Bank Trust Modelling

First Order linear differential equation is in form of

$$y' + P(x)y = Q(x)$$

solve for $u(x) = e^{\int P(x)\,dx}$ in order to solve $P(x)$ & $Q(x)$

$\Rightarrow$ general solution is $y(x) = \frac{1}{u(x)} \int Q(x)\,u(x)\,dx$

Now, Consider a bank account that earns 8% interest compounded continuously has an initial balance of zero. Money is deposited into the account at a constant rate of \$1000 per year (about \$2.74/day)

What is the balance in the account after 20 years ?

$$\Rightarrow \frac{dy}{dt} = (0.08)y + 1000$$

$$\Rightarrow \frac{dy}{0.08y + 1000} = dt.$$

$$\Rightarrow \int \frac{dy}{0.08y + 1000} = \int dt \Rightarrow \frac{1}{0.08} \ln|0.08y + 1000| = t + C$$

$$\Rightarrow \frac{1}{\frac{8}{100}} \ln|0.08y + 1000| = t + C \Rightarrow \frac{100}{8} \ln|0.08y + 1000| = t + C$$

$$\Rightarrow \ln|0.08y + 1000| = t\left(\frac{2}{25}\right) + \left(C \cdot \frac{2}{25}\right)$$

$$= \frac{2t}{25} + C_2$$

$$\Rightarrow 0.08y + 1000 = e^{\left(\frac{2t}{25} + C_2\right)}$$

$$\Rightarrow 0.08y = e^{\frac{2}{25}t + C_2} - 1000$$

$$\Rightarrow y = \left(e^{\frac{2t}{25} + C_2} - 1000\right)\frac{25}{2}$$

$$= 12.5e^{\frac{2t}{25} + C_2} - 12500$$

$\therefore$ $y(t) = 12.5e^{\frac{2t}{25} + C_2} - 12500$, now at $t = 0$ $y(0) = 0$

$$\therefore y(0) = 0 = 12.5e^{C_2} - 12500$$

$$\Rightarrow 12500 = 12.5e^{C_2}.$$

$$\Rightarrow \frac{12500}{12.5} = e^{C_2}$$

$$\Rightarrow C_2 = \ln\left|\frac{12500}{12.5}\right|$$

$$\therefore \quad y(t) = 12.5\, e^{\frac{2t}{25} + C_2} - 12500$$

$$\Rightarrow y(t) = 12.5 \left(e^{\frac{2t}{25}} \; e^{\ln\left|\frac{12500}{12.5}\right|} \right) - 12500$$

$$= 12.5 \cdot \frac{12500}{12.5} \, e^{\frac{2t}{25}} - 12500$$

$$= 12500\, e^{\frac{2t}{25}} - 12500$$

$$\boxed{\therefore \quad y(t) = 12500\, e^{\frac{2t}{25}} - 12500}$$

$$\therefore \quad y(20) \simeq \$ \; 49143.$$

- For Numerical analysis

Basic form : $\frac{dy}{dt} = ky + D$

where $k = 0.08$ $\quad t_{max} = 20$
$D = 1000$
$y_0 = 0$

Case 2 : $k = 10\%$

$$\therefore \quad \frac{dy}{dt} = 0.1\, y + 1000$$

$$\Rightarrow \quad 10 \ln|0.1y + 1000| = t + C$$

$$\Rightarrow \quad \ln|0.1y + 1000| = \frac{t}{10} + C_2$$

$$\Rightarrow \quad 0.1y + 1000 = e^{\frac{t}{10} + C_2}$$

$$\Rightarrow \quad y = \left(e^{\frac{t}{10} + C_2} - 1000\right) \times 10$$

$$= 10 e^{\frac{t}{10} + C_2} - 10000$$

$$\therefore \quad y(0) = 0 = 10 e^{C_2} - 10000$$

$$10000 = 10 e^{C_2}$$

$$C_2 = \ln\left|\frac{10000}{10}\right|$$

$$\therefore \quad y(t) = 10000\, e^{\frac{t}{10}} - 10000$$

$$\therefore \quad \text{at} \quad y(20) = \$ \; 63891.$$

By observation

$$\frac{10-8}{8} = \frac{2}{8} = \frac{1}{4} = 0.25$$

$\Rightarrow$ $0.25 \times 100\% = 25\%$

By 25% increase bank balance

But by final result from the simulated model

$$\frac{\$63891 - \$49413}{\$49413} \approx 29.3\%.$$

$\Rightarrow$ $29.3\% \neq 25\%$

$\therefore$ linear scaling of the result of the first simulation can't be applied to get correct result for second simulation case.