RMIT University

OENG-1116: MODELLING & SIMUALTION OF ENGINEERING SYSTEMS

Week 6

Plotting Surfaces with MATLAB. Axial Response of the Rod, using FEM.

Tutorial-3

MATLAB:

PLOTTING 3D SURFACES

(“meshgrid” and “surf”)

MATLAB:

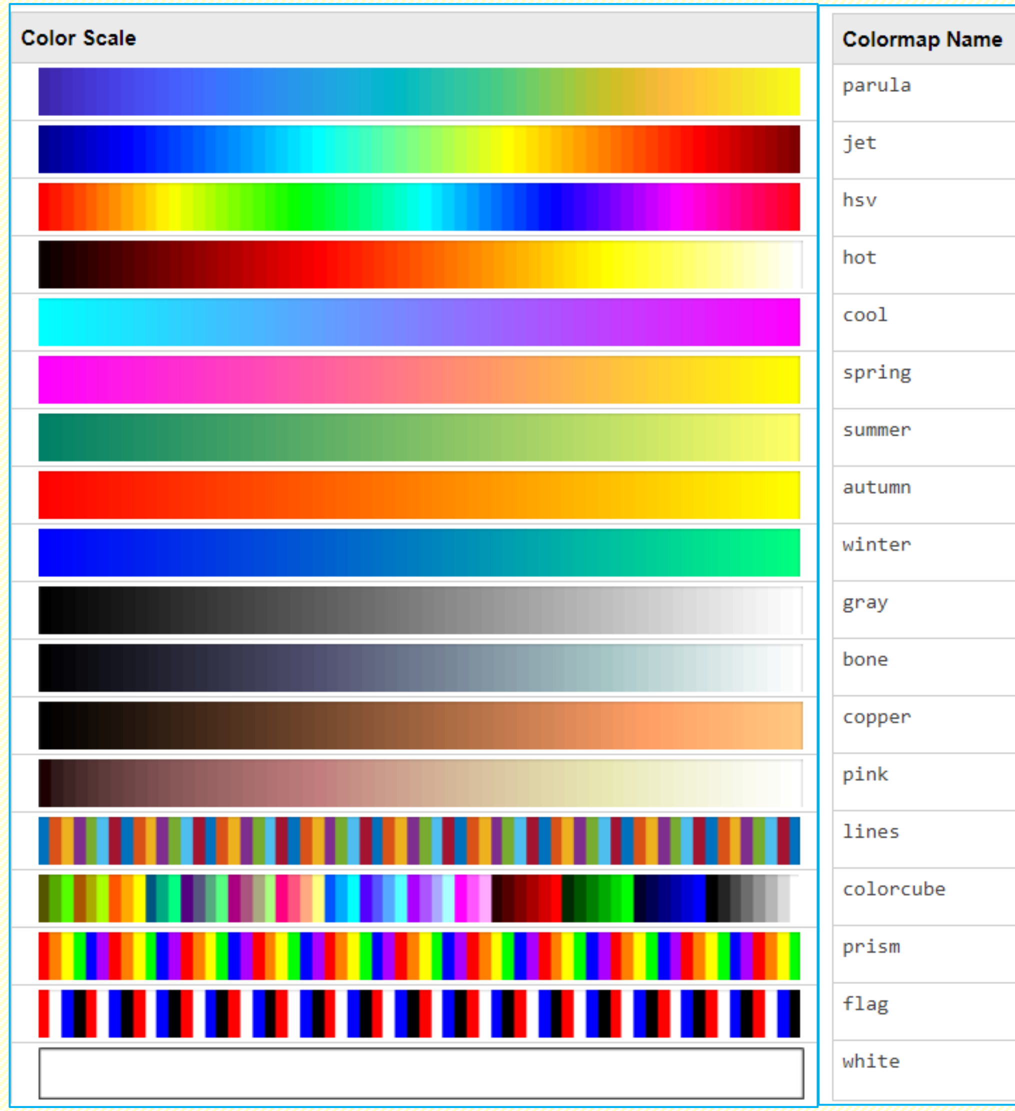
Understanding Colormaps (examples of the same plot, presented with different colormaps)

MATLAB: BUILT-IN COLORMAPS

What Is a Colormap?

A colormap is matrix of values between 0 and 1 that define the colors for graphics objects such as surface, image, and patch objects. MATLAB® draws the objects by mapping data values to colors in the colormap.

Colormaps can be any length, but must be three columns wide. Each row in the matrix defines one color using an RGB triplet. An RGB triplet is a three-element row vector whose elements specify the intensities of the red, green, and blue components of the color. The intensities must be in the range [0, 1]. A value of 0 indicates no color and a value of 1 indicates full intensity.



Curtesy: https://au.mathworks.com/help/matlab/ref/colormap.html

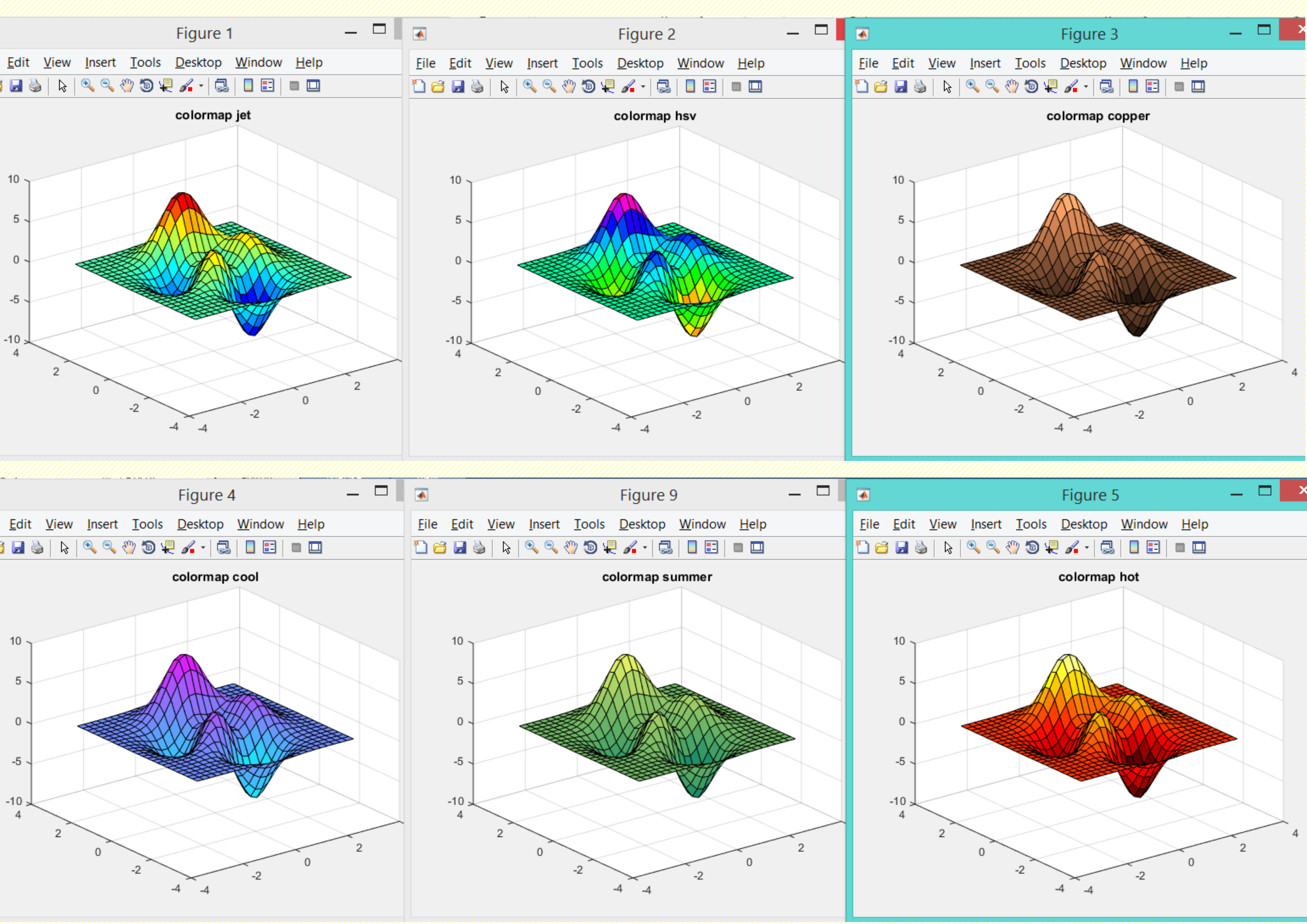


Figure 1
colormap jet
Figure 2
colormap hsv
Figure 3
colormap copper
Figure 4
colormap cool
Figure 9
colormap summer
Figure 5
colormap hot

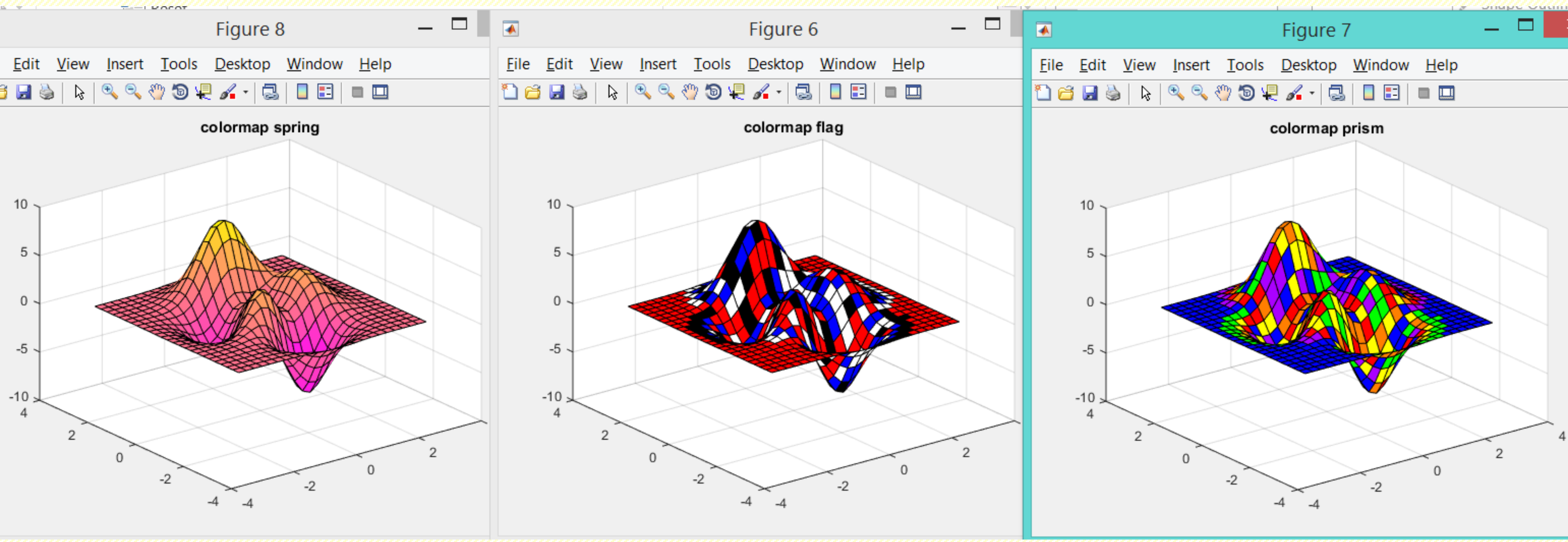


Figure 8
Edit View Insert Tools Desktop Window Help
colormap spring
Figure 6
File Edit View Insert Tools Desktop Window Help
colormap flag
Figure 7
File Edit View Insert Tools Desktop Window Help
colormap prism
10
5
0
-5
-10
4
2
0
-2
-4

MATLAB Script for the “colormap” examples

```
%% OENG1116-S1-2019
% Designed by Prof P.M.Trivailo (C) 2019
%----------------------------
figure;
[X,Y,Z] = peaks(25);
surf(X,Y,Z);
colormap jet;
title('colormap jet')
pause(1)
%------------------
figure;
surf(X,Y,Z);
colormap hsv;
title('colormap hsv')
pause(1)
%------------------
figure;
surf(X,Y,Z);
colormap copper;
title('colormap copper')
pause(1)
%------------------
figure;
surf(X,Y,Z);
colormap cool;
title('colormap cool')
pause(1)
%------------------
```

```
%------------------
figure;
surf(X,Y,Z);
colormap hot;
title('colormap hot')
pause(1)
%------------------
figure;
surf(X,Y,Z);
colormap summer;
title('colormap summer')
pause(1)
%------------------
figure;
surf(X,Y,Z);
colormap spring;
title('colormap spring')
pause(1)
%------------------
figure;
surf(X,Y,Z);
colormap flag;
title('colormap flag')
pause(1)
%------------------
figure;
surf(X,Y,Z);
colormap prism;
title('colormap prism')
```

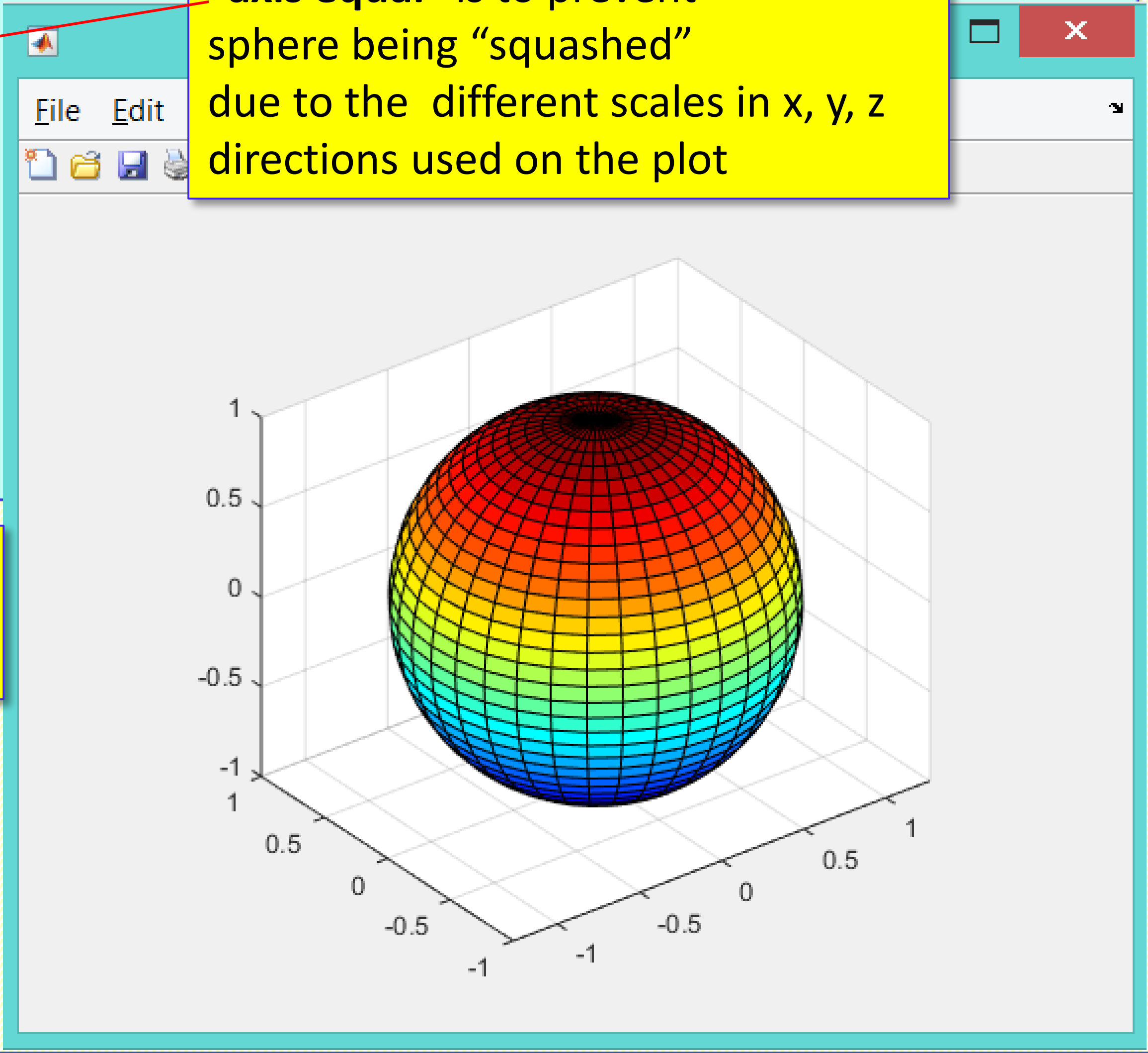
MATLAB:

Plotting Sphere – Standard Surface

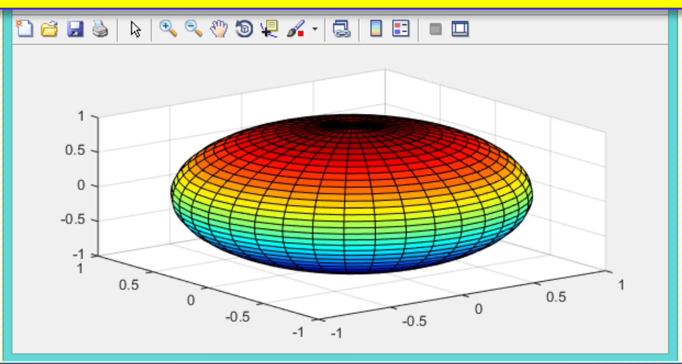
MATLAB: "sphere" & "surf"

```
%%
clear; close all; clc;
figure; hold on;
axis equal; grid on

colormap jet;
[x,y,z]=sphere(36);
surf(x,y,z);
view(3);
```

"**axis equal**" is to prevent sphere being "squashed" due to the different scales in x, y, z directions used on the plot



Without "**axis equal**" sphere can be seen as "squashed" due to the different scales in x, y, z directions used on the plot



MATLAB:

Plotting Non-Standard Surfaces; Understanding “meshgrid”

MATLAB: MESHGRID

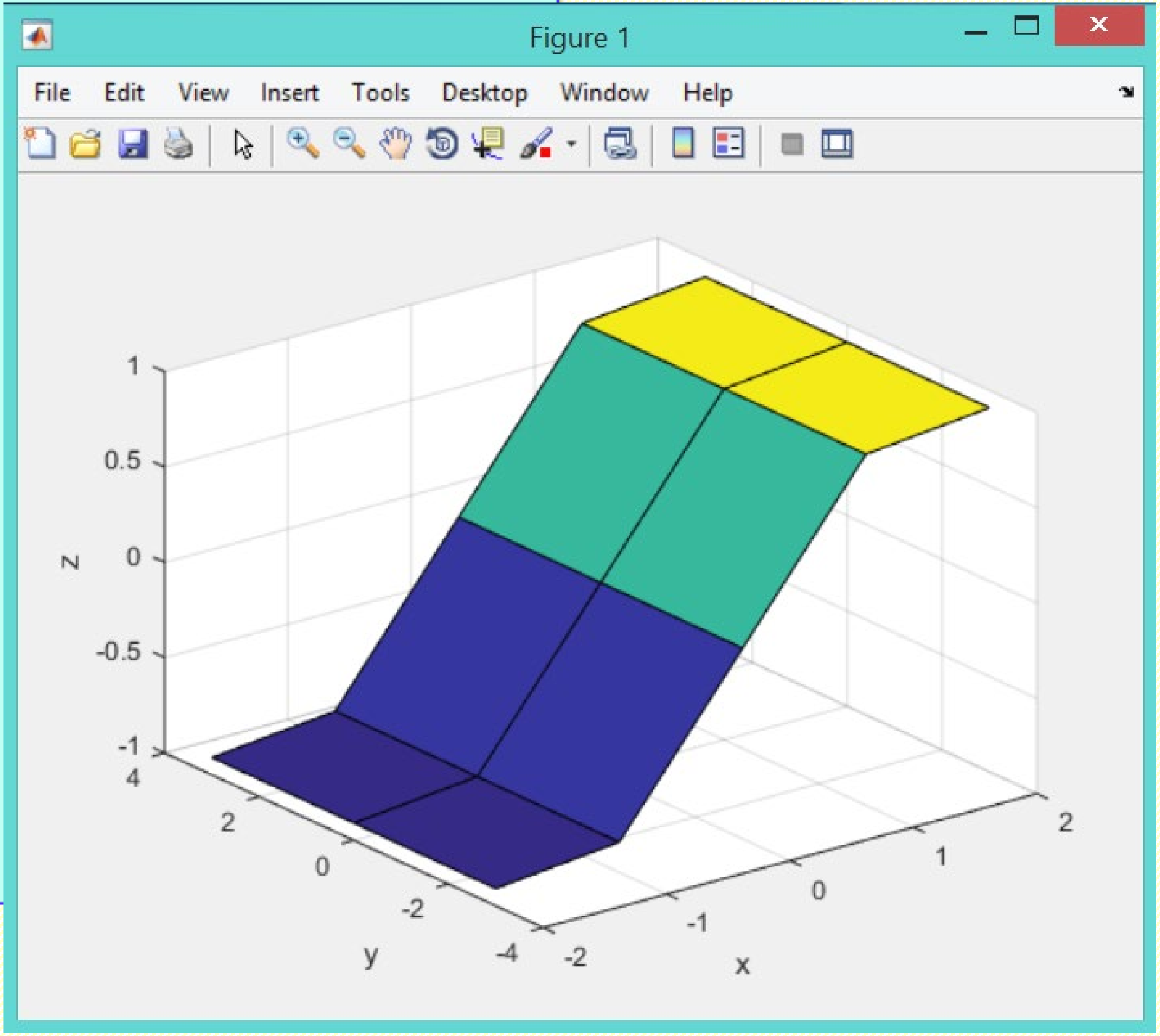
Understanding inputs and outputs of meshgrid

```
>> [X,Y] = meshgrid(-2:1:2, -3:3:3)

X =

    -2    -1     0     1     2
    -2    -1     0     1     2
    -2    -1     0     1     2


Y =

    -3    -3    -3    -3    -3
     0     0     0     0     0
     3     3     3     3     3
```

```
>> size(X)

ans =

     3     5

>> size(Y)

ans =

     3     5
```

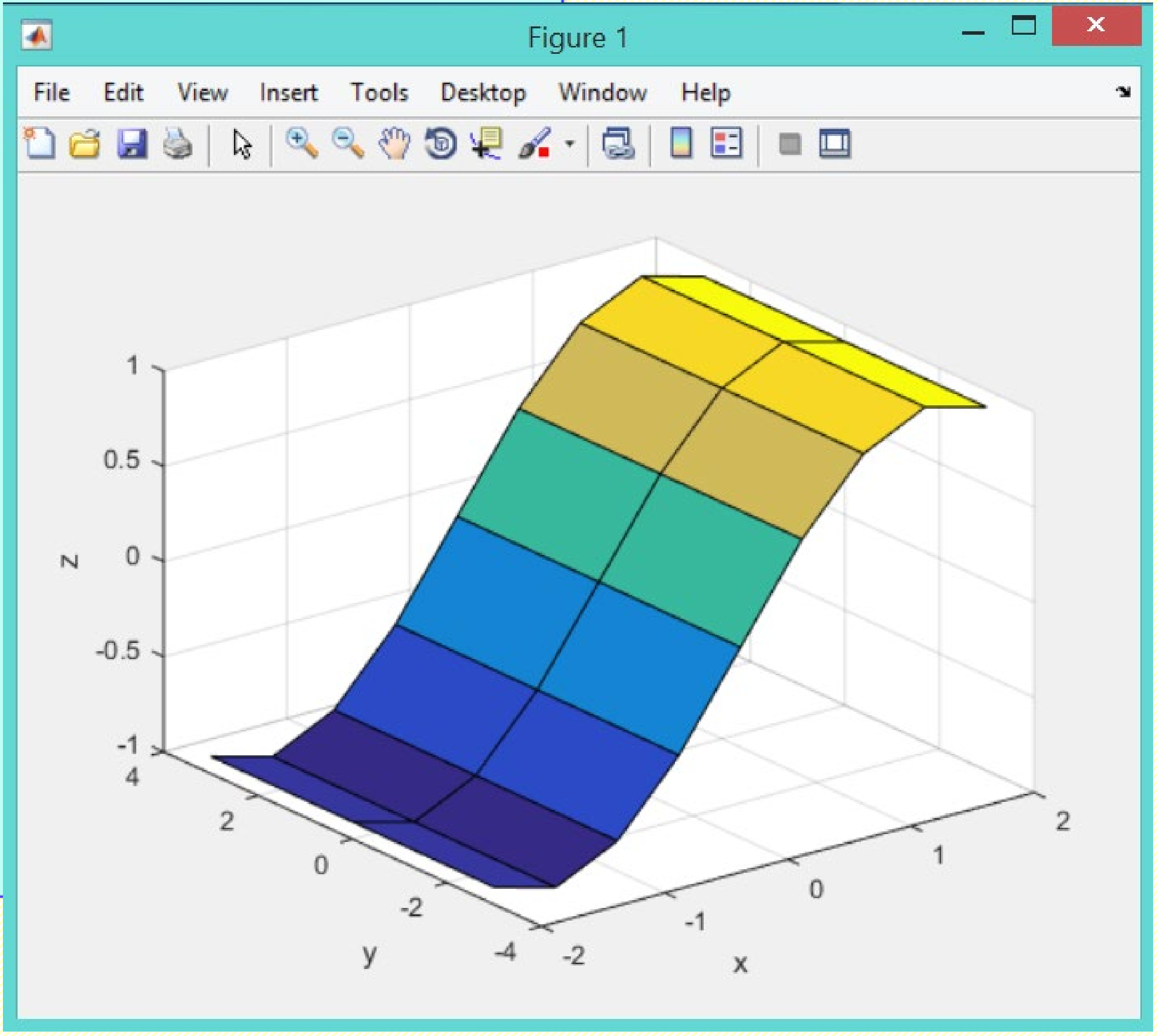
MATLAB: MESHGRID: DISCUSSION

```
[X,Y] = meshgrid(-2: 1 : 2,     -3:3:3);

Z = sin(X);

surf(X,Y,Z);

xlabel('x');
ylabel('y');
zlabel('z');
```



MATLAB: MESHGRID: DISCUSSION

```
[X,Y] = meshgrid(-2: 0.5 : 2,   -3:3:3);

Z = sin(X);

surf(X,Y,Z);

xlabel('x');
ylabel('y');
zlabel('z');
```

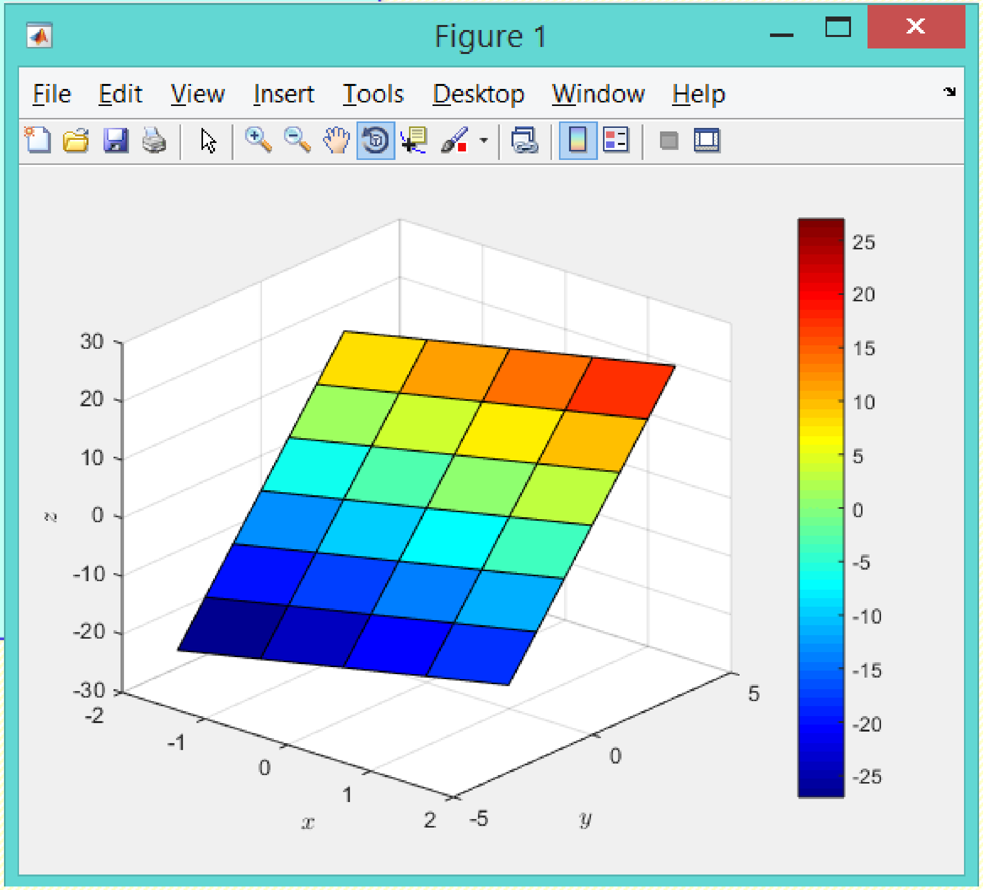


MATLAB:

Plotting 3D Plane, using “surf”

MATLAB: Surface Plot of a Plane

```
%% --- Basic "surf" plot ---
figure; hold on; grid on; colorbar;

[X,Y] = meshgrid(-2: 1 : 2,    -3:1:3);
Z=3*X+7*Y;
surf(X,Y,Z); view(40,25);

xl=xlabel('$x$');
yl=ylabel('$y$');
zl=zlabel('$z$');
set([xl,yl,zl],…
   'Interpreter','LaTeX');
colormap jet;
rotate3d on
```

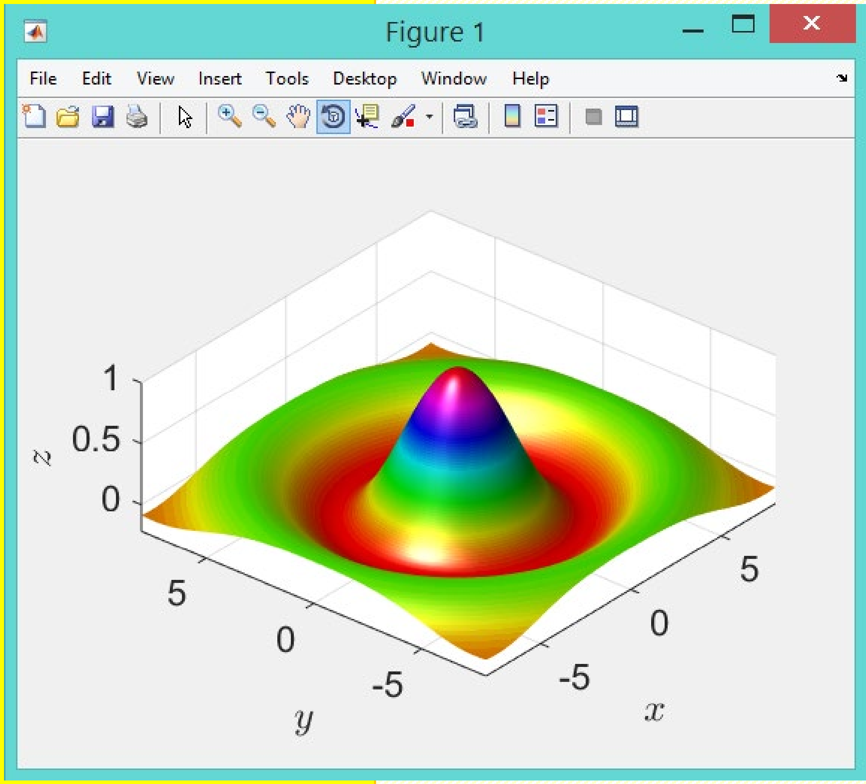


MATLAB:

Other examples of plotting 3D surfaces

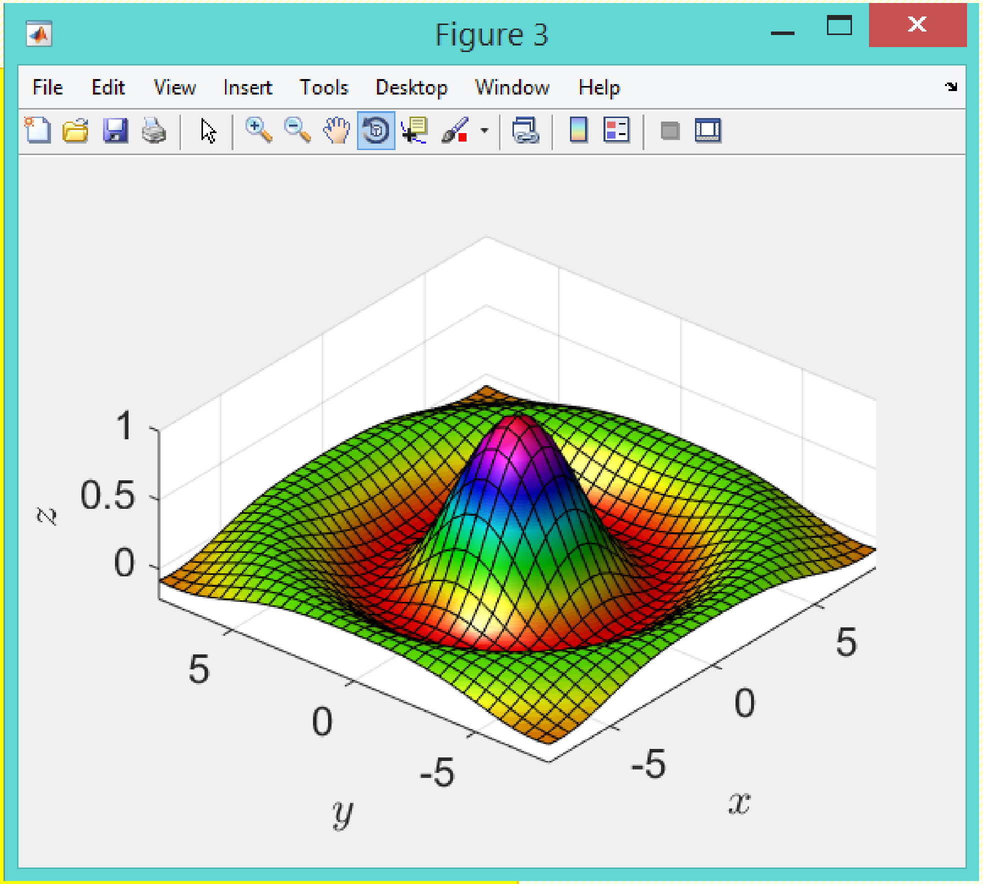
MANIPULATIONS WITH SURFACES: Class Exercises (lighting, rotate3d)

```
clear; close a[X,Y] = meshgrid(-8:.1:8);
R = sqrt(X.^2 + Y.^2) + eps;
Z = sin(R)./R;

figure
colormap hsv
surf(X,Y,Z,'FaceColor','interp',...
   'EdgeColor','none',...
   'FaceLighting','gouraud')
daspect([5 5 1])
axis tight
view(-50,30)
camlight left
xl=xlabel('$x$');
yl=ylabel('$y$');
zl=zlabel('$z$');
set([xl,yl,zl],'Interpreter','LaTeX');
set(gca,'FontSize',18);
rotate3d on
```



MANIPULATIONS WITH SURFACES: Class Exercises (lighting, rotate3d)

```
figure
colormap hsv
surf(X,Y,Z,…
   'FaceColor','interp',...
   'FaceLighting','gouraud')
%   'EdgeColor','none',...

daspect([5 5 1])
axis tight
view(-50,30)
camlight left
xl=xlabel('$x$');
yl=ylabel('$y$');
zl=zlabel('$z$');
set([xl,yl,zl],'Interpreter','LaTeX');
set(gca,'FontSize',18);
rotate3d on
```



FEM:

For more materials (with emphasis on analytical aspects), please, refer to the Separate File

https://drive.google.com/open?id=1-6Sh6mctXu4kYy5Sp8oOBTo-hd2u1mdU

This Chapter on FEM is extracted from the following Book:

REFERENCE: **Trivailo P.M.** (2008), Vibrations: Theory & Aerospace Applications, Vol.1&2, - (The textbook for senior undergraduate and graduate aerospace students). - Melbourne: RMIT Publisher - 2008. - 247pp +348pp=595 pp., 355 ill, 4 software programs.

Study Case:

Excitation of Axial Vibrations of Rods

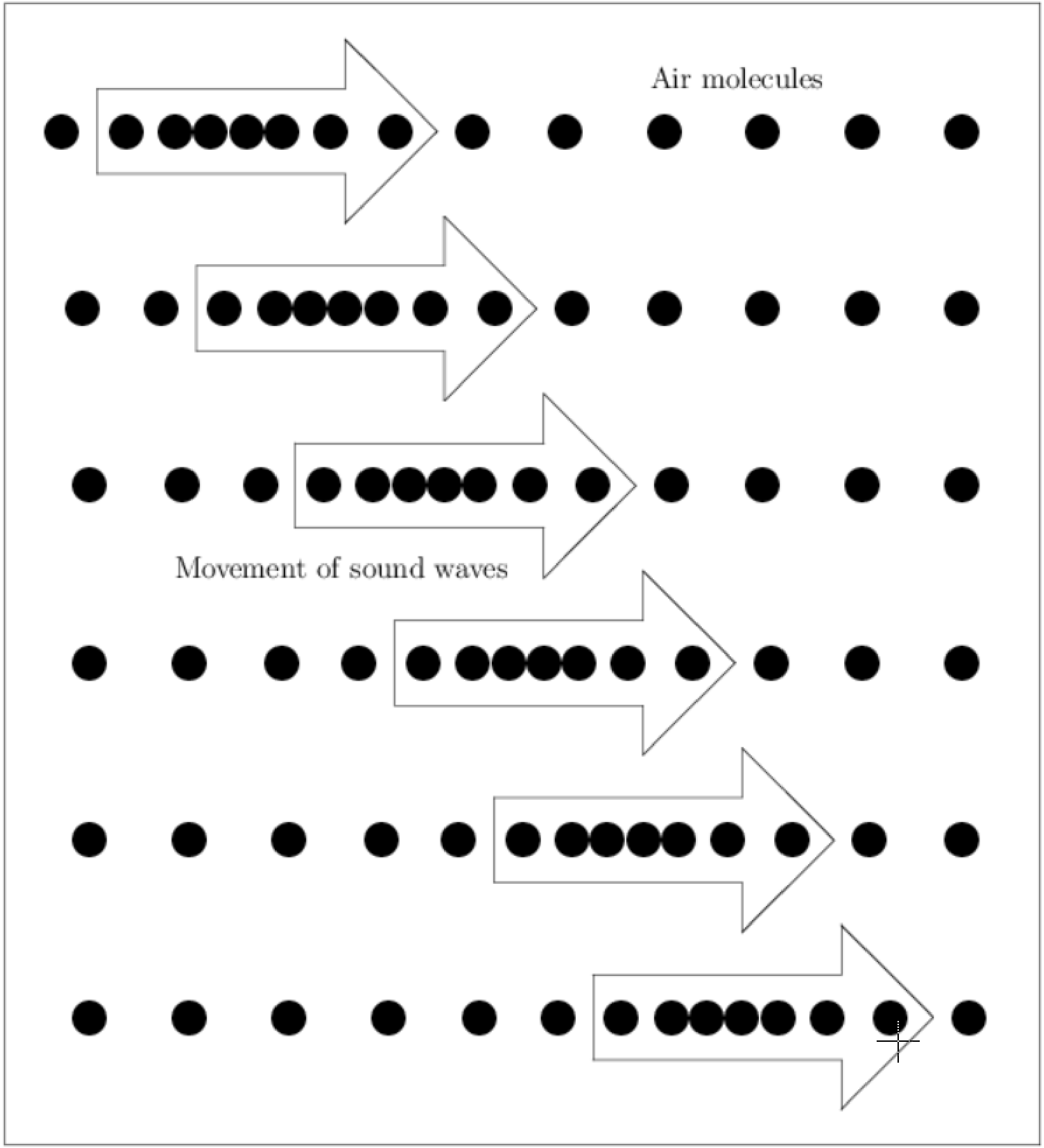


Figure 8.3: Analogy with a sound wave.

- It is easy to send longitudinal waves along a coiled spring: pull end of spring and out.

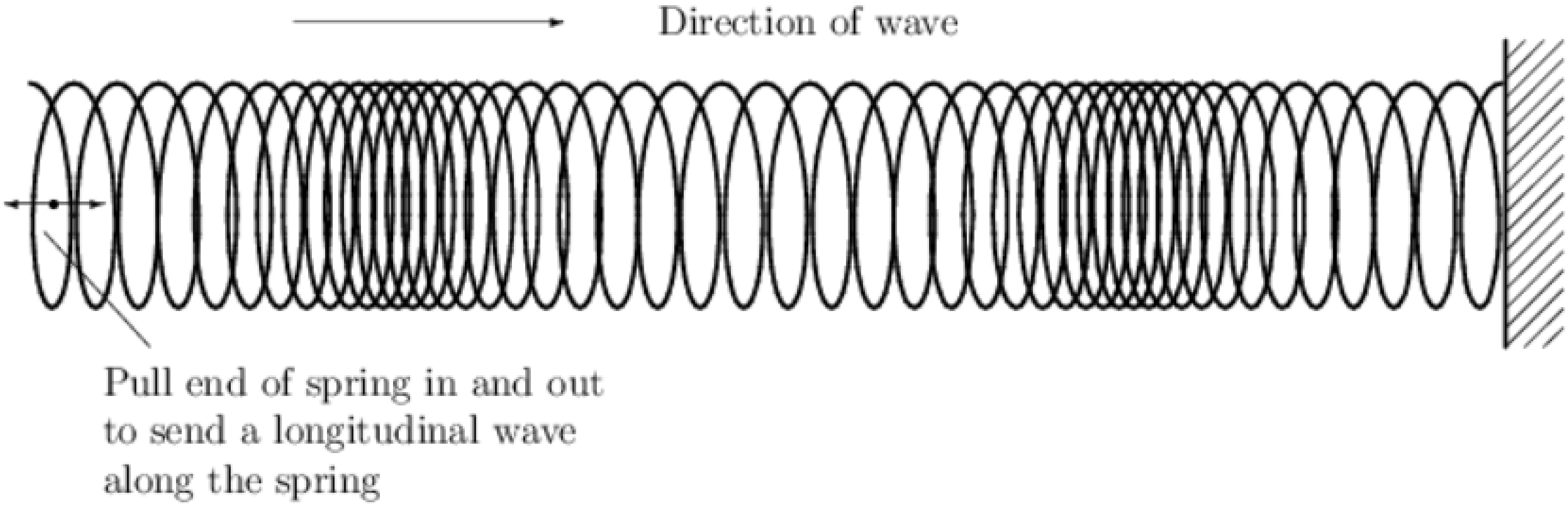
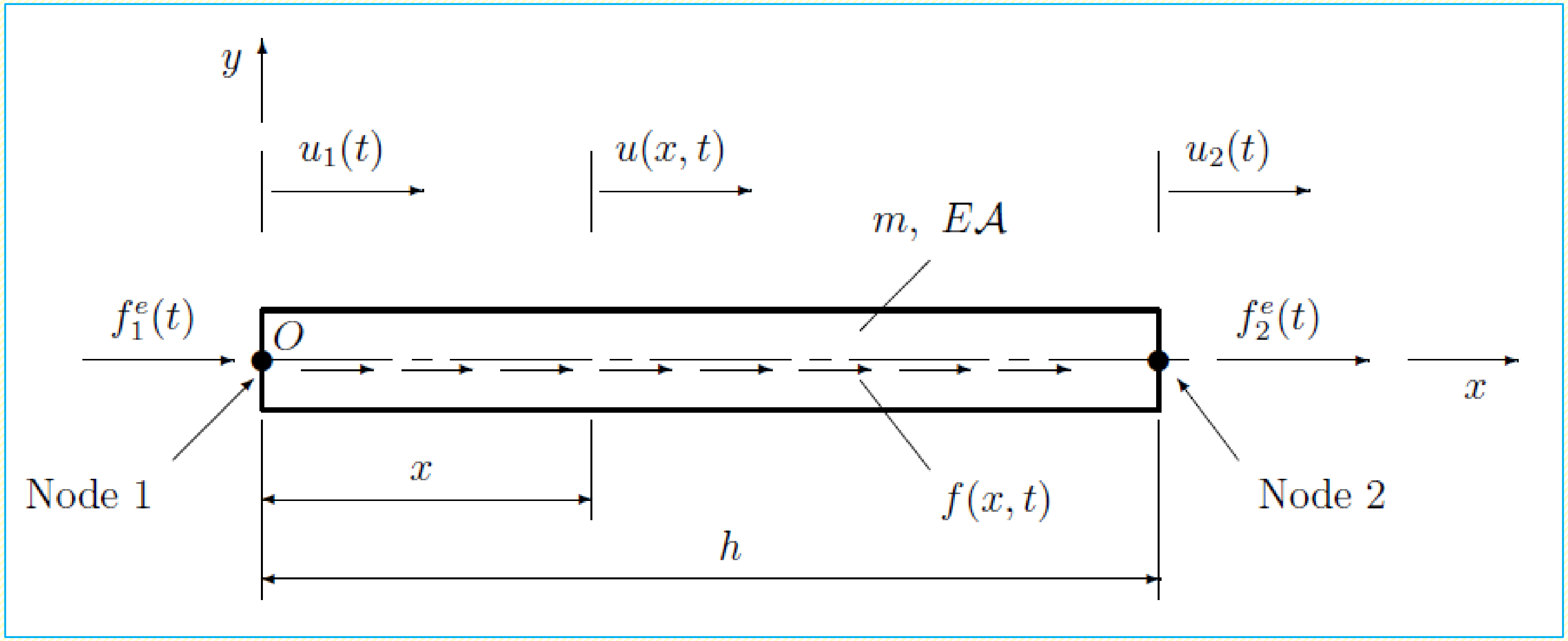


Figure 8.4: Analogy with transverse wave in a spring.

FEM: Truss Element:

Shape Functions; $[m^e]$ & $[k^e]$ matrices; interpolation of axial displacements (example)

FEM: TRUSS ELEMENT



$$u(x,t) = \left(1 - \frac{x}{h}\right) u_1(t) + \frac{x}{h} u_2(t).$$

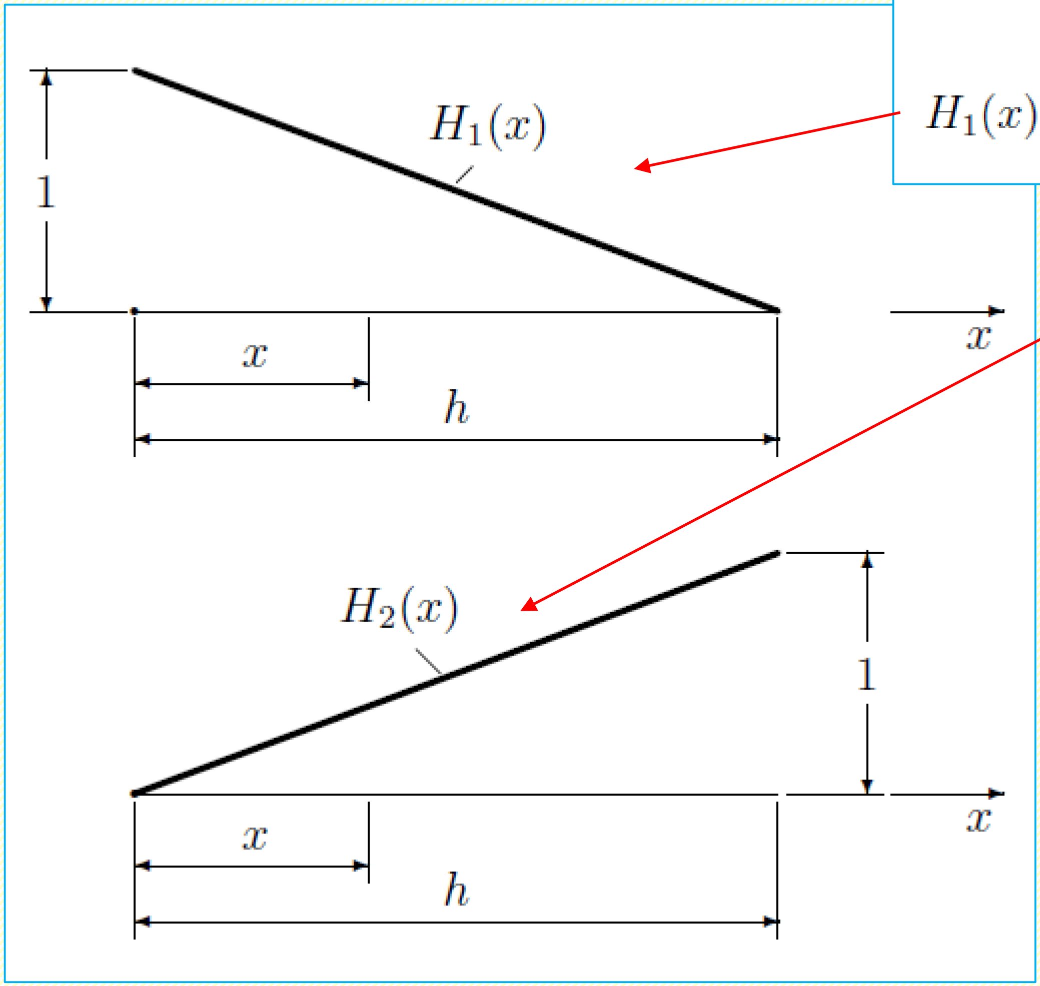
$$u(x,t) = H_1(x)\, u_1(t) + H_2(x)\, u_2(t), \qquad \text{where}$$

$$H_1(x) = 1 - \frac{x}{h}; \qquad H_2(x) = \frac{x}{h}$$

FEM: SHAPE FUNCTIONS for TRUSS ELEMENT

$$u(x,t) = H_1(x)\,u_1(t) + H_2(x)\,u_2(t), \qquad \text{where}$$

$$H_1(x) = 1 - \frac{x}{h}; \qquad H_2(x) = \frac{x}{h}$$



$$u(x,t) = \left(1 - \frac{x}{h}\right) u_1(t) + \frac{x}{h}\, u_2(t).$$

FEM: STIFFNESS & MASS MATRICES

Stiffness Matrix for a 1D Truss FE:
(Global and FE Local Coordinate Systems)

$$[k^e] = \frac{E\mathcal{A}}{h}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

Consistent Mass Matrix for a 1D Truss FE:
(Global and FE Local Coordinate Systems)

$$[m^e] = \frac{mh}{6}\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$

FEM: Truss Element:

Static case of the Rod modelled with only 1 Finite Element:

ANALYSIS of DISPLACEMENTS

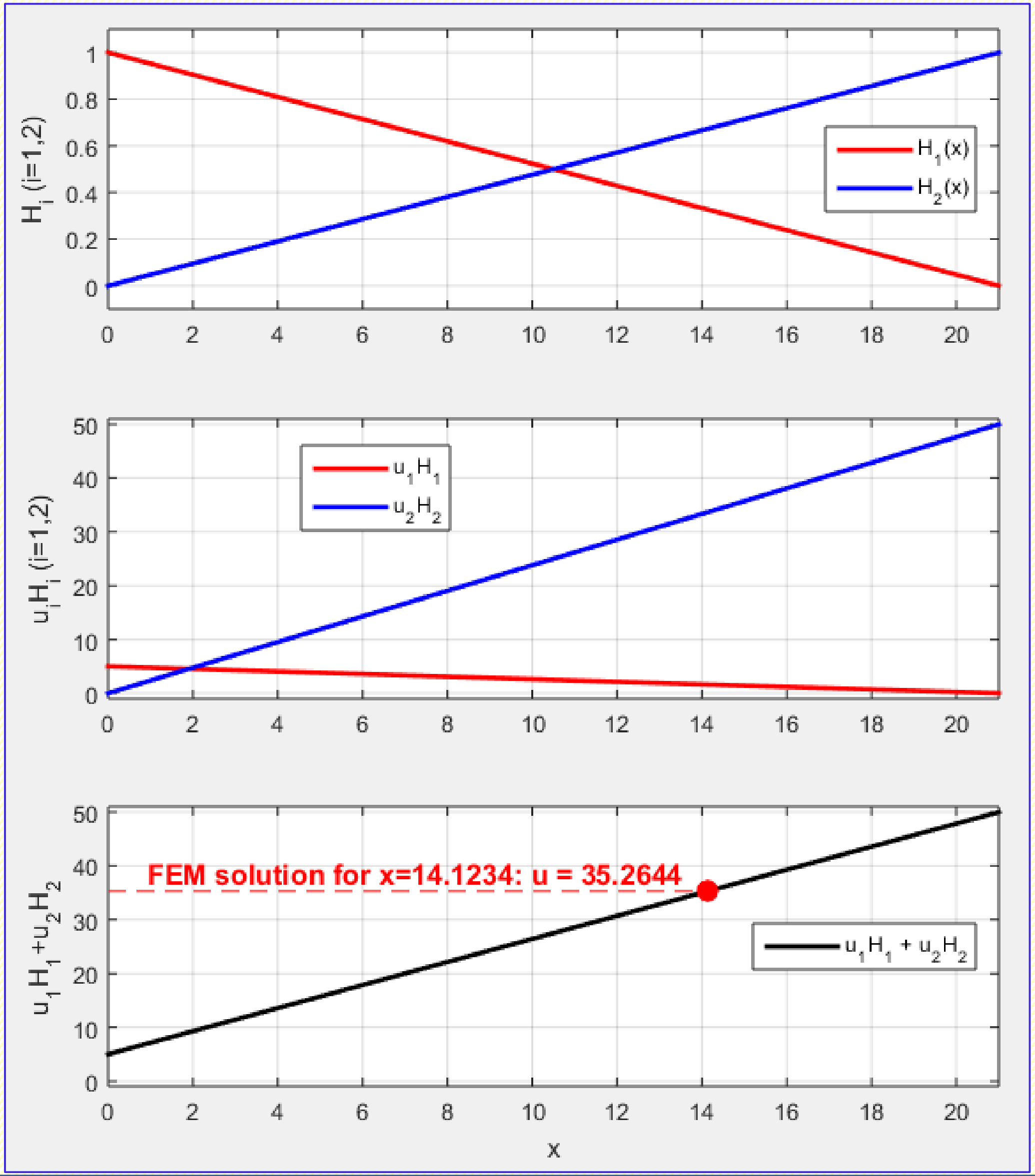
FEM: EXAMPLE-1 with TRUSS ELEMENT

$$h = 21 \quad [\mathrm{m}]$$

$$u_1 = 5 \quad [\mathrm{m}]$$

$$u_2 = 50 \quad [\mathrm{m}]$$

$$u(14.1234) = ?$$



FEM: MATLAB Script for the EXAMPLE-1

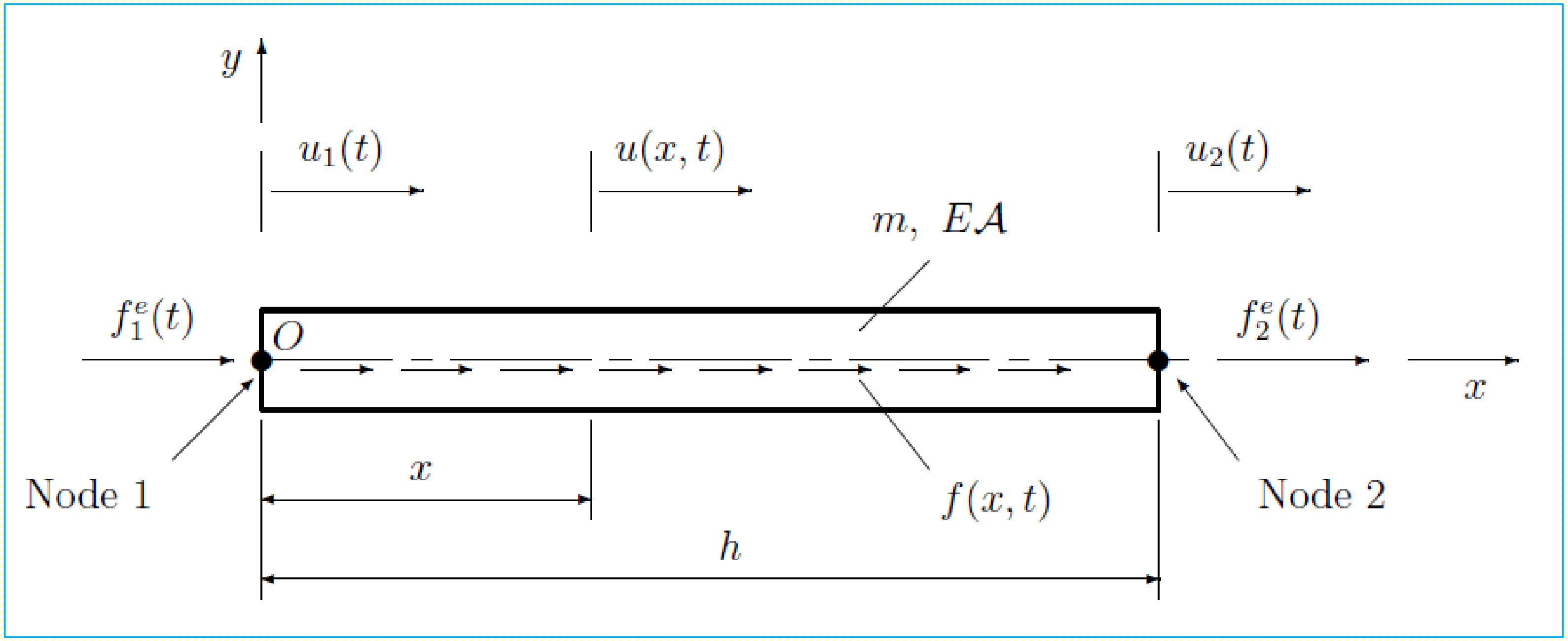
```
%% OENG1116-S1-2020
% Designed by Prof P.M.Trivailo (C) 2020
% FEM example for the 2DOF Axial Rod
%---------------------------

clear; clc; close('all')
h=21; u1=5; u2=50; % m

x=[0:0.01:1]*h;

H1x = 1-x/h; H2x = x/h;

figure;
%--------------------------
subplot(3,1,1)
plot(x,H1x,'r', x,H2x,'b','LineWidth',2);
h_legend=legend('H_1(x)','H_2(x)','Location','East');
ylabel('H_i (i=1,2)')
grid on
axis([0 h -.1 1.1])
```

```
% Continuation of the Script
%--------------------------
subplot(3,1,2)
plot(x,u1*H1x,'r',  x,u2*H2x,'b','LineWidth',2);
legend('u_1H_1','u_2H_2','Location','best');
ylabel('u_iH_i (i=1,2)')
grid on
axis([0 h -1 51])
%--------------------------
subplot(3,1,3)
u = u1*H1x + u2*H2x;
plot(x,u,'k','LineWidth',2);
legend('u_1H_1 + u_2H_2','Location','East');
grid on
axis([0 h -1 51])
ylabel('u_1H_1+u_2H_2')
xlabel('x')
xC=14.1234; %Just an example: we would like to know u at this point
yC=interp1(x,u,xC);
line('XData',xC, 'YData',yC,'Marker','o','MarkerSize',8,'Color',[1 0 0])
set(gcf,'Position', [488 47 649 733])
```

FEM: Truss Element:

Static case of the Rod modelled with only 1 Finite Element: ANALYSIS of FORCES

FEM: TRUSS ELEMENT: STATIC EQUATION



Model of the Rod, having only One Finite Element:

$\left[k^e\right]\{u\} = \{F\}$ (matrix equation), or,

$$\frac{EA}{h}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} = \begin{Bmatrix} F_1 \\ F_2 \end{Bmatrix}$$ (the same, but in the expanded format)

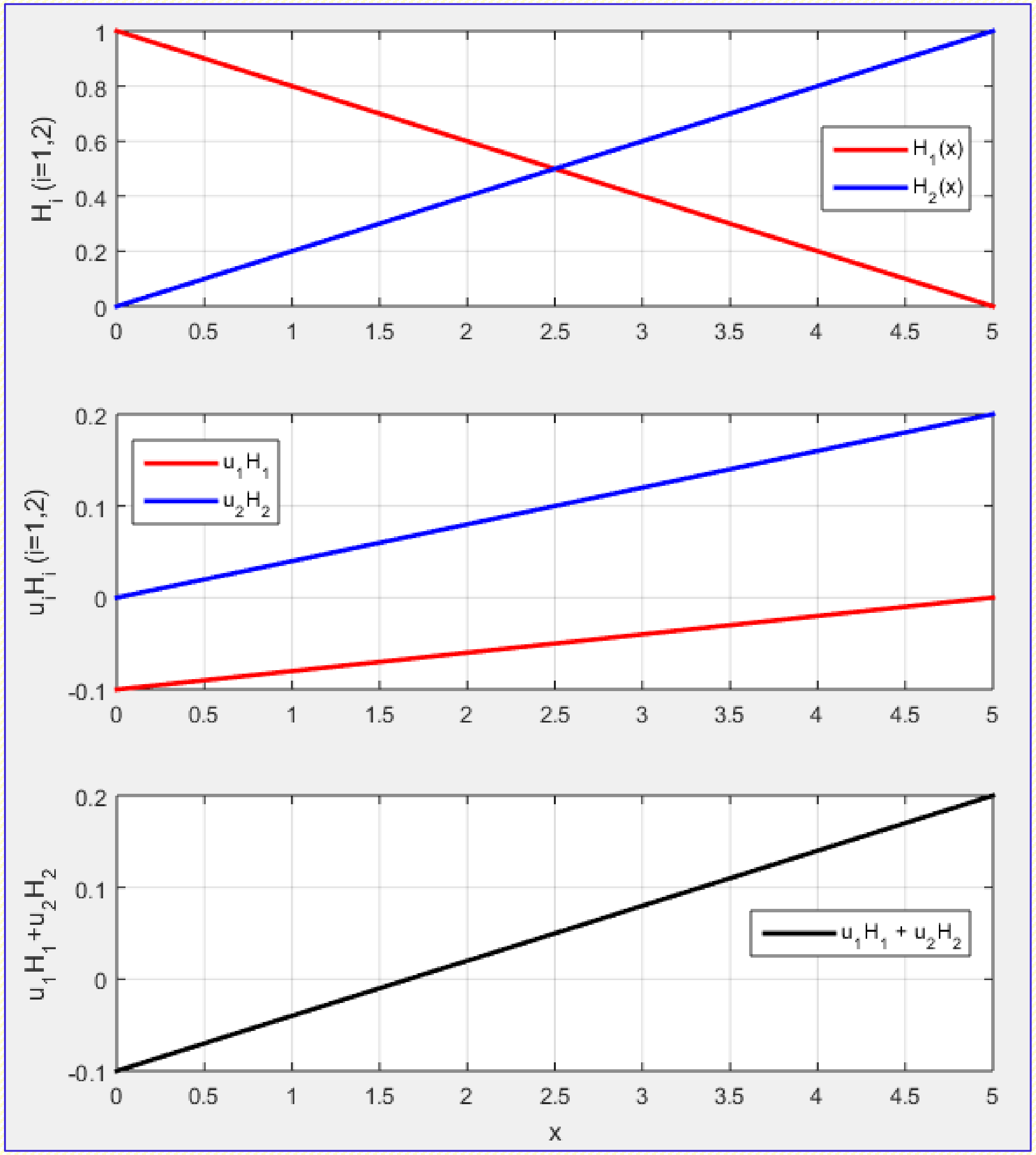
FEM: EXAMPLE-2 with TRUSS ELEMENT

$$h = 5 \quad [\mathrm{m}]$$

$$u_1 = -0.1 \quad [\mathrm{m}]$$

$$u_2 = 0.2 \quad [\mathrm{m}]$$

$$F = \begin{Bmatrix} F_1 \\ F_2 \end{Bmatrix} = ?$$



FEM: MATLAB Script for the EXAMPLE-2

```
%% OENG1116-S1-2020
% Designed by Prof P.M.Trivailo (C) 2020
% FEM Example-2 for the 2DOF Axial Rod
%----------------------------
clear; clc; close('all')
h=21;
x=[0:0.01:1]*h; H1x = 1-x/h; H2x = x/h; % Shape Functions
figure; a=0.08; b=0.08; h=5; u1=-0.1; u2=0.2; % Data in [m]
E=0.01*10^9;                             % Young Modulus [Pa]
%---------------------------
subplot(3,1,1)
plot(x,H1x,'r', x,H2x,'b','LineWidth',2);
h_legend=legend('H_1(x)','H_2(x)','Location','East');
ylabel('H_i (i=1,2)'); grid on; axis tight;
%---------------------------
subplot(3,1,2)
plot(x,u1*H1x,'r',  x,u2*H2x,'b','LineWidth',2);
legend('u_1H_1','u_2H_2','Location','best');
ylabel('u_iH_i (i=1,2)'); grid on; axis tight;
%---------------------------
subplot(3,1,3)
u = u1*H1x + u2*H2x;
plot(x,u,'k','LineWidth',2);
legend('u_1H_1 + u_2H_2','Location','East');
grid on; axis tight; ylabel('u_1H_1+u_2H_2'); xlabel('x');
set(gcf,'Position', [488    47    649    733])
```

FEM: MATLAB Script for the EXAMPLE-2

```
% Continuation of the Script for Example-2
%---------------------------
%
% --- ANALYSING AXIAL FORCES ---
A=a*b; EA=E*A;
% --- Calculate Forces using FEM ---
ke=(EA/h)*[1 -1; -1 1];
F=ke*[u1;u2];
disp(sprintf('=== Case: u1=%g; u2=%g; h=%g; E=%g; a=%g;
b=%g',u1,u2,h,E,a,b))
disp(sprintf('F1=%12.2f [N]; F2=%12.2f [N]; ',F(1),F(2)))

% --- Checking Axial Forces using Hooke's Law ---
Fchk=(u2-u1)*EA/h;
disp(sprintf('Check by Hooke''s Law: |F|=%12.2f [N]',Fchk))

commandwindow
```

$$\left[k^e\right]\{u\} = \{F\}$$

Command Window

New to MATLAB? See resources for Getting Started.

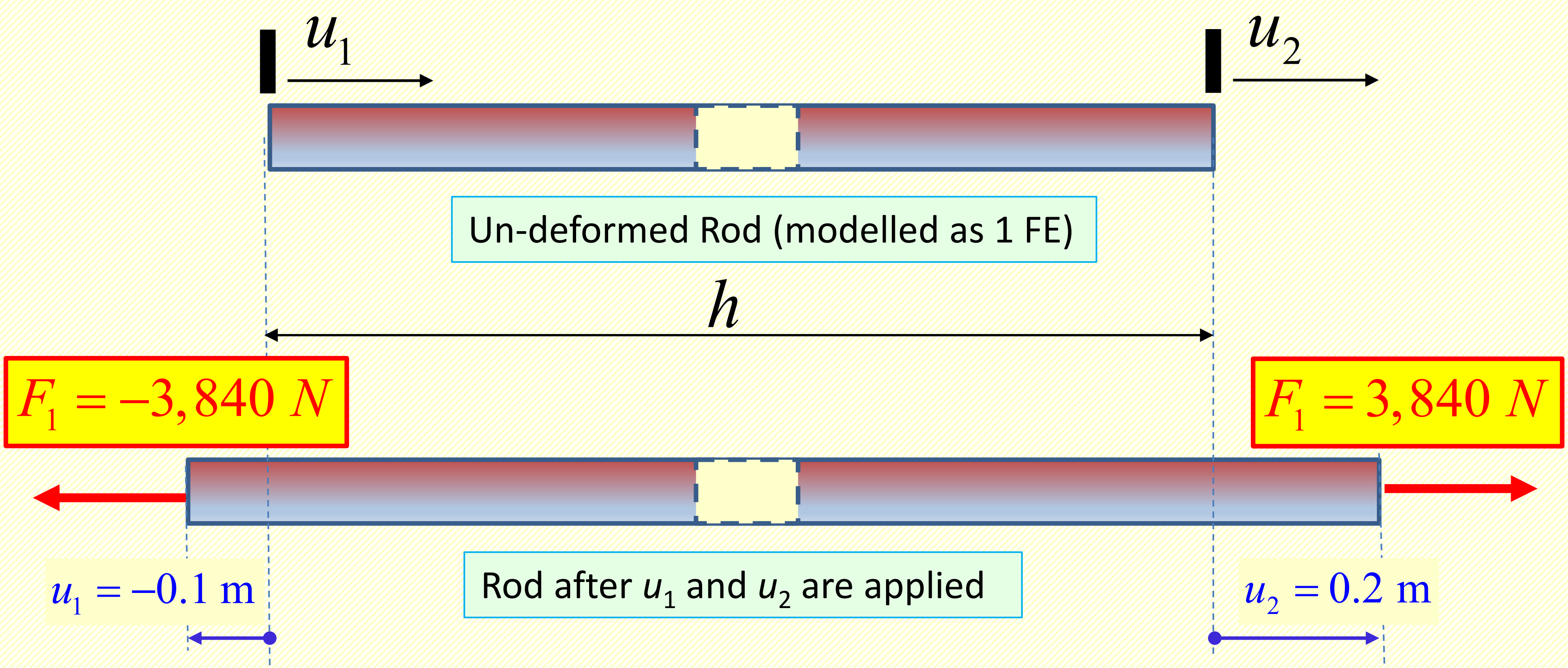
```
=== Case: u1=-0.1; u2=0.2; h=5; E=1e+07; a=0.08; b=0.08
F1=     -3840.00 [N]; F2=      3840.00 [N];
Check by Hooke's Law: |F|=      3840.00 [N]
```

FEM: Interpretation of Results for EXAMPLE-2

Command Window

New to MATLAB? See resources for Getting Started.

```
=== Case: u1=-0.1; u2=0.2; h=5; E=1e+07; a=0.08; b=0.08
F1=     -3840.00 [N]; F2=      3840.00 [N];
Check by Hooke's Law: |F|=      3840.00 [N]
```



FEM: Truss Element:

Comparison of frequencies, calculated with FEM and exact analytical expressions (for 50 FEs model)

FEM: MATLAB Script for the 50 FE Rod

```
%% ----- EXAMPLE: FEM modelling of axial rod
% Designed by Prof P.M.Trivailo (C) 2020

%=== ENTERING DATA =========
clear; clc; close('all');
L=1; E=0.01*10^9; % Pa
rho=1.2*10^3; % kg/m^3
a=0.08; b=0.08; A=a*b; EA=E*A;
c=sqrt(E/rho);
NumFE=50;
h=L/NumFE; m=A*1*rho;

%=== BUILDING [M] & [K] global matrices =========
ke=(EA/h)*[1 -1; -1 1];
me=(m*h/6)*[2 1; 1 2];
M=zeros([1,1]*(NumFE+1)); K=zeros([1,1]*(NumFE+1));
for ii=1:NumFE
    idx=[1 2]+(ii-1);
    M(idx, idx)=M(idx, idx)+me;    K(idx, idx)=K(idx, idx)+ke;
end
```

$$[k^e] = \frac{EA}{h}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$[m^e] = \frac{mh}{6}\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$

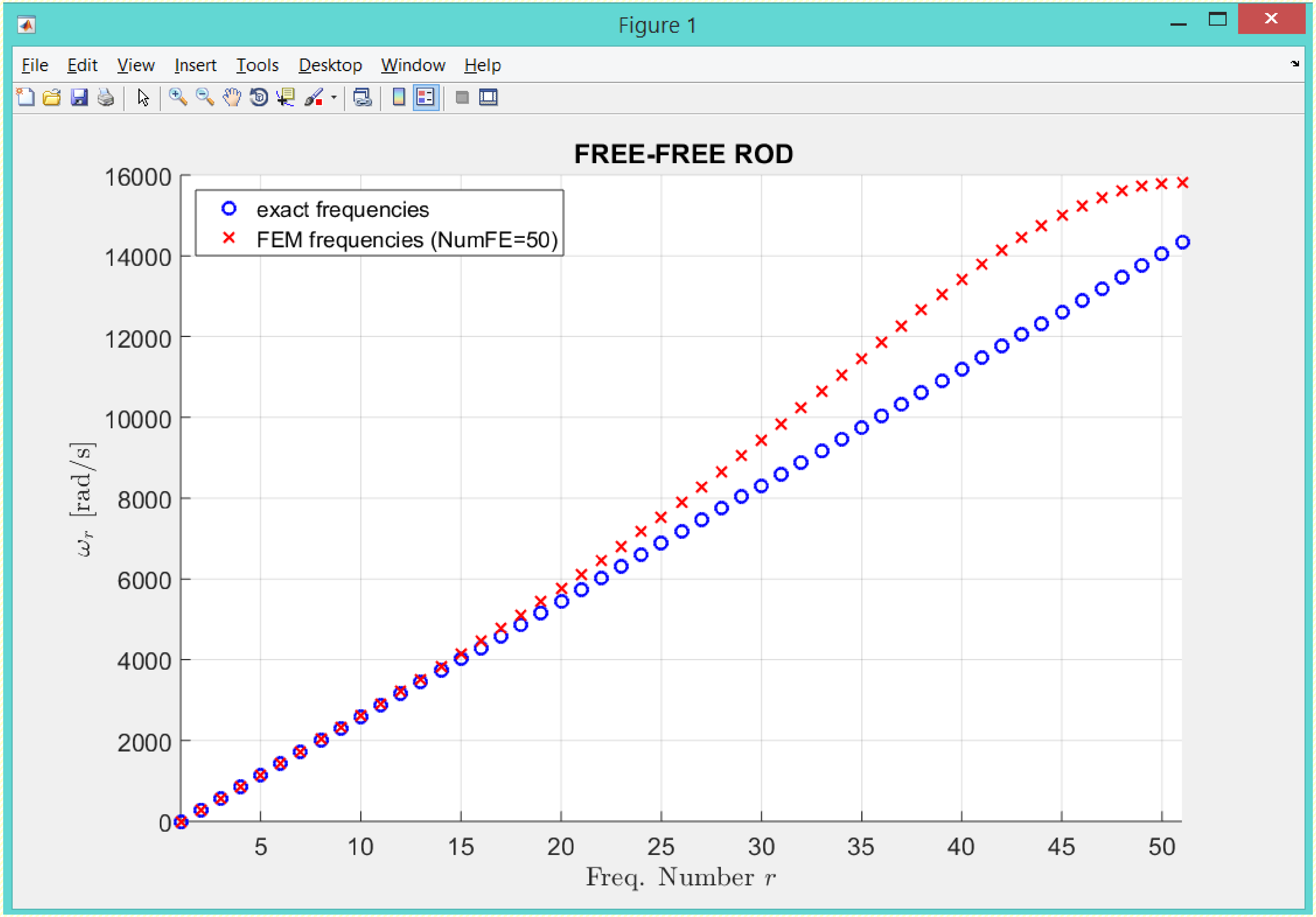
FEM: MATLAB Script(Continued-1)

```
%=== Solving Eigenvalue Problem =========
[U,D]=eig(K,M);

%=== Plotting FEM and exact frequencies =========
w1_exact=1*pi*sqrt(E/(rho*L^2));

figure; grid on; hold on;
for i=1:NumFE+1
    w_exact=(i-1)*w1_exact;
    w_FEM=sqrt(abs(diag(D(i,i))));
    disp(sprintf('w_%2i = %7.2f rad/s   w_exact_%2i = %7.2f rad/s',i,w_FEM,i,w_exact))
    plot(i,w_exact,'ob','MarkerSize',8); % plot i-th exact frequency
    plot(i,w_FEM,'xr','MarkerSize',8);    % plot i-th FEM frequency
end
```

FEM: MATLAB Script(Continued-2)

```
%=== "Decorations": adding legend, labels and title =========
str=sprintf('FEM frequencies (NumFE=%i)',NumFE);
legend('exact frequencies',str,'Location','NorthWest');
xlabel('Freq. Number $r$','Interpreter','LaTeX');
ylabel('$\omega_r$ [rad/s]','Interpreter','LaTeX');
title('\bf FREE-FREE ROD');
%
xlim([1 NumFE+1]);
set(gca,'FontSize',16);
set(gcf,'Position', [65 14 1107  680]);
if NumFE<10, set(gca,'XTick',[1:NumFE+1]); end
commandwindow
```

50 FE Model: Analytical & FEM Frequencies



FEM: MATLAB Command Window Output

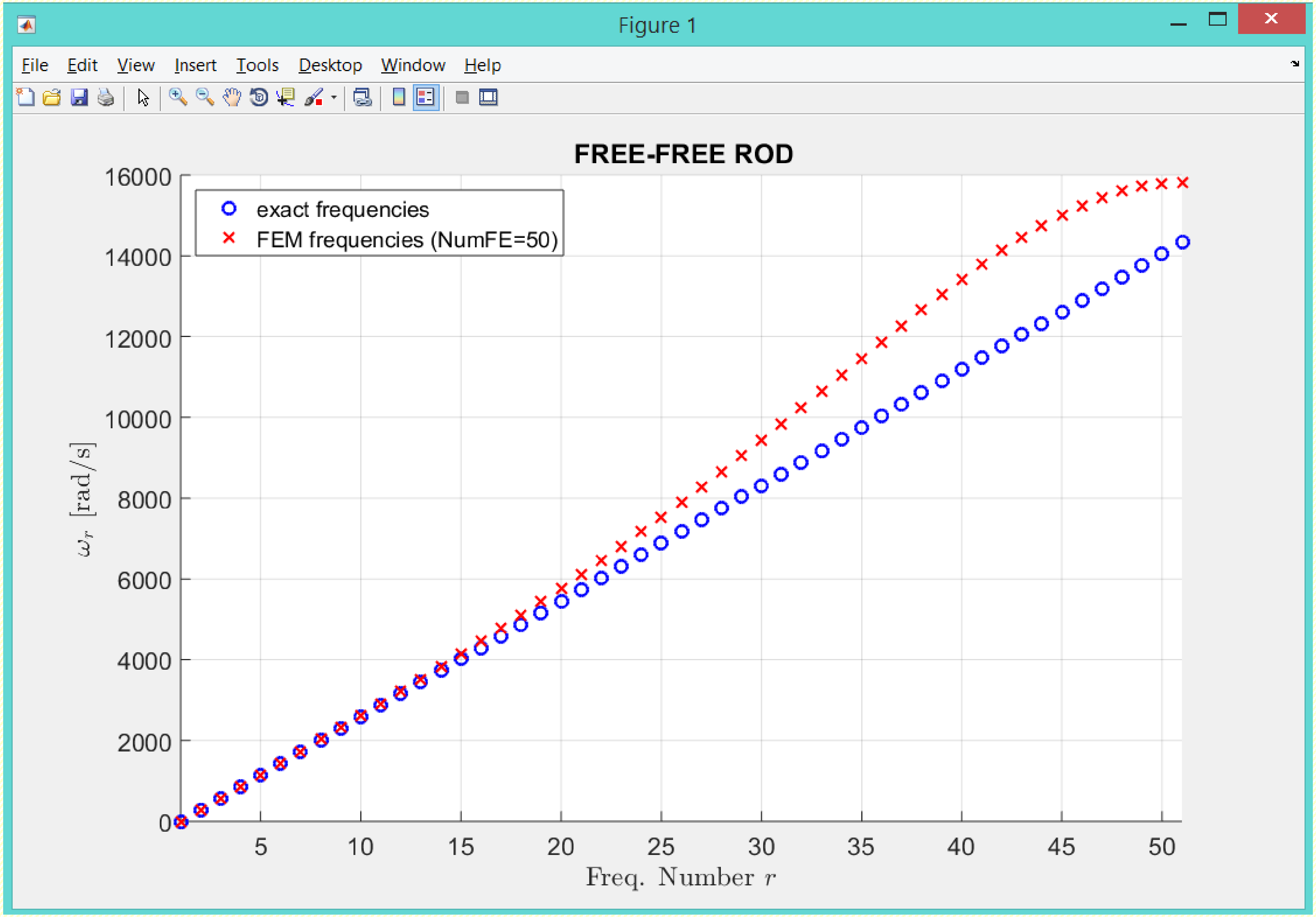
```
w_ 1 =    0.00 rad/s   w_exact_ 1 =    0.00 rad/s
w_ 2 =  286.83 rad/s   w_exact_ 2 =  286.79 rad/s
w_ 3 =  573.95 rad/s   w_exact_ 3 =  573.57 rad/s
w_ 4 =  861.63 rad/s   w_exact_ 4 =  860.36 rad/s
w_ 5 = 1150.17 rad/s   w_exact_ 5 = 1147.15 rad/s
w_ 6 = 1439.84 rad/s   w_exact_ 6 = 1433.93 rad/s
w_ 7 = 1730.93 rad/s   w_exact_ 7 = 1720.72 rad/s
w_ 8 = 2023.73 rad/s   w_exact_ 8 = 2007.51 rad/s
w_ 9 = 2318.52 rad/s   w_exact_ 9 = 2294.29 rad/s
w_10 = 2615.60 rad/s   w_exact_10 = 2581.08 rad/s
w_11 = 2915.25 rad/s   w_exact_11 = 2867.87 rad/s
w_12 = 3217.76 rad/s   w_exact_12 = 3154.66 rad/s
w_13 = 3523.43 rad/s   w_exact_13 = 3441.44 rad/s
w_14 = 3832.54 rad/s   w_exact_14 = 3728.23 rad/s
w_15 = 4145.38 rad/s   w_exact_15 = 4015.02 rad/s
w_16 = 4462.24 rad/s   w_exact_16 = 4301.80 rad/s
w_17 = 4783.38 rad/s   w_exact_17 = 4588.59 rad/s
w_18 = 5109.09 rad/s   w_exact_18 = 4875.38 rad/s
w_19 = 5439.62 rad/s   w_exact_19 = 5162.16 rad/s
w_20 = 5775.22 rad/s   w_exact_20 = 5448.95 rad/s
w_21 = 6116.11 rad/s   w_exact_21 = 5735.74 rad/s
w_22 = 6462.49 rad/s   w_exact_22 = 6022.52 rad/s
w_23 = 6814.53 rad/s   w_exact_23 = 6309.31 rad/s
w_24 = 7172.37 rad/s   w_exact_24 = 6596.10 rad/s
w_25 = 7536.08 rad/s   w_exact_25 = 6882.88 rad/s
```

```
w_26 = 7905.69 rad/s   w_exact_26 = 7169.67 rad/s
w_27 = 8281.15 rad/s   w_exact_27 = 7456.46 rad/s
w_28 = 8662.31 rad/s   w_exact_28 = 7743.25 rad/s
w_29 = 9048.92 rad/s   w_exact_29 = 8030.03 rad/s
w_30 = 9440.63 rad/s   w_exact_30 = 8316.82 rad/s
w_31 = 9836.89 rad/s   w_exact_31 = 8603.61 rad/s
w_32 = 10237.04 rad/s   w_exact_32 = 8890.39 rad/s
w_33 = 10640.16 rad/s   w_exact_33 = 9177.18 rad/s
w_34 = 11045.16 rad/s   w_exact_34 = 9463.97 rad/s
w_35 = 11450.64 rad/s   w_exact_35 = 9750.75 rad/s
w_36 = 11854.97 rad/s   w_exact_36 = 10037.54 rad/s
w_37 = 12256.18 rad/s   w_exact_37 = 10324.33 rad/s
w_38 = 12651.99 rad/s   w_exact_38 = 10611.11 rad/s
w_39 = 13039.78 rad/s   w_exact_39 = 10897.90 rad/s
w_40 = 13416.61 rad/s   w_exact_40 = 11184.69 rad/s
w_41 = 13779.19 rad/s   w_exact_41 = 11471.47 rad/s
w_42 = 14123.97 rad/s   w_exact_42 = 11758.26 rad/s
w_43 = 14447.18 rad/s   w_exact_43 = 12045.05 rad/s
w_44 = 14744.89 rad/s   w_exact_44 = 12331.84 rad/s
w_45 = 15013.13 rad/s   w_exact_45 = 12618.62 rad/s
w_46 = 15248.04 rad/s   w_exact_46 = 12905.41 rad/s
w_47 = 15445.96 rad/s   w_exact_47 = 13192.20 rad/s
w_48 = 15603.63 rad/s   w_exact_48 = 13478.98 rad/s
w_49 = 15718.34 rad/s   w_exact_49 = 13765.77 rad/s
w_50 = 15788.02 rad/s   w_exact_50 = 14052.56 rad/s
w_51 = 15811.39 rad/s   w_exact_51 = 14339.34 rad/s
```

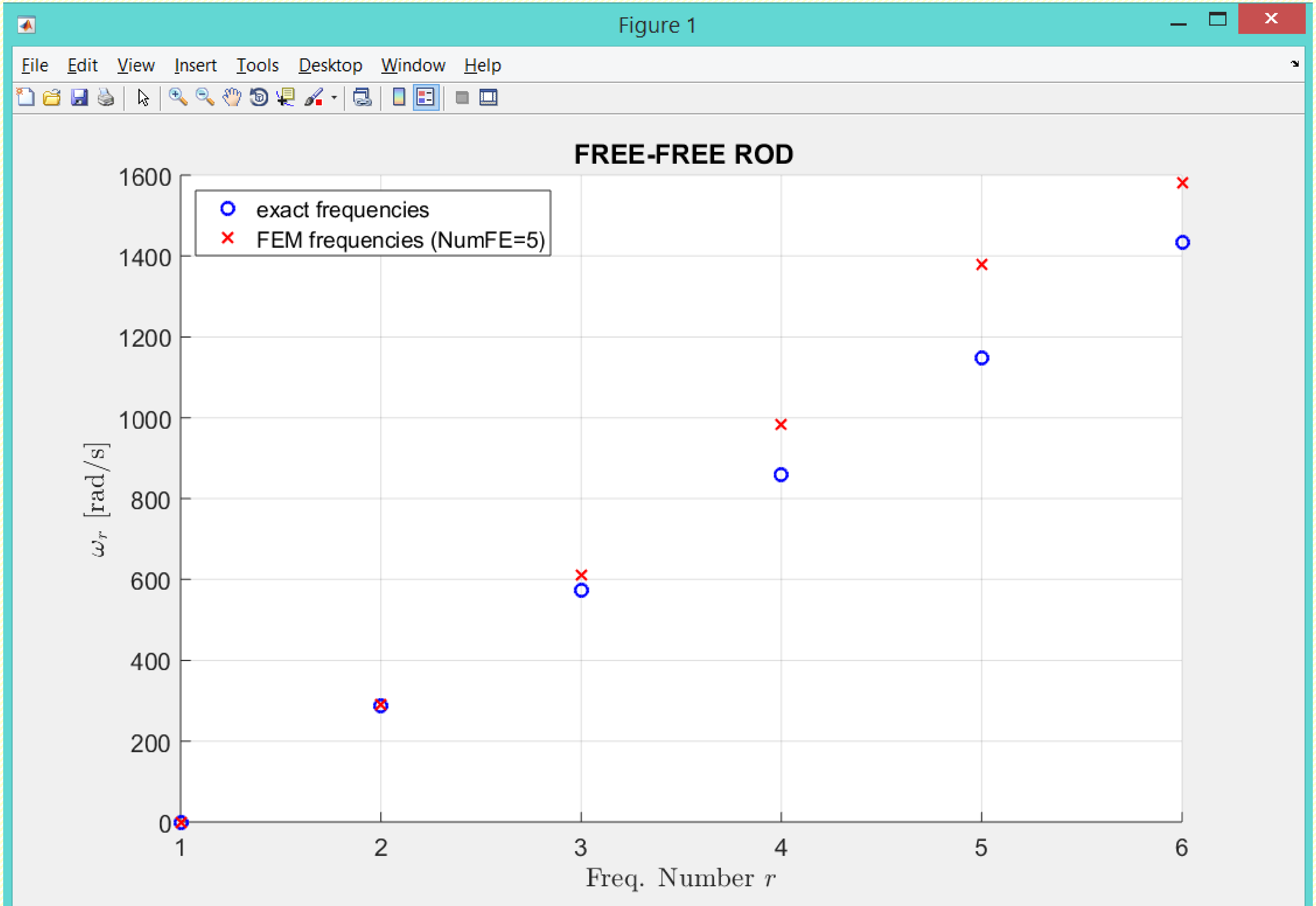
FEM: Truss Element:

Comparison of frequencies, calculated with FEM and exact analytical expressions (for 50, 5, 2 & 1 FEs model)

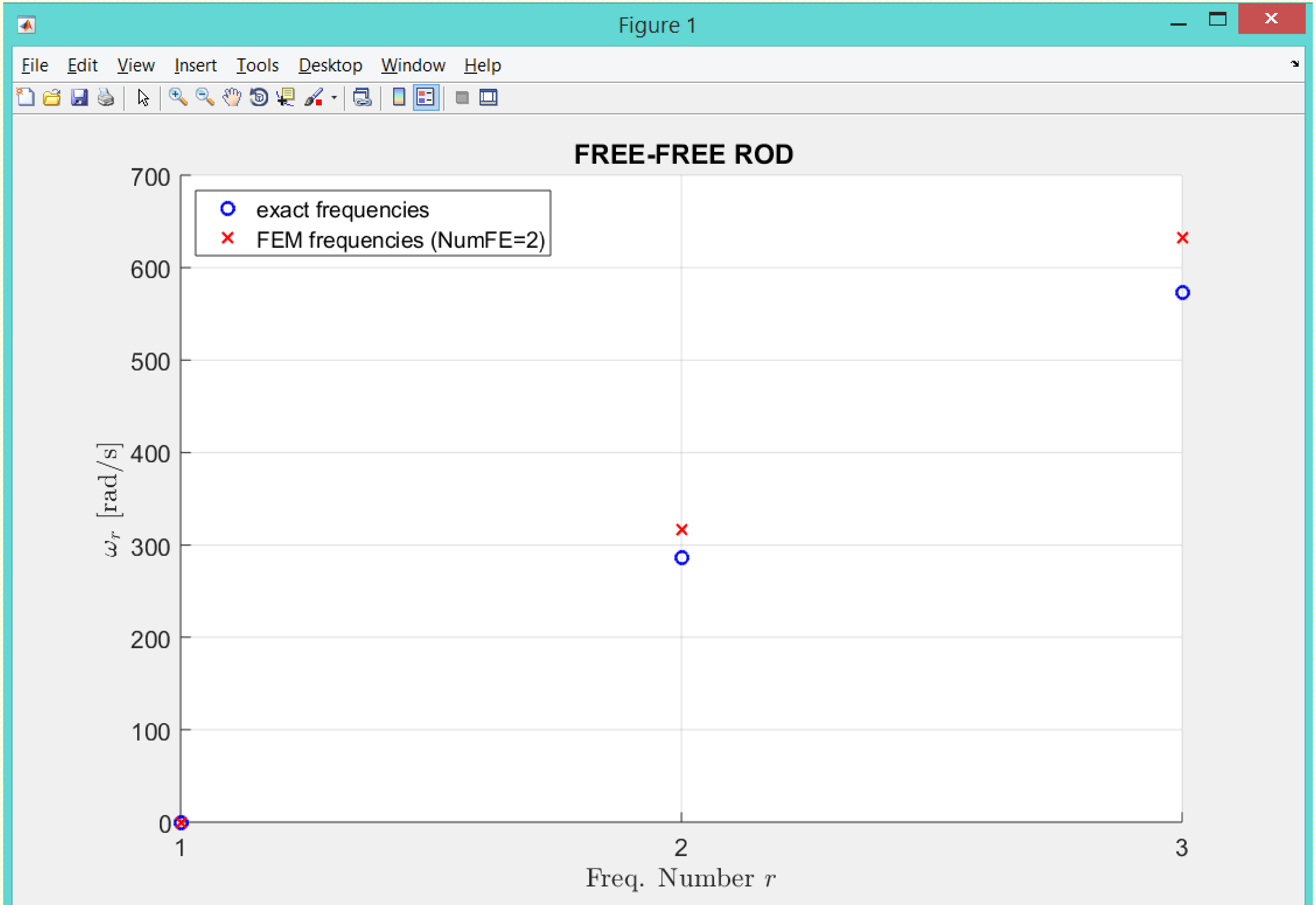
50 FE Model: Analytical & FEM Frequencies



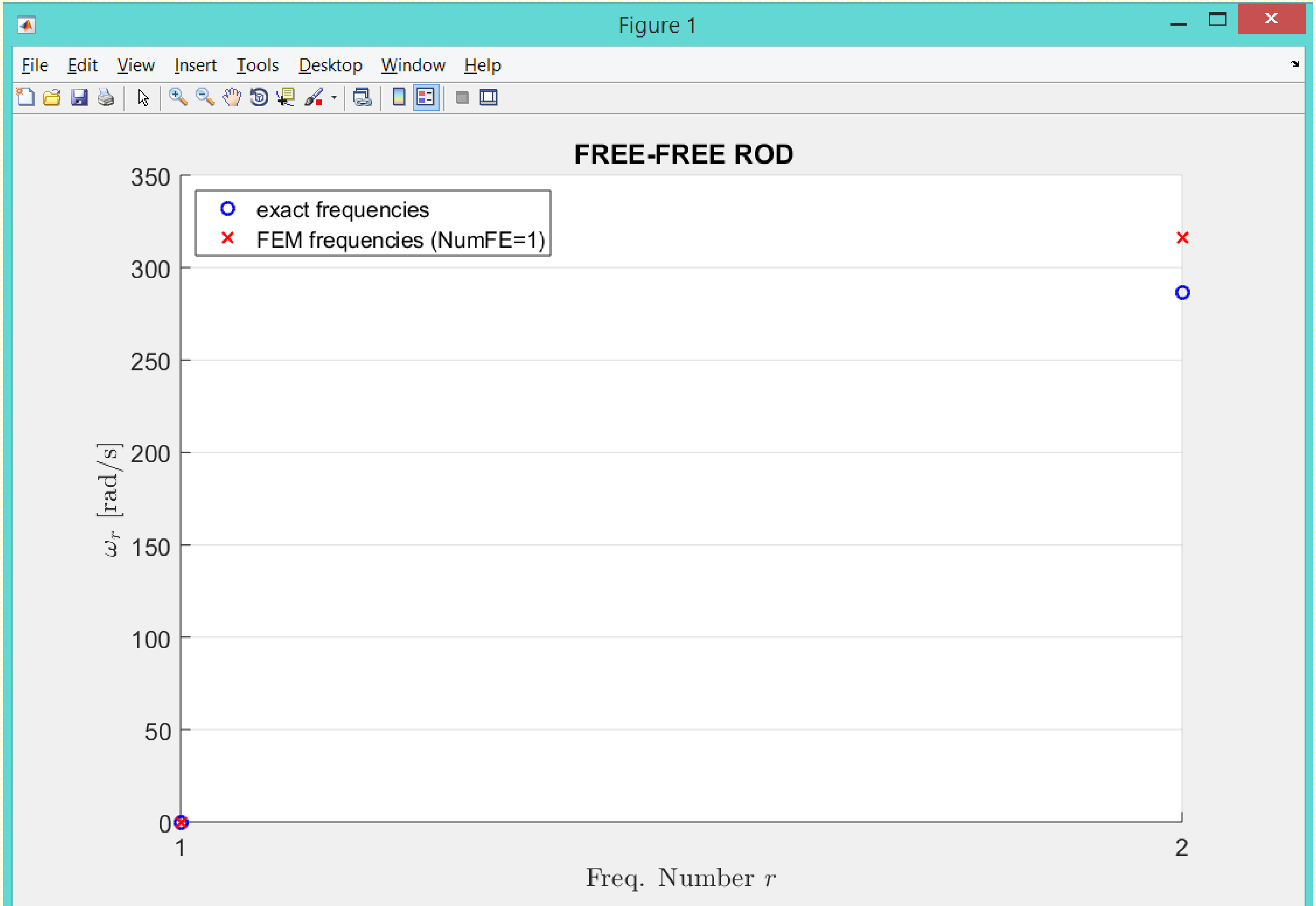
5 FE Model: Analytical & FEM Frequencies



2 FE Model: Analytical & FEM Frequencies



1 FE Model: Analytical & FEM Frequencies



FEM: Truss Element:

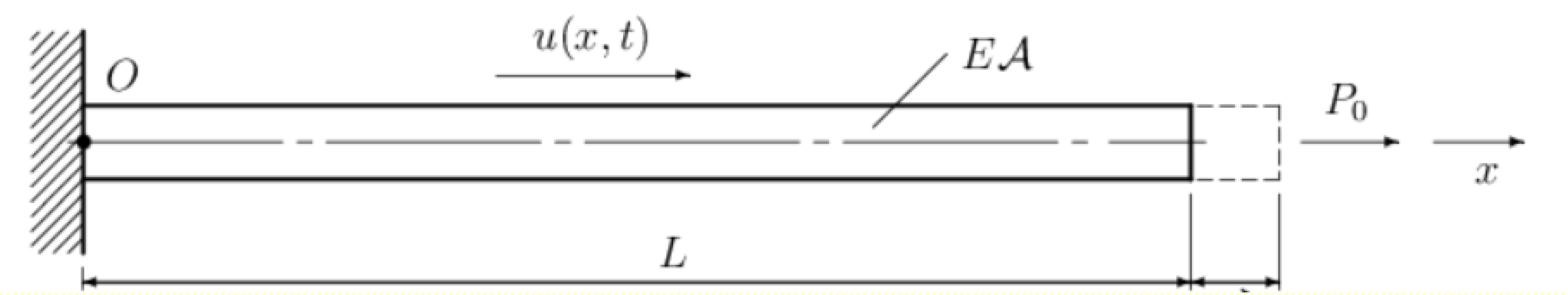
Static case of the Rod modelled with only 1 Finite Element:

RESPONSE TO INITIAL CONDITIONS

EXAMPLE-2: Axial Vibrations & FEM

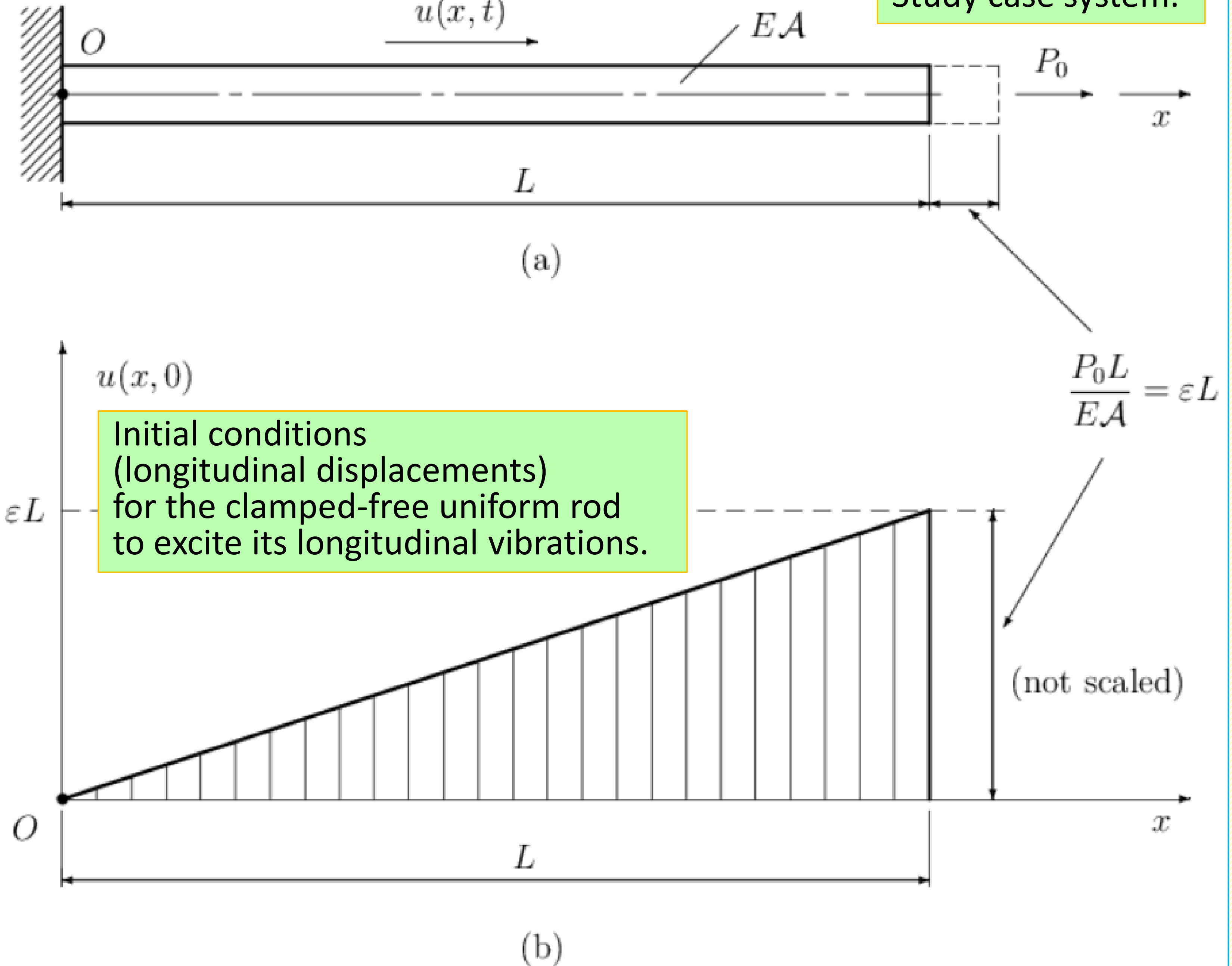
PROBLEM: As an exercise, we will use only ONE Finite Element to model rod, excited to vibrate in axial direction by application of the initial conditions (and no external force).

TASK: A uniform rod, with one end fixed and other free, is stretched under a static load, as shown in Figure, and suddenly released from rest at time t=0. From these initial conditions, determine the longitudinal displacements $\underline{u}(x,t)$.



REFERENCE: **Trivailo P.M.** (2008), Vibrations: Theory & Aerospace Applications, Vol.1&2, - (The textbook for senior undergraduate and graduate aerospace students). - Melbourne: RMIT Publisher - 2008. - 247pp +348pp=595 pp., 355 ill, 4 software programs.

Study case system.



EQUATIONS OF MOTION

$$\frac{mh}{6}\begin{bmatrix}2 & 1\\ 1 & 2\end{bmatrix}\begin{Bmatrix}\ddot{u}_1\\ \ddot{u}_2\end{Bmatrix}+\frac{EA}{h}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}\begin{Bmatrix}u_1\\ u_2\end{Bmatrix}=\begin{Bmatrix}0\\ 0\end{Bmatrix}\ \text{(for supplementary FREE-FREE ROD)}$$

Applying Boundary Conditions and FIXING left end (u_1=0)
by crossing out first equation and removing first rows on the remaining [m^e] and [k^e] matrices, we reduce previous equation to the following:

$$\frac{mh}{6}[2]\{\ddot{u}_2\}+\frac{EA}{h}[1]\{u_2\}=\{0\}\ \text{(for FIXED-FREE ROD) or}$$

$$[M]\,\ddot{u}_2+[K]\,u_2=[0]\quad\left(\text{here } [M]=\frac{mh}{6}[2] \text{ and } [K]=\frac{EA}{h}[1]\right)$$

This matrix EQ can be re-written using state-spaces concept. Assume 2 states: $\mathbf{x}=\begin{Bmatrix}u_2\\ \dot{u}_2\end{Bmatrix}$ then

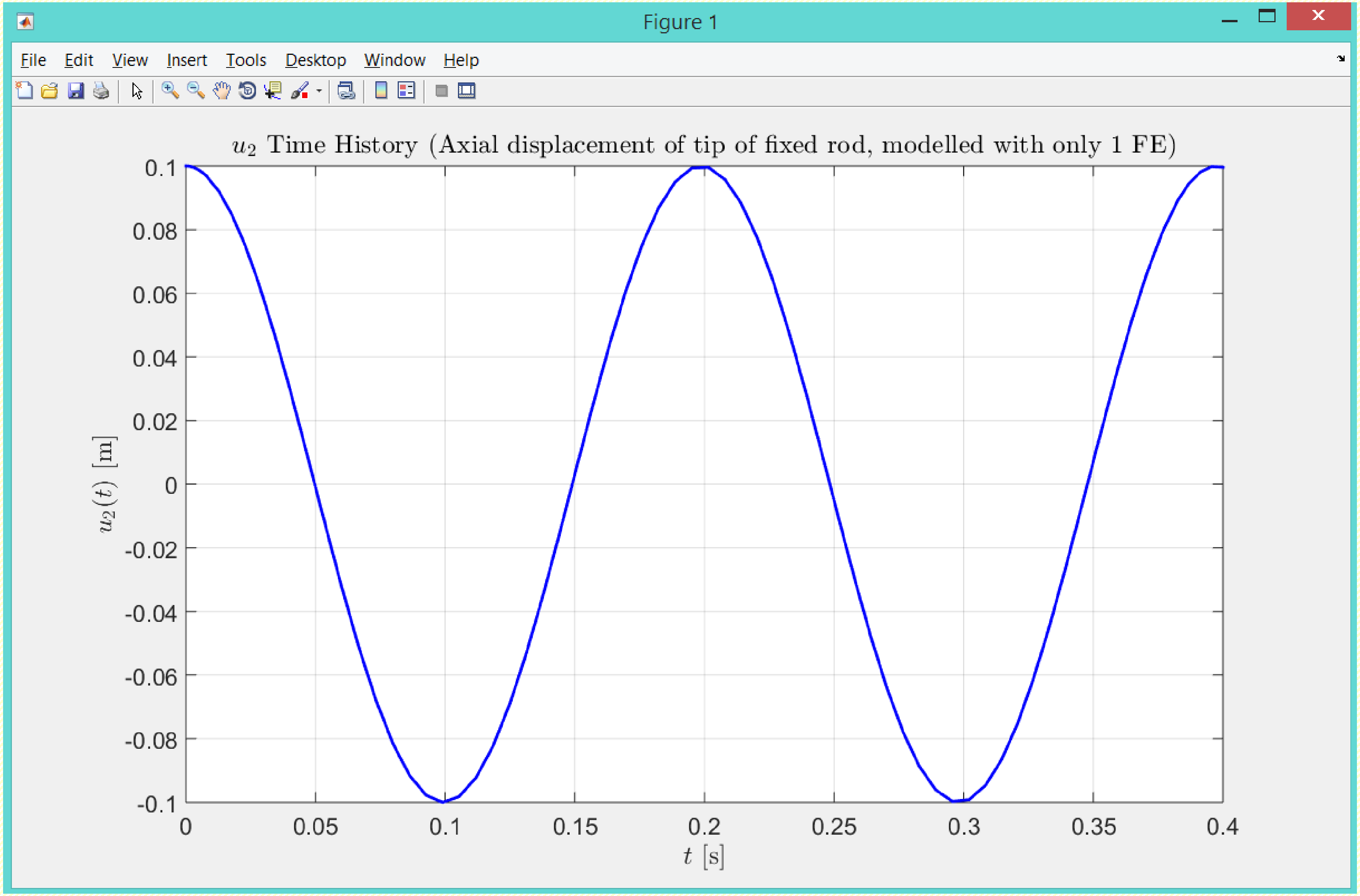
$$\begin{Bmatrix}\dot{x}_1\\ \dot{x}_2\end{Bmatrix}=\begin{Bmatrix}x_2\\ -\mathrm{inv}([M])*[K]*x_1\end{Bmatrix}$$

MATLAB SCRIPT (Solving Response to Initial Conditions):

Main Commands

```
%% OENG1116-S1-2020
% Designed by Prof P.M.Trivailo (C) 2020
%----------------------------
clear; clc; close('all')
a=0.08; b=0.08; h=5; e=0.1; % Data in [m]
E=0.01*10^9;                % Young Modulus [Pa]
rho=1.2*10^3;               % density [kg/m^3]
tmax=0.4;                   % Simulation time [s]
A=a*b; EA=E*A; m=rho*a*b*1;

x=[0:0.01:1]*h; H1x = 1-x/h; H2x = x/h; % Shape Functions
% --- SUPPLEMENTARY SYSTEM: Modelling rod with only 1 FE ---
ke=(EA/h)*[1 -1; -1 1]; me=(m*h/6)*[2 1; 1 2];
% --- MAIN SYSTEM: Constraining Left Boundary ---
me(:,1)=[]; me(1,:)=[]; ke(:,1)=[]; ke(1,:)=[];

invMxK=inv(me)*ke; x0=[e; 0]';
FEM_xdot_anon = @(t,x) [x(2); -invMxK*x(1) ];
[tt,xx] = ode45(FEM_xdot_anon,[0 tmax],x0);
figure; plot(tt,xx(:,1),'b','LineWidth',2); grid on;

xl=xlabel('$t$  [s]'); yl=ylabel('$u_2(t)$   [m]')
str=sprintf('$u_2$ Time History (Axial displacement of tip of fixed rod, modelled with only 1 FE)');
ti=title(str,'FontWeight','bold');
set([xl,yl,ti],'Interpreter','LaTeX');
set(gca,'FontSize',16);
set(gcf,'Position',[120 30  1200  700])
```

Plot *x-t-u* with *"u"* contour lines



Plotting Response Results for Clamped-Free Rod, modelled with only 1 FE:

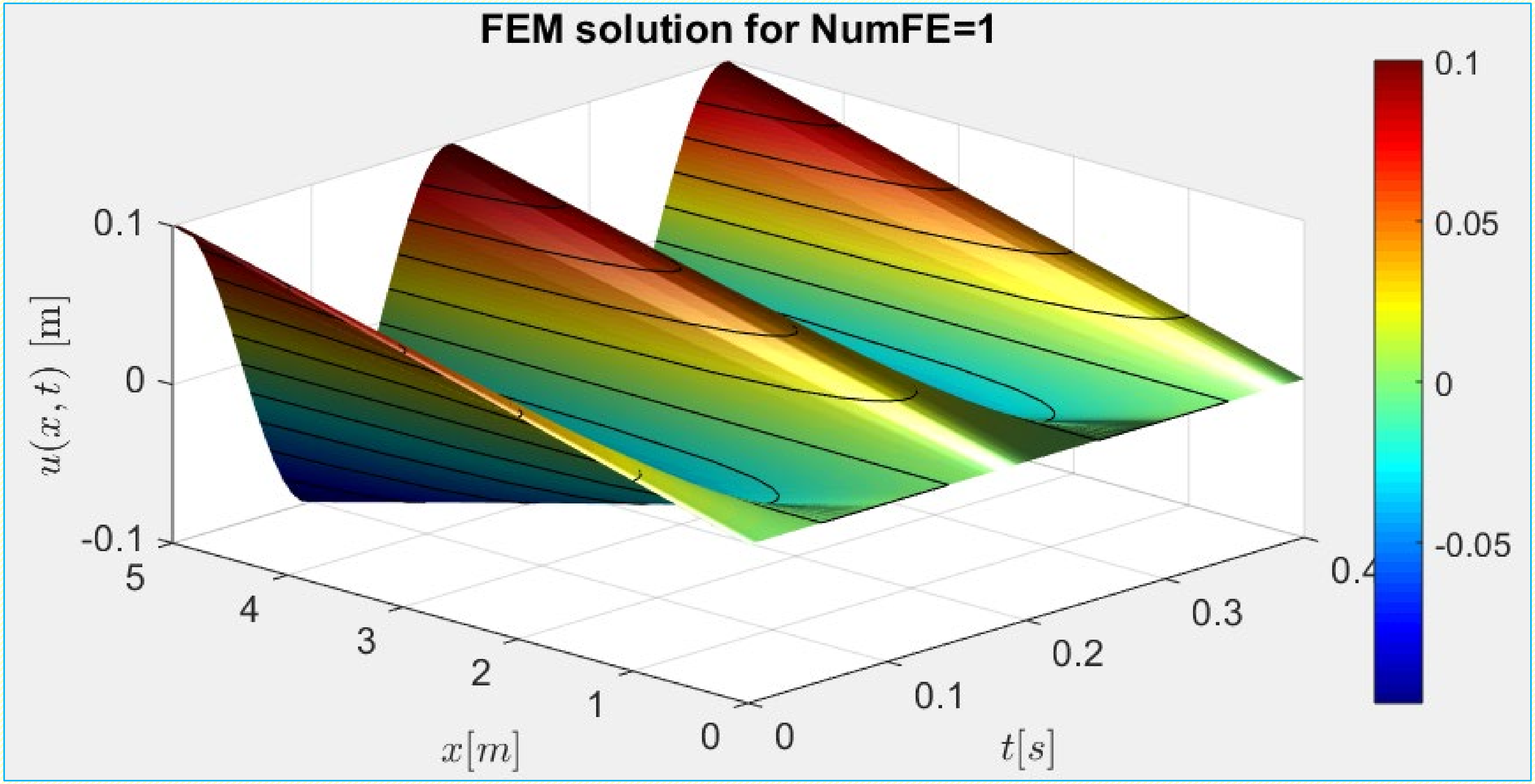
The same data is plotted, using different order of axes for 3D surface plot & "slicing" surface along each of the axes

MATLAB SCRIPT
(Plot *x-t-u* with *"u"* contour lines):

```
%% OENG1116-S1-2020
% Designed by Prof P.M.Trivailo (C) 2020
%---------------------------
%%
figure; hold on; grid on; rotate3d on;
[TT,XX]=meshgrid(tt,[0 h]);
ZZ=[0*xx(:,1)'; xx(:,1)'];
surf(TT,XX,ZZ);
colormap jet; lighting phong; shading interp;
contour3(TT,XX,ZZ,[-1:0.2:1]*e,'k');
view([-46, 36]);
%--- "Decorations" ---
colorbar; hh=camlight;
set(hh,'Position',[0.09,0.04,0.03]);
xl=xlabel('$t [s]$'); yl=ylabel('$x [m]$');
zl=zlabel('$u(x,t)$ [m]');
set([xl,yl,zl],'Interpreter','LaTeX');
str=sprintf('FEM solution for NumFE=1');
title(str,'FontWeight','bold');
set(gca,'FontSize',16);
set(gcf,'Position',[120   300   920   450]);
```

Main Commands

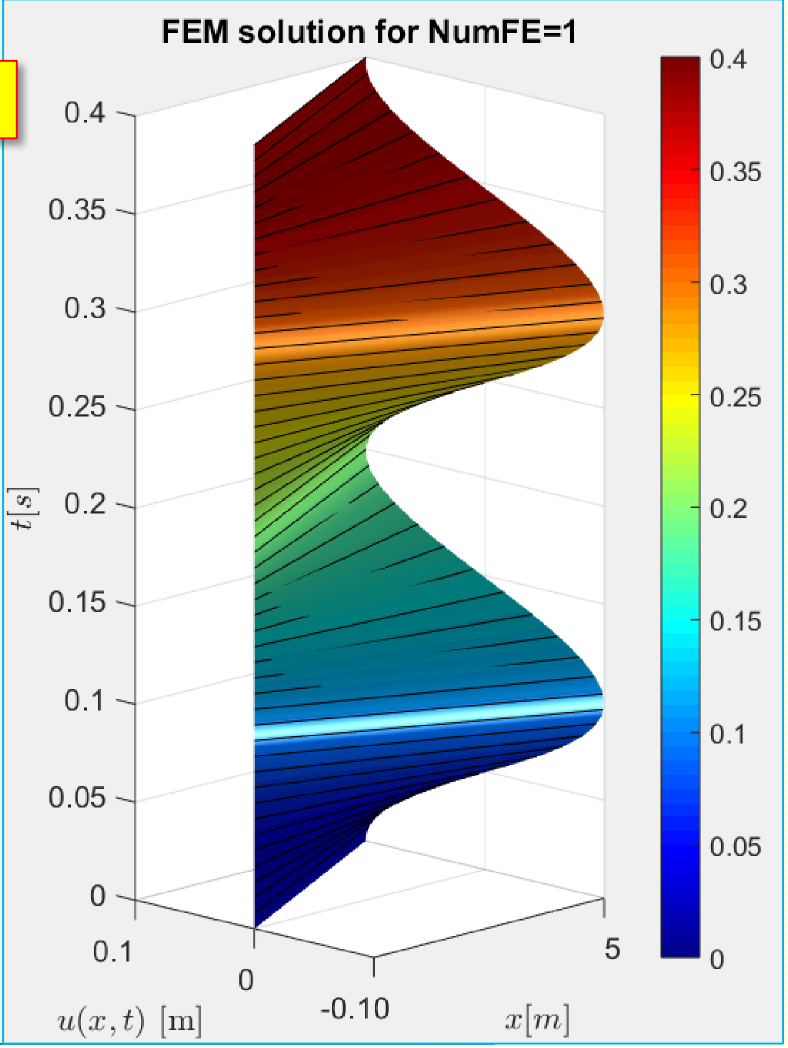
Plot *x-t-u* with *"u"* contour lines



MATLAB SCRIPT
(Plot *x-u-t* with *"t"* contour lines):

```
% Continuation of the Script
% Designed by Prof P.M.Trivailo (C) 2020
%-----------------------------
%%
figure; hold on; grid on; rotate3d on;
surf(XX,ZZ,TT);
colormap jet;
lighting phong; shading interp;
contour3(XX,ZZ,TT,[0:0.02:1]*tmax,'k');
view([-46, 6]);

%--- "Decorations"
colorbar; hh=camlight;
set(hh,'Position',[2,-4, 0.11]);
zl=zlabel('$t [s]$'); xl=xlabel('$x [m]$');
yl=ylabel('$u(x,t)$ [m]');
set([xl,yl,zl],'Interpreter','LaTeX');
str=sprintf('FEM solution for NumFE=1');
title(str,'FontWeight','bold');
set(gca,'FontSize',16);
set(gcf,'Position',[488   10   560   814]);
```

Main Commands

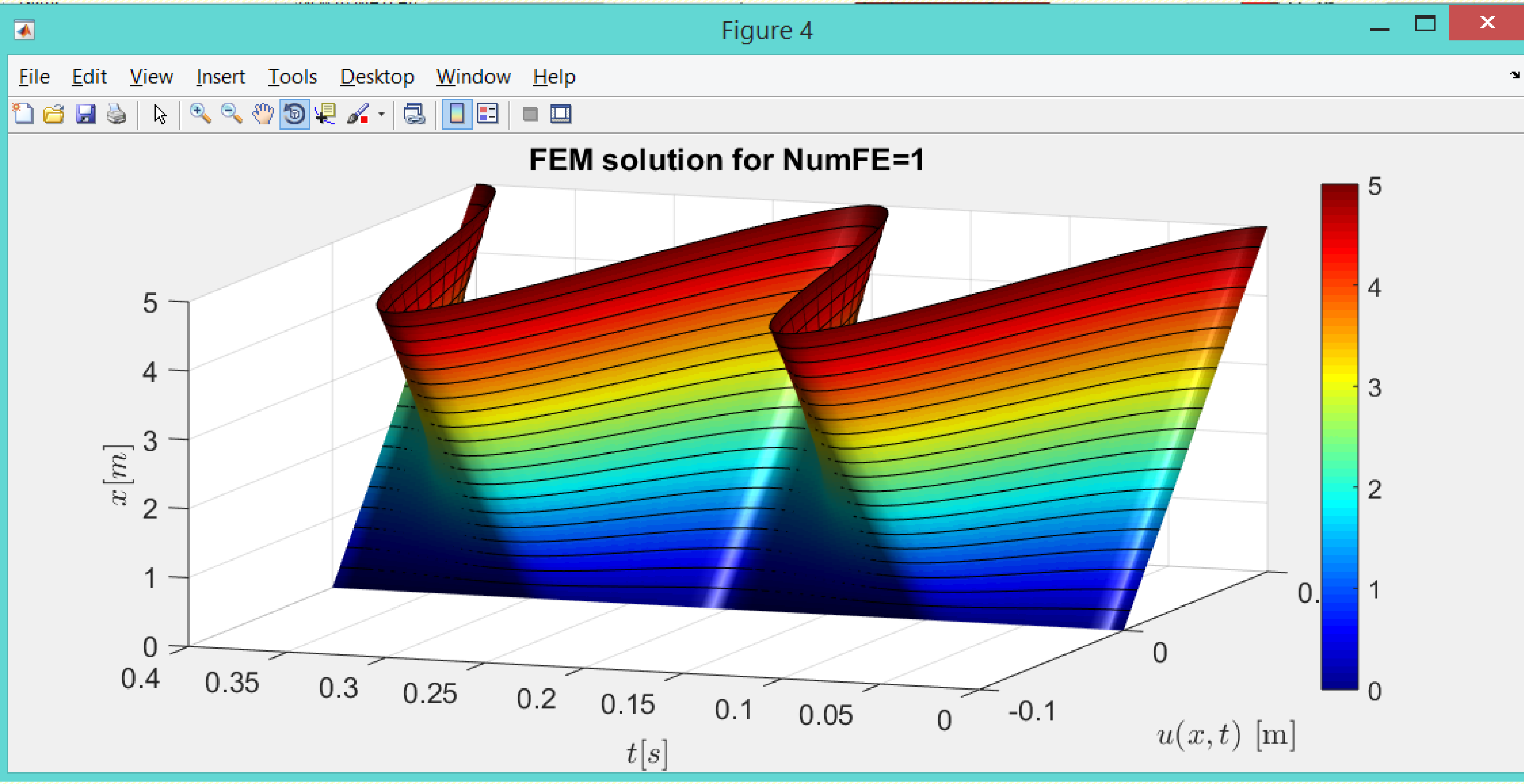


MATLAB SCRIPT (Plot *u-t-x* with *"x"* contour lines):

```
% Continuation of the Script
% Designed by Prof P.M.Trivailo (C) 2020
%--------------------------
%%
figure; hold on; grid on; rotate3d on;
surf(ZZ,TT,XX);
colormap jet;
lighting phong; shading interp;
contour3(ZZ,TT,XX,[0:0.05:1]*h,'k');
view([-70, 20]);

%--- "Decorations"
colorbar; hh=camlight;
set(hh,'Position',[-0.4, -1.2, 2]);
yl=ylabel('$t [s]$'); zl=zlabel('$x [m]$');
xl=xlabel('$u(x,t)$ [m]');
set([xl,yl,zl],'Interpreter','LaTeX');
str=sprintf('FEM solution for NumFE=1');
title(str,'FontWeight','bold');
set(gca,'FontSize',16);
set(gcf,'Position',[40 80  1120   470]);
```

Main Commands

MATLAB SCRIPT
(Plot *u-t-x* with *"x"* contour lines):



END OF WEEK-6 TUTORIAL SLIDES