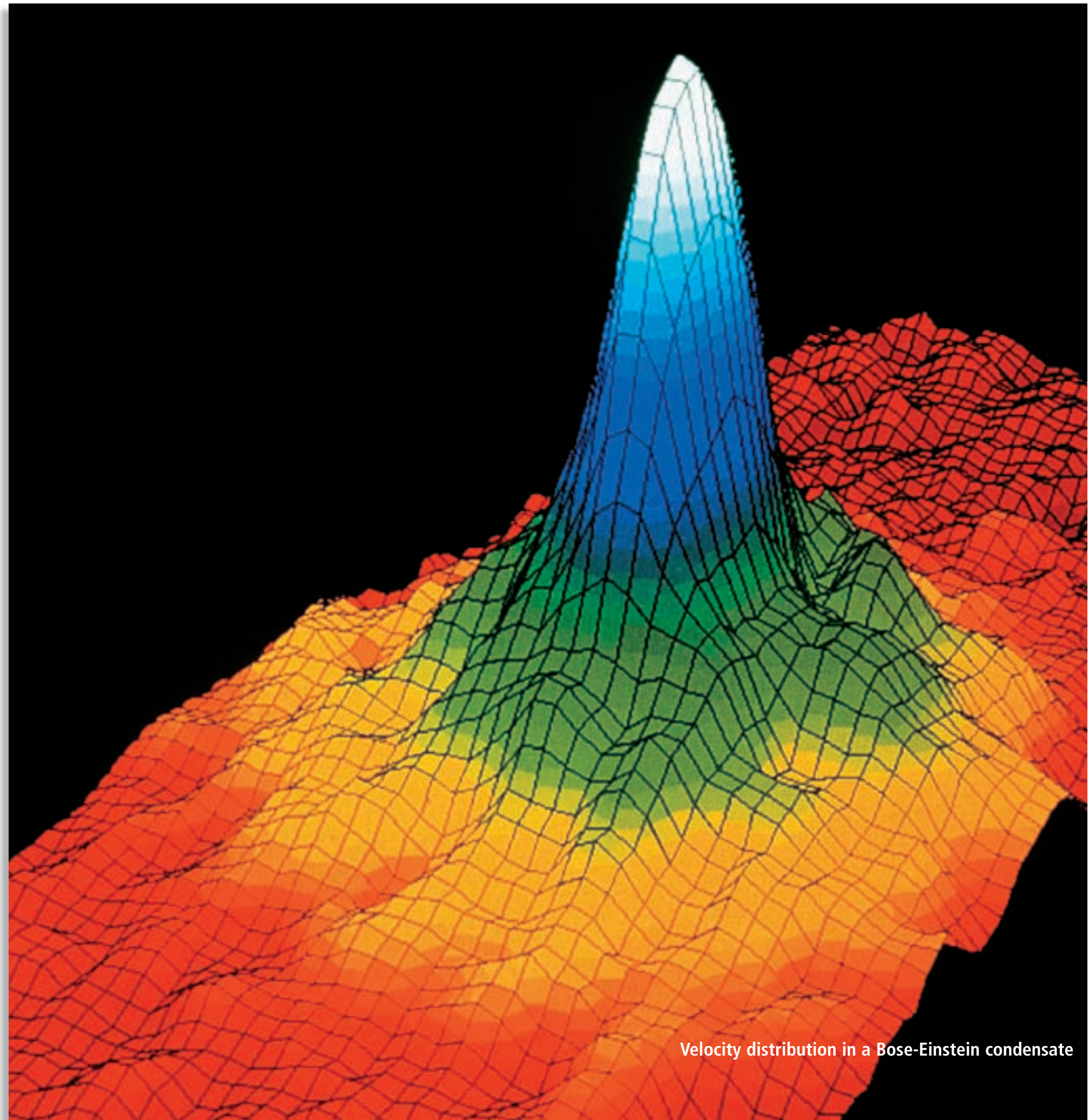
CHAPTER 2

Measurements and Calculations

Quantitative measurements are fundamental to chemistry.



Velocity distribution in a Bose-Einstein condensate

Scientific Method

SECTION 1

Objectives

- Describe the purpose of the scientific method.
- Distinguish between qualitative and quantitative observations.
- Describe the differences between hypotheses, theories, and models.

Sometimes progress in science comes about through accidental discoveries. Most scientific advances, however, result from carefully planned investigations. The process researchers use to carry out their investigations is often called the scientific method. *The* **scientific method** *is a logical approach to solving problems by observing and collecting data, formulating hypotheses, testing hypotheses, and formulating theories that are supported by data.*

Observing and Collecting Data

Observing is the use of the senses to obtain information. Observation often involves making measurements and collecting data. The data may be descriptive (qualitative) or numerical (quantitative) in nature. Numerical information, such as the fact that a sample of copper ore has a mass of 25.7 grams, is *quantitative*. Non-numerical information, such as the fact that the sky is blue, is *qualitative*.

Experimenting involves carrying out a procedure under controlled conditions to make observations and collect data. To learn more about matter, chemists study systems. *A* **system** *is a specific portion of matter in a given region of space that has been selected for study during an experiment or observation.* When you observe a reaction in a test tube, the test tube and its contents form a system.

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FIGURE 1 These students have designed an experiment to determine how to get the largest volume of popped corn from a fixed number of kernels. They think that the volume is likely to increase as the moisture in the kernels increases. Their experiment will involve soaking some kernels in water and observing whether the volume of the popped corn is greater than that of corn popped from kernels that have not been soaked.

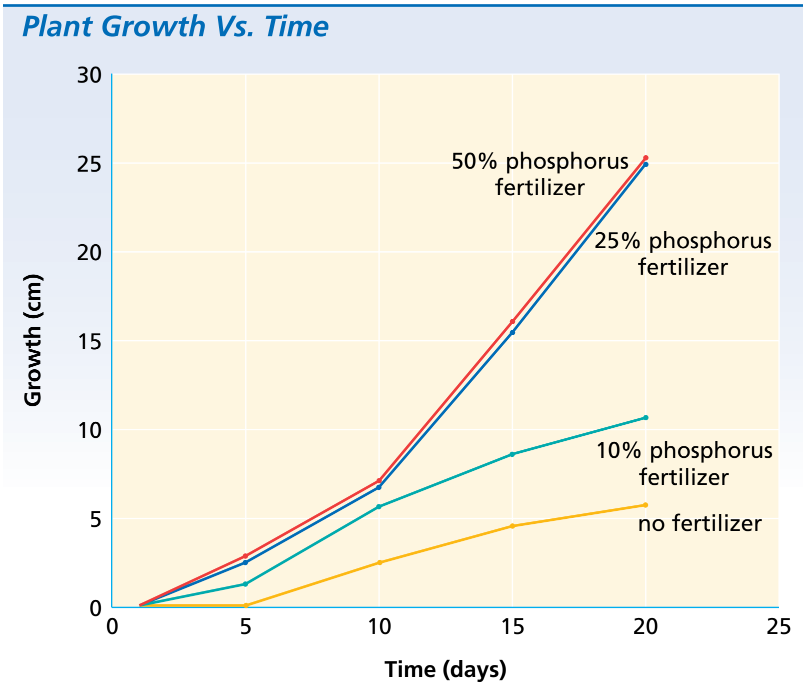


FIGURE 2 A graph of data can show relationships between two variables. In this case the graph shows data collected during an experiment to determine the effect of phosphorus fertilizer compounds on plant growth. The following is one possible hypothesis: *If* phosphorus stimulates corn-plant growth, *then* corn plants treated with a soluble phosphorus compound should grow faster, under the same conditions, than corn plants that are not treated.

Formulating Hypotheses

As scientists examine and compare the data from their own experiments, they attempt to find relationships and patterns—in other words, they make generalizations based on the data. Generalizations are statements that apply to a range of information. To make generalizations, data are sometimes organized in tables and analyzed using statistics or other mathematical techniques, often with the aid of graphs and a computer.

Scientists use generalizations about the data to formulate a **hypothesis,** *or testable statement*. The hypothesis serves as a basis for making predictions and for carrying out further experiments. Hypotheses are often drafted as "if-then" statements. The "then" part of the hypothesis is a prediction that is the basis for testing by experiment. **Figure 2** shows data collected to test a hypothesis.

Testing Hypotheses

Testing a hypothesis requires experimentation that provides data to support or refute a hypothesis or theory. During testing, the experimental conditions that remain constant are called *controls,* and any condition that changes is called a *variable.* Any change observed is usually due to the effects of the variable. If testing reveals that the predictions were not correct, the hypothesis on which the predictions were based must be discarded or modified.

STAGES IN THE SCIENTIFIC METHOD

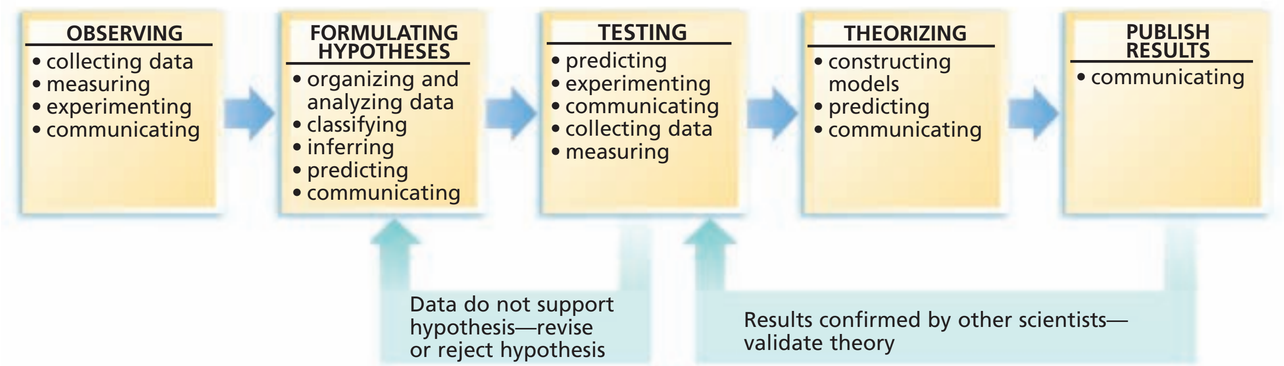


FIGURE 3 The scientific method is not a single, fixed process. Scientists may repeat steps many times before there is sufficient evidence to formulate a theory. You can see that each stage represents a number of different activities.

Theorizing

When the data from experiments show that the predictions of the hypothesis are successful, scientists typically try to explain the phenomena they are studying by constructing a model. *A* **model** *in science is more than a physical object; it is often an explanation of how phenomena occur and how data or events are related*. Models may be visual, verbal, or mathematical. One important model in chemistry is the atomic model of matter, which states that matter is composed of tiny particles called atoms.

If a model successfully explains many phenomena, it may become part of a theory. The atomic model is a part of the atomic theory, which you will study in Chapter 3. *A* **theory** *is a broad generalization that explains a body of facts or phenomena*. Theories are considered successful if they can predict the results of many new experiments. Examples of the important theories you will study in chemistry are kinetic-molecular theory and collision theory. **Figure 3** shows where theory fits in the scheme of the scientific method.

SECTION REVIEW

1. What is the scientific method?
2. Which of the following are quantitative?
   a. the liquid floats on water
   b. the metal is malleable
   c. the liquid has a temperature of 55.6°C
3. How do hypotheses and theories differ?
4. How are models related to theories and hypotheses?

Critical Thinking

5. **INTERPRETING CONCEPTS** Suppose you had to test how well two types of soap work. Describe your experiment by using the terms *control* and *variable.*

Chemistry in Action

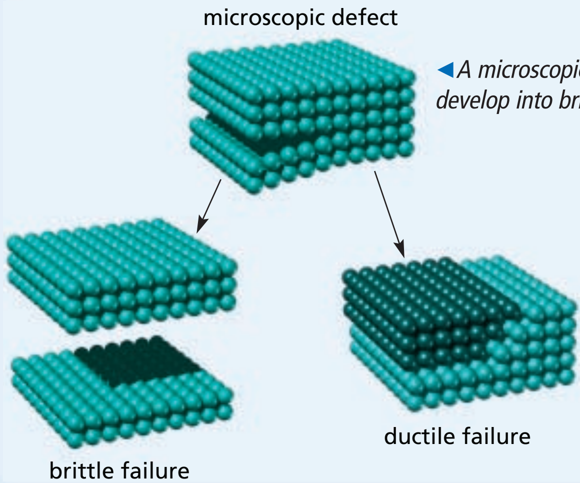
Breaking Up Is Easy To Do

It may seem obvious that chemistry is important in the making of materials, but chemistry is also vital to the study of how materials break. Everyday items have to be made to withstand various types of force and pressure or they cannot be used. For example, scientists and engineers work to ensure that highway bridges do not collapse.

When excessive force is applied to an object, the material that the object is made of will break. The object breaks because the force creates stress on the bonds between the atoms of the material and causes the bonds to break. This creates microscopic cracks in the material. When a material breaks, it is said to have undergone *failure.* Materials typically break in one of two ways: *ductile failure* and *brittle failure.* Both types of failure start with microscopic cracks in the material. However, the way a material eventually breaks depends how its atoms are organized.

Shattering glass undergoes brittle failure. Glass shatters when the bonds between the two layers of atoms that are along the initial crack break. This breakage causes the layers to pull apart, which separates the material into pieces. This type of failure is common in materials that do not have a very orderly arrangement of atoms.

When a car bumper crumples, ductile failure happens. This type of failure tends to happen in materials such as metals, that have a regular, ordered arrangement of atoms. This arrangement of atoms is known as a *crystal structure.* Ductile failure happens when the bonds in the material break across many layers of atoms that are not in the same plane as the original crack. Rather than splitting apart, the layers slip past each other into new positions. The atoms form new chemical bonds, between them and the material stays in one piece; only the shape has changed.



◀*A microscopic crack in a material can develop into brittle or ductile failure.*

In addition to the type of material influencing breakage, the quality of the material also influences breakage. All objects contain microscopic defects, such as bubbles in plastic pieces. A material will tend to undergo failure at its defect sites first. Careful fabrication procedures can minimize, but not completely eliminate, defects in materials.

Even though materials are designed to withstand a certain amount of force, the normal wear and tear that materials experience over their lifetimes creates defects in the material. This process is referred to as *fatigue.* If fatigue were to go undetected, the microscopic cracks that form could then undergo brittle or ductile failure. It would be catastrophic if the materials in certain products, such as airplane parts, failed. To avoid such a failure, people monitor materials that are exposed to constant stress for signs of fatigue. The defects in the metal parts of airplanes can be detected with nondestructive techniques, such as electromagnetic analysis.

Questions

1. Can you name some ways in which metal or plastic parts might obtain defects caused by chemical reactions?
2. Does a ceramic dinner plate undergo brittle or ductile failure when it is dropped and breaks?

SECTION 2

Units of Measurement

Objectives

- Distinguish between a quantity, a unit, and a measurement standard.
- Name and use SI units for length, mass, time, volume, and density.
- Distinguish between mass and weight.
- Perform density calculations.
- Transform a statement of equality into a conversion factor.

Measurements are quantitative information. A measurement is more than just a number, even in everyday life. Suppose a chef were to write a recipe listing quantities such as 1 salt, 3 sugar, and 2 flour. The cooks could not use the recipe without more information. They would need to know whether the number 3 represented teaspoons, tablespoons, cups, ounces, grams, or some other unit for sugar.

Measurements *represent* quantities. *A* **quantity** *is something that has magnitude, size, or amount.* A quantity is not the same as a measurement. For example, the quantity represented by a teaspoon is volume. The teaspoon is a unit of measurement, while volume is a quantity. A teaspoon is a measurement standard in this country. Units of measurement compare what is to be measured with a previously defined size. Nearly every measurement is a number plus a unit. The choice of unit depends on the quantity being measured.

Many centuries ago, people sometimes marked off distances in the number of foot lengths it took to cover the distance. But this system was unsatisfactory because the number of foot lengths used to express a distance varied with the size of the measurer's foot. Once there was agreement on a standard for foot length, confusion as to the real length was eliminated. It no longer mattered who made the measurement, as long as the standard measuring unit was correctly applied.

SI Measurement

Scientists all over the world have agreed on a single measurement system called *Le Système International d'Unités*, abbreviated **SI.** This system was adopted in 1960 by the General Conference on Weights and Measures. SI now has seven base units, and most other units are derived from these seven. Some non-SI units are still commonly used by chemists and are also used in this book.

SI units are defined in terms of standards of measurement. The standards are objects or natural phenomena that are of constant value, easy to preserve and reproduce, and practical in size. International organizations monitor the defining process. In the United States, the National Institute of Standards and Technology (NIST) plays the main role in maintaining standards and setting style conventions. For example, numbers are written in a form that is agreed upon internationally. The number seventy-five thousand is written 75 000, not 75,000, because the comma is used in other countries to represent a decimal point.

TABLE 1 SI Base Units

Quantity	Quantity symbol	Unit name	Unit abbreviation	Defined standard
Length	l	meter	m	the length of the path traveled by light in a vacuum during a time interval of 1/299 792 458 of a second
Mass	m	kilogram	kg	the unit of mass equal to the mass of the international prototype of the kilogram
Time	t	second	s	the duration of 9 192 631 770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the cesium-133 atom
Temperature	T	kelvin	K	the fraction 1/273.16 of the thermodynamic temperature of the triple point of water
Amount of substance	n	mole	mol	the amount of substance of a system which contains as many elementary entities as there are atoms in 0.012 kilogram of carbon-12
Electric current	I	ampere	A	the constant current which, if maintained in two straight parallel conductors of infinite length, of negligible circular cross section, and placed 1 meter apart in vacuum, would produce between these conductors a force equal to 2×10^{-7} newton per meter of length
Luminous intensity	I_v	candela	cd	the luminous intensity, in a given direction, of a source that emits monochromatic radiation of frequency 540×10^{12} hertz and that has a radiant intensity in that direction of 1/683 watt per steradian

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SI Base Units

The seven SI base units and their standard abbreviated symbols are listed in **Table 1.** All the other SI units can be derived from the fundamental units.

Prefixes added to the names of SI base units are used to represent quantities that are larger or smaller than the base units. **Table 2** lists SI prefixes using units of length as examples. For example, the prefix *centi-*, abbreviated c, represents an exponential factor of 10^{-2}, which equals 1/100. Thus, 1 centimeter, 1 cm, equals 0.01 m, or 1/100 of a meter.

Mass

As you learned in Chapter 1, mass is a measure of the quantity of matter. The SI standard unit for mass is the kilogram. The standard for mass defined in **Table 1** is used to calibrate balances all over the world.

TABLE 2 *SI Prefixes*

Prefix	Unit abbreviation	Exponential factor	Meaning	Example
tera	T	10^{12}	1 000 000 000 000	1 terameter (Tm) = 1×10^{12} m
giga	G	10^{9}	1 000 000 000	1 gigameter (Gm) = 1×10^{9} m
mega	M	10^{6}	1 000 000	1 megameter (Mm) = 1×10^{6} m
kilo	k	10^{3}	1000	1 kilometer (km) = 1000 m
hecto	h	10^{2}	100	1 hectometer (hm) = 100 m
deka	da	10^{1}	10	1 dekameter (dam) = 10 m
		$\mathbf{10^{0}}$	**1**	**1 meter (m)**
deci	d	10^{-1}	1/10	1 decimeter (dm) = 0.1 m
centi	c	10^{-2}	1/100	1 centimeter (cm) = 0.01 m
milli	m	10^{-3}	1/1000	1 millimeter (mm) = 0.001 m
micro	μ	10^{-6}	1/1 000 000	1 micrometer (μm) = 1×10^{-6} m
nano	n	10^{-9}	1/1 000 000 000	1 nanometer (nm) = 1×10^{-9} m
pico	p	10^{-12}	1/1 000 000 000 000	1 picometer (pm) = 1×10^{-12} m
femto	f	10^{-15}	1/1 000 000 000 000 000	1 femtometer (fm) = 1×10^{-15} m
atto	a	10^{-18}	1/1 000 000 000 000 000 000	1 attometer (am) = 1×10^{-18} m

The mass of a typical textbook is about 1 kg. The gram, g, which is 1/1000 of a kilogram, is more useful for measuring masses of small objects, such as flasks and beakers. For even smaller objects, such as tiny quantities of chemicals, the milligram, mg, is often used. One milligram is 1/1000 of a gram, or 1/1 000 000 of a kilogram.

Mass is often confused with weight because people often express the weight of an object in grams. Mass is determined by comparing the mass of an object with a set of standard masses that are part of the balance. **Weight** *is a measure of the gravitational pull on matter.* Unlike weight, mass does not depend on gravity. Mass is measured on instruments such as a balance, and weight is typically measured on a spring scale. Taking weight measurements involves reading the amount that an object pulls down on a spring. As the force of Earth's gravity on an object increases, the object's weight increases. The weight of an object on the moon is about one-sixth of its weight on Earth.

Length

The SI standard unit for length is the meter. A distance of 1 m is about the width of an average doorway. To express longer distances, the kilometer, km, is used. One kilometer equals 1000 m. Road signs in the United States sometimes show distances in kilometers as well as miles. The kilometer is the unit used to express highway distances in most other countries of the world. To express shorter distances, the centimeter

CROSS-DISCIPLINARY

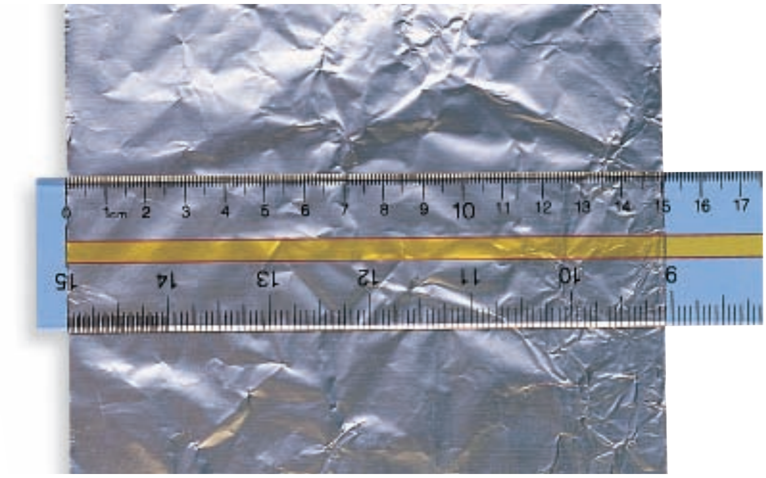
Some Handy Comparisons of Units

To become comfortable with units in the SI system, try relating some common measurements to your experience.

A meter stick is a little longer than a yardstick. A millimeter is about the diameter of a paper clip wire, and a centimeter is a little more than the width of a paper clip.

One gram is about the mass of a paper clip. A kilogram is about 2.2 pounds (think of two pounds plus one stick of butter). And there are about five milliliters in a teaspoon.

FIGURE 4 The meter is the SI unit of length, but the centimeter is often used to measure smaller distances. What is the length in cm of the rectangular piece of aluminum foil shown?



is often used. From **Table 2,** you can see that one centimeter equals 1/100 of a meter. The width of this book is just over 20 cm.

Derived SI Units

Many SI units are combinations of the quantities shown in **Table 1.** *Combinations of SI base units form* **derived units.** Some derived units are shown in **Table 3.**

Derived units are produced by multiplying or dividing standard units. For example, area, a derived unit, is length times width. If both length and width are expressed in meters, the area unit equals meters times meters, or square meters, abbreviated m^2. The last column of

TABLE 3 *Derived SI Units*

Quantity	Quantity symbol	Unit	Unit abbreviation	Derivation
Area	A	square meter	m^2	length × width
Volume	V	cubic meter	m^3	length × width × height
Density	D	kilograms per cubic meter	$\frac{kg}{m^3}$	$\frac{\text{mass}}{\text{volume}}$
Molar mass	M	kilograms per mole	$\frac{kg}{mol}$	$\frac{\text{mass}}{\text{amount of substance}}$
Molar volume	V_m	cubic meters per mole	$\frac{m^3}{mol}$	$\frac{\text{volume}}{\text{amount of substance}}$
Energy	E	joule	J	force × length

Table 3 shows the combination of fundamental units used to obtain derived units.

Some combination units are given their own names. For example, pressure expressed in base units is the following.

$$\mathrm{kg/m{\bullet}s^2}$$

The name *pascal,* Pa, is given to this combination. You will learn more about pressure in Chapter 11. Prefixes can also be added to express derived units. Area can be expressed in cm^2, square centimeters, or mm^2, square millimeters.

FIGURE 5 The speed that registers on a speedometer represents distance traveled per hour and is expressed in the derived units kilometers per hour or miles per hour.

Volume

Volume *is the amount of space occupied by an object.* The derived SI unit of volume is cubic meters, m^3. One cubic meter is equal to the volume of a cube whose edges are 1 m long. Such a large unit is inconvenient for expressing the volume of materials in a chemistry laboratory. Instead, a smaller unit, the cubic centimeter, cm^3, is often used. There are 100 centimeters in a meter, so a cubic meter contains 1 000 000 cm^3.

$$1\ \mathrm{m}^3 \times \frac{100\ \mathrm{cm}}{1\ \mathrm{m}} \times \frac{100\ \mathrm{cm}}{1\ \mathrm{m}} \times \frac{100\ \mathrm{cm}}{1\ \mathrm{m}} = 1\ 000\ 000\ \mathrm{cm}^3$$

When chemists measure the volumes of liquids and gases, they often use a non-SI unit called the liter. The liter is equivalent to one cubic decimeter. Thus, a liter, L, is also equivalent to 1000 cm^3. Another non-SI unit, the milliliter, mL, is used for smaller volumes. There are 1000 mL in 1 L. Because there are also 1000 cm^3 in a liter, the two units—milliliter and cubic centimeter—are interchangeable.

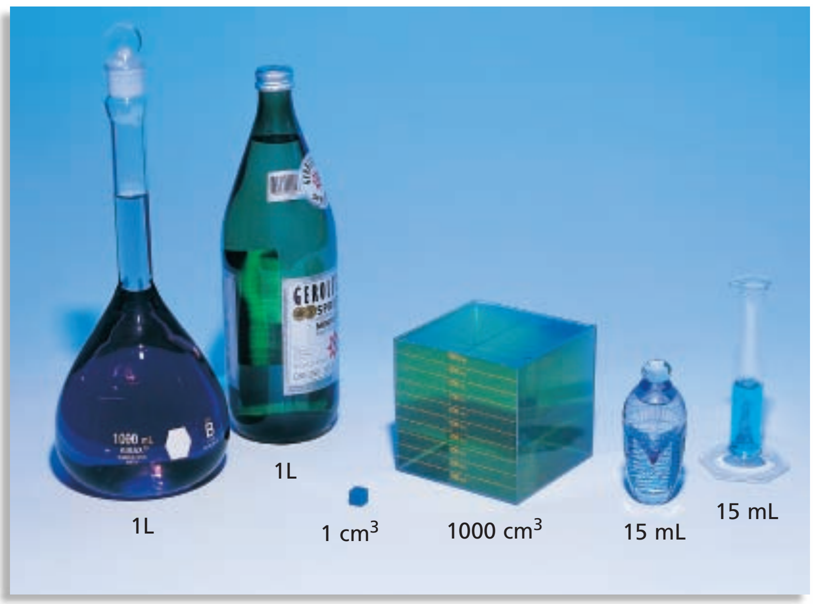


FIGURE 6 The relationships between various volumes are shown here. One liter contains 1000 mL of liquid, and 1 mL is equivalent to 1 cm^3. A small perfume bottle contains about 15 mL of liquid. The volumetric flask (far left) and graduated cylinder (far right) are used for measuring liquid volumes in the lab.

FIGURE 7 Density is the ratio of mass to volume. Both water and copper shot float on mercury because mercury is more dense.

Density

An object made of cork feels lighter than a lead object of the same size. What you are actually comparing in such cases is how massive objects are compared with their size. This property is called density. **Density** *is the ratio of mass to volume, or mass divided by volume*. Mathematically, the relationship for density can be written in the following way.

$$density = \frac{mass}{volume} \text{ or } D = \frac{m}{V}$$

The quantity m is mass, V is volume, and D is density.

The SI unit for density is derived from the base units for mass and volume—the kilogram and the cubic meter, respectively—and can be expressed as kilograms per cubic meter, kg/m^3. This unit is inconveniently large for the density measurements you will make in the laboratory. You will often see density expressed in grams per cubic centimeter, g/cm^3, or grams per milliliter, g/mL. The densities of gases are generally reported either in kilograms per cubic meter, kg/m^3, or in grams per liter, g/L.

Density is a characteristic physical property of a substance. It does not depend on the size of the sample because as the sample's mass increases, its volume increases proportionately, and the ratio of mass to volume is constant. Therefore, density can be used as one property to help identify a substance. **Table 4** shows the densities of some common materials. As you can see, cork has a density of only 0.24 g/cm^3, which is less than the density of liquid water. Because cork is less dense than water, it floats on water. Lead, on the other hand, has a density of 11.35 g/cm^3. The density of lead is greater than that of water, so lead sinks in water.

Note that **Table 4** specifies the temperatures at which the densities were measured. That is because density varies with temperature. Most objects expand as temperature increases, thereby increasing in volume. Because density is mass divided by volume, density usually decreases with increasing temperature.

TABLE 4 ***Densities of Some Familiar Materials***

Solids	Density at 20°C (g/cm^3)	Liquids	Density at 20°C (g/mL)
cork	0.24*	gasoline	0.67*
butter	0.86	ethyl alcohol	0.791
ice	$0.92^{\dagger}$	kerosene	0.82
sucrose	1.59	turpentine	0.87
bone	1.85*	water	0.998
diamond	3.26*	sea water	1.025**
copper	8.92	milk	1.031*
lead	11.35	mercury	13.6

† measured at 0°C
* typical density

** measured at 15°C

QuickLAB

Wear safety goggles and an apron.

Density of Pennies

Materials

- balance
- 100 mL graduated cylinder
- 40 pennies dated before 1982
- 40 pennies dated after 1982
- water

Procedure

1. Using the balance, determine the mass of the 40 pennies minted prior to 1982. Repeat this measurement two more times. Average the results of the three trials to determine the average mass of the pennies.

2. Repeat step 1 with the 40 pennies minted after 1982.

3. Pour about 50 mL of water into the 100 mL graduated cylinder. Record the exact volume of the water. Add the 40 pennies minted before 1982. CAUTION: Add the pennies carefully so that no water is splashed out of the cylinder. Record the exact volume of the water and pennies. Repeat this process two more times. Determine the volume of the pennies for each trial. Average the results of those trials to determine the average volume of the pennies.

4. Repeat step 3 with the 40 pennies minted after 1982.

5. Review your data for any large differences between trials that could increase the error of your results. Repeat those measurements.

6. Use the average volume and average mass to calculate the average density for each group of pennies.

7. Compare the calculated average densities with the density of the copper listed in Table 4.

Discussion

1. Why is it best to use the results of three trials rather than a single trial for determining the density?

2. How did the densities of the two groups of pennies compare? How do you account for any difference?

3. Use the results of this investigation to formulate a hypothesis about the composition of the two groups of pennies. How could you test your hypothesis?

SAMPLE PROBLEM A

A sample of aluminum metal has a mass of 8.4 g. The volume of the sample is 3.1 cm^3. Calculate the density of aluminum.

SOLUTION

Given: mass (m) = 8.4 g

volume (V) = 3.1 cm^3

Unknown: density (D)

$$density = \frac{mass}{volume} = \frac{8.4\ \text{g}}{3.1\ \text{cm}^3} = 2.7\ \text{g/cm}^3$$

PRACTICE *Answers in Appendix E*

1. What is the density of a block of marble that occupies 310. cm^3 and has a mass of 853 g?
2. Diamond has a density of 3.26 g/cm^3. What is the mass of a diamond that has a volume of 0.351 cm^3?
3. What is the volume of a sample of liquid mercury that has a mass of 76.2 g, given that the density of mercury is 13.6 g/mL?

extension

Go to **go.hrw.com** for more practice problems that ask you to calculate density.

Keyword: HC6MEAX

Conversion Factors

A **conversion factor** *is a ratio derived from the equality between two different units that can be used to convert from one unit to the other.* For example, suppose you want to know how many quarters there are in a certain number of dollars. To figure out the answer, you need to know how quarters and dollars are related. There are four quarters per dollar and one dollar for every four quarters. Those facts can be expressed as ratios in four conversion factors.

$$\frac{4 \text{ quarters}}{1 \text{ dollar}} = 1 \qquad \frac{1 \text{ dollar}}{4 \text{ quarters}} = 1 \qquad \frac{0.25 \text{ dollar}}{1 \text{ quarter}} = 1 \qquad \frac{1 \text{ quarter}}{0.25 \text{ dollar}} = 1$$

Notice that each conversion factor equals 1. That is because the two quantities divided in any conversion factor are equivalent to each other—as in this case, where 4 quarters equal 1 dollar. Because conversion factors are equal to 1, they can be multiplied by other factors in equations without changing the validity of the equations. You can use conversion factors to solve problems through dimensional analysis. **Dimensional analysis** *is a mathematical technique that allows you to use units to solve problems involving measurements.* When you want to use a conversion factor to change a unit in a problem, you can set up the problem in the following way.

$$\text{quantity sought} = \text{quantity given} \times \text{conversion factor}$$

For example, to determine the number of quarters in 12 dollars, you would carry out the unit conversion that allows you to change from dollars to quarters.

$$\text{number of quarters} = 12 \text{ dollars} \times \text{conversion factor}$$

Next you would have to decide which conversion factor gives you an answer in the desired unit. In this case, you have dollars and you want quarters. To eliminate dollars, you must divide the quantity by dollars. Therefore, the conversion factor in this case must have dollars in the denominator and quarters in the numerator. That factor is 4 quarters/1 dollar. Thus, you would set up the calculation as follows.

$$? \text{ quarters} = 12 \text{ dollars} \times \text{conversion factor}$$

$$= 12 \cancel{\text{dollars}} \times \frac{4 \text{ quarters}}{1 \cancel{\text{dollar}}} = 48 \text{ quarters}$$

Notice that the dollars have divided out, leaving an answer in the desired unit—quarters.

Suppose you had guessed wrong and used 1 dollar/4 quarters when choosing which of the two conversion factors to use. You would have an answer with entirely inappropriate units.

$$? \text{ quarters} = 12 \text{ dollars} \times \frac{1 \text{ dollar}}{4 \text{ quarters}} = \frac{3 \text{ dollars}^2}{\text{quarter}}$$

It is always best to begin with an idea of the units you will need in your final answer. When working through the Sample Problems, keep track of the units needed for the unknown quantity. Check your final answer against what you've written as the unknown quantity.

Deriving Conversion Factors

You can derive conversion factors if you know the relationship between the unit you have and the unit you want. For example, from the fact that *deci-* means "1/10," you know that there is 1/10 of a meter per decimeter and that each meter must have 10 decimeters. Thus, from the equality (1 m = 10 dm), you can write the following conversion factors relating meters and decimeters. In this book, when there is no digit shown in the denominator, you can assume the value is 1.

$$\frac{1 \text{ m}}{10 \text{ dm}} \text{ and } \frac{0.1 \text{ m}}{\text{dm}} \text{ and } \frac{10 \text{ dm}}{\text{m}}$$

The following sample problem illustrates an example of deriving conversion factors to make a unit conversion.

SAMPLE PROBLEM B

Express a mass of 5.712 grams in milligrams and in kilograms.

SOLUTION

Given: 5.712 g
Unknown: mass in mg and kg

The expression that relates grams to milligrams is

$$1 \text{ g} = 1000 \text{ mg}$$

The possible conversion factors that can be written from this expression are

$$\frac{1000 \text{ mg}}{\text{g}} \text{ and } \frac{1 \text{ g}}{1000 \text{ mg}}$$

To derive an answer in mg, you'll need to multiply 5.712 g by 1000 mg/g.

$$5.712\ \cancel{g} \times \frac{1000\ \text{mg}}{\cancel{g}} = 5712\ \text{mg}$$

This answer makes sense because milligrams is a smaller unit than grams and, therefore, there should be more of them.

The kilogram problem is solved similarly.

$$1\ \text{kg} = 1000\ \text{g}$$

Conversion factors representing this expression are

$$\frac{1\ \text{kg}}{1000\ \text{g}} \text{ and } \frac{1000\ \text{g}}{\text{kg}}$$

To derive an answer in kg, you'll need to multiply 5.712 g by 1 kg/1000 g.

$$5.712\ \cancel{g} \times \frac{1\ \text{kg}}{1000\ \cancel{g}} = 0.005712\ \text{kg}$$

The answer makes sense because kilograms is a larger unit than grams and, therefore, there should be fewer of them.

PRACTICE

Answers in Appendix E

1. Express a length of 16.45 m in centimeters and in kilometers.
2. Express a mass of 0.014 mg in grams.

extension

Go to **go.hrw.com** for more practice problems that ask you to perform unit conversions.

Keyword: HC6MEAX

SECTION REVIEW

1. Why are standards needed for measured quantities?
2. Label each of the following measurements by the quantity each represents. For instance, a measurement of 10.6 kg/m^3 represents density.
   - **a.** 5.0 g/mL
   - **b.** 37 s
   - **c.** 47 J
   - **d.** 39.56 g
   - **e.** 25.3 cm^3
   - **f.** 325 ms
   - **g.** 500 m^2
   - **h.** 30.23 mL
   - **i.** 2.7 mg
   - **j.** 0.005 L
3. Complete the following conversions.
   - **a.** 10.5 g = _____ kg
   - **b.** 1.57 km = _____ m
   - **c.** 3.54 µg = _____ g
   - **d.** 3.5 mol = _____ µmol
   - **e.** 1.2 L = _____ mL
   - **f.** 358 cm^3 = _____ m^3
   - **g.** 548.6 mL = _____ cm^3
4. Write conversion factors for each equality.
   - **a.** 1 m^3 = 1 000 000 cm^3
   - **b.** 1 in. = 2.54 cm
   - **c.** 1 µg = 0.000 001 g
   - **d.** 1 Mm = 1 000 000 m
5. **a.** What is the density of an 84.7 g sample of an unknown substance if the sample occupies 49.6 cm^3?
   b. What volume would be occupied by 7.75 g of this same substance?

Critical Thinking

6. **INFERRING CONCLUSIONS** A student converts grams to milligrams by multiplying by the conversion factor $\frac{1\ \text{g}}{1000\ \text{mg}}$. Is the student performing this calculation correctly?

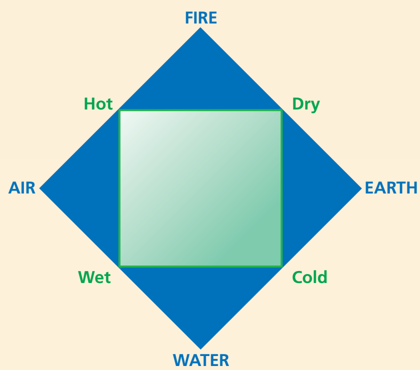
HISTORICAL CHEMISTRY

Classical Ideas About Matter

The Greeks were among the many ancient cultures that sought to understand the nature of matter. One group of Greek philosophers, called the *atomists,* believed that matter could be broken down into pieces of a minute size. These pieces, called *atoms* or *atomos* which means "indivisible," possessed intrinsic, unchanging qualities. Another group of Greeks believed that matter could be divided an infinite number of times and could be changed from one type of matter into another.

Between 500 and 300 BCE, the Greek philosophers Leucippus and Democritus formulated the ideas that the atomists held. Leucippus and Democritus believed that all atoms were essentially the same but that the properties of all substances arose from the unique characteristics of their atoms. For example, solids, such as most metals, were thought to have uneven, jagged atoms. Because the atoms were rough, they could stick together and form solids. Similarly, water was thought to have atoms with smooth surfaces, which would allow the atoms to flow past one another. Though atomists did not have the same ideas about matter that we have today, they did believe that atoms were constantly in motion, even in objects that appeared to be solid.

Some Greek philosophers who studied matter between 700 and 300 BCE described matter in a way that differed from the way atomists described it. They attempted to identify and describe a fundamental substance from which all other matter was formed. Thales of Miletus (640–546 BCE) was among the first to suggest the existence of a basic element. He chose water, which exists as liquid, ice, and steam. He interpreted water's changeability to mean that water could transform into any other substance. Other philosophers suggested that the basic element was air or fire. Empedokles (ca. 490–ca. 430 BCE) focused on four elements: earth, air, fire, and water. He thought that these elements combined in various proportions to make all known matter.



▲ *This diagram shows Aristotle's belief about the relationship between the basic elements and properties.*

Aristotle (384–322 BCE), a student of Plato, elaborated on the earlier ideas about elements. He argued that in addition to the four elements that make up all matter, there were four basic properties: hot, cold, wet, and dry. In Aristotle's view, the four elements could each have two of the basic properties. For example, water was wet and cold, while air was wet and hot. He thought that one element could change into another element if its properties were changed.

For more than 2,000 years, Aristotle's classical ideas dominated scientific thought. His ideas were based on philosophical arguments, not on the the scientific process. It was not until the 1700s that the existence of atoms was shown experimentally and that the incredible intuition of the atomists was realized.

Questions

1. In Aristotle's system of elements, fire opposes water. Why do you think that he chose this relationship?
2. Use the ideas of the atomists to describe the atoms of the physical phases of matter—solid, liquid, and gas.

SECTION 3

Objectives

- Distinguish between accuracy and precision.
- Determine the number of significant figures in measurements.
- Perform mathematical operations involving significant figures.
- Convert measurements into scientific notation.
- Distinguish between inversely and directly proportional relationships.

Using Scientific Measurements

If you have ever measured something several times, you know that the results can vary. In science, for a reported measurement to be useful, there must be some indication of its reliability or uncertainty.

Accuracy and Precision

The terms *accuracy* and *precision* mean the same thing to most people. However, in science their meanings are quite distinct. **Accuracy** *refers to the closeness of measurements to the correct or accepted value of the quantity measured*. **Precision** *refers to the closeness of a set of measurements of the same quantity made in the same way*. Thus, measured values that are accurate are close to the accepted value. Measured values that are precise are close to one another but not necessarily close to the accepted value.

Figure 8 can help you visualize the difference between precision and accuracy. A set of darts thrown separately at a dartboard may land in various positions, relative to the bull's-eye and to one another. The

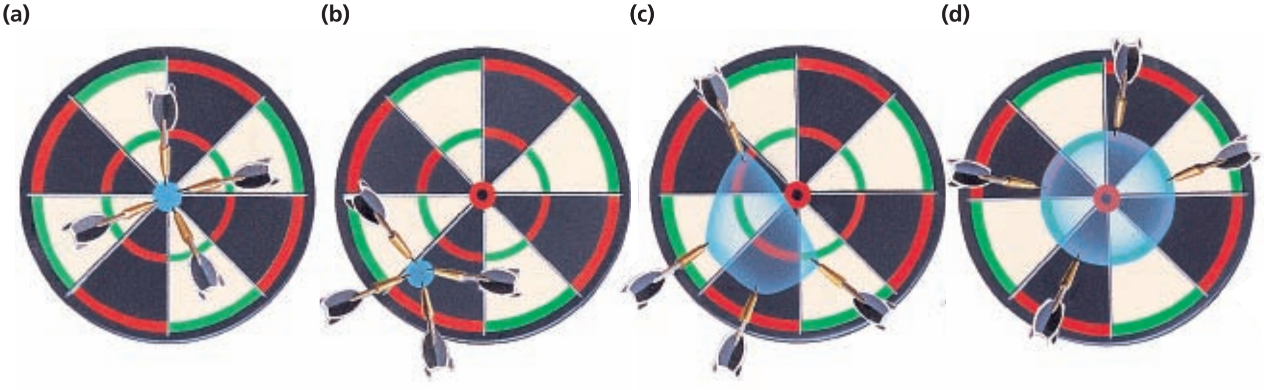


FIGURE 8 The sizes and locations of the areas covered by thrown darts illustrate the difference between precision and accuracy.

(a) Darts within small area = High precision. Area centered on bull's-eye = High accuracy

(b) Darts within small area = High precision. Area far from bull's-eye = Low accuracy

(c) Darts within large area = Low precision. Area far from bull's-eye = Low accuracy

(d) Darts within large area = Low precision. Area centered around bull's-eye = High accuracy (on average) closer the darts land to the bull's-eye, the more accurately they were thrown. The closer they land to one another, the more precisely they were thrown. Thus, the set of results shown in **Figure 8a** is both accurate and precise because the darts are close to the bull's-eye and close to each other. In **Figure 8b,** the set of results is inaccurate but precise because the darts are far from the bull's-eye but close to each other. In **Figure 8c,** the set of results is both inaccurate and imprecise because the darts are far from the bull's-eye and far from each other. Notice also that the darts are not evenly distributed around the bull's-eye, so the set, even considered on average, is inaccurate. In **Figure 8d,** the set on average is accurate compared with the third case, but it is imprecise. That is because the darts are distributed evenly around the bull's-eye but are far from each other.

extension

Chemistry in Action

Go to **go.hrw.com** for a full-length article on using measurements to determine a car's pollution rating.

Keyword: HC6MEAX

Percentage Error

The accuracy of an individual value or of an average experimental value can be compared quantitatively with the correct or accepted value by calculating the percentage error. **Percentage error** *is calculated by subtracting the accepted value from the experimental value, dividing the difference by the accepted value, and then multiplying by 100.*

$$\text{Percentage error} = \frac{Value_{experimental} - Value_{accepted}}{Value_{accepted}} \times 100$$

Percentage error has a negative value if the accepted value is greater than the experimental value. It has a positive value if the accepted value is less than the experimental value. The following sample problem illustrates the concept of percentage error.

SAMPLE PROBLEM C

A student measures the mass and volume of a substance and calculates its density as 1.40 g/mL. The correct, or accepted, value of the density is 1.30 g/mL. What is the percentage error of the student's measurement?

SOLUTION

$$\text{Percentage error} = \frac{Value_{experimental} - Value_{accepted}}{Value_{accepted}} \times 100$$

$$= \frac{1.40\ \cancel{\text{g/mL}} - 1.30\ \cancel{\text{g/mL}}}{1.30\ \cancel{\text{g/mL}}} \times 100 = 7.7\%$$

PRACTICE

Answers in Appendix E

1. What is the percentage error for a mass measurement of 17.7 g, given that the correct value is 21.2 g?
2. A volume is measured experimentally as 4.26 mL. What is the percentage error, given that the correct value is 4.15 mL?

extension

Go to **go.hrw.com** for more practice problems that ask you to calculate percentage error.

Keyword: HC6MEAX

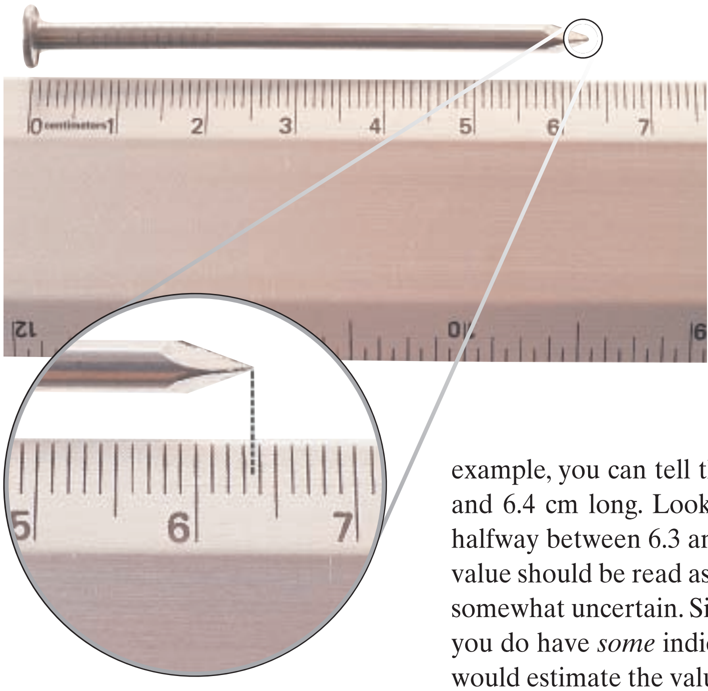


FIGURE 9 What value should be recorded for the length of this nail?

Error in Measurement

Some error or uncertainty always exists in any measurement. The skill of the measurer places limits on the reliability of results. The conditions of measurement also affect the outcome. The measuring instruments themselves place limitations on precision. Some balances can be read more precisely than others. The same is true of rulers, graduated cylinders, and other measuring devices.

When you use a properly calibrated measuring device, you can be almost certain of a particular number of digits in a reading. For example, you can tell that the nail in **Figure 9** is definitely between 6.3 and 6.4 cm long. Looking more closely, you can see that the value is halfway between 6.3 and 6.4 cm. However, it is hard to tell whether the value should be read as 6.35 cm or 6.36 cm. The hundredths place is thus somewhat uncertain. Simply leaving it out would be misleading because you do have *some* indication of the value's likely range. Therefore, you would estimate the value to the final questionable digit, perhaps reporting the length of the nail as 6.36 cm. You might include a plus-or-minus value to express the range, for example, 6.36 cm ± 0.01 cm.

Significant Figures

In science, measured values are reported in terms of significant figures. **Significant figures** *in a measurement consist of all the digits known with certainty plus one final digit, which is somewhat uncertain or is estimated*. For example, in the reported nail length of 6.36 cm discussed above, the last digit, 6, is uncertain. All the digits, including the uncertain one, are significant, however. All contain information and are included in the reported value. Thus, the term *significant* does not mean *certain*. In any correctly reported measured value, the final digit is significant but not certain. Insignificant digits are never reported. As a chemistry student, you will need to use and recognize significant figures when you work with measured quantities and report your results, and when you evaluate measurements reported by others.

Determining the Number of Significant Figures

When you look at a measured quantity, you need to determine which digits are significant. That process is very easy if the number has no zeros because all the digits shown are significant. For example, in a number reported as 3.95, all three digits are significant. The significance of zeros in a number depends on their location, however. You need to learn and follow several rules involving zeros. After you have studied the rules in **Table 5,** use them to express the answers in the sample problem that follows.

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Topic: Significant Figures
Code: HC61392

TABLE 5 *Rules for Determining Significant Zeros*

Rule	Examples
1. Zeros appearing between nonzero digits are significant.	a. 40.7 L has three significant figures. b. 87 009 km has five significant figures.
2. Zeros appearing in front of all nonzero digits are not significant.	a. 0.095 897 m has five significant figures. b. 0.0009 kg has one significant figure.
3. Zeros at the end of a number and to the right of a decimal point are significant.	a. 85.00 g has four significant figures. b. 9.000 000 000 mm has 10 significant figures.
4. Zeros at the end of a number but to the left of a decimal point may or may not be significant. If a zero has not been measured or estimated but is just a placeholder, it is not significant. A decimal point placed after zeros indicates that they are significant.	a. 2000 m may contain from one to four significant figures, depending on how many zeros are placeholders. **For measurements given in this text, assume that 2000 m has one significant figure.** b. 2000. m contains four significant figures, indicated by the presence of the decimal point.

SAMPLE PROBLEM D

For more help, go to the Math Tutor at the end of Chapter 1.

How many significant figures are in each of the following measurements?

a. 28.6 g
b. 3440. cm
c. 910 m
d. 0.046 04 L
e. 0.006 700 0 kg

SOLUTION Determine the number of significant figures in each measurement using the rules listed in **Table 5.**

a. 28.6 g
There are no zeros, so all three digits are significant.

b. 3440. cm
By rule 4, the zero is significant because it is immediately followed by a decimal point; there are 4 significant figures.

c. 910 m
By rule 4, the zero is not significant; there are 2 significant figures.

d. 0.046 04 L
By rule 2, the first two zeros are not significant; by rule 1, the third zero is significant; there are 4 significant figures.

e. 0.006 700 0 kg
By rule 2, the first three zeros are not significant; by rule 3, the last three zeros are significant; there are 5 significant figures.

PRACTICE *Answers in Appendix E*

1. Determine the number of significant figures in each of the following.
   a. 804.05 g
   b. 0.014 403 0 km
   c. 1002 m
   d. 400 mL
   e. 30 000. cm
   f. 0.000 625 000 kg

2. Suppose the value "seven thousand centimeters" is reported to you. How should the number be expressed if it is intended to contain the following?
   a. 1 significant figure
   b. 4 significant figures
   c. 6 significant figures

extension
Go to **go.hrw.com** for more practice problems that ask you to determine significant figures.
Keyword: HC6MEAX

Rounding

When you perform calculations involving measurements, you need to know how to handle significant figures. This is especially true when you are using a calculator to carry out mathematical operations. The answers given on a calculator can be derived results with more digits than are justified by the measurements.

Suppose you used a calculator to divide a measured value of 154 g by a measured value of 327 mL. Each of these values has three significant figures. The calculator would show a numerical answer of 0.470948012. The answer contains digits not justified by the measurements used to calculate it. Such an answer has to be rounded off to make its degree of certainty match that in the original measurements. The answer should be 0.471 g/mL.

The rules for rounding are shown in **Table 6.** The extent of rounding required in a given case depends on whether the numbers are being added, subtracted, multiplied, or divided.

TABLE 6 ***Rules for Rounding Numbers***

If the digit following the last digit to be retained is:	then the last digit should:	Example (rounded to three significant figures)
greater than 5	be increased by 1	42.68 g $\longrightarrow$ 42.7 g
less than 5	stay the same	17.32 m $\longrightarrow$ 17.3 m
5, followed by nonzero digit(s)	be increased by 1	2.7851 cm $\longrightarrow$ 2.79 cm
5, not followed by nonzero digit(s), and preceded by an odd digit	be increased by 1	4.635 kg $\longrightarrow$ 4.64 kg (because 3 is odd)
5, not followed by nonzero digit(s), and the preceding significant digit is even	stay the same	78.65 mL $\longrightarrow$ 78.6 mL (because 6 is even)

Addition or Subtraction with Significant Figures

Consider two mass measurements, 25.1 g and 2.03 g. The first measurement, 25.1 g, has one digit to the right of the decimal point, in the tenths place. There is no information on possible values for the hundredths place. That place is simply blank and cannot be assumed to be zero. The other measurement, 2.03 g, has two digits to the right of the decimal point. It provides information up to and including the hundredths place.

Suppose you were asked to add the two measurements. Simply carrying out the addition would result in an answer of 25.1 g + 2.03 g = 27.13 g. That answer suggests there is certainty all the way to the hundredths place. However, that result is not justified because the hundredths place in 25.1 g is completely unknown. The answer must be adjusted to reflect the uncertainty in the numbers added.

When adding or subtracting decimals, the answer must have the same number of digits to the right of the decimal point as there are in the measurement having the fewest digits to the right of the decimal point. Comparing the two values 25.1 g and 2.03 g, the measurement with the fewest digits to the right of the decimal point is 25.1 g. It has only one such digit. Following the rule, the answer must be rounded so that it has no more than one digit to the right of the decimal point. The answer should therefore be rounded to 27.1 g. When working with whole numbers, the answer should be rounded so that the final significant digit is in the same place as the leftmost uncertain digit. (For example, 5400 + 365 = 5800.)

Multiplication and Division with Significant Figures

Suppose you calculated the density of an object that has a mass of 3.05 g and a volume of 8.47 mL. The following division on a calculator will give a value of 0.360094451.

$$density = \frac{mass}{volume} = \frac{3.05\ \text{g}}{8.47\ \text{mL}} = 0.360094451\ \text{g/mL}$$

The answer must be rounded to the correct number of significant figures. The values of mass and volume used to obtain the answer have only three significant figures each. The degree of certainty in the calculated result is not justified. *For multiplication or division, the answer can have no more significant figures than are in the measurement with the fewest number of significant figures*. In the calculation just described, the answer, 0.360094451 g/mL, would be rounded to three significant figures to match the significant figures in 8.47 mL and 3.05 g. The answer would thus be 0.360 g/mL.

SAMPLE PROBLEM E *For more help, go to the* ***Math Tutor*** *at the end of Chapter 1.*

Carry out the following calculations. Express each answer to the correct number of significant figures.

a. 5.44 m − 2.6103 m

b. 2.4 g/mL × 15.82 mL

SOLUTION Carry out each mathematical operation. Follow the rules in **Table 5** and **Table 6** for determining significant figures and for rounding.

a. The answer is rounded to 2.83 m, because for subtraction there should be two digits to the right of the decimal point, to match 5.44 m.

b. The answer is rounded to 38 g, because for multiplication there should be two significant figures in the answer, to match 2.4 g/mL.

PRACTICE *Answers in Appendix E*

1. What is the sum of 2.099 g and 0.05681 g?

2. Calculate the quantity 87.3 cm − 1.655 cm.

3. Calculate the area of a rectangular crystal surface that measures 1.34 μm by 0.7488 μm. (Hint: Recall that *area* = *length* × *width* and is measured in square units.)

4. Polycarbonate plastic has a density of 1.2 g/cm^3. A photo frame is constructed from two 3.0 mm sheets of polycarbonate. Each sheet measures 28 cm by 22 cm. What is the mass of the photo frame?

extension

Go to **go.hrw.com** for more practice problems that ask you to calculate using significant figures.

Keyword: HC6MEAX

Conversion Factors and Significant Figures

Earlier in this chapter, you learned how conversion factors are used to change one unit to another. Such conversion factors are typically exact. That is, there is no uncertainty in them. For example, there are exactly 100 cm in a meter. If you were to use the conversion factor 100 cm/m to change meters to centimeters, the 100 would not limit the degree of certainty in the answer. Thus, 4.608 m could be converted to centimeters as follows.

$$4.608 \text{ m} \times \frac{100 \text{ cm}}{\text{m}} = 460.8 \text{ cm}$$

The answer still has four significant figures. Because the conversion factor is considered exact, the answer would not be rounded. Most exact conversion factors are defined, rather than measured, quantities. Counted numbers also produce conversion factors of unlimited precision. For example, if you counted that there are 10 test tubes for every student, that would produce an exact conversion factor of 10 test tubes/student. There is no uncertainty in that factor.

Scientific Notation

In **scientific notation,** *numbers are written in the form* M × *10*^{n}, *where the factor* M *is a number greater than or equal to 1 but less than 10 and* n *is a whole number.* For example, to write the quantity 65 000 km in scientific notation and show the first two digits as significant, you would write the following.

$$6.5 \times 10^4 \text{ km}$$

Writing the *M* factor as 6.5 shows that there are exactly two significant figures. If, instead, you intended the first three digits in 65 000 to be significant, you would write 6.50×10^4 km. When numbers are written in scientific notation, only the significant figures are shown.

Suppose you are expressing a very small quantity, such as the length of a flu virus. In ordinary notation this length could be 0.000 12 mm. That length can be expressed in scientific notation as follows.

$$0.000\,12 \text{ mm} = 1.2 \times 10^{-4} \text{ mm}$$

Move the decimal point four places to the right, and multiply the number by 10^{-4}.

1. Determine *M* by moving the decimal point in the original number to the left or the right so that only one nonzero digit remains to the left of the decimal point.

2. Determine *n* by counting the number of places that you moved the decimal point. If you moved it to the left, *n* is positive. If you moved it to the right, *n* is negative.

Mathematical Operations Using Scientific Notation

1. *Addition and subtraction* These operations can be performed only if the values have the same exponent (*n* factor). If they do not, adjustments must be made to the values so that the exponents are equal. Once the exponents are equal, the *M* factors can be added or subtracted. The exponent of the answer can remain the same, or it may then require adjustment if the *M* factor of the answer has more than one digit to the left of the decimal point. Consider the example of the addition of 4.2×10^4 kg and 7.9×10^3 kg.

We can make both exponents either 3 or 4. The following solutions are possible.

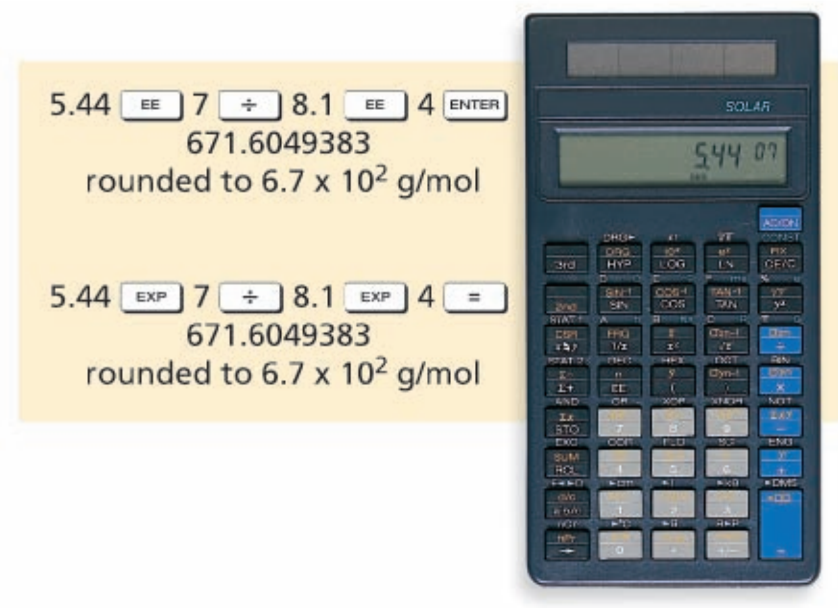
$$\begin{array}{r}4.2 \times 10^4 \text{ kg}\\ +0.79 \times 10^4 \text{ kg}\\ \hline 4.99 \times 10^4 \text{ kg}\end{array} \text{ rounded to } 5.0 \times 10^4 \text{ kg}$$

or

$$\begin{array}{r}7.9 \times 10^3 \text{ kg}\\ +42 \times 10^3 \text{ kg}\\ \hline 49.9 \times 10^3 \text{ kg}\end{array} = 4.99 \times 10^4 \text{ kg rounded to } 5.0 \times 10^4 \text{ kg}$$

Note that the units remain kg throughout.

FIGURE 10 When you use a scientific calculator to work problems in scientific notation, don't forget to express the value on the display to the correct number of significant figures and show the units when you write the final answer.



2. *Multiplication* The *M* factors are multiplied, and the exponents are added algebraically.

Consider the multiplication of 5.23×10^6 µm by 7.1×10^{-2} µm.

$$\begin{aligned}(5.23 \times 10^6 \ \mu\text{m})(7.1 \times 10^{-2} \ \mu\text{m}) &= (5.23 \times 7.1)(10^6 \times 10^{-2}) \\ &= 37.133 \times 10^4 \ \mu\text{m}^2 \text{ (adjust to two significant digits)} \\ &= 3.7 \times 10^5 \ \mu\text{m}^2\end{aligned}$$

Note that when length measurements are multiplied, the result is area. The unit is now µm^2.

3. *Division* The *M* factors are divided, and the exponent of the denominator is subtracted from that of the numerator. The calculator keystrokes for this problem are shown in **Figure 10.**

$$\begin{aligned}\frac{5.44 \times 10^7 \text{ g}}{8.1 \times 10^4 \text{ mol}} &= \frac{5.44}{8.1} \times 10^{7-4} \text{ g/mol} \\ &= 0.6716049383 \times 10^3 \text{ (adjust to two significant figures)} \\ &= 6.7 \times 10^2 \text{ g/mol}\end{aligned}$$

Note that the unit for the answer is the ratio of grams to moles.

Using Sample Problems

Learning to analyze and solve such problems requires practice and a logical approach. In this section, you will review a process that can help you analyze problems effectively. Most Sample Problems in this book are organized by four basic steps to guide your thinking in how to work out the solution to a problem.

Analyze

The first step in solving a quantitative word problem is to read the problem carefully at least twice and to analyze the information in it. Note any important descriptive terms that clarify or add meaning to the problem. Identify and list the data given in the problem. Also identify the unknown—the quantity you are asked to find.

Plan

The second step is to develop a plan for solving the problem. The plan should show how the information given is to be used to find the unknown. In the process, reread the problem to make sure you have gathered all the necessary information. It is often helpful to draw a picture that represents the problem. For example, if you were asked to determine the volume of a crystal given its dimensions, you could draw a representation of the crystal and label the dimensions. This drawing would help you visualize the problem.

Decide which conversion factors, mathematical formulas, or chemical principles you will need to solve the problem. Your plan might suggest a single calculation or a series of them involving different conversion factors. Once you understand how you need to proceed, you may wish to sketch out the route you will take, using arrows to point the way from one stage of the solution to the next. Sometimes you will need data that are not actually part of the problem statement. For instance, you'll often use data from the periodic table.

Compute

The third step involves substituting the data and necessary conversion factors into the plan you have developed. At this stage you calculate the answer, cancel units, and round the result to the correct number of significant figures. It is very important to have a plan worked out in step 2 before you start using the calculator. All too often, students start multiplying or dividing values given in the problem before they really understand what they need to do to get an answer.

Evaluate

Examine your answer to determine whether it is reasonable. Use the following methods, when appropriate, to carry out the evaluation.

1. Check to see that the units are correct. If they are not, look over the setup. Are the conversion factors correct?
2. Make an estimate of the expected answer. Use simpler, rounded numbers to do so. Compare the estimate with your actual result. The two should be similar.
3. Check the order of magnitude in your answer. Does it seem reasonable compared with the values given in the problem? If you calculated the density of vegetable oil and got a value of 54.9 g/mL, you should know that something is wrong. Oil floats on water; therefore, its density is less than water, so the value obtained should be less than 1.0 g/mL.
4. Be sure that the answer given for any problem is expressed using the correct number of significant figures.

Look over the following quantitative Sample Problem. Notice how the four-step approach is used, and then apply the approach yourself in solving the practice problems that follow.

SAMPLE PROBLEM F

Calculate the volume of a sample of aluminum that has a mass of 3.057 kg. The density of aluminum is 2.70 g/cm^3.

SOLUTION

1 ANALYZE

Given: mass = 3.057 kg, density = 2.70 g/cm^3

Unknown: volume of aluminum

2 PLAN

The density unit in the problem is g/cm^3, and the mass given in the problem is expressed in kg. Therefore, in addition to using the density equation, you will need a conversion factor representing the relationship between grams and kilograms.

$$1000 \text{ g} = 1 \text{ kg}$$

Also, rearrange the density equation to solve for volume.

$$density = \frac{mass}{volume} \quad \text{or} \quad D = \frac{m}{V}$$

$$V = \frac{m}{D}$$

3 COMPUTE

$$V = \frac{3.057 \cancel{\text{kg}}}{2.70 \cancel{\text{g}}/\text{cm}^3} \times \frac{1000 \cancel{\text{g}}}{\cancel{\text{kg}}} = 1132.222 \ldots \text{cm}^3 \text{ (calculator answer)}$$

The answer should be rounded to three significant figures.

$$V = 1.13 \times 10^3 \text{ cm}^3$$

4 EVALUATE

The unit of volume, cm^3, is correct. An order-of-magnitude estimate would put the answer at over 1000 cm^3.

$$\frac{3}{2} \times 1000$$

The correct number of significant figures is three, which matches that in 2.70 g/cm^3.

PRACTICE

Answers in Appendix E

1. What is the volume, in milliliters, of a sample of helium that has a mass of 1.73×10^{-3} g, given that the density is 0.178 47 g/L?
2. What is the density of a piece of metal that has a mass of 6.25×10^5 g and is 92.5 cm × 47.3 cm × 85.4 cm?
3. How many millimeters are there in 5.12×10^5 kilometers?
4. A clock gains 0.020 second per minute. How many seconds will the clock gain in exactly six months, assuming exactly 30 days per month?

extension

Go to **go.hrw.com** for more practice problems that ask you to calculate using scientific notation.

Keyword: HC6MEAX

Direct Proportions

Two quantities are **directly proportional** *to each other if dividing one by the other gives a constant value*. For example, if the masses and volumes of different samples of aluminum are measured, the masses and volumes will be directly proportional to each other. As the masses of the samples increase, their volumes increase by the same factor, as you can see from the data in **Table 7.** Doubling the mass doubles the volume. Halving the mass halves the volume.

When two variables, x and y, are directly proportional to each other, the relationship can be expressed as $y \propto x$, which is read as "y is *proportional* to x." The general equation for a directly proportional relationship between the two variables can also be written as follows.

$$\frac{y}{x} = k$$

The value of k is a constant called the proportionality constant. Written in this form, the equation expresses an important fact about direct proportion: the ratio between the variables remains constant. Note that using the mass and volume values in **Table 7** gives a mass-volume ratio that is constant (neglecting measurement error). The equation can be rearranged into the following form.

$$y = kx$$

The equation $y = kx$ may look familiar to you. It is the equation for a special case of a straight line. If two variables related in this way are graphed versus one another, a straight line, or linear plot that passes through the origin (0,0), results. The data for aluminum from **Table 7** are graphed in **Figure 11.** The mass and volume of a pure substance are directly proportional to each other. Consider mass to be y and volume to be x. The constant ratio, k, for the two variables is density. The slope of the line reflects the constant density, or mass-volume ratio, of aluminum,

TABLE 7 ***Mass-Volume Data for Aluminum at 20°C***

Mass (g)	Volume (cm^3)	$\frac{m}{V}$ (g/cm^3)
54.7	20.1	2.72
65.7	24.4	2.69
83.5	30.9	2.70
96.3	35.8	2.69
105.7	39.1	2.70

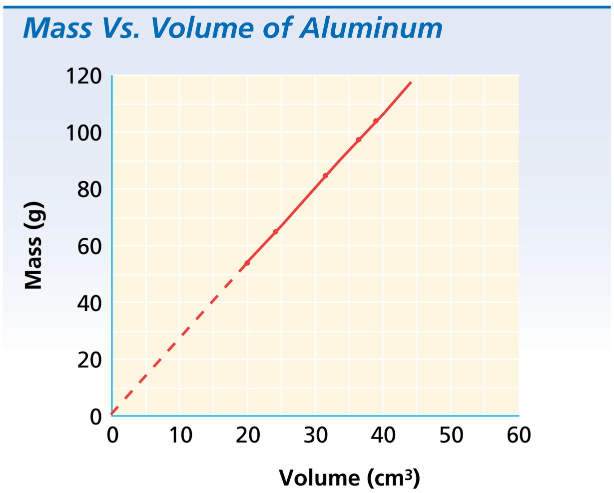


FIGURE 11 The graph of mass versus volume shows a relationship of direct proportion. Notice that the line is extrapolated to pass through the origin.

which is 2.70 g/cm^3 at 20°C. Notice also that the plotted line passes through the origin. All directly proportional relationships produce linear graphs that pass through the origin.

Inverse Proportions

Two quantities are **inversely proportional** *to each other if their product is constant*. An example of an inversely proportional relationship is that between speed of travel and the time required to cover a fixed distance. The greater the speed, the less time that is needed to go a certain fixed distance. Doubling the speed cuts the required time in half. Halving the speed doubles the required time.

When two variables, x and y, are inversely proportional to each other, the relationship can be expressed as follows.

$$y \propto \frac{1}{x}$$

This is read "y is *proportional* to 1 divided by x." The general equation for an inversely proportional relationship between the two variables can be written in the following form.

$$xy = k$$

In the equation, k is the proportionality constant. If x increases, y must decrease by the same factor to keep the product constant.

A graph of variables that are inversely proportional produces a curve called a hyperbola. Such a graph is illustrated in **Figure 12.** When the temperature of the gas is kept constant, the volume (V) of the gas sample decreases as the pressure (P) increases. Look at the data shown in **Table 8.** Note that $P \times V$ gives a reasonably constant value. The graph of this data is shown in **Figure 12.**

TABLE 8 ***Pressure-Volume Data for Nitrogen at Constant Temperature***

Pressure (kPa)	Volume (cm^3)	$P \times V$
100	500	50 000
150	333	50 000
200	250	50 000
250	200	50 000
300	166	49 800
350	143	50 100
400	125	50 000
450	110	49 500