PID Design for First Order Plus Delay Model (1)

With the polynomial approximation (called first order Pade approximation), the transfer function model becomes

$$G(s) = \frac{K_p e^{-ds}}{\tau_p s + 1} \approx \frac{K_p(-ds + 2)}{(\tau_p s + 1)(ds + 2)} \tag{36}$$

$$G(s) = \frac{b_1 s + b_0}{(s + \alpha_1)(s + \alpha_2)} \tag{37}$$

where $b_1 = -\frac{K_p}{\tau_p}$, $b_0 = \frac{2K_p}{\tau_p d}$. If $\frac{1}{\tau_p} < \frac{2}{d}$, then we choose $\alpha_1 = \frac{1}{\tau_p}$ and $\alpha_2 = \frac{2}{d}$. In the case of a plant with a dominant time delay when $\frac{1}{\tau_p} > \frac{2}{d}$, we let $\alpha_1 = \frac{2}{d}$ and $\alpha_2 = \frac{1}{\tau_p}$.

PID Design for First Order Plus Delay Model (2)

Because the zero is unstable located at $s = \frac{2}{d}$, this zero should not be cancelled in the controller design. However, we will cancel the pole $-\alpha_2$, which has a faster dynamics response. The PID controller structure has the transfer function form,

$$C(s) = \frac{c_2 s^2 + c_1 s + c_0}{s(s + l_0)} \tag{38}$$

which is an ideal PID controller with a filter. The filter pole $-l_0$ will be used in the design. We also assume that the PID with filter has the zeros located at $-\gamma_1$ and $-\alpha_2$, where α_2 corresponds to one of the poles in the model. The open- loop transfer function is the quantity,

$$\begin{aligned} L(s) = G(s)C(s) &= \frac{b_1 s + b_0}{(s + \alpha_1)(s + \alpha_2)} \frac{c_2(s + \gamma_1)(s + \alpha_2)}{s(s + l_0)} \\ &= \frac{c_2(b_1 s + b_0)(s + \gamma_1)}{s(s + \alpha_1)(s + l_0)} \end{aligned} \tag{39}$$

PID Design for First Order Plus Delay Model (3)

The closed-loop transfer function is

$$T(s) = \frac{L(s)}{1 + L(s)} = \frac{c_2(b_1 s + b_0)(s + \gamma_1)}{s(s + \alpha_1)(s + l_0) + c_2(b_1 s + b_0)(s + \gamma_1)} \tag{40}$$

Note that the denominator of (40) is a third order polynomial and it has three unknown controller parameters, l_0, c_2 and γ_1. Thus, the desired closed-loop polynomial $A_{cl}(s)$ must be a third order polynomial with its order to match the denominator of (40). To this end, we select

$$A_{cl}(s) = (s^2 + 2\xi w_n s + w_n^2)(s + \lambda_1) \tag{41}$$

where $\lambda_1 > 0$ is a positive parameter.

PID Design for First Order Plus Delay Model (4)

- Same as before, we select the damping coefficient $\xi = 0.707$ and the natural frequency w_n to reflect the design requirements such as closed-loop response time and bandwidth. The extra pole located at $-\lambda_1$ is often chosen to be away from the pair of dominate poles $-\xi w_n \pm j\xi w_n$.
- The Diophantine equation is expressed as

$$s(s+\alpha_1)(s+l_0)+c_2(b_1 s+b_0)(s+\gamma_1)=(s^2+2\xi w_n s+w_n^2)(s+\lambda_1) \qquad (42)$$

where the left-hand side of this equation is the closed-loop polynomial and the right-hand side is the desired closed-loop polynomial.

PID Design for First Order Plus Delay Model (5)

$$
\begin{aligned}
s^3 & +(\alpha_1 + l_0 + c_2 b_1)s^2 + (\alpha_1 l_0 + c_2(b_0 + b_1\gamma_1))s + c_2 b_0 \gamma_1 \\
s^3 & +(2\xi w_n + \lambda_1)s^2 + (w_n^2 + 2\lambda_1 \xi w_n)s + \lambda_1 w_n^2
\end{aligned} \tag{43}
$$

By comparing the coefficients of the both sides of the polynomials, three linear equations are obtained,

$$
s^2 \quad : \quad \alpha_1 + l_0 + c_2 b_1 = 2\xi w_n + \lambda_1 \tag{44}
$$

$$
s \quad : \quad \alpha_1 l_0 + c_2 b_0 + c_2 b_1 \gamma_1 = w_n^2 + 2\lambda_1 \xi w_n \tag{45}
$$

$$
s^0 \quad : \quad c_2 b_0 \gamma_1 = \lambda_1 w_n^2 \tag{46}
$$

PID Design for First Order Plus Delay Model (6)

We solve for $c_2\gamma_1$ based on (46):

$$c_2\gamma_1 = \frac{\lambda_1 w_n^2}{b_0} \tag{47}$$

Then, the value of $c_2\gamma_1$ is substituted into (45), which becomes,

$$\alpha_1 l_0 + c_2 b_0 = w_n^2 + 2\lambda_1 \xi w_n - \frac{b_1 \lambda_1 w_n^2}{b_0} \tag{48}$$

Note that both (44) and (48) contain the same pair of unknown variables (α_1, c_2), so we will solve these two together using these two equations. From (44), we find the value of l_0,

$$l_0 = -c_2 b_1 + 2\xi w_n + \lambda_1 - \alpha_1 \tag{49}$$

PID Design for First Order Plus Delay Model (7)

Substituting this l_0 into (48) and collecting the terms, we find c_2 as

$$c_2 = \frac{-2\xi w_n \alpha_1 - \lambda_1 \alpha_1 + \alpha_1^2 + w_n^2 + 2\lambda_1 \xi w_n - \frac{b_1 \lambda_1 w_n^2}{b_0}}{b_0 - \alpha_1 b_1} \tag{50}$$

where we assume that $b_0 - \alpha_1 b_1 \neq 0$. The value of l_0 is found using (49) with c_2 given by (50). From c_2, we also find γ_1 as

$$\gamma_1 = \frac{\lambda_1 w_n^2}{b_0 c_2} \tag{51}$$

PID Design for First Order Plus Delay Model (8)

With the parameters c_2, γ_1 and l_0 calculated, the PID controller with filter is re-constructed as,

$$C(s) = \frac{c_2(s+\gamma_1)(s+\alpha_2)}{s(s+l_0)} \tag{52}$$

where α_2 corresponds to the location of the pole in the model that we chose to cancel. Equivalently, (52) is expressed in the more general form,

$$C(s) = \frac{c_2 s^2 + c_1 s + c_0}{s(s+l_0)} \tag{53}$$

where c_2 is calculated using (50), $c_1 = c_2(\gamma_1 + \alpha_2)$ and $c_0 = c_2\alpha_2\gamma_1$.

Example

Given a first order plus delay system with the transfer function

$$G(s) = \frac{10e^{-5s}}{10s + 1} \tag{54}$$

find the PID controller parameters using pole assignment design technique. The desired closed-loop performance is specified by $\xi = 0.707$, $\lambda_1 = 1$. To understand that the approximation of time delay using the transfer function model causes error between the actual plant and the model used for the design, find the PID controller parameters for $w_n = 0.4$ and then reducing it to $w_n = 0.2$, and simulate the closed-loop performance with a unit step set-point signal and disturbance rejection of step signal with amplitude 0.2.

Solution I

The first order plus delay model is approximated using Pade approximation, leading to

$$G(s) \approx \frac{-s + 0.4}{(s + 0.1)(s + 0.4)} \tag{55}$$

In the design, we cancel the pole from the time delay, and assign the values of α_1 and α_2 as $\alpha_1 = 0.1$ and $\alpha_2 = 0.4$. Also from (55), we find $b_1 = -1$ and $b_0 = 0.4$.

Solution II

In the calculation, we first use $w_n = 0.4$. We calculate the value of c_2 as

$$c_2 = \frac{-2\xi w_n \alpha_1 - \lambda_1 \alpha_1 + \alpha_1^2 + w_n^2 + 2\lambda_1 \xi w_n - \frac{b_1 \lambda_1 w_n^2}{b_0}}{b_0 - \alpha_1 b_1} = 1.9581 \tag{56}$$

l_0 as

$$l_0 = -c_2 b_1 + 2\xi w_n + \lambda_1 - \alpha_1 = 3.4237 \tag{57}$$

$c_1 = c_2(\gamma_1 + \alpha_2) = 1.1832$ and $c_0 = c_2 \alpha_2 \gamma_1 = 0.16$. From these parameters, we calculate the PID controller parameters using (31) -(34):

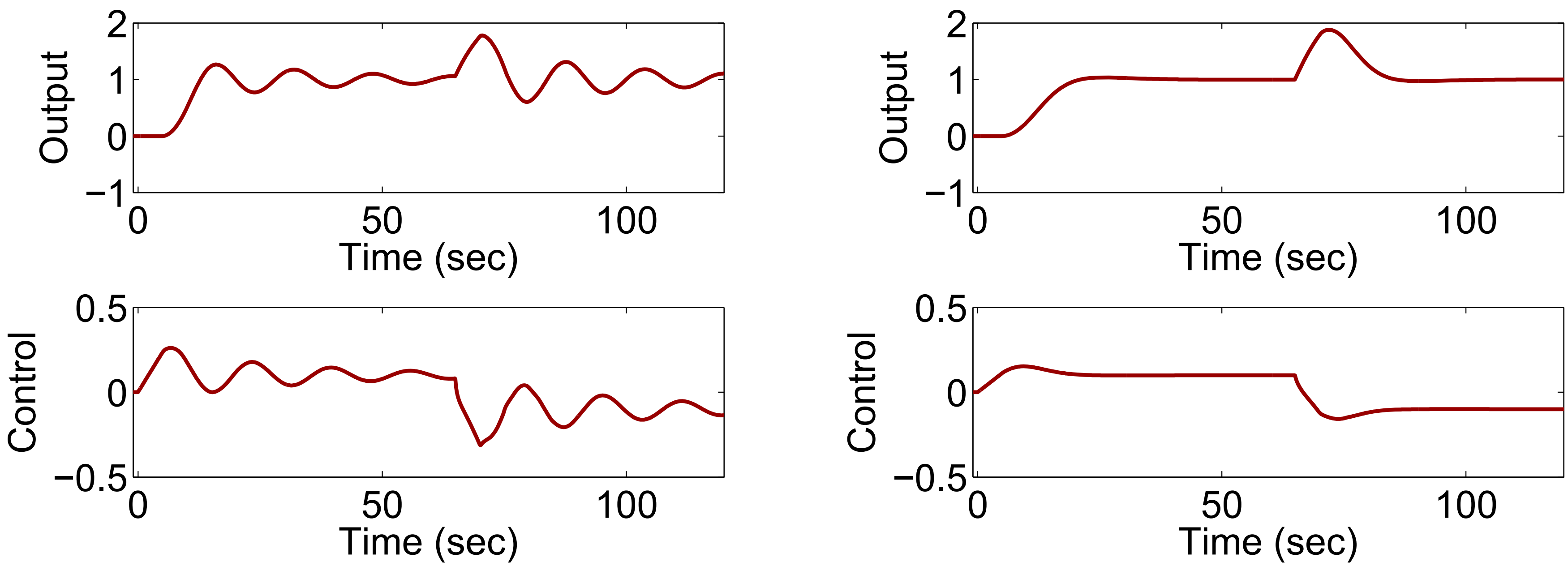
$$K_c = 0.332; \;\; \tau_I = 7.1; \;\; \tau_D = 1.43; \;\; \tau_f = 0.292.$$

We obtain the PID controller parameters for $w_n = 0.2$, as

$$K_c = .1793; \;\; \tau_I = 8.0323; \;\; \tau_D = 1.3375; \tau_f = 0.5581.$$

Response

Figure 5a shows the closed-loop response. It is seen that both control signal and the plant output signal are oscillatory, which is due to the modelling error introduced by the approximation of the time delay. Figure 5b shows the closed-loop response with this reduced w_n. It is seen indeed that the closed-loop oscillation is eliminated.



(a) PID control with $\xi = 0.707$, $\lambda_1 = 1$ and $w_n = 0.4$

(b) PID control with $\xi = 0.707$, $\lambda_1 = 1$ and $w_n = 0.2$

Figure 5: Closed-loop response of PID control system