Tutorial 05 ACS Notes

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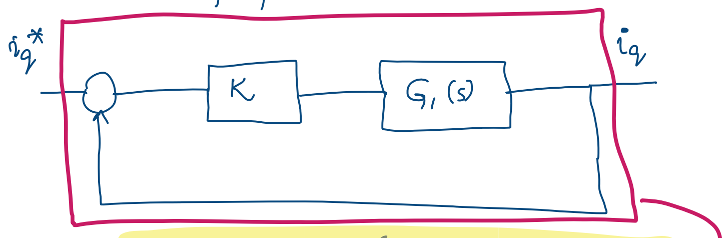
Question Q.1

Design a cascade Control structure P + PI for the system given below

$$G_1(s) = \frac{1}{s+1} \quad ; \quad G_2(s) = \frac{0.1}{s+0.1}$$

In a cascade system, we design the inner loop first.



Block diagram for inner loop only

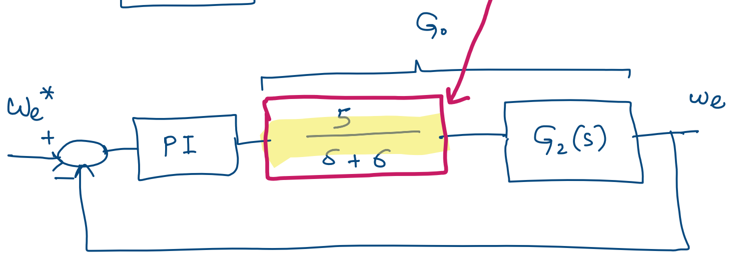
$$1 + KG = 0$$

$$\Rightarrow \quad 1 + \frac{K}{s+1} = 0$$

$$s + 1 + K = 0$$

$$1 + K = 6$$

$$\boxed{K = 5}$$



$$G_o(s) = \left(\frac{5}{s+6}\right)\left(\frac{0.1}{s+0.1}\right)$$

$$G_o(s) = \frac{0.5}{(s+6)(s+0.1)}$$

Reduce to first-order

$$G_o(s) \simeq \frac{0.5/6}{s+0.1}$$

$$1 + CG$$

$$1 + \left(\frac{C_1 s + C_0}{s}\right)\left(\frac{0.5/6}{s+0.1}\right) = 0$$

$$s^2 + 0.1s + \frac{0.5}{6}C_1 s + \frac{0.5}{6}C_0 = 0$$

$$s^2 + \left(0.1 + \frac{0.5}{6}C_1\right)s + \frac{0.5}{6}C_0 = 0$$

$$(s+\lambda)^2 = 0 \quad \text{Compare with}$$

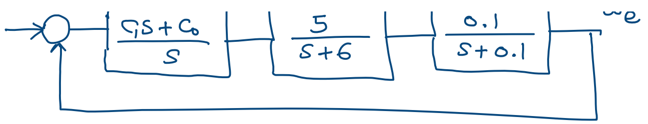
$$0.1 + \frac{0.5}{6}C_1 = 2\lambda \qquad \Bigg| \qquad \frac{0.5}{6}C_0 = \lambda^2$$

$$\Rightarrow C_1 = \left(\frac{2\lambda - 0.1}{0.5}\right)6 \qquad \Bigg| \qquad \boxed{C_0 = 12\lambda^2}$$

$$\boxed{C_1 = 12(2\lambda - 0.1)}$$

we haven't been given a value of "λ" so, we just write the equations for now.

Q 0.2



$$\frac{\omega_e}{\omega_e^*} = \frac{Y(s)}{R(s)} \leftarrow \text{In conventional terms}$$

closed-loop polynomial for the whole system

$$\Rightarrow 1 + C(s)G_o(s)$$

$$1 + \left(\frac{c_1 s + c_0}{s}\right)\left(\frac{0.5}{(s+6)(s+0.1)}\right) = 0$$

$$s(s+6)(s+0.1) + 0.5c_1 s + 0.5c_0 = 0$$

$$\boxed{1}s^3 + \boxed{6.1}s^2 + \boxed{(0.6 + 0.5c_1)}s + \boxed{0.5c_0} = 0$$

Construct a RH-table

s^3	1	$0.6+0.5c_1$	0
s^2	6.1	$0.5c_0$	0
s^1	x	0	0
s^0	y	0	0

$$x = -\frac{1}{6.1}\begin{vmatrix} 1 & 0.6+0.5c_1 \\ 6.1 & 0.5c_0 \end{vmatrix}$$

$$x = -\frac{1}{6.1}\left(0.5c_0 - 6.1(0.6+0.5c_1)\right)$$

$$x = 0.6 + 0.5c_1 - \frac{0.5}{6.1}c_0$$

To avoid an unstable pole, $x > 0$

$$x = 0.6 + 6(2\lambda - 0.1) - \frac{6\lambda^2}{6.1}$$

$$\cancel{0.6} + 12\lambda \cancel{-0.6} - \frac{6\lambda^2}{6.1} > 0$$

$$\left(12 - \frac{6\lambda}{6.1}\right)\lambda > 0$$

$\lambda > 0$; $12 - \frac{6\lambda}{6.1} > 0$

$$12 > \frac{6\lambda}{6.1}$$

$$12.2 > \lambda$$

$$\lambda < 12.2$$

$\boxed{0 < \lambda < 12.2}$ Condition from $x > 0$

$$y = -\frac{1}{x}\begin{vmatrix} 6.1 & 0.5c_0 \\ x & 0 \end{vmatrix}$$

$$y = 0.5c_0$$

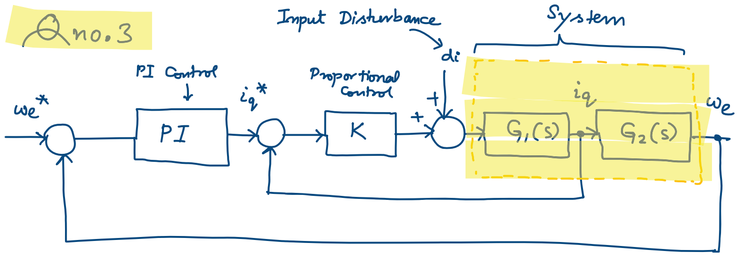
$$0.5 \times 12\lambda^2 > 0$$

$$6\lambda^2 > 0$$

$\boxed{\lambda > 0} \leftarrow$ condition from $y > 0$

Overall, λ must be between 0 & 12.2 in order for the system to be stable.

Q no.3



$$i_q = G_1\left(d_i + K(i_q^* - i_q)\right)$$

Multiply both sides with "G_2"

$$\underbrace{G_2 i_q}_{w_e} = G_1 G_2\left(d_i + K(i_q^* - i_q)\right)$$

$$w_e = G_1 G_2 d_i + K G_1 G_2 i_q^* - \boxed{K G_1 w_e}$$

$$w_e\left[1 + K G_1\right] = G_1 G_2 d_i + K G_1 G_2 i_q^*$$

$$w_e = \frac{G_1 G_2}{1 + K G_1} d_i + \frac{K G_1 G_2}{1 + K G_1} i_q^*$$

$$i_q^* = C(w_e^* - w_e)$$

$$w_e = \frac{G_1 G_2}{1 + K G_1} d_i + \frac{K G_1 G_2 C}{1 + K G_1} w_e^* - \boxed{\frac{K G_1 G_2 C w_e}{1 + K G_1}}$$

$$w_e\left[1 + \frac{K G_1 G_2 C}{1 + K G_1}\right] = \frac{G_1 G_2}{1 + K G_1} d_i + \frac{K G_1 G_2 C w_e^*}{1 + K G_1}$$

$$w_e\left[\frac{1 + K G_1 + K G_1 G_2 C}{1 + K G_2}\right] = \frac{G_1 G_2}{1 + K G_2} d_i + \frac{K G_1 G_2 C \; w_e^*}{1 + K G_2}$$

set all sources to zero except "d_i"

$$\frac{w_e}{d_i} = \frac{G_1 G_2}{1 + K G_1 + K G_1 G_2 C}$$

$$\frac{w_e}{d_i} = \frac{\left(\frac{1}{s+1}\right)\left(\frac{0.1}{s+0.1}\right)}{1 + \frac{5}{s+1} + \frac{5}{s+1} \times \frac{0.1}{s+0.1} \times \frac{C_1 s + C_0}{s}}$$

$$\frac{w_e}{d_i} = \frac{0.1 s}{s(s+0.1)(s+1) + 5s(s+0.1) + 0.5C_1 s + 0.5C_0}$$

$$\boxed{\frac{w_e}{d_i} = \frac{0.1 s}{s^3 + 6.5 s^2 + s(0.6 + 0.5C_1) + 0.5C_0}}$$

Apply final value theorem

$$\underbrace{\lim_{s \to 0} s\, w_e(s)} = 0 \quad \text{due to disturbance}$$

Consider LHS

$$\lim_{s \to 0} s \frac{w_e(s)}{d_i(s)} \times d_i(s)$$

$$\lim_{s \to 0} s\left(\frac{0.1 \cancel{s}}{s^3 + \square s^2 + \square s + \square}\right) \times \frac{1}{\cancel{s}}$$

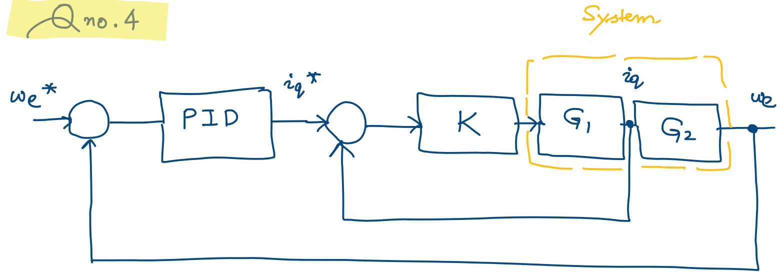
Apply limit $s = 0$

$$0\left(\square\right) = 0$$

so a unit step disturbance

will be rejected successfully.

Q no. 4



$$G_1 = \frac{-s+10}{(s+10)(s+3)} \quad \Bigg| \quad G_2 = \frac{0.1}{s(s+2)}$$

Consider inner loop only

$$1 + KG_1 = 0$$

$$1 - \frac{(Ks - 10K)}{(s+10)(s+3)} = 0$$

$$(s+10)(s+3) - Ks + 10K = 0$$

$$s^2 + 13s + 30 - Ks + 10K = 0$$

Find "K" such that the closed loop system has identical real poles.

$$s^2 + (13-K)s + 30 + 10K = 0$$

$$s_1, s_2 = \frac{-(13-K) \pm \sqrt{(13-K)^2 - 4(30+10K)}}{2}$$

this should be zero to give a pair of identical real poles.

So,

$$(13-K)^2 - 4(30+10K) = 0$$

$$169 + K^2 - 26K - 120 - 40K = 0$$

$$K^2 - 66K + 49 = 0$$

find "K"

$$K_1, K_2 = \frac{+66 \pm \sqrt{66^2 - 4(49)}}{2}$$

$$K_1, K_2 = \frac{66 \pm 64.5}{2}$$

$K_1 = 65.25$; $K_2 = 0.75$

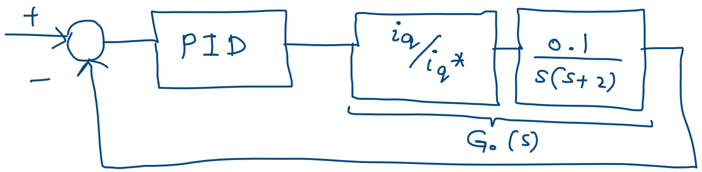
Putting $K = K_1$ gives unstable poles so we will take $K = K_2$

$$s_1, s_2 = \frac{K-13}{2} = \frac{0.75-13}{2} = \frac{-12.25}{2}$$

$$\boxed{s_1, s_2 = -6.125}$$

So, K for inner loop will be taken as 0.75

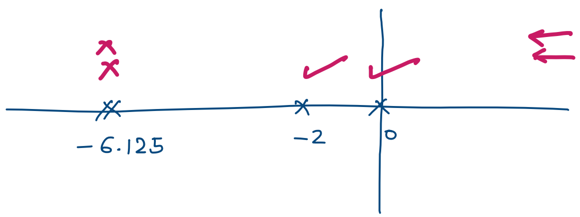
$$\frac{i_q}{i_q^*} = \frac{K G_1}{1 + K G_1} = \frac{0.75(-s+10)}{(s+6.125)^2}$$



$$G_o = \frac{0.75(-s+10)}{(s+6.125)^2} \times \frac{0.1}{s(s+2)}$$

$$G_o = \frac{0.075(-s+10)}{s(s+2)(s+6.125)(s+6.125)}$$

for PID, we want a second-order system



$$G_o = \frac{0.075(-s+10)}{s(s+2)(6.125)^2}$$

$$G_o(s) = \frac{\overset{\alpha}{\boxed{\frac{0.075}{(6.125)^2}}}(-s+10)}{s(s+2)}$$

$$G_o(s) = \frac{\alpha(-s+10)}{\ldots}$$

$s(s+2)$

$$C(s) = \frac{c_2(s+\gamma_1)(s+\gamma_2)}{s}$$

Use pole-zero cancellation

$$C(s) = \frac{c_2(s+\gamma_1)(s+2)}{s}$$

$$1 + CG_o = 0$$

$$1 + \frac{\alpha c_2(s+\gamma_1)(-s+10)}{s^2} = 0$$

$$s^2 + \alpha c_2(s+\gamma_1)(-s+10) = 0$$

$$s^2 + \alpha c_2(-s^2 + (10-\gamma_1)s + 10\gamma_1) = 0$$

$$s^2[1-\alpha c_2] + \alpha c_2(10-\gamma_1)s + 10\gamma_1\alpha c_2 = 0$$

$$s^2 + \frac{\alpha c_2(10-\gamma_1)s}{1-\alpha c_2} + \frac{10\gamma_1\alpha c_2}{1-\alpha c_2} = 0$$

$$(s+\lambda)^2 \Rightarrow (s+1)^2 = s^2 + 2s + 1$$

$$\frac{\alpha c_2(10-\gamma_1)}{1-\alpha c_2} = 2 \qquad \Bigg| \qquad \frac{10\gamma_1\alpha c_2}{1-\alpha c_2} = 1$$

$$10\gamma_1\alpha c_2 = 1 - \alpha c_2$$

$$\alpha c_2(10-\gamma_1) = 20\gamma_1\alpha c_2$$

$$10\alpha C_2 - r_1\alpha C_2 = 20 r_1 \alpha C_2$$

$$10\,\cancel{\alpha}\,\cancel{C_2} = 21\, r_1 \cancel{\alpha}\,\cancel{C_2}$$

$$\boxed{r_1 = 10/21}$$

$$10 r_1 \alpha C_2 = 1 - \alpha C_2$$

$$C_2\left[10 r_1 \alpha + \alpha\right] = 1$$

$$C_2 = \frac{1}{\alpha\left[10 r_1 + 1\right]}$$

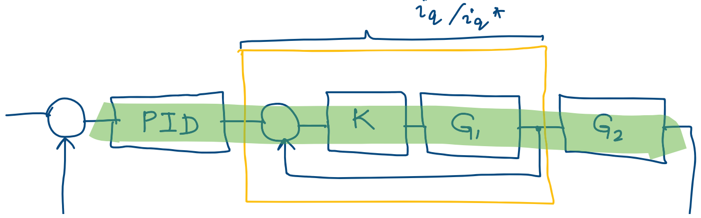
$$C_2 = \frac{1}{\alpha\left[\frac{100}{21} + 1\right]}$$

$$\boxed{C_2 = \frac{21}{121\alpha}}$$

$$\boxed{C(s) = \frac{\frac{21}{121\alpha}\left(s + \frac{10}{21}\right)(s+2)}{s}}$$

Nyquist plot

find forward path gain



The multiplication of the green highlighted transfer functions is forward path gain

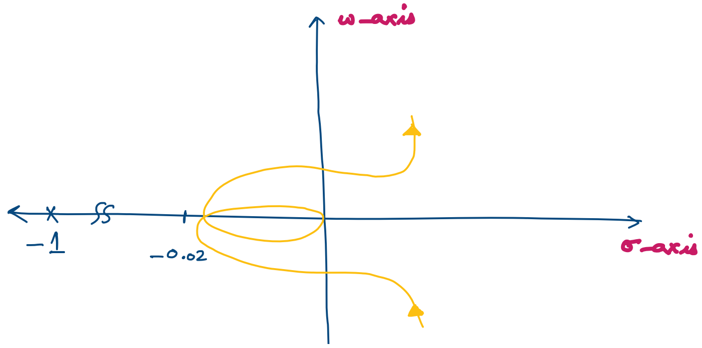
$$F(s) = \frac{\frac{21}{121\alpha}\left(s+10/21\right)(s+2)}{s} \times \frac{0.75(-s+10)}{(s+6.125)^2} \times \frac{0.1}{s(s+2)}$$

put $s = j\omega$ in F

$$F(j\omega) = \sigma + j\omega$$

where

σ = real part of $F(j\omega)$ & ω = imaginary part to $F(j\omega)$



if the curve does not encircle (-1,0) point, the system is stable!

Question no. 5

$$G_1(s) = \frac{1}{s+1} \quad ; \quad G_2 = \frac{0.1}{s+0.1}$$

PI for inner loop & PI for outer loop

$$1 + C_1 G_1 = 0$$

$$1 + \left(\frac{C_1 s + C_0}{s}\right)\left(\frac{1}{s+1}\right) = 0$$

$$s(s+1) + C_1 s + C_0 = 0$$

$$s^2 + (1 + C_1)s + C_0 = 0$$

$$(s+6)^2 = s^2 + 12s + 36$$ place poles at -6

$$1 + C_1 = 12$$

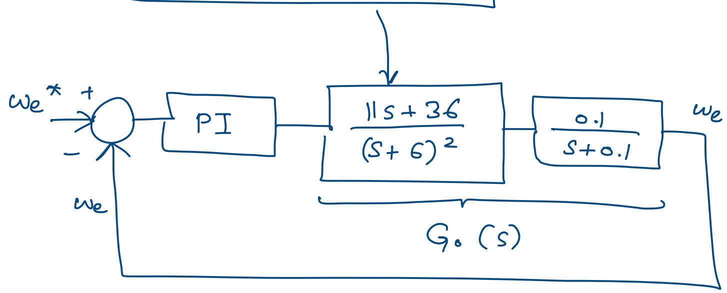
$$\boxed{C_1 = 11}$$

$$\boxed{C_0 = 36}$$

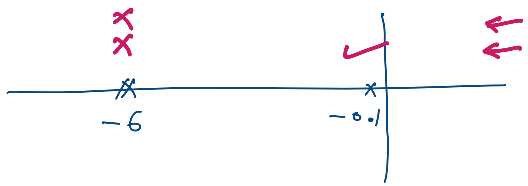
$$\frac{i_q}{\;} = \frac{C_1 G_1}{\;}$$

$$i_q^* \qquad 1 + C_1 G_1$$

$$\boxed{\frac{i_q}{i_q^*} = \frac{11s + 36}{(s+6)^2}}$$



$$G_o(s) = \frac{11s+36}{(s+6)^2} \times \frac{0.1}{s+0.1}$$



$$G_o(s) \simeq \frac{(11s+36)\,0.1}{6^2\,(s+0.1)}$$

$$G_o(s) \simeq \frac{\alpha\,(11s+36)}{s+0.1} \qquad \boxed{\alpha = 0.1/6^2}$$

$$C_2 = \frac{C_1 s + C_0}{s}$$

$$1 + C_2 G_o = 0$$

$$1 + \alpha(C_1 s + C_0)(11s + 36)$$

$$\frac{\qquad}{s(s+0.1)} = 0$$

$$s^2 + 0.1s + \alpha(c_1 s + c_0)(11s + 36) = 0$$

$$s^2 + 0.1s + \alpha(11c_1 s^2 + (11c_0 + 36c_1)s + 36c_0) = 0$$

$$s^2 + 0.1s + 11c_1\alpha s^2 + \alpha(11c_0 + 36c_1)s + 36\alpha c_0 = 0$$

$$s^2 + \frac{0.1 + \alpha(11c_0 + 36c_1)}{1 + 11c_1\alpha} + \frac{36\alpha c_0}{1 + 11c_1\alpha} = 0$$

place poles at -0.6

$$(s + 0.6)^2 = s^2 + 1.2s - 0.036$$

Compare coeffs & solve for c_1 & c_0