Part IV: Disturbance Observer Based PID and Resonant Control

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Outline

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Motivation

PID controllers

- Because the integral control has embedded a marginally stable mode in the controller structure, this could cause the problem of integrator windup when the control signal reaches its saturation limits.
- The PID control system implementation requires modification to overcome this problem.

Resonant controller

- Similar implementation problems to a worse degree are faced by the resonant controllers.
- It is much harder to derive the implementation scheme for resonant controller with an anti-windup mechanism because it has at least two poles on the imaginary axis.

Learning Objectives

- How to introduce the integral mode and resonant modes through disturbance estimation.
- How to implement disturbance observer based PID and resonant controllers with anti-windup mechanisms.
- How to analyze disturbance observer based PID controllers.

Outline

First Order Model with Disturbance

We assume that there is a constant input disturbance $d(t)$, which is unknown. So the differential equation used to describe a first order system is given by

$$\dot{y}(t) = -ay(t) + b(u(t) + d(t)) \quad (1)$$

where a and b are model coefficients, $u(t)$ and $y(t)$ are the input and output variables.

System Diagram

Figure 1 illustrate the mathematical model to be used for the estimator based PI controller design.

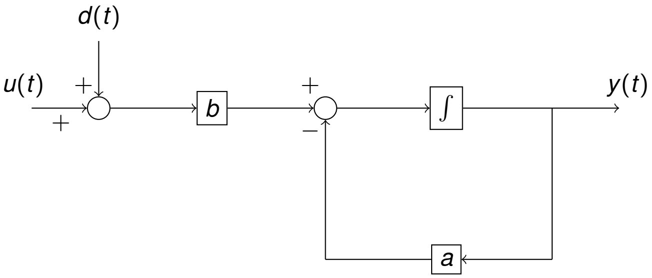


Figure 1: Block diagram of the system for disturbance observer-based PI controller

Proportional Controller

The proportional control

Firstly, we define:

$$\tilde{u}(t) = u(t) + d(t) \tag{2}$$

Then (1) becomes

$$\dot{y}(t) = -ay(t) + b\tilde{u}(t) \tag{3}$$

The intermediate control signal $\tilde{u}(t)$ is

$$\tilde{u}(t) = -K_1 y(t)$$

Proportional controller gain K_1

The transfer function is

$$\frac{Y(s)}{\tilde{U}(s)} = \frac{b}{s+a}$$

The proportional controller K_1 is

$$K_1 = \frac{\alpha_1 - a}{b} \tag{4}$$

where $-\alpha_1$ is the desired closed-loop pole.

Estimating Disturbance

Motivation

There will be a steady-state error for the proportional control system. Instead of using an integrator, the steady-state error is estimated and subtracted from the control system.

Disturbance estimation

Because of the assumption that $d(t)$ is a constant, we have

$$\dot{d}(t) = 0 \tag{5}$$

Extracting the disturbance information from (1) leads to

$$bd(t) = \dot{y}(t) + ay(t) - bu(t) \tag{6}$$

Comments

- One might attempt to directly calculate the unknown disturbance $d(t)$ using (6) and compensate it in the control signal.
- However, it can be easily verified that such an approach fails to produce the control signal required because of uncertainties in the model parameters and other imperfections in practical applications.
- We will estimate the disturbance signal $d(t)$ with compensation on the error.

Closed-loop Estimation of Disturbance

The error

Let $\hat{d}(t)$ denote the estimated disturbance signal. The error, between what is given and what is to be estimated, is described by,

$$\epsilon(t) = bd(t) - b\hat{d}(t) = \dot{y}(t) + ay(t) - bu(t) - b\hat{d}(t) \tag{7}$$

The estimation equation

With the gain K_2 weighted on the error $\epsilon(t)$, together with the assumption $\dot{d}(t) = 0$, we construct the estimation $\hat{d}(t)$ as

$$\frac{d\hat{d}(t)}{dt} = K_2(\dot{y}(t) + ay(t) - bu(t) - b\hat{d}(t)) \tag{8}$$

Choice of K_2

Choice of K_2

We will choose the gain K_2 such that the error $\tilde{d}(t) = d(t) - \hat{d}(t)$ converges to zero.

The closed-loop error system

Note that

$$\frac{d\tilde{d}(t)}{dt} = -K_2 b\tilde{d}(t) \tag{9}$$

then, the parameter K_2 is chosen such that $-K_2 b = -\alpha_2$. This then leads to

$$K_2 = \frac{\alpha_2}{b}$$

Hence,

$$\frac{d\tilde{d}(t)}{dt} = -\alpha_2 \tilde{d}(t) \tag{10}$$

For any given initial condition $|\tilde{d}(0)| < \infty$ and $\alpha_2 > 0$, the estimation error $|\tilde{d}(t)| \to 0$ as $t \to \infty$.

Calculation of Control Signal

Now, to calculate the control signal with compensation on the steady-state error, the unknown disturbance $d(t)$ in (2) is replaced by the estimated $\hat{d}(t)$ from (8), leading to the control signal calculated as

$$u(t) = -K_1 y(t) - \hat{d}(t) \tag{11}$$

The Desired Closed-loop Poles

One pole from proportional control

$-\alpha_1$ is the desired closed-loop pole for the proportional control ($K_1 = \frac{\alpha_1 - a}{b}$).

One pole from the disturbance estimation

$-\alpha_2$ is the desired closed-loop pole for the disturbance estimation ($K_2 = \frac{\alpha_2}{b}$).

Implementation

Not implementable

$$\frac{d\hat{d}(t)}{dt} = K_2(\dot{y}(t) + ay(t) - bu(t) - b\hat{d}(t))$$

Because the estimation equation contains the derivative of the output signal $y(t)$, direct discretization requires information of $y(t_{i+1})$ at sampling time t_i, which is not available to us.

Choosing another variable

Let us define a variable $\hat{z}(t)$ as

$$\hat{z}(t) = \hat{d}(t) - K_2 y(t) \tag{12}$$

Then, substituting this variable into the estimation equation yields

$$\begin{aligned} \frac{d\hat{z}(t)}{dt} &= -K_2 b\hat{z}(t) - (K_2^2 b - K_2 a)y(t) - K_2 bu(t) \\ &= -\alpha_2 \hat{z}(t) - K_2(\alpha_2 - a)y(t) - \alpha_2 u(t) \end{aligned} \tag{13}$$

Anti-windup Implementation

1. Calculate the estimated disturbance signal at sample t_i as
$$\hat{d}(t_i) = \hat{z}(t_i) + K_2(y(t_i) - r(t_i))$$
2. Calculate the control signal using the following equation
$$u(t_i) = -K_1\,(y(t_i) - r(t_i)) - \hat{d}(t_i)$$
3. Implement the control signal saturation:
$$u(t_i) = \begin{cases} u^{min} & if\ u(t_i) < u^{min} \\ u(t_i) & if\ u^{min} \le u(t_i) \le u^{max} \\ u^{max} & if\ u(t_i) > u^{max} \end{cases}$$
4. Update the estimation of $\hat{z}(t_{i+1})$ for the next sampling instant as
$$\hat{z}(t_{i+1}) = \hat{z}(t_i) - (\alpha_2\hat{z}(t_i) + \frac{\alpha_2(\alpha_2 - a)}{b}(y(t_i) - r(t_i)) + \alpha_2 u(t_i))\Delta t$$
5. send the control signal $u(t_i)$ for implementation. When the next sampling period arrives, the new measurement of the output is taken and the computation is repeated from Step 1.

Embedded Anti-windup Mechanism

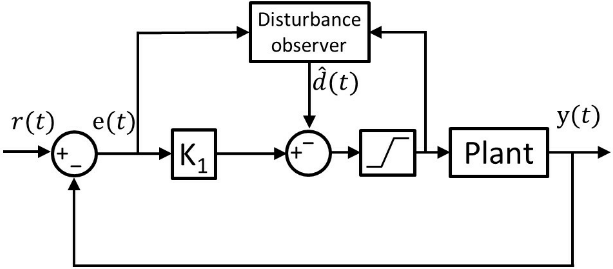


Figure 2: Block diagram of control system using disturbance observer

Equivalence to PI Controller

- The equivalent PI controller is revealed as

$$\begin{aligned} C(s) &= \frac{K_1(s+\alpha_2)}{s} + \frac{K_2 s + K_2 a}{s} \\ &= K_1 + K_2 + \frac{(K_1\alpha_2 + K_2 a)}{s} \end{aligned} \tag{14}$$

- The PI controller parameters are

$$K_c = K_1 + K_2$$

$$\frac{K_c}{\tau_I} = K_1\alpha_2 + K_2 a$$

where $K_1 = \frac{\alpha_1 - a}{b}$ and $K_2 = \frac{\alpha_2}{b}$.

- We can verify that the closed-loop poles are at $-\alpha_1$ and $-\alpha_2$, which was the design specification.

Example

A continuous-time system is approximated by the following first order model:

$$G(s) = \frac{0.1}{T_1 s + 1} \tag{15}$$

where T_1 is 10 sec. It is known that the system has a variable delay of T_d which has a maximum value of 1 sec and a neglected time constant T_2 with a maximum value of 5 sec. Design an estimator based PI controller for this system and simulate the closed-loop control performance for unit step reference change and rejection of input disturbance having amplitude of 20.

Solution (i)

- Because this system has neglected time delay and time constant, this model uncertainty will limit the specification of desired closed-loop performance.
- A good starting point is to select the dominant closed-loop pole equal to the known pole of system, which is at -0.1. Hence, $\alpha_1 = 0.1$.
- The second desired closed-loop pole $-\alpha_2$ is determined using closed-loop simulations.

Solution (ii)

- With $a = 0.1$, and $b = 0.01$, and $\alpha_1 = 0.1$, the parameter K_1 is calculated as

$$K_1 = \frac{\alpha_1 - a}{b} = 0$$

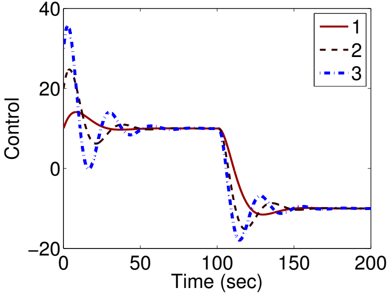
- The parameter K_2 is calculated as

$$K_2 = \frac{\alpha_2}{b}$$

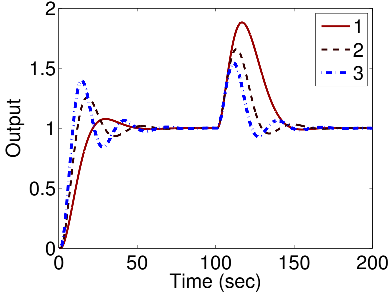
- With the parameter α_2 being selected as 0.1, 0.2 and 0.3, three values of K_2 are calculated as 10, 20 and 30.

Simulation Studies (i)

$$G(s) = \frac{0.1e^{-s}}{(10s+1)(5s+1)}$$



(a) Control signal

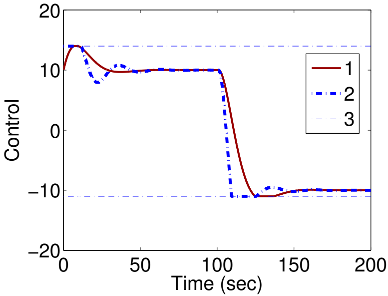


(b) Output

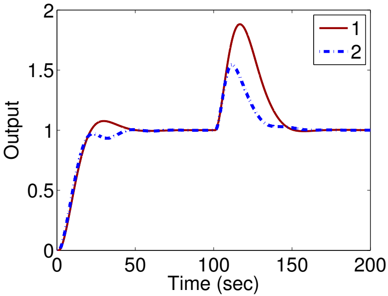
Figure 3: Comparison of closed-loop control performance using estimator based PI controller with different α_2 values. Key: line (1) $\alpha_2 = 0.1$; line (2) $\alpha_2 = 0.2$; line (3) $\alpha_2 = 0.3$

Simulation Studies (ii)

$$-11 \leq u(t) \leq 14 \tag{16}$$



(a) Control signal



(b) Output

Figure 4: Comparison of closed-loop control performance using estimator based PI controller with different α_2 values. Key: line (1) $\alpha_2 = 0.1$; line (2) $\alpha_2 = 0.3$; and line (3) the limits of the control signal.

Outline

Assumption on Model

Second Order Model

For PID controller design, the dynamic model needs to be a second order, which has the transfer function:

$$G(s) = \frac{b}{s^2 + a_1 s + a_0} = \frac{Y(s)}{U(s)} \tag{17}$$

Differential equation

$$\ddot{y}(t) = -a_1 \dot{y}(t) - a_0 y(t) + bu(t).$$

In matrix form, it is expressed as

$$\begin{bmatrix} \dot{y}(t) \\ \ddot{y}(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -a_0 & -a_1 \end{bmatrix} \begin{bmatrix} y(t) \\ \dot{y}(t) \end{bmatrix} + \begin{bmatrix} 0 \\ b \end{bmatrix} u(t). \tag{18}$$

Proportional Plus Derivative Control (i)

PD controller design

The feedback control signal $u(t)$ is

$$u(t) = -\begin{bmatrix} K_1 & K_2 \end{bmatrix} \begin{bmatrix} y(t) \\ \dot{y}(t) \end{bmatrix}. \tag{19}$$

which has the transfer function

$$U(s) = -(K_1 + K_2 s)Y(s)$$

PD Controller transfer function

The controller transfer function is

$$C(s) = K_1 + K_2 s$$

PD Controller Parameters

As before, the pole assignment controller design is used. We have the actual closed-loop polynomial:

$$(s^2 + a_1 s + a_0) + b(K_1 + K_2 s)$$

and we choose the second order desired closed-loop polynomial as

$$s^2 + 2\xi w_n s + w_n^2$$

We compare the actual closed-loop polynomial with the desired one to obtain:

$$K_1 = \frac{w_{\mathrm{n}}^2 - a_0}{b} \tag{20}$$

and the derivative control gain:

$$K_2 = \frac{2\xi w_{\mathrm{n}} - a_1}{b}. \tag{21}$$

PD Controller Filter Selction)

Determining the derivative filter

$$C(s) = (K_1 + K_2 s) = K_1(1 + \frac{K_2}{K_1} s)$$

Clearly, the derivative gain is $\tau_D = \frac{K_2}{K_1}$.

PD controller with filter

$$C(s) = K_1(1 + \frac{\tau_D s}{0.1\tau_D s + 1})$$

Introducing Integral Control

Similar to the PI controller design via estimation, we will write the unknown disturbance term as

$$bd(t) = \ddot{y}(t) + a_1\dot{y}(t) + a_0 y(t) - bu(t). \tag{22}$$

Together with the assumption that $\dot{d}(t) = 0$, the estimation of $d(t)$ is constructed as

$$\frac{\mathrm{d}\hat{d}(t)}{\mathrm{d}t} = K_3(\ddot{y}(t) + a_1\dot{y}(t) + a_0 y(t) - bu(t) - b\hat{d}(t)). \tag{23}$$

We choose $\alpha_3 > 0$ and determine the estimator's gain K_3 using

$$K_3 = \frac{\alpha_3}{b}.$$

Estimation

Because Equation (23) has double derivative and derivative of output signal $y(t)$, it is not suitable for computational purposes. To this end, we define a new variable

$$\hat{z}(t) = \hat{d}(t) + K_3\dot{y}(t)$$

and rewrite Equation (23) as function of $\hat{z}(t)$:

$$\frac{\mathrm{d}\hat{z}(t)}{\mathrm{d}t} = -\alpha_3\hat{z}(t) + K_3(a_1 - \alpha_3)\dot{y}(t) + K_3 a_0 y(t) - \alpha_3 u(t). \tag{24}$$

Anti-windup Implementation

1. Update the estimated disturbance signal. An initial condition on $\hat{z}(t_0)$ will be given as the start up of the control algorithm.

$$\hat{d}(t_i) = \hat{z}(t_i) - K_3\dot{y}(t_i).$$

2. Calculate the control signal

$$u(t_i) = -K_1(y(t_i) - r(t_i)) - K_2\dot{y}(t_i) - \hat{d}(t_i).$$

3. Implement control signal saturation:

$$u(t_i) = \begin{cases} u^{\min} & \text{if } u(t_i) < u^{\min} \\ u(t_i) & \text{if } u^{\min} \leq u(t_i) \leq u^{\max} \\ u^{\max} & \text{if } u(t_i) > u^{\max} \end{cases}.$$

4. Update the disturbance estimator for t_{i+1}:

$$\begin{aligned} \hat{z}(t_{i+1}) &= \hat{z}(t_i) + \Delta t\,(-\alpha_3\hat{z}(t_i) + K_3(a_1 - \alpha_3)\dot{y}(t_i)) \\ &\quad + \Delta t\,(K_3 a_0(y(t_i) - r(t_i)) - \alpha_3 u(t_i)) \end{aligned}.$$

5. When the next sample period arrives, repeat the computation at step 1.

Example (i)

An unstable system is described by the transfer function:

$$G(s) = \frac{0.1\mathrm{e}^{-0.05s}}{(s+2)(s-2)} \tag{25}$$

where the small time delay is due to the dynamics from the actuator. Design a estimator based PID controller with the pair of dominant poles at -2 and the estimator pole at -10. Evaluate the control system performance for reference following and disturbance rejection in the presence of control amplitude constraints.

Example (ii)

Choosing $\xi = 1$ and $w_\mathrm{n} = 2$ positions the closed-loop poles at -2. The model (25) gives the parameters $a_1 = 0$, $a_0 = -4$, and $b = 0.1$. The PD controller parameters are calculated as

$$K_1 = \frac{w_\mathrm{n}^2 - a_0}{b} = 80$$

and the derivative control gain

$$K_2 = \frac{2\xi w_\mathrm{n} - a_1}{b} = 40.$$

With $\alpha_3 = 10$, the parameter K_3 is calculated as

$$K_3 = \frac{\alpha_3}{b} = 100.$$

Example (iii)

Derivative filter

A derivative filter time constant is calculated as

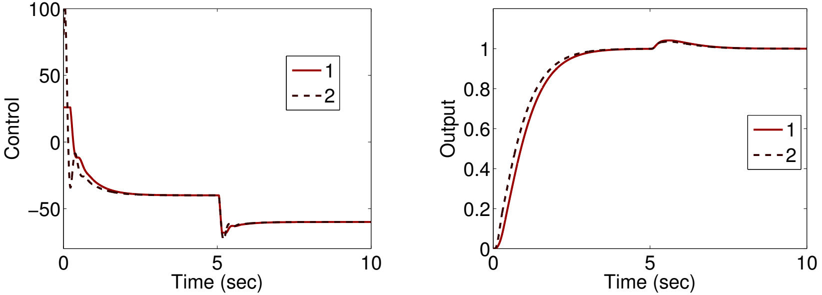
$$\tau_{\mathrm{f}} = \frac{0.01K_2}{K_1} = 0.005.$$

The filter time constant used is quite small because it introduces additional dynamics in the system.

Simulation conditions

Choosing the sampling interval $\Delta t = 0.001$ (sec), in the simulation, a unit step reference signal enters the closed-loop system at $t = 0$ and an input step disturbance with amplitude of 20 enters the system at half of the simulation time. The control signal amplitude is constrained between -68 and 26.

Simulation Results



(a) Control signal (b) Output

Figure 5: Closed-loop control performance using disturbance observer-based PID controller with control signal amplitude constraints. Key: line (1) constrained responses; line (2) unconstrained responses

Outline

Assumption on Input Disturbance

Dynamic model

$$\dot{y}(t) = -ay(t) + b(u(t) + d(t)) \tag{26}$$

where a and b are the coefficients; $u(t)$ and $y(t)$ are the input and output signals; $d(t)$ is the input disturbance signal.

Periodic disturbance

In particular, we assume that $d(t)$ is a sinusoidal signal with known frequency ω_0, but unknown amplitude d_m and phase angle ψ_0, which is expressed as

$$d(t) = d_m \sin(\omega_0 t + \psi_0)$$

Resonant Control Law

Control signal

The resonant control law is expressed as

$$u(t) = -K_1(y(t) - r(t)) - \hat{d}(t)$$

where $\hat{d}(t)$ is an estimate of the unknown disturbance $d(t)$.

Proportional controller gain

By choosing the desired closed-loop pole at $-\alpha_1$ and $\alpha_1 > 0$, the proportional feedback control gain K_1 is calculated as

$$K_1 = \frac{\alpha_1 - a}{b}$$

Modelling Periodic Disturbance

- The derivative of this disturbance signal is

$$\dot{d}(t) = d_m \omega_0 \cos(\omega_0 t + \psi_0)$$

  and its second derivative is

$$\ddot{d}(t) = -d_m \omega_0^2 \sin(\omega_0 t + \psi_0) = -\omega_0^2 d(t)$$

- Now, we choose $x_1(t) = d(t)$ and $x_2(t) = \dot{d}(t)$.
- With these choices, the following differential equations are used to describe the sinusoidal disturbance signal:

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\omega_0^2 & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} \tag{27}$$

Estimation of Periodic Disturbance (i)

The estimated variables $\hat{x}_1(t)$ and $\hat{x}_2(t)$ are constructed as

$$\begin{bmatrix} \frac{d\hat{x}_1(t)}{dt} \\ \frac{d\hat{x}_2(t)}{dt} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\omega_0^2 & 0 \end{bmatrix} \begin{bmatrix} \hat{x}_1(t) \\ \hat{x}_2(t) \end{bmatrix} + \begin{bmatrix} \gamma_1 \\ \gamma_2 \end{bmatrix} (\dot{y}(t) + ay(t) - bu(t) - \begin{bmatrix} b & 0 \end{bmatrix} \begin{bmatrix} \hat{x}_1(t) \\ \hat{x}_2(t) \end{bmatrix}) \quad (28)$$

where γ_1 and γ_2 are the estimator gains chosen for the design.

Estimation of Periodic Disturbance (ii)

$$\begin{aligned} \gamma_1 &= \frac{2\xi w_n}{b}; \\ \gamma_2 &= \frac{w_n^2 - \omega_0^2}{b} \end{aligned} \tag{29}$$

In the applications, the damping parameter ξ is chosen to be 0.707 and the parameter w_n is adjusted for how fast we would like to see the errors converge to zero.

Resonant Controller Implementation

Not implementable

The calculation of the estimated input disturbance using (28) requires the derivative of the output signal $\dot{y}(t)$, which is not desirable in the implementation.

Implementation

To overcome the problem, we define a pair of new variables:

$$\hat{z}_1(t) = \hat{x}_1(t) - \gamma_1 y(t); \;\; \hat{z}_2(t) = \hat{x}_2(t) - \gamma_2 y(t)$$

Then, from (28), the following two equations are obtained:

$$\frac{d\hat{z}_1(t)}{dt} = -2\xi w_n \hat{z}_1(t) + \hat{z}_2(t) + (a\gamma_1 + \gamma_2 - 2\xi w_n \gamma_1)y(t) - b\gamma_1 u(t) \quad (30)$$

$$\frac{d\hat{z}_2(t)}{dt} = -w_n^2 \hat{z}_1(t) + (a\gamma_2 - w_n^2 \gamma_1)y(t) - b\gamma_2 u(t) \quad (31)$$

Discretization and Anti-windup Mechanism (i)

Assume that the control signal $u(t)$ is limited to u^{min} and u^{max}, that is

$$u^{min} \leq u(t) \leq u^{max}$$

Choosing the initial conditions for $\hat{z}_1$ and $\hat{z}_2$, the control signal is calculated iteratively according to the following steps, where $r(t_i)$ is the reference signal at sampling time t_i.

Discretization and Anti-windup Mechanism (ii)

1. Calculate the estimated sinusoidal disturbance $\hat{d}(t_i)$ as
$$\hat{d}(t_i) = \hat{z}_1(t_i) + \gamma_1(y(t_i) - r(t_i))$$
2. Calculate the control signal $u(t_i)$ as
$$u(t_i) = -K_1(y(t_i) - r(t_i)) - \hat{d}(t_i)$$
3. Implement the saturations on the control signal using
$$u(t_i) = \begin{cases} u^{min} & \text{if } u(t_i) < u^{min} \\ u(t_i) & \text{if } u^{min} \leq u(t_i) \leq u^{max} \\ u^{max} & \text{if } u(t_i) > u^{max} \end{cases}$$
4. Update the estimated disturbance signals using the following equations:
$$\begin{aligned} \hat{z}_1(t_{i+1}) &= \hat{z}_1(t_i) + \Delta t(-2\xi w_n \hat{z}_1(t_i) + \hat{z}_2(t_i)) \\ &\quad + \Delta t((a\gamma_1 + \gamma_2 - 2\xi w_n \gamma_1)(y(t_i) - r(t_i)) - b\gamma_1 u(t_i)) \\ \hat{z}_2(t_{i+1}) &= \hat{z}_2(t_i) + \Delta t(-w_n^2 \hat{z}_1(t_i) + (a\gamma_2 - w_n^2\gamma_1)(y(t_i) - r(t_i)) - b\gamma_2 u(t_i)) \end{aligned}$$
5. When the next sampling period arrives, repeat the computation from Step 1.

Example (i)

An electrical system is approximated by the following first order plus time delay model:

$$G(s) = \frac{0.3e^{-0.0015s}}{0.001s + 1} \tag{32}$$

where the delay is used to describe the neglected time constants from the other electronic components in the system. The control objective is for the output of the system to follow a sinusoidal reference signal with frequency $\omega_0 = 2\pi \times 50$ rad/s. Design a resonant controller and simulate the closed-loop output with sampling interval $\Delta t = 0.00001$ (sec).

Example (ii)

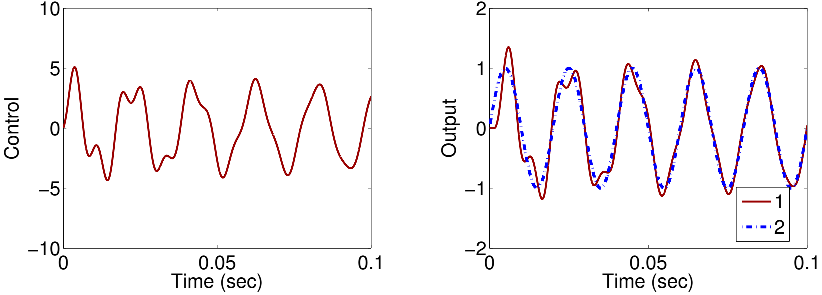
- The resonant controller is designed using the first order model, which gives the parameters $a = 1/0.001 = 1000$ and $b = 0.3/1000 = 3000$.
- A good starting point is to select the desired closed-loop pole $-\alpha_1$ for the controller gain K_1 equal to the model pole $-a$, leading to

$$K_1 = \frac{\alpha_1 - a}{b} = 0$$

- We select the parameters for the estimator to satisfy the closed-loop stability and performance in the presence of unmodelled time delay.
- By choosing $\xi = 0.707$, the parameter w_n is used for adjusting the closed-loop response speed and robustness.

Example (iii) ($w_n = 500$)

$$\gamma_1 = \frac{2\xi w_n}{b} = 2.3567; \quad \gamma_2 = \frac{w_n^2 - \omega_0^2}{b} = 504.3465 \tag{33}$$

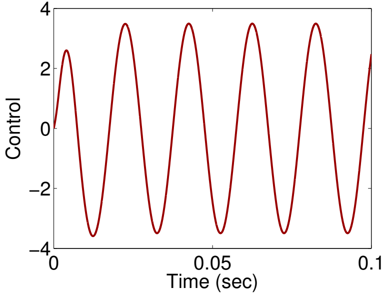


(a) Control signal (b) Output

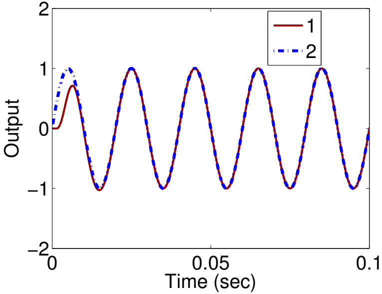
Figure 6: Closed-loop control response using estimator based resonant controller ($w_n = 500$). Key: line (1) output response; line (2) reference signal

Example (iv) ($w_n = 300$)

$$\gamma_1 = \frac{2\xi w_n}{b} = 1.4140; \quad \gamma_2 = \frac{w_n^2 - \omega_0^2}{b} = -28.9868 \tag{34}$$



(a) Control signal

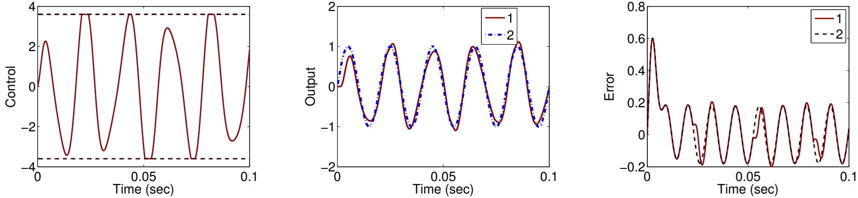


(b) Output

Figure 7: Closed-loop control response using estimator based resonant controller ($\alpha_1 = 1000, w_n = 300$). Key: line (1) output response; line (2) reference signal

Constraints on Control Signal

$$-3.6 \leq u(t) \leq 3.6$$



(a) Solid line- control signal, dashed line -control signal limits

(b) Output

(c) Error. Key: line (1) constrained control; line (2) unconstrained control

Figure 8: Closed-loop control response using estimator based resonant controller in the presence of control signal constraints ($\alpha_1 = 1000, w_n = 300$)