Chapter 9
Exhaust System Acoustic Modeling

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Keywords Exhaust system • Sound • Acoustic filter • Physical networks • Transfer function • Power spectrum • Bode plot

9.1 Introduction

An exhaust system removes chemical and acoustic pollutions generated by a combustion engine. Like all other subsystems in automotive engineering, exhaust systems have evolved for all this years of automotive history, from simple cast iron manifold, with pipes and a silencer to complex systems that include catalytic converters, supertone silencer, muffler, and tailpipe tips. One of the key system components is a catalytic converter which converts poison gasses pollution by transforming it into carbon dioxide and water. System often has few exhaust pipes and silencers.

All components of the system, regardless of their primary role of chemical or noise pollution control, can be seen as a kind of acoustic filters. Mufflers are particularly designed in such a way that acoustic waves, travelling through, are maximally attenuated, obstructed, and subjected to self-interference. A study on acoustic modeling and testing of exhaust and intake system components is presented in Elnemr (2011). In addition to that, research of automotive exhaust system hanger location, for the best prevention of vibration transfer to the vehicle body, is conducted as presented in Jianwang et al. (2008). In our research, at this stage, we have concentrated on the acoustic side of the system.

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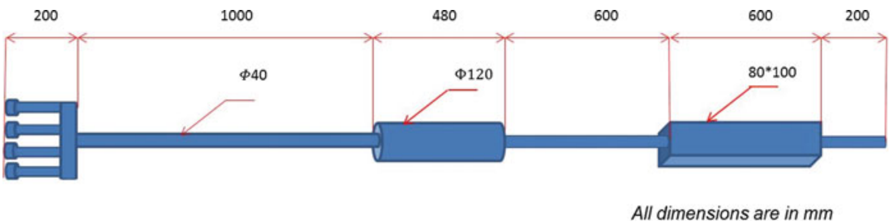


Fig. 9.1 An exhaust system structure

There is a large variety of system constructions for various vehicles. A generic exhausted system is shown in Fig. 9.1. Dimensions are given as they are used in our modeling. Some systems may have everything doubled, or only mufflers, or just tailpipe. This is used to increase power and torque, i.e., to introduce less attenuation of those key performance engine characteristics. For example, an investigation on exhaust system parameters for fuel economy improvement of small gasoline engine is presented in Peng et al. (2009). In addition to that there is still energy that can be extracted from the exhaust system as shown in Capel et al. (2013). We have measured the real signal, i.e., sound pressure, from the vehicle and designed exhaust system model starting from that.

Our investigation of the system acoustics is based on the Physical Networks' approach. Any engineering system can be expressed as a physical network (Sanford 1965). It can incorporate physical quantities of the same type, like mechanical system with translator motion, or just electrical system. Modern engineering systems are mainly comprehensive, including physical subsystems with different types of quantities, like electromechanical, or more than that, mechatronics (Silva 2005). Physical networks approach, modeling and simulations of acoustic, hydraulic, and mechanical systems, using Electronics Circuits Analysis Program (ECAP) are presented in Simic et al. (1978).

In this paper, the basic physical network principles are explained in Sect. 9.1. Components of the exhaust physical network are introduced in Sect. 9.2. The whole system model and simulation by a large ladder filter topology are presented in Sect. 9.3. Transfer function, i.e., bode diagram of the model is given in the next section. Finally frequency characteristic of the measured sound, i.e., power spectrum of the sound from the real vehicle is compared to the model performances and presented in the final section of this chapter.

9.2 Physical Networks

Long time ago, in the nineteenth century, scientists, physicists, and philosophers have stated that the unity of the nature is expressed in the amazing analogies of the differential equations used to represent various physical phenomena. The same equations can be used for the study of hydrodynamics and for the electrical potential theory. The theory of turbulence in liquids and the theory of friction in gases show extreme analogies with the electromagnetic theory.

Our, nonlinear, exhaust system, as shown in Fig. 9.1, can be presented as a cascaded structure of linear, one-dimensional physical subsystems with constant parameters. Those building components, called physical networks, consist of ideal linear elements with two terminals. Network elements represent mathematical relationships between two dependent system variables in a physical system. Often, real systems are extremely complex, so that they cannot be easily simplified, but many systems are in that category, or have linear subsystems as their integral parts.

An equivalent physical network represents a sort of linear graph associated to the system equations. The basic definitions, principles, and rules for solving system equations represented by the network are independent of physical system that the particular network is representing.

9.2.1 Network Variables

Physical networks have two basic time dependent variables: flow, $\boldsymbol{f}$, and potential, $\boldsymbol{p}$.

- Flow is a variable that flows through network elements and connection lines
- Potential is a variable measured across a network element, or between two network points
- Potential of a point in the network depends on the chosen referent point.

Examples of physical variables are electrical *current* and *potential*. Current is directly related to the flow of electrons as $i = \frac{dq}{dt}$. Potential difference is called voltage. Then we have *force* and *velocity* in a mechanical system with translation, or *torque* and *angular* speed in a mechanical system with rotation. In a hydraulic system we have *flow* and *pressure*. All those systems can be modeled using the same type of ordinary differential equations (ODE).

9.2.2 Ordinary Differential Equations

We can use ODEs with constant coefficients for modeling various one-dimensional physical systems. An ODE is a relation between two variables, t and $y = y(t)$, and the derivatives of y, $\frac{dy}{dt}$, $\frac{d^2y}{dt^2}, \ldots, \frac{d^ny}{dt^n}$.

$$\boldsymbol{A_n \frac{d^n y}{dt^n} + A_{n-1} \frac{d^{n-1} y}{dt^{n-1}} + \cdots + A_1 \frac{dy}{dt} + A_0 y = f(t)} \tag{9.1}$$

Equation (9.1) is called ordinary because only one independent variable exists, which is usually time, and it is linear because only the first exponent of dependent variable, or its derivatives is present. General solution, $y_G(t)$, of an ODE is given as

$$y_G(t) = y_H(t) + y_P(t),$$

where $y_H(t)$ is general solution of equation when $f(t) = 0$, and $y_P(t)$ is any solution of the Eq. (9.1).

9.2.3 Network Elements

Physical network is a linear graph that includes network elements and connecting lines between them. There are two types of elements: active and passive. Active elements are flow, or potential sources, whose operations are expressed as functions of time: $\boldsymbol{F} = \boldsymbol{F}(\boldsymbol{t})$ and $\boldsymbol{P} = \boldsymbol{P}(\boldsymbol{t})$. They supply energy to the system. Passive elements are two connection points elements with defined relationship between *flow* through and *potential* across elements' terminals. They cannot supply more energy to the system than what was already accumulated. That is expressed through initial conditions. There are three types of passive elements used to express three different relationships between network variables, *flow* and *potential*. Those basic relationships are *proportion*, *integration*, and *differentiation*.

Examples of all three types of passive elements in generic physical network, as well as in electrical and mechanical system with translation are given in the Table 9.1. Expressions for the accumulated energy and power dissipation in each of those systems are also given. In a generic physical system, two network quantities relations as integration, proportion, and differentiation are associated to elements labeled as A, B, and C, as shown in Table 9.1. The general name for all elements, A, B, and C is impedance, Z, or admittance Y. Similarly, analog to those quantities, in an electrical system we have inductivity, L, conductivity G, i.e., resistivity, $R = 1/G$, and capacity C. Network variables are current, i, and voltage, u, which is electrical potential difference. Finally for the translation we have k for stiffness, B for friction, m for mass of the object. Network variables here are force, F, and velocity v.

Table 9.1 Analogies relationships

Relationship	Prototype	Electrical	Translation
Integration	$f = A \int pdt$	$i = \frac{1}{L} \int udt$	$F = k \int vdt$
Accumulated energy	$\frac{f^2}{2A}$	$\frac{Li^2}{2}$	$\frac{F^2}{2k}$
Proportion	$\mathrm{f} = \mathrm{Bp}$	$i = Gu = \frac{1}{R}u$	$\mathrm{f} = \mathrm{Bv}$
Power dissipation	$fp = \frac{f^2}{B} = p^2 B$	$iu = i^2 R = \frac{u^2}{R}$	$Fv = \frac{F^2}{B} = v^2 B$
Differentiation	$f = C \frac{dp}{dt}$	$i = C \frac{du}{dt}$	$F = m \frac{dv}{dt}$
Accumulated energy	$\frac{Cp^2}{2}$	$\frac{Cu^2}{2}$	$\frac{mv^2}{2}$

There are also other systems like thermal, fluids, or rotation where analogies could be easily established as given in Sanford (1965) and Silva (2005).

9.2.4 Mono and Multi-dimensional Systems

Apart from already presented mono-dimensional systems, real physical systems are often multi-dimensional. In that case, they have to be represented by partial differential equations including more than one independent variable. Systems that deal with the physical fields' effects in 3D space, like gravity, or electromagnetic field are multi-dimensional. Even when energy is distributed through one dimension, but that transfer is the function of time, we have a multi-dimensional system.

In mono-dimensional dynamic systems, where independent variable is time, we can assume that, the energy associated to the dependent variables is transferred instantly through the system, to all remote parts. Accordingly, dependent variables are just functions of time and not the distance from the energy source. We should mention here that, there are also static mono-dimensional systems, where independent variable is not time, but could be distance x. Examples of such systems could be found in Statics, a branch of Mechanics.

The issue of system multi-dimensionality, caused by the energy transition time, could appear in electrical systems of the small dimensions, when the emitting frequency of the energy source is high, or when we have long distance energy transmissions. While electrical energy travels relatively quickly many other forms of energy are for the order of magnitude slower than that. Examples are acoustic waves, forces, and waves in liquids, and especially heat energy transfer in solids. Those systems have to be treated as multi-dimensional even when low frequencies are involved and physical dimensions of the systems are small.

In order to get objective criteria for the best estimation of the system dimensionality, when harmonic energy sources are applied, we should look at the source signal, i.e., energy wavelength, λ. It is given as

$$\lambda = \frac{\mathrm{c}}{\mathrm{f}} \tag{9.2}$$

where c is the transfer speed and f is the frequency defined by the energy source. Wavelength must be larger than the longest dimension in the physical system so that the system can be treated as mono-dimensional. When that is not the case, then system has to be treated as multi-dimensional since impedance is not concentrated in one point only. It is distributed along the length of the system and following that, partial differential equations have to be used, with two independent variables, time, and the distance.

9.3 Acoustic Elements

Exhaust system is a nonlinear, multi-dimensional physical network. Since we are now concentrated on the acoustics of the system, only, it is for us an acoustic physical network. Nonlinearities come from the changes of the temperature and the gas pressure along the system, as well as the physical dimensions and the shape of the system structure.

9.3.1 From Multi to Mono-Dimensional System Model

Let us start with the dimensionality problem first. The longest dimension in the systems is the length, which is 3,800 mm. The speed of sound, in the air, at 20 °C is equal to $c_{20} = 343$ m/s. Temperature of the gasses is higher than the temperature of the environment, which influence the speed, but for the initial estimations we will take the speed as $c = c_{20} = 343$ m/s. Now we need the frequencies of the emitted sound. In order to get that, sound is recorded and power spectrum is extracted using the program shown in Fig. 9.2.

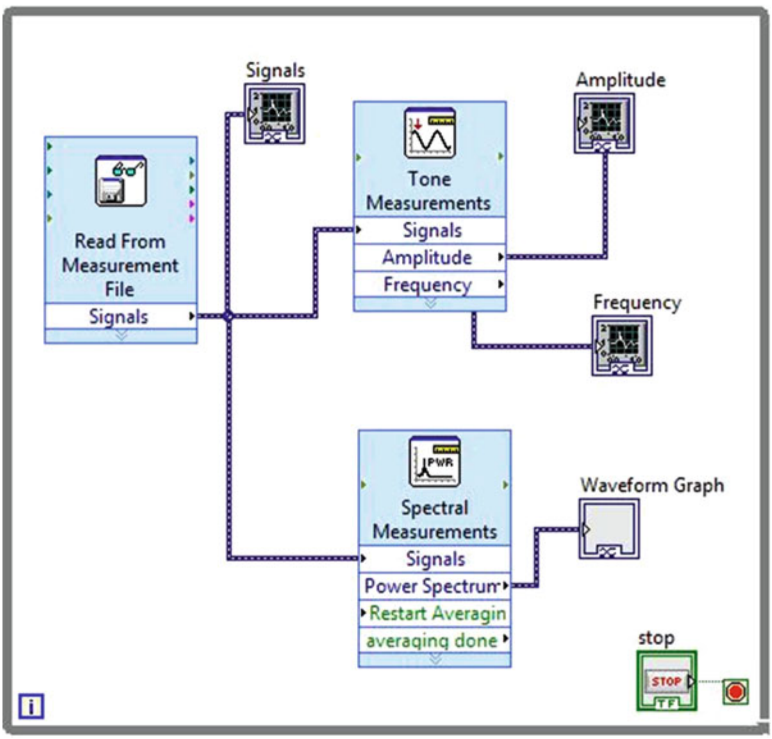


Fig. 9.2 Power spectrum measurement program

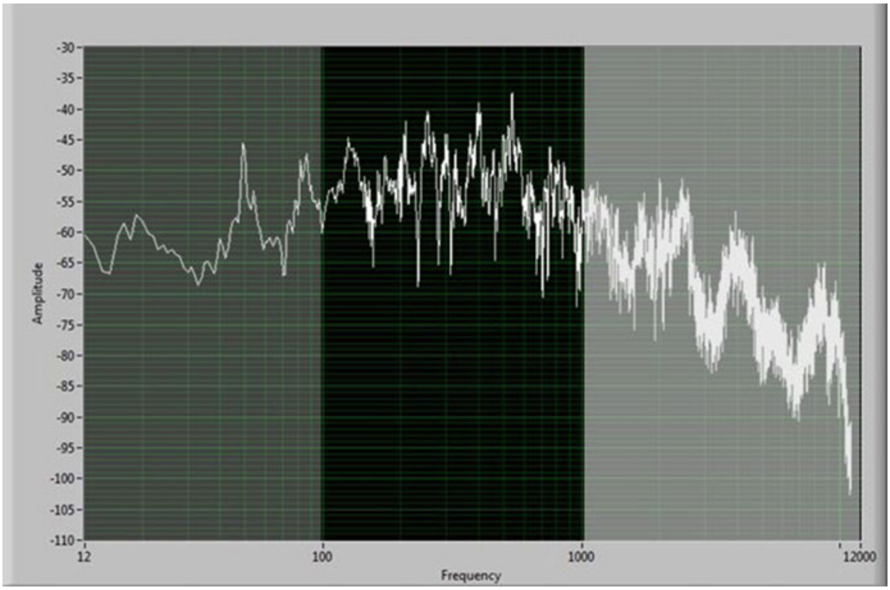


Fig. 9.3 Exhaust system sound power spectrum

Powers spectrum of the sound is shown in Fig. 9.3. We can see that the majority of energy is transferred by the frequency components in the rage of 10–1,000 Hz. The maximum energy is carried by the components around 500 Hz.

Following that we can calculate wavelengths associated to those frequencies:

$$\lambda_1 = \frac{c}{f_1} = \frac{343}{10} = 34.3m\ \lambda_2 = \frac{c}{f_2} = \frac{343}{1,000} = 0.343m \tag{9.3}$$

We will now split the system into the subsystems whose maximum linear dimensions are small compared to the minimum wavelength in the range of interest, which is λ_2. In the first simulation we will adapt the subsystems' lengths of 80, 160, and 200 mm as given by the vector ***V***. Vector components, associated to the exhaust system from the Fig. 9.1, are all given in mm. After comparing simulation results by the real measurements, given in the Fig. 9.3, we will see if further subdivisions are needed.

$$V = (200,\ 200,\ 200,\ 200,\ 200,\ 200,\ 80,\ 80,\ 80,\ 80,\ 80,\ 80,\ 200,\ 200,\ 200,\ 200,\ 200,\ 200,\ 200,\ 200) \tag{9.4}$$

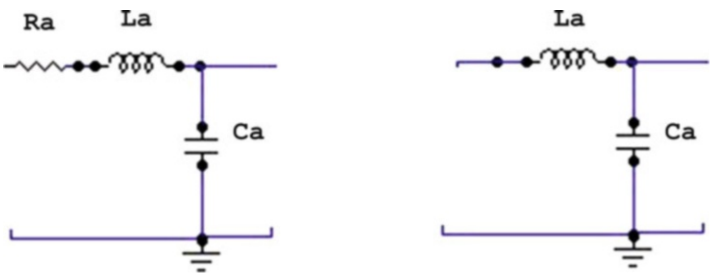


Fig. 9.4 Basic acoustic subsystem

9.3.2 Acoustic Network Elements Linearization

Exhaust system, shown in the Fig. 9.1, can be represented as a cascaded network of $R_aL_aC_a$ subsystems, shown in Fig. 9.4, where R_a is a symbol for the acoustic resistance, L_a is a symbol for the acoustic inductance, and C_a represents acoustic capacity.

Acoustic resistivity is an element that dissipates acoustic energy in the system. It incorporates all sound attenuation components, as found along the whole exhaust, especially in catalytic converters and mufflers. For the simplicity of the initial modeling we will neglect effects of the resistivity as a separate component and represent basic subsystem without proportional element, i.e., resistivity, as shown in the Fig. 9.4. At the same time we are going to include attenuation through the effects of more rapid temperature decrease in the converter and mufflers chambers. The fast temperature change is caused by the gas molecules kinetic energy drop, when colliding with obstacles on the path through the various system components.

Acoustic capacity and inductivity are calculated using the following expressions:

$$C_a = \frac{Sl}{\varrho c^2} \quad L_a = \varrho \frac{l}{S} \tag{9.5}$$

where c is the transfer speed, as before, S is the cross section surface area, l is the length of the subsystem, and ρ is the gas density (Sanford 1965; Simic et al. 1978). Parameter l (*length*) takes the values from the components of the vector $\boldsymbol{V}$, given by Eq. (9.4).

The next nonlinearity issue to challenge is the temperature dependency of the whole system that can be represented as changes in network elements L_a and C_a, as the function of the temperature. Let us start with the ideal gas low equation

$$\begin{gathered} \frac{pV}{T} = Const \\ \frac{p_1V_1}{T_1} = \frac{p_0V_0}{T_0} \end{gathered} \tag{9.6}$$

where

$$p_0 = 760mmHg = 0.1 * 10^6 N/m^2 = 10^6 \mu bar$$

is normal atmospheric pressure, while V_0 is gas volume at the temperature of T_0,

$$T_0 = 273.15K = 0°C.$$

Let us assume that the pressure difference at the borders of the subsystems is $\triangle p = 200\ \mu bar$. Then we have the following:

$$p_1 = p_0 + \Delta p$$

$$\frac{p_1}{p_0} = \frac{10^6 + 200}{10^6} = 1.0002 \approx 1$$

Following that we have simplified Eq. (9.6) as given below,

$$\frac{V_1}{T_1} = \frac{V_0}{T_0} \quad or \quad \frac{V_0}{V_1} = \frac{T_0}{T_1} \tag{9.7}$$

Gas density is expressed as

$$\varrho_0 = \frac{m}{V_0}\ for\ T = T_0$$

$$\varrho_1 = \frac{m}{V_1}\ for\ T = T_1$$

Dividing last two equations we have

$$\frac{\varrho_1}{\varrho_0} = \frac{V_0}{V_1} \tag{9.8}$$

From the Eqs. (9.7) and (9.8) we derive

$$\varrho_1 = \varrho_0 \frac{V_0}{V_1} = \varrho_0 \frac{T_0}{T_1} \tag{9.9}$$

Finally expression for the acoustic inductivity becomes

$$L_a = \varrho \frac{l}{S} = \varrho_0 \frac{l}{S} \frac{T_0}{T_1} \tag{9.10}$$

Specific gas density is labeled as ρ_0 at the temperature of $T_0 = 0\,°C = 273.15\ °K$. Working temperature is labeled as T_1. Gas density is another nonlinear quantity, which depends on the engine cycle, but we will assume that it is constant and equal to the air density.

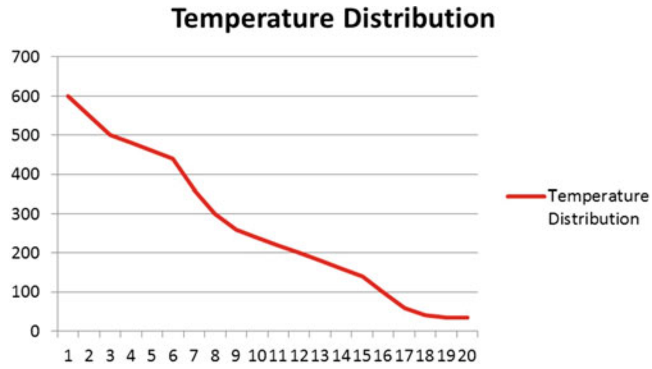


Fig. 9.5 Gas temperature distribution along the system

Finally expression for the acoustic capacity would be:

$$C_a = \frac{Sl}{\varrho c^2} = \frac{Sl}{\varrho_0 \frac{T_0}{T_1}\left(c_0\sqrt{\frac{T_1}{T_0}}\right)^2} = \frac{Sl}{\varrho_0 c_0{}^2} \tag{9.11}$$

We can see that the acoustic capacity is not the function of the temperature.

Gas burning temperature depends on the engine type but it is around 650 °C generally. At the exit, out of the tailpipe, temperature is still above ambient temperature and we will adapt the value of 35 °C. Temperature distribution is given in the Fig. 9.5.

We can see that there are 20 subsystems, with the length distribution as given by the vector $\boldsymbol{V}$, Eq. (9.4), and with the temperature distribution as per Fig. 9.5, from 600 °C at the start of the cascade down to 35 °C at the end. In addition to that, each subsystem has its cross section surface area as specified by Fig. 9.1. We have circles with given diameters as $\Phi 1 = 40$ and $\Phi 2 = 120$ and a rectangle with 80×100, all in *mm*.

In the subsystems which have higher temperatures, sound speed is higher too, and accordingly the wavelength as per Eq. (9.2). As a consequence, the approximation of the physical system is more accurate because of the better relationship between wavelength and the maximum liner dimensions of the system.

9.4 Ladder Topology

We can now design the whole system. Figure 9.6 shows a segment of the ladder network that consists of the subsystems as given in the Fig. 9.4.

In this physical network we have *flow* and *pressure* as network variables.

An Excel program is used to calculate distributed parameters. A fragment from the calculations is demonstrated in Fig. 9.7.

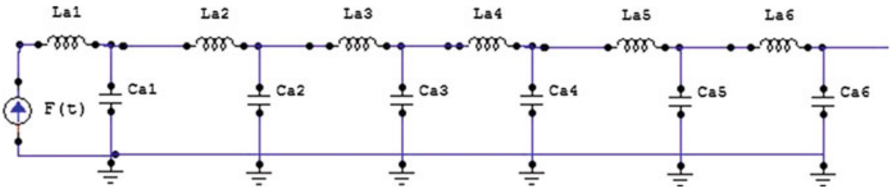
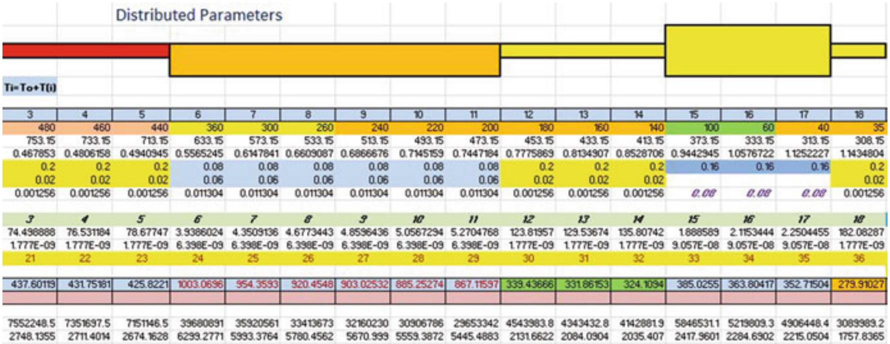


Fig. 9.6 A segment from the acoustic ladder network



3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
480	460	440	360	300	260	240	220	200	180	160	140	100	60	40	35
753.15	733.15	713.15	633.15	573.15	533.15	513.15	493.15	473.15	453.15	433.15	413.15	373.15	333.15	313.15	308.15
0.467853	0.4806158	0.4940945	0.5565245	0.6147841	0.6609087	0.6866676	0.7145159	0.7447184	0.7775869	0.8134907	0.8528706	0.9442945	1.0576722	1.1252227	1.1434804
0.2	0.2	0.2	0.08	0.08	0.08	0.08	0.08	0.08	0.2	0.2	0.2	0.16	0.16	0.16	0.2
0.02	0.02	0.02	0.06	0.06	0.06	0.06	0.06	0.06	0.02	0.02	0.02				0.02
0.001256	0.001256	0.001256	0.011304	0.011304	0.011304	0.011304	0.011304	0.011304	0.001256	0.001256	0.001256	0.08	0.08	0.08	0.001256
3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
74.498888	76.531184	78.67747	3.9386024	4.3509136	4.6773443	4.8596436	5.0567294	5.2704768	123.81957	129.53674	135.80742	1.888589	2.1153444	2.2504455	182.08287
1.777E-09	1.777E-09	1.777E-09	6.398E-09	6.398E-09	6.398E-09	6.398E-09	6.398E-09	6.398E-09	1.777E-09	1.777E-09	1.777E-09	9.057E-08	9.057E-08	9.057E-08	1.777E-09
21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36
437.60119	431.75181	425.8221	1003.0696	954.3593	920.4548	903.02532	885.25274	867.11597	339.43666	331.86153	324.1094	385.0255	363.80417	352.71504	279.91027
7552248.5	7351697.5	7151146.5	39680891	35920561	33413673	32160230	30906786	29653342	4543983.8	4343432.8	4142881.9	5846531.1	5219809.3	4906448.4	3089989.2
2748.1355	2711.4014	2674.1628	6299.2771	5993.3764	5780.4562	5670.999	5559.3872	5445.4883	2131.6622	2084.0904	2035.407	2417.9601	2284.6902	2215.0504	1757.8365

Fig. 9.7 Distributed parameters calculations

We have to clarify just one more design solution. Header, as shown in Figs. 9.1 and 9.8, consists of four extractors with the diameter of 40 mm and the length of 200 mm. Those four pipe segments can be represented in the physical network as four parallel connected acoustic inductors and four parallel connected acoustic capacitors, as shown in Fig. 9.8. Energy sources are presented as series connection with acoustic inductivities. Equivalent circuit or network structure is given in Fig. 9.8. This subsystem is presented as parallel connection of elements because we have about the same pressure difference on each of them and the total flow down the pipe, i.e., the whole exhaust system is equal to the sum of four flows. This qualifies such a system as a parallel structure in the physical networks. According to that, total equivalent inductivity L_{ae} and capacity C_{ae} of the first subsystem are given as

$$L_{ae} = \frac{L_a}{4} \quad and \quad C_{ae} = 4C_a \tag{9.12}$$

To finalize this discussion we have to mention that the exhaust system behaves as a sound source at the end of tailpipe. We need to know impedance of the open pipe. It can be show that it is predominantly inductive. We also have to know that the gas has a one-directional motion component, apart from oscillations, and that it moves outside of the exhaust forming a hot gas cylinder. By observation we concluded that this cylinder has the length of around 200 mm as the most of our subsystems. We put that in the calculations together with the simulation of the open space by the larger values for the last subsystem dimensions.

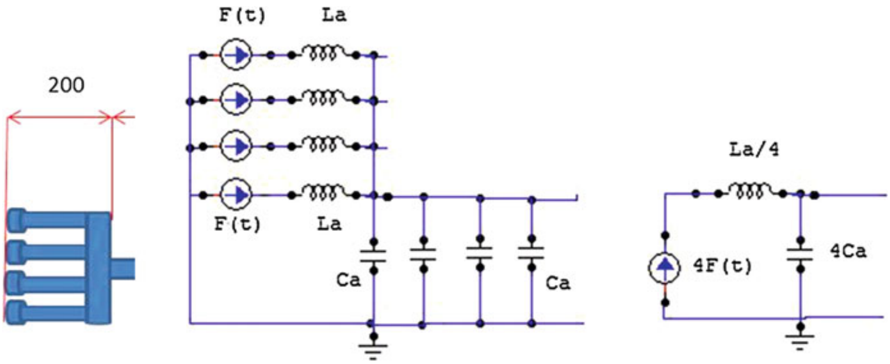


Fig. 9.8 Header and its equivalent physical network: Flow sources F(t) are represented by arrows, acoustic inductors as La and capacitors as Ca. Equivalent circuit is given on the right where equivalent source flow is 4F(t) and equivalent components are as given in the text and on the figure

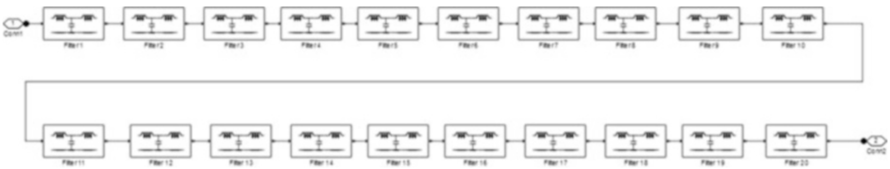


Fig. 9.9 Simulink model of the cascaded acoustic filter

We have used various modeling environments in the different stages of the design. Modern technology enables easy integration and combination of results from the different programming platforms. Our model in Simulink is shown in Fig. 9.9. Ladder network shown in Fig. 9.6 is built on *L half* sections, while the other one from the Fig. 9.9 is based on *T section* filter subsystems integration.

9.5 Transfer Function

Let us look at the basic L Half section of our ladder network as shown in Fig. 9.4. Transfer function in Laplace domain of this $L_a C_a$ filter is given as

$$T = \frac{u_{out}}{u_{in}} = \frac{\frac{1}{Cs}}{\frac{1}{Cs} + Ls} = \frac{1}{1 + LCs^2} \tag{9.13}$$

Applying this to the whole ladder network we come up with the acoustic transfer function of the exhaust system.

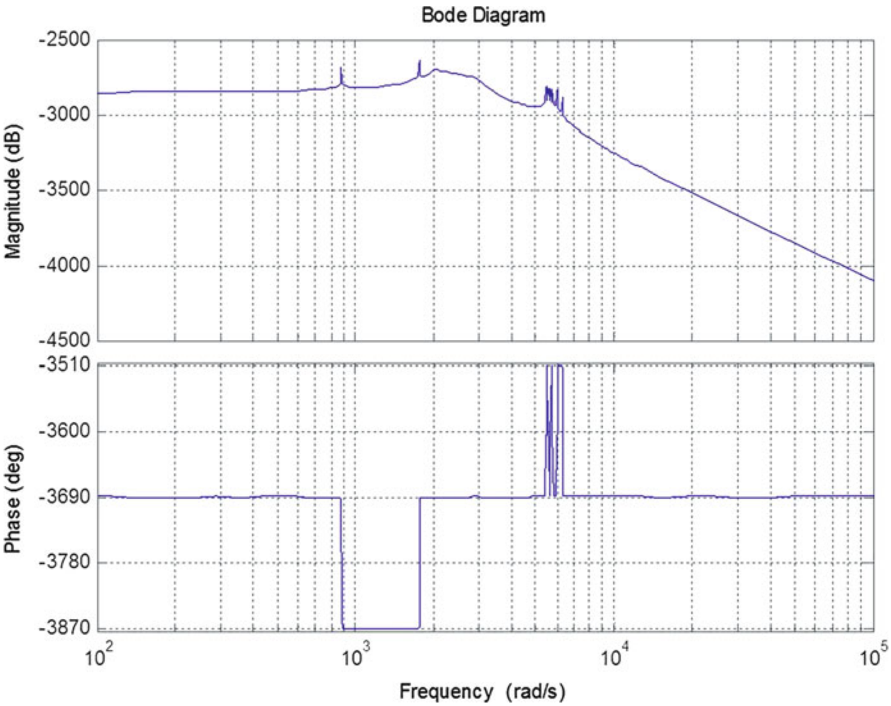


Fig. 9.10 Bode plots of the exhaust system

$$T_{all} = \frac{1}{\prod_{i=1}^{i=20} (1 + L_{ai} C_{ai} s^2)} \tag{9.14}$$

Bode plot of the transfer function is calculated using MATLAB and the magnitude and phase shift graphs are shown in the Fig. 9.10.

9.6 Conclusion

Comparing bode plot graph from the Fig. 9.9 and the power spectrum diagram from the real exhaust system, shown on the Fig. 9.3, we can see that our model simulate exhaust system with extremely high precision. Figure 9.9 also presents phase response of our acoustic ladder filter. Both transfer function diagrams have frequency expressed in *rad/s* shown on *x* axis in logarithmic scale. Phase shift plot is shown in *degrees*, while magnitude is presented in decibels of power. The range presented is from 10^2 to 10^5 rad/s. Since we know that $w[rad/s] = 2\pi f[Hz]$, the frequency range covered by the diagrams in Fig. 9.9, in *Hz* is from 16 to 1,600 Hz. This was our range of interest according to the real, measured data from the vehicle, Fig. 9.3.

This model is now a tool for the further investigation in acoustic properties and design of the new exhaust systems. We can easily increase number of subsystems and so achieve better relationship between wavelength and the maximum dimension in the subsystem. In addition to that we could place temperature sensors along the system to get more accurate temperature distribution. Microphones, pressure sensors, infrared, and other could be also used to collect more real data (Ba et al. 2010).

Key Symbols

A, B, C	Network elements
A_i	Ordinary Differential Equation (ODE) Coefficient i, $i = 0, 1, \ldots, n$
c	Wave transfer speed
C	Capacity
C_a	Acoustic capacity
C_{ae}	Equivalent acoustic capacity
$\frac{d^i y}{dt}$	ith derivative of y, $i = 1, 2, \ldots, n$
dp	An infinitesimal change in potential
dq	An infinitesimal change in electric charge
$\frac{dq}{dt}$	First derivative of charge with respect to time
dt	An infinitesimal change in time
du	An infinitesimal change in voltage
dv	An infinitesimal change in velocity
E	*E*quivalent sound source
f	Flow
f	Frequency
$f(t)$	Function of time
$F, F(t)$	Flow source
F	Force
G	Conductivity
i	Electric current
k	Spring stiffness
l	Subsystem length
L	Inductivity
L_a	Acoustic inductance
L_{ae}	Equivalent acoustic inductance
m	Mass
p	Potential
$P, P(t)$	Potential source
q	Electric charge
R	Resistivity
R_a	Acoustic resistivity
s	Complex argument

S	Cross section surface area
t	Time
T, T_0	Temperature
T	Transfer function
u	Voltage
v	Velocity
w	Angular frequency
V, V_0	Gas volume
$\mathbf{V}$	Lengths vector
y	Variable
$y_G(t)$	ODE general solution
$y_H(t)$	Homogenous ODE solution
$y_P(t)$	A particular solution of an ODE
Y	Admittance
Z	Impedance
$\triangle p$	Pressure difference
λ	Wavelength
π	pi constant
$\prod_{i=1}^{i=20}$	Product from i = 1 to i = 20
ρ	Gas density
Φ	Diameter

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