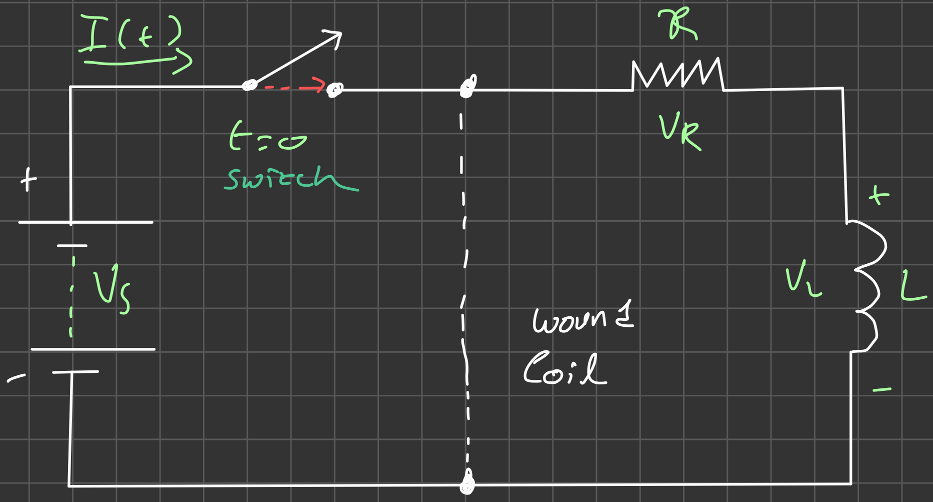
DC Modelling



When electromotive force (emf) is removed from a circuit containing inductance and resistance but no capacitors, and rate of decrease of current is proportional to the current.

If the initial current is 30 A but decays to 11A after 0.01 seconds find the expression for the current

Rate of decrease of current $\frac{dI}{dt} = -kI$

where k is the constant of proportionality.

$$\Rightarrow \quad \frac{dI}{dt} = -kI$$

$$\Rightarrow \frac{dI}{I} = -k\,dt \Rightarrow \int \frac{dI}{I} = \int -k\,dt \Rightarrow \ln|I| = -kt + C$$

$$\Rightarrow \ln|I| = -kt + C$$

$$\Rightarrow \log_e I = -kt + C$$

$$\Rightarrow e^{(\log_e I)} = e^{(-kt+c)}$$

$$\Rightarrow \therefore \; I(t) = e^{-kt+c}$$

Since at $t = 0$, $I(0) = 30$ A

$$\therefore \; I(0) = 30 = e^{-0+c}$$

$$\Rightarrow 30 = e^{c} \Rightarrow e^{c} = 30$$

$$\therefore \; I(t) = e^{-kt} e^{c} = e^{-kt}(30)$$

$$\boxed{\therefore \; I(t) = 30e^{-kt}}$$

Since $I(t) = 30e^{-Rt}$

and $I(0.01) = 11\,A$

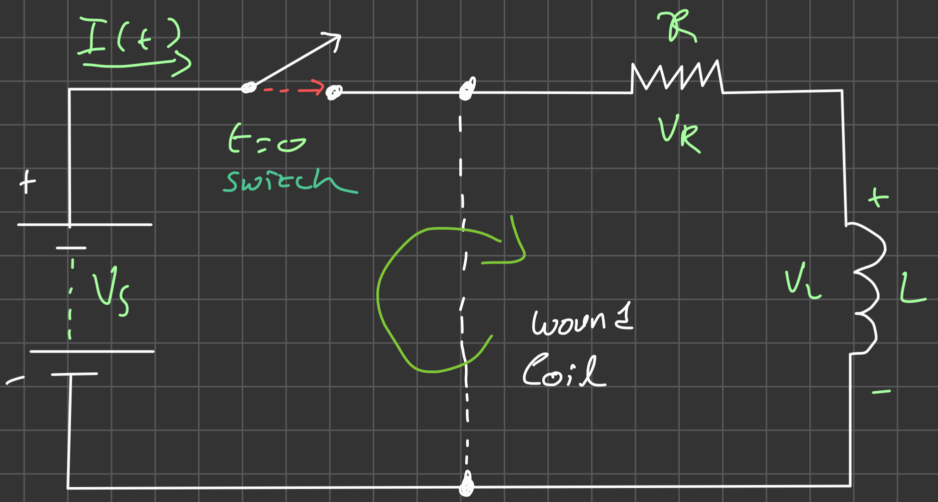
$\therefore \quad I(t) = 30e^{-Rt}$.

$$\begin{aligned}\ln|I(t)| &= \ln|30e^{-Rt}| \\ &= \ln|30| + \ln|e^{-Rt}| \\ &= \ln|30| - Rt(1)\end{aligned}$$

$\therefore \quad Rt = \ln|30| - \ln|I(t)|$

$$\therefore \quad R = \frac{\ln\left|\frac{30}{I(t)}\right|}{t} = \frac{\ln\left|\frac{30}{11}\right)}{0.01} = 100.3302$$

$$\boxed{\therefore \; I(t) = 30e^{-100.3302t}}$$



LR series circuit is connected constant voltage source (battery) and a switch.

Assume that the switch, S_1, is open until it is closed at a time $t=0$, then remains permanently closed producing a "step response" type voltage input.

$$\Sigma V = 0 \Rightarrow -V_s + V_R + V_L = 0 \Rightarrow V_s - (V_R + V_L) = 0$$

$$\Rightarrow V_s - \left\{ IR + L\frac{dI}{dt} \right\} = 0$$

$$\Rightarrow V_s - IR - L\frac{dI}{dt} = 0$$

$$\Rightarrow L\frac{dI}{dt} = V_s - IR$$

$$\therefore \quad \frac{dI}{V_s - IR} = \frac{1}{L}dt \Rightarrow \int \frac{dI}{V_s - IR} = \int \frac{1}{L}dt$$

$$\Rightarrow \frac{1}{\frac{d}{dI}(V_s - IR)} \ln|V_s - IR| = \frac{1}{L}t + C$$

$$= -\frac{1}{R}\ln|V_s - IR| = \frac{t}{L} + C$$

Continue .... $\Longrightarrow$

$$-\frac{1}{R}\ln|V_s - IR| = \frac{t}{L} + C$$

$$\Rightarrow \ln|V_s - IR| = -\frac{R}{L}t + C$$

$$\Rightarrow V_s - IR = e^{(-\frac{R}{L}t + c)}$$

$$\Rightarrow IR = V_s - e^{(-\frac{R}{L}t + c)}$$

$$\Rightarrow \therefore I(t) = \frac{1}{R}\left[V_s - e^{(\frac{R}{L}t + c)}\right]$$

at $t = 0$ switch is opened

$\therefore I(0) = 0$

$$I(0) = 0 = \frac{1}{R}\left[V_s - e^{(0+c)}\right]$$

$\therefore 0 = V_s - e^c \Rightarrow e^c = V_s$

$$\therefore I(t) = \frac{1}{R}\left[V_s - \{e^{-\frac{R}{L}t}\, e^c\}\right]$$

$$= \frac{1}{R}\left[V_s - (e^{-\frac{R}{L}t} \cdot V_s)\right]$$

$$= \frac{1}{R}\left[V_s - V_s e^{-\frac{R}{L}t}\right]$$

$$= \frac{V_s}{R}\left(1 - e^{-\frac{R}{L}t}\right)$$

$$\therefore I(t) = \frac{V_s}{R}\left(1 - e^{-\frac{R}{L}t}\right)$$

In this cause $\frac{dI}{dt} = -r(I - I_f)$