Week 11 – Linear & Nonlinear Control of Manipulators

Advanced Robotic Systems – MANU2453

Dr Ehsan Asadi, School of Engineering
RMIT University, Victoria, Australia
Email: ehsan.asadi@rmit.edu.au

Lectures

Wk	Date	Lecture (NOTE: video recording)	Maths Difficulty	Hands-on Activity	Related Assessment
1	24/7	• Introduction to the Course • Spatial Descriptions & Transformations			
2	31/7	• Spatial Descriptions & Transformations • Robot Cell Design			Robot Cell Design Assignment
3	7/8	• Forward Kinematics • Inverse Kinematics			
4	14/8	• ABB Robot Programming via Teaching Pendant • ABB RobotStudio Offline Programming		ABB RobotStudio Offline Programming	Offline Programming Assignment
5	21/8	• Jacobians: Velocities and Static Forces			
6	28/8	• Manipulator Dynamics			
7	11/9	• Manipulator Dynamics		MATLAB Simulink Simulation	
8	18/9	• Robotic Vision		MATLAB Simulation	Robotic Vision Assignment
9	25/9	• Robotic Vision II		MATLAB Simulation	
10	2/10	• Trajectory Generation			
11	9/10	• Linear & Nonlinear Control		MATLAB Simulink Simulation	
12	16/10	• Introduction to I4.0 • Revision			Final Exam

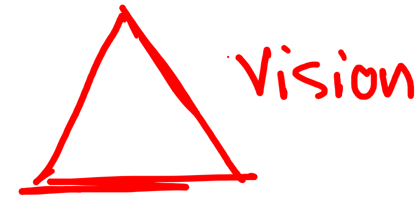
Content

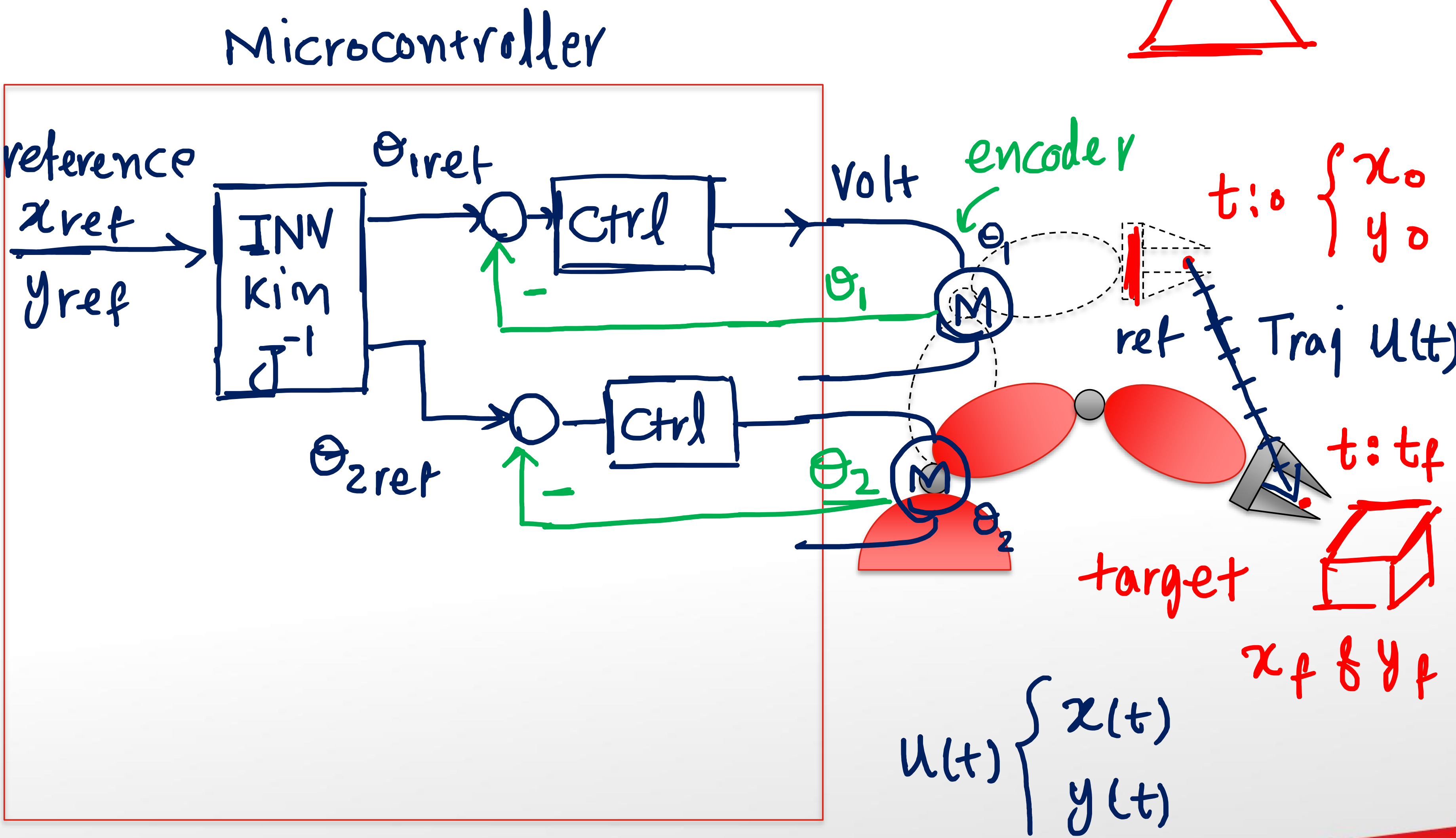
- Introduction
- Control of Second Order Linear Systems
- Control-Law Partitioning
- Trajectory Following
- Nonlinear and Time-Varying Systems
- Nonlinear Control of Multi-Input-Multi-Output System
- Matlab Simulink Simulation

Content

- Introduction
- Control of Second Order Linear Systems
- Control-Law Partitioning
- Trajectory Following
- Nonlinear and Time-Varying Systems
- Nonlinear Control of Multi-Input-Multi-Output System
- Matlab Simulink Simulation

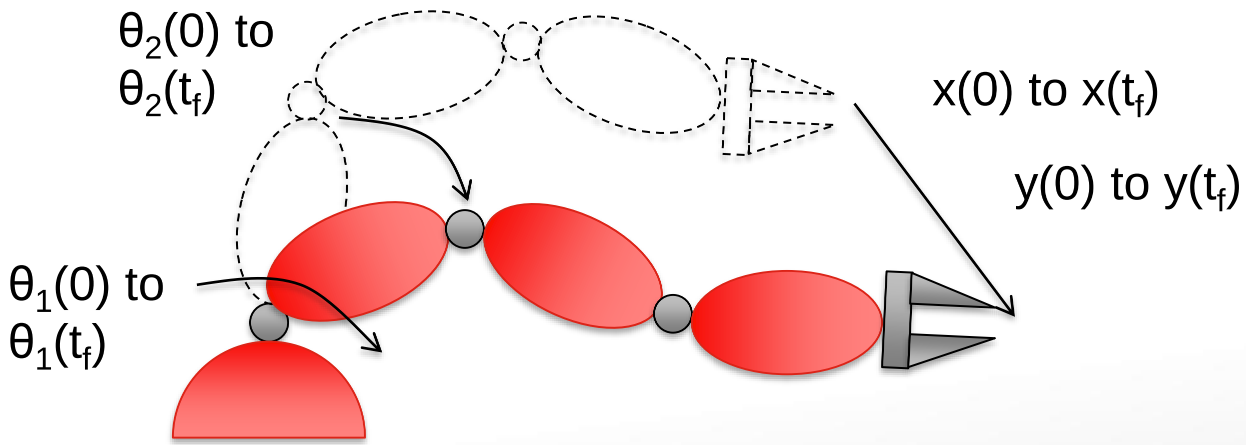
Introduction





Introduction

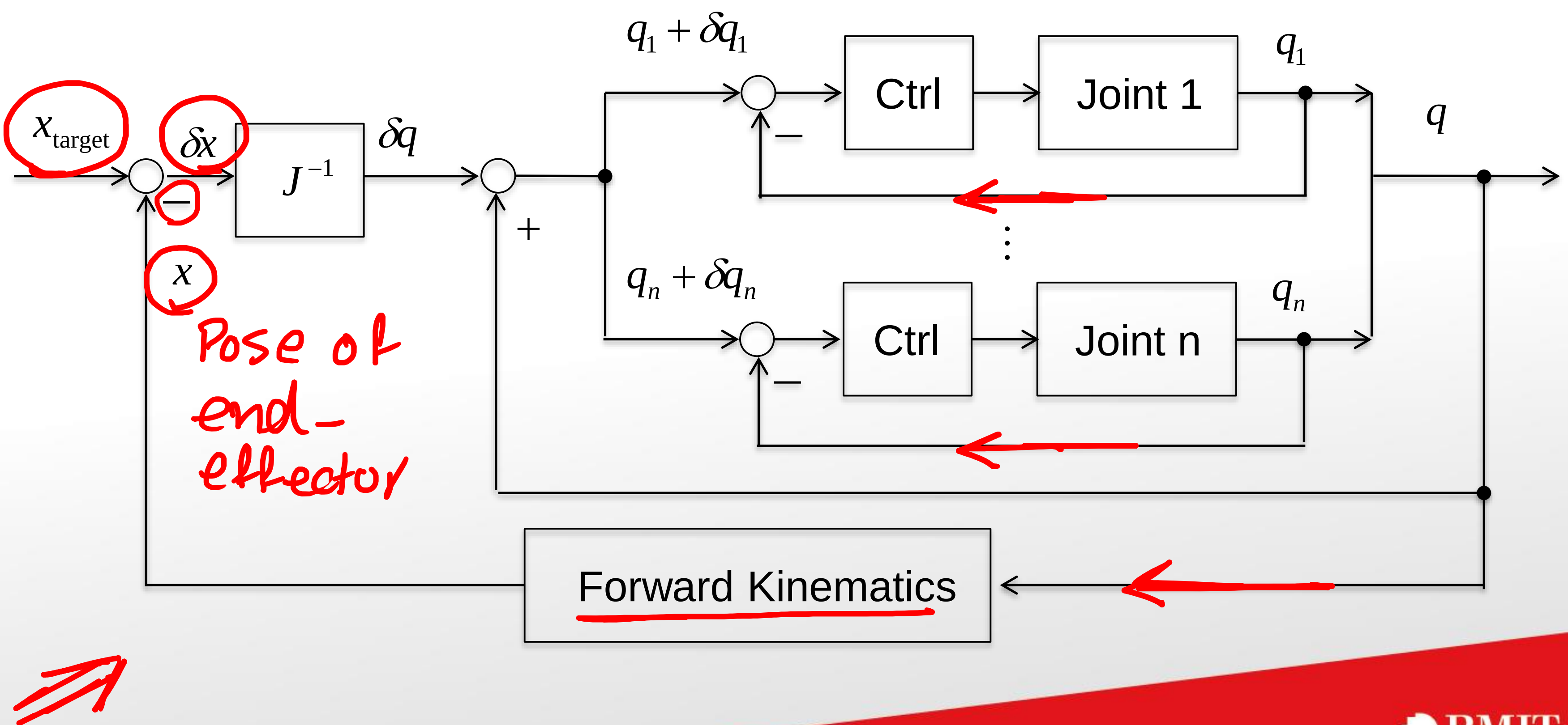
- Last week, we discussed about the trajectory which the robot is required to follow.



- In today’s lecture, we will study how we can control the robot (or joints) so that they follow the desired trajectory.

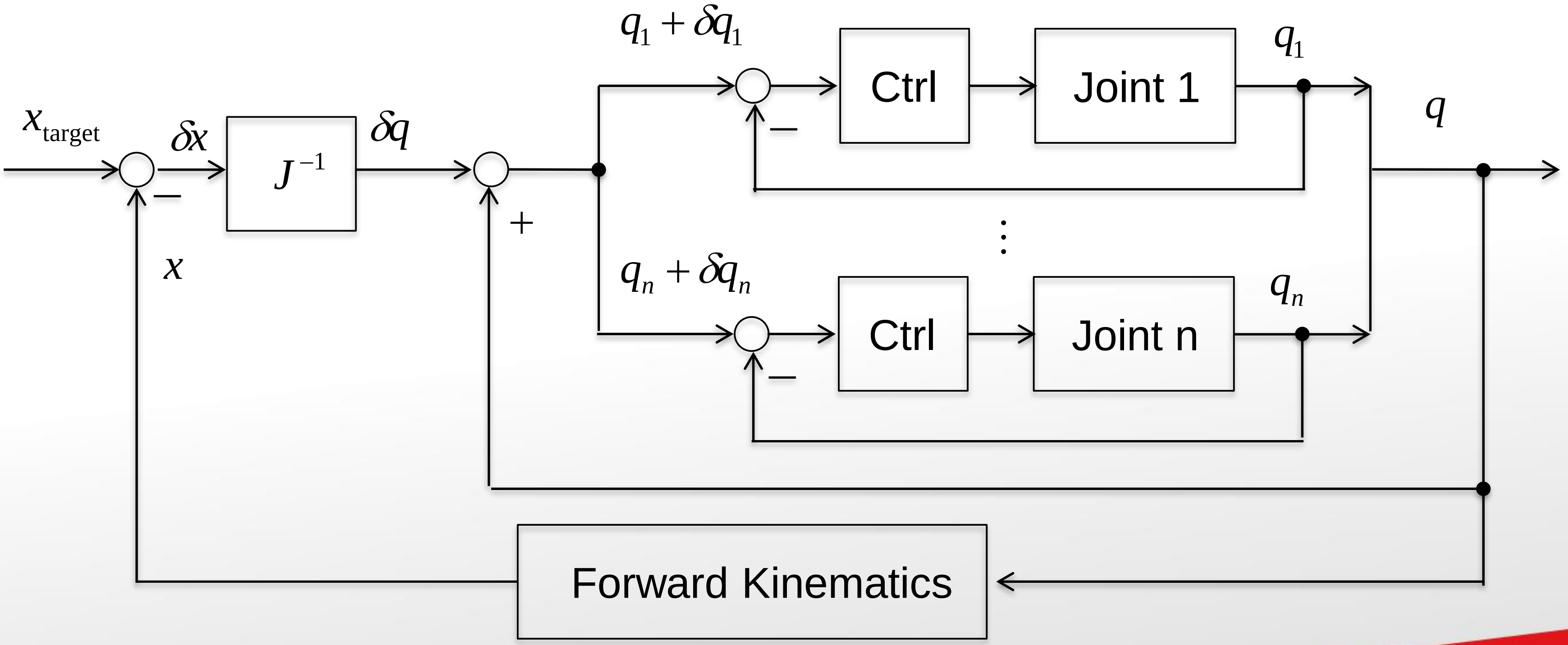
Introduction – Linear Control

- We will first explore linear control techniques.
- That is, we assume or approximate the nonlinear, coupled manipulator as a few linear and decoupled joints/links, and we control each joints individually.



Introduction – Linear Control

- The controller of each joint only cares about bringing that particular joint to reach a goal, or to track a trajectory,
- while ignoring coupling effects from all other links or just treat them as disturbances.



Introduction – Linear Control

- While this method seems crude, it is in fact quite widely used in industrial robotic manipulators.
- Advantage:
  - Simple
  - Acceptable performance.
- Disadvantage:
  - Performance not as good as using nonlinear control.
  - Performance may vary at different configurations.

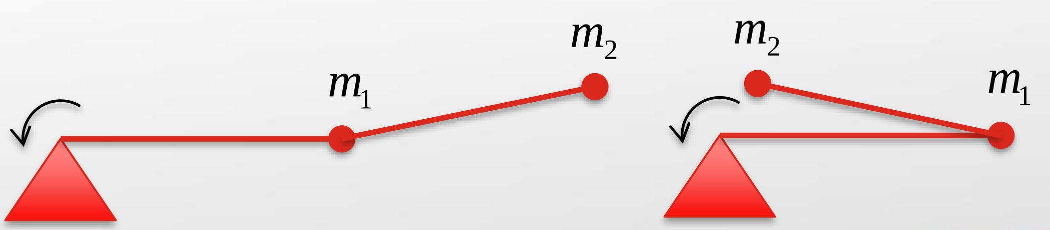
Introduction – Nonlinear Control

- The disadvantages of linear control method are due to the following reasons:
  - The joints or links are highly coupled.
  - The inertia (and other) matrices are NOT constant.

$$\underbrace{\begin{bmatrix}(m_1+m_2)L_1^2+m_2L_2^2+2m_2L_1L_2c_2 & m_2L_2^2+m_2L_1L_2c_2\\ m_2L_2^2+m_2L_1L_2c_2 & m_2L_2^2\end{bmatrix}}_{M(q)}\begin{bmatrix}\ddot{\theta}_1\\ \ddot{\theta}_2\end{bmatrix}$$

$$+\underbrace{\underbrace{\begin{bmatrix}-m_2L_1L_2s_2\dot{\theta}_2^2\\ m_2L_1L_2s_2\dot{\theta}_1^2\end{bmatrix}}_{\text{Centrifugal}}+\underbrace{\begin{bmatrix}-2m_2L_1L_2s_1\dot{\theta}_1\dot{\theta}_2\\ 0\end{bmatrix}}_{\text{Coriolis}}}_{V(q,\dot{q})}+\underbrace{\begin{bmatrix}m_2gL_2c_{12}+(m_1+m_2)gL_1c_1\\ m_2gL_2c_{12}\end{bmatrix}}_{G(q)}=\begin{bmatrix}\tau_1\\ \tau_2\end{bmatrix}$$

large inertia seen from joint 1



small inertia seen from joint 1

Introduction – Nonlinear Control

- The use of linear control will therefore lead to undesirable results.
  - E.g. the damping will NOT be uniform throughout the workspace.
- Thus, we will also learn about some nonlinear control techniques to achieve better performance.
- Using nonlinear techniques, we will design the controller for the robot as a multi-input-multi-output system, instead of individual joints.

Introduction – Open Loop Control

- We have the dynamic equation of the robot:

$$M(q)\ddot{q} + V(q,\dot{q}) + G(q) = \tau$$

- We also have the desired trajectories for position, speed and acceleration.

$$q_d, \dot{q}_d, \ddot{q}_d$$

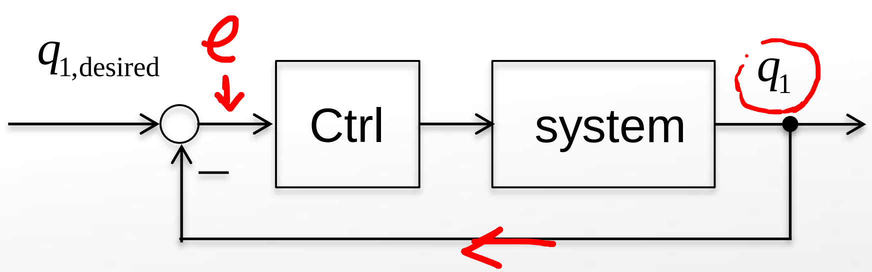
- In an ideal world where there is no modeling error or disturbance, then designing the joint torques as:

$$\tau = M(q_d)\ddot{q}_d + V(q_d, \dot{q}_d) + G(q_d)$$

  - could make the robot follow the desired trajectories!
- → Open Loop Control.
- Unfortunately, real world system definitely has modelling error and disturbances.
  - Therefore the robot will deviate from the desired trajectory.

Introduction – Feedback Control

- To make sure the robot joint actually follows the desired trajectory, we need feedback control.
  - Use sensors to measure joint angles and velocities.
  - If there are errors (difference between desired and actual trajectory), then provide corrective actions (increase or reduce torque) so that the actual trajectory moves back towards the desired trajectory.

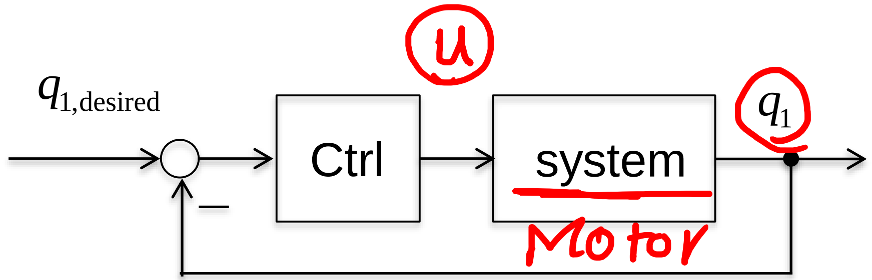


  - We need to ensure the stability of such closed-loop systems.

Content

Control of only one joint/link

- To control a single joint with feedback control.

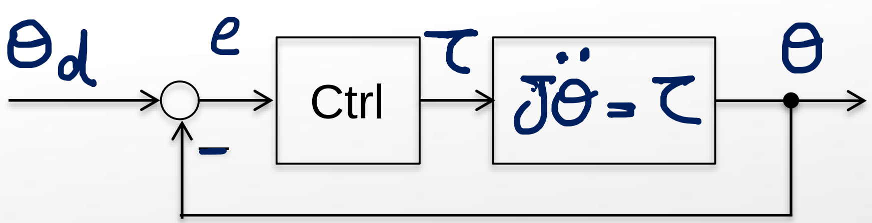


For the Motor: [u(input) = Voltage]

q_1 (output) = angle θ

assumption voltage ~ Torque

u(input) = τ



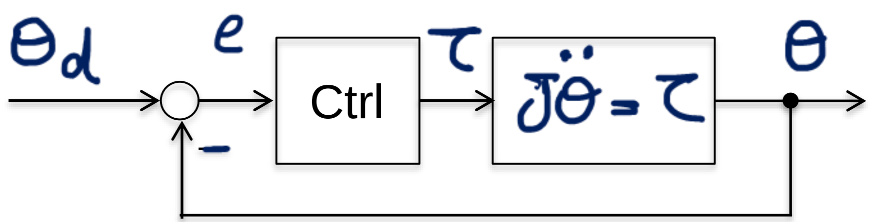
Simple model for Motor

$J\ddot{\theta} = \tau$

Control of only one joint/link

- P-Control, assuming desired Theta is zero . $\theta_d = 0$

P means τ is proportional to error (e)



Motor Model : $J\ddot{\theta} = \tau$ ①

Control signal: $\tau = K_p \cdot e$

$= K_p(\cancel{\theta_d}^{\,0} - \theta) = -K_p\theta$ ②

- To analyse the closed loop system, to put ② into ①

$\Rightarrow J\ddot{\theta} = \tau = -K_p\theta$

$$\boxed{J\ddot{\theta} + K_p\theta = 0}$$

Natural Systems

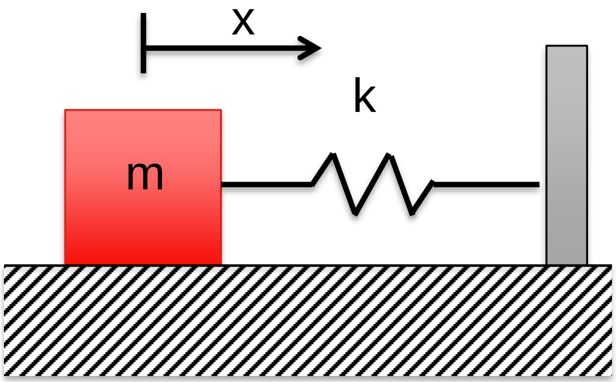
$J\ddot{\theta} + K_p\theta = 0$

- let's have an understanding of natural systems first.
- Imagine you have a mass-spring-system on a frictionless surface:

$m\ddot{x} = -kx$

or

$m\ddot{x} + kx = 0$

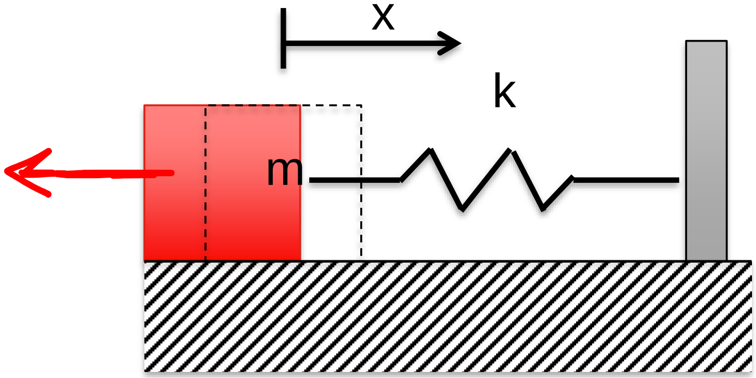


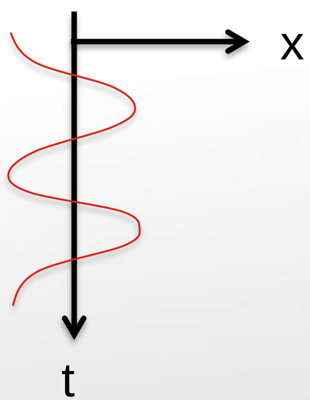
$m\ddot{x} + kx = 0$

- Let the equilibrium position be x = 0.

Natural Systems

- If you perturb the mass from its equilibrium position, and then release it, the mass will swing back and forth continuously.





How to Predict this response?

$$m\ddot{x} + kx = 0$$

need to solve 2nd order ODE

characteristic equation:

$$m\lambda^2 + k = 0 \rightarrow m\lambda^2 = -k$$

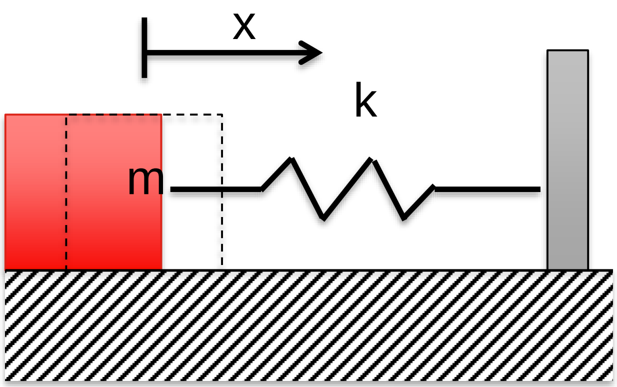
root: $\lambda^2 = -k/m$

$$\lambda = \pm\sqrt{\frac{-k}{m}} = \pm\sqrt{\frac{k}{m} i^2} = \pm\sqrt{\frac{k}{m}}\, i$$

Natural Systems

- Mathematically, the differential equation (Newton's Law) for the mass-spring system on frictionless surface is:

$$m\ddot{x} = -kx \quad \text{or} \quad m\ddot{x} + kx = 0$$



Natural Frequency

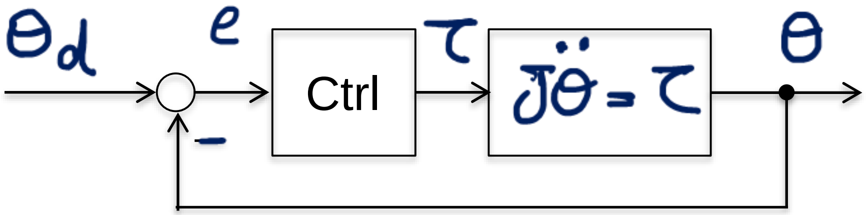
$$\omega_n = \sqrt{\frac{k}{m}}$$

- Solving the differential equation gives:

$$x = C_1 \cos\left(\sqrt{\frac{k}{m}}t\right) + C_2 \sin\left(\sqrt{\frac{k}{m}}t\right)$$

Control of only one joint/link

- P-Control, assuming desired Theta is zero .



- To analyse the closed loop system

$$J\ddot{\theta} + K_p\theta = 0$$

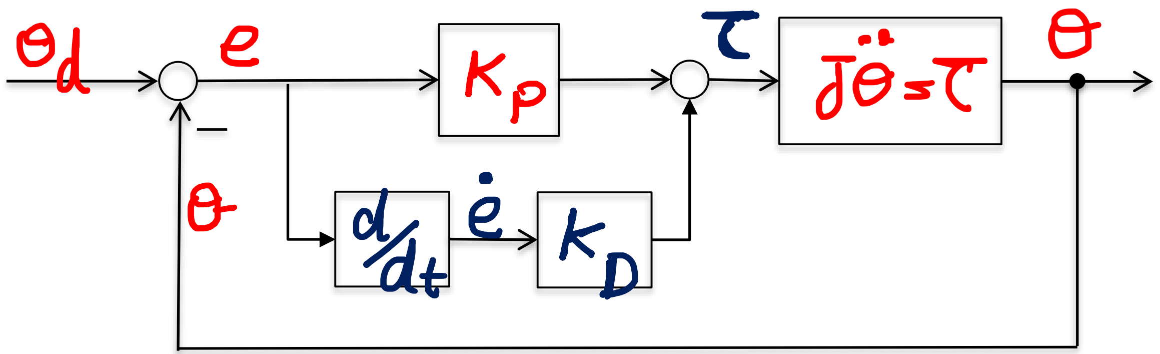
Motor will swing back & forth

$\Rightarrow$ Not good enough

Control of only one joint/link

- PD-Control, assuming desired Theta is zero . $\theta_d = 0$

D: means Proportional to the Derivative of error(e)



Motor : $J\ddot{\theta} = \tau$ ③

control : $\tau = K_p e + K_D \dot{e} = K_p(\cancel{\theta_d} - \theta) + K_D(\cancel{\dot{\theta}_d} - \theta)$

$= -K_p \theta - K_D \dot{\theta}$ ④

- To analyse the closed loop system

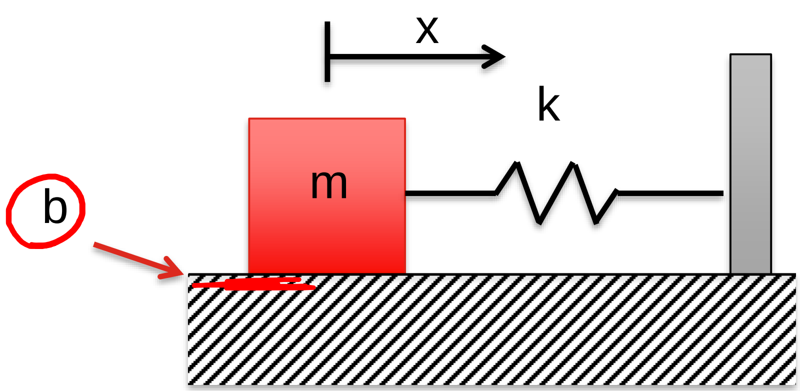
Put ④ into ③

$$\boxed{J\ddot{\theta} + K_D\dot{\theta} + K_p\theta = 0}$$

Natural Systems

$J\ddot{\theta} + K_D\dot{\theta} + K_P\theta = 0$

- Now, imagine the surface is not frictionless anymore.
- Instead it has a viscous friction b.



- Let the equilibrium position be x = 0.

Natural Systems

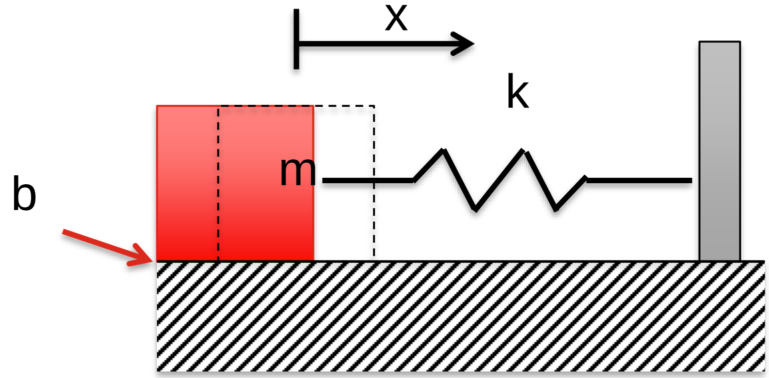
$m\ddot{x} + b\dot{x} + Kx = 0$

- Mathematically, the differential equation (Newton's Law) for the mass-spring system on surface with viscous friction is:

$$m\ddot{x} = -b\dot{x} - kx$$

or

$$m\ddot{x} + b\dot{x} + kx = 0$$



- This is also equivalent to a mass-spring-damper system.
- The roots of the characteristic equation $ms^2 + bs + k = 0$ are:

$$\Rightarrow \quad s_1 = \frac{-b + \sqrt{b^2 - 4mk}}{2m} \qquad s_2 = \frac{-b - \sqrt{b^2 - 4mk}}{2m}$$

$$\begin{cases} b^2 > 4mk \\ b^2 < 4mk \\ b^2 = 4mk \end{cases}$$

- Depending on the relationship between b and k, we have one of the three possible solutions:

Natural Systems

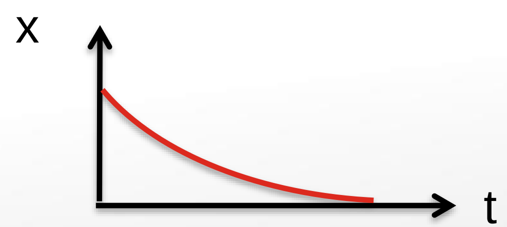
- If $b^2 > 4mk$: Real and unequal roots

$$s_{1,2} = \frac{-b \pm \sqrt{b^2 - 4mk}}{2m} = \lambda \pm \mu i$$

s_1 & s_2

- Overdamped response.
- Decreases to equilibrium position (0) very slowly.

$$x(t) = c_1 e^{s_1 t} + c_2 e^{s_2 t}$$



overdamped

Natural Systems

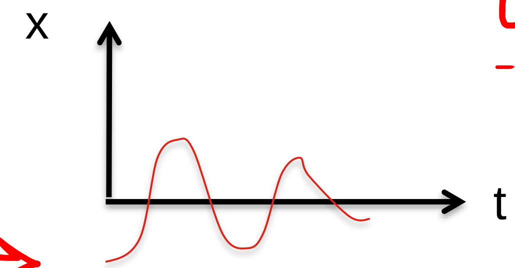
- If $b^2 < 4mk$: Complex roots

$$s_{1,2} = \frac{-b \pm \sqrt{b^2 - 4mk}}{2m} = \lambda \pm \mu i$$

- Oscillatory response, with reducing amplitude.

underdamped

$$x(t) = c_1 e^{\lambda t} \cos(\mu t) + c_2 e^{\lambda t} \sin(\mu t)$$



- Another way to write 2nd order system is: $s^2 + 2\xi\omega_n s + \omega_n^2 = 0$
- where ξ is the damping ratio.
- Comparing all equations, we have: $\lambda = -\xi\omega_n$ $\quad \mu = \omega_n\sqrt{1-\xi^2}$

$$\xi = \frac{b}{2\sqrt{km}} \qquad \omega_n = \sqrt{\frac{k}{m}}$$

Damped natural frequency

Natural Systems

- Finally, if b² = 4mk:

$$s_1 = s_2 = -\frac{b}{2m}$$

$$x(t) = (c_1 + c_2 t)e^{s_1 t}$$

- Critically damped response.

- System reaches equilibrium position rapidly and without oscillation!
- Highly desirable!
- In this case:

$$\xi = \frac{b}{2\sqrt{km}} = \frac{b}{2\sqrt{b^2/4}} = 1$$

Damping ratio for critically damped system

Example (1)

- Find the response of the mass-spring-damper system for m =1, b = 5, k = 6, when the block is released from position x = -1.
- Answer:
  - b^2 (= 25) > 4mk (= 24).
  - Roots of characteristic equation: $s_{1,2} = \dfrac{-b \pm \sqrt{b^2 - 4mk}}{2m} = \dfrac{-5 \pm 1}{2} = -3 \,\&\, -2$
  - Thus:

$$x(t) = c_1 e^{-3t} + c_2 e^{-2t} \qquad \dot{x}(t) = -3c_1 e^{-3t} - 2c_2 e^{-2t}$$

  - To find c_1 and c_2, use initial conditions:

$$x(0) = c_1 + c_2 = -1 \qquad \dot{x}(0) = -3c_1 - 2c_2 = 0$$

  - This gives: $c_1 = 2$ $\quad c_2 = -3$
  - Thus the complete solution is: $x(t) = 2e^{-3t} - 3e^{-2t}$

Example (2)

- Find the response of the mass-spring-damper system for m =1, b = 1, k = 1, when the block is released from position x = -1.
- Answer:
  - b^2 (= 1) < 4mk (= 4).
  - Roots of characteristic equation: $s_{1,2} = \dfrac{-b \pm \sqrt{b^2 - 4mk}}{2m} = \dfrac{-1 \pm \sqrt{1-4}}{2} = \dfrac{-1 \pm \sqrt{3}i}{2}$
  - Thus: $x(t) = c_1 e^{-\frac{1}{2}t}\cos\left(\dfrac{\sqrt{3}}{2}t\right) + c_2 e^{-\frac{1}{2}t}\sin\left(\dfrac{\sqrt{3}}{2}t\right) = e^{-\frac{1}{2}t}\left(c_1\cos\left(\dfrac{\sqrt{3}}{2}t\right) + c_2\sin\left(\dfrac{\sqrt{3}}{2}t\right)\right)$

$$\dot{x}(t) = -\frac{1}{2}e^{-\frac{1}{2}t}\left(c_1\cos\left(\frac{\sqrt{3}}{2}t\right) + c_2\sin\left(\frac{\sqrt{3}}{2}t\right)\right) + e^{-\frac{1}{2}t}\left(-\frac{\sqrt{3}}{2}c_1\sin\left(\frac{\sqrt{3}}{2}t\right) + \frac{\sqrt{3}}{2}c_2\cos\left(\frac{\sqrt{3}}{2}t\right)\right)$$

  - To find c_1 and c_2, use initial conditions:

$$x(0) = c_1 = -1 \qquad \dot{x}(0) = -\frac{1}{2}c_1 + \frac{\sqrt{3}}{2}c_2 = \frac{1}{2} + \frac{\sqrt{3}}{2}c_2 = 0$$

  - This gives: $x(t) = e^{-\frac{1}{2}t}\left(-\cos\left(\dfrac{\sqrt{3}}{2}t\right) - \dfrac{1}{\sqrt{3}}\sin\left(\dfrac{\sqrt{3}}{2}t\right)\right)$

RMIT UNIVERSITY

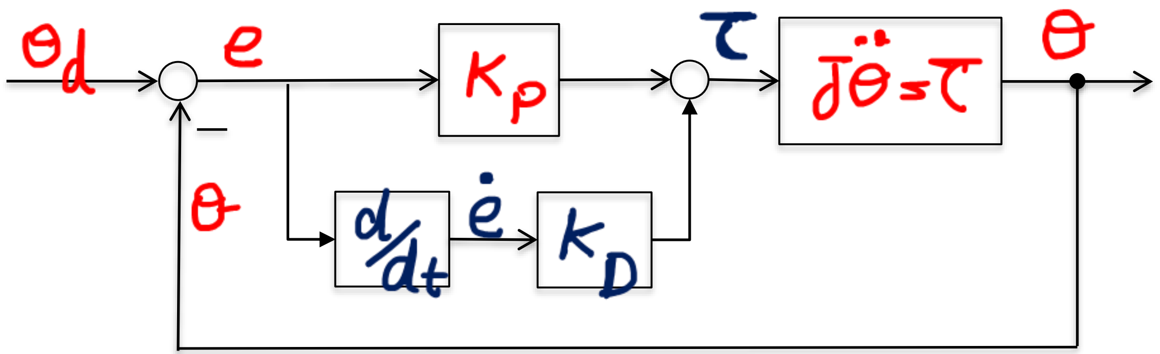
Example (3)

- Find the response of the mass-spring-damper system for m =1, b = 4, k = 4, when the block is released from position x = -1.
- Answer:
  - b^2 (= 16) < 4mk (= 16).
  - Roots of characteristic equation: $$s_{1,2} = \frac{-b \pm \sqrt{b^2 - 4mk}}{2m} = \frac{-4 \pm \sqrt{16-16}}{2} = -2$$
  - Thus: $$x(t) = (c_1 + c_2 t)e^{-2t} \qquad \dot{x}(t) = -2(c_1 + c_2 t)e^{-2t} + c_2 e^{-2t}$$
  - To find c_1 and c_2, use initial conditions: $$x(0) = c_1 = -1 \qquad \dot{x}(0) = -2c_1 + c_2 = 0$$
  - This gives: $$x(t) = (-1 - 2t)e^{2t}$$

Control of only one joint/link

$$J\ddot{\theta} + k_D\dot{\theta} + k_p\theta = 0$$

- PD-Control, assuming desired Theta is zero .



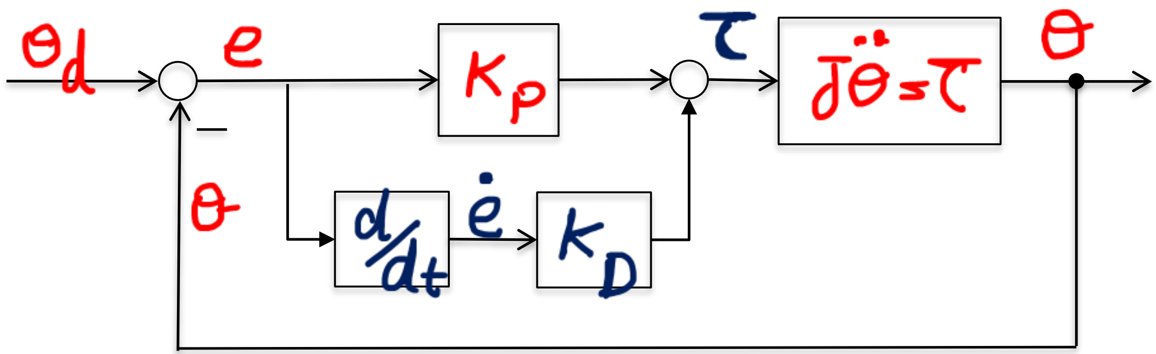
- We learnt that if the relationship between b, k and m is $b^2 = 4mk$, then we have the critically damped response which is fast and non-oscillatory.
- Therefore, we decide on a good k_p (this determines the stiffness of the mass), and then let: 1

$$k_D^2 = 4Jk_p \quad \Rightarrow \quad k_D = 2\sqrt{Jk_p}$$

2

Control of only one joint/link

- PD-Control, assuming desired Theta is zero .



- Problems of this method
- It relies on model $J\ddot{\theta} = \tau$; In real world, J may change : e.g. if robot pick an object
- If model is not $J\ddot{\theta} = \tau$, then the solution is not optimal. e.g. system has some friction.

$$J\ddot{\theta} + \underbrace{b\dot{\theta}}_{\text{friction}} = \tau$$

Content

- Introduction
- Control of Second Order Linear Systems
- Control-Law Partitioning
- Trajectory Following
- Nonlinear and Time-Varying Systems
- Nonlinear Control of Multi-Input-Multi-Output System
- Matlab Simulink Simulation

Tutorial Assignments

- **Question 1:**
  - Determine the motion of a mass-spring-damper system if parameter values are $m = 2$, $b = 6$ and $k = 4$, and the mass (initially at rest) is released from the position $x = 1$.

Tutorial Assignments

- **Question 2:**
  - Determine the motion of a mass-spring-damper system if parameter values are $m = 1$, $b = 2$ and $k = 1$, and the mass (initially at rest) is released from the position $x = 4$.

Tutorial Assignments

- **Question 3:**
  - Determine the motion of a mass-spring-damper system if parameter values are $m = 1$, $b = 4$ and $k = 5$, and the mass (initially at rest) is released from the position $x = 2$.

Tutorial Assignments

- **Question 4:**
  - Consider a mass-spring-damper system with parameter values $m = 1$, $b = 4$ and $k = 5$.
  - The system is known to possess an unmodeled resonance at $\omega_{res} = 6$ rad/sec.
  - Determine the gains k_v and k_p which will critically damp the system with as high a stiffness as reasonable.

Tutorial Assignments

- **Question 5:**
  - Give the nonlinear control equations for the system:

$$\left(2\sqrt{\theta}+1\right)\ddot{\theta}+3\dot{\theta}^{2}-\sin(\theta)=\tau$$

  - Choose gains so that this system is always critically damped with closed-loop stiffness K_{CL} of 10.

RMIT UNIVERSITY

Tutorial Assignments

- **Question 6:**
  - Give the nonlinear control equations for the system:

$$2\ddot{\theta} + 5\theta\dot{\theta} - 13\dot{\theta}^3 + 5 = \tau$$

  - Choose gains so that this system is always critically damped with closed-loop stiffness K_{CL} of 10.

Tutorial Assignments

- **Question 7:**
  - Design a trajectory-following control system for a system with the following dynamic equations:

$$
\begin{aligned}
m_1 l_1^2 \ddot{\theta}_1 + m_1 l_1 l_2 \dot{\theta}_1 \dot{\theta}_2 &= \tau_1 \\
m_2 l^2 \left(\ddot{\theta}_1 + \ddot{\theta}_2 \right) + v_2 \dot{\theta}_2 &= \tau_2
\end{aligned}
$$

Thank you!

Have a good evening.