Question 1

part (a):

$$(\phi \rightarrow \psi) \land \neg(\neg p \lor \psi)$$

$\equiv (\neg\phi \lor \psi) \land \neg(\neg p \lor \psi)$ $\quad (A \rightarrow B \equiv \neg A \lor B)$

$\equiv (\neg\phi \lor \psi) \land (\neg\neg p \land \neg\psi)$ $\quad$ (De Morgan's law)

$\equiv (\neg\phi \lor \psi) \land (p \land \neg\psi)$ $\quad (\neg\neg A \equiv A)$

$\equiv (\neg\phi \land (p \land \neg\psi)) \lor (\psi \land (p \land \neg\psi))$ $\quad$ (Distributivity of $\land$ over $\lor$)

$\equiv (\neg\phi \land p \land \neg\psi) \lor (\psi \land p \land \neg\psi)$ $\quad$ (Associativity of $\land$)

$\equiv (\neg\phi \land p \land \neg\psi) \lor (\psi \land \neg\psi \land p)$ $\quad$ (com $A \land B \equiv B \land A$)

$\equiv (\neg\phi \land p \land \neg\psi) \lor (\bot \land p)$ $\quad (\neg A \land A \equiv \bot)$

$\equiv (\neg\phi \land p \land \neg\psi) \lor \bot$ $\quad (\bot \land A \equiv \bot)$

$\equiv (\neg\phi \land p \land \neg\psi)$ $\quad (A \lor \bot \equiv A)$

part (b): It is sound.

Informally, $\neg\land I$ is just $\lor I$ but with De Morgan's Law applied to it. i.e.

$$\neg(\neg A \land \neg B) \equiv A \lor B.$$

we could also show it is a derived rule.

Q1. part (c):

~~i) ∀~~

(i) "x is smallest value in xs"
becomes

$\forall y : Nat\ \forall i : Nat\ ((xs\,!!\,i) = y \rightarrow x \leq y)$

(ii) "the value x occurs at least twice in xs"
becomes

$\exists i : Nat\ \exists j : Nat\ [\neg(i = j) \wedge ((xs\,!!\,i) = x) \wedge ((xs\,!!\,j) = x)]$

~~(iii) "All values in xs occur at least twice in ys"~~

$\cancel{\forall x : Nat\ \forall i : Nat\ [(xs\,!!\,i) = x \longrightarrow \exists a : Nat\ \exists b : Nat\ [\neg(a = b) \wedge ((ys\,!!\,a)}$

(iii) "All values in xs occur at least twice in ys"
becomes

$\forall x : Nat\ \forall i : Nat\ [(xs\,!!\,i) = x \rightarrow twice(x, ys)]$

(iv) "There are exactly two values that are repeated in xs"

$\exists x : Nat\ \exists y : Nat\ [\neg(x = y) \wedge twice(x, xs) \wedge twice(y, xs)$
$\wedge (\forall z : Nat\ [twice(z, xs) \longrightarrow (z = x) \vee (z = y)])$
$]$