

Developing a Navigation System for Mobile Robots

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Abstract

Design solution of a novel mobile robot navigation system, presented here, is used to control robot's locomotion across slippery surfaces. Usually, motion control strategies, are based on assumption of sufficient traction between tyres and the road. Motion across slippery surfaces can endanger the robot and its surroundings. Our solution combines Light Detection and Ranging (LIDAR) measurements with odometry data. It performs well on any surface, regardless of sensing, localization and navigation errors, within an indoor environment, in real-time. An accelerated feature detection method is used to improve LIDAR localization update rate and improve localization accuracy. Experiments conducted validate proposed approach.

Keywords

Real-time Localization LIDAR Hough transform Gyroscope Motion

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Developing a Navigation System for Mobile Robots

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Abstract. Design solution of a novel mobile robot navigation system, presented here, is used to control robot's locomotion across slippery surfaces. Usually, motion control strategies, are based on assumption of sufficient traction between tyres and the road. Motion across slippery surfaces can endanger the robot and its surroundings. Our solution combines Light Detection and Ranging (LIDAR) measurements with odometry data. It performs well on any surface, regardless of sensing, localization and navigation errors, within an indoor environment, in real-time. An accelerated feature detection method is used to improve LIDAR localization update rate and improve localization accuracy. Experiments conducted validate proposed approach.

Keywords: Real-time. Localization. LIDAR. Hough Transform. Gyroscope. Motion

1 Introduction

Majority of mobile robots move on various surfaces with limited knowledge of the surface properties, such as friction coefficient. Wheel slipping can cause poor localization results [1] and could lead to task failure and collision. Often, techniques employed in mobile robotics lack to provide reliable data for safe autonomous navigation. Traditionally, robots perform pre-programmed sequences of operations in constrained and predicted environments, like path following, and are not able to operate in new environments, or to face unexpected dynamic situations. There is an emerging need for truly autonomous systems. Applications include intelligent service robots for offices, hospitals and factory floors. In such conditions, an additional capability is needed, to maintain a sufficiently accurate onboard position estimate.

Several technologies for indoor localization [2] were suggested, such as inertial measurement [3], visual odometry, LIDAR [4] and dead-reckoning. However, none of the existing technologies, by itself, could perform reliable solutions. LIDAR is an attractive technology due to its high accuracy in ranging, its wide-area view and low data-processing requirements. Unfortunately, LIDAR can be inaccurate with respect to turns and rotational movements. On the other hand, inertial odometry, based on accelerometer and gyroscope readings, is intended for the short distances, and so it suffers from drift. The distance traveled is obtained by double integration of the accelerometer sensor signal. Bias offset drift exhibited on the acceleration signal is accumulative and the accuracy of the distance measurement can deteriorate with time due to the integration. A novel method is used to solve the problem by correcting raw inertial odometry with LIDAR measurements, which allow robot to navigate accurately on any indoor slippery surface.

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2 Environment and Task

We assume that the vehicle is mostly travelling in the horizontal plane [5]. The test field was set up, as shown in Figure 1, to simulate a food manufacturing workshop, separated into two, $16 m^2$ areas. The left workshop (area I) had dynamic obstacles and slippery surface, and the right workshop (area II) had icy surface similar to cold working environment. A vertical lift, auto roller shutter Door (G), jointed the two areas together. No guiding lines were available in both workshops. Mobile robot had 3 tasks:

- Run from A (charging point) to loading area B by wireless instruction order.
- Then deliver parts to nine robot arm work cell, $W_i(i=1-9)$.
- Finally, mobile robot transports the collected products from robot arm work stations back to charging point.

Navigation system aims to enable the robots to move safely and avoid all unknown obstacles in the area I and robot arm work stations in the area II. It has to park at loading area B accurately for automatic loading, and transport parts to all work stations.

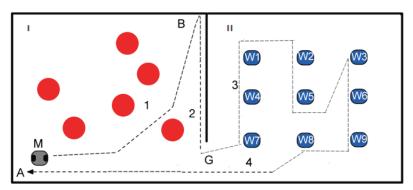


Figure 1.Test environment and mission path layout

In the Figure 1, letter *M* represents mobile robot, *A* stands for the charging, start and finishing point, *G* is the automatic door gate, red dots represent unknown obstacles in the area I, and W1-W9 are stationary robots' working cells.

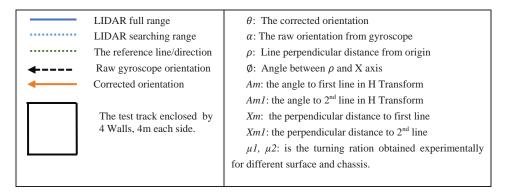
3 System Components

The National Instruments MyRio-1900 [6] was chosen as on board microprocessor for its capabilities, that comes from the concurrent use of the microcontroller, for processing and Field Programmable Gate Area (FPGA) for fast input / output (IO). It has good Electrostatic Discharge (ESD) and over-voltage protection, with many external interfaces: analog/digital IO, Ethernet, and USB. Hokuyo LIDAR URG-04LX-UG01 was selected as it has optimal performances suitable for the localization applications in indoor environments. An appropriate off shelf gyroscope is equipped on board too.

4 LIDAR and gyroscope based motion control solution

This section presents detailed algorithms for LIDAR based motion control solution in LabView environment. The measurement parameters are listed in the **Table 1**. Solution enables a precise, fast update for robust localization, with the focus on: two walls LIDAR based detection, Hough Transform (HT) [7] localization, wall alignment, wall following, and obstacle avoidance.

Table 1 Measurement parameters



4.1 Obstacle free environment localization

The perpendicular lines from LIDAR to walls were used to update robot with precise localization data (*X*, *Y*, and heading direction) in wall enclosure scenario, without obstacles. The coarse heading direction, shown on Figure 2, as a tick, black arrow, from either preset scenario, or from gyroscope; the perpendicular distances to two walls which were most exposed to LIDAR, would be the *X* and *Y* in global coordinate. As each LIDAR's output was a paired data of object's magnitude and angle related to LIDAR. The difference between angle paired with *X* and LIDAR *0* degree would be the exact orientation of LIDAR also mobile robot in global coordinate.

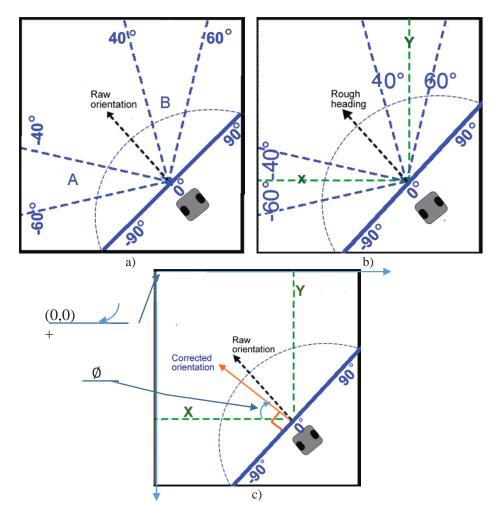


Figure 2 LIDAR self-localization in obstacle free and wall enclosure

In figure 2(a), based on rough heading, entire LIDAR scan region has been minimized into 2 sub-regions, each was 20 degrees range. The shortest distance magnitude in sub-regions **A** and **B** would be X and Y; see Figure 2(b). In Figure 2(c), \emptyset , the angular difference between X's angle and LIDAR's θ degree would be the exact mobile robot's orientation in the global coordinate.

In the programming code example given in Figure 3, the preset angle was set as 45° , and LIDAR range was set from -60° to 60° . In the region between -40° to -60° , the shortest distance i.e. magnitude was selected. Meanwhile, the angle between robot's current heading and the shortest magnitude to left wall could be calculated. Then we were able to find exact heading direction.

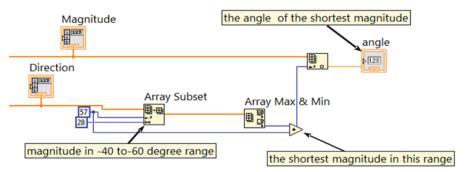


Figure 3 LabVIEW code that calculates the perpendicular distance to a wall in obstacle free scenario

4.2 Unknown cluttered environment localization

In this senario, the perpendicular directions were blocked by obstacles. The previous method could not be reliably utilized for solving localisation and heading direction calculation. The Hough transform (TH) [7], also called Standard Hough Transform (SHT), was applied as it is capable of detecting straight line segments in the presence of noise

Standard LabVIEW library, used for static images, could not be applied into real time target like MyRio. Subsequently, a real time, wall detection method, needed to be developed to work within unknown obstacle senario.

After the Hough Transform method was implemented as shown in Figure 4 the precise X, Y coordinates and heading could be obtained in obstacle scenario, Figure 5 and Figure 6. On the other hand, due to noisey measurements and false readings of LIDAR, the Hough Transform becomes more time consuming. Using hardware, as already defined for this research, it takes 7 seconds to calculate location, based on the two walls. For a mobile robot moving at $2 \, m/s$ speed, in the real-time scenario, localization time cost should be under $0.5 \, second$ for each update.

To accelerate SHT performace, the abundant literature about the HT for straight line have been researched. HT could be classified into two large groups depending on the parameterization used for expressing the lines [8].

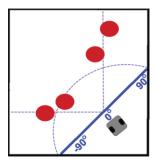


Figure 4 Scenario of Hough Transform applications with obstacles

On one hand, the most abundant group corresponds to those using the parameters ρ and θ , where ρ is the distance from the line to the origin and θ is the angle from a vector which was normal to the line to the abscissas axis. In this group is included the work about the combinatorial Hough transform (CHT) [9] and the piecewise-linear Hough function (PLHT) [10]. Another group used the parameters m and c to express the lines, where m was the slope and c was the point of intersection with the ordinate axis. In the second group we have randomized Hough transform (RHT) [11], and fast Hough transform (FHT)[12].

In the case like Figure 6, there were no complex image or noise, therefore all those algrithom would not improve SHT to be fast enough to achieve the mobile robot real-time localization requirments. The solution for the problem of recognition, presented here, is in using a method that minimize dectecting range in Hough domain (an accumulator array) by combining estimation orientation from gyroscope and given indoor coordinate to save computation load.

There were 5 methods applied to achieve improvement:

- Reduce searching range in Hough domain using gyroscope, or pre-calculation. Raw orientation ±10 degree searching range for 1st wall and ±5 degree searching range for 2nd wall were applied instead of searching in full Hough domain of 180 degrees twice.
- Use median filter and screen to reduce the noise and false readings. Median filters were used for removing noise in image processing. They set each pixel value to the median, in a two-dimensional neighborhood mesh. This allowed us to remove spike noise.
- 3. Increase the bin size. The time to fill and search into the Hough domain is proportional to the size of Hough domain [13]. Considering the mobile robot's size was 300mm*300mm and the test field was 4000mm*4000mm, centimeter was the most suitable minimum length unit to represent robot's localization.
- 4. Calculate with fix point instead floating point arithmetic.
- 5. Rewrite program into FPGA. In contrast to traditional processors, programming an FPGA rewires the chip itself to implement algorithms in hardware.

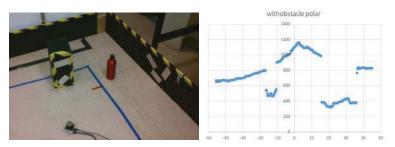


Figure 5 HOUGH experiment scenario (left) and raw reading (right)

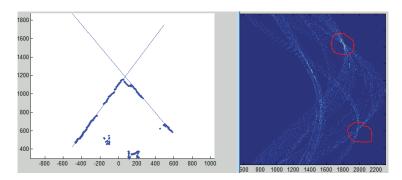


Figure 6 HOUGH transform results in Matlab. Left: Cartesian and Right: Hough Domain

Results Comparison

The least squares method was used to measure the accuracy of the proposed filter. The coefficient of determination compares the estimated and actual values, where a perfect correlation would be I and no correlation would be 0. As the wall was a straight line, the accuracy of the applying different HOUGH domain sizes and filters were compared using the coefficient of determination, r^2 . In Figure 7 (a) r^2 =0.47 which was smaller than Figure 7 (b)'s r^2 =0.56. This indicated that 180*560 (degrees * centimeters) HOUGH domain's bin size gave much better representation of the wall and therefore it improved the localization of the robot.

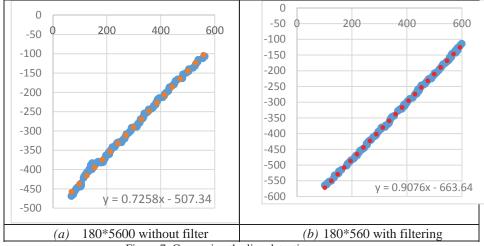


Figure 7. Comparing the line detection accuracy

Table 2 explained results obtained by applying method (1-5) one by one. It was clear that the line detection time was improved, when system required fewer computational operations. Meanwhile, the accuracy of line detection is not affected, as the proper filter and HOUGH domain have been applied. There were 15 successful

runs to evaluate the accuracy and efficiency of the obtained localization data. An instance after 1.5 seconds is given in **Error! Reference source not found.**. LIDAR localization with all 5 methods applied, has improved the update rate to *10Hz*. This was fast enough to allow real-time localisation.

Table 2. Time results

Method	Time (seconds) to detect walls			Improvement	
	First wall	Second wall	Total	Se-	Per-
				conds	centage
Baseline	3	4	7	-	-
3	2	3	5	2	28%
2	1.5	2.5	4	1	20%
1	1	1	2	2	50%
4	0.5	0.5	1	1	50%
5	0.05	0.05	0.1	0.9	90%

5 Supplementary Files

The robot demonstration video could be visited by: https://www.youtube.com/watch?v=-4t5nPGBhHU https://www.youtube.com/watch?v=fcTDMds9Mmg

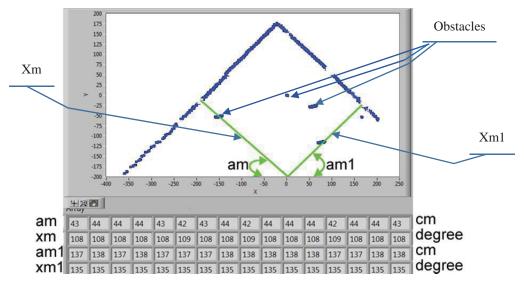


Figure 8 Results of localization acquired in 1.5 seconds

6 Conclusion

A motion control approach, for mobile robots' locomotion across slippery surfaces, is presented here. Novel algorithm developed and tested, is efficient and tolerant to noise in the scenario without obstacles on the road, as well as, in the case with obstacles. The methods of minimizing the detecting range, by applying raw orientation, extracted from gyroscope, were applied in both environments. After course direction is obtained, the LIDAR, i.e. robot's (X,Y) coordinates in global positioning system were obtained finding the shortest magnitudes through Hough Transform application. Various HT transforms were considered. Finally exact orientations were calculated from the difference between perpendicular angle to the reference wall and 0° angle of LIDAR scan range. Those two localization solutions removed much of the bias on the single gyroscope method, including the drifting at low speed conditions. Following that, a novel motion control over slippery surfaces was established and tested comprehensively.

In practice, the algorithm presented here was efficient and tolerant to noise. The Hough transform iterates, over points of interest, in a given data cloud, only in a reduced range of $2*20^0$ instead of full range of $2*180^0$, which requires less amount of memory and is fast enough to be used in real-time applications with the given time constraints as defined by the project requirements.

Following all of that, we have better motion control over slippery surfaces. Due to the intense computational loads, the proposed method was limited to speeds below 2m/s. Under the time constraints, inside that given speed limit, test robot, with the hardware available, was easily performing real time operations. Future work includes a more powerful processor and algorithms, to deal with the intense computational loads that would be required for real-time operation if the motion is performed at the higher speeds.

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