Vehicle Modelling Using Physical Networks

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Abstract - This paper presents a contribution in vehicle modelling that could be used in investigation and search for flat ride and beter passangers' comfort. Modelling approach, given here, is based on physical networks, i.e. analogies among various physical systems, in this case mechanical and electrical. Vibrations around pitch axis are anaylised and a novel model designed for simulations. Presented modelling method could be used for mechanical design improvements, for smart, active, suspension systems' design, or other applications, like energy recovery and harvesting in hybrid, or electrical vehicles.

Keywords - Physical networks, Smart suspension, Flat ride, Passenger comfort, Vehicle vibrations, Vehicle dynamics, Suspension design, Energy recovery.

I. INTRODUCTION

A moving vehicle is subjected to motion and vibrations along three axes: roll, yaw and pitch. It is happening regardless of the environment, i.e. it affects ground, air, water and underwater vehicles. Subjects of this study are ground vehicles performing translator motion and vibrations induced around pitch axis as shown in Fig. 1.

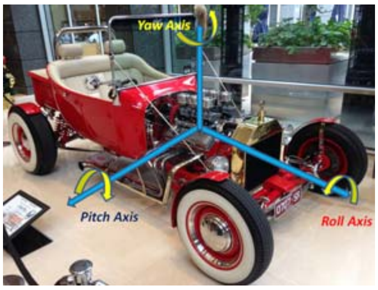

Fig. 1. Axes and vibration directions acting on a vehicle when performing translatory motion

An ordinary car, with four wheels, as a complex mobile system, if often presented using simplified, two-wheel bicycle model. This representation is used for vibration studies and flat ride investigations, as shown in [1], as well as for the path planning of the autonomous vehicles to achive the maximum ride comfort [2]. It is also used for autodriver algorithm development [3, 4]. Depending on the application, different views and parameters of the bicycle model are considered. Further simplification, used in vibration studies, is to *decouple* that car bicycle model. Following that, we assume that the front wheel excitation does not affect subsystem at the rear axle and vice versa.

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By using *physical networks* approach it is possible to conduct more comprehensive vibration studies while taking into account mutual influences from forces acting on front or rear axles. Physical networks express amusing analogies of ordinary differential equations used to represent various physical and engineering systems. More on those modelling solutions could be found in [5-7].

II. BICYCLE MODEL

Bicycle model used in this investigation is given in Fig. 2. Vehicle is represented as a beam of mass *m,* equally distributed along the length $l=a_1+a_2$ and with a moment of inertia *I*. It is a two degree-of-freedom (DOF) system where mass center is located in C and it is centre of rotation, i.e. bounce and pitch motion, with angular speed of *w* and angle Θ. Interfacing to mechanical translational reference, i.e. road, is realised by two sets of springs, K_f, K_r, and friction elements, B_f, B_r. We will investigate car body upward velocities (v_f, v_c, v_r) and displacements (x_f, x_c, x_r) along *x* axes.

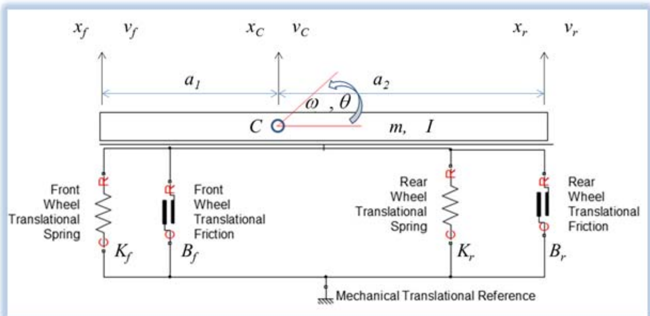

Fig. 2. Model 1: The bicycle model of a car as a beam of mass *m* and moment of inertia *I,* sitting on two sets of springs and friction elements representing two wheels

The translational coordinate x_C and the rotational coordinate θ are the usual generalized coordinates used to measure beam kinematics. Driving over the bumps on the road generates vertical forces that are acting on the two sides of the vehicle body, i.e. mass beam as shown in the Fig. 2. That action causes rotation represented by an angle θ. Moment of inertia of a beam with mass *m* is given as shown in Eq. (1):

$$I = \frac{1}{12} ml^2 \tag{1}$$

Radius of the beam rotation is given by Eq. (2):

$$R = \sqrt{\frac{I}{m}} \tag{2}$$

Vehicle on the road is subject to vertical forces acting on the front, F_f, and rear axle, F_r. We can add them, as they act together, in reference to the total body mass centre as given by Eq. (3):

$$F_f + F_r = m\frac{dv_C}{dt} \tag{3}$$

Adding moments of inertia, in reference to the mass centre, is presented by Eq. (4):

$$I\frac{dw}{dt} = mR^2\frac{dw}{dt} = F_r a_2 - F_f a_1 \tag{4}$$

Totall mass of the vehicle, as shown in Fig. 2, can be seen as two masses that correspond to the front, m_f, and the rear, m_r, part of the body, together with a *mutual mass,* m_m, between them, as given in the Fig. 3. Forces acting on the front and rear end of the vehicle are shown in the figure as ideal force sources. Mutual mass express influences of the forces acting on one part of the vehicle to the other and vice versa.

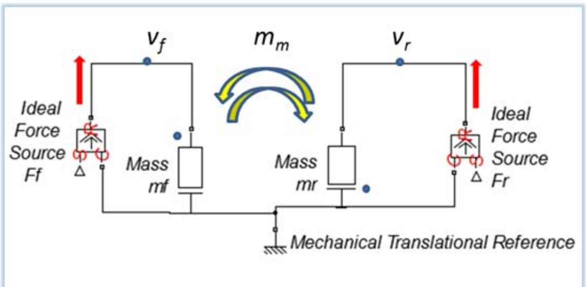

Fig. 3. Model 2: Mass represented as a coupled element

As it is show in [6], all three masses can be easily determined through calculations given by the set of equations Eq. (5-7):

$$m_f = m\frac{a_2^2 + R^2}{l^2} \tag{5}$$

$$m_r = m\frac{a_1^2 + R^2}{l^2} \tag{6}$$

$$m_m = m\frac{a_1 a_2 - R^2}{l^2} \tag{7}$$

From the Eq.(7) we can see that the decoupling condition is when m_m=0. It is given as expression in Eq. (8). In that case and we can treat parts of the vehicle's body separately.

$$R^2 = a_1 a_2 \tag{8}$$

Finally, we can write equations of motion as following:

$$m_f\frac{dv_f}{dt} + m_m\frac{dv_r}{dt} = F_f \tag{9}$$

$$m_m\frac{dv_f}{dt} + m_r\frac{dv_r}{dt} = F_r \tag{10}$$

These equations correspond to the mechanical systems diagram, or physical network, as shown in Fig. 3. The polarities of the mutual mass influences are given by dots. As in any other coupled physical network, polarities depend on the defined positive direction on the diagram.

III. Physical Networks

Physical network represents a class of linear graph associated to the particular physical system equations. Basic definitions, principles and rules for solving system equations, represented by the network, are independent of physical system. In a physical network there are two basic time dependent variables: flow, f, and potential, p. Flow is a variable that streams through network elements and connection lines, while potential is a variable manifested and measured across network elements, between two network points. Potential of a point in the network depends on the chosen referent point.

Examples of physical variables are electrical *current* and *potential*. Current is directly related to the mechanical motion of the electrical charges. Potential difference is called voltage. We have *force* and *velocity* in a mechanical system with translation, or *torque* and *angular speed* in a mechanical system with rotation. There are *flow* and *pressure* in hydraulic systems. Modelling of various systems can be implemented using the same type of ordinary differential equations (ODE), or physical network diagrams. Comparing electrical and mechanical system with translation, we can see analogies as given in Table I.

TABLE I
Electrical / Mechanical Analogies

Relation \ System	Electrical	Translation
Proportion	$i = \frac{1}{R}u$	$F = Bv$
Integration	$i = \frac{1}{L}\int udt$	$F = K\int vdt$
Differentiation	$i = C\frac{du}{dt}$	$F = m\frac{dv}{dt}$

IV. SYSTEM MODELLING STEPS

In this section we will perform modelling of the vehicle together with the road conditions and their interaction. The scenario is shown in Fig. 4. Vehicle is riding over the road bump. That will cause pitch motion, which should fade into the bounce motion, as quick as possible, for the comfortable flat ride. Road imperfection will generate two upward forces acting on front and then on the rear axle delayed in time. Time delay is defined by the translatory speed of the vehicle.

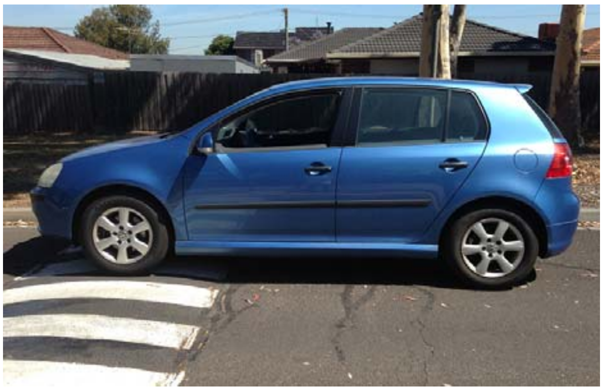

Fig. 4. Riding over a bump on the road

Road bumps can have various shapes, high and lengths. Sometimes step, or pulse functions could be used, as in flat ride studies and design [1]. In our case, as shown in Fig. 4, we are dealing with a harmonic type road bump with the amplitude of A=0.05m and the length of 1m.

Kinodinamic characteristics of the vehicle are given as following:

- Vehicle mass is m=1500,
- the length is l=4m,
- front and rear interfacing springs are the same and equal to K_f=K_r=2000N/m,
- while friction components are also the same and have value of B_f=B_r=10.

Translatory speed of the vehicle is v_t=10m/s.

We will assume that the centre of the mass is in the middle of the beam. Following that, the radius of rotation can be found using equations Eq. (1) and Eq. (2). It is equal to $R=\frac{2\sqrt{3}}{3}$. Using equations Eq. (5-7) values for the masses m_f, m_m, m_r are calculated and presented in Eq. (11).

$$\begin{aligned} m_r &= m_r = 500 \\ m_m &= 250 \end{aligned} \tag{11}$$

Through the whole document MKS units are used, unless otherwise specified. Vehicle model that corresponds to this case is given in Fig. 3, while motion equations are specified by Eq. (9) and Eq. (10). For the simplicity of the modelling process we will now transfer our system into and electrical circuit as given in Fig. 5.

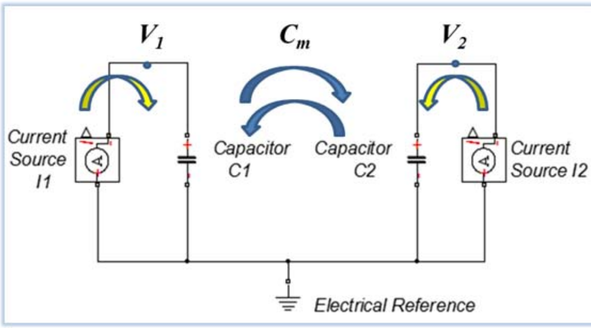

Fig. 5. Model 3: Vehicle model as an electrical network, where current corresponds to force and voltage to velocity

We can now write equations of motion, converted to electrical circuit, as following:

$$C_1\frac{dv_1}{dt}+C_m\frac{dv_2}{dt}=I_1 \tag{12}$$

$$C_m\frac{dv_1}{dt}+C_2\frac{dv_2}{dt}=I_2 \tag{13}$$

Further model transformation is to simplify it as shown in the Fig. 6. We can now write system equations and establish correspondence between two circuits.

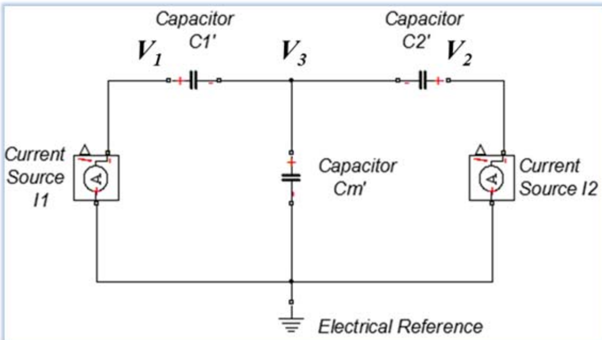

Fig. 6. Model 4: Simplified electrical network

Following equations are related to model 4:

$$C_1^{'}(\frac{dv_1}{dt}-\frac{dv_3}{dt})=I_1 \tag{14}$$

$$C_2^{'}(\frac{dv_2}{dt}-\frac{dv_3}{dt})=I_2 \tag{15}$$

$$C_m^{'}\frac{dv_3}{dt}=I_1+I_2 \tag{16}$$

By comparing equations Eq. (12) and (13) with equations set of Eq. (14) to (16) we can derive expressions for model 4 circuit elements C_1', C_2' and C_m'. Finally, when we put numerical values for our particular system we can get the following:

$$
\begin{aligned}
C_1' &= C_2' = 250 \\
C_m' &= -750
\end{aligned}
\tag{17}
$$

By looking at the Table I we can see that the mass corresponds to capacity, spring stiffness to inductivity and friction to resistivity. When MKS used there is direct correspondence between units for physical quantities in various physical network systems. This means that if mass is expressed in *kg*, then the corresponding capacity will be expressed in *F*. Beam of mass m=1500kg, from the bicycle model as given in Fig. 2, is represented by model 6 where capacitor values are given by Eq. (17) in Farads.

V. Comprehensive Model

In previous section we have performed modelling of the vehicle body including effects of the mutual interferences between vertical motions caused by the forces acting one axle of the vehicle to the whole body. In the following section we will include interfacing between body and the road. We will refer to the whole bicycle model as shown in Fig. 2. Since vehicle body representation, given by model 4, is now an electrical circuit, we will convert the whole system to electrical. Model 5 is shown in Fig. 7.

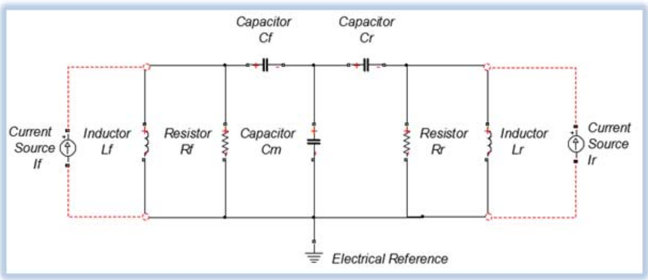

Fig. 7. Model 5: Electrical circuit as a representation for the vehicle bicycle model shown in Fig. 2

In Fig. 7 set of capacitors represent mass of the vehicle body, from model 4, while resistors and inductors correspond to the wheels parameters, i.e. friction and stiffness interfacing components to that road. Road is represented by electrical ground. Models 1-5 are not functional models. They are just used to demonstrate steps and principles in whole system design. Models, that follow in this report, are full functional models, running in Simulink environment, based on MATLAB R2012b.

Riding over the road bumps is an isolated event, i.e. two linked events: bump under the front wheels and then, after a delay, bump under the rear wheels. That causes vertical forces on front and rear axle of the car. We can model force sources as controlled current sources. Road bump of amplitude A will generate force F given by the Hooke's law:

$$F = kx = KA \tag{18}$$

Since the bump follows a harmonic waveform we need to find its period, i.e. the frequency, to be able to perform modelling. Since the velocity of the horizontal translator motion is equal to v=10m/s travelling time over the bump is 0.1s, which is the half of the sinusoidal signal period. So we have T=0.2s and f=5Hz. Our force sources are positive half-periods of the waveform as given by Eq. (19).

$$F = kx\sin(2\pi f) = 1000\sin(10\pi) \tag{19}$$

We have K=k=20000N/m and x=A=0.05m. Simulink model of the force source F_f is given in the Fig. 8.

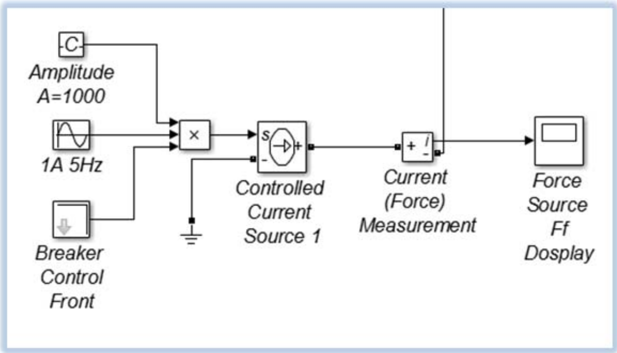

Fig. 8. Simulation of the force source generated by the road bump acting on front wheels

Model for the force source acting on the rear wheels is the same, but with a delay implemented. Delay depends on the vehicle speed. Force sources F_f and F_r graphs are given in Fig. 9 and Fig. 10 respectively

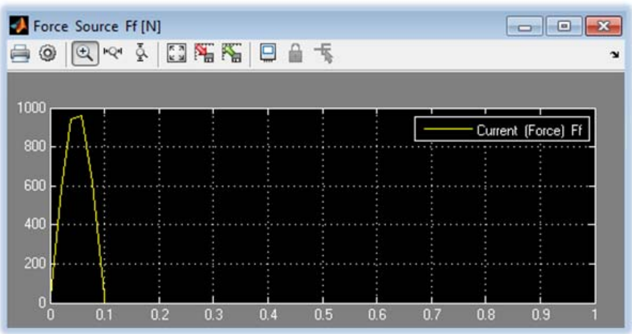

Fig. 9. Vertical force F_f generated by a bump on the road. Axes x represent time in *sec*.

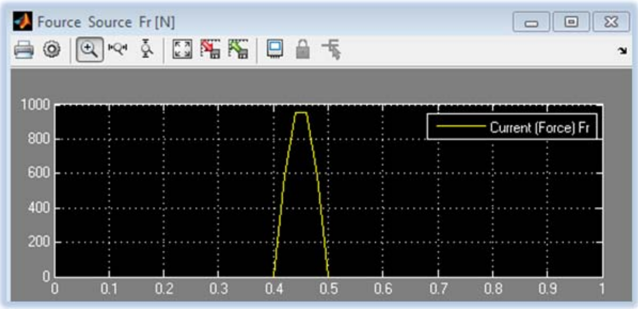

Fig. 10. Vertical force F_r generated by a bump on the road. Axes x represents time in *sec*.

Complete model of the vehicle on the road, including the scenario of the ride over the road bump, is given in Fig. 15. In addition to vehicle system components, already described and explained comprehensively, measurement and display devices are shown. For example, since vertical displacement is integral of the vertical velocity a circuit for integration is added, consisting of capacitor with the value C=1. Front vertical velocity is shown in Fig 11, while all displacements are presented in Fig 12.

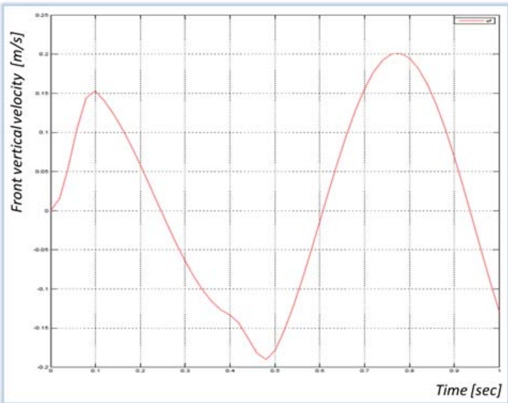

Fig. 11. Front vertical velocity generated by a bump on the road expressed in *m/s*. Axes x represents time in *sec*.

From the Fig. 12 we can see that the *front* is first going up, while the *back* end is going down. This is the result of the mutual mass interaction and can be observed in the real life scenario. At the same time mass centre is exposed to the minim disruption, which is contributing to the ride comfort when the passenger is sitting close to it.

In addition to the research capabilities opened by this modelling approach, in the areas of *flat ride* and *smart suspension* design, this model can be used for the green energy, or energy recovery investigation. Solar and thermal energy harvesting, for the automotive applications, are already subjects of intensive research, but recovery of the kinetic energy dissipated while driving over the bumpy road, could be subject to another interesting study.

Instant power in mechanical systems is product of force and velocity. Powers from the front and from the back of the vehicle are calculated and shown in Fig. 13.

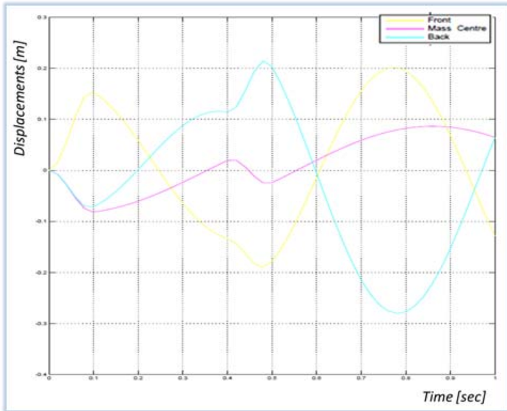

Fig. 12. Displacements from the three key body points: front, centre of mass and back of the vehicle

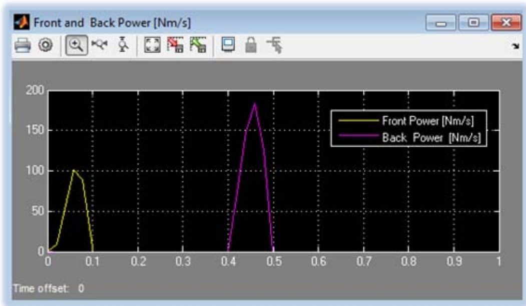

Fig. 13. Study of power dissipation. Axes x represents time in *sec*.

V. Experimental Testing

Analytical, as well as, simulation and modelling investigations in vehicle vibrations, are accompanied by lab experiments. RMIT University School of Engineering vibration lab is shown in Fig. 14. The whole lab setup was conducted by students and it is subject to constant improvements so that more comprehensive research can take place. One of the next steps is implementation of the hardware in the loop testing and smart suspension.

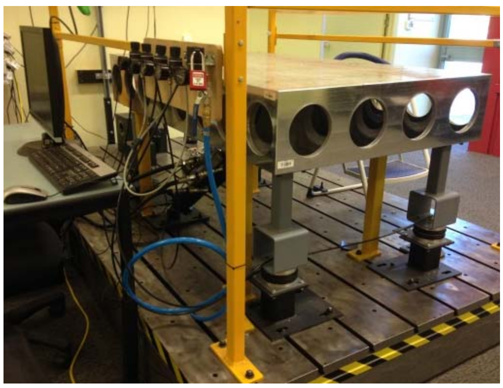

Fig. 14. Vehicle vibration lab at RMIT University

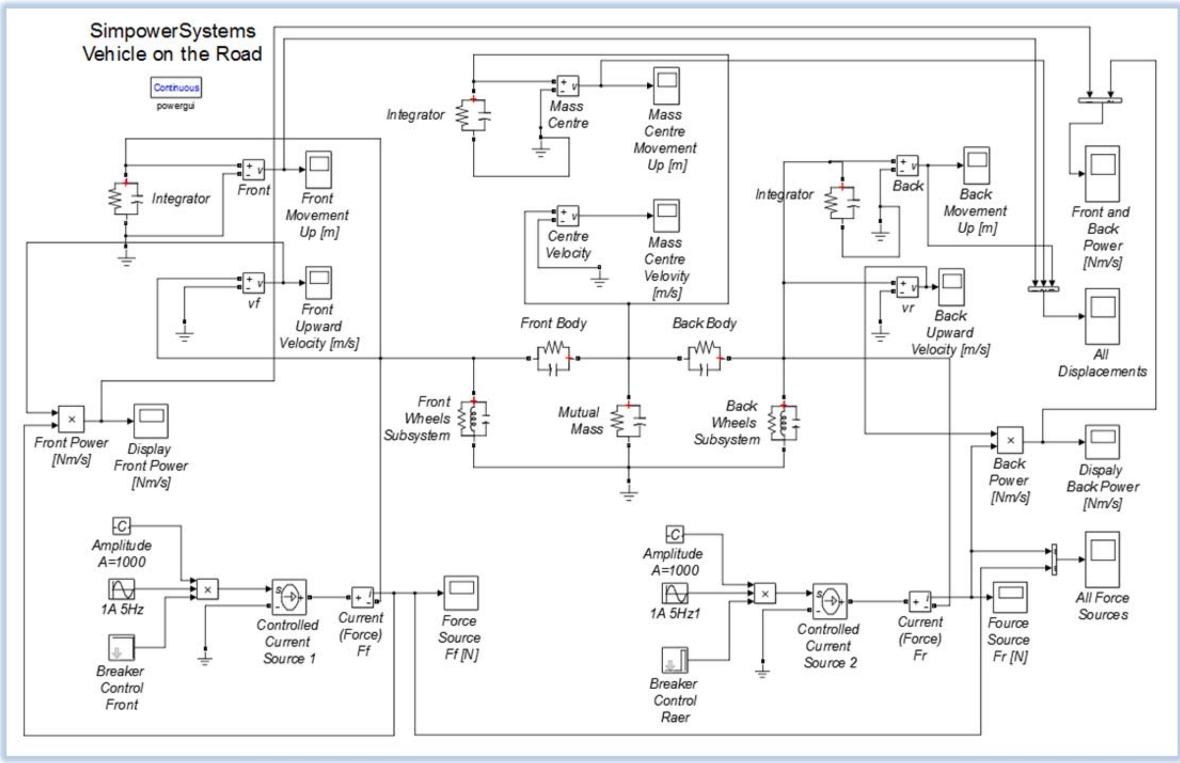

Fig. 15. Simulink model of the vehicle on the road

VI. Conclusion

In this paper a novel modelling of a vehicle and its road interaction is presented. It is based on physical networks and analogies between electrical and mechanical systems. There is a wide range of possible applications. We could investigate vehicle vibrations and ride comfort. In addition to that we could analyse other system quantities and parameters. Forces, power and energy could easily be monitored. Further applications of the model will be in the hardware in the loop testing for the research in smart suspensions.

Acknowledgement

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