$$x(n) = \sum_{n=-\infty}^{\infty} x(n)\, z^{-n}$$

$$x(n) = \cos\omega_0 n\, u(n)$$

$$X(z) = \frac{z(z-\cos\omega_0)}{z^2 - 2z\cos\omega_0 + 1}$$

Properties:

1. Linearity:

If $Z\{x_1(n)\} \rightarrow X_1(z)$

$Z\{x_2(n)\} \rightarrow X_2(z)$

$$Z\{a x_1(n) + b x_2(n)\} = a X_1(z) + b X_2(z)$$

eg: $x(n) = [\underbrace{3(2^n)}_{x_1(n)} - \underbrace{4(3^n)}_{x_2(n)}]\, u(n)$

$x_1(n) = 3(2^n)u(n)$ $\qquad$ $x_2(n) = 4(3^n)u(n)$

$$X_1(z) = Z\{3(2^n)u(n)\}$$

$$= 3\, Z\{2^n u(n)\} = 3 \cdot \frac{z}{z-2}$$

$a^n u(n) \rightarrow \frac{z}{z-a}$ $\qquad$ ROC: $|z| > 2$

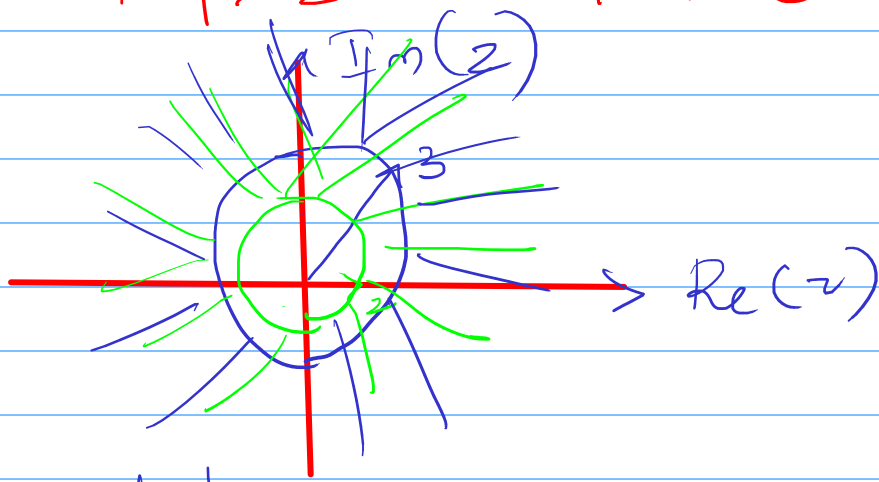
$|z| > a$

$$X_2(z) = Z\{4 \cdot 3^n u(n)\} = 4\, Z\{3^n u(n)\}$$

$$= 4 \cdot \frac{z}{z-3}, \quad ROC: |z| > 3$$

$$X(z) = \frac{3z}{z-2} - \frac{4z}{z-3}.$$

$$|z| > 2 \qquad |z| > 3$$



$$ROC: |z| > 3$$

2. Time Shifting property (Translation)

If $X(z) = Z\{x(n)\}$, then

$$Z\{x(n-m)\} = z^{-m} X(z).$$

$x(n) \longrightarrow x(n-m)$ $\quad m \to$ integer

$x(n) \longrightarrow x(n-3)$

eg. $x(n) = u(n-2)$

$$Z\{u(n)\} = \frac{z}{z-1}, \quad ROC: |z| > 1$$

$$Z\{u(n-2)\} = z^{-2} \cdot \frac{z}{z-1} = \frac{z^{-1}}{z-1}$$

$$= \frac{1}{z(z-1)}$$

ROC: $|z| > 1$

(5) Qn: $x(n) = u(n) - u(n-3)$

3. Time Reversal:

If $X(z) = Z\{x(n)\}$, then

$$Z\{x(-n)\} = X(z^{-1})$$

ROC of $x(n)$ is $|z| > |a|$ then
ROC of $x(-n)$ is $|z| < |1/a|$

eg: $x(n) = u(-n)$.

$$Z\{u(-n)\} = \frac{1}{1-z}$$

$u(n)$ $|z| > 1$

ROC: $|z| < 1$.

4. Multiplication of $x(n)$ by n:

If $Z\{x(n)\} = X(z)$, then

$$Z\{n \cdot x(n)\} = -z \frac{d X(z)}{dz}$$

Proof
Assign

$$x(n) = n \cdot \underbrace{a^n u(n)}$$

$$a^n u(n) \longrightarrow \frac{z}{z-a}$$

$$\mathcal{Z}\{n \cdot a^n u(n)\} = -z \frac{d}{dz} X(z) = -z \frac{d}{dz}\left(\frac{z}{z-a}\right)$$

$$= -z \left[\frac{(\cancel{z}-a) \times 1 - \cancel{z} \cdot 1}{(z-a)^2}\right] = -z \times \frac{-a}{(z-a)^2}$$

$$= \frac{az}{(z-a)^2}, \quad \text{ROC: } |z| > a.$$

5. Scaling property

If $Z\{x(n)\} = X(z)$, then

$$Z\{a^n x(n)\} = X(a^{-1}z).$$

eg: $x(n) = 2^n u(n)$

$$Z\{u(n)\} = \frac{z}{z-1}$$

$$Z\{2^n u(n)\} = \left.\frac{z}{z-1}\right|_{z = 2^{-1}z} = \frac{2^{-1}z}{2^{-1}z - 1}$$

$$= \frac{z/2}{z/2 - 1} = \frac{z}{z-2}.$$

ROC: $|z| > 2$

Qn: $x(n) = 2^n u(n-2)$.

$X(z) = Z\{u(n-2)\} = z^{-2} \cdot \frac{z}{z-1} = \frac{z^{-1}}{z-1}$

$Z\{2^n u(n-2)\} = \frac{z^{-1}}{z-1} \Big|_{z = 2^{-1}z}$

$= \frac{\frac{1}{2^{-1}z}}{2^{-1}z-1} = \frac{4}{z(z-2)}$

6. Convolution Property.

$Z\{x_1(n)\} \rightarrow X_1(z)$

$Z\{x_2(n)\} \rightarrow X_2(z)$

$Z\{x_1(n) * x_2(n)\} = X_1(z) \cdot X_2(z)$

$X_1(z)$ & $X_2(z)$ after common on.

If $x_1(n) = \{\underset{\uparrow}{1}, 2, 3, 1\}$ $x_2(n) = \{\underset{\uparrow}{1}, 1, 1, 2\}$

$x_1(n) * x_2(n)$ using z-Tr?

$X_1(z)$ x $X_2(z)$

$X_1(z) = Z\{x_1(n)\} = \sum_{n=-\infty}^{\infty} x(n) z^{-n}$

$\{\underset{\uparrow}{1}, 2, 3, 1\}$

$$= 1\cdot z^{0} + 2\cdot z^{-1} + 3\cdot z^{-2} + 1\cdot z^{-3}$$

$$= 1 + 2z^{-1} + 3z^{-2} + z^{-3}$$

$$X_2(z) = 1 + z^{-1} + z^{-2} + 2z^{-3}$$

$$X_1(z)\cdot X_2(z) =$$

$$\left(1+2z^{-1}+3z^{-2}+z^{-3}\right)\left(1+z^{-1}+z^{-2}+2z^{-3}\right)$$

Assignment :-

1. $x(n) = 2^n u(n) + 3\left(\frac{1}{2}\right)^n u(n)$
2. $x(n) = n^2 a^n u(n)$
3. $x(n) = \frac{1}{2}\delta(n) + \delta(n-1) - \frac{1}{3}\delta(n-2)$
4. $x(n) = \left(\frac{1}{3}\right)^{n-1} u(n-1)$

$$X_1(z)\cdot X_2(z) = 1 + 3z^{-1} + 6z^{-2} + 8z^{-3} + 8z^{-4} + 7z^{-5} + 2z^{-6}$$

$$x_1(n) * x_2(n) = \{\underset{\uparrow}{1}, 3, 6, 8, 8, 7, 2\}$$

$$X(z) = \sum_{n=-\infty}^{\infty} x(n)\, z^{-n}$$