OENG1116 – Modelling and Simulation of Engineering Systems

Non Linear Regression

(NLR)

Course Lecturer:
Dr Hamid Khayyam
Office: 251-02-34
Phone: 03 9925 4630
Email: hamid.khayyam@rmit.edu.au

Project Based Learning (PBL)

- **Pavel and I** are always trying to improve the course so would be very happy to hear any suggestions. The course has been run by using Project Based Learning (PBL) method.

  - **https://www.youtube.com/watch?v=LMCZvGesRz8**

  - **https://www.youtube.com/watch?v=Nr0sQCPqkOI**

  - **https://www.youtube.com/watch?v=EuzgJIqzjFw**

About the Course

I remind you that: :

(1) The course is run in its current configuration for the first time; the number of contact face-to-face hours has been dramatically increased. With the minimum required 12x3=36 hrs, we actually delivered more hours than this minimum: 4x3 + 8x4 = 12+32 = 44 hrs.

(2) Our qualified Team has designed a modern and relevant context for this Course;

About the course

(3) We have presented universal techniques to solve variety of problems; at the same time representative examples were provided from a range of engineering applications (mechanical, automotive, electrical, environmental engineering, etc.);

(4) We have introduced Computer-based hand-on tutorials and involved the Team of best expert Tutors;

(5) We worked hard to take into account student's feedback and listen to the students suggestions;

(6) We value your feedback and CES (it is not claim from us) is important to us to improve the course,

About the course in this year

(7) Write 3 text books (book chapters) for using the related materials for your course.

(4) Record tutorial for the first time.

(5) Self assessment.

CES

10 Minutes to complete your feedback and CES please!!

OENG1116 – Modelling and Simulation of Engineering Systems 2019

Non Linear Regression

(NLR)

Course Lecturer:

Dr Hamid Khayyam
Office: 251-02-34
Phone: 03 9925 4630
Email: hamid.khayyam@rmit.edu.au

RMIT UNIVERSITY

Outline

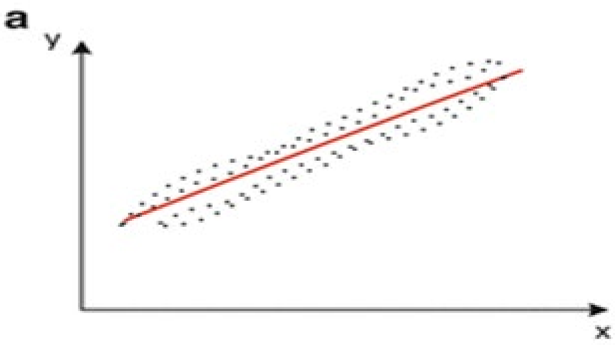
- Linear and Non-linear Regression?
- Non-Linear Regression Models
- Exponential Model
- Polynomial Model
- Transformation
- Comparison
- References.

Outline

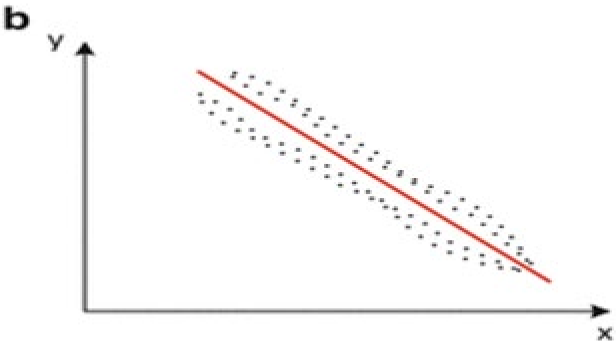
- Linear and Non-linear regression?
- Non-Linear Regression Models
- Exponential Model
- Polynomial Model
- Transformation
- Comparison
- References

Linear and Non-linear regression models:

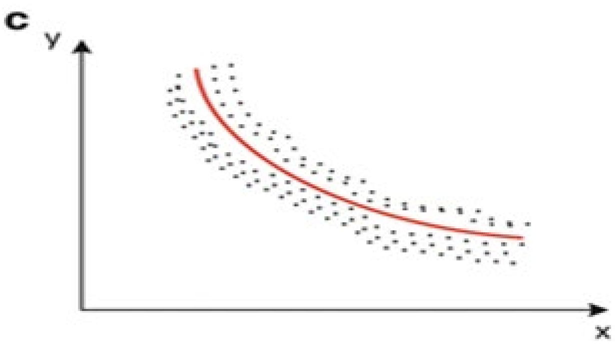
Regression models describe the relationship (correlation) between a response (**output**) variable, and one or more predictor (**input**) variables:



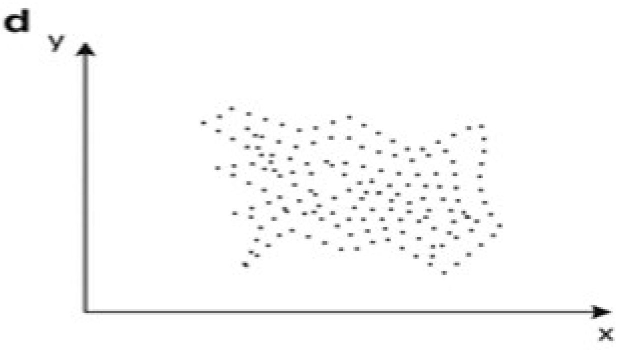
(**a**) Positive linear correlation



(**b**) negative linear correlation



(**c**) nonlinear correlation

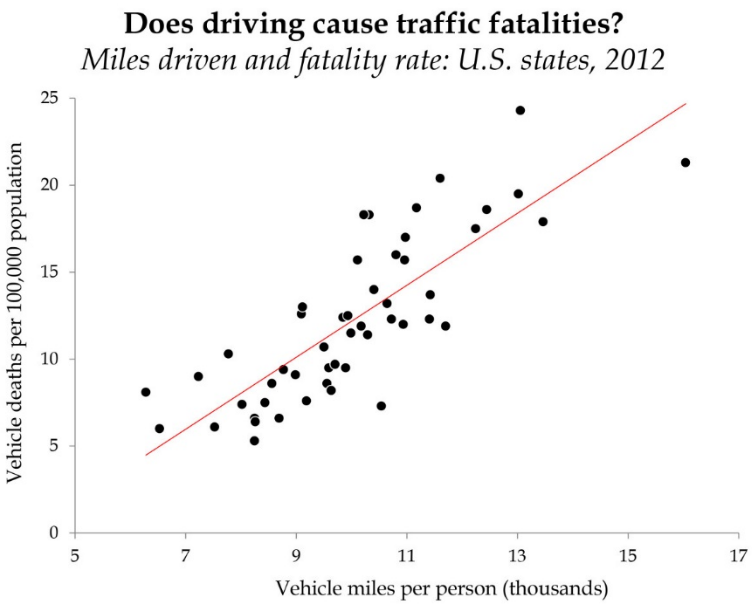


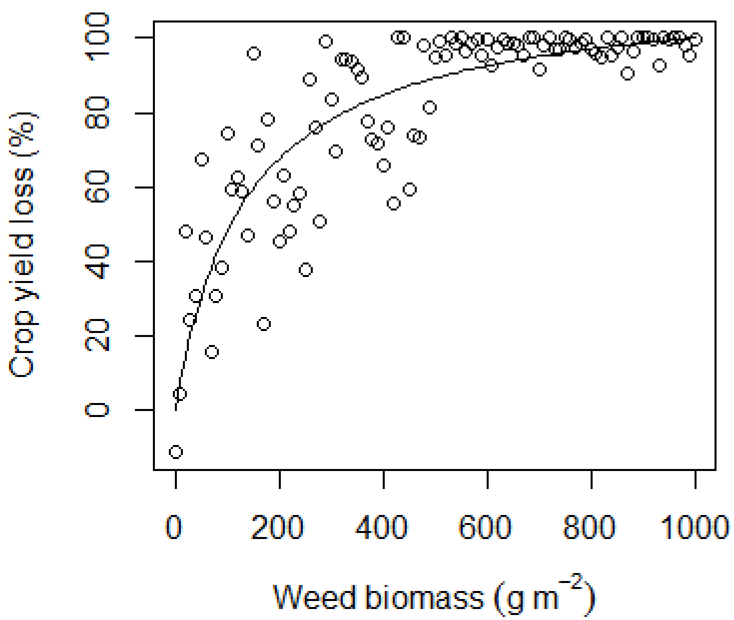
(**d**) no correlation

Linear and Non-linear Regression Models

Regression models:

- Linear Model
- Non-linear model



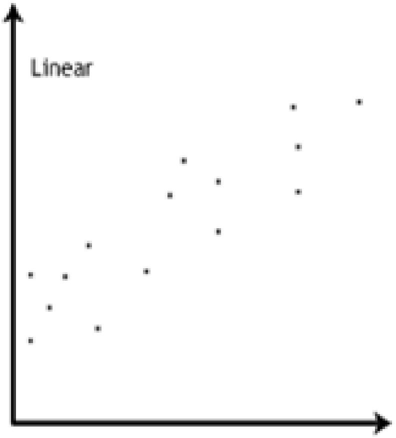


Linear Model

Nonlinear model

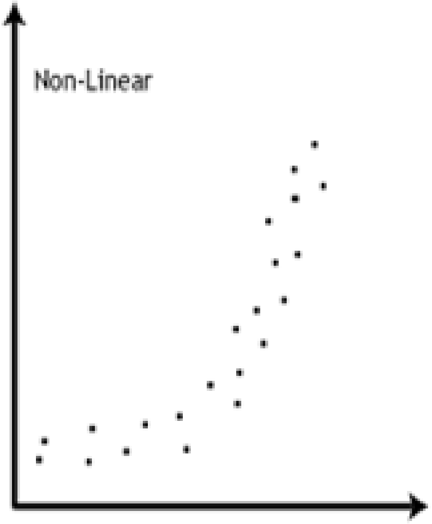
Linear vs Non-Linear

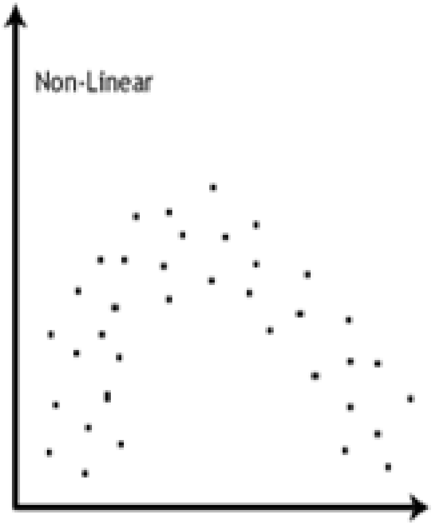
Linear



- Linear scatter plot
- No curves in residual plot
- Correlation between variable is significant

Non-linear





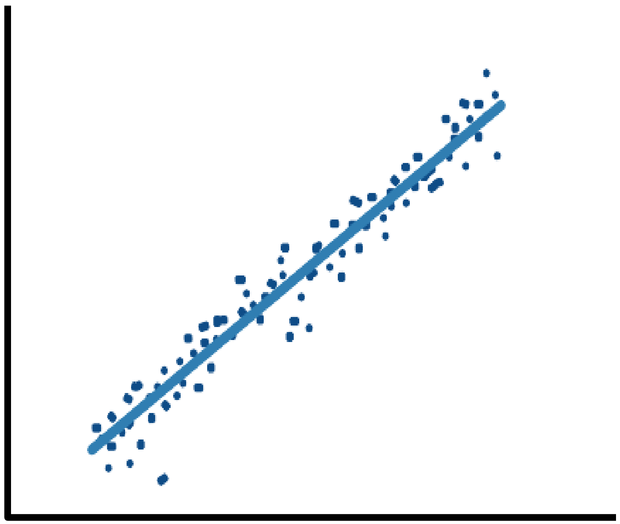
- Curves in scatter plot
- Curves in residual plot
- No significant correlation between variables

Linear Regression

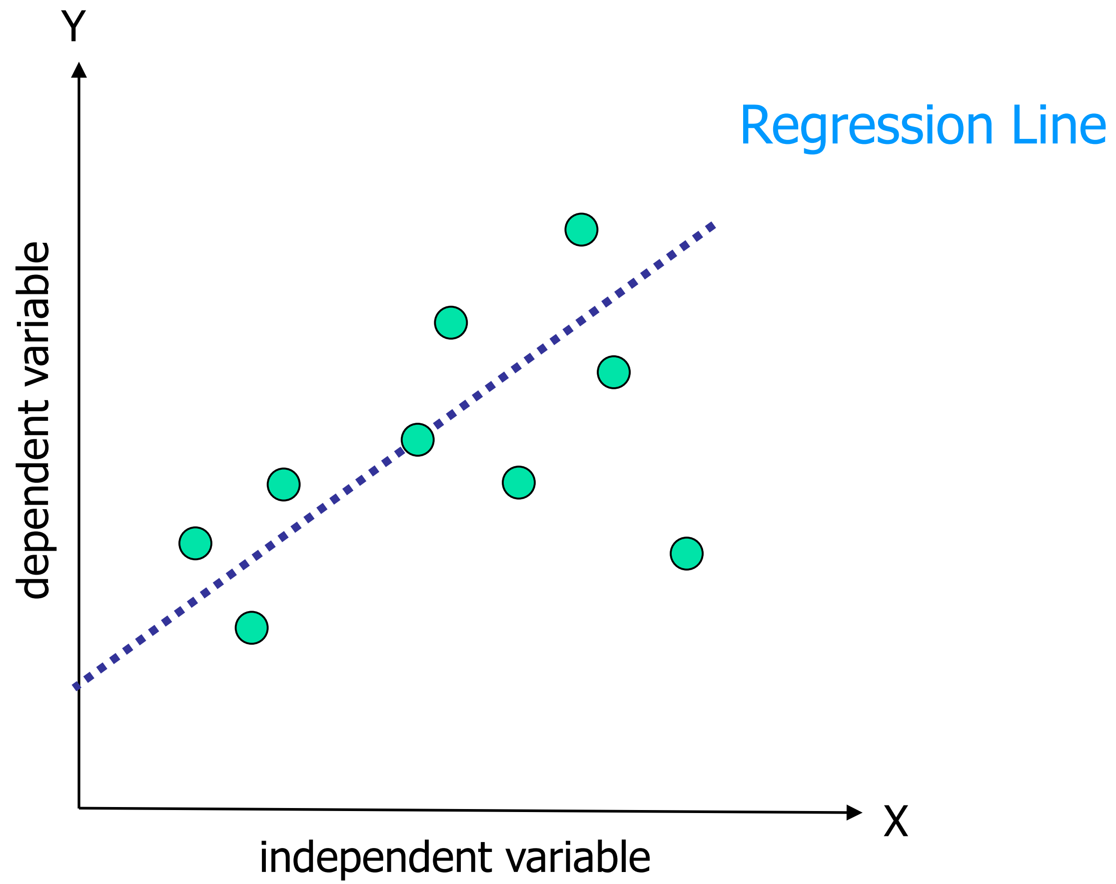
Linear regression is a statistical modeling technique used to describe a continuous response variable as a linear function of one or more predictor variables. Because linear regression models are simple to interpret and easy to train, they are often the first model to be fitted to a new dataset.

Best Used...

- When you need an algorithm that is easy to interpret and fast to fit
- As a baseline for evaluating other, more complex, regression models

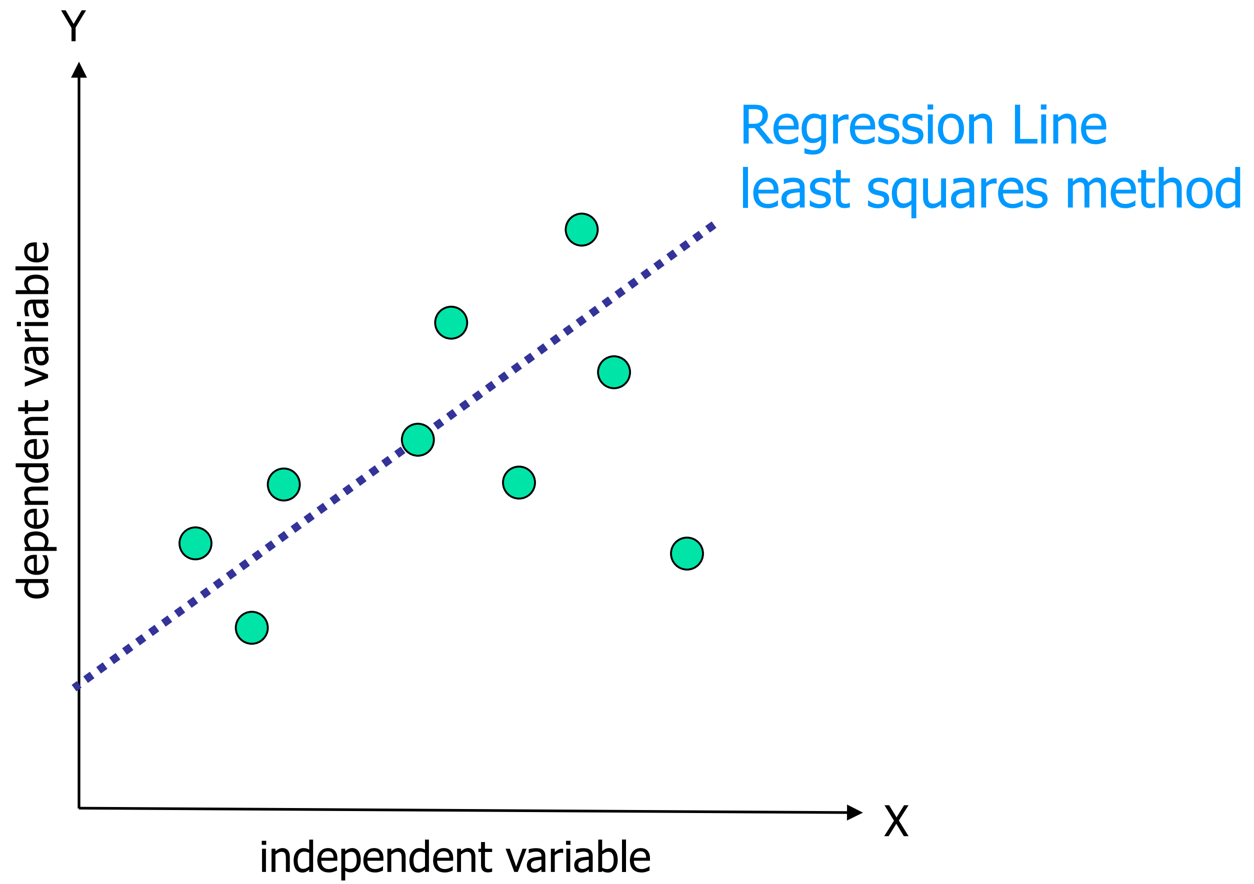


Linear regression model

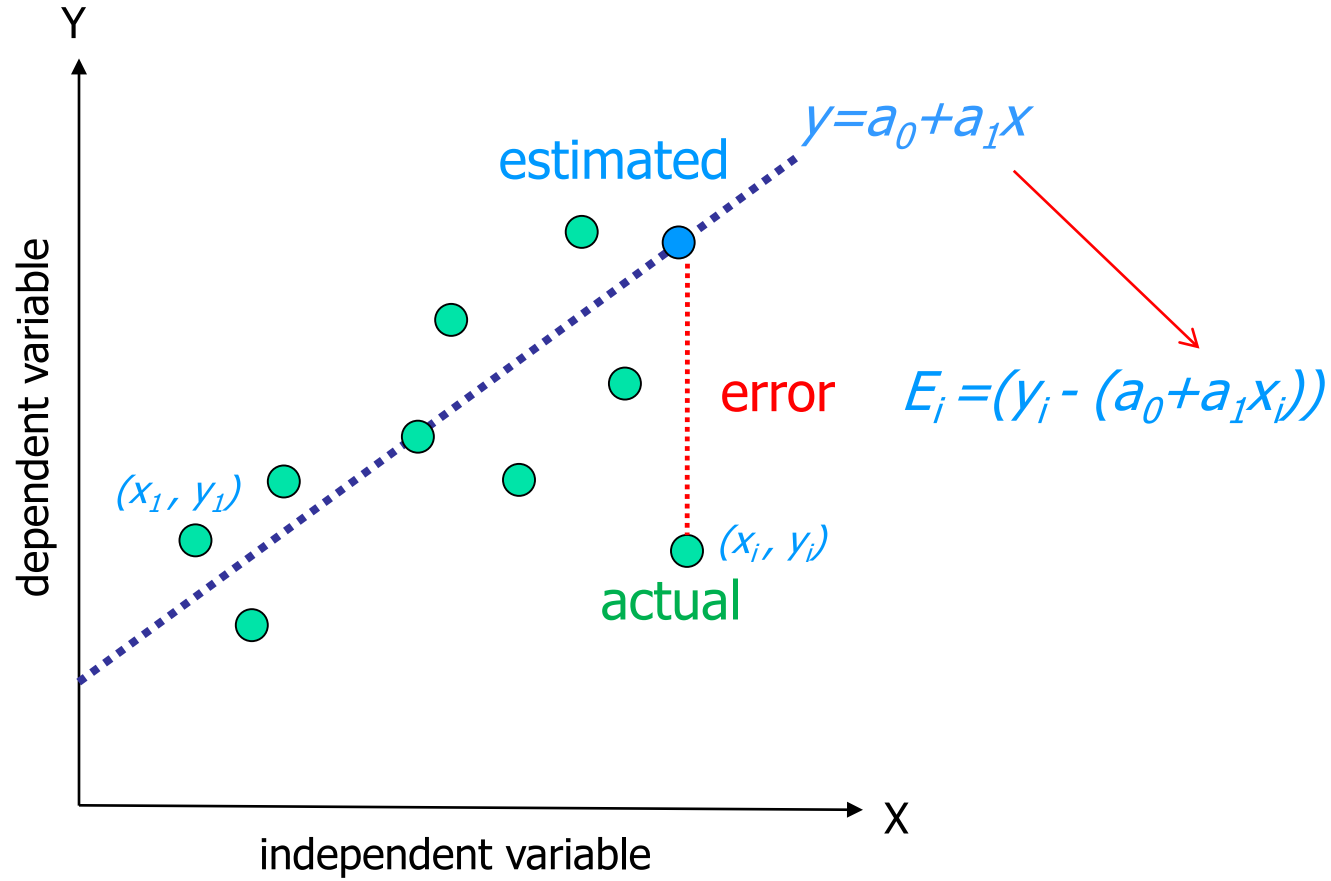


Linear Regression Model

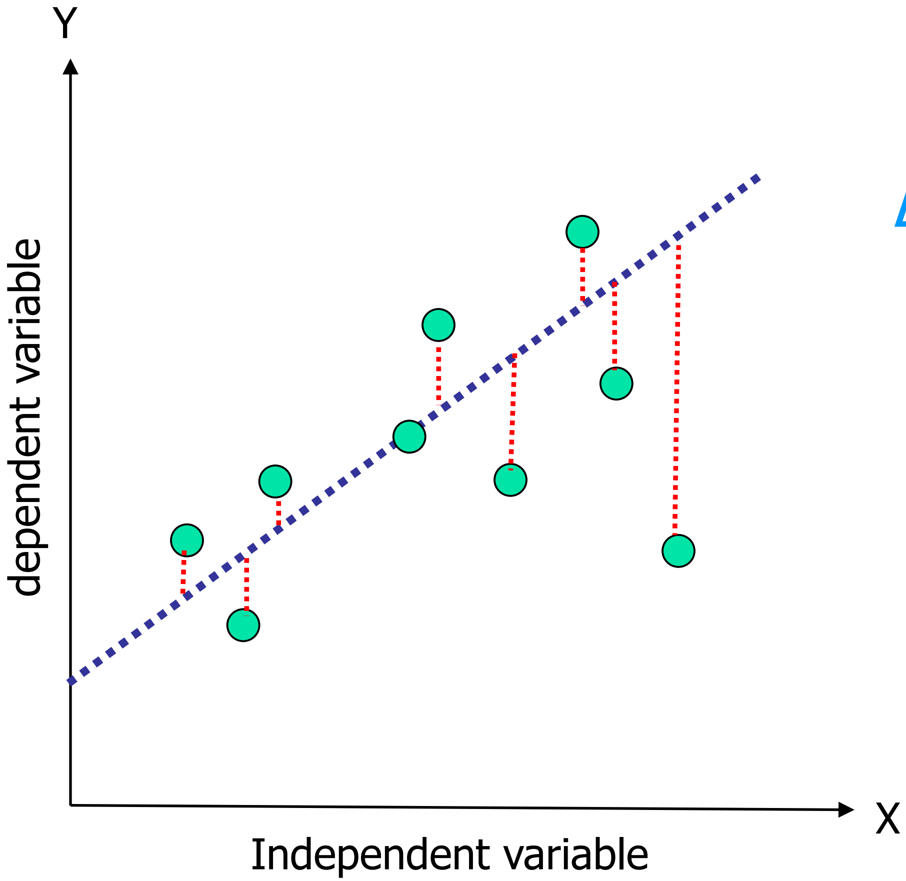
The method of **least squares** is a standard approach in regression analysis to approximate the solution of overdetermined systems credited to Carl Friedrich Gauss.



Linear Regression Models



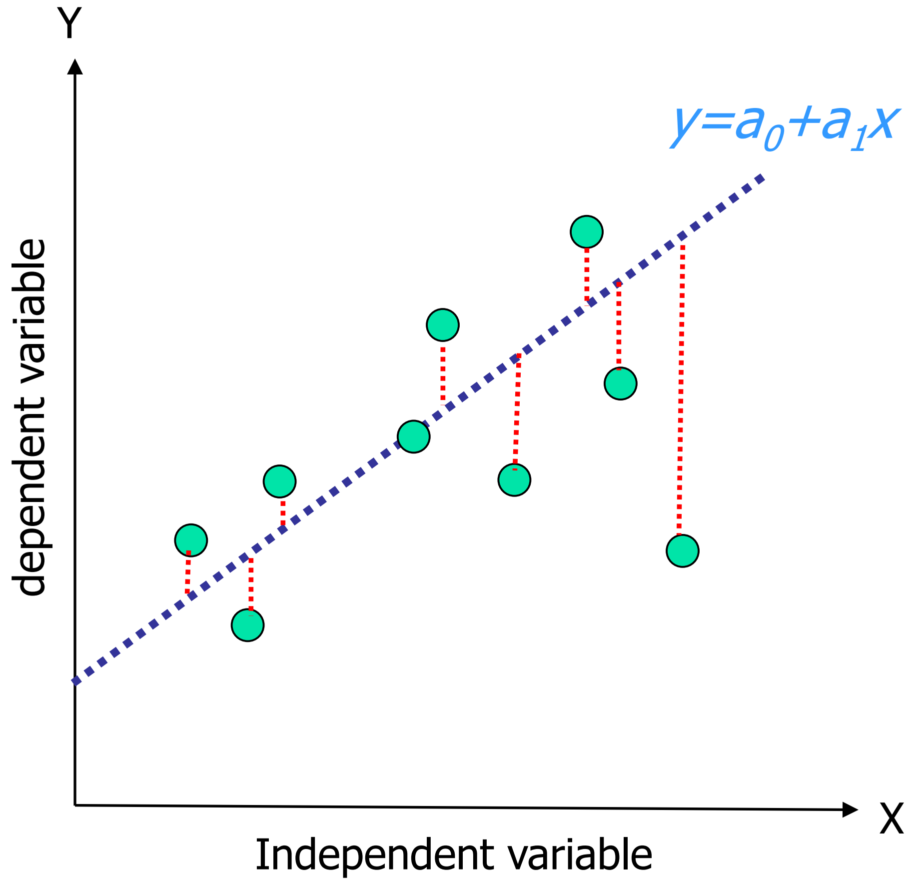
Linear Regression Models



$$E_i = (y_i - (a_0 + a_1 x_i))$$

minimize errors

Linear Regression Models



$$\min \quad Sr = \sum_{i=1}^{n} E_i^2$$

$$a_1 = \frac{n\sum_{i=1}^{n} x_i y_i - \sum_{i=1}^{n} x_i \sum_{i=1}^{n} y_i}{n\sum_{i=1}^{n} x_i^2 - \left(\sum_{i=1}^{n} x_i\right)^2}$$

$$a_0 = \bar{y} - a_1 \bar{x}$$

Outline

- Linear and Non-Linear Regression?
- Non-Linear Regression Models
- Exponential Model
- Polynomial Model
- Transformation
- Comparison
- References.

Non-Linear Regression

- Nonlinear regression is a method of finding a nonlinear model of the relationship between the dependent variable and a set of independent variables.

- Unlike traditional linear regression, which is restricted to estimating linear models, nonlinear regression can estimate models with arbitrary relationships between independent and dependent variables. This is accomplished using iterative estimation algorithms.

Note: this procedure is not necessary for simple polynomial models of the form Y = A + BX**2. By defining W = X**2, we get a simple linear model, Y = A + BW, which can be estimated using traditional methods such as the Linear Regression procedure.

Non-linear regression

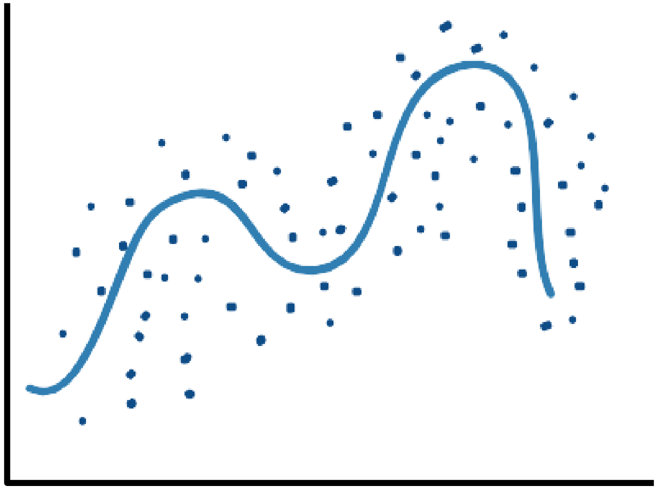
- Nonlinear regression is a statistical modelling technique that helps describe nonlinear relationships in experimental data.

- Nonlinear regression models are generally assumed to be parametric, where the model is described as a nonlinear equation.

- "Nonlinear" refers to a fit function that is a nonlinear function of the parameters.

Best Used...

- When data has strong nonlinear trends and cannot be easily transformed into a linear space
- For fitting custom models to data



Non-Linear Regression

- Non linear regression arises when predictors and response follows particular function form.

$$y = f(\beta, x) + \varepsilon$$

<u>Examples:</u>

$y = \beta^2 x + \varepsilon$ - non linear

$y = \beta x^2 + \varepsilon$ - linear

$y = \frac{1}{\beta} x + \varepsilon$ - non linear

$y = \beta \frac{1}{x} + \varepsilon$ - linear

$y = e^{\beta x} + \varepsilon$ - non linear

$y = \beta \ln x + \varepsilon$ - linear

$y = \frac{1}{1+\beta x} + \varepsilon$ - non linear

Non-Linear Regression Models:

- Exponential
- Power
- Saturation growth
- Polynomial
- Arc Sine
- Arc Tangent
- Cosine
- Integer part
- Log base e
- Log base 10
- SQR
- TAN

Some Popular Non-Linear Regression Models:

1. Exponential model: $(y = ae^{bx})$

2. Power model: $(y = ax^{b})$

3. Saturation growth model: $\left(y = \dfrac{ax}{b+x}\right)$

4. Polynomial model: $(y = a_0 + a_1 x + \ldots + a_m x^m)$

Non-Linear Regression (NLR)?

Given n data points $(x_1, y_1), (x_2, y_2), \ldots, (x_n, y_n)$ best fit $y = f(x)$ to the data, where $f(x)$ is a non-linear function of x.

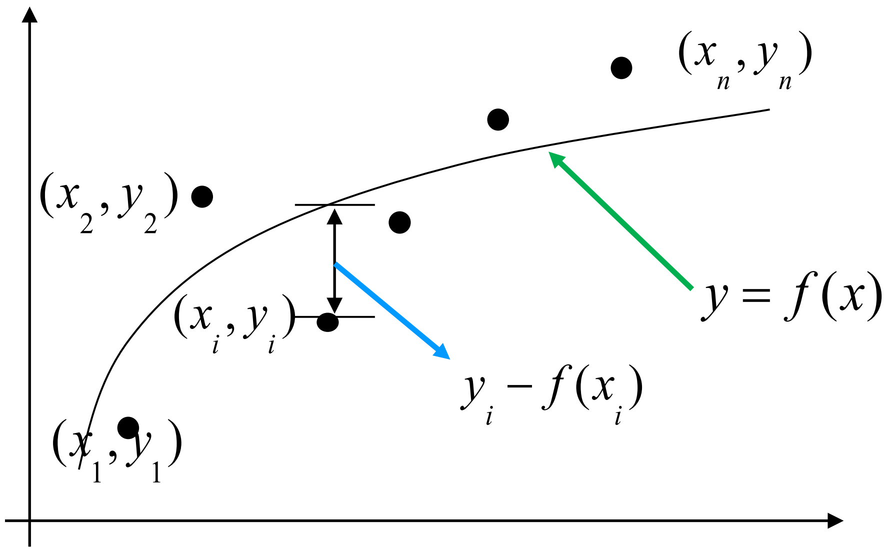


Figure. Non-linear regression model for discrete y vs. x data

Outline

- Linear and Non-linear regression?
- Non-linear regression models
- Exponential model
- Polynomial model
- Transformation
- Comparison
- References.

Regression Exponential Model

Given $(x_1, y_1), (x_2, y_2), \dots, (x_n, y_n)$ best fit $y = ae^{bx}$ to the data.

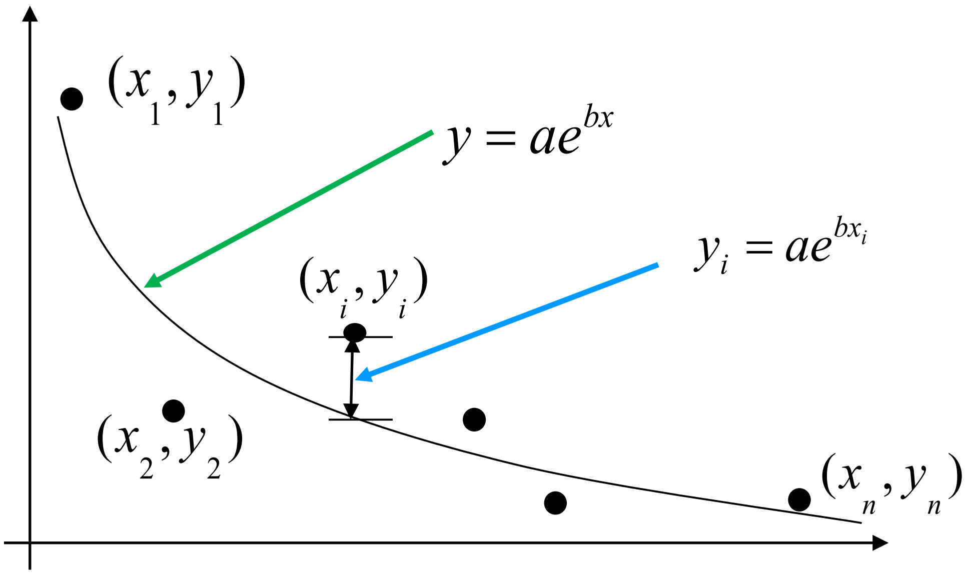


Figure. Exponential model of nonlinear regression for y vs. x data

Finding Constants of Exponential Model

The sum of the square of the residuals is defined as

$$S_r = \sum_{i=1}^{n} \left(y_i - a e^{b x_i} \right)^2$$

Differentiate with respect to *a* and *b*

$$\frac{\partial S_r}{\partial a} = \sum_{i=1}^{n} 2\left(y_i - a e^{b x_i} \right)\left(- e^{b x_i} \right) = 0$$

$$\frac{\partial S_r}{\partial b} = \sum_{i=1}^{n} 2\left(y_i - a e^{b x_i} \right)\left(- a x_i e^{b x_i} \right) = 0$$

https://www.youtube.com/watch?v=G1Podn687-4&list=PL1D3520FC200FA4AE

Finding Constants of Exponential Model

Rewriting the equations, we obtain

$$-\sum_{i=1}^{n} y_i e^{bx_i} + a\sum_{i=1}^{n} e^{2bx_i} = 0$$

$$\sum_{i=1}^{n} y_i x_i e^{bx_i} - a\sum_{i=1}^{n} x_i e^{2bx_i} = 0$$

Finding Constants of Exponential Model

Solving the first equation for a yields

$$a = \frac{\sum_{i=1}^{n} y_i e^{bx_i}}{\sum_{i=1}^{n} e^{2bx_i}}$$

Substituting a back into the previous equation

$$\sum_{i=1}^{n} y_i x_i e^{bx_i} - \frac{\sum_{i=1}^{n} y_i e^{bx_i}}{\sum_{i=1}^{n} e^{2bx_i}} \sum_{i=1}^{n} x_i e^{2bx_i} = 0$$

The constant b can be found through numerical methods such as Bisection method.

The Bisection Method

- Bisection Method = a numerical method in Mathematics to find a root of a given *function*

- Root of a function $f(x)$ = a value a such that:

  - $f(a) = 0$

The Bisection Method: Example

Function: $f(x) = x^2 - 4$

Roots: $x = -2, \; x = 2$

Because:

$$f(-2) = (-2)^2 - 4 = 4 - 4 = 0$$
$$f(2) = (2)^2 - 4 = 4 - 4 = 0$$

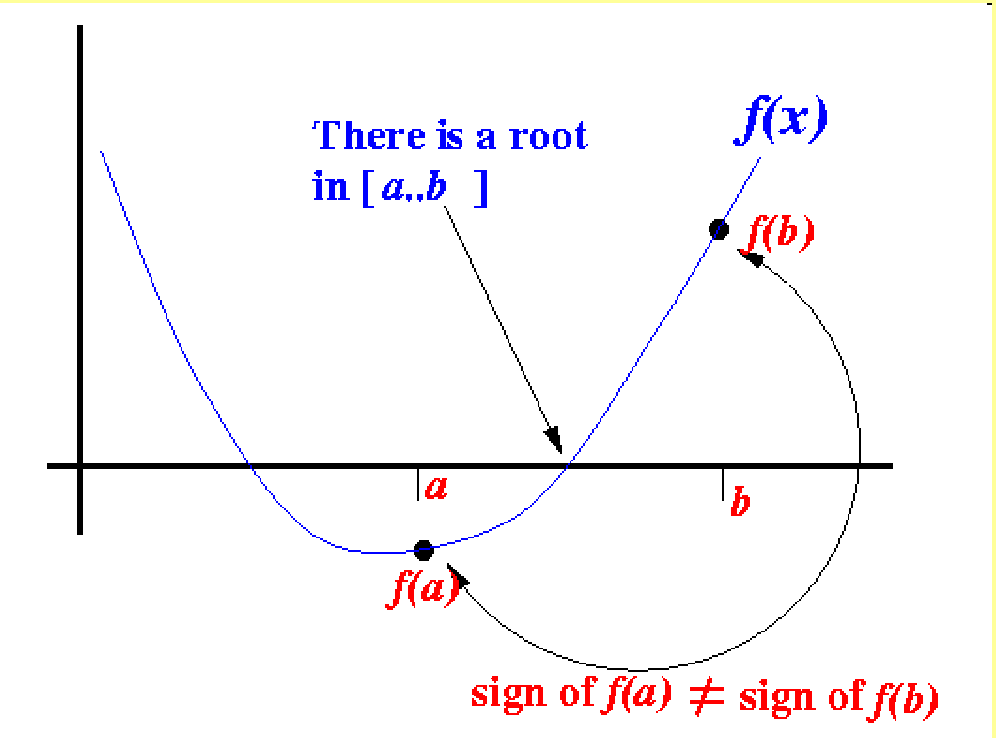
The Bisection Method

- Well-known Mathematical Property:

• If a function $f(x)$ is continuous on the interval $[a..b]$ and sign of $f(a) \neq$ sign of $f(b)$, then

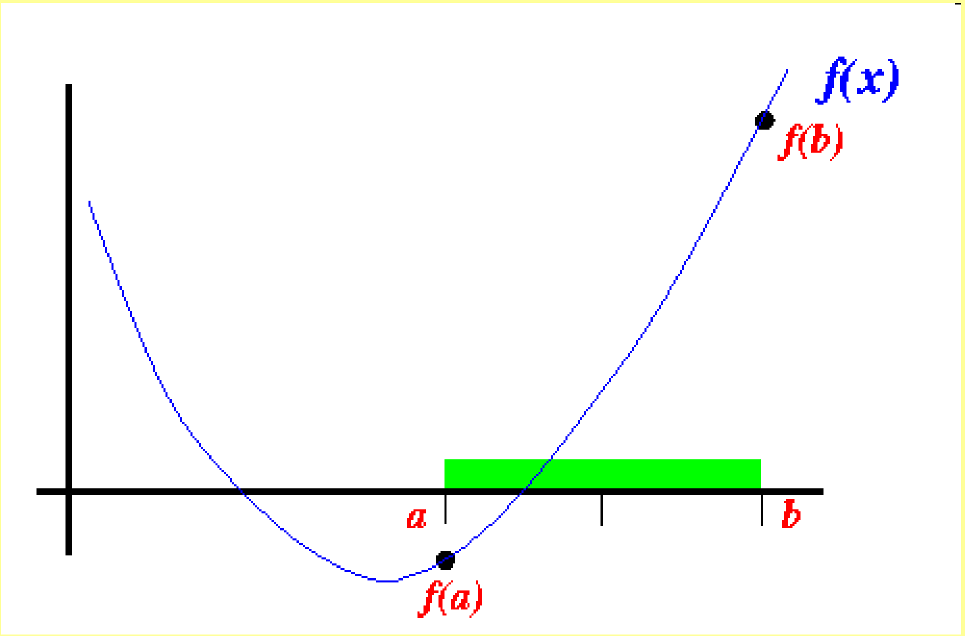
> • There is a value $c \in [a..b]$ such that: $f(c) = 0$
> I.e., there is a root c in the interval $[a..b]$

The Bisection Method: Example



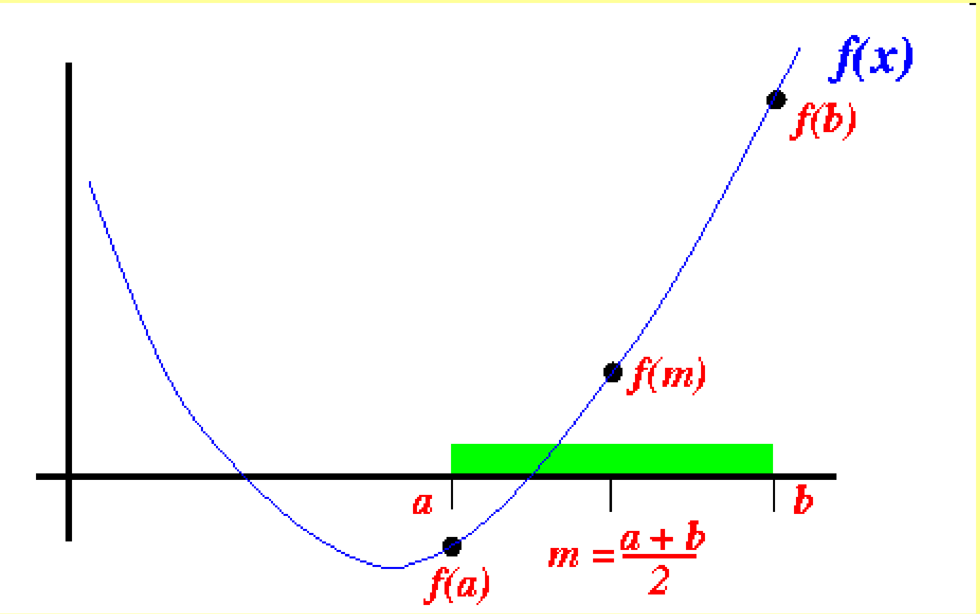
The Bisection Method: Example

- Suppose the interval [*a*..*b*] is as follows:



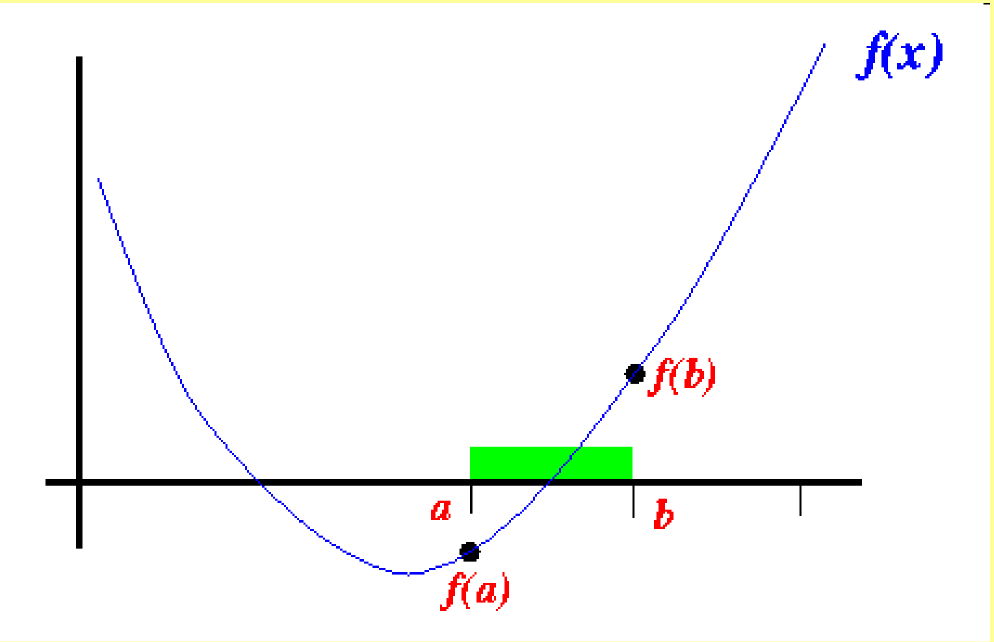
The Bisection Method: Example

- We cut the interval [$a..b$] in the middle: $m = (a+b)/2$



The Bisection Method: Example

- Because sign of $f(m) \neq$ sign of $f(a)$, we *proceed* with the search in the *new* interval $[a..b]$:



The Bisection Method: Example

We can use this statement to change to the new interval:

```
b = m;
```

- In the above example, we have changed the end point *b* to obtain a smaller interval that still contains a root
- In other cases, we may need to changed the end point *b* to obtain a smaller interval that still contains a root

The Bisection Method

- The Bisection Method is a *successive* approximation method that narrows down an interval that contains a root of the function $f(x)$
- The Bisection Method is *given* an initial interval $[a..b]$ that contains a root (We can use the property sign of $f(a) \neq$ sign of $f(b)$ to find such an initial interval)
- The Bisection Method will *cut the interval* into 2 halves and check which half interval contains a root of the function
- The Bisection Method will keep *cut the interval* in halves until the resulting interval is extremely small

  The root is then *approximately equal* to *any value* in the final (very small) interval.

Example 1-Exponential Model

Many patients get concerned when a test involves injection of a radioactive material.

For example for scanning a gallbladder, a few drops of Technetium-99m isotope is used. Half of the Technetium-99m would be gone in about 6 hours. It, however, takes about 24 hours for the radiation levels to reach what we are exposed to in day-to-day activities. Below is given the relative intensity of radiation as a function of time.

Table. Relative intensity of radiation as a function of time.

t(hrs)	0	1	3	5	7	9
γ	1.000	0.891	0.708	0.562	0.447	0.355

Example 1-Exponential Model cont.

The relative intensity is related to time by the equation

$$\gamma = Ae^{\lambda t}$$

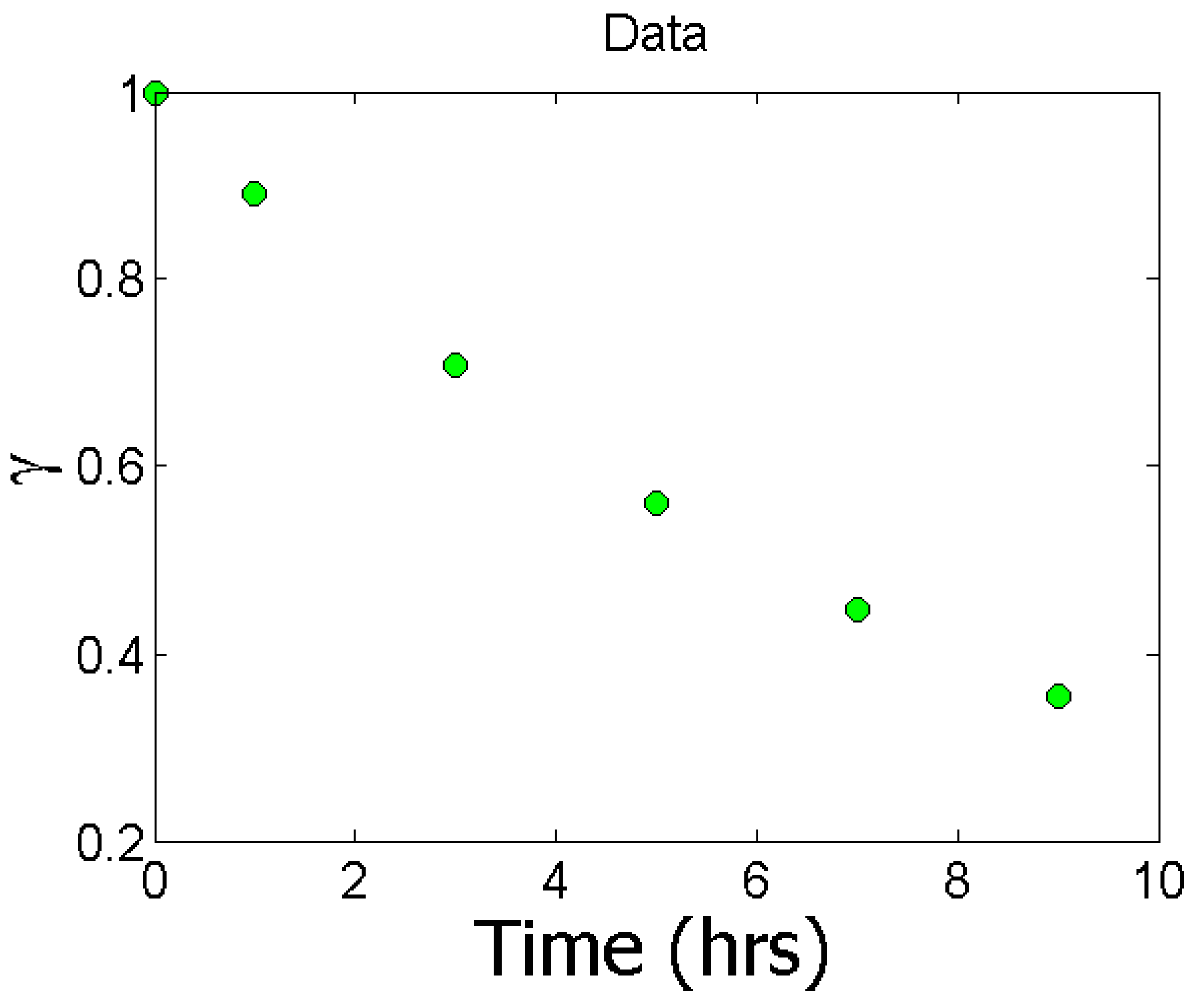
Find:

a) The value of the regression constants A and λ

b) The half-life of Technetium-99m

c) Radiation intensity after 24 hours

Plot of data



Constants of the Model

$$\gamma = Ae^{\lambda t}$$

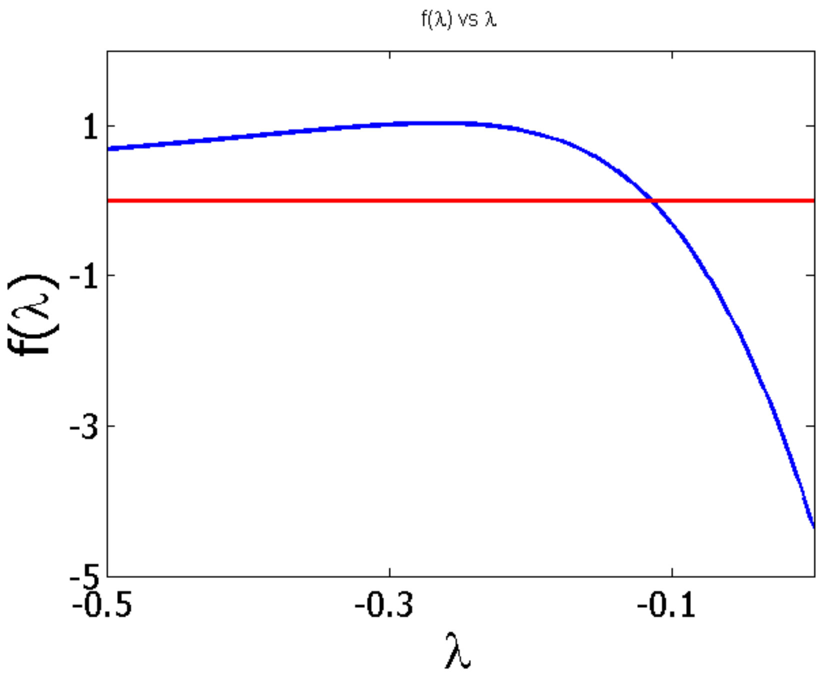
The value of λ is found by solving the nonlinear equation:

$$f(\lambda) = \sum_{i=1}^{n} \gamma_i t_i e^{\lambda t_i} - \frac{\sum_{i=1}^{n} \gamma_i e^{\lambda t_i}}{\sum_{i=1}^{n} e^{2\lambda t_i}} \sum_{i=1}^{n} t_i e^{2\lambda t_i} = 0$$

$$A = \frac{\sum_{i=1}^{n} \gamma_i e^{\lambda t_i}}{\sum_{i=1}^{n} e^{2\lambda t_i}}$$

Setting up the equation in MATLAB

$$f(\lambda) = \sum_{i=1}^{n} \gamma_i t_i e^{\lambda t_i} - \frac{\sum_{i=1}^{n} \gamma_i e^{\lambda t_i}}{\sum_{i=1}^{n} e^{2\lambda t_i}} \sum_{i=1}^{n} t_i e^{2\lambda t_i} = 0$$



t (hrs)	0	1	3	5	7	9
γ	1.000	0.891	0.708	0.562	0.447	0.355

Setting up the Equation in MATLAB

$$f(\lambda)=\sum_{i=1}^{n}\gamma_i t_i e^{\lambda t_i}-\frac{\sum_{i=1}^{n}\gamma_i e^{\lambda t_i}}{\sum_{i=1}^{n}e^{2\lambda t_i}}\sum_{i=1}^{n}t_i e^{2\lambda t_i}=0$$

$$\lambda=-0.1151$$

```
t=[0 1  3  5  7  9]
gamma=[1  0.891  0.708  0.562  0.447  0.355]
syms lamda
sum1=sum(gamma.*t.*exp(lamda*t));
sum2=sum(gamma.*exp(lamda*t));
sum3=sum(exp(2*lamda*t));
sum4=sum(t.*exp(2*lamda*t));
f=sum1-sum2/sum3*sum4;
```

Calculating the Other Constant

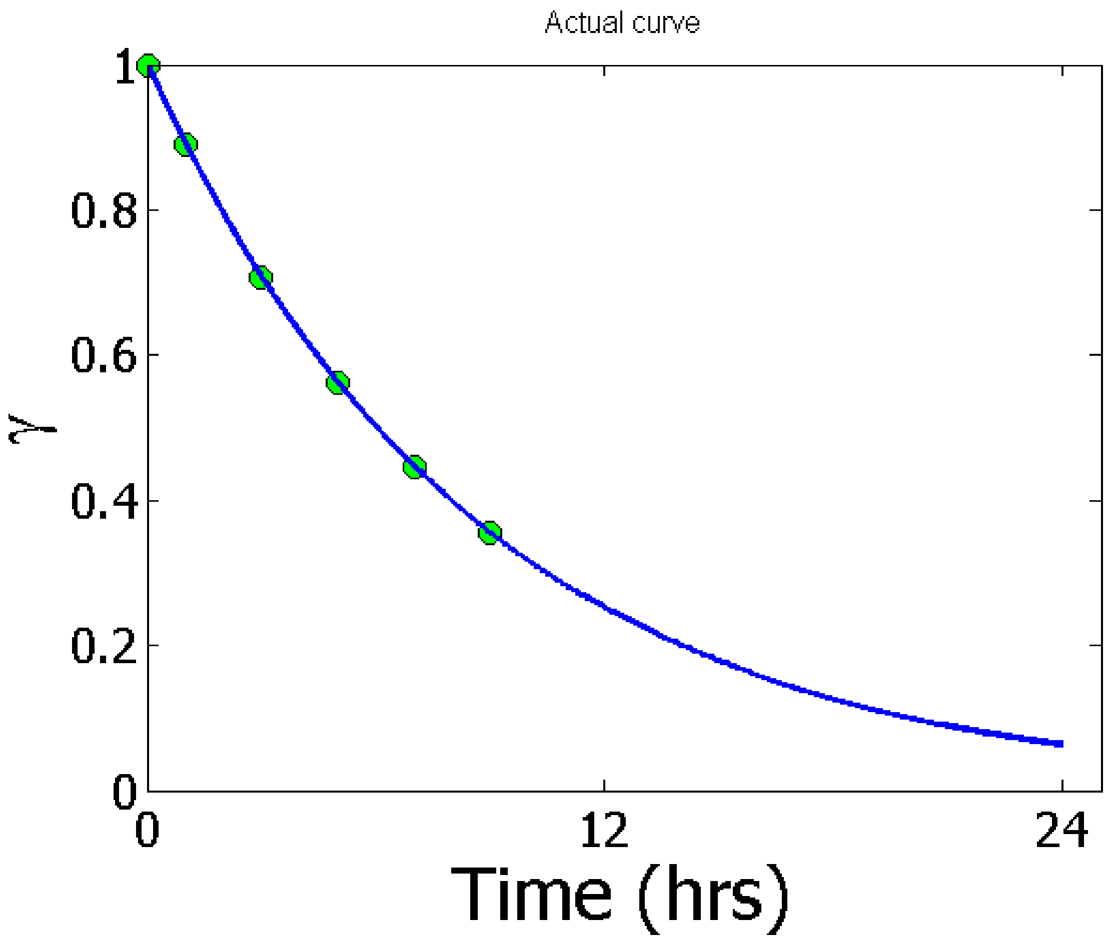
The value of *A* can now be calculated

$$A = \frac{\sum_{i=1}^{6} \gamma_i e^{\lambda t_i}}{\sum_{i=1}^{6} e^{2\lambda t_i}} = 0.9998$$

The exponential regression model then is

$$\gamma = 0.9998\, e^{-0.1151 t}$$

Plot of data and regression curve



Relative Intensity After 24 hrs

The relative intensity of radiation after 24 hours

$$\gamma = 0.9998 \times e^{-0.1151(24)}$$

$$= 6.3160 \times 10^{-2}$$

This result implies that only

$$\frac{6.316 \times 10^{-2}}{0.9998} \times 100 = 6.317\%$$

radioactive intensity is left after 24 hours.

Outline

- Linear and Non-linear regression?
- Non-linear regression models
- Exponential model
- Polynomial model
- Transformation
- Comparison
- References.

Polynomial Model

Given $(x_1, y_1), (x_2, y_2), \ldots, (x_n, y_n)$ best fit $y = a_0 + a_1 x + \ldots + a_m x^m$ $(m \leq n-2)$ to a given data set.

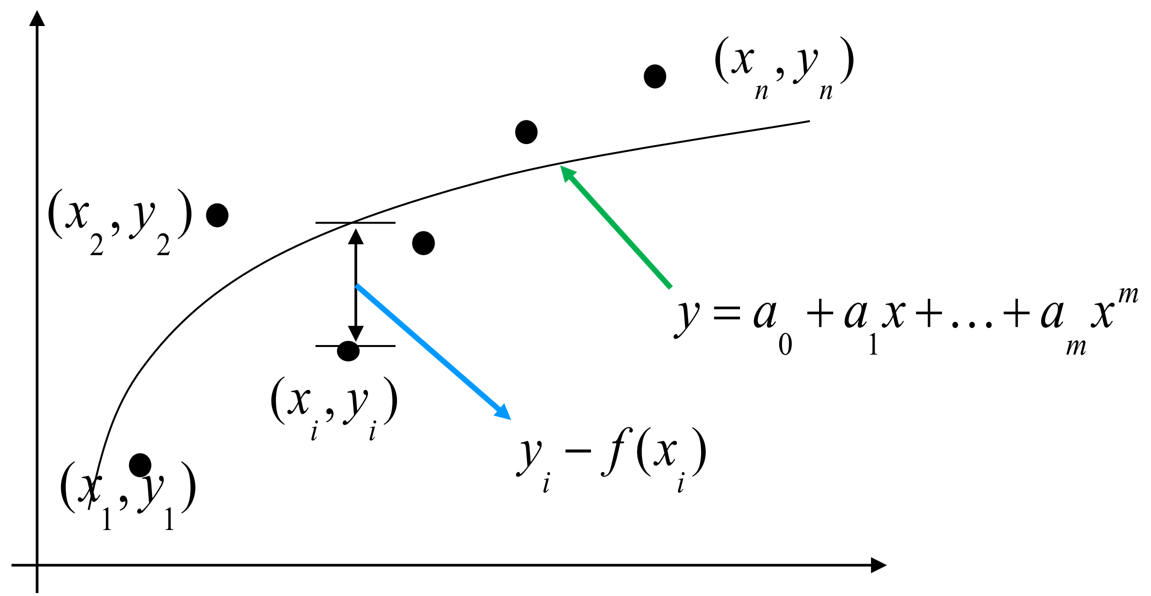


Figure. Polynomial model for nonlinear regression of y vs. x data

Polynomial Model cont.

The residual at each data point is given by

$$E_i = y_i - a_0 - a_1 x_i - . \; . \; . - a_m x_i^m$$

The sum of the square of the residuals then is

$$S_r = \sum_{i=1}^{n} E_i^2$$

$$= \sum_{i=1}^{n} \left(y_i - a_0 - a_1 x_i - . \; . \; . - a_m x_i^m \right)^2$$

Polynomial Model cont.

To find the constants of the polynomial model, we set the derivatives with respect to:

a_i where $i = 1, \ldots m,$ equal to zero.

$$\frac{\partial S_r}{\partial a_0} = \sum_{i=1}^{n} 2.\left(y_i - a_0 - a_1 x_i - . \ . \ . - a_m x_i^m\right)(-1) = 0$$

$$\frac{\partial S_r}{\partial a_1} = \sum_{i=1}^{n} 2.\left(y_i - a_0 - a_1 x_i - . \ . \ . - a_m x_i^m\right)(-x_i) = 0$$

$$\vdots \qquad \vdots \qquad \vdots \qquad \vdots$$

$$\frac{\partial S_r}{\partial a_m} = \sum_{i=1}^{n} 2.\left(y_i - a_0 - a_1 x_i - . \ . \ . - a_m x_i^m\right)(-x_i^m) = 0$$

Polynomial Model cont.

These equations in matrix form are given by:

$$\begin{bmatrix} n & \left(\sum_{i=1}^{n} x_i\right) & \cdot & \cdot & \cdot & \left(\sum_{i=1}^{n} x_i^m\right) \\ \left(\sum_{i=1}^{n} x_i\right) & \left(\sum_{i=1}^{n} x_i^2\right) & \cdot & \cdot & \cdot & \left(\sum_{i=1}^{n} x_i^{m+1}\right) \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \left(\sum_{i=1}^{n} x_i^m\right) & \left(\sum_{i=1}^{n} x_i^{m+1}\right) & \cdot & \cdot & \cdot & \left(\sum_{i=1}^{n} x_i^{2m}\right) \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ \cdot\ \cdot\ \cdot \\ a_m \end{bmatrix} = \begin{bmatrix} \sum_{i=1}^{n} y_i \\ \sum_{i=1}^{n} x_i\, y_i \\ \cdot\ \cdot\ \cdot \\ \sum_{i=1}^{n} x_i^m y_i \end{bmatrix}$$

The above equations are then solved for $a_0, a_1, \ldots, a_m$

Example 2-Polynomial Model

Regress the thermal expansion coefficient vs. temperature data to a second order polynomial.

Table. Data points for temperature vs α

Temperature, T (ºF)	Coefficient of thermal expansion, α (in/in/ºF)
80	6.47×10^{-6}
40	6.24×10^{-6}
−40	5.72×10^{-6}
−120	5.09×10^{-6}
−200	4.30×10^{-6}
−280	3.33×10^{-6}
−340	2.45×10^{-6}

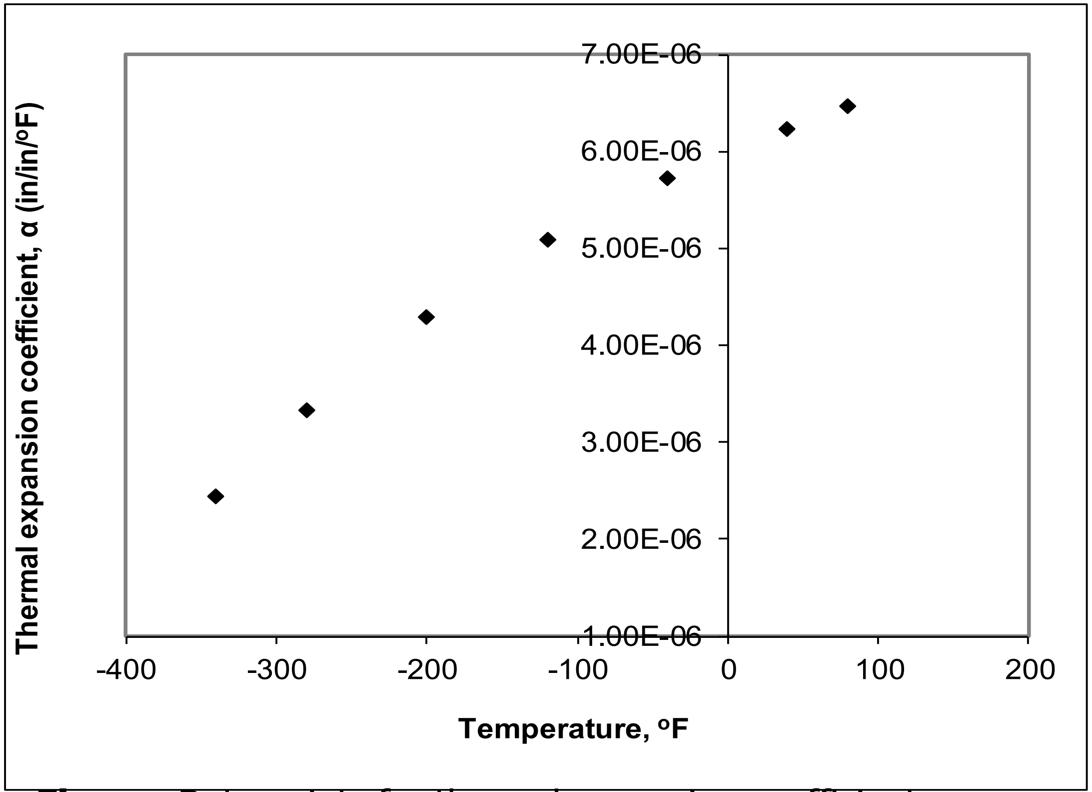


Figure. Data points for thermal expansion coefficient vs temperature.

Example 2-Polynomial Model cont.

We are to fit the data to the polynomial regression model

$$\alpha = a_0 + a_1 T + a_2 T^2$$

The coefficients a_0, a_1, a_2 are found by differentiating the sum of the square of the residuals with respect to each variable and setting the values equal to zero to obtain

$$\begin{bmatrix} n & \left(\sum_{i=1}^{n} T_i\right) & \left(\sum_{i=1}^{n} T_i^2\right) \\ \left(\sum_{i=1}^{n} T_i\right) & \left(\sum_{i=1}^{n} T_i^2\right) & \left(\sum_{i=1}^{n} T_i^3\right) \\ \left(\sum_{i=1}^{n} T_i^2\right) & \left(\sum_{i=1}^{n} T_i^3\right) & \left(\sum_{i=1}^{n} T_i^4\right) \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} \sum_{i=1}^{n} \alpha_i \\ \sum_{i=1}^{n} T_i \, \alpha_i \\ \sum_{i=1}^{n} T_i^2 \alpha_i \end{bmatrix}$$

Example 2-Polynomial Model cont.

The necessary summations are as follows:

Table. Data points for temperature vs. α

Temperature, T (°F)	Coefficient of thermal expansion, α (in/in/°F)
80	6.47×10^{-6}
40	6.24×10^{-6}
−40	5.72×10^{-6}
−120	5.09×10^{-6}
−200	4.30×10^{-6}
−280	3.33×10^{-6}
−340	2.45×10^{-6}

$$\sum_{i=1}^{7} T_i^2 = 2.5580\times10^{5}$$

$$\sum_{i=1}^{7} T_i^3 = -7.0472\times10^{7}$$

$$\sum_{i=1}^{7} T_i^4 = 2.1363\times10^{10}$$

$$\sum_{i=1}^{7} \alpha_i = 3.3600\times10^{-5}$$

$$\sum_{i=1}^{7} T_i\alpha_i = -2.6978\times10^{-3}$$

$$\sum_{i=1}^{7} T_i^2\alpha_i = 8.5013\times10^{-1}$$

Example 2-Polynomial Model cont.

Using these summations, we can now calculate a_0, a_1, a_2

$$\begin{bmatrix} 7.0000 & -8.6000\times10^{2} & 2.5800\times10^{5} \\ -8.600\times10^{2} & 2.5800\times10^{5} & -7.0472\times10^{7} \\ 2.5800\times10^{5} & -7.0472\times10^{7} & 2.1363\times10^{10} \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} 3.3600\times10^{-5} \\ -2.6978\times10^{-3} \\ 8.5013\times10^{-1} \end{bmatrix}$$

Solving the above system of simultaneous linear equations we have:

$$\begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} 6.0217\times10^{-6} \\ 6.2782\times10^{-9} \\ -1.2218\times10^{-11} \end{bmatrix}$$

The polynomial regression model is then:

$$\begin{aligned} \alpha &= a_0 + a_1 T + a_2 T^2 \\ &= 6.0217\times10^{-6} + 6.2782\times10^{-9}\,\mathrm{T} - 1.2218\times10^{-11}\,\mathrm{T}^2 \end{aligned}$$

Outline

- Linear and Non-linear regression?
- Non-linear regression models
- Exponential model
- Polynomial model
- Transformation
- Comparison
- References.

Transformation

- Some nonlinear regression problems can be moved to a linear domain by a suitable transformation of the model formulation.

- Four common transformations to induce linearity are:

  (i) logarithmic transformation,
  (ii) square root transformation,
  (iii) inverse transformation
  (iv) the square transformation

<u>Examples:</u>

- $y = e^{\beta x} \Longrightarrow \ln y = \beta x \quad$ **if** $y \geq 0$

- $y = \frac{1}{1+\beta x} \Longrightarrow \frac{1}{y} - 1 = \beta x \quad$ **if** $y \neq 0$

Transformation

As shown in the previous example, many chemical and physical processes are governed by the equation,

$$y = ae^{bx}$$

Taking the natural log of both sides yields,

$$\ln y = \ln a + bx$$

Let $z = \ln y$ and $a_0 = \ln a$

We now have a linear regression model where $z = a_0 + a_1 x$

(implying) $a = e^{a_o}$ with $a_1 = b$

Transformation

$$a_1 = \frac{n\sum_{i=1}^{n} x_i z_i - \sum_{i=1}^{n} x_i \sum_{i=1}^{n} z_i}{n\sum_{i=1}^{n} x_i^2 - \left(\sum_{i=1}^{n} x_i\right)^2}$$

$$a_0 = \bar{z} - a_1 \bar{x}$$

Once a_o, a_1 are found, the original constants of the model are found as

$$b = a_1$$

$$a = e^{a_0}$$

Example 3-Transformation of data

Many patients get concerned when a test involves injection of a radioactive material. For example for scanning a gallbladder, a few drops of Technetium-99m isotope is used. Half of the Technetium-99m would be gone in about 6 hours. It, however, takes about 24 hours for the radiation levels to reach what we are exposed to in day-to-day activities. Below is given the relative intensity of radiation as a function of time.

Table. Relative intensity of radiation as a function of time

t(hrs)	0	1	3	5	7	9
γ	1.000	0.891	0.708	0.562	0.447	0.355

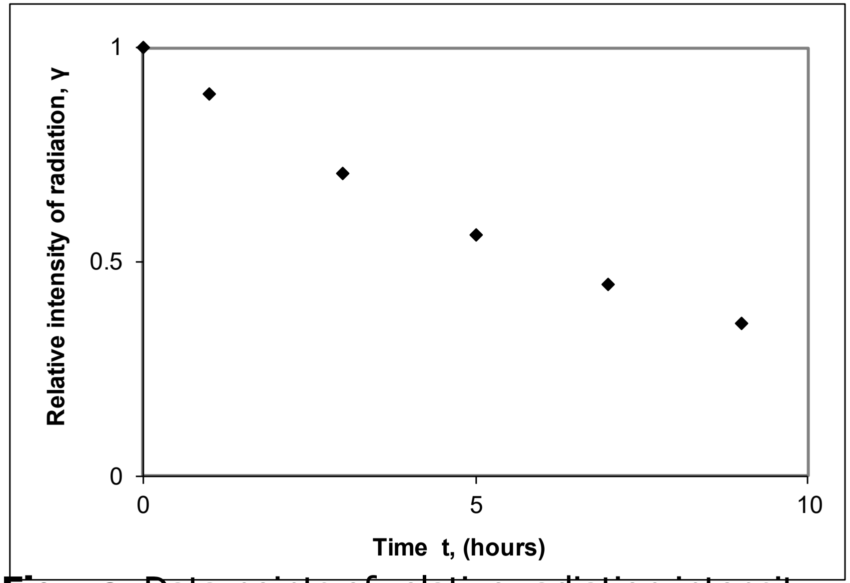


Figure. Data points of relative radiation intensity vs. time

Example 3-Transformation of data cont.

Find:

a) The value of the regression constants A and λ

b) The half-life of Technetium-99m

c) Radiation intensity after 24 hours

The relative intensity is related to time by the equation:

$$\gamma = Ae^{\lambda t}$$

Example 3-Transformation of data cont.

Exponential model given as,

$$\gamma = Ae^{\lambda t}$$

$$\ln(\gamma) = \ln(A) + \lambda t$$

Assuming $z = \ln \gamma$, $a_o = \ln(A)$ and $a_1 = \lambda$ we obtain

$$z = a_0 + a_1 t$$

This is a linear relationship between z and t

Example 3-Transformation of data cont.

Using this linear relationship, we can calculate a_0, a_1 where

$$a_1 = \frac{n\sum_{i=1}^{n} t_i z_i - \sum_{i=1}^{n} t_i \sum_{i=1}^{n} z_i}{n\sum_{i=1}^{n} t_1^2 - \left(\sum_{i=1}^{n} t_i\right)^2}$$

$$a_0 = \bar{z} - a_1 \bar{t}$$

and

$$\lambda = a_1$$

$$A = e^{a_0}$$

Example 3-Transformation of Data cont.

Summations for data transformation are as follows:

Table. Summation data for Transformation of data model

i	t_i	γ_i	$z_i = \ln \gamma_i$	$t_i z_i$	t_i^2
1	0	1	0.00000	0.0000	0.0000
2	1	0.891	−0.11541	−0.11541	1.0000
3	3	0.708	−0.34531	−1.0359	9.0000
4	5	0.562	−0.57625	−2.8813	25.000
5	7	0.447	−0.80520	−5.6364	49.000
6	9	0.355	−1.0356	−9.3207	81.000
$\sum$	25.000		−2.8778	−18.990	165.00

With $n = 6$

$$\sum_{i=1}^{6} t_i = 25.000$$

$$\sum_{i=1}^{6} z_i = -2.8778$$

$$\sum_{i=1}^{6} t_i z_i = -18.990$$

$$\sum_{i=1}^{6} t_i^2 = 165.00$$

Example 3-Transformation of Data cont.

Calculating a_0, a_1

$$a_1 = \frac{6(-18.990) - (25)(-2.8778)}{6(165.00) - (25)^2} \qquad = -0.11505$$

$$a_0 = \frac{-2.8778}{6} - (-0.11505)\frac{25}{6} \qquad = -2.6150 \times 10^{-4}$$

Since

$$a_0 = \ln(A)$$

$$A = e^{a_0}$$

$$= e^{-2.6150 \times 10^{-4}} = 0.99974$$

also

$$\lambda = a_1 = -0.11505$$

Example 3-Transformation of Data cont.

Resulting model is $\gamma = 0.99974 \times e^{-0.11505t}$

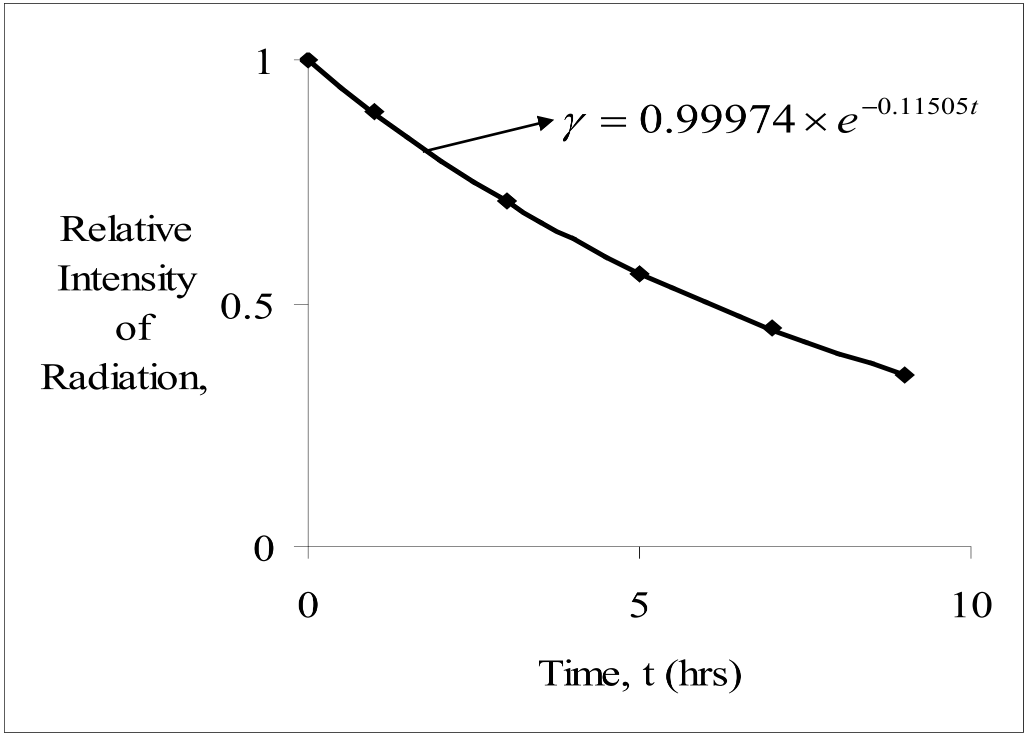


Figure. Relative intensity of radiation as a function of temperature using transformation of data model.

Example 3-Transformation of Data cont.

The regression formula is then

$$\gamma = 0.99974 \times e^{-0.11505t}$$

b) Half life of Technetium-99m is when $\gamma = \frac{1}{2}\gamma\Big|_{t=0}$

$$0.99974 \times e^{-0.11505t} = \frac{1}{2}(0.99974)e^{-0.11505(0)}$$

$$e^{-0.11508t} = 0.5$$

$$-0.11505t = \ln(0.5)$$

$$t = 6.0248 \; hours$$

Example 3-Transformation of Data cont.

c) The relative intensity of radiation after 24 hours is then

$$\gamma = 0.99974e^{-0.11505(24)}$$
$$= 0.063200$$

This implies that only $\dfrac{6.3200\times10^{-2}}{0.99983}\times100 = 6.3216\%$ of the radioactive material is left after 24 hours.

Outline

- Linear and Non-linear regression?
- Non-linear regression models
- Exponential model
- Polynomial model
- Transformation
- Comparison
- References.

Comparison

Comparison of exponential model with and without data Transformation:

Table. Comparison for exponential model with and without data Transformation.

	With data Transformation (Example 3)	Without data Transformation (Example 1)
A	0.99974	0.99983
λ	-0.11505	-0.11508
Half-Life (hrs)	6.0248	6.0232
Relative intensity after 24 hrs.	6.3200×10^{-2}	6.3160×10^{-2}

References

[1] H. Khayyam, G. Golkarnarenji, R. N Jazar “Limited data modelling approaches for engineering applications” Book Nonlinear approaches in Engineering Applications, Pages 345-379 Publisher Springer (2018)

[2] Burges, C. “A Tutorial on Support Vector Machines for Pattern Recognition.” Bell Labs. 1998

[3] Law, Martin. “A Simple Introduction to Support Vector Machines.” Michigan State University. 2006

[4] Prabhakar, K. “An Introduction to Support Vector Machines”

[5] www.dsl-lab.org/ml_tutorial/Presentation/file4.ppt

[6] http://en.wikipedia.org/wiki/Support_vector_machine

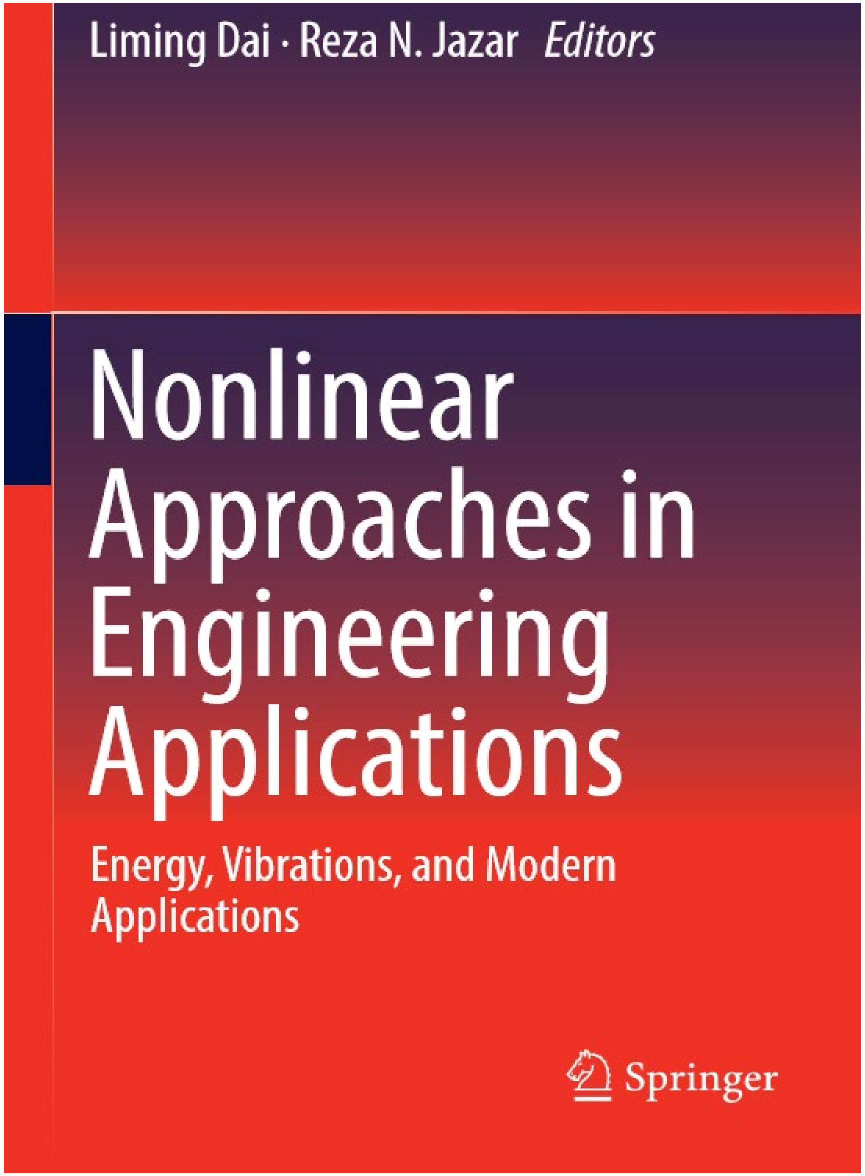
[7] http://www.saedsayad.com/

[8] http://www.robots.ox.ac.uk/~az/lectures/ml/lect3.pdf

[9] www.public.asu.edu/~huanliu/dmml_presentation/.../SVM%20for%20Regression.ppt

[10] Build Machine Learning Models with a MATLAB Trial

References



Chapter 12
Limited Data Modelling Approaches for Engineering Applications

Hamid Khayyam, Gelayol Golkarnarenji, and Reza N. Jazar

12.1 Introduction

Over the past several years, the study of various complex systems has been of great interest to researchers and scientists. Complex systems and problems are very pervasive and appear in different application areas including education, healthcare, medicine, finance, marketing, homeland security, defense, and environmental management, among others. In these systems, many components are involved with nonlinear interactions. Forecasting the future state of a complex system and designing such a system are very costly, time consuming, and compute intensive due to project times and technical constraints in industry. To overcome these complexities and save considerable amount of cost, time, and energy, modelling can be utilized. Modelling is generally defined as mathematical realization and computerized analysis of abstract representation of real systems. It helps achieve comprehensive insight into the functionality of the modelled systems, investigate the performance and behavior of processes, and finally optimize the process control. Mathematical modelling is an inexpensive and a powerful paradigm to deal with real-world complex problems. It comprises a wide range of computational methods. This technique can lower the costs by reducing the number of experiments and increasing the safety by forecasting the events, the results of laboratory tests, or the industrial data (Dobre and Sanchez Marcano 2007; Pham 1998; Rodrigues and Minceva 2005).

H. Khayyam (✉) • R.N. Jazar
School of Engieering, RMIT University, Melbourne, VIC, Australia
e-mail: hamid.khayyam@rmit.edu.au

G. Golkarnarenji
Institute for Frontier Materials, Carbon Nexus, Deakin University, Waurn Ponds, VIC, Australia

© Springer International Publishing AG 2018 345
L. Dai, R.N. Jazar (eds.), *Nonlinear Approaches in Engineering Applications*,
https://doi.org/10.1007/978-3-319-69480-1_12

Resources

http://www.kernel-machines.org

http://www.support-vector.net/

http://www.support-vector.net/icml-tutorial.pdf

http://www.kernel-machines.org/papers/tutorial-nips.ps.gz

http://www.clopinet.com/isabelle/Projects/SVM/applist.html

http://www.cs.cornell.edu/People/tj/

http://www.idi.ntnu.no/emner/it3704/lectures/papers/Bennett_2000_ Support.pdf

http://aya.technion.ac.il/karniel/CMCC/SVM-tutorial.pdf

http://www.ecs.soton.ac.uk/~srg/publications/pdf/SVM.pdf

http://en.wikipedia.org/wiki/Support_vector_machine

http://numericalmethods.eng.usf.edu

https://www.google.com/search?rlz=1C1CHBF_en&ei=HgDIXJLBLpjZz7sPg7mzyAE&q=the+bisection+method%3Appt&oq=the+bisection+method%3Appt&gs_l=psy-ab.3...5456.6447..6879...0.0..0.341.1069.0j1j2j1......0....1..gws-wiz.......0i71j0j0i22i30.2WQnw50QHfc

Resources in Matlab

- Mathworks “Train support vector machine classifier”. <http://www.mathworks.com/help/toolbox/bioinfo/ref/svmtrain.html> (4/6/2011)

- “Support vector machine”. <http://en.wikipedia.org/wiki/Support_vector_machine> (4/6/2011)

- Jason Weston, “Support Vector Machine (and Statistical Learning Theory) Tutorial”, NEC Labs America.