Tutorial # 4.

Overview: Disturbance observer for PI controller.

In the design, we choose two desired closed-loop poles to calculate the controller and estimator gain.

$-\alpha_1$ is the closed-loop pole for proportional control.

$$K_1 = \frac{\alpha_1 - a}{b}$$

$-\alpha_2$ is the closed-loop pole for the estimator:

$$K_2 = \frac{\alpha_2}{b}$$

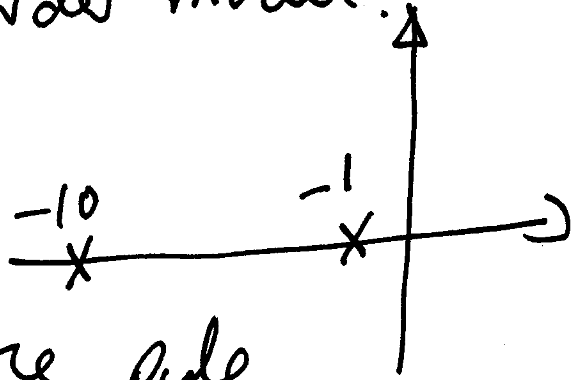
model:

$$G(s) = \frac{b}{s+a}.$$

Q1: $$G(s) = \frac{e^{-0.001s}}{(s+1)(s+10)}.$$

In order to design the PI controller, for disturbance observer, we simplify the model to get a first order model.

Two poles:



we will neglect the pole at -10. We write

$$G(s) = \frac{e^{-0.001s}}{(s+1)\times 10\left(\frac{1}{10}s+1\right)}$$

We approximate $\frac{1}{\frac{1}{10}s+1} \approx 1$ and $e^{-0.001s} \approx 1$

The first order model becomes

$$G(s) \approx \frac{0.1}{s+1}.$$

$a = 1, \quad b = 0.1.$

$$K_1 = \frac{\alpha_1 - a}{b} = \frac{2a - a}{b} = \frac{a}{b} = 10.$$

$$K_2 = \frac{\alpha_2}{b} = \frac{3a}{b} = 30$$

We evaluate the closed-loop performance through simulation. Note that in the simulation, we need to use the original system.

We simulate the closed-loop response without constraints first to find the

$u^{max} \Rightarrow 40$

$u^{min} \Rightarrow -12.$

then we simulate again with the constraints: $u^{max} = 40 * 0.85$

$u^{min} = -12 * 0.85.$

Q2: $G(s) = \dfrac{2e^{-0.5s}}{(s+0.1)(s+10)^2}$ (4)

The system has three poles and a delay. The dominant pole is -0.1. The time delay is small, relative to the dominant time constant which is $10 \approx \frac{1}{0.1}$. We neglect the two poles at -10 and the time delay.

$$\frac{2e^{-0.5s}}{(s+0.1)(s+10)^2} = \frac{2e^{-0.5s}}{100(s+0.1)(\frac{1}{10}s+1)^2}$$

$$\approx \frac{0.02}{s+0.1} \qquad e^{-0.5s} \approx 1$$

$$\frac{1}{(\frac{1}{10}s+1)^2} \approx 1.$$

$$K_1 = \frac{\alpha_1 - a}{b}$$

$$= \frac{2a-a}{b} = \frac{a}{b} = \frac{0.1}{0.02} = 5$$

$$K_2 = \frac{3a}{b} = \frac{0.1 \times 3}{0.02} = 15$$

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without constraints,

$u^{max} \Rightarrow 22$

$u^{min} \Rightarrow -16$.

For disturbance rejection, the u^{min} is ~~almost~~ required to achieve zero steady-state error compensation. We can experiment and find out if we constrain the $u^{min} = -16 * 0.85$, then the disturbance rejection is not good. However, the system has no oscillation.

Overview PID controller with disturbance observer. / second order model

$$G(s) = \frac{b}{s^2 + a_1 s + a_0}$$

Design for PD controller. The closed-loop performance is specified with the polynomial: $s^2 + 2\xi\omega_n s + \omega_n^2$

$\xi = 0.707$. ω_n is selected by the user.

$$K_1 = \frac{\omega_n^2 - a_0}{b}, \quad K_2 = \frac{2\xi\omega_n - a_1}{b}.$$

the derivative filter is

$$\tau_f = \beta \tau_D = 0.1 \frac{K_2}{K_1}$$

$\beta = 0.1$ for implementation.

The estimator gain K_3 is the same as before,

$$K_3 = \frac{\alpha_3}{b}.$$

where $-\alpha_3$ is the desired closed-loop pole.

Implementation.

$$\hat{d}(t) = \hat{z}(t) - K_3\left(\dot{y}_f(t)\right)$$

$$u(t) = -K_1\left(y(t) - r(t)\right) - K_2\dot{y}_f(t) - \hat{d}(t).$$

Saturate $u(t)$ if it exceeds limit.

$$\dot{\hat{z}}(t) = -\alpha_3\hat{z}(t) + K_3(a_1 - \alpha_3)\dot{y}(t)$$
$$+ K_3 a_0\left(y(t) - r(t)\right) - \alpha_3 u(t)$$

Q3: $G(s) = \dfrac{2e^{-0.01s}}{(s-1)(s+1)} \approx \dfrac{2}{s^2-1}$.

$a_1 = 0, \quad a_0 = -1, \quad b = 2. \qquad e^{-0.01s} \approx 1$

$\xi = 0.707, \quad \omega_n = 3.$

$$K_1 = \frac{\omega_n^2 - a_0}{b} = \frac{9+1}{2} = 5$$

$$K_2 = \frac{2 \times 0.707 \times 3 - 0}{b} = 4.242.$$

$$K_3 = \frac{\alpha_3}{b} = \frac{4}{2} = 2.$$

$$\tau_D = \frac{K_2}{K_1} = \frac{4.242}{5} = 0.8484.$$

$$\tau_f = \tau_D * 0.1 = 0.08484.$$