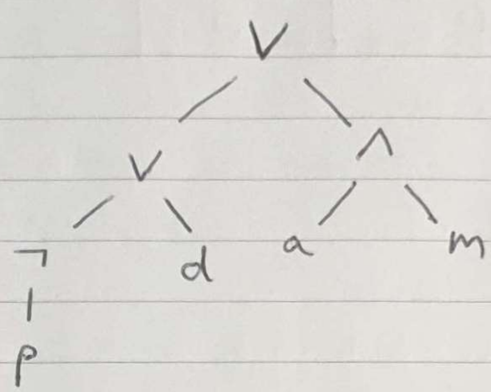
Logic

2a) i) (A)

a	means card is activated
m	means there is money in the account
p	means pin code is requested
d	means payment is declined

$(\neg p \vee d) \vee (a \wedge m)$

(B)



Subformulas:

- p
- d
- a
- m
- $\neg p$
- $\neg p \vee d$
- $a \wedge m$
- $(\neg p \vee d) \vee (a \wedge m)$

(C) $|p|_v = ff$, $|d|_v = tt$, $|a|_v = ff$, $|m|_v = ff$

This causes the formula to be true as $v(\neg p) = tt$ and so $v(\neg p \vee d) = tt$. By definition of $\vee$ we then have $v((\neg p \vee d) \vee (a \wedge m)) = tt$.

ii)

P	Q	R	$P \to \neg(Q \vee R)$	$R \to \neg P$	$(P \to \neg(Q \vee R)) \vee (R \to \neg P)$
0	0	0	1	1	1
0	0	1	1	1	1
0	1	0	1	1	1
0	1	1	1	1	1
1	0	0	1	1	1
1	0	1	0	0	0
1	1	0	0	1	1
1	1	1	0	0	0

Clauses

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$\neg P \vee Q \vee \neg R$

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$\neg P \vee \neg Q \vee \neg R$

So the formula in CNF is:

$$(\neg P \vee Q \vee \neg R) \wedge (\neg P \vee \neg Q \vee \neg R)$$

iii) First assume that $\vDash \phi \leftrightarrow \psi$ holds. So in an arbitrary situation either ϕ and ψ are both true or both false. So if we take the LHS of the disjunction we have $\phi \wedge \neg\psi$ which evaluates to false as the negation of ψ means we have $\top \wedge \bot$ or $\bot \wedge \top$. By a similar argument, the RHS of the disjunction evaluates to false. So the formula is false by definition of $\vee$. Since this is an arbitrary situation we therefore have that the formula is always false, therefore unsatisfiable.

In the other direction, assuming $(\emptyset \wedge \neg\psi) \vee (\neg\emptyset \wedge \psi)$ is unsatisfiable, this means the formula always evaluates to false. Therefore we have that on the LHS, which evaluates to false, $\emptyset \wedge \neg\psi$ so we have both $\emptyset$ is false and $\neg\psi$ is true. If $\neg\psi$ was false then on the RHS we would have the conjunction evaluate to true which isn't possible since I have assumed that the formula is unsatisfiable. Therefore $\emptyset$ is false and ψ false or $\emptyset$ is true and ψ is true. This is the same assignment as in the other direction so $\models \emptyset \leftrightarrow \psi$ holds.

b) i) (A)

i) This says every circle that doesn't have an outgoing arrow or every circle that doesn't have an incoming arrow, so:

$x = \{6, 3, 1\}$

Using De Morgan's this means every circle that doesn't have an incoming and outgoing arrow, so only 6, 3 and 1 satisfy this.

ii) This says all black circles that have an outgoing arrow, which have at most 2 outgoing arrows.

So $x = \{6\}$.

The other black circles have more than 2 outgoing arrows.

iii) This is saying that there are at least two outgoing arrows that connect to white circles from x.

So $x = \{4, 2, 6\}$

As these have outgoing arrows that connect to at least two white circles.

(B) ~~The sentence says that all circles~~ For the formula to hold we must have $F(x)$ is true for all x or the LHS of the implication to be false. Therefore take different cases.

Take $x = 1$: x has no out going arrows so $R(x, y)$ is false meaning the formula is true

Take $x = 2$: