

COMP245: Probability and Statistics 2016 - Problem Sheet 5

Discrete Random Variables

- Q1) An experiment involves tossing two unbiased coins.
- (a) What is the sample space of this experiment?
 - (b) What is the probability mass function of the random variable X , which takes value 2 if two heads show, 1 if one head shows, and 0 if no heads show?
 - (c) What is the probability mass function of the random variable Y , which takes the value 3 if at least one head shows and 1 if no head shows?
- Q2) Suppose that two fair dice are thrown and define a random variable X as the total number of spots showing. Make a table showing the probability mass function, $p(x)$ of X and plot a graph of $p(x)$.
- Q3) In tossing a fair coin four times, what is the probability that one will obtain
- (a) four heads;
 - (b) three heads;
 - (c) at least two heads;
 - (d) not more than one head?
- Q4) An urn holds 5 white and 3 black marbles.
- (a) If two marbles are drawn at random without replacement and X denotes the number of white marbles
 - i. find the probability mass function of X , and
 - ii. plot the cumulative distribution function of X .
 - (b) Repeat 4a if the marbles are drawn with replacement.
- Q5) The probability that a student will pass a particular course is 0.4. Find the probability that, out of 5 students
- (a) none pass;
 - (b) one passes;
 - (c) at least one passes.
- Q6) (a) If each student in a class of 110 has the same probability, 0.8, of passing an examination, what is

- i. the expected number of passes?
 - ii. the standard deviation of the number of passes?
 - (b) If each student in a college of 11000 has the same probability of graduating, what is
 - i. the expected number of graduates?
 - ii. the standard deviation of the number of graduates?
- Q7) An insurance salesman sells policies to 5 computer companies. The probability that each of these companies will make a claim over the next five years is $\frac{1}{5}$. Find the probability that, over the next five years
- (a) all companies will claim;
 - (b) at least three companies will claim;
 - (c) only two will claim;
 - (d) at least one will not claim.
- Q8) Compute the mean, sd, and the skewness for the following binomial distributions, and comment on the results:
- (a) Binomial(100, 0.9);
 - (b) Binomial(100, 0.7);
 - (c) Binomial(100, 0.5);
 - (d) Binomial(1000, 0.9);
 - (e) Binomial(1000, 0.7);
 - (f) Binomial(1000, 0.5).
- Q9) In a class of 20 students taking an examination,
- 2 have probability 0.4 of passing;
 - 4 have probability 0.6 of passing;
 - 5 have probability 0.7 of passing;
 - 7 have probability 0.8 of passing;
 - 2 have probability 0.9 of passing.
- (a) What is the expected number of passes?
 - (b) What is the standard deviation of the number of passes?
- Q10) (Geometric distribution.)
- A computer class has a limited number of terminals available for use. A student notices that, on average, there is a 0.4 chance that there will be a free terminal each time he tries to use a machine.
- (a) What is the average number of times he will have to try use a machine until he is successful?

- (b) What is his chance of being successful the first time he tries?
- (c) What is his probability of being successful the first time on each of three different occasions?

- Q11) (a) What is the mean and variance of a sum of n independent Bernoulli random variables, each with parameter p ?
- (b) What if they have different parameters, (p_1, p_2, \dots, p_n) ?
 - (c) What can you say if I now tell you that they are not independent?

- Q12) The random variable N takes values in the non-negative integers. Show that N has expectation satisfying

$$E(N) = \sum_{j=0}^{\infty} P(N > j).$$