S_1C_{23} & -S_1S_{23} & -C_1 & L_1S_1 + L_2S_1C_2 \\ S_{23} & C_{23} & 0 & L_2S_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

The explicit method state that

$$ {}^{0}J_{\omega} = \left[\begin{array}{c:c:c} \text{3rd column} & \text{3rd column} & \text{3rd column} \\ \text{of } {}^{0}_{1}T & \text{of } {}^{0}_{2}T & \text{of } {}^{0}_{3}T \end{array} \right] $$

and if any joint is Prismatic the replace that column with zero

No prismatic joint in this example

$$
{}^{0}J_{\omega} = \begin{bmatrix} 0 & S_1 & S_1 \\ 0 & -C_1 & -C_1 \\ 1 & 0 & 0 \end{bmatrix}
$$