Tutorial One

0.1. Write down the control signal $u(t)$ in relation to the set-point signal $r(t)$ and output signal $y(t)$ for the following PID controllers, where the proportional gain K_c, integral time constant τ_I and derivative gain τ_D are given as

1. $K_c = 0.5$, $\tau_I = 0.6$ and $\tau_D = 0.1$;
2. $K_c = -1.5$, $\tau_I = 6$ and $\tau_D = 1$;
3. $K_c = 15$, $\tau_I = 1$ and $\tau_D = 3$.

0.2. Given that $K_c = -1.5$, $\tau_I = 6$ and $\tau_D = 1$, find the Laplace transform $U(s)$ for PID controllers with following cases.

1. the ideal PID controller given by Problem 0.1;
2. the PID controller with derivative control implemented on the output only and with filter time constant $\beta\tau_D$, where $\beta = 0.1$;
3. the PID controller with both proportional control and derivative control implemented on output only, in addition to a derivative filter with time constant $\beta\tau_D$, where $\beta = 0.1$.

0.3. Draw the diagram of a closed-loop feedback control system, where the plant transfer function is $G(s)$, the controller transfer function is $C(s)$ and the transfer function for the sensor is $H(s)$. On the diagram, mark set-point signal $R(s)$, control signal $U(s)$ and output signal $Y(s)$, feedback error $E(s)$. Find the following closed-loop transfer functions:

$$\frac{Y(s)}{R(s)}; \quad \frac{E(s)}{R(s)}; \quad \frac{U(s)}{R(s)}$$

0.4. Continue from Problem 0.3. Add input disturbance signal $D_i(s)$, output disturbance signal $D_o(s)$ and measurement noise signal $D_m(s)$ to the diagram from Problem 0.3. Find the following closed-loop transfer functions:

$$\frac{Y(s)}{D_i(s)}; \quad \frac{Y(s)}{D_o(s)}; \quad \frac{Y(s)}{D_m(s)}$$

0.5. Consider the following transfer functions:

1. $G(s) = \frac{-s+1}{(s+10)(s^2+2s+2)}$;
2. $G(s) = \frac{1}{(s-1)(s+2)(s+10)}$;
3. $G(s) = \frac{(s+1)(s-1)}{(s+3)^2(s-3)}$

For each transfer function, find its poles and zeros, and mark the locations of the poles and zero on the complex plane. Is the transfer function stable? Why?

0.6. Assume that a system is described by the following transfer function

$$G(s) = \frac{-s+3}{s^2+3s+1}$$

and a proportional controller $K_c > 0$ is used. Will the closed-loop system be stable for any choice of $K_c > 0$? If not, what is the value of K_c that will lead to a pair of poles on the imaginary axis? Can we apply final value theorem to determine the steady-state error of the closed-loop system when $K_c = 10$?

0.7. Suppose that the transfer function model of a system is given by $G(s) = \frac{0.01}{s+0.1}$, and a controller is chosen to have the structure

$$C(s) = \frac{K_c(s+0.1)}{s}.$$

1. What K_c value do we have to use in order to have a closed-loop system with a closed-loop pole at -0.5?
2. Supposing that the reference signal is a step signal with amplitude of 0.1, show that the steady-state error $\lim_{t\to\infty}(r(t) - y(t)) = 0$ as long as K_c is positive.

0.8. Assume that a closed-loop control system has the transfer function

$$\frac{Y(s)}{R(s)} = \frac{s+K}{s^3+2s^2+4s+K}.$$

Determine the minimum and maximum values of K such that the closed-loop system is stable by using Routh-Hurwitz criterion.

0.9. Use Routh-Hurwitz criterion to determine the range of the proportional controller K_c that will stabilize the systems with the following transfer functions.

1. $G(s) = \frac{0.1}{(s+1)(s+3)}$
2. $G(s) = \frac{(-5s+1)}{(s+2)^2(s+10}$
3. $G(s) = \frac{s+0.1}{(s-3)(s+6)(s+1)}$
4. $G(s) = \frac{-s+3}{(s+3)(s^2+s+5)}$