SMA2307 Discrete Mathematics

Problem sheet-1

1. Which of the following sentences are propositions? What are the truth values of those that are propositions?

   (a) $2 + 3 = 7$.

   (b) Open the door.

   (c) $5 + 7 < 10$.

   (d) The moon is a satellite of the earth.

   (e) $x + 5 = 7$.

   (f) $x + 5 > 9$ for every real number x.

   (g) The integer 36 is even.

   (h) Is the integer $3^{15} - 8$ even?

   (i) The product of 3 and 4 is eleven.

   (j) if $x > 2$ then $x^2 \geq 3$(Assume x as a real number).

2. Explain why the following sentences are not propositions:

   (a) $x + 1 = 2$.

   (b) $x - y = y - x$.

   (c) $A^2 = 0$ implies $A = 0$.

3. State the converse, inverse and contrapositive of each of the following implications.

   (a) If two angles are congruent, then they have the same measure.

   (b) If a quadrilateral is a rectangle, then it has two pairs of parallel sides.

   (c) If it snows today, I will stay home.

   (d) We play the game if it is sunny.

   (e) If a positive integer is a prime then it has no divisors other than 1 and itself.

4. Construct a truth table for each of the following compound propositions.

   (a) $p \wedge \neg q$

   (b) $(p \vee \neg q) \rightarrow q$

   (c) $(p \rightarrow q) \leftrightarrow (\neg q \rightarrow \neg p)$

5. Verify the following logical equivalences using truth table and by developing a series of logical equivalences

   (a) $(p \vee \neg q) \wedge (\neg p \vee \neg q) \equiv \neg q$

   (b) $p \rightarrow q \equiv \neg p \vee q$

   (c) $\neg(p \rightarrow q) \equiv p \wedge \neg q$

   (d) $\neg(p \vee \neg q) \vee (\neg p \wedge \neg q) \equiv \neg p$

   (e) $(p \wedge (\neg(\neg p \vee q))) \vee (p \wedge q) \equiv p$

   (f) $(p \wedge \neg q) \vee p \equiv p$