1.c) Assuming the daily stats are poisson distributed with parameter λ.

$$f(X) = \frac{e^{-\lambda} \lambda^{X}}{X!}$$

An exponential prior is assumed on the parameters ~~prior~~ λ.

$$\beta \text{ (mean of prior)} = \lambda_{MME} \quad - (1)$$

W.K.T, for exponential distribution, the mean is ~~given~~ as follows -

$$\beta = E[X] = \frac{1}{\lambda}$$

Given $\beta = \lambda_{MME}$ from (1).

$$\therefore \lambda_{MME} = \frac{1}{\lambda} \quad \text{or} \quad \lambda_{prior} = \frac{1}{\lambda_{MME}}$$

Applying Bayesian Inference -

Posterior $\propto$ Likelihood $\times$ Prior

$$= \prod_{i=1}^{n} \left(\frac{e^{-\lambda} \lambda^{x_i}}{x_i!} \right) \times \left(\frac{e^{-\lambda / \lambda_{MME}}}{\lambda_{MME}} \right)$$

$$\propto \frac{e^{-n\lambda} \cdot \lambda^{\sum x_i}}{\prod_{i=1}^{n} x_i!} \times \frac{e^{-\lambda/\lambda_{MME}}}{\lambda_{MME}}$$

$$= c\left(e^{-\lambda\left(n+\frac{1}{\lambda_{MME}}\right)} \cdot \lambda^{\sum x_i}\right)$$

$$\left(\begin{array}{l}\lambda_{MME}, \text{ the first moment -} \\ E[X] = \bar{X} = \lambda_{MME}\end{array}\right)$$

$$= c\, e^{-\lambda\left(n+\frac{1}{\bar{X}}\right)} \cdot \lambda^{\sum x_i}$$

This is a gamma distribution

$$\text{Gamma}(\alpha, \beta) = \frac{x^{\alpha-1} e^{-\beta x} \beta^{\alpha}}{\Gamma(\alpha)}$$

$$\alpha = \sum_{i=1}^{n} x_i + 1 \qquad \beta = n + \frac{1}{\bar{X}} = n + \frac{1}{\lambda_{MME}}$$

The above is for 1^{st} week calculation.

For 2^{nd} week calculation:

$$\text{posterior} \propto \prod_{i=n+1}^{2n} \left(\frac{e^{-\lambda} \lambda^{X_i}}{X_i!} \right) \times c \left(e^{-\lambda\left(n + \frac{1}{\lambda_{MME}}\right)} \cdot \lambda^{\sum_{i=1}^{n} X_i} \right)$$

$$\propto \left(\frac{e^{-n\lambda} \lambda^{\sum_{i=n+1}^{2n} X_i}}{\prod_{i=n+1}^{2n} X_i!} \right) \times e^{-\lambda\left(n + \frac{1}{\lambda_{MME}}\right)} \times \lambda^{\sum_{i=1}^{n} X_i}$$

$$\propto e^{-\left(2n\lambda + \frac{\lambda}{\lambda_{MME}}\right)} \cdot \lambda^{\sum_{i=1}^{2n} X_i}$$

$$\propto e^{-\lambda\left(2n + \frac{1}{\lambda_{MME}}\right)} \cdot \lambda^{\sum_{i=1}^{2n} X_i}$$

This is gamma distribution with -

$$\alpha = \sum_{i=1}^{2n} X_i + 1, \quad \beta = 2n + \frac{1}{\lambda_{MME}}$$

Similarly for 3^{rd} week -

$$\alpha = \sum_{i=1}^{3n} X_i + 1, \quad \beta = 3n + \frac{1}{\lambda_{MME}}$$

Similarly for 4^{th} week -

$$\alpha = \sum_{i=1}^{4n} X_i + 1, \quad \beta = 4n + \frac{1}{\lambda_{MME}}$$