IMPERIAL COLLEGE OF SCIENCE, TECHNOLOGY AND MEDICINE

EXAMINATIONS 2017

BEng Honours Degree in Computing Part I
MEng Honours Degrees in Computing Part I
for Internal Students of the Imperial College of Science, Technology and Medicine

This paper is also taken for the relevant examinations for the Associateship of the City and Guilds of London Institute

PAPER C145

MATHEMATICAL METHODS

Thursday 18 May 2017, 10:00
Duration: 80 minutes

Answer ALL TWO questions

Paper contains 2 questions
Calculators required

Always provide justifications and show any intermediate work for your answers. A correct but unsupported answer may not receive any marks. Pay attention to mathematical formalism.

1 a Consider the sequence

$$a_n = \frac{(-1)^n n}{e^n}.$$

Determine (using any appropriate method) whether the sequence converges, and if so, to what value. Show your work.

b Use an appropriate test to determine whether the series

$$S = \sum_{n=1}^{\infty} \frac{1+n!}{(1+n)!}$$

converges. Explain which test you use and show your work.

c Compute the Maclaurin series for the following function and find its radius of convergence:

$$f(x) = \ln \frac{1+x}{1-x}.$$

Hint: Simplify the equation.

d Compute the value of the series

$$S = \sum_{n=1}^{\infty} \frac{n}{3^n}.$$

Hint: Manipulate the formula for geometric series $\sum_{n=0}^{\infty} x^n$.

The four parts carry equal marks.

2a i) Show that $(\mathbb{R}\backslash\{-1\}, \star)$ is an Abelian group, where

$$a \star b := ab + a + b, \qquad a, b \in \mathbb{R}\backslash\{-1\} \tag{1}$$

ii) Find all solutions x with

$$3 \star x \star x = 15$$

in the Abelian group $(\mathbb{R}\backslash\{-1\}, \star)$, where $\star$ is defined in (1).

b Consider an endomorphism $\Phi : \mathbb{R}^4 \to \mathbb{R}^4$ with transformation matrix

$$\boldsymbol{A} = \begin{bmatrix} 0 & 4 & -8 & 4 \\ 3 & 1 & 1 & 0 \\ 2 & -2 & 6 & -2 \\ 1 & -1 & 3 & 0 \end{bmatrix}$$

and a vector $\boldsymbol{x} = [1, 0, 0, 0]^\top$.

i) Compute all eigenvalues of $\boldsymbol{A}$

ii) Determine a basis change matrix $\boldsymbol{S}$ and a matrix $\boldsymbol{D}$, such that $\boldsymbol{D} = \boldsymbol{S}^{-1}\boldsymbol{A}\boldsymbol{S}$ is diagonal and $d_{11} \geq d_{22} \geq d_{33} \geq d_{44}$.

iii) Using the standard scalar product in $\mathbb{R}^4$, determine the distance of $\boldsymbol{x}$ from its orthogonal projection $\pi_U(\boldsymbol{x})$ onto the subspace U spanned by the eigenvector associated with the greatest eigenvalue of $\boldsymbol{A}$.

iv) Consider an affine subspace $T = \boldsymbol{p} + U \subset \mathbb{R}^2, \dim(U) = 1$. How would you project a point $\boldsymbol{x} \in \mathbb{R}^2$ onto the affine subspace? Briefly describe your approach (no computations required).
Hint: Instead of trying to solve this directly, it may be easier to reduce the problem to something that you know better. A diagram may be helpful.

The two parts carry, respectively, 40% and 60% of the marks.