COMP245: Probability and Statistics 2016 - Problem Sheet 3

Probability

Q1) For two events E and F, show that

$$\mathrm{P}(E \cup F) = \mathrm{P}(E) + \mathrm{P}(F) - \mathrm{P}(E \cap F).$$

Q2) Suppose two events E and F are mutually exclusive. State the precise conditions under which they may also be independent.

Q3) What is the probability that a single roll of a die will give an odd number if

(a) no other information is given;

(b) you are told that the number is less than 4.

Q4) (a) What's the probability of getting two sixes with two dice?

(b) What's the probability of getting a total of 3 with two dice?

Q5) Two students try to solve a problem they've been set. Student A has a probability of $\frac{2}{5}$ of being able to solve the problem, and student B has a probability of $\frac{1}{3}$. If both try it independently, what is the probability that the problem is solved?

Q6) A straight AB line of unit length is divided internally at a point X, where X is equally likely to be any point of AB. What is the probability that $AX.XB < \frac{3}{16}$?

Q7) (a) In one spin of a European roulette wheel (which has pockets numbered 0, 1, 2, up to and including 36) what is the probability that the outcome is odd?

(b) An urn contains x red balls and y green ones (both larger than 2). You remove them, without replacing them, one at a time.

i. What is the chance that the first is red?

ii. What is the chance that the second is red?

iii. What is the chance that the first two are red?

iv. What is the chance that the last but one is red?

Q8) (a) An experiment consists of tossing a fair coin and rolling a fair die. What is the probability of the joint event 'heads with an odd number of spots'?

(b) In a particular class, 30% were female, and 90% of the males and 80% of the females passed the examination. What percentage of the class passed the examination altogether?

Q9) On any day the chance of rain is 25%. The chance of rain on two consecutive days is 10%.

(a) Does this mean that the events of rain on two consecutive days are independent or dependent events?

(b) Given that it is raining today, what is the chance of rain tomorrow?

(c) Given that it will rain tomorrow, what is the chance of rain today?

Q10) A university lecturer leaves his umbrella behind with probability $\frac{1}{4}$ every time he visits a shop (and, once he has left it, he does not collect it again).

(a) If he sets out with his umbrella to visit four different shops, what is the probability that he will leave it in the fourth shop?

(b) If he arrives home without his umbrella, what is the probability that he left it in the fourth shop?

(c) If he arrives home without it, and was seen to be carrying it after leaving the first shop, what is the probability that he left it in the fourth shop?

Q11) A warehouse contains packs of electronic components. Forty percent of the packs contain components of low quality for which the probability that any given component will prove satisfactory is 0.8; forty percent contain components of medium quality for which this probability is 0.9; and the remaining twenty percent contain high quality components which are certain to be satisfactory.

(a) If a pack is chosen at random and one component from it is tested, what is the probability that this component is satisfactory?

(b) If a pack is chosen at random and two components from it are tested, what is the probability that exactly one of the components tested is satisfactory?

(c) If it was found that just one of the components tested was satisfactory, what is the probability that the selected pack contained medium quality components?

(d) If both components were found to be satisfactory, what is the probability that the selected pack contained high quality components?

Q12) Prove that if $\mathrm{P}(A) > \mathrm{P}(B)$ then $\mathrm{P}(A|B) > \mathrm{P}(B|A)$.