IMPERIAL COLLEGE LONDON

TIMED REMOTE ASSESSMENTS 2021-2022

BEng Honours Degree in Computing Part I
MEng Honours Degrees in Computing Part I
for Internal Students of the Imperial College of Science, Technology and Medicine

This paper is also taken for the relevant assessments for the Associateship of the City and Guilds of London Institute

PAPER COMP40018A

DISCRETE MATHEMATICS, LOGIC AND REASONING PART 1

Thursday 5 May 2022, 10:00
Writing time: 80 minutes
Upload time: 25 minutes

Answer ALL TWO questions
Open book assessment

Paper contains 2 questions

1 a Let $V = \{\{a\},\{b\},\{c\}\}$ and $W = \{\{a\},b,c\}$. Determine $V \cap W$, $V \cup W$, $\wp W$, $V \cap \wp W$, and $V \triangle W$.

b Let $A = \{a,b,c,d,e,f\}$ and let R be a binary relation on A. If $\langle a,b\rangle$, $\langle a,f\rangle$, $\langle c,d\rangle$, $\langle f,e\rangle \in R$ and R is reflexive, symmetric, and transitive, what (other) ordered pairs must belong to R?

c Let R denote a binary relation on $\mathbb{N}^2$, the set of pairs of natural numbers, defined by $\langle p,q\rangle \, R \, \langle r,s\rangle \triangleq \exists\, n,m \in \mathbb{N} \setminus \{0\} \; (n \times p = r \wedge m \times q = s)$. Prove that R is a partial order.

d Let V and W be arbitrary sets with $|V| = 2$ and $|W| = 3$. How many functions are there from $\wp V$ to $\wp W$? How many partial functions are there? Make sure to motivate your answer.

e Prove that $\mathbb{Q}^2$ is countable; clearly state if you refer to any results shown in the lectures and prove your additional statements explicitly.

The five parts carry equal marks.

2 a i) Consider the following sentence:

Unless the card is activated and there is money in the account, the pin code is not requested, or the payment is declined.

(A) Translate the above sentence into *propositional logic*. (Use brackets to disambiguate your formalisation.) Give the meaning of each propositional atom used in your formalisation.

(B) Draw the formation tree for the formula constructed in (A), and give all its subformulas.

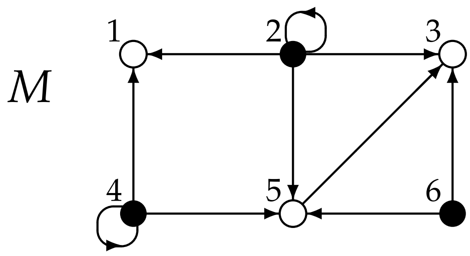
(C) Give an atomic evaluation function that makes the formula constructed in (A) true. Justify your answer.

ii) Let P, Q and R be atomic propositional formulas. Rewrite the formula below into CNF.

$$(P \to \neg(Q \vee R)) \vee (R \to \neg P)$$

iii) Show, using direct argument, that the statement $\models \phi \leftrightarrow \psi$ holds if and only if $(\phi \wedge \neg\psi) \vee (\neg\phi \wedge \psi)$ is unsatisfiable.

b i) Consider a signature L composed of a unary relation symbol C and a binary relation symbol R. Below is a diagram of an L-structure M whose domain has six objects. The interpretation of R is shown by the arrows, and the black dots are the objects satisfying C, e.g., $M \models R(2,1) \wedge \neg R(1,4) \wedge C(4)$.



The formula $C(x)$ is true in M for $x = 2,4,6$ only as the diagram shows.

(A) Write down all values of x for which the following formulae are true in M. Explain your answer.

i) $\forall y \neg R(y,x) \vee \forall y \neg R(x,y)$

ii) $\exists y \exists z \forall w (C(w) \wedge R(w,x) \to w = y \vee w = z))$

iii) $\forall y \exists z (\neg C(z) \wedge R(x,z) \wedge y \neq z)$

(B) Consider an additional unary relation symbol F. State what objects in M must be included in the interpretation of F for the sentence $\forall x(\forall y(R(x,y) \to F(y)) \to F(x))$ to be true in M.

ii) Using natural deduction, show that

$$\exists x \forall y(P(y) \leftrightarrow y = x) \vdash \forall y((\neg \exists x P(x) \vee Q(y)) \to Q(y))$$

The two parts carry equal marks.