Advanced Control Tutorial # 01

Motivation of control systems:—

→ Industrial automation

  ↳ conveying an item at a conveyor belt, mobile phone chargers

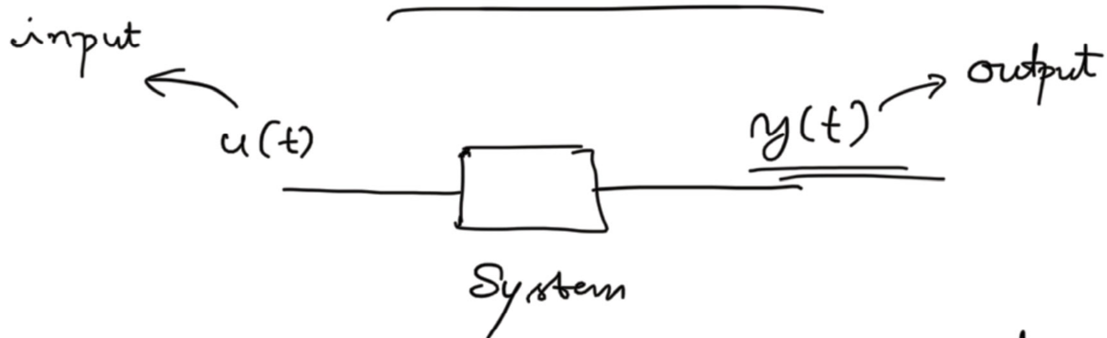
  ↳ opening & closing of dam shutters

  → Atomic reactors ← control rods

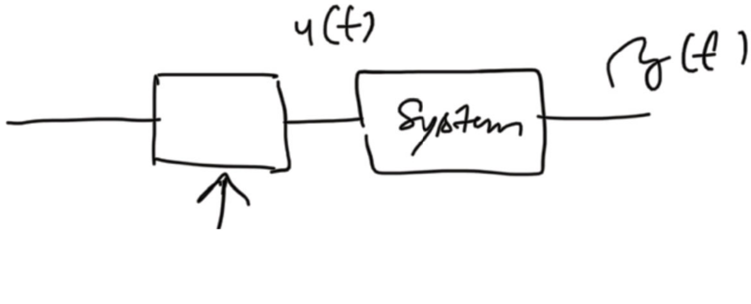
→ Automobiles

  ↳ power steering

  → throttle control



One way to control this plant is to manually change u(t) across the full range & see when you get your desired y(t). Sounds tiring, right?



Control

PID $\longrightarrow$ Proportional + integral + derivative

class of controller

P + PI + PD + PID

$\longrightarrow$ Resonant controller

$\longrightarrow$ Model Predictive Controllers

$\longrightarrow$ Non linear controllers

This course stops here!

$$e(t) = y(t) - r(t)$$

$$u(t) = K_c\, e(t) + \frac{K_c}{\tau_i}\int e(t)\, dt + \frac{K_c \tau_D\, de(t)}{dt}$$

Q #0.2

① $$U(s) = K_c E(s) + \frac{K_c}{\tau_i}\frac{E(s)}{s} + K_c \tau_D\, s E(s)$$

② $$u(t) = K_c e(t) + \frac{K_c}{\tau_i}\int_{-\infty}^{t} e(t)\, dt + K_c \tau_D \dot{e}(t)$$

$$\downarrow$$

$$K_c \tau_D \frac{d(r-y)}{dt}$$

$$\downarrow$$

$$K_c \tau_D \left(\frac{dr}{dt} - \frac{dy}{dt}\right)$$

0

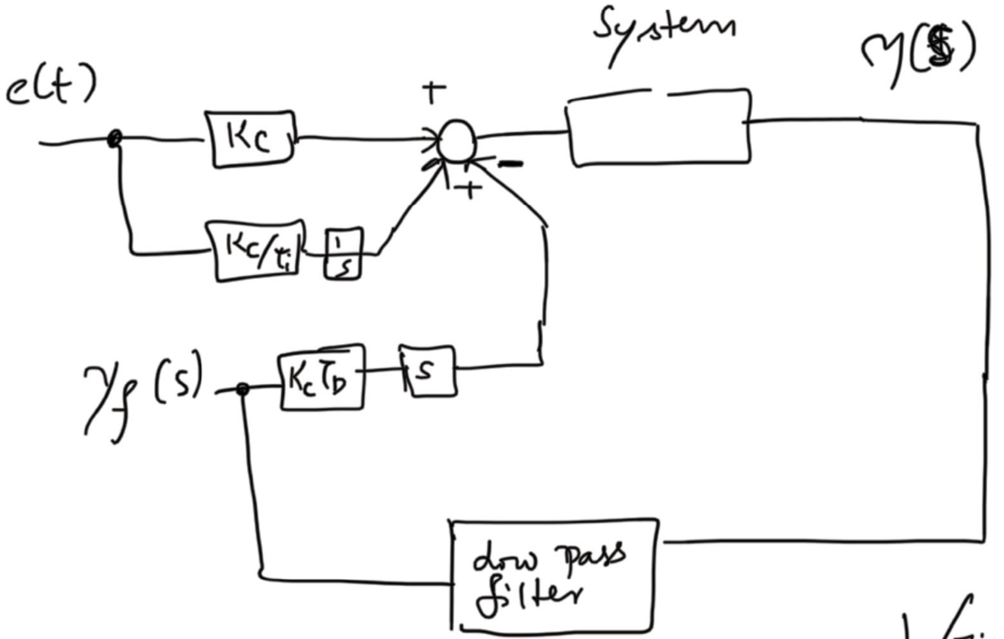
$$-K_c \tau_D \frac{dy_f}{dt}$$ $\leftarrow$ (Noise) derivative amplifies

So, we use filter   noise

$$-K_c \tau_D \frac{dy_f}{dt}$$

$$u(s) = K_c E(s) + \frac{K_c}{\tau_i s} E(s) - K_c \tau_D \, s Y_f(s)$$

→ is not $Y(S)$



$$\frac{1/\text{Time constt}}{s + 1/\text{time constt}}$$

$$Y_f(s) = \frac{Y(s)/\text{time constt}}{s + \frac{1}{\text{Time constt}}}$$

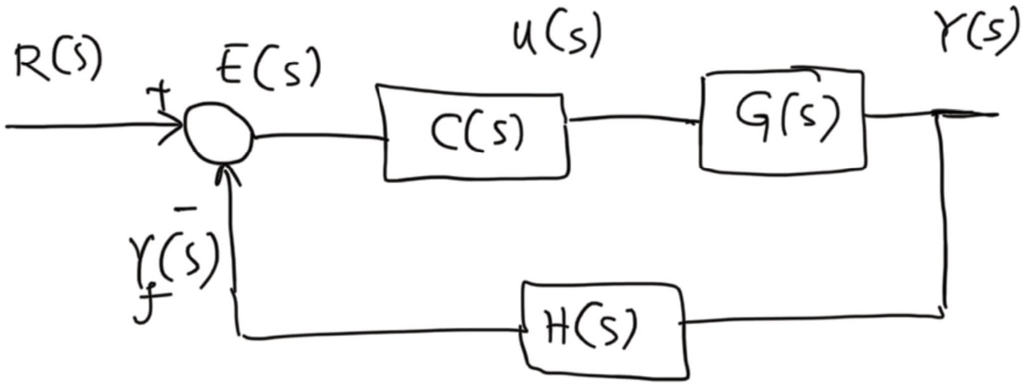
$$Y_f(s) = \frac{Y(s)}{\beta \tau_D s + 1}$$

$K_c E(s) \quad K_c \quad E(s) \quad K_c \tau_D \; s \; Y(s)$

$$K_c E(s) + \frac{K_c}{\tau_i s} E(s) - \frac{K_c \tau_D s Y(s)}{\beta \tau_D s + 1} = U(s)$$

plug-in the values of $K_c, \tau_i, \tau_D, \beta$ etc.

Q#03



$$\longrightarrow Y(s) = G(s)U(s)$$
$$= G(s)C(s)E(s)$$

$$Y = GC(R - HY)$$

$$Y = GCR - GCHY$$

$$Y(1 + GCH) = GCR$$

$$\boxed{Y/R = \frac{GC}{1 + GCH}}$$

$$\longrightarrow \frac{E(s)}{R(s)}$$

$$E(s) = R - HY$$

$$E = R - HECG$$

$$E(1 + HGC) = R$$

$$\boxed{\frac{E}{R} = \frac{1}{1 + CGH}}$$

$$\longrightarrow \frac{U(s)}{R(s)} =$$

$$U(s) = C(s)\,E(s)$$

$$= C(s)\,(R - HY)$$

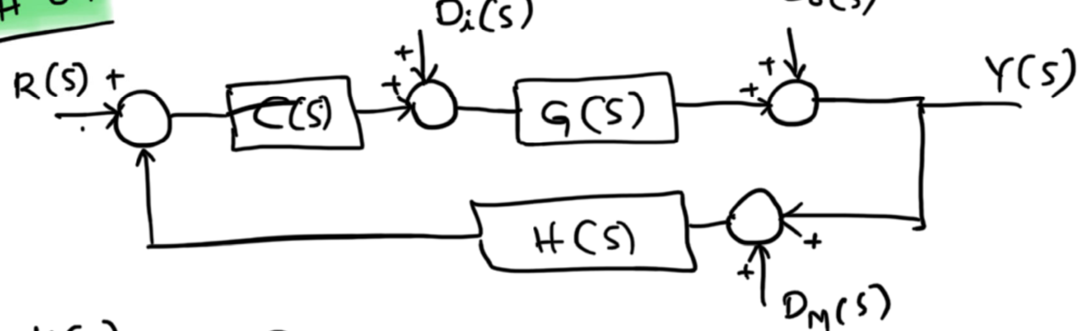
$$U = C\,(R - HGU)$$

$$U = RC - CHGU$$

$$U(1 + CHG) = RC$$

$$\boxed{\frac{U}{R} = \frac{C}{1 + CHG}} \longleftarrow \frac{U(s)}{R(s)}$$

Q # 04



$$\frac{Y(s)}{D_i(s)} = ?$$

$\longrightarrow$ Principle of super position $\longleftarrow$

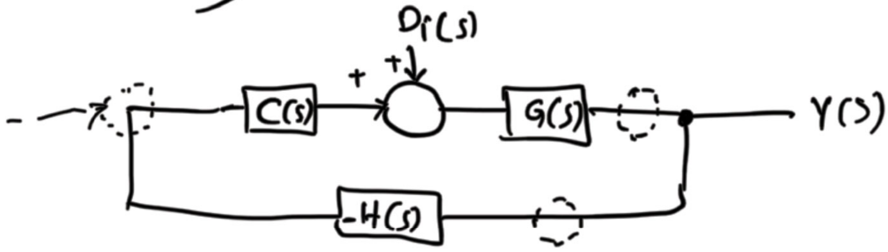
4 sources $=0$

$\cancel{R(s)} \longrightarrow Y(s)$

$D_i(s) \longrightarrow Y(s)$

$\cancel{D_o(s)} \overset{=0}{\longrightarrow} Y(s)$

$\cancel{D_m(s)} \overset{=0}{\longrightarrow} Y(s)$



$$Y(s) = G(s)\,U(s)$$

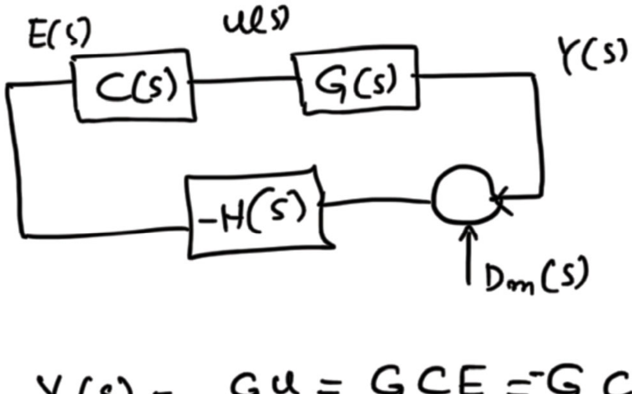
$$= G(CE + D_i)$$

$$Y = GC(-HY) + GD_i$$

$$Y(1 + GCH) = GD_i$$

$$\boxed{\frac{Y}{D_i} = \frac{G}{1+GCH}}$$

$$\frac{Y(s)}{D_m(s)} =$$



$$Y(s) = GU = GCE = -GCH(+Y + D_m)$$

$$Y(s) = -GCHY - GCHD_m$$

$$\boxed{\frac{Y(s)}{D_m(s)} = \frac{-GCH}{1 + GCH}}$$

Q no. 5

$$\frac{-s+1}{(s+10)(s^2+2s+2)}$$

$-s+1$ — Numerator

$(s+10)(s^2+2s+2) \longrightarrow$ Denominator

$-s+1=0 \longrightarrow \boxed{s=1}$

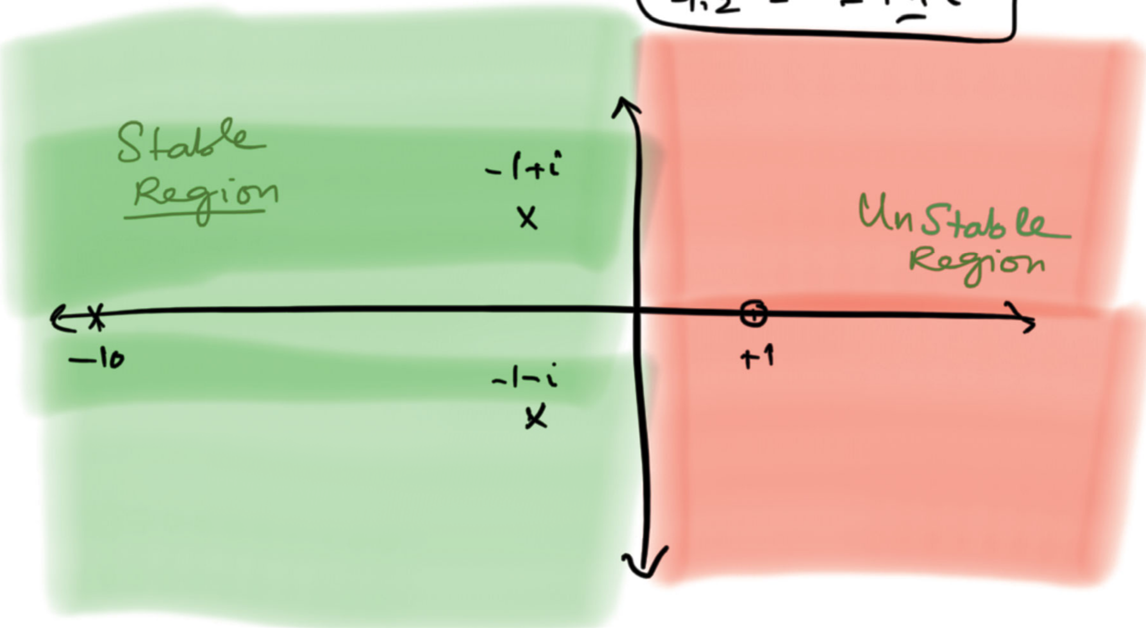
$s+10=0 \quad ; \quad s^2+2s+2=0$

$\boxed{s=-10}$

$$s_{1,2} = \frac{-2 \pm \sqrt{4-8}}{2}$$

$$= \frac{-2 \pm 2i}{2}$$

$$\boxed{s_{1,2} = -1 \pm i}$$



Q no. 6

$$G(s) = \frac{-s+3}{s^2+3s+1}$$

$$C(s) = K_c \quad ; \quad G(s) = \text{bla. bla.}$$

$$\frac{Y(s)}{R(s)} = \frac{GCH}{1+GC}$$

$$= \frac{Gc}{1+GC}$$

$$= \frac{K_c(-s+3)/s^2+3s+1}{1+K_c(-s+3)/s^2+3s+1}$$

$$\frac{Y}{R} = \frac{K_c(3-s)}{s^2+3s+1+K_c(3-s)}$$

<u>closed loop poles</u>

Den: $s^2+3s+1+3K_c-K_cs$

$s^2+(3-K_c)s+(1+3K_c)$

$$s_{1,2} = \frac{(K_c-3) \pm \sqrt{(3-K_c)^2 - 4(1+3K_c)}}{2} \quad \text{——— (I)}$$

Open loop poles

$s^2+3s+1=0$

$-3 \pm \sqrt{9-4}$

$$s_{1,2} = \frac{(K_c-3) \pm \sqrt{(3-K_c)^2 - 4(1+3K_c)}}{2} \quad \text{——— (I)}$$

Open loop poles

$s^2 + 3s + 1 = 0$

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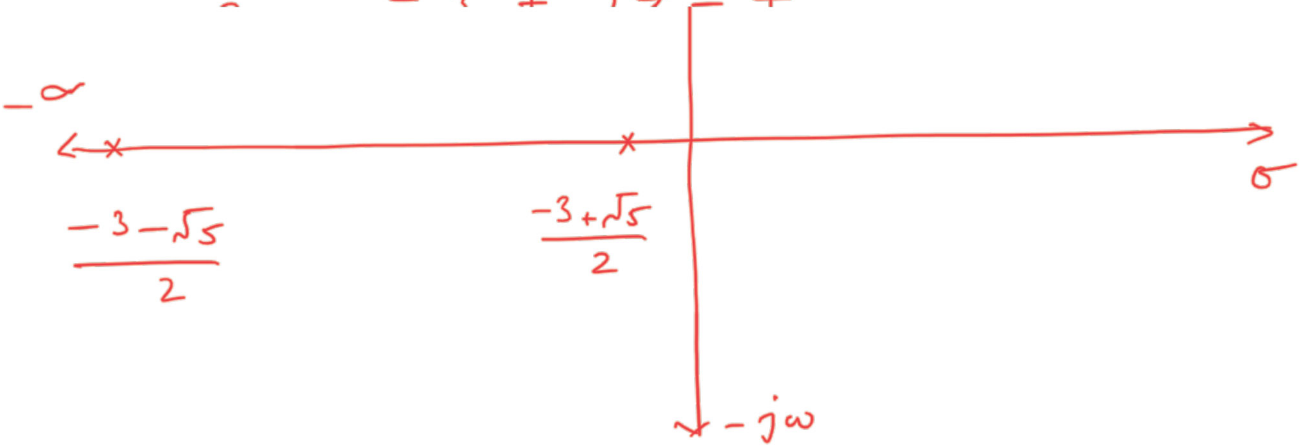


Now we know that root locus is the map of poles when K_c changes.

→ How will the root locus look like in our case?

Well! let's try and figure that out.

let's find out the values of K_c for which the system will have two recurring (on same location) poles.

Separate the real & imaginary parts of (I)

$$s_{1,2} = \frac{K_3 - 3}{2} \pm \frac{\sqrt{(K_c - 3)^2 - 4(1 + 3K_c)}}{2}$$

Assume

$\sqrt{(K_c - 3)^2 - 4(1 + 3K_c)} = 0$

$$\frac{\sqrt{(K_c-3)^2 - 4(1+3K_c)}}{2} = 0$$

$$K_c^2 + 9 - 6K_c - 4 - 12K = 0$$

$$K_c^2 - 18K_c + 5 = 0$$

$$K_{c_{1,2}} = \frac{+18 \pm \sqrt{324 - 20}}{2}$$

$$K_{c_{1,2}} = \frac{18 \pm \sqrt{304}}{2}$$

$$= \frac{18 \pm 2\sqrt{76}}{2}$$

$$K_{c_{1,2}} = 9 \pm \sqrt{76}$$

This tells us that the poles will be recurring for two values of K_c.

Those poles are given by

$$s_{1,2} = \frac{9 + \sqrt{76} - 3}{2}$$

$$s_{1,2} = \frac{6 + \sqrt{76}}{2}$$

$$s_{1,2} = 3 + \sqrt{19}$$

$$s_{1,2} = \frac{9 - \sqrt{76} - 3}{2}$$

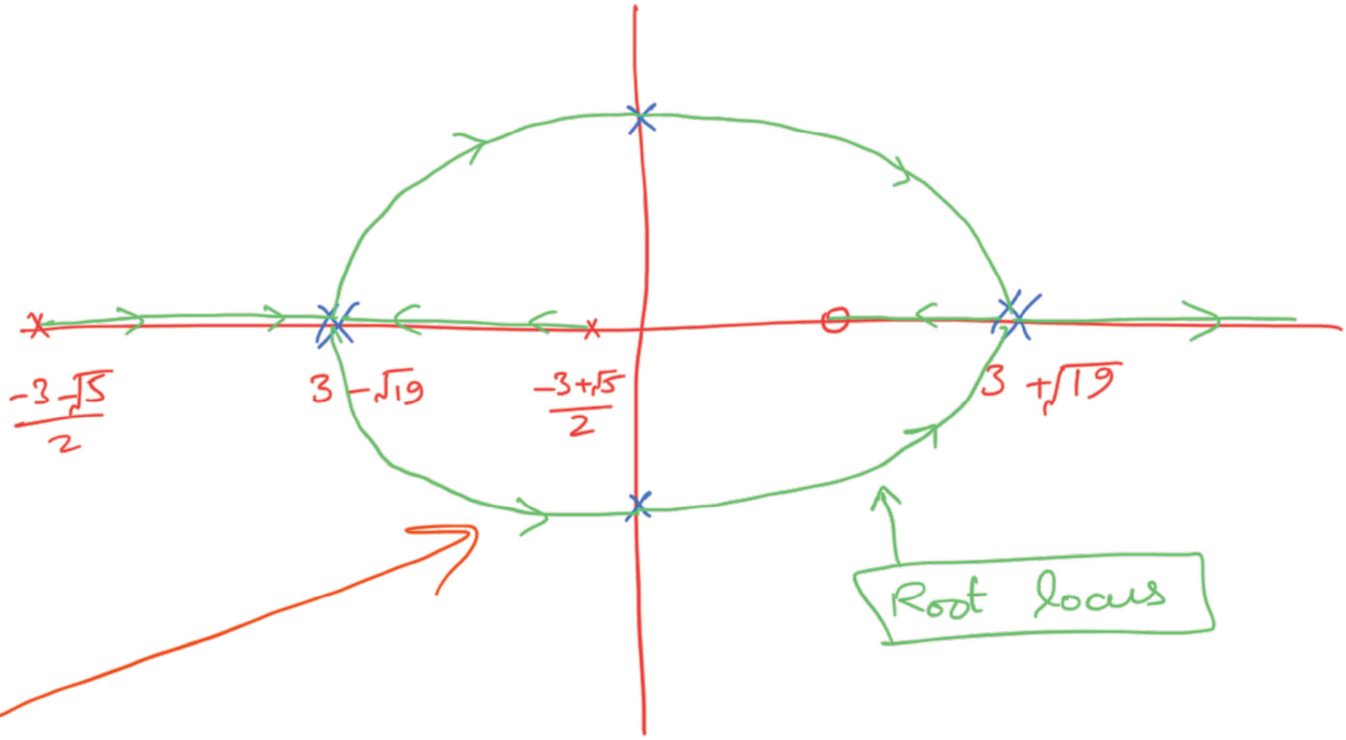
$$s_{1,2} = \frac{6 - \sqrt{76}}{2}$$

$$s_{1,2} = 3 - \sqrt{19}$$

$K_c = 9 + \sqrt{76}$ gives unstable poles

$K_c = 9 - \sqrt{76}$ gives stable poles

Mark these poles on the pole-zero map.



Now

Assume the real part of I equal to zero.

$$\frac{K_c - 3}{2} = 0$$

$$\boxed{K_c = 3}$$

put $K_c = 3$ in I

$$s_{1,2} = \frac{0 \pm \sqrt{0 - 4(1+9)}}{2}$$

$$s_{1,2} = \pm \sqrt{10}\,j$$

Mark these poles on pole-zero map.

we can see that $K_c = 3$ gives a pair of imaginary poles. Any value of $K_c > 3$ will result in unstable poles.

So stable value for K_c is

$$\boxed{0 < K_c \leq 3}$$

In the next part it is required to find the steady state error for $K_c = 10$. However, steady state error can only be found if closed loop system is stable. For $K_c = 10$, the system is not stable.

Q no. 7

$$G(s) = \frac{0.01}{s + 0.1}$$

$$C(s) = \frac{K_c(s+0.1)}{s}$$

→ closed-loop poles:–

$$1 + GC = 0$$

$$1 + \frac{0.01\,K_c\cancel{(s+0.1)}}{s\cancel{(s+0.1)}} = 0$$

$$s + 0.01K_c = 0$$

$$\boxed{s = -0.01K_c}$$

The closed loop pole is given at -0.5

$$-0.01K_c = -0.5$$

$$\boxed{K_c = 50}$$

K_c value has to be set at 50 to give a closed-loop poles at -0.5.

Steady-state error is the error in the controlled variable (output) after a reasonable time has been allowed for the system to settle.

$$S.S.E = \lim_{t \to \infty} \left(r(t) - y(t) \right)$$

$\downarrow$

In laplace domain

$$S.S.E = \lim_{s \to 0} s \left(R(s) - Y(s) \right)$$

$$S.S.E = \lim_{s \to 0} s \left(1 - \frac{Y(s)}{R(s)} \right) R(s)$$

$$S.S.E = \lim_{s \to 0} s \left(1 - \frac{0.01K_c}{s + 0.01K_c} \right) R(s)$$

$$S.S.E = \lim_{s \to 0} s \left(\frac{s}{s + 0.01K_c} \right) R(s)$$

$R(s) = 0.1/s$ (step signal of 0.1)

$$S.S.E = \lim_{s \to 0} s \left(\frac{\cancel{s}}{s + 0.01K_c} \right) \frac{0.1}{\cancel{s}}$$

$$S.S.E = \lim_{s \to 0} \left(\frac{0.1s}{s + 0.01K_c} \right)$$

Operate the limit.

$$S.S.E = \frac{0.1(0)}{0 + 0.01K_c} = 0$$

So, in this system S.S.E will be zero after enough time has passed. The value of K_c has to be positive for this analysis to be valid because otherwise the system will be unstable.

Q#0.8

$$\frac{Y(s)}{R(s)} = \frac{s+K}{s^3+2s^2+4s+K}$$

Routh-Hurwitz criterion is often used to check the stability of the closed loop system & can also be used to design proportional gain.

R-H table for Q#0.8

Start from maximum power of denominator.

s^3	1	4	0
s^2	2	K	0

Populate only the first two rows with

s^1	$\boxed{x}$	$\boxed{0}$
s^0	y	

$$-\frac{1}{\square}\begin{vmatrix} 1 & 4 \\ 2 & K \end{vmatrix}$$

x is given as

$$x = -\frac{1}{2}\begin{vmatrix} 1 & 4 \\ 2 & K \end{vmatrix}$$

$$x = -\frac{1}{2}(K-8) = \frac{8-K}{2}$$

$$y = -\frac{1}{x}\begin{vmatrix} 2 & K \\ x & 0 \end{vmatrix}$$

$$y = \frac{-Kx}{-x} = K$$

The first column of RH table must have no sign inversion in order for the system to be stable.

1st column.

		Sign
s^3	1	+

$8-K > 0$

s^2	2	$+$
s^1	$\frac{8-K}{2}$	$?$
s^0	K	$?$

$$\frac{\quad}{2}$$

$$8-K>0$$

$$8>K$$

$$K>0$$

This gives us a limit on K_c for a stable system.

$$\boxed{0<K<8}$$

Q no. 9

Part 3

$$G(s) = \frac{s+0.1}{(s-3)(s+6)(s+1)}$$

$$C(s) = K$$

closed loop poles

$$1+GC = 0$$

$$1+K\frac{(s+0.1)}{(s-3)(s+6)(s+1)} = 0$$

$$(s-3)(s+6)(s+1) + Ks + 0.1K = 0$$

$$(s-3)(s^2+7s+6) + Ks + 0.1K = 0$$

$$s^3+7s^2+6s-3s^2-21s-18+Ks+0.1K=0$$

$$s^3+4s^2+(K-15)s+0.1K-18=0$$

s^3	1	$K-15$	0
s^2	4	$0.1K-18$	0
	$4K-60-0.1K+18$		0

$$
\begin{array}{c|cc}
s^1 & \frac{\ \cdots\ }{4} & 0 \\
s^0 & 0.1K-18 &
\end{array}
$$

$$4K - 60 - 0.1K + 18 > 0$$

$$3.9K > 42$$

$$\boxed{K > 42/3.9}$$

$$0.1K - 18 > 0$$

$$\boxed{K > 180}$$

This condition is more stringent.

Therefore $K > 180$ in order for the

K