bi. $\text{valid}(f) \iff \int_{-\infty}^{\infty} f = 1 \quad \wedge \quad f[-\infty, \infty] = 1$

$\forall k \in \mathbb{N} \,.\, \forall x > 0 \,.\, x^{-k-1} > 0$ satisfies right conjunct

$$\int_a^b C(k)\, x^{-k-1}\, dx \,.$$

$$= -\frac{C(k)}{k}\left[\; x^{-k} \;\right]_a^b$$

$$= \frac{-C(k)}{k}\left(b^{-k} - a^{-k} \right) = 1$$

$$C(k) = - \frac{k}{b^{-k} - a^{-k}}$$

ii. $C(1) = \dfrac{1}{a-b}$

$$c^2 = \frac{Var(X)}{E(\alpha)^2}$$

$$= \frac{E(X^2) - E(X)^2}{E(X)^2}$$

$$= \frac{E(X^2)}{E(X)^2} - 1$$

$$E(X) = \int_a^b x \cdot \left(C(1)\, x^{-2}\right) dx$$

$$= C(1) \int_a^b x^{-1}\, dx$$

$$= C(1) \left[\ln|x| \right]_a^b$$

$$= \frac{1}{a-b}\left(\ln|b| - \ln|a|\right)$$

$$= C(1) \quad \ln b/a$$

$$\mathbb{E}(X^2) = C(1)\int_a^b x^2 \, x^{-2} \, dx$$

$$= \frac{1}{a-b}\,(b-a) = -1$$

$$c^2 = \frac{-1}{C(1)^2 (\ln b/a)^2} \quad -1$$

$$= \frac{-(a-b)^2}{(\ln b/a)^2} \quad -1$$

$$= -\left(\frac{(a-b)}{\ln b/a}\right)^2 \quad -1$$

iii. $k=1$ $\quad b=4a$

$$E[X \mid X > 2a] = \frac{1}{a-b}\int_{2a}^{4a} x^{-1} \, dx$$

$$= \frac{1}{a-b}\left(\ln 4a - \ln 2a\right)$$

$$= \frac{\ln 2}{a-b}$$