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MATLAB Assignment

Week 1:

Question:
Solve the matrix by using gaussian elimination:

x+2y+z=3,2x+y-2z=3,-3x+y+z=-6

Code:

```
C = [1 2 -1; 2 1 -2; -3 1 1]
b= [3 3 -6]' A = [C b]; n=
size(A,1); x = zeros(n,1);
for i=1:n-1
    for j=i+1:n
        m = A(j,i)/A(i,i)
        A(j,:) = A(j,:) -
m*A(i,:) end end
x(n) =
A(n,n+1)/A(n,n) for
i=n-1:-1:1 summ = 0
```

```
    for j=i+1:n
        summ = summ + A(i,j)*x(j,:)
    x(i,:) = (A(i,n+1) - summm)/A(i,i)
    end
end
```

Output:

```
C =

     1     2    -1
     2     1    -2
    -3     1     1

b =

     3
     3
    -6

m =
     2
A =
```

```
   1     2    -1 ------------------------------------------------- 2

   0    -3     0 ------------------------------------------------- 3

    -3     1     1 ----------------------------------------------- 5


m =

   -3

A =

     1     2    -1     3
     0    -3     0    -3
     0     7    -2     3

m =

  -2.3333
A =

     1     2    -1     3
     0    -3     0    -3
     0     0    -2    -4
```

```
x =

   0 0
     2

summm =

     0

summm =

     0

x =

     0
     1
     2

     summm

=

     0
```

```
summm =

     2

x =

     1
     1
     2

summ =

     0

x =

     3
     1
     2
```
—--

Practice Problems:

Question:

```
C = [1 1 1; 2 -6 -1; 3 4 2]

b= [11 0 0]
```

<u>Output:</u>

```
C =

     1     1     1
     2    -6    -1
     3     4     2

b =

    11
     0
     0

m =

     2
A =

     1     1     1    11
```

```
     0    -8    -3   -22
     3     4     2     0

m =

     3

A =

     1     1     1    11
     0    -8    -3   -22
     0     1    -1   -33

m =

   -0.1250

A =
    1.0000    1.0000    1.0000   11.0000
         0   -8.0000   -3.0000  -22.0000
         0         0   -1.3750  -35.7500
x =

     0
```

```
     0
    26

summm =

     0

summm =

-78 x =

     0
    -7 26
summm =

     0
summm =

    -7

x =

    18
```

```
    -7
    26

summm =

    19

x =

    -8
    -7
    26
```

<u>Question:</u>

```
C = [2 1 -1; 2 5 7; 1 1 1]
b= [0 52 9]
```

<u>Output:</u>

```
C =

     2     1    -1
     2     5     7
     1     1     1
```

```
b =

     0
    52
     9

m =

     1



A =

     2     1    -1     0
     0     4     8    52
     1     1     1     9
m =

    0.5000

A =

    2.0000    1.0000   -1.0000         0
         0    4.0000    8.0000   52.0000
```

```
         0    0.5000    1.5000    9.0000


m =


    0.1250


A =
    2.0000    1.0000   -1.0000         0
         0    4.0000    8.0000   52.0000
         0         0    0.5000    2.5000
x =


     0
     0
     5
summm =


     0


summm =


40 x =


     0
```

```
     3
     5

summm =

     0

summm =

     3
x =

   -1.5000
    3.0000
    5.0000

summm =

    -2

x =

     1
     3
```

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<u>Question:</u>

Find by Gauss-Jordan method:

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 4 & 3 & -1 \\ 3 & 5 & 3 \end{bmatrix}$$

<u>Code:</u>

```
A     = [1,1,1;4,3,-
1;3,5,3];n =
length(A(1,:)); Aug =
[A,eye(n,n)] for j = 1:n-1
for i = j+1:n
        Aug(i,j:2*n) = Aug(i,j:2*n) -
    Aug(i,j)/Aug(j,j)*Aug(j,j:2*n) end
end
for j = n:-1:2
    Aug(1:j-1,:) = Aug(1:j-1,:) - Aug(1:j-1,j)/Aug(j,j)*Aug(j,:)
end
for j = 1:n
    Aug(j,:) = Aug(j,:)/Aug(j,j)
end
```

B =

Aug(:,n+1:2*n)<u>Output:</u>

Aug =

```
     1     1     1     1     0     0
     4     3    -1     0     1     0
     3     5     3     0     0     1
```

Aug =

```
     1     1     1     1     0     0
     0    -1    -5    -4     1     0
     3     5     3     0     0     1
```

Aug =

```
     1     1     1     1     0     0
     0    -1    -5    -4     1     0
     0     2     0    -3     0     1
```

Aug =

```
     1     1     1     1     0     0
```

```
     0    -1    -5    -4     1     0
     0     0   -10   -11     2     1

Aug =

  Columns 1 through 5
    1.0000    1.0000         0   -0.1000    0.2000
         0   -1.0000         0    1.5000         0
         0         0  -10.0000  -11.0000    2.0000 Column 6

    0.1000
   -0.5000
    1.0000

Aug =

  Columns 1 through 5

    1.0000         0         0    1.4000    0.2000
         0   -1.0000         0    1.5000         0
         0         0  -10.0000  -11.0000    2.0000

  Column 6
```

```
   -0.4000
   -0.5000
    1.0000

Aug =
  Columns 1 through 5

    1.0000         0         0    1.4000    0.2000
         0   -1.0000         0    1.5000         0
         0         0  -10.0000  -11.0000    2.0000

  Column 6

   -0.4000
   -0.5000
    1.0000

Aug =

  Columns 1 through 5

    1.0000         0         0    1.4000    0.2000
         0    1.0000         0   -1.5000         0
         0         0  -10.0000  -11.0000    2.0000
```

```
  Column 6
   -0.4000
    0.5000
    1.0000
Aug =

  Columns 1 through 5

    1.0000         0         0    1.4000    0.2000
         0    1.0000         0   -1.5000         0
         0         0    1.0000    1.1000   -0.2000

  Column 6

   -0.4000
    0.5000
   -0.1000

B =

    1.4000    0.2000   -0.4000
   -1.5000         0    0.5000
    1.1000   -0.2000   -0.1000
```

—---

Practice Problems:

Question:

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 2 & 4 \\ 1 & 1 & 5 \end{bmatrix}$$

Output:

Aug =

1	2	3	1	0	0
1	2	4	0	1	0
1	1	5	0	0	1

Aug =

1	2	3	1	0	0
0	0	1	-1	1	0
1	1	5	0	0	1

Aug =

1	2	3	1	0	0
0	0	1	-1	1	0

```
     0    -1     2    -1     0     1

Aug =

     1     2     3     1     0     0
     0     0     1    -1     1     0
     0   NaN   Inf  -Inf   Inf   NaN

Aug =

     1   NaN   NaN   NaN   NaN   NaN
     0   NaN   NaN   NaN   NaN   NaN
     0   NaN   Inf  -Inf   Inf   NaN

Aug =

   NaN   NaN   NaN   NaN   NaN   NaN
     0   NaN   NaN   NaN   NaN   NaN
     0   NaN   Inf  -Inf   Inf   NaN

Aug =

   NaN   NaN   NaN   NaN   NaN   NaN
     0   NaN   NaN   NaN   NaN   NaN
```

```
     0   NaN   Inf  -Inf   Inf   NaN
Aug =

   NaN   NaN   NaN   NaN   NaN   NaN
   NaN   NaN   NaN   NaN   NaN   NaN
     0   NaN   Inf  -Inf   Inf   NaN

Aug =

   NaN   NaN   NaN   NaN   NaN   NaN
   NaN   NaN   NaN   NaN   NaN   NaN
     0   NaN   NaN   NaN   NaN   NaN

B =

   NaN   NaN   NaN
   NaN   NaN   NaN
   NaN   NaN   NaN
```

—---

Question:

$$A = \begin{bmatrix} -1 & 2 & 6 \\ -1 & -2 & 4 \\ -1 & 1 & 5 \end{bmatrix}$$

<u>Output:</u>

Aug =

```
   -1     2     6     1     0     0
   -1    -2     4     0     1     0
   -1     1     5     0     0     1
```

Aug =

```
   -1     2     6     1     0     0
    0    -4    -2    -1     1     0
    1     1     5     0     0     1
```

Aug =

```
   -1     2     6     1     0     0
    0    -4    -2    -1     1     0
    0    -1    -1    -1     0     1
```

Aug =

```
   -1.0000    2.0000    6.0000    1.0000         0         0
         0   -4.0000   -2.0000   -1.0000    1.0000         0
         0         0   -0.5000   -0.7500   -0.2500    1.0000
```

Aug =

```
   -1.0000    2.0000         0   -8.0000   -3.0000   12.0000
         0   -4.0000         0    2.0000    2.0000   -4.0000
         0         0   -0.5000   -0.7500   -0.2500    1.0000
```

Aug =

```
   -1.0000         0         0   -7.0000   -2.0000   10.0000
         0   -4.0000         0    2.0000    2.0000   -4.0000
         0         0   -0.5000   -0.7500   -0.2500    1.0000
```

Aug =

```
    1.0000         0         0    7.0000    2.0000  -10.0000
         0   -4.0000         0    2.0000    2.0000   -4.0000
         0         0   -0.5000   -0.7500   -0.2500    1.0000
```

Aug =

```
    1.0000         0         0    7.0000    2.0000  -10.0000
```

```
         0    1.0000         0   -0.5000   -0.5000    1.0000
         0         0   -0.5000   -0.7500   -0.2500    1.0000
Aug =

    1.0000         0         0    7.0000    2.0000  -10.0000
         0    1.0000         0   -0.5000   -0.5000    1.0000
         0         0    1.0000    1.5000    0.5000   -2.0000

B =

    7.0000    2.0000  -10.0000
   -0.5000   -0.5000    1.0000
    1.5000    0.5000   -2.0000
```

—--

Question:

Find LU decomposition of A = [1, 1, -1; 3, 5, 6; 7, 8, 9]

Code:

```
Ab = [1 1 -1;3 5 6;7 8 9];
n= length(A); L =
```

eye(n); for i =2:3 alpha =
Ab(i,1)/Ab(1,1); L(i,1) =
alpha;
Ab(i,:) = Ab(i,:) -alpha*Ab(1,:);
end i=3;
alpha = Ab(i,2)/Ab(2,2);
L(i,2) = alpha
Ab(i,:) = Ab(i,:) -alpha*Ab(2,:);

U = Ab(1:n,1:n) <u>Output:</u>

```
L =

    1.0000         0         0
    3.0000    1.0000         0
    7.0000    0.5000    1.0000


U =

    1.0000    1.0000   -1.0000
         0    2.0000    9.0000
         0         0   11.5000
```

—---

<u>Practice Problems:</u>

Question:

$$A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 2 & 4 \\ 1 & 1 & 5 \end{bmatrix}$$

Output:

L =

1	0	0
1	1	0
1	0	1

U =

1	1	3
0	1	1
0	0	2

—---

$$A = \begin{bmatrix} -1 & 4 & 6 \\ 0 & -2 & 4 \\ 0 & 0 & 5 \end{bmatrix}$$

Question:

Output:

L =

```
     1     0     0
     0     1     0
     0     0     1
```

U =

```
    -1     4     6
     0    -2     4
     0     0     5
```

—---

Week 2:

Find the fundamental subspaces for the matrix A

= [2,3,4;4,3,8;1,3,2] Code:

```
A=[2,3,4;4,3,8;1,3,2]
[V,pivot] = rref(A) r
= length(pivot)
```

Question:

```
colspace = A(:,pivot)
nullspace = null(A,'r')
rowspace = V(1:r,:)'
leftns = null(A','r')
```

Output: A =

```
     2     3     4
     4     3     8
     1     3     2
V =

     1     0     2
     01     0
     00     0
```

pivot =

12 r =

2colspac

e =

2 3
4 3
13

nullspace =

-2
0
1

rowspace = 1 0
0 1
20 leftns =

-1.5000
0.5000
1.0000

—---

Week 3:

Question:

Find the projection of the matrix

$$A = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{pmatrix} \; ; x = \begin{pmatrix} u \\ v \end{pmatrix} \text{ and}$$

$$b = \begin{pmatrix} 1 \\ 3 \\ 4 \end{pmatrix}$$

Code:

```
A=[1,0;0,1;1,1]
b=[1;3;4] x =
```

lsqr(A,b)

Output:

A =

```
     1     0
     0     1
     1     1
```

b =

```
     1
     3
     4
```

lsqr converged at iteration 2 to a solution with relative residual 6.7e-17.

x =

```
    1.0000
    3.0000
```

—--

Question:

Find the projection for the matrix

$$A = \begin{pmatrix} 1 & 0 \\ 0 & 2 \\ 3 & 1 \end{pmatrix} ; x = \begin{pmatrix} u \\ v \end{pmatrix}$$

$$b = \begin{pmatrix} 1 \\ 0 \\ 4 \end{pmatrix}$$

Code:

```
A=[1,0;0,2;3,1]
b=[1;0;4] x =
lsqr(A,b)
```

Output:

```
A =

     1     0
     0     2
     3     1
```

b =

1
0
4

lsqr converged at iteration 2 to a solution with relative residual 0.076.

x =

1.2927
0.0244

Question:

Find a point on the plane x+y+z = 0 that is closest to (2,1,0)

Code:

```
syms c
P=[2,1,0]+c*[1,1,-1]
s=1*(c+2)+1*(c+1)-1*(-c)==0
```

```
s1=solve(s,c)
p=[2,1,0]+s1*[1,1,-1]
```

Output:

```
P =
[c + 2, c + 1, -c]
s = 3*c + 3

== 0 s1 = -1

p =

[1, 0, 1]
```

—---

Question:

Find a point on the plane 3x+4y+z=1 that is closest to (1,0,1)

Code:

```
syms c
P=[1,0,1]+c*[3,4,1]
s=3*(1+3*c)+4*(4*c)+(1+c)==1
```

s1=solve(s,c)
p=[1,0,1]+s1*[3,4,1]
<u>Output:</u>

P =

[3*c + 1, 4*c, c + 1]

s =

26*c + 4 == 1

s1 = -3/26

p =
[17/26, -6/13, 23/26]
—--
<u>Question:</u>

Let $u = \begin{bmatrix} 1 \\ 7 \end{bmatrix}$ onto $v = \begin{bmatrix} -4 \\ 2 \end{bmatrix}$ and find P, the matrix that will project any matrix onto the vector v. Use the result to find projection of v on u

Code:

```
u=[1;7] v=[-
4;2]
P=(v*transpose(v))/(transpose(v)*v)
P*u
```

Output:

```
u =

     1
     7

v =
    -4
     2

P =
    0.8000   -0.4000
   -0.4000    0.2000
ans =
    -2
     1
```

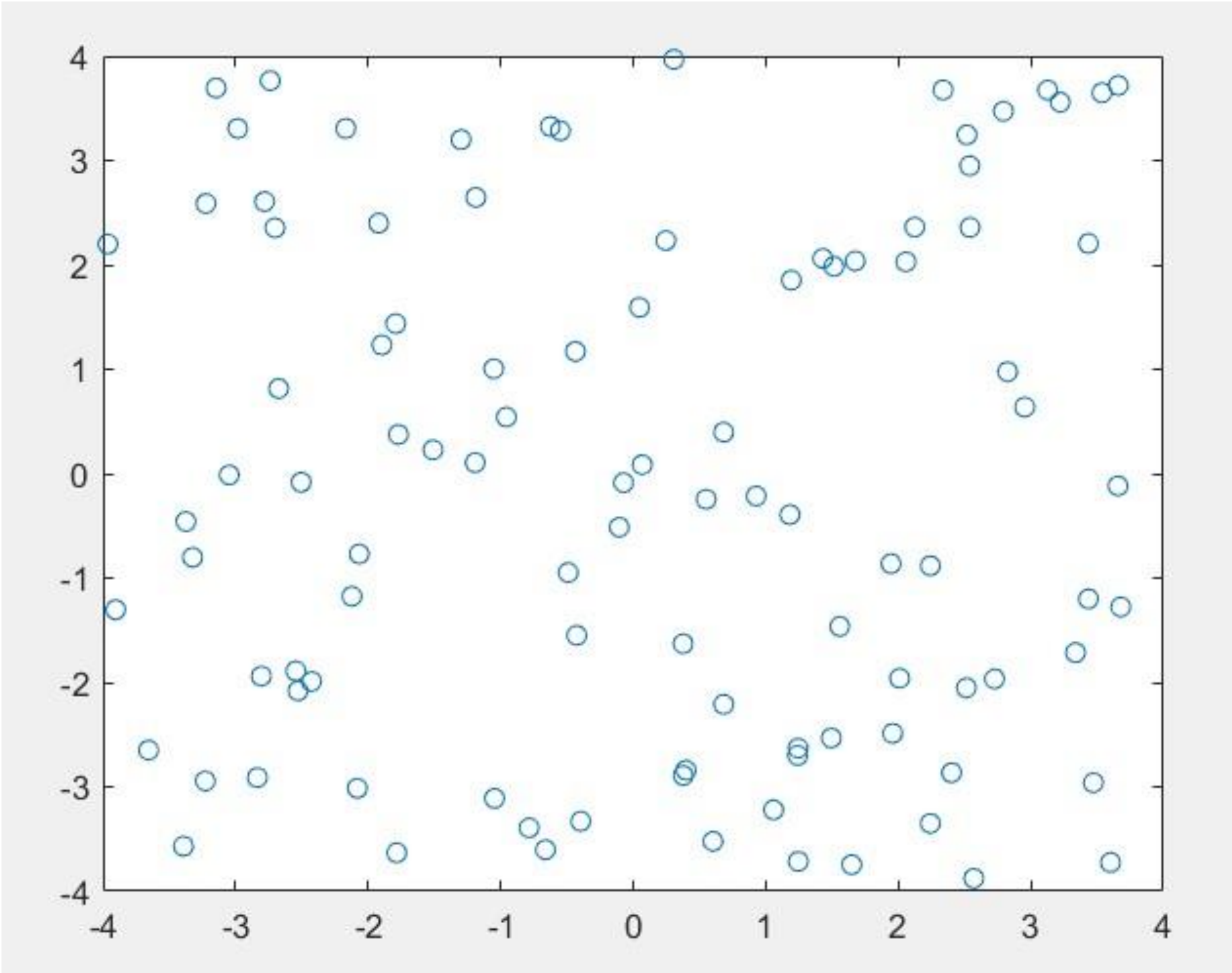
Question:

Projecting a lot of vectors on a single vector

Code:

```
u=8*rand(2,100)-4;
x=u(1,: ) y=u(2,: )
plot(x,y,'o')
```

Output:



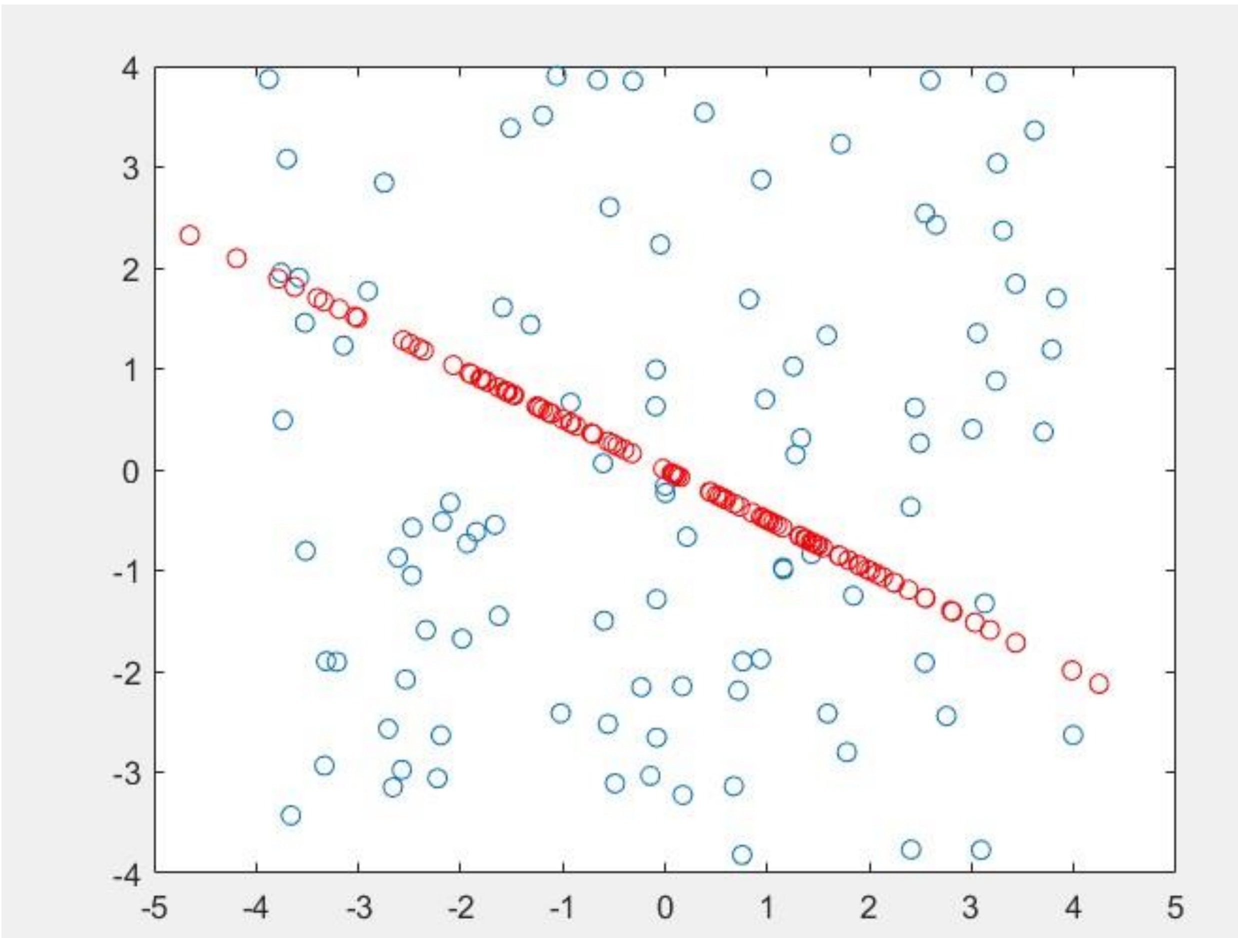
—---

Code 2:

P=[0.8,-0.4;-0.4,0.2]
Pu=P*u;
x=Pu(1,:)
y=Pu(2,:) hold on

plot(x,y,'ro')

<u>Output:</u>



—---

<u>Question:</u>

Find the least square fit for the system

$$x + 2y = 3$$
$$3x + 2y = 5$$
$$x + y = 2.09$$

Code:

```
A=[1,2;3,2;1,1]
b=[3;5;2.09] x
= lsqr(A,b)
```

Output:

```
A =

     1     2
     3     2
     1     1

b =

    3.0000
    5.0000
    2.0900
lsqr converged at iteration 2 to a solution with relative
residual 0.014.
```

x =

    1.0000
    1.0100

—--

<u>Question:</u>

Find the point on the plane 13x+4y+z=1 that is closest to (1,-1,1)

<u>Code:</u>

```
syms c
P=[1,-1,1]+c*[13,4,1]
s=13*(13*c+1)+4*(4*c-1)+1*(c+1)==0
s1=solve(s,c) p=[1,-1,1]+s1*[13,4,1]
```

<u>Output:</u>

P =

[13*c + 1, 4*c - 1, c + 1]

s =

186*c + 10 == 0

s1 = -5/93 p =

[28/93, -113/93, 88/93]

—--

Question:

Find the least square fit for this system

x + 2y + z = 3

3x + 2y - 2z = 5 x

+ y + 7z = 21.09

Code:

```
A=[1,2,1;3,2,-2;1,1,7]
b=[3;5;21.09] x =
lsqr(A,b)
```

Output:

A =

```
     1     2     1
     3     2    -2
```

```
     1     1     7

b =

    3.0000
    5.0000
   21.0900
```

lsqr converged at iteration 3 to a solution with relative residual 4.9e-15.

```
x =

    4.9497
   -2.2914
    2.6331
```

—---

Week 4:

Question:

Apply the Gram-Schmidt process to the vectors(1,0,1),

(1,0,0) and (2,1,0) to produce a set of orthonormal vectors

Code:

```
A=[1,1,2;0,0,1;1,0,0]
Q=zeros(3)
R=zeros(3)
for j=1:3
v=A(: , j)
for i=1:j-1
        R(i,j)=Q(:,i)'*A(:,j) v=v-
        R(i,j)*Q(:,i)
    end
    R(j,j)=norm(v)
    Q(:,j)=v/R(j,j)
end
```

Output:

```
A =

     1     1     2
     0     0     1
     1     0     0
```

Q =

```
     0     0     0
     0     0     0
     0     0     0
```

R =

```
     0     0     0
     0     0     0
     0     0     0
```

v =

```
     1
     0
     1
```

R =

```
    1.4142         0         0
         0         0         0
         0         0         0
```

Q =

```
    0.7071         0         0
         0         0         0
    0.7071         0         0

v =

     1
     0
     0

R =

    1.4142    0.7071         0
         0         0         0
         0         0         0
v =

    0.5000
         0
   -0.5000

R =

    1.4142    0.7071         0
```

```
         0    0.7071         0
         0         0         0

Q =

    0.7071    0.7071         0
         0         0         0
    0.7071   -0.7071         0

v =

     2
     1
     0
R =

    1.4142    0.7071    1.4142
         0    0.7071         0
         0         0         0

v =

    1.0000
    1.0000
```

```
   -1.0000


R =


    1.4142    0.7071    1.4142
         0    0.7071    1.4142
         0         0         0


v =


   -0.0000
    1.0000
    0.0000
R =


    1.4142    0.7071    1.4142
         0    0.7071    1.4142
         0         0    1.0000


Q =


    0.7071    0.7071   -0.0000
         0         0    1.0000
    0.7071   -0.7071    0.0000
```

—---

Question:

Apply the Gram-Schmidt process to the vectors a = (0,1,1,1), b = (1,1,-1,0) and c = (1,0,2,-1) Code:

```
A=[0,1,1;1,1,0;1,-1,2;1,0,-1]
Q=zeros(4,3)
R=zeros(3)
for j=1:3
v=A(: , j)
    for i=1:j-1
    R(i,j)=Q(:,i)'*A(:,j)
    v=v-R(i,j)*Q(:,i)
    end
    R(j,j)=norm(v)
    Q(:,j)=v/R(j,j)
end
```

Output:

```
A =

     0     1     1
     1     1     0
```

1	-1	2
1	0	-1

Q =

0	0	0
0	0	0
0	0	0
0	0	0

R =

0	0	0
0	0	0
0	0	0

v =

0
1
1
1

R =

1.7321	0	0
0	0	0
0	0	0

Q =

0	0	0
0.5774	0	0
0.5774	0	0
0.5774	0	0

v =

1
1
-1
0

R =

1.7321	0	0
0	0	0
0	0	0

v =

```
     1
     1
    -1
     0

R =
    1.7321         0         0
         0    1.7321         0
         0         0         0
Q =

         0    0.5774         0
    0.5774    0.5774         0
    0.5774   -0.5774         0
    0.5774         0         0

v =

     1
     0
     2
    -1

R =
```

1.7321	0	0.5774
0	1.7321	0
0	0	0

v =

1.0000
-0.3333
1.6667
-1.3333

R =

1.7321	0	0.5774
0	1.7321	-0.5774
0	0	0

v =

1.3333
0
1.3333
-1.3333

R =

1.7321	0	0.5774

```
        0    1.7321   -0.5774
        0         0    2.3094
```

Q =

```
        0    0.5774    0.5774
   0.5774    0.5774         0
   0.5774   -0.5774    0.5774
   0.5774         0   -0.5774
```

Question:

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

Find QR factorization of the matrix

Code:

```
A=[1,1,0;1,0,1;0,1,1];
[Q,R]=qr(A)
```

Output:

Q =

```
  -0.7071    0.4082   -0.5774
  -0.7071   -0.4082    0.5774
```

```
         0    0.8165    0.5774
```

R =

```
   -1.4142   -0.7071   -0.7071
         0    1.2247    0.4082
         0         0    1.1547
```

<u>Question:</u>

QR factorization of Pascal Matrix

<u>Code:</u>

```
A = sym(pascal(3))
[Q,R] = qr(A)
isAlways(A == Q*R)
```

<u>Output:</u>

A =

```
[1, 1, 1]
[1, 2, 3]
[1, 3, 6]
```

Q =

[3^(1/2)/3, -2^(1/2)/2, 6^(1/2)/6]
[3^(1/2)/3,   0, -6^(1/2)/3]
[3^(1/2)/3, 2^(1/2)/2, 6^(1/2)/6] R =

[3^(1/2), 2*3^(1/2), (10*3^(1/2))/3]
[0,   2^(1/2), (5*2^(1/2))/2]
[0,   0,   6^(1/2)/6]

ans =

  3×3 logical array

   1   1   1
   1   1   1
   1   1   1

—---

Question:

QR decomposition to solve matrix equation of the form Ax = b

Code:

```
A = sym(invhilb(5))
b = sym([1:5]')
[C,R] = qr(A,b); X =
R\C isAlways(A*X
== b)
```

<u>Output:</u>

```
A =

[   25,   -300,    1050,   -1400,    630]
[ -300,   4800,  -18900,   26880, -12600]
[ 1050, -18900,   79380, -117600,  56700]
[-1400,  26880, -117600,  179200, -88200]

[  630, -12600,   56700,  -88200,  44100] b

=

1
2
3
4
5

X =
        5
```

```
   71/20
  197/70
 657/280
1271/630
ans =

  5×1 logical array

   1
   1
   1
   1
   1
```

—---

Practice Problems:

Question:

Find QR decomposition (Gram Schmidt Method) using MATLAB command

$$\begin{bmatrix} 1 & -1 & 4 \\ 1 & 4 & -2 \\ 1 & 4 & 2 \\ 1 & -1 & 0 \end{bmatrix}$$

Code:

A = [1,-1 4;1 4 -2;1 4 2;1 -1
0] b = sym([1:4]') [C,R] =
qr(A,b); X = R\C
isAlways(A*X == b) Output:

A =

```
    1    -1     4
    1     4    -2
    1     4     2
    1    -1     0
```

b =

1
2

```
3
4
Warning: Solution does not exist because the system is
inconsistent.
> In symengine
In sym/privBinaryOp (line 1136)
In  \  (line 497)
In QR_decomposition (line 4)

X =

Inf
Inf

Inf

ans =

  4×1 logical array

   0
   0
   0
   0
```

—--

<u>Question:</u>

Find QR decomposition (Gram Schmidt Method) using MATLAB command

$$\begin{bmatrix} 3 & 2 & 4 \\ 2 & 0 & 2 \\ 4 & 2 & 3 \end{bmatrix}$$

<u>Output:</u>

A =

```
     3     2     4
     2     0     2
     4     2     3
```

b =

1
2
3
X =

3/2
-3/4 -

1/2 ans

=

3×1 logical array

1
1
1

—--

Question:

Find QR decomposition (Gram Schmidt Method) using MATLAB command

$$\begin{bmatrix} 1 & 2 & 4 \\ 3 & 8 & 9 \\ 5 & 7 & 3 \end{bmatrix}$$

Output:

A =

```
     1     2     4
     3     8     9
     5     7     3
```

b =

```
1
2
3
```

X =

```
 37/43
-11/43
  7/43
```

ans =

```
  3×1 logical array 1
   1
   1
```