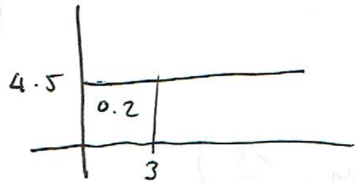
Statistics 2012-2013

1 i)



$0.2 \times 5 = 1$

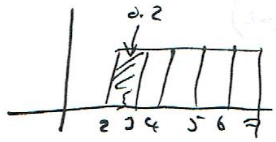
$\Rightarrow$ there are 5 'items'

$\frac{2+7}{2} = 4.5$

$7-2=5$

$\frac{1}{5} = 0.2$

$P(X<3) = 0.2$



d

ii)

Observed (O)

	RH	LH	
RF	148	43	191
LF	40	21	61
	188	64	252

Expected (E)

	RH	LH	
RF	$\frac{188 \times 191}{252} = 142.49$	$\frac{64 \times 191}{252} = 48.51$	191
LF	$\frac{188 \times 61}{252} = 45.51$	$\frac{64 \times 61}{252} = 15.49$	61
	188	64	252

	$O - E_i$	$(O-E_i)^2/E_i$
RF, RH	5.51	0.213
RF, LH	-5.51	0.626
LF, RH	-5.51	0.667
LF, LH	5.51	1.96
	χ^2	3.50

a

iii)

10%	20%	70%
E	G	O/W
DDos 10%	25% DDos	20% DDos

Total DDos $= 0.1 \times 0.1 + 0.2 \times 0.25 + 0.7 \times 0.2 = 0.2$

E DDos $= 0.1 \times 0.1 = 0.01$

$\Rightarrow \frac{0.01}{0.2} = 0.05 \Rightarrow$ a

iv) $P(A\_six \geq 1) = 1 - P(\neg A\_six)$

$= 1 - \left(\frac{5}{6}\right)^6$

$= 0.67$ <u>d</u>

v) Since they are <u>identically</u> distributed

$X \sim N(n\mu, n\sigma^2)$ $X \sim N(\mu, \frac{\sigma^2}{n})$

↖ when not identically distributed.

<u>b</u>

2 i a) $E(X) = \frac{-11.57 + 3.43 + 13.99 + -8.45 + 37.06 + 10.34, -0.91 + 24.08 - 6.42 + 1.08 + \ldots}{12}$

$= 2.51$

$S^2_{n-1} = \frac{1}{n-1}\left(\sum x_i^2 - \frac{1}{n}(\sum x_i)^2\right)$

$= \frac{1}{11}\left(\frac{(-11.57^2 + 3.43^2 + \ldots (-18.03)^2}{} - \frac{1}{12}\left(-11.57 + 3.43 + \ldots + -18.03\right)^2\right)$

$= 202.5$

b) $\left(\bar{x} - t_{n-1, 0.975}\frac{\sigma}{\sqrt{n}}, \bar{x} + t_{n-1, 0.975}\frac{\sigma}{\sqrt{n}}\right)$

Need to estimate value from t table since value is not given to us

$\Rightarrow t_{11, 0.975}$

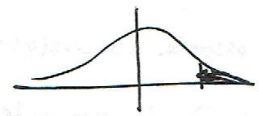
$\left(2.51 - 2.20\sqrt{\frac{202.5}{12}}, 2.51 + 2.20\sqrt{\frac{202.5}{12}}\right)$

roughly in the middle

$(-6.53, 11.55)$

c) No, since 0 is in our confidence interval.

ii) a) test statistic $\Rightarrow Z = \dfrac{\bar{x} - \mu}{\sigma}$



one sided test

$\Phi^{-1}(0.95) = 1.645$

$\Rightarrow 1.645 = \dfrac{\bar{x} - \mu}{1}$

$1.645 = \bar{x} - \mu$

$\therefore$ rejection region $\Rightarrow (1.645, \infty)$

b) $X \sim N(1,1)$

$Z = \bar{x} - 1 \sim N(0,1)$

$P(X \in R) = P(X > z_{0.95}) = P(Z > z_{0.95} - 1 = \Phi^{-1}(z_{0.95} - 1)$

probability x is in rejection region

$= \Phi^{-1}(1.645 - 1)$

$= \Phi^{-1}(0.645)$

$= 0.740$

Power of test $= 1 - 0.740$

$= 0.26$

3 i) $$\sum_{x=0}^{\infty} P_X(x) = \sum_{x=0}^{\infty} \frac{\lambda^x e^{-\lambda}}{x!}$$

$$= e^{-\lambda} \sum_{x=0}^{\infty} \frac{\lambda^x}{x!}$$

$$= e^{-\lambda} e^{\lambda}$$

$$= 1$$

ii) $$\frac{P_X(x)}{P_X(x-1)} = \frac{\lambda^x e^{-\lambda}}{x!} \Big/ \frac{\lambda^{x-1} e^{-\lambda}}{(x-1)!} = \frac{\lambda}{x}$$

$\frac{\lambda}{x} > 1$ iff $\lambda > x$ which is the case when $x \in \{1, 2, \ldots \lfloor \lambda \rfloor\}$ since $\lfloor \lambda \rfloor < \lambda$

iii) Mode - most common value.

Since $P_X(x)$ is non-decreasing in x until $x = \lfloor \lambda \rfloor$, and is decreasing thereafter, $\lfloor \lambda \rfloor$ will always provide a maximum of P_X

$P_X(1) = \dfrac{\lambda^1 e^{-\lambda}}{1!} \Rightarrow$ increasing x $1 \to \lfloor \lambda \rfloor$

$P_X(\lambda) = \dfrac{\lambda^{\lambda} e^{-\lambda}}{\lambda!} \Rightarrow$ decreasing.

iv) :/
The mode is unique when λ is not an integer

when λ is an integer, then both λ & $(\lambda-1)$ are maxima of p_X, since the ratio of their pmf values will be 1.

v) $$P(Z=z) = \sum_{x=0}^{z} P_X(x) P_Y(z-x)$$

$$= \sum_{x=0}^{z} \frac{\lambda^x e^{-\lambda}}{x!} \frac{\mu^{z-x} e^{-\mu}}{(z-x)!}$$

$$= e^{-(\lambda+\mu)} \sum_{x=0}^{z} \frac{\lambda^x \mu^{z-x}}{x!\,(z-x)!}$$

$$= e^{-(\lambda+\mu)} \sum_{x=0}^{z} \binom{z}{x} \lambda^x \mu^{z-x}$$

$$= \frac{e^{-(\lambda+\mu)} (\lambda+\mu)^z}{z!}$$ Binomial theorem

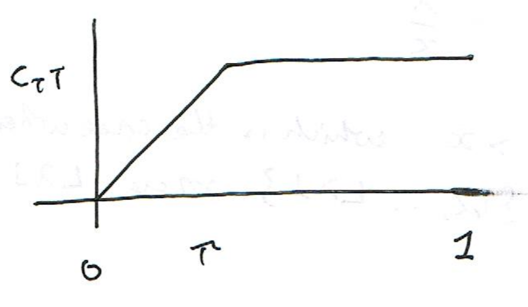
$$\therefore P(Z=z) = \frac{e^{-(\lambda+\mu)} (\lambda+\mu)^z}{z!} = P(\lambda+\mu)$$

4 i) For f to be a pdf

1) $f(x) \geq 0 \quad \forall x \in \mathbb{R}$

2) $\int_{-\infty}^{\infty} f(x)\,dx = 1$

ii)



iii) $1 = \int_0^t c_T x \, dx + \int_t^1 c_T T \, dx$

$= \left[\frac{c_T}{2} x^2 \right]_0^t + \left[c_t \cancel{t} x \right]_t^1$

$= \frac{c_t t^2}{2} - 0 + c_t T - c_t t^2$

$= -\frac{1}{2} c_t t^2 + c_t t$

$1 = c_t \left(-\frac{1}{2} t^2 + t \right)$

$2 = c_t (-t^2 + 2t)$

$c_t = 2 / (2t - t^2)$

iv) $C_T = 2(2T - T^2)^{-1}$

$\frac{d}{dT} C_T = 2 \times -1 (2T - T^2)^{-2} \times (2 - 2t)$

$= \frac{-2(2-2t)}{(2T - T^2)^2}$

when $T = 0$ $=0$ so cant have denominator of 0.

Let $g(t) = 2t - t^2$

$g(t)' = 2 - 2t$ whis is positive for $t \in (0,1)$ implying that g is increasing ~~incresing~~ & hence c_t is decreasing in t

could ato do.

$-2(2-2t) = 0$

$2 - 2t = 0$

$C_T T = \frac{2t}{2T - T^2} = \frac{2}{2-T}$

$C_T T' = 2(2-T)^{-2} \times -1 \times -1$

$= \frac{2}{(2-T)^2}$

$\Rightarrow$ decreasing in t.

$\frac{2}{(2-T)^2} = 0$

v) For $t \geq x$ we have seen that c_T is decreasing in t, so for t within $(x, 1)$, $f(x)$ is maximised by $t = x$. For $t \leq x$, we have seen $C_T T$ is increasing in T and so it is again maximised by $t = x$. Hence $\hat{t} = x$ is a global maximum i.e. MLE for θ