Tutorial Three

0.1. For a system with the transfer function

$$G(s) = \frac{b_0}{s^2 + a_1 s + a_0}$$

design a controller with the structure

$$C(s) = \frac{c_2 s^2 + c_1 s + c_0}{s}$$

where all three desired closed-loop poles are chosen to be $-\lambda$. Convert this controller into an ideal PID controller that has the structure $K_c + \frac{K_c}{\tau_I s} + K_c \tau_D s$ so to find the proportional gain K_c, integral time constant τ_I and the derivative time τ_D. Use final value theorem to show that for a step reference signal with amplitude R_0, the closed-loop output response is R_0 as $t \to \infty$.

1. $G(s) = \frac{1}{s(s+2)}$, $\lambda = 3$, $R_0 = 1$.
2. $G(s) = \frac{-3}{s^2+3^2}$, $\lambda = 6$, $R_0 = -3$
3. $G(s) = \frac{1}{s^2-1}$, $\lambda = 1$, $R_0 = 2$

0.2. For a system with the transfer function

$$G(s) = \frac{b}{s + a}$$

design a resonant controller with the structure

$$C(s) = \frac{c_2 s^2 + c_1 s + c_0}{s^2 + \omega_0^2}$$

where all three desired closed-loop poles are chosen to be $-\lambda$. Supposing that the reference signal is a sinusoidal signal $r(t) = sin(\omega_0 t)$, show that as $t \to \infty$, the feedback error $r(t) - y(t) \to 0$.

1. $G(s) = \frac{-1}{2s+1}$, $\omega_0 = 1$, and $\lambda = 2$.
2. $G(s) = \frac{0.5}{s}$, $\omega_0 = 0.1$, $\lambda = 0.5$.
3. $G(s) = \frac{1}{s-1}$, $\omega_0 = 2$, $\lambda = 1$.

0.3. In order for the closed-loop system to track a reference signal $r(t) = sin(\omega_0 t) + R_0$, the controller needs to contain the factors s and $s^2 + \omega_0^2$ in its denominator. For a system with the transfer function

$$G(s) = \frac{b}{s + a}$$

design a resonant controller plus integral action with the structure

$$C(s) = \frac{c_3 s^3 + c_2 s^2 + c_1 s + c_0}{s(s^2 + \omega_0^2)}$$

where all four desired closed-loop poles are chosen to be $-\lambda$. Supposing that the reference signal is a sinusoidal signal $r(t) = sin(\omega_0 t) + 1$, show that as $t \to \infty$, the feedback error $r(t) - y(t) \to 0$.

1. $G(s) = \frac{0.1}{s+0.1}$, $\omega_0 = 1$, $\lambda = 1$.
2. $G(s) = \frac{1}{5s+3}$, $\omega_0 = 0.1$, $\lambda = 2$.
3. $G(s) = \frac{2}{s-2}$, $\omega_0 = 2$, $\lambda = 2$

0.4. Assume that a physical system has the second order transfer function $G(s) = \frac{1}{s^2}$ and pole-assignment controller design technique will be used to design a feedback controller for the purpose of reference signal following and disturbance rejection.

1. Design a resonant control system such that the output of the robot arm will follow a sinusoidal reference signal $r(t) = sin(2t + \frac{1}{3}\pi)$ without steady-state error. All desired closed-loop poles are positioned at -1.
2. Will the closed-loop control system reject an input disturbance signal $d_i(t) = sin(2t - \pi)$ without steady-state error? Explain your answer.