Newton Heating & Cooling Model

- Rate at which temperature of an object changes is proportional to the difference between the temperature of the object and the temperature of its surrounding.

$$\frac{dT}{dt} = k(T - T_s)$$

$T(t)$ = temperature of the object at time t

$T_s(t)$ = temperature of the surrounding at time t.

Now, consider a large temperature difference means rapid when a coffee is much hotter than the air, its temperature dropped by 8°C in one minute.

Coffee has cooled to 30°C.
Temperature changes by only 1.2°C / per minute.

Find the expression of $T(t)$ of the coffee.

Now $\frac{dT}{dt} = k(T - T_s)$

$$\Rightarrow \frac{dT}{T - T_s} = k\,dt \Rightarrow \int \frac{dT}{T - T_s} = \int k\,dt$$

$$\Rightarrow \frac{1}{1} \ln|T - T_s| = kt + C$$

$$\Rightarrow \ln|T - T_s| = kt + C \quad \therefore \quad T - T_s = e^{kt + C}$$

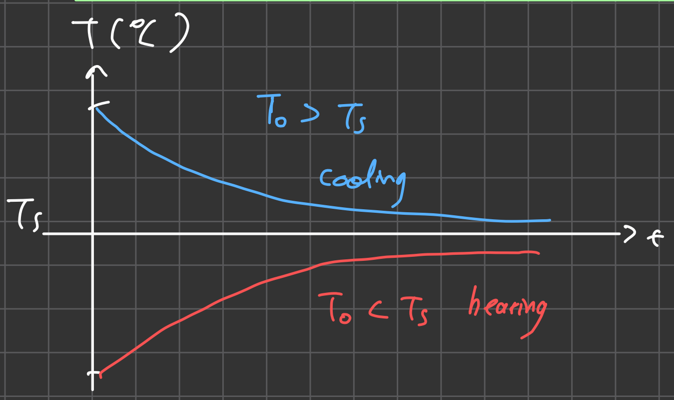
Now let T_0 be $T(0) = T_0$

$$\therefore \quad T(t) - T_s = e^{kt + C}$$

$$\Rightarrow T(t) = T_s + e^{kt + C} \Rightarrow T(0) = T_0 = T_s + e^0 e^C \Rightarrow e^C = T_0 - T_s$$

$$\therefore \quad T(t) = T_s + (e^{kt} e^C)$$
$$= T_s + (e^{kt}(T_0 - T_s))$$

$$\boxed{\therefore \quad T(t) = T_s + (T_0 - T_s)e^{kt}}$$



• Case :

A small metal bar whose temperature is 30°C is dropped into a container of 75°C water.

After 1 second the temperature of the bar has increased by 1°C

a) How long will it take for the temperature of the bar to reach 70°C ?

b) How long will it reach 74

a) $T_0 = 30$ $\qquad T_s = 75$

$$T(t) = T_s + (T_0 - T_s)e^{kt}$$

at $T(1) = 31 = 75 + (30 - 75)e^{kt}$

$$\Rightarrow -44 = -45e^{k}$$

$$\Rightarrow e^{k} = \frac{44}{45}$$

$$\Rightarrow k = \ln\left|\frac{44}{45}\right| = -0.0225$$

$$\therefore T(t) = 75 - 45e^{-0.0225t}$$

$$\therefore T = 75 - 45e^{-0.0225t}$$

$$\Rightarrow 45e^{-0.0225t} = 75 - T$$

$$\Rightarrow e^{-0.0225t} = \frac{75 - T}{45}$$

$$\Rightarrow -0.0225t = \ln\left|\frac{75 - T}{45}\right|$$

$$\Rightarrow t = \frac{\ln\left|\frac{75 - T}{45}\right|}{-0.0225} = 97.7 \text{ seconds} \quad \text{as } T = 70°C$$

b) at $T = 74°C$

$$\therefore t = \frac{\ln\left|\frac{75 - 74}{45}\right|}{-0.0225} = 169.2 \text{ seconds}$$