Tutorial 04 Notes by (Junaid Saeed)

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Background

$$G(s) = \frac{b}{s+a}$$

— PI Control (Classical Approach)

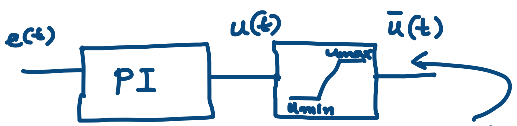
$$C(s) = \frac{c_1 s + c_0}{s}$$

OR

$$C(s) = K_c + \frac{K_c}{T_i s}$$

✓ — Easy to design
✓ — Easy to implement
✗ — A bit of additional work is required to implement "anti wind up".

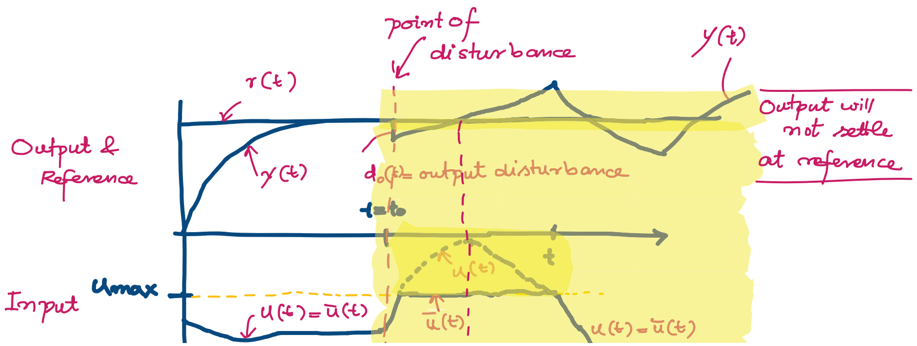
What is wind up??

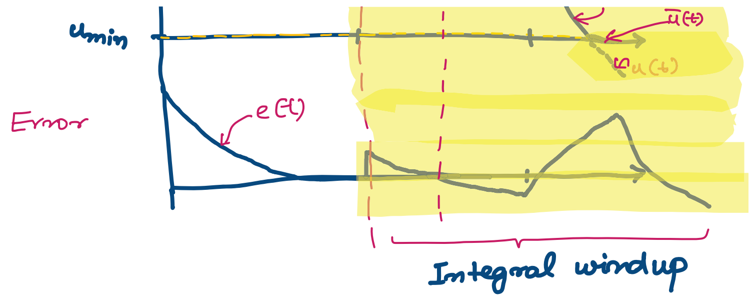


Is this anti wind up??

— NO!! —

(Graphical Analysis)





Anti windup technique is where integration of error is stopped when u(t) hits umax or umin.

$$u(t) = K_c\, e(t) + \frac{K_c}{T_i}\int e(t)\, dt$$

if $u(t) > u_{max}$ or $u(t) < u_{min}$

$$u(t) = K_c\, e(t)$$

Disturbance Observer Control is a modern solution to this problem. It comes with embedded anti windup structure. Moreover, disturbance observer control is not limited to the class of PID controllers. You can change it to resonant controller by merely changing the nature of the disturbance.

If the so-called disturbance is constant (nature of error in the system), the controller will emulate the behaviour of a PI or PID control.

If this disturbance is sinusoidal, The disturbance observer controller will emulate the behaviour of a resonant controller.

So, This control is more like a plug and play type of approach.

$$G(s) = \frac{b}{s+a}$$

$$\frac{Y(s)}{U(s)} = \frac{b}{s+a}$$

$$Y(s)[s+a] = b\,U(s)$$

$$sY(s) + aY(s) = bU(s)$$

Inverse Laplace-transform

$$\dot{y}(t) + a y(t) = b u(t)$$

$$\dot{y}(t) = -a y(t) + b u(t)$$

We assume That $u(t)$ is corrupted with a disturbance "$d(t)$" such that

$$\dot{y}(t) = -a y(t) + b\,\underbrace{(u(t) + d(t))}_{\tilde{u}(t)} \quad \text{——} \quad (\mathrm{I})$$

Now the corrupted signal $\tilde{u}(t)$ is our new control input.

$$\dot{y}(t) = -a y(t) + b\tilde{u}(t) \quad \text{——} \quad (\mathrm{II})$$

A simple proportional control

$$\tilde{u}(t) = \boxed{K_1}(-y(t))$$

$$u(t) = -\boxed{K_1}\,y(t)$$

We will discuss later about how to design this!

Note that the proportional controller has given a corrupted control signal $\tilde{u}(t)$ instead of the actual signal $u(t)$.

Using (I)

$$u(t) = -K_1 y(t) \underbrace{- d(t)}_{\text{unknown disturbance}} \quad \text{——} \quad (\mathrm{III})$$

→ What we know

Nature of disturbance

$$d(t) = \text{constt.}$$

$$\dot{d}(t) = 0 \quad \leftarrow \text{derivative of constt.}$$

from (I)

$$b\,d(t) = \dot{y}(t) + a y(t) - b u(t) \quad \text{——} \quad \mathrm{IV}$$

→ what we don't know

the value of disturbance!

We have to have a value of $d(t)$ so we can find out the actual "$u(t)$" using III.

We can't measure $d(t)$ because it is not a physical quantity. Therefore, we estimate it.

Estimator

Estimator

An estimator/observer is an algorithm that is used to estimate a quantity that we can't or do not want to measure.

Estimators are generally used in state feedback, LQR & MPC control techniques.

let $\hat{d}(t)$ be the estimate of the disturbance.

Recall, $bd(t) = \dot{y}(t) + ay(t) - bu(t)$

and

$$\text{V} \quad \dot{d}(t) = 0 \Rightarrow b\dot{d}(t) = 0$$

→ Observer Equation

$$\frac{d}{dt}(\text{Quantity to be observed}) = \text{original equation} + \text{correction term proportional to estimate error.}$$

$$\frac{d}{dt}\hat{d}(t) = 0 + K_2\big(\overbrace{bd(t) - b\hat{d}(t)}^{\text{Estimation Error}}\big) \quad \text{VI}$$

$$\dot{\hat{d}}(t) = K_2(\dot{y} + ay - bu - b\hat{d}(t))$$

$$\epsilon(t) = d(t) - \hat{d}(t)$$

$$\dot{\epsilon}(t) = \underset{\text{from V}}{\dot{d}(t)} - \underset{\text{from VI}}{\dot{\hat{d}}(t)}$$

$$\dot{\epsilon}(t) = -K_2 b\,\epsilon(t)$$

Laplace

$$s\epsilon(s) - \epsilon(0) = -K_2 b\,\epsilon(s)$$

$$\epsilon(s)[s + K_2 b] = \epsilon(0)$$

$$\epsilon(s) = \frac{\epsilon(0)}{s + K_2 b} \quad \text{VII}$$

Inverse Laplace

$$\epsilon(t) = \epsilon(0)\, e^{-K_2 b t}$$

Note the negative exponential
This will decay to zero for $K_2 b > 0$.

This means that over time (as $t \to \infty$) $\epsilon(t)$ (i.e. the estimation error) will converge to zero.

Also note from VII that the pole of the observer is "$-K_2 b$". This means if you are given a value on the observer pole, you can find "K_2" very easily.

→ How to design proportional gain K_1?

Consider II

$$\dot{y} = -ay + b\tilde{u}$$

&

$$u = -K_1 y$$

$$\Rightarrow \dot{y} = -ay - bK_1 y$$

$$\dot{y} = -(a + bK_1)y$$

Laplace: $$sY(s) - y(0) = -(a + bK_1)Y(s)$$

$$Y(s)\left[s + (a + bK_1)\right] = y(0)$$

$$Y(s) = \frac{y(0)}{s + (a + bK_1)}$$

Inverse laplace: $$y(t) = y(0)\, e^{-(a+bK_1)t}$$

As long as $(a + bK_1) > 0$, the $y(t)$ will go to its reference (zero).

Note the pole of $Y(s)$ is $-(a + bK_1)$. If you are given the value of the pole for proportional control, you can easily find "K_1".

Implementation

In order to be practically realizable, the estimator equation ideally, should not have a derivative of a physical measurement on RHS.

However, VII has $\dot{y}(t)$ on RHS. So, we need to work around this term

$$\dot{\hat{d}} = K_2(\dot{y} + ay - bu - b\hat{d})$$

$$\dot{\hat{d}} = K_2\dot{y} + aK_2 y - bK_2 u - bK_2\hat{d}$$

$$\Rightarrow \dot{\hat{d}} - K_2\dot{y} = aK_2 y - bK_2 u - bK_2\hat{d}$$

Add & substract $bK_2^2 y$ on RHS

$$\Rightarrow \dot{\hat{d}} - K_2\dot{y} = aK_2 y - bK_2 u - bK_2\hat{d} + bK_2^2 y - bK_2^2 y$$

$$\Rightarrow \dot{\hat{d}} - K_2\dot{y} = aK_2 y - bK_2 u - bK_2(\hat{d} - K_2 y) - bK_2^2 y$$

let $\hat{z} = \hat{d} - K_2 y$

$\dot{\hat{z}} = \dot{\hat{d}} - K_2\dot{y}$

IX — $$\dot{\hat{z}} = -bK_2\hat{z} - K_2(bK_2 - a)y - bK_2 u$$

Note that this term does not have a derivative on right hand side and hence is practically realizable.

Lets take
$$\dot{\hat{z}} = \frac{\hat{z}(k+1) - \hat{z}(k)}{\Delta t}$$

put in IX

$$\hat{z}(k+1) = \hat{z}(k) + \Delta t\left(-bK_2\hat{z}(k) - K_2(bK_2 - a)y(k) - bK_2 u(k)\right)$$

$\boxed{\text{Algorithm}}$

① Calculate $\hat{d}(k)$

① Calculate $u(k)$

$$\hat{d}(k) = \hat{z}(k) - K_2(r(k) - y(k))$$

②

$$u(k) = -K_1(r(k) - y(k)) - \hat{d}(k)$$

③

limit "u" between u_{min} & u_{max}
(Anti windup)

④ $\hat{z}(k+1)$

$$= \hat{z}(k) + \Delta t\left(-bK_2\hat{z}(k) - K_2(bK_2 - a)(y(k) - r(k)) - bK_2 u(k)\right)$$

⑤ Go to step-1 (In step-1 $\hat{z}(k+1)$ becomes $\hat{z}(k)$)

Question 0.1 (1)

$$G(s) = \frac{e^{-0.01s}}{(s+1)(s+10)}$$

neglect delay $\quad e^{-0.01s} \simeq 1$

$$G_A(s) = \frac{1}{(s+1)(\cancel{s}^{=0}+10)}$$

$$G_A(s) = \frac{0.1}{s+1}$$

Most dominant pole "-1"

—(Controller Design)—

$$-(a + bK_1) = -2a \qquad (a = 1 \;\&\; b = 0.1)$$

$$-(1 + 0.1K_1) = -2$$

$$1 + 0.1K_1 = 2$$

$$0.1K_1 = 1$$

$$\boxed{K_1 = 10}$$

—(Observer Design)—

$$-K_2 b = -3a$$

$$K_2 b = 3a$$

$$K_2 = \frac{3}{0.1} = 30$$

$$\boxed{K_2 = 30}$$

Question 0.1 (2)

$$G(s) = \frac{2e^{-0.5s}}{(s+0.1)(s+10)^2}$$

$$e^{-0.5s} \simeq 1$$

Most dominant pole $\longrightarrow -0.1$

least " " $\longrightarrow -10, -10$

$$G_A(s) = \frac{2(1)}{(s+0.1)(\cancel{s}^{=0}+10)^2}$$

$$\boxed{G_A(s) = \frac{\overbrace{0.02}^{\to b}}{s + \underbrace{0.1}_{\to a}}}$$

—(Controller)—

$$-(a + bK_1) = -2a$$

$$a + bK_1 = 2a$$

$$K_1 = \frac{2a - a}{b}$$

$$K_1 = a/b = \frac{0.1}{0.02} = 5$$

$$\boxed{K_1 = 5}$$

— (Observer) —

$$-bK_2 = -3a$$

$$bK_2 = 3a$$

$$K_2 = \frac{3a}{b}$$

$$K_2 = \frac{3(0.1)}{0.2} = 15$$

$$\boxed{K_2 = 15}$$

Question 0.1(3)

$$G(s) = \frac{0.5e^{-2s}}{(s+0.01)(s+1)^3}$$

$$e^{-2s} \simeq 1$$

least dominant pole(s) : $-1, -1, -1$
most " " : -0.01

$$G_A(s) = \frac{0.5 \rightarrow b}{(s+0.01) \rightarrow a}$$

Q 0.2(1)

$$G(s) = \frac{2e^{-0.01s}}{(s \ldots 1)(s \ldots 1)}$$

$$\boxed{e^{-0.01s} \simeq 1}$$

$$(s-1)(s+1)$$

$$G_A(s) = \frac{2}{s^2 - 1}$$

$$G_A(s) = \frac{b}{s^2 + a_1 s + a_0}$$

$$b = 2 \; ; \; a_1 = 0 \; ; \; a_0 = -1$$

— (Controller) —

$$K_1 = \frac{\omega_n^2 - a_0}{b}$$

$$K_1 = \frac{9+1}{2}$$

$\omega_n = 3$
$\zeta = 0.707$

$$\boxed{K_1 = 5}$$

$$K_2 = \frac{2\zeta\omega_n - a_1}{b}$$

$$K_2 = \frac{2(0.707)(3) - 0}{2}$$

$$\boxed{K_2 = 2.121}$$

— (Observer) —

$$K_3 = \frac{\alpha_3}{b}$$

$$K_3 = \frac{4}{2} = 2$$

$$\boxed{K_3 = 2}$$

$$T_D = \frac{K_2}{K_1}$$

$$T_D = \ldots \times T = 2.121 \times 0.1$$

$$T_f = 0.1 \times T_D = \frac{\ldots}{5}$$

$$\boxed{T_f = 0.04242}$$

(Algorithm for disturbance observer PID)

① First, calculate filtered derivative of the output "y_{df}"

$$y_{df}(k) = \frac{T_f}{T_f + \Delta t}\, y_{df}(k-1) + \left(\frac{y(k) - y(k-1)}{T_f + \Delta t} \right)$$

② Calculate $\hat{d}(k)$

$$\hat{d}(k) = \hat{z}(k) + K_3\, y_{df}(k)$$

③ Calculate $u(k)$

$$u(k) = -K_1\big(y(k) - r(k)\big) - K_2\, y_{df}(k) - \hat{d}(k)$$

④ Limit $u(k)$ between u_{min} & u_{max}

⑤

$$\hat{z}(k+1) = \hat{z}(k) + \Delta t\Big(-\alpha_3 \hat{z}(k) + K_3\,(a_1 - \alpha_3)\, y_{df}(k) + K_3 a_0\big(y(k) - r(k)\big) - \alpha_3\, u(k)\Big)$$

⑥ Goto step-1

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