FINAL

IMPERIAL COLLEGE OF SCIENCE, TECHNOLOGY AND MEDICINE

EXAMINATIONS 2017

BEng Honours Degree in Computing Part I
MEng Honours Degrees in Computing Part I
for Internal Students of the Imperial College of Science, Technology and Medicine

This paper is also taken for the relevant examinations for the Associateship of the City and Guilds of London Institute

PAPER C142

DISCRETE MATHEMATICS

Tuesday 16 May 2017, 10:00
Duration: 80 minutes

Answer ALL TWO questions

Paper contains 2 questions
Calculators not required

1 a Let $A = \{1,4,5,7\}$ and $B = \{1,3,7\}$. Write down explicit sets for

i) $A \cup B$ and $A \cap B$;

ii) $A \setminus B$ and $B \setminus A$;

iii) $A \triangle B$;

iv) $A \times \phi$ and $A \times (B \setminus A)$;

b i) Let R be a binary relation on A. State the three formal properties that R should satisfy in order to be called, respectively, reflexive, symmetric, or transitive.

ii) Give examples of relations on $\{1,2,3,4\}$ having the following properties:

A) reflexive, symmetric, not transitive.

B) reflexive, not symmetric, not transitive.

C) symmetric, transitive, not reflexive.

and explain your answers briefly.

c i) Let $(A, \leq)$ be a partial order and $a \in A$. Give the formal definition for the notions *minimal*, *least*, *maximal*, respectively *greatest* element of A.

ii) Consider the set $F \triangleq \{2,3,4,5,6,10,12,15,20,30,60\}$ and consider the relation $<$ on F defined by: $n < m \triangleq \exists k \in \mathbb{N} \setminus \{1\} \, (k \times n = m)$.

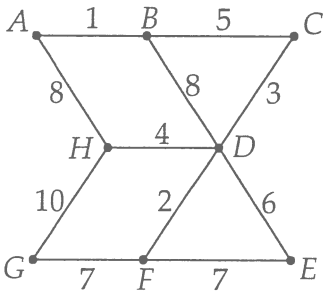
Give the Hasse diagram for $\langle F, < \rangle$.

iii) Does $\langle F, < \rangle$ have minimal, least, maximal, or greatest elements? If yes, state which element(s), and show why each satisfies the criterion.

d Give the definition of the relation $\sim$ between sets. Show that $\{0,1\}^V \sim \wp(V)$, for any set V.

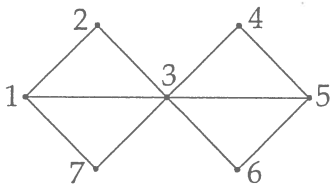
The four parts carry, respectively, 20%, 30%, 30%, and 20% of the marks.

2a i) Use Prim's algorithm to find a minimum spanning tree (MST) for the following weighted graph. Give the MST in the form of a diagram. Also state the order in which the nodes are added to the MST.



ii) Does the graph from part (i) have a *unique* MST? Explain your answer.

b Use depth-first search starting from node 3 in the following graph to obtain a spanning tree. Assume that where there is a choice the numerically least node is chosen. Give your answer in the form of a diagram. Also state the order in which the nodes are visited.



c A node x in a connected simple undirected graph is said to be an *articulation point* if its removal (together with all arcs incident on x) causes the resulting graph to be disconnected.

i) Which nodes of the graph in part (b) are articulation points?

ii) Give an example of a connected simple graph with five nodes and exactly two articulation points. Indicate on your diagram which nodes are the articulation points.

iii) Let G be any connected simple graph. Suppose that we traverse G using depth-first search (DFS) to obtain a spanning tree T. Show that if y is a leaf node of T (i.e. a node of degree one in T) then y is not an articulation point of G.

iv) Suppose that the DFS in part (iii) starts at node x (the root of T). Show that x is an articulation point iff it has degree ≥ 2 in T.

The three parts carry, respectively, 35%, 20%, and 45% of the marks.