IMPERIAL COLLEGE OF SCIENCE, TECHNOLOGY AND MEDICINE

EXAMINATIONS 2016

BEng Honours Degree in Computing Part I
MEng Honours Degrees in Computing Part I
for Internal Students of the Imperial College of Science, Technology and Medicine

This paper is also taken for the relevant examinations for the Associateship of the City and Guilds of London Institute

PAPER C142

DISCRETE MATHEMATICS

Friday 13 May 2016, 10:00
Duration: 80 minutes

Answer ALL TWO questions

Paper contains 2 questions
Calculators not required

1 a Let $A = \{2,4,5,7\}$ and $B = \{2,3,7\}$. Write down explicit sets for

i) $A \cup B$ and $A \cap B$;

ii) $A \setminus B$ and $B \setminus A$;

iii) $A \triangle B$;

iv) $A \times \phi$ and $A \times (B \setminus A)$;

b Let R be a binary relation on A.

i) State the formal property that R should satisfy in order to be called: reflexive, symmetric, or transitive.

Show that:

ii) R is reflexive if and only if $id_A \subseteq R$.

iii) R is symmetric if and only if $R = R^{-1}$.

iv) R is transitive if and only if $R \circ R \subseteq R$.

c Let $f : A \to B$ and $g : B \to C$ be total functions.

i) State the formal property that f should satisfy in order to be called: injective, surjective, or bijective.

ii) Give a specific example of f and g such that $g \circ f$ is one-to-one but g is not.

iii) Prove that if $g \circ f$ is onto then so is g.

d Take $A = \{1,2,3,4\}$, and let $\leq_L$ denote the lexicographic order on A^2 that is the natural extension of $\leq_A$ on A, defined by $\{\langle 1,2\rangle, \langle 2,3\rangle, \langle 3,4\rangle\}$..

i) Define what property a relation $\leq_L$ should satisfy in order to be a lexicographic order on A with respect to $\leq_A$.

ii) Find all pairs in A^2 which are less than $\langle 2,3\rangle$ with respect to $\leq_L$.

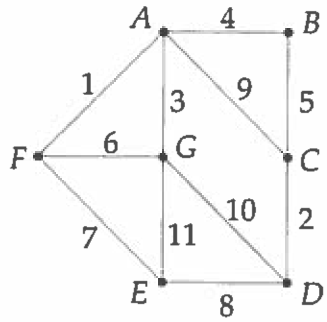
iii) Find all pairs in A^2 which are greater than $\langle 3,1\rangle$ with respect to $\leq_L$.

iv) Draw the Hasse diagram of the order $(A^2, \leq_L)$.

The four parts carry, respectively, 20%, 40%, 20%, and 20% of the marks.

2 a i) Use Kruskal's algorithm to find a minimum spanning tree (MST) for the following weighted graph.

Give the MST as a diagram and also state the order in which the arcs are added.



ii) Does the graph from part (i) have a unique MST?

Justify your answer briefly.

b Show (by induction or otherwise) that for any $n \geq 1$, if an (unweighted) graph G has n nodes and at least n arcs then G has a cycle.

c An arc a of a connected graph is said to be a *bridge* if removing a would disconnect the graph, i.e. the graph with a removed (but retaining all nodes and all other arcs) is disconnected.

i) Give a example of a connected graph with four nodes and no bridges.

ii) Give an example of a connected graph with four nodes and exactly one bridge.

iii) Let G be a connected graph and let a be an arc of G with endpoints x and y. Show that if a is a bridge then a does not belong to any cycle of G.

iv) Let G be a connected graph with n nodes ($n \geq 1$). Show that G has no more than $n - 1$ bridges.

The three parts carry, respectively, 35%, 25%, and 40% of the marks.

