

Active Suspension Investigation Using Physical Networks

Milan Simic

Abstract Active suspension systems consist of mechanical, electrical, electronics and other subsystems. Different types of physical signals and forms of energy circulate between subsystems which make system design a challenging task. Various physical systems can be expressed using the same type of ordinary differential equations. This gives us a common platform for the complex systems' design. Using physical networks' approach modeling of an active suspension system is conducted and evaluated for various on road scenarios. Active suspension contributes to the ride comfort, safety, better and easier car control both by the human driver, or autonomous system.

Keywords Active suspension • Flat ride • Passenger comfort • Vehicle vibrations • Roll vibrations • Pitch vibrations • Physical network • Simulation

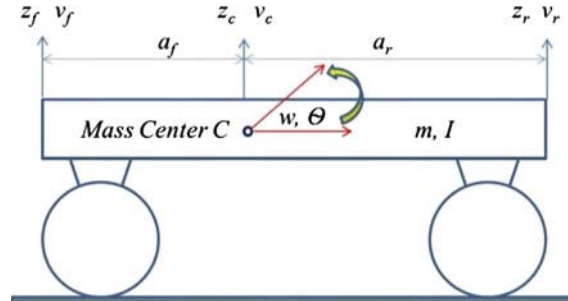
1 Introduction

While performing translatory motion, passenger car is subjected to road imperfections of different type, size and shapes, resulting in unwanted vibrations. They may cause serious damages to the vehicle. Main objective of this investigation is to ensure flat ride over road imperfections, by performing modeling of the moving vehicle and its suspension system. Suspension is interfacing between vehicle and the road. Systems could be relatively simple, i.e. built using passive spring type components. Car body movement, as reaction to the road conditions, is determined by the road and fixed suspension parameters: stiffness, mass and friction.

When adaptive, i.e. semi-active suspension is applied, suspension parameters are controlled by onboard computer. With active suspension implemented, movements of the wheels, along yaw axis, are detected by sensors and controlled by computer system via actuators mounted on the wheels. Actions performed improve ride

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Fig. 1 Bicycle model of the car as a beam of mass m and moment of inertia I



comfort and enable better car control. Modern active suspension systems minimize roll and pitch vibrations. It is important when the vehicle performs a curvature trajectory, or when it increases, or decreases translatory motion speed. All vibrations, around pitch, roll, or yaw axis, disturb ride comfort in driver operated vehicles and even more in autonomous vehicles where passengers are subjected to loss of controllability experience, as presented in [1].

A conventional passenger vehicle, with four wheels, is a complex mechatronics system, but, it is often presented using simplified, two-wheel bicycle model as shown in Fig. 1. There is a large number of factors to consider when developing new suspension systems [2]. This model is used for vibration studies and flat ride investigations, as given in [3]. It is also used for autonomous vehicles' path planning in order to achieve maximum ride comfort [1]. Bicycle model is again used for autodrivers algorithm development and implementation as explained in [4, 5]. Depending on the application, different views, descriptions and parameters of the bicycle model are considered. There are other models, like multi-body system where wheels are the end effectors on the mobile chassis [6]. On the other side we have quarter-car active suspension approach [7, 8]. One of the interesting approaches is the evaluation of active, energy regenerative suspension systems, using optimal control [9]. Bicycle model is used in our research to present the concept. It can easily be extended to full body model, or used for energy harvesting investigation. Model selection depends on the research objective targeted.

In our model, from Fig. 1, vehicle is represented as a beam of mass m , equally distributed along its length, $l = a_f + a_r$, and with a moment of inertia I . Length portion associated to the front is denoted as a_f while the length portion associated to the rear of the vehicle is denoted as a_r . This representation is a two degree-of-freedom (DOF) system. Mass center is located in C and it is center of rotation, i.e. bounce and pitch motion, with angular speed of ω and angle θ . We will investigate car body vertical velocities in three key points, front, mass center and rear of the vehicle. They are labeled as v_f, v_c, v_r , while displacements are z_f, z_c, z_r along vertical, z axes. Interfacing between the vehicle body and the road is conducted through suspension system and tires. Those components of the system will be presented in the next, revised, mechanical model of the vehicle.

In addition to that, further simplification, used in flat ride vibration studies, is to decouple bicycle model, i.e. separate systems equations that describe behavior of the front and the rear of the vehicle. Application of physical networks enables us to conduct more comprehensive vibration studies while taking into account mutual influences from forces acting on front, or rear vehicle's axles. Physical network modeling is possible thanks to the widespread analogies of ordinary differential equations used to represent various physical systems. Short introduction on physical network modeling of mechanical systems is given in the next section, while more on various engineering systems' modeling solutions could be found in [10–13].

2 Physical Networks

Physical systems are classified based on the type of equations used to express system behavior for the given input. Systems equations' coefficients are defined by physical system's parameters. Having in mind difficulties to solve some types of differential equations, we usually restrict our approach to investigations of systems which could be described by ordinary, linear differential equations with constant coefficients.

An ordinary differential equation (ODE) is a relation between two variables, an independent variable t , for time, dependent variable $y = y(t)$, together with the derivatives of y , being $\frac{dy}{dt}$, $\frac{d^2y}{dt^2}$, \dots , $\frac{d^ny}{dt^n}$, as shown in Eq. (1):

$$A_n \frac{d^ny}{dt^n} + A_{n-1} \frac{d^{n-1}y}{dt^{n-1}} + \dots + A_1 \frac{dy}{dt} + A_0 y = f(t) \quad (1)$$

Equation (1) is ordinary because only one independent variable exists, which is time. It is linear because only first exponent of dependent variable, or its derivatives, is present. Physical network is a class of linear graphs that represent physical system equations. Basic definitions, principles and mathematical rules for solving system of differential equations are independent of the physical system. There are two time dependent variables in each physical network: flow, f , and potential, p . Flow is a variable that streams through network elements and connection lines. Potential is a variable established and measured across network elements, or between any two network points. Potential of a point in the network depends on the chosen referent point. Each network has a basic reference i.e. ground point.

There are two types of network elements interconnected in a physical network: *active* and *passive*. Active elements are flow, or potential sources, whose operations are expressed as functions of time: $F = F(t)$ and $P = P(t)$. They supply energy to the system. Passive elements, with two connection points, define relationships between basic network variables: *flow* and *potential*. They cannot supply more energy to the system than what was already accumulated, if they can store energy.

Table 1 Physical network elements

| Description | Generic | Electrical | Translation |
|-------------------|------------------------------|------------------------------|------------------------------|
| Trough variable | <i>Flow—f</i> | <i>Current—i</i> | <i>Force—F</i> |
| Across variable | <i>Potential—p</i> | <i>Voltage—u</i> | <i>Velocity—v</i> |
| Element | A | Inductivity L | Stiffness k |
| Integration | $f = A \int p dt$ | $i = \frac{1}{L} \int u dt$ | $F = k \int v dt = kx$ |
| Stored energy | $\frac{f^2}{2A}$ | $\frac{Li^2}{2}$ | $\frac{F^2}{2k}$ |
| Element | B | Conductivity G/Resistivity R | Damping constant B |
| Proportion | $f = Bp$ | $i = Gu = \frac{1}{R}u$ | $F = Bv$ |
| Power dissipation | $fp = \frac{f^2}{B} = p^2 B$ | $iu = i^2 R = \frac{u^2}{R}$ | $Fv = \frac{F^2}{B} = v^2 B$ |
| Element | C | Capacity C | Mass m |
| Differentiation | $f = C \frac{dp}{dt}$ | $i = C \frac{du}{dt}$ | $F = m \frac{dv}{dt}$ |
| Stored energy | $\frac{Cp^2}{2}$ | $\frac{Cu^2}{2}$ | $\frac{mv^2}{2}$ |

That is expressed through initial conditions. There are three types of passive elements which specify relationships between network variables. Those are *proportion*, *integration* and *differentiation*.

Examples of all three types of passive elements in a generic physical network are given in column 1 of the Table 1. Examples for an electrical network, i.e. electrical circuit, and a mechanical system with translation are given in the following columns of the same table. We could illustrate analogies in other systems, like mechanical with rotation, acoustical [12] and hydraulic.

Expressions for the accumulated energy and power dissipation in each of the systems are also given. In a generic system, elements that define relations between two network variables, as integration, proportion and differentiation are labeled as A, B and C, respectively, as shown in the table. Common names for all passive network elements are impedance, Z, or admittance Y. In electrical systems we have electrical current and voltage as network variables. Their relationships are expressed through inductivity, L, conductivity G, i.e. resistivity, $R = 1/G$, and capacity C. Inductivity is associated to integration, conductivity to proportion and capacity to differentiation.

For a mechanical system with translation we have k for stiffness, related to the integration, B for friction, as a proportional element, and m for the mass of the object expressing differentiation. Basic network variables in translatory systems are force, F, and velocity v. Equations for stored energy, by elements that perform integration, or differentiation of network variables, and power losses for the proportional elements, are also presented.

3 Modeling

3.1 Mechanical Model

Referring to the basic bicycle model from Fig. 1 we will analyze vehicle behavior when driving over a bump on the road. Road bump of amplitude A will generate vertical forces F_f and F_r , defined by the Hooke's law, acting on the front and the rear of the vehicle. In our study we assume that they are the same, given by Eq. (2):

$$F_f = F_r = F = kA \quad (2)$$

Vehicle is shown as a beam of mass m and moment of inertia I , sitting on two sets of springs and friction elements representing two wheels. Vertical forces will cause rotation represented by an angle θ and angular speed w . Moment of inertia of a beam with mass m and length l is given by Eq. (3):

$$I = \frac{ml^2}{12} \quad (3)$$

Beam rotation radius, R , is given by Eq. (4):

$$R = \sqrt{\frac{I}{m}} \quad (4)$$

Vertical forces acting on the front axle, F_f , and on the rear axle, F_r , are together moving body mass center as defined by Eq. (5):

$$F_f + F_r = m \frac{dv_C}{dt} \quad (5)$$

For the further study we have to include moments of inertia, in reference to the mass center, as given by Eq. (6):

$$F_r a_r - F_f a_f = I \frac{dw}{dt} = m R^2 \frac{dw}{dt} \quad (6)$$

Total mass of the vehicle can be split into masses that correspond to the front, m_f , and the rear part of the body, m_r , together with a mutual mass, m_m , between them, as shown in the Fig. 2. Forces acting on the front and rear end of the vehicle are shown as ideal force sources. Mutual mass is introduced to express influences of the forces acting on one part of the vehicle to the other and vice versa. It is presented in [14], that all three masses can be determined through calculations given by the set of following Eqs. (7)–(9):

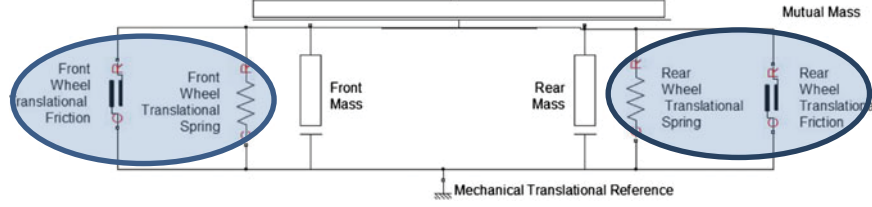


Fig. 2 Mechanical model showing coupled system through mutual mass

$$m_f = m \frac{a_r^2 + R^2}{l^2} \quad (7)$$

$$m_r = m \frac{a_f^2 + R^2}{l^2} \quad (8)$$

$$m_m = m \frac{a_f a_r - R^2}{l^2} \quad (9)$$

From the last equation, when $m_m = 0$, we have decoupling condition. In that case, specified by Eq. (10), we can treat parts of the vehicle's body separately.

$$R^2 = a_f a_r \quad (10)$$

The same condition could be derived by the evaluation of motion vibration equations, as shown in [3]. This study is more comprehensive and we will not decouple the system.

System description as given by Eq. (11). We concentrate on mass of the vehicle modeling, because of the mutual influences. Other elements, circled in Fig. 2 will be included later.

$$\begin{bmatrix} m_f & m_m \\ m_m & m_r \end{bmatrix} \begin{bmatrix} \frac{dv_f}{dt} \\ \frac{dv_r}{dt} \end{bmatrix} = \begin{bmatrix} F_f \\ F_r \end{bmatrix} \quad (11)$$

We will perform modeling of the vehicle, together with the road conditions and their interaction. In our scenario vehicle is riding, with a constant speed, over a road bump. That will generate two upward forces acting on front and on the rear axle. Actions are delayed in time, defined by the translatory speed of the vehicle. In this study we have assumed that a pulse function is good representation of the bump. It has amplitude of $A = 0.05$ m and the length of 1 m. Kinodynamic characteristics of the vehicle are given as following: vehicle mass is $m = 1500$ kg, length is $l = 4$ m, front and rear interfacing springs are the same type and equal to $K_f = K_r = K = 2000$ N/m, friction components are also the same and have value of $B_f = B_r = 10$. Translatory speed of the vehicle is $v_t = 10$ m/s.

We assume that the center of the mass is in the middle of the beam. Radius of rotation can be found using Eqs. (3) and (4). It is equal to $R = l/(2\sqrt{3}) = 2\sqrt{3}/3$. In physical networks *meter*, *kilogram* and *second* (MKS) system of units is used. That simplifies transitions between systems. Using Eqs. (7)–(9) values for the masses m_f , m_m , m_r are calculated and presented by a vector given in Eq. (12).

$$\begin{bmatrix} m_f \\ m_m \\ m_r \end{bmatrix} = \begin{bmatrix} 500 \\ 250 \\ 500 \end{bmatrix} \quad (12)$$

3.2 Electrical Model

As a next step, for the simplicity of modeling, we have transferred our system into an electrical circuit as given in Fig. 3a. Mechanical force corresponds to electrical current, while velocity corresponds to voltage. Stiffness k corresponds to inductivity, L , as $k \sim 1/L$. Damping constant B corresponds to conductivity G , i.e. resistivity R , as $B \sim G = 1/R$. Finally, mass m (kg), corresponds to capacity $C(F)$, as $m \sim C$. Numerical values for elements and variables are the same across various networks, all in MKS system. Forces, acting on axles, are represented as electrical current sources, while three mass elements are given as capacitors.

Equation (13) define electrical circuit from Fig. 3a:

$$\begin{bmatrix} C_f & C_m \\ C_m & C_r \end{bmatrix} \begin{bmatrix} \frac{dV_f}{dt} \\ \frac{dV_r}{dt} \end{bmatrix} = \begin{bmatrix} I_f \\ I_r \end{bmatrix} \quad (13)$$

Next model transformation is to simplify it as shown in the Fig. 3b. New system equations are (14) to (16). We have to find correspondence between two circuits.

$$C_f \left(\frac{dV_f}{dt} - \frac{dV_m}{dt} \right) = I_f \quad (14)$$

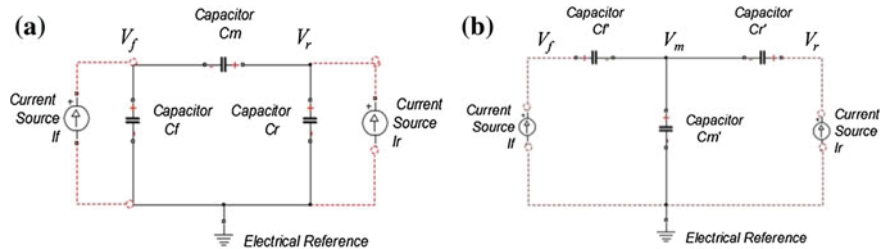


Fig. 3 a Vehicle as an equivalent electrical network. b Simplified electric circuit

$$C'_r \left(\frac{dV_r}{dt} - \frac{dV_m}{dt} \right) = I_r \quad (15)$$

$$C'_m \frac{dV_m}{dt} = I_f + I_r \quad (16)$$

By comparing Eq. (13) with Eqs. (14)–(16) we can derive analytical expressions for circuit elements from Fig. 3b, as given in [14]. Corresponding numerical values for our particular system are given by Eq. (17).

$$C'_f = C'_r = 250; \quad C'_m = -750 \quad (17)$$

4 Final Model for Vibration Studies

Model, which now includes wheel parameters, is given in Fig. 4. We have friction and stiffness interfacing components to the road. Road is represented as electrical ground. Stiffness is given as its analog quantity, inductivity ($L = 1/k$). Friction is represented with its analog quantity, resistivity ($R = 1/B$). Mass is given as capacity ($m = C$).

Mass of the wheels is represented as C_{wf} and C_{wr} . Energy is supplied to the system by front and rear controlled current sources that correspond to forces acting on the axles. Force sources, as per Hooke's law, are given by Eq. (2). Models presented up to this point are not functional models. They are used to demonstrate steps and concepts in the system design. Following models are fully functional, running in Simulink environment, based on MATLAB R2012b.

Complete vehicle model, including scenario of the ride over the road bump, is given in Fig. 5. In addition to system components, already described, data acquisition (DAQ) and display devices are also presented. Integrators are added to calculate displacements based on vertical velocities.

Front vertical velocity is shown in Fig. 6, while displacements are presented in Fig. 7.

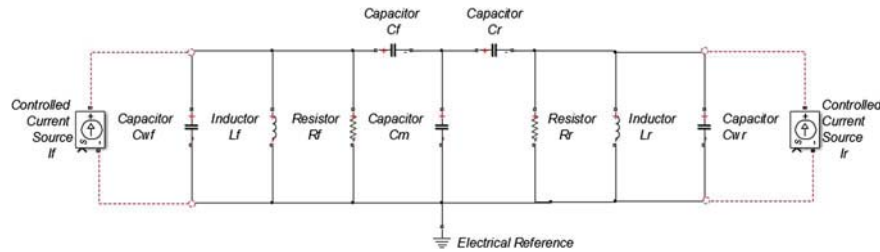


Fig. 4 Model of the vehicle on the road represented as an equivalent electrical circuit

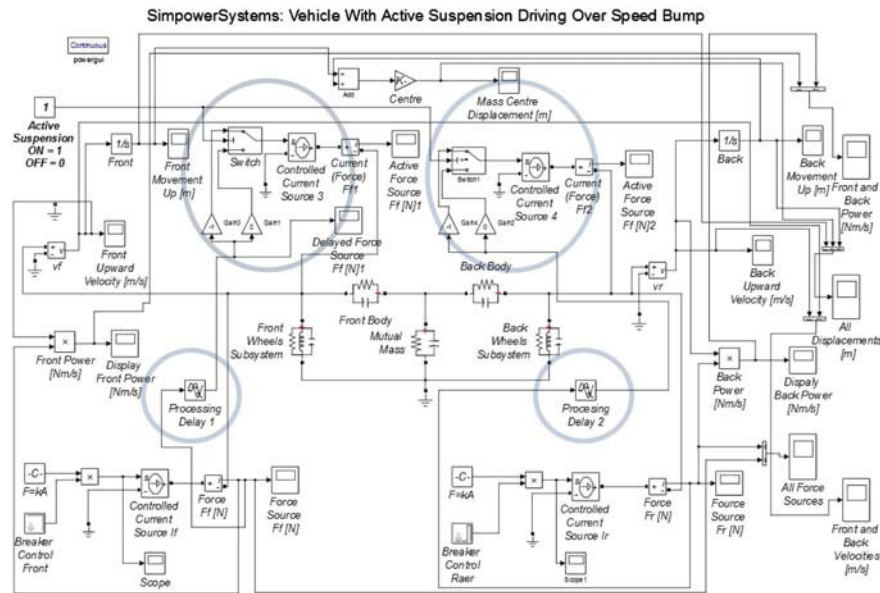


Fig. 5 Final model of the vehicle with active suspension components circled

Figures 6 and 7 show behavior of a vehicle with standard passive suspension system. Model need fine tuning and testing with sets of real suspension data. Our next experiment is to apply active suspension by detecting the bump and applying counter force, using actuators. Elements of the active suspension system are circled in Fig. 5. Signal is coming from the DAQ current sensor. It goes through delay block simulating DAQ electronics processing delay. Switch is used for the selection of the suspension mode of operation. After the switch, controlled force is applied to

Fig. 6 Vertical velocity at the front of the vehicle generated by a bump on the road

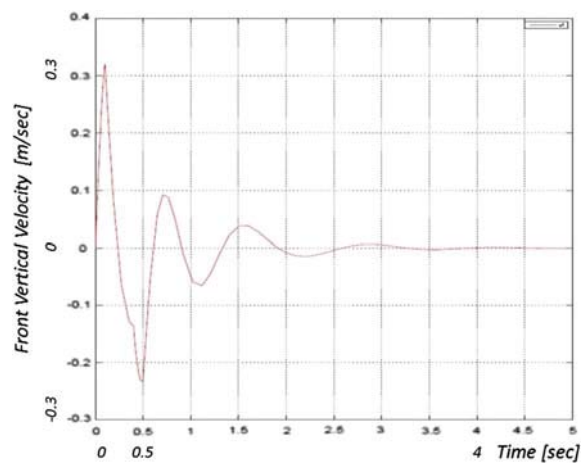
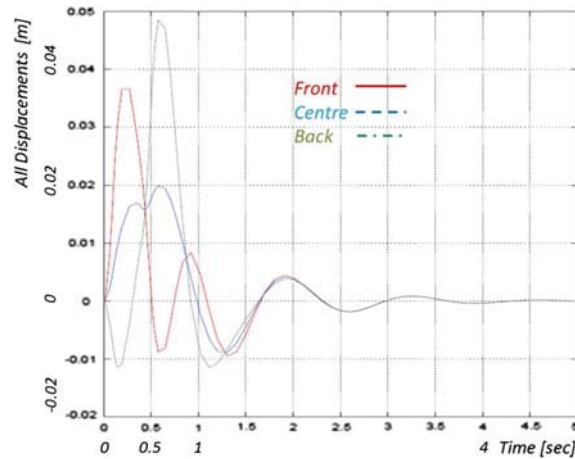
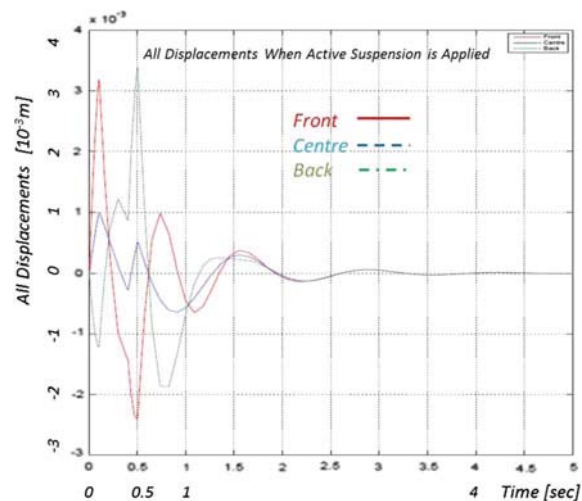


Fig. 7 Displacements from the key locations: front, center and back of the vehicle



the actuator. Actuators should be placed on each wheel. Figure 8 shows displacements from the key locations when the active system is ON. When we compare vibrations shown on Fig. 7, when active suspension systems is OFF and the vibrations from the Fig. 8, when active suspension systems is ON, we can see that the shapes of the signals are similar, but the main difference is in the magnitude. With the active suspension displacements are 10 times smaller than without it. We assumed that the DAQ processing delay and actuator delay are in the range of *ms*. For quicker responses, i.e. less vibrations, we need shorter delays. It is interesting to mention here that sliding mode control uses inertial delay for active suspension with full car model [15]. On the other side quarter-car active suspension model is used for adaptive tracking control of vehicle suspensions with actuator saturations [7].

Fig. 8 Displacements with active suspension engaged are for the order of magnitude smaller



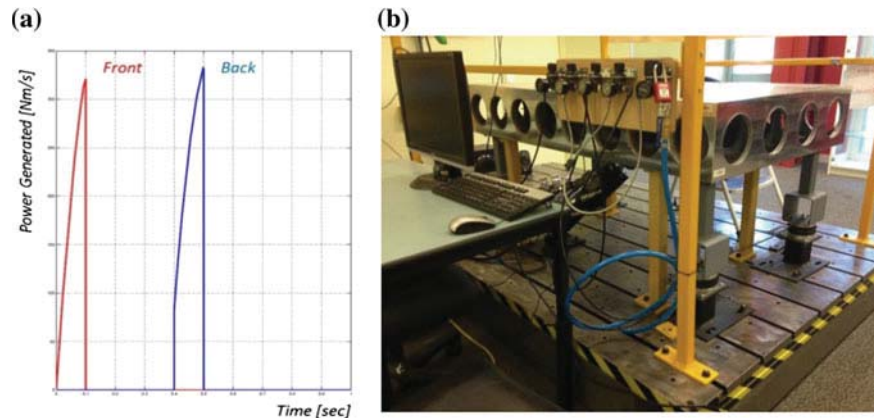


Fig. 9 a Power generated by riding over a road bump b Vehicle vibration lab test rig

Solar and thermal energy harvesting are already subjects of intensive research [16], but recovery of the energy dissipated while driving over the bumpy road, will be subject to another investigation. Modeling approach presented here gives us a powerful tool for that research as well. Instant power in mechanical systems is product of force and velocity. Power from any location on the vehicle body, can easily be calculated and presented for the investigation [10]. Power generated as result of the driving over the road bump, from front and from the back of the vehicle is shown in Fig. 9a.

Figure 9b shows School of engineering vehicle vibration lab test rig. It will be used for further research and our modeling and simulation methodology evaluation, ride comfort studies and power harvesting.

5 Conclusion

Engineering systems' modeling using physical networks' is a new approach with widespread potential. Vehicle suspension systems, which include various physical systems, can easily be represented by electrical circuits' models. This modeling approach covers both passive and active systems. Active systems design is based on the latest development in sensing and actuation technology. Model enables investigations with simulated dips, or bumps on the road. Various scenarios can easily be examined, with, or without active suspension engaged. Further investigations will be realized by conducted hardware in the loop testing and tuning in the RMIT University vibration lab. Flat ride is important for driver operated vehicle, but probably even more for autonomous vehicles where passengers develop loss of controllability experience.

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