Part VI: Cascade Control System Design

Liuping Wang

School of Engineering
Royal Melbourne Institute of Technology University
Australia

Outline

Outline

Learning Objectives

- Case study of permanent magnetic synchronous motor (PMSM) control
- Configuration of a cascade feedback control system
    - selection of inner-loop and out-loop systems
    - design of inner-loop control system (secondary control system)
    - design of outer-loop control system (primary control system)
- Use of multiple PID controllers for multi-input and multi-output systems
    - neglecting the interactions between the inputs and outputs
    - feedforward using the interactions

Key Reference

Chapter two in "PID and Predictive Control of Electrical Drives and Power Converters using MATLAB/Simulink" (Wiley-IEEE Press).

Outline

System suitable for cascade control

- A typical system suitable for cascade control is shown in Figure 1.
- The variable between the transfer functions, $x_1(t)$, is measurable.

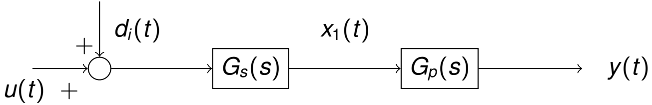


Figure 1: Block diagram for a system suitable for cascade control

Cascade control structure

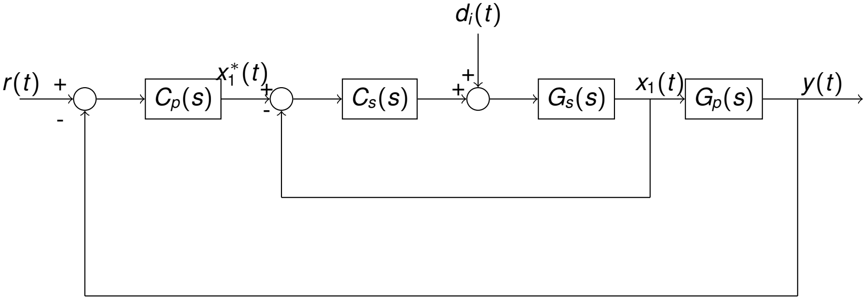


Figure 2: Block diagram of a cascade control system

Secondary and primary systems

The inner-loop system is the secondary system and the outer-loop system is the primary system. The link between these two loops is the reference signal $x_1^*(t)$.

Design Steps

Subsystems

A complex system is decomposed into a series of first order or second order subsystems based on the considerations of physical relationships and availability of measurements.

Subsystem controllers

Design P, PI, PID, PD for each of the subsystems depending on the requirements. In general, the outer- loop systems are required to contain integral action for eliminate steady-state errors.

Design procedures

In the design process, the inner-loop control system is designed first and the closed-loop transfer function for the inner-loop system is obtained. The outer-loop control system is designed based on the outer-loop system, where the relatively small time constants resulted from the inner closed- loop system are neglected, but its steady-state gain is taken into account in the outer-loop model.

Stability and Performance Analysis

- Robust stability and performance analysis are performed, and closed-loop performances are adjusted using the bandwidths of the inner-loop and outer-loop systems.
- This step is important because there are neglected dynamics in the cascade control system.
- In principle, the bandwidth of the inner closed-loop control should be much wider than the one used in the outer closed-loop control. Namely, the inner-loop control system should have a much faster response speed for obtaining the closed-loop stability of the cascade control system.
- In the implementation, a wider bandwidth for the secondary closed-loop system is desired and also achieved by putting proportional control K_c on the feedback error.

Outline

Design Example: PI +PI

$$G_s(s) = \frac{5}{s+10}; \quad G_p(s) = \frac{0.005}{s+0.05}$$

Design a cascade control system with two PI controllers. For simplicity, we select the damping coefficient $\xi = 0.707$ for both inner and outer-loop control systems and use the bandwidths w_{ns} and w_{np} as the tuning parameters of the inner (secondary) and outer-loop (primary) systems respectively.

Solution I

For the inner-loop control system, we choose $w_{ns} = 5 \times 10 = 50$ leading to a pair of closed-loop poles at $-35.35 \pm j35.3607$, and for the outer-loop system, we choose $w_{np} = 4 \times 0.05 = 0.2$ leading to a pair of closed-loop poles at $-0.1414 \pm j0.1414$. These selections give us the ratio of inner-loop bandwidth to outer-loop bandwidth of 250.

The inner-loop control system

Controller parameters

$$K_{cs} = \frac{2\xi w_{ns} - a}{b} = \frac{2\xi w_{ns} - 10}{5} = 12.14;$$
$$\tau_{Is} = \frac{2\xi w_{ns} - a}{w_{ns}^2} = \frac{2\xi w_{ns} - 10}{w_{ns}^2} = 0.0243$$

Closed-loop transfer function

The closed-loop transfer function between the reference signal $X_1^*(s)$ and the output signal $X_1(s)$ is calculated as

$$\frac{X_1(s)}{X_1^*(s)} = \frac{(2\xi w_{ns} - 10)s + w_{ns}^2}{s^2 + 2\xi w_{ns} s + w_{ns}^2} \quad (1)$$

The outer-loop control system

To design the outer-loop controller, we consider the transfer function between $X_1^*(s)$ and the output $Y(s)$, which is

$$\frac{Y(s)}{X_1^*(s)} = \frac{(2\xi w_{ns} - 10)s + w_{ns}^2}{s^2 + 2\xi w_{ns} s + w_{ns}^2} \frac{0.005}{s + 0.05} \tag{2}$$

We neglect the inner-closed-loop system by considering

$$\frac{X_1(s)}{X_1^*(s)} = \frac{\frac{(2\xi w_{ns} - 10)}{w_{ns}^2} s + 1}{\frac{1}{w_{ns}^2} s^2 + \frac{2\xi}{w_{ns}} s + 1} \approx 1 \tag{3}$$

$$K_{cp} = \frac{2\xi w_{np} - 0.05}{0.005} = 46.56; \quad \tau_{Ip} = \frac{2\xi w_{np} - 0.05}{w_{np}^2} = 5.82$$

where $w_{np} = 0.2$.

Closed-loop poles

- One can verify that there are four closed-loop poles with the following values: $-35.2335 \pm j35.4441$ and $-0.1415 + \pm j0.1415$.
- The pair of dominant closed-loop poles are almost equal to the performance specifications from the outer-loop control system and the remaining pair is close to the performance specification from the inner-loop control system.

Design example: P+ PID

Secondary system

The secondary system in a cascade control system is a motor which has the transfer function

$$G_s(s) = \frac{0.03}{s(s+30)} \tag{4}$$

where the output of the motor is angular position.

Primary system

The primary system is an undamped oscillator with the transfer function

$$G_p(s) = \frac{0.6}{s^2+1} \tag{5}$$

Specification

Design a cascade control system with inner-loop proportional and outer-loop PID control. The outer-loop control system is specified with $\xi = 0.707$ and $w_{np} = 1$ and the remaining poles are placed at -2.

Solution

Secondary P controller design

The secondary system is approximated by the following integral model: $G_s(s) \approx \frac{0.001}{s}$ where the stable mode is neglected.

Controller parameter

Because the primary control system is required to have the natural frequency $w_{np} = 1$, we select the closed-loop pole for the secondary control system at -10, leading to the proportional controller $K_{cs} = 10000$.

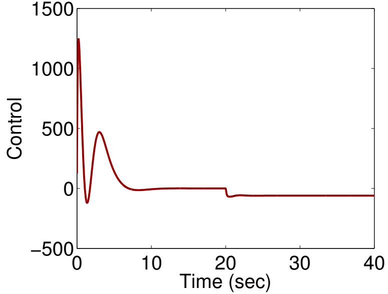
Primary Controller Design

- The PID controller with filter is designed using the MATLAB function pidplace.m.
- In the design of PID controller, the desired closed-loop polynomial is selected as

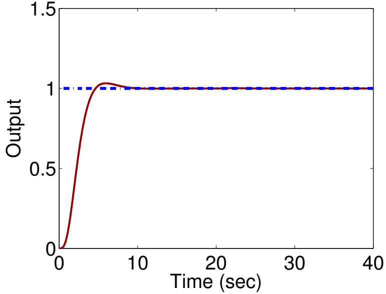
$$A_{cl}(s) = (s^2 + 2\xi w_{np} s + w_{np}^2)(s+2)^2$$

  where $w_{np} = 1$ and $\xi = 0.707$.
- $$K_{cp} = 1.0784; \;\; \tau_{Ip} = 0.8758; \;\; \tau_{Dp} = 2.5717; \;\; \tau_{fp} = 0.1847$$

Closed-loop control results



(a) Control signal



(b) Output

Figure 3: Cascade closed-loop response signals (Primary controller PID and secondary controller P).

Outline

Block Diagram for Cascade Control

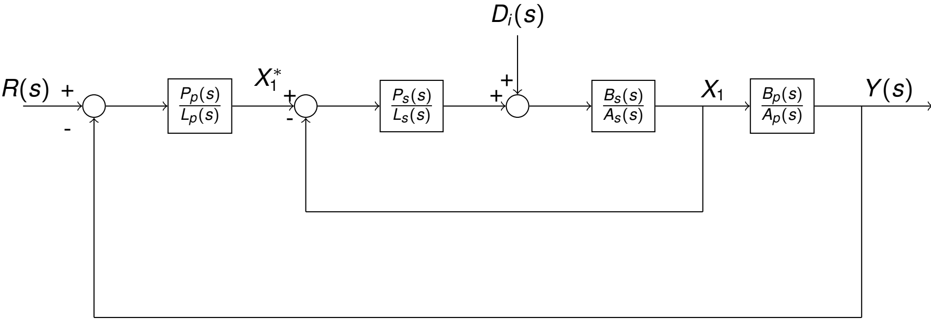


Figure 4: Closed-loop cascade control system

Closed-loop Transfer Function For Disturbance Rejection (i)

- To examine the effectiveness of disturbance rejection, we calculate the closed-loop transfer function between the disturbance $D_i(s)$ and the output $Y(s)$ as shown in Figure 4. Here, we assume the reference signal $R(s) = 0$.
-

$$X_1(s) = \overbrace{\frac{B_s(s)P_s(s)}{A_s(s)L_s(s) + B_s(s)P_s(s)}}^{T(s)_s} X_1(s)^* + \overbrace{\frac{B_s(s)L_s(s)}{A_s(s)L_s(s) + B_s(s)P_s(s)}}^{S_i(s)_s} D_i(s) \tag{6}$$

$$X_1(s) = T(s)_s X_1(s)^* + S_i(s)_s D_i(s) \tag{7}$$

Closed-loop Transfer Function For Disturbance Rejection (ii)

The primary output $Y(s)$ is expressed as

$$\begin{aligned} Y(s) &= \frac{B_p(s)}{A_p(s)} X_1(s) \\ &= \frac{B_p(s)}{A_p(s)} (T(s)_s X_1(s)^* + S_i(s)_s D_i(s)) \end{aligned} \quad (8)$$

With the control signal $X_1(s)^*$ generated from the primary controller as

$$X_1(s)^* = -\frac{P_p(s)}{L_p(s)} Y(s)$$

we obtain the closed-loop transfer function from the input disturbance $D_i(s)$ to the output $Y(s)$:

$$\frac{Y(s)}{D_i(s)} = \frac{G_p(s) S_i(s)_s}{1 + G_p(s) C_p(s) T(s)_s} \quad (9)$$

Example

Consider the position control of a DC motor in the presence of unknown load T_L. The relationship between the input voltage $V(s)$ and the angular velocity of the motor $\Omega(s)$ is described by the normalized Laplace transfer function:

$$\frac{\Omega(s)}{V(s)} = \frac{e^{-ds}}{s+1} \tag{10}$$

where a small time delay $d = 0.0016$ (sec) is used to model the delay induced by the sensing and actuation devices. The angular position $\Theta(s)$ is related to the angular velocity through integration:

$$\frac{\Theta(s)}{\Omega(s)} = \frac{1}{s}$$

Design a cascade control system for the position control of the DC motor and show its advantage in terms of disturbance rejection of the unknown load.

Solution (i)

- For the cascade control system design, the secondary transfer function is

$$G_s(s) = \frac{e^{-ds}}{s+1}$$

- By neglecting the time delay, from the pole-assignment controller design, the proportional controller gain and the integral time constant are

$$K_{cs} = 2\xi w_{ns} - 1 = 34.35; \quad \tau_{Is} = \frac{2\xi w_{ns} - 1}{w_{ns}^2} = 0.0550$$

where $\xi = 0.707$ and $w_{ns} = 25$.

- The primary transfer function is

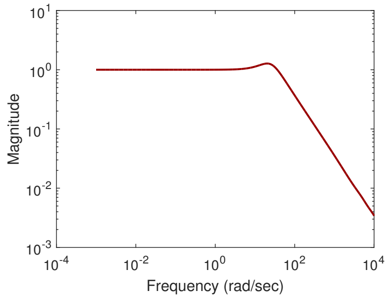
$$G_p(s) = \frac{1}{s}$$

and the proportional controller gain and the integral time constant are

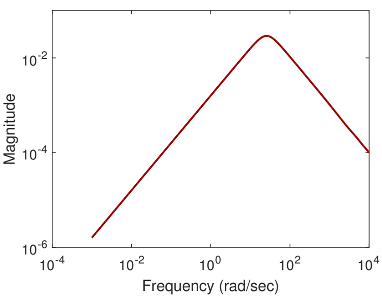
$$K_{cp} = 2\xi w_{np} = 3.535; \quad \tau_{Ip} = \frac{2\xi}{w_{np}} = 0.5656$$

where $\xi = 0.707$ and $w_{np} = 2.5$

Sensitivity Analysis (i)



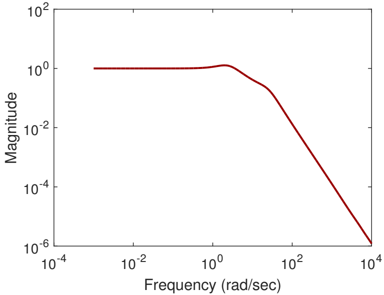
(a) Complementary sensitivity (secondary)



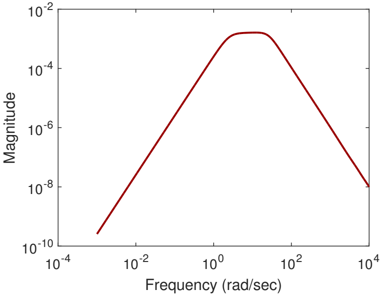
(b) Input sensitivity (secondary)

Figure 5: Sensitivity functions for the secondary control system

Sensitivity Analysis (ii)



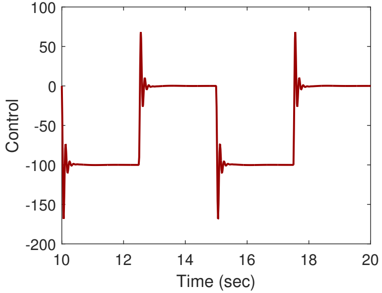
(a) Complementary sensitivity (primary)



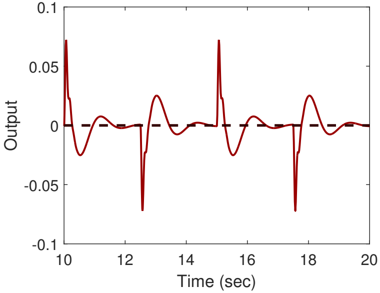
(b) Input sensitivity (primary)

Figure 6: Sensitivity functions for the cascade control system

Closed-loop Simulation Results



(a) Control signal

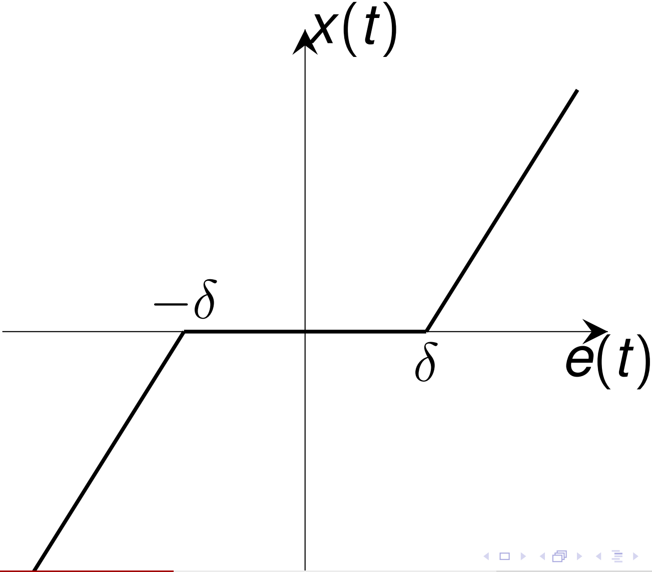


(b) Output signal

Figure 7: Cascade closed-loop response to square wave disturbance signal with amplitude 100 and period of 10.

Outline

Actuator with Deadzone



Deadzone Nonlinearity

Deadzone nonlinearity for an actuator, which is due to wearing and tearing, is described by the following equations:

$$x(t) = \begin{cases} e(t) - \delta & e(t) > \delta \\ 0 & -\delta \leq e(t) \leq \delta \\ e(t) + \delta & e(t) < \delta \end{cases} \tag{11}$$

Example

The actuator for a physical system is described by the transfer function

$$G_s(s) = \frac{0.5}{s+15},$$

which is secondary plant. The primary plant is described by the transfer function:

$$G_p(s) = \frac{0.8}{(0.1s+1)(s+0.1)}. \tag{12}$$

There is a deadzone associated with the actuator.

Ignoring Actuator Dynamics

By neglecting this small time constant and taking consideration of the steady-state gain from the actuator, which is $\frac{0.5}{15}$, we obtain the approximate model for the PI controller design as

$$G(s) = \frac{0.5}{15}\frac{0.8}{s+0.1} = \frac{b}{s+a}$$

With $a = 0.1$, $b = 0.0267$, $w_n = 1$ and $\xi = 0.707$, we calculate the PI controller parameters as

$$K_c = \frac{2\xi w_n - a}{b} = 49.275; \quad \tau_I = \frac{2\xi w_n - a}{w_n^2} = 1.314.$$

Closed-loop Response

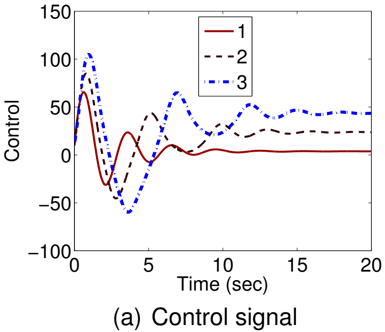


Figure 9: Closed-loop control response by neglecting actuator dynamics. Key: line (1) response without deadzone; line (2) response with deadzone ($\delta = 20$); line (3) response with deazone ($\delta = 40$)

Cascade Control

We continue from this example. Instead of neglecting the actuator dynamics, we use a PI controller to control the actuator and a PI controller for the primary plant.

Inner-loop control

We select the natural frequency for the secondary control system as $w_{ns} = 20$, which is 20 times of that used for the primary control system. With this selection, the PI controller parameters are

$$K_{cs} = \frac{2 \times 0.707 \times 20 - 15}{0.5} = 26.56; \tau_{Is} = \frac{2 \times 0.707 \times 20 - 15}{400} = 0.0332$$

Outer-loop control

In the design of primary controller, the inner-loop dynamics are neglected. Therefore, the PI controller is designed using the transfer function (12) for the primary plant, leading to

$$K_{cp} = \frac{2 \times 0.707 \times 1 - 0.1}{0.8} = 1.6425; \tau_{Ip} = \frac{2 \times 0.707 \times 1 - 0.1}{1} = 1.314.$$

Simulation Program

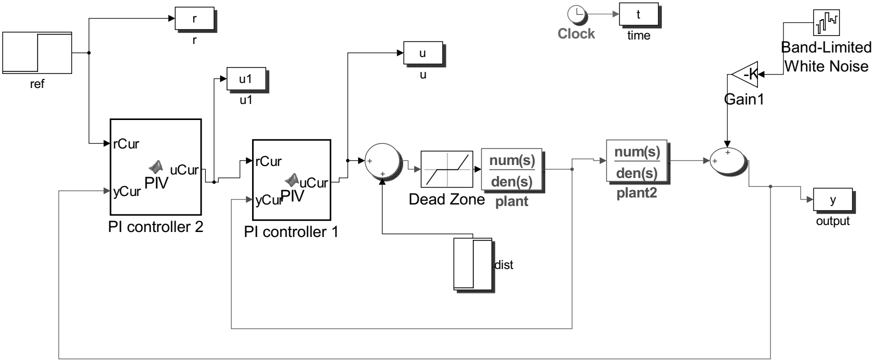


Figure 10: Simulink simulation program for the cascade control system with deadzone nonlinearity in the actuator.

Simulation Results

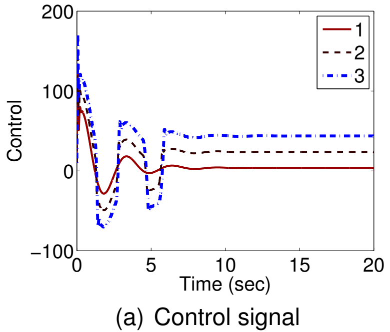


Figure 11: Closed-loop control response using cascade control ($w_{ns} = 20$, $w_{np} = 1$). Key: line (1) response without deadzone; line (2) response with deadzone ($\delta = 20$); line (3) response with deadzone ($\delta = 40$)

Outline

Quantization Errors

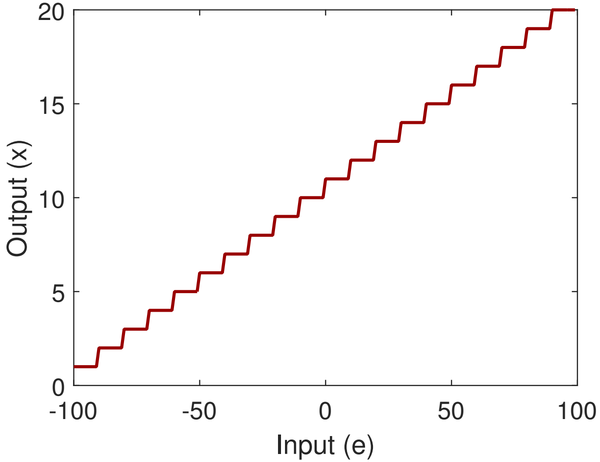


Figure 12: Illustration of a quantization of signal $e(t)$ with quantization interval $q = 1$.

Example

Consider the system used in Slaughter's original paper (1964) for study of quantization error, which has the following continuous-time transfer function:

$$G(s) = \frac{4500}{s(s+10)(s+20)}. \tag{13}$$

Design a PI controller for this system with the natural frequency $w_n = 1$ and $\xi = 0.707$. Choosing sampling interval $\Delta t = 0.01$ (sec), simulate the effect of quantization errors on the closed-loop performance.

Solution

Approximation

Although the system is of third order, it can be approximated by the following first order transfer function by neglecting the two small time constants, which gives the design model for the PI controller.

$$G(s) = \frac{4500}{s(s+10)(s+20)} = \frac{22.5}{s(0.1s+1)(0.05s+1)} \approx \frac{22.5}{s}$$

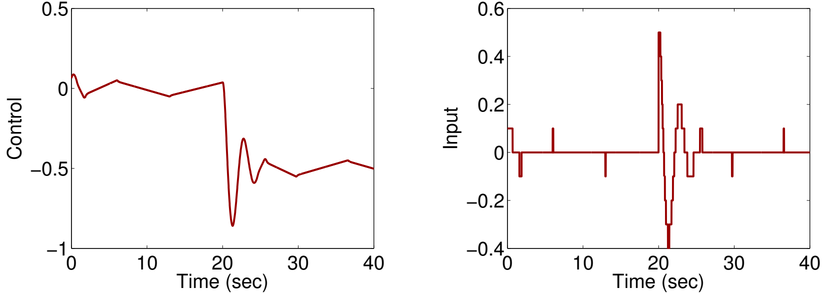
PI controller

With the choice of $w_n = 1$ and $\xi = 0.707$, the PI controller parameters are found as

$$K_c = \frac{2\xi w_n - a}{b} = \frac{2 \times 0.707}{22.5} = 0.0628; \quad \tau_I = \frac{2\xi w_n - a}{w_n^2} = 2 \times 0.707 = 1.414.$$

where the natural frequency $w_n = 1$, $\xi = 0.707$, $a = 0$, $b = 22.5$.

Simulation Results (i)



(a) Control signal (b) Input signal

Figure 13: Closed-loop control response with quantization on input signal ($q = 0.1$)

Simulation Results (ii)

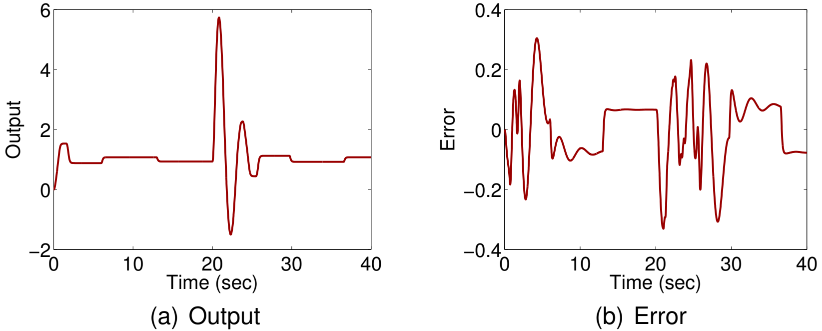


Figure 14: Closed-loop control response with quantization on input signal ($q = 0.1$)

Cascade Control

- In the next example, we assume that the system in the previous example can be decomposed into an actuator and a plant, and the output from the actuator can be measured to form a feedback control for the actuator.
- Then, we examine how the cascade control structure improves the closed-loop control performance with the quantization.
- We assume that the actuator for the secondary plant has the transfer function

$$G_s(s) = \frac{45}{s+10}$$

  and the primary plant has the transfer function:

$$G_p(s) = \frac{100}{s(s+20)}. \tag{14}$$

Cascade Controller Design

Secondary controller

With the selection of the natural frequency $w_{ns} = 30$ for the secondary control system, the PI controller parameters are calculated as

$$K_{cs} = \frac{2 \times 0.707 \times 30 - 10}{45} = 0.7204; \tau_{Is} = \frac{2 \times 0.707 \times 30 - 10}{900} = 0.0360.$$

Primary controller

The inner-loop dynamics are neglected in the primary controller design, however, there is an approximation introduced to obtain the integrator model with gain equal to 5 (see (14)). We calculate the PI controller parameters for the outer-loop system, with $w_{np} = 1$ and $\xi = 0.707$, as

$$K_{cp} = \frac{2 \times 0.707}{5} = 0.2828; \tau_{Ip} = 2 \times 0.707 = 1.414.$$

Simulation Program

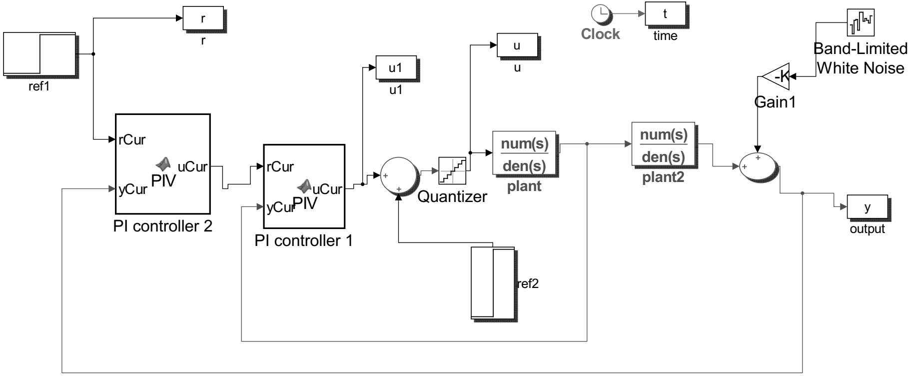
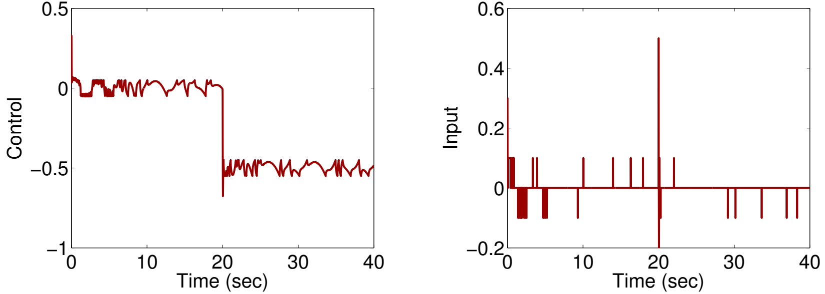


Figure 15: Simulink simulation program for the cascade control with actuator quantization errors

Simulation Results (i)



(a) Control signal (b) Input signal

Figure 16: Cascade closed-loop control response with quantization on input signal ($q = 0.1$)

Simulation Results (ii)

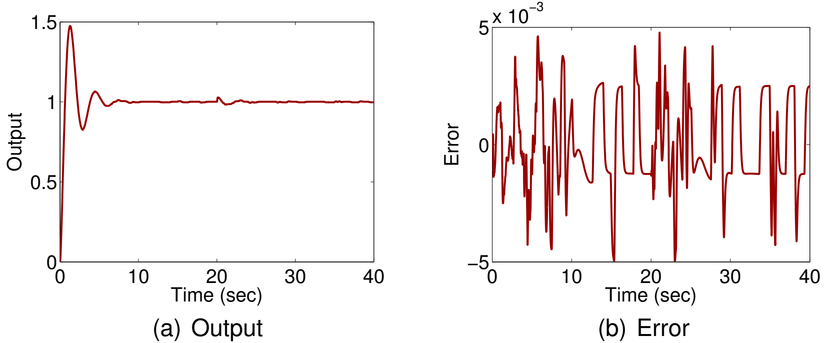
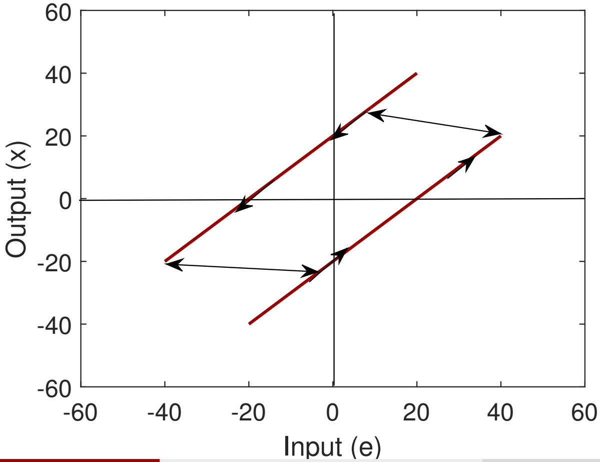


Figure 17: Cascade closed-loop control response with quantization on input signal ($q = 0.1$)

Outline

Backlash Nonlinearity

Assuming a dead-band Δ for the backlash nonlinearity, with the input signal to the backlash as $e(t)$ and output as $x(t)$, Figure 18 illustrates a backlash nonlinearity with gain ($k = 1$) and deadzone ($\Delta = 40$).



Mathematical Description

As illustrated in the figure, following the path with the arrows pointed upwards, the output signal is described by

$$x(t) = k(e(t) - \frac{\Delta}{2})$$

and following the path with the arrows pointed downwards, the output signal is described by

$$x(t) = k(e(t) + \frac{\Delta}{2})$$

The output signal $x(t)$ can switch between these two pathes.

Example

In this example, the secondary system corresponding to the actuator dynamics is described by the following transfer function:

$$G_s(s) = \frac{0.5}{s + 15}. \tag{15}$$

The primary system is an integrator with a small time delay described by the transfer function:

$$G_p(s) = \frac{0.01e^{-0.3}}{s}. \tag{16}$$

The actuator has a backlash nonlinearity with deadband width $\Delta = 60$.

PI Controller Design

Approximation

When the actuator dynamics is neglected, its steady-state value is still considered in the design of PI controller. For this purpose, the approximate model used for the design of PI controller becomes:

$$G(s) \approx \frac{3.333 \times 10^{-4}}{s}$$

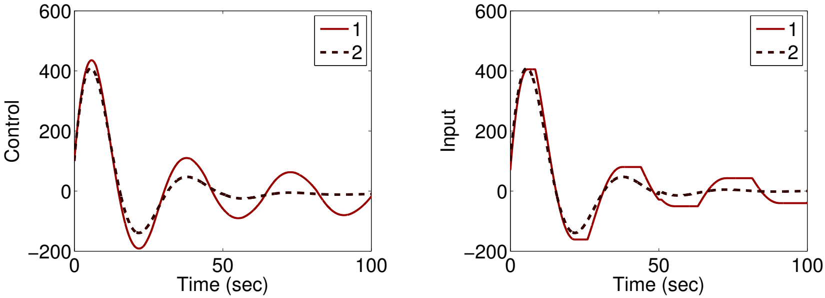
where the time delay is also neglected in the design.

PI controller

With $w_n = 0.1$ and $b = 3.333 \times 10^{-4}$, the PI controller parameters become:

$$K_c = \frac{2 \times 0.707 w_n}{b} = 424.2; \tau_I = \frac{2 \times 0.707 w_n}{w_n^2} = 14.14.$$

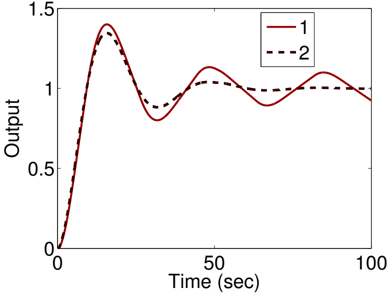
Simulation Studies(i)



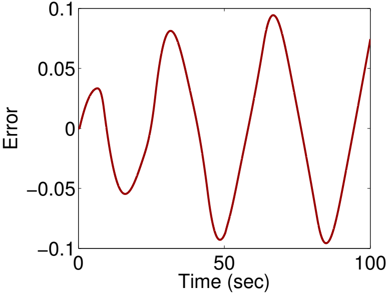
(a) Control signal (b) Input signal

Figure 19: The effect of backlash on closed-loop performance ($w_n = 0.1$). Key: line (1) control system with backlash; line (2) control system without backlash.

Simulation Studies(ii)



(a) Output



(b) Error

Figure 20: The effect of backlash on closed-loop performance ($w_n = 0.1$). Key: line (1) control system with backlash; line (2) control system without backlash.

Cascade Control

Inner-loop control

From the transfer function model (15), we have $b = 0.5$ and $a = 15$. With the desired natural frequency $w_{ns} = 20$, the PI controller for the actuator has the following proportional gain and integral time constant,

$$K_{cs} = \frac{2\xi w_{ns} - a}{b} = 26.56; \quad \tau_{Is} = \frac{2\xi w_{ns} - a}{w_{ns}^2} = 0.0332.$$

Outer-loop control

For the primary plant, with $w_{np} = 0.1$, $a = 0$ and $b = 0.01$, the PI controller parameters are calculated as

$$K_{cp} = \frac{2\xi w_{np}}{b} = 14.14; \quad \tau_{Ip} = \frac{2\xi}{w_{np}} = 14.14,$$

where the time-delay is neglected in design and the steady-state gain of the inner-loop system is taken to be one.

Simulink Simulation

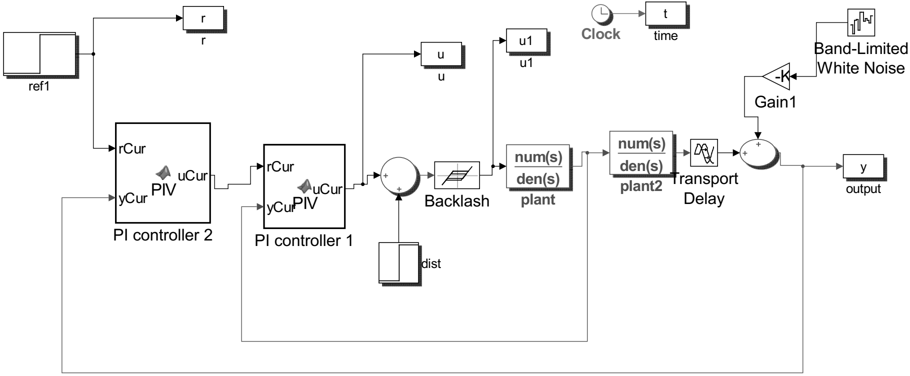
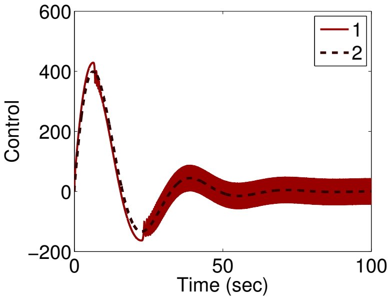
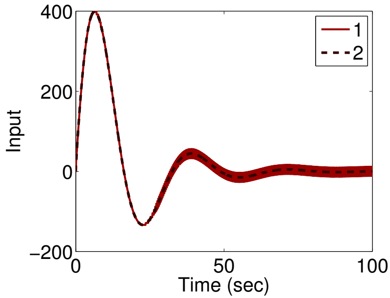


Figure 21: Simulink simulation program for the cascade control with backlash nonlinearity in the actuator

Simulation Results (i)



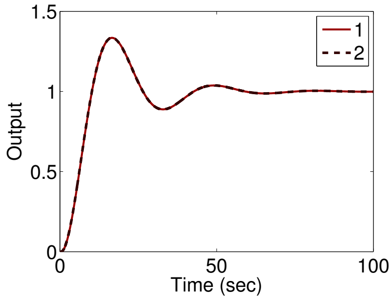
(a) Control signal



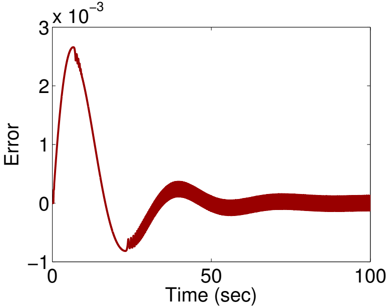
(b) Input signal

Figure 22: The effect of backlash on cascaded closed-loop performance ($w_{ns} = 20$ and $w_{np} = 0.1$). Key: line (1) control system with backlash; line (2) control system without backlash.

Simulation Results (ii)



(a) Output



(b) Error

Figure 23: The effect of backlash on cascaded closed-loop performance ($w_{ns} = 20$ and $w_{np} = 0.1$). Key: line (1) control system with backlash; line (2) control system without backlash.

Outline

Mathematical Model of PMSM

- PMS machine is described by the differential equations in the d-q rotating reference frame

$$\frac{di_d(t)}{dt} = \frac{1}{L_d}(v_d(t) - Ri_d(t) + \omega_e(t)L_q i_q(t)) \tag{17}$$

$$\frac{di_q(t)}{dt} = \frac{1}{L_q}(v_q(t) - Ri_q(t) - \omega_e(t)L_d i_d(t) - \omega_e(t)\phi_{mg}) \tag{18}$$

$$\frac{d\omega_e(t)}{dt} = \frac{p}{J}(T_e - \frac{B}{p}\omega_e(t) - T_L) \tag{19}$$

$$T_e = \frac{3}{2}p[\phi_{mg} i_q + (L_d - L_q)i_d(t)i_q(t)] \tag{20}$$

- v_d and v_q represent the stator voltages in the d-q frame, i_d and i_q represent the stator currents in this frame, and T_L is load torque that is assumed to be zero if no load is attached to the motor.
- The electromagnetic torque T_e consists of two parts: that produced by the flux of the permanent magnet ϕ_{mg} and that by i_d and i_q, respectively.

Velocity Control of PMSM

- ω_e is the electrical speed and is related to the rotor speed by $\omega_e = p\omega_m$ with p denoting the number of pole pairs. Thus, the output for the velocity control problem is ω_e (or ω_m).
- v_d and v_q are the manipulated variables or the input variables for this control problem.
- T_L is the input disturbance to the system. In addition to tracking the reference signal of the velocity, the closed-loop control system will also maintain its operation at the steady-state when the load torque T_L changes.

Outline

How Should the Control System Be Configured? (i)

- In the mathematical model, Equations (17) and (18) describe the dynamics of the electrical part of the machine.
- Equation (19) describes the dynamics of the mechanical part of the machine.
- Equation (20) presents the link between the electrical system and the mechanical system.
- It is reasonable to assume that the response times of the electrical system are much faster than the mechanical counter-part, namely, $\frac{R}{L_d} >> \frac{B}{J}$, $\frac{R}{L_q} >> \frac{B}{J}$.

How Should the Control System Be Configured? (ii)

- Because there are large differences between the time constants between the electrical systems and the mechanical system and because the d-axis and q-axis currents are measurable, this system is a candidate for a cascade feedback control system.
- We will choose the systems with faster dynamics as the inner-loop systems. Here the electrical systems are the inner-loop systems.
- The systems with slower dynamics are chosen as the outer-loop systems. Here the mechanical system is the outer-loop system.

Outline

Advantages of Cascade Control Systems (i)

- In the design of cascade control system, the inner-loop control system will have a much wider bandwidth than the outer-loop control system because it has a smaller time constant to begin with. Namely, the inner-loop control system must have a much faster closed-loop response time.
- As a result, the disturbances occur at the inner current loop will be rejected in a much faster speed.
- The configuration of cascade control system allows the designer to use different sampling intervals Δt for the implementation. For instance, sampling interval for the inner-loop current control can be selected as 50μs and the outer-loop velocity control can be 200μs.

Advantages of Cascade Control Systems (ii)

- Simplification of control system design for a higher order or a complex system because the higher order dynamics components are decomposed into a series of lower order units. For instance, the AC motor velocity control design problem is converted into control problems of current and velocity, which are two first order systems suited to PI controllers.
- For a nonlinear system, the inner-loop control will lead to a linearized system so that the overall nonlinear system is better controlled.

Disadvantages of Cascade Control Systems

- We need to have sensors to measure all secondary variables for the inner-loop feedback control.
- For some applications, these sensors are not available or too expensive so that the configuration of a cascade control system is not possible.

Design of a Cascade Control System

- We will design the inner-loop control systems first.
- The outer-loop control systems are designed based on the outer-loop dynamics and the dynamics from the inner-loop feedback control system.
- So, the inner-loop control systems need to be considered when we design the outer-loop control system.

Outline

Inner-loop Current Controller Design (i)

- There are nonlinear cross-coupling terms in (17) and (18) by $\omega_e i_q$, $\omega_e i_d$ and ω_e.
- These cross-coupling terms can be eliminated using a technique called feedforward linearization and also decoupling in this application.

How to Design Feedforward Control

The central idea in the feedforward control is to use auxiliary variables $\hat{v}_d$ and $\hat{v}_q$ such that

$$\frac{1}{L_d}\hat{v}_d = \frac{1}{L_d}(v_d + \omega_e L_q i_q) \tag{21}$$

$$\frac{1}{L_q}\hat{v}_q = \frac{1}{L_q}(v_q - \omega_e L_d i_d - \omega_e \phi_{mg}) \tag{22}$$

Inner-loop Current Controller Design (ii)

- By substituting these equations into (17) and (18), we obtain the first order models for the electrical part of the machine dynamics as

$$\frac{di_d}{dt} = -\frac{R}{L_d} i_d + \frac{1}{L_d} \hat{v}_d \tag{23}$$

$$\frac{di_q}{dt} = -\frac{R}{L_q} i_q + \frac{1}{L_q} \hat{v}_q \tag{24}$$

- Based on (23) and (24), two feedback controllers can be designed for the stator current control by manipulating the auxiliary stator voltages in the d-q frame.
- These are two first order models. So we can use model based design methods.

Inner-loop Current Controller Design (iii)

Once $\hat{v}_d$ and $\hat{v}_q$ are calculated, the true stator voltages in the d-q frame are computed through (21) and (22):

$$v_d = \hat{v}_d - \omega_e L_q i_q \tag{25}$$

$$v_q = \hat{v}_q + \omega_e L_d i_d + \omega_e \phi_{mg} \tag{26}$$

Choices of Inner-loop Current Controllers

- We can use the proportional controller for q-axis current control and use PI controller for d-axis current control. This is
  - because there will be an outer-loop PI controller for the velocity via the q-axis current. In the case of cascade control, the accuracy of inner-loop control system at the steady-state is less important than the consideration of response speed and robustness of the closed-loop system against parameter variations;
  - there is no outer-loop control for the d-axis (flux) current. Therefore, in order to maintain the correct steady-state value of the flux current (i_d), we need to use PI controller for the d-axis current control.
- The industrial controllers have PI for the inner-loop control systems.
- If the steady-state operation of the inner-loop control system is very important in the design, then PI controller is better suited.

Block Diagram

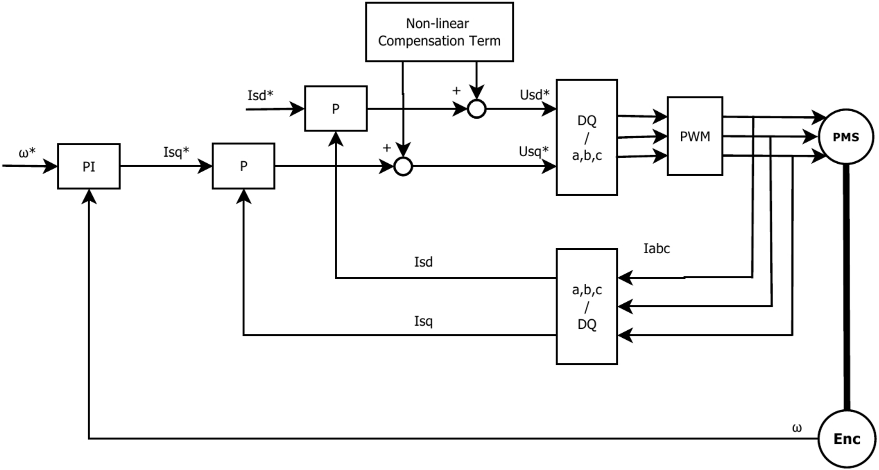


Figure 24: Nonlinear cascade control of PMS motor

Design for i_d Current Control

- By assuming a damping coefficient $\xi(= 0.707)$ and a natural frequency w_n, the PI controller parameters for the control of d-axis current are calculated using the pole-assignment control method:

$$\begin{aligned} K_c^d &= \frac{2\xi w_n - \frac{R}{L_d}}{\frac{1}{L_d}} \\ &= 2\xi w_n L_d - R \qquad (27) \\ \tau_I^d &= \frac{2\xi w_n - \frac{R}{L_d}}{w_n^2} \\ &= \frac{2\xi w_n L_d - R}{L_d w_n^2} \qquad (28) \end{aligned}$$

Implementation of i_d Current Control

Using the relationship between $\hat{v}_d$ and v_d, the d-axis and q-axis voltages is calculated:

$$v_d(t) = K_c^d(i_d^*(t) - i_d(t)) + \frac{K_c^d}{\tau_I^d}\int_0^t (i_d^*(\tau) - i_d(\tau))d\tau - \omega_e(t)L_q i_q(t)$$

Design for i_q Current Control (P) (i)

- For proportional gain K_c^q, the closed-loop transfer function between the set-point signal $I_q^*(s)$ and the actual current $I_q(s)$ are written as

$$T^{iq}(s) = \frac{I_q(s)}{I_q^*(s)} = \frac{\frac{K_c^q}{L_q}}{s + \frac{R}{L_q} + \frac{K_c^q}{L_q}} \tag{29}$$

- The closed-loop pole for the q-axis current control is at $-\frac{R}{L_q} - \frac{K_c^q}{L_q}$.
- The larger K_c^q is, the faster the inner-loop current responses will be.
- The steady-state gains of the current control-loops are calculated by setting $s = 0$ (29) as $\frac{\frac{K_c^q}{L_q}}{\frac{R}{L_q} + \frac{K_c^q}{L_q}}$ for the q-axis.

Design for i_q Current Control (P) (ii)

- The factors affecting the choice of the proportional gain for the current control loop include the dynamic response speed, the closed-loop steady-state gain, and the noise level in the system.
- On one hand, we desire a faster closed-loop response speed and a higher closed-loop steady-state gain, and on the other hand, we will try to avoid amplification of the noise in the inner-loop system which will be the consequence of higher gain and faster response speed.
- Because the steady-state gain in the inner-loop control systems will be used in the design of outer-loop control system, it is convenient to directly specify their desired values, then incorporate them later on in the design.

Design for i_q Current Control (P) (iii)

- For this purpose, for the q-axis current control, we let the parameter $0 < \alpha < 1$ represent the steady-state gain for the current control loop, so that

$$\alpha = \frac{\frac{K_c^q}{L_q}}{\frac{R}{L_q} + \frac{K_c^q}{L_q}} \qquad (30)$$

- By solving this steady-state equations, we obtain the proportional gain for the q-axis current control loop:

$$K_c^q = \frac{\alpha}{1-\alpha} R \qquad (31)$$

where $\alpha \neq 1$.

Design for i_q Current Control (P) (iv)

- By substituting the proportional controller gain into the closed-loop transfer function, we obtain

$$T^{iq}(s) = \frac{I_q(s)}{I_q^*(s)} = \frac{\frac{\alpha}{1-\alpha}\frac{R}{L_q}}{s + \frac{1}{1-\alpha}\frac{R}{L_q}} \tag{32}$$

Implementation of Inner-loop P Controller

- Upon deciding the values of the proportional gain, the control signal for the q-axis current-loop is calculated using the feedback and feedforward configurations:

$$v_q(t) = K_c^q(i_q^*(t) - i_q(t)) + \omega_e(t)L_d i_d(t) + \omega_e(t)\phi_{mg} \tag{33}$$

Outline

Design for the Outer-loop Control System (i)

- The design of outer-loop control system is based on the equations (19-20) that have been used in describing the mechanical part of the system and the link between the mechanical and electrical systems.
- By substituting (20) into (19), we obtain

$$\begin{aligned} \frac{d\omega_e(t)}{dt} &= \frac{p}{J}T_e(t) - \frac{B}{J}\omega_e(t) - \frac{P}{J}T_L \\ &= \frac{3}{2}\frac{p^2\phi_{mg}}{J}i_q(t) + \frac{3}{2}\frac{p^2}{J}(L_d - L_q)i_d(t)i_q(t) - \frac{B}{J}\omega_e(t) - \frac{p}{J}T_L \quad (34) \end{aligned}$$

- Note that the second term on the right-hand side of (34) is bilinear and contains a factor $L_d - Lq$.
- For the class of surface mounted PMS machines, $L_d = L_q$, thus this bilinear term vanishes. However, if $L_d \neq L_q$, the set-point signal for the current control of d-axis is chosen to be zero in the majority of the applications, namely $i_d^* = 0$, then in the steady-state, this term equals zero. Therefore, in the control system design for the outer-loop system, this bilinear term is neglected.

Design for the Outer-loop Control System (ii)

- The fourth term in (34) is proportional to the load torque, which is considered as a disturbance in control system design and should be rejected by the outer-loop control system as long as it is a constant or varies in a step signal manner.
- It is worthwhile to emphasis that because of the existence of load torque, without exception, the outer-loop controller should contain an integrator in order to completely reject the disturbance caused by the load torque.
- By neglecting the bilinear term, we re-write (34) in a first order differential equation:

$$\frac{d\omega_e(t)}{dt} = -\frac{B}{J}\omega_e(t) + \frac{3}{2}\frac{p^2\phi_{mg}}{J}i_q(t) - \frac{p}{J}T_L \tag{35}$$

Design for the Outer-loop Control System (iii)

- From control system design point of view, the output variable is $\omega_e(t)$ and the input variable is current $i_q(t)$. However, because $i_q(t)$ is the output variable for the inner-loop control system, it is not available for the manipulation needed for the outer-loop. What is available and free is the set-point signal i_q^* to the inner-loop control of the q-axis current.
- The relationship between i_q and i_q^* is characterized by the inner-loop control of the q-axis current and is, in Laplace transform,

$$I_q(s) = \frac{\frac{\alpha}{1-\alpha}\frac{R}{L_q}}{s + \frac{1}{1-\alpha}\frac{R}{L_q}} I_q^*(s) \tag{36}$$

- The Laplace transform of (35) in regarding the relationship between $\Omega_e(s)$ and $I_q(s)$ is

$$(s + \frac{B}{J})\Omega_e(s) = \frac{3}{2}\frac{p^2\phi_{mg}}{J} I_q(s) \tag{37}$$

Design for the Outer-loop Control System (iv)

- By substituting (36) into (37), we obtain the transfer function between $\Omega_e(s)$ and $I_q^*(s)$ as,

$$\Omega_e(s) = \frac{\frac{3}{2}\frac{p^2\phi_{mg}}{J}}{s+\frac{B}{J}} \frac{\frac{K_c^q}{L_q}}{s+\frac{R}{L_q}+\frac{K_c^q}{L_q}} I_q^*(s) \tag{38}$$

$$= \frac{\frac{3}{2}\frac{p^2\phi_{mg}}{B}}{\frac{J}{B}s+1} \frac{\alpha}{(1-\alpha)\frac{L_q}{R}s+1} I_q^*(s) \tag{39}$$

- This is the model for the design of the outer-loop velocity control system. Because it is a second order, a PID controller could be appropriate.
- However, if we closely examine the model, then we find that the closed-loop time constant for the electrical system $\frac{L_q}{R}$ is far smaller than the time constant for the mechanical system $\frac{J}{B}$.

Design for the Outer-loop Control System (v)

- In addition, with the proportional feedback control gain K_c^q being large (see $K_c^q = \frac{\alpha}{1-\alpha}R$), the dynamics from the inner-loop control of the q-axis current is ensured to be much faster than the dynamics from the mechanical system $((1-\alpha)\frac{L_q}{R} >> \frac{J}{B}$.
- Therefore, second order model (38) is simplified to a first order system by neglecting the dynamics from the inner-loop control of q-axis current by letting $(1-\alpha)\frac{L_q}{R} = 0$, which is

$$
\begin{aligned}
\frac{\Omega_e(s)}{I_q^*(s)} &= \frac{\frac{3}{2}\frac{p^2\phi_{mg}}{B}\alpha}{\frac{J}{B}s+1} \\
&= \frac{\frac{3}{2}\frac{p^2\phi_{mg}}{J}\alpha}{s+\frac{B}{J}}
\end{aligned}
\tag{40}
$$

Design for the Outer-loop Control System (vi)

- With this first order model, the design of a PI controller leads to analytical solution of the controller parameters using the technique of pole-assignment controller design.
- To simplify the notation, we let

$$a = \frac{B}{J}; \quad b = \frac{3}{2}\frac{p^2\phi_{mg}}{J}\alpha$$

- Here, by choosing a pair of desired closed-loop poles $-\xi w_n \pm w_n j\sqrt{1-\xi^2}$, where the damping coefficient $\xi = 0.707$, the proportional gain K_c is calculated as

$$K_c = \frac{2\xi w_n - a}{b} \tag{41}$$

and the integral time constant is calculated as

$$\tau_I = \frac{2\xi w_n - a}{w_n^2} \tag{42}$$

Implementation of Outer-loop Controller

- The control signal $i_q^*(t)$ is calculated using the PI controller as

$$i_q^*(t) = K_c(\omega_e^*(t) - \omega_e(t)) + \frac{K_c}{\tau_I}\int_0^t (\omega_e^*(\tau) - \omega_e(\tau))d\tau \tag{43}$$

  where $\omega_e^*(t)$ is the set-point signal for the electrical velocity.
- Mechanical velocity $\omega_m(t)$ is related to $\omega_e^*(t)$ by the relationship: $\omega_e = p\omega_m$.

Simulation Results

In this simulation example, the parameters for the nonlinear model are given as $\phi_{mg} = 0.125$, $L_d = 7e-3$, $L_q = 7e-3$, $R = 2.98$, $B = 11e-5$, $p = 2$, $J = 0.47e-4$. The closed-loop steady-state gain is at $\alpha = 0.9$ for the inner-loop.

Output Response Plots

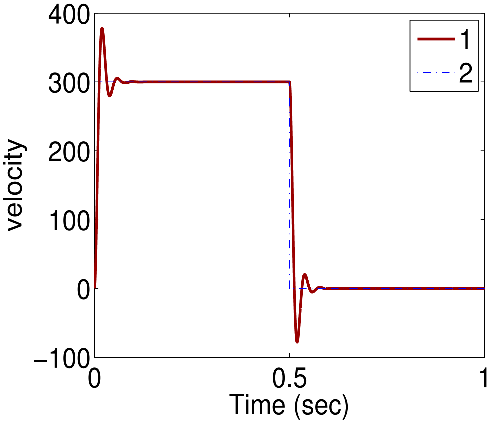


Figure 25: Closed-loop response of the angular electrical velocity. Key: line (1) the actual velocity; line (2) the reference velocity.

Control Signal Plots

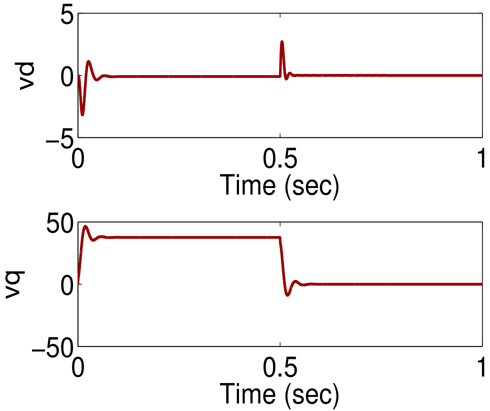


Figure 26: Closed-loop control signal responses (the d-axis and q-axis voltages)

Examples

Summary of This Lecture

- Configuration of control systems in terms of cascade control system;
- Inner-loop control system design;
- Outer-loop control system design;
- Feedforward compensation.