# COMP245: Probability and Statistics 2016 - Problem Sheet 9

## Hypothesis Testing

Q1) To decide from which manufacturer to purchase 2000 PCs for its undergraduates, a University decided to carry out a test. It bought 50 PCs from manufacturer A and 50 from manufacturer B. Over the course of a six month period 6 of those from manufacturer A experienced problems, and 10 of those from manufacturer B. This suggested that, to be on the safe side, the University should by machines from manufacturer A. However, A's machines were more expensive than B's. Before making the decision to buy from A, the University wanted to be confident that the difference was a real one, and was not merely due to chance fluctuation. Carry out a test to investigate this.

- You may wish to use the extract from chi-squared tables given below:

| Degrees of | Upper tail area | | |
|---|---|---|---|
| freedom | .10 | .05 | .01 |
| 1 | 2.706 | 3.841 | 6.635 |
| 2 | 4.605 | 5.991 | 9.210 |
| 3 | 6.251 | 7.815 | 11.345 |
| 4 | 7.779 | 9.488 | 13.277 |
| 5 | 9.236 | 11.071 | 15.086 |

Q2) In an ESP experiment, a subject in one room is asked to state the colour (red or blue) of 100 cards randomly selected, with replacement, from a pack of 25 red and 25 blue cards by someone in another room. If the subject gets 34 right, determine whether the results are significant at the 1% level. Clearly write down your null hypothesis, alternative hypothesis, the test statistic you use, the rejection region, and interpret your conclusions.

- Hint: you may regard 100 as a large sample. You may wish to use the following extract form a standard normal distribution, showing the upper tail area from a N(0,1) distribution for certain points $x$.

| $x$ | Upper tail area |
|---|---|
| 1.28 | 0.010 |
| 1.64 | 0.050 |
| 1.96 | 0.025 |
| 2.33 | 0.001 |
| 2.58 | 0.005 |

Q3) Charles Darwin measured differences in height for 15 pairs of plants of the species *Zea mays*. (Each plant had parents grown from the same seed - one plant in each pair was the progeny of a cross-fertilisation, the other of a self-fertilisation. Darwin's measurements were the differences in height between cross-fertilised and self-fertilised progeny.) The data, measured in eighths of an inch, are given below.

49, -67, 8, 16, 6, 23, 28, 41, 14, 29, 56, 24, 75, 60, -48.

(a) Supposing that the observed differences $\{d_i | i = 1, \ldots, 15\}$ are independent observations on a normally distributed random variable $D$ with mean $\mu$ and variance $\sigma^2$, state appropriate null and alternative hypotheses for a two-sided test of the hypothesis that there is no difference between the heights of progeny of cross-fertilised and self-fertilised plant, and state the null distribution of an appropriate test statistic.

(b) Obtain the form of the rejection region for the test you defined in part (a), assuming a 10% significance level.

(c) Calculate the value of the test statistic for this data set, and state the conclusions of your test.

- You may want to use the following extract from a t-distribution, giving the point $x$ with the specified area under the upper tail beyond $x$ of a t-distribution with df degrees of freedom.

| df | 5% | 10% |
|---|---|---|
| 13 | 1.7709 | 1.3502 |
| 14 | 1.7613 | 1.3450 |
| 15 | 1.7531 | 1.3406 |

Q4) Some of 'Student's' original experiments involved counting the numbers of yeast cells found on a microscope slide. The results of one experiment are given in the table below, which shows the number of small squares on a slide which contain 0, 1, 2, 3, 4, or 5 cells. We want to use these data to see if the mean number of cells per square is 0.6, using the 5% significance level.

| Number of cells in square | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| Frequency | 213 | 128 | 37 | 18 | 3 | 1 |

(a) State the null hypothesis and the alternative hypothesis.

(b) The distribution of the numbers of cells is far from normal; it can take only positive integer values, it is very far from symmetric, and dies away very quickly. Which familiar distribution might be appropriate as a model for these data?

(c) Estimate the mean and the variance of the distribution you suggested in 4b.

(d) Explain why a critical region with the form

$$\left\{\overline{x} < \mu - 1.96\sqrt{\frac{\mu}{n}}\right\} \cup \left\{\overline{x} > \mu + 1.96\sqrt{\frac{\mu}{n}}\right\}$$

would be a reasonable region, making sure you explain the 1.96, the term $\sqrt{\frac{\mu}{n}}$ and the implications of the union. What is the test statistic in mind?

(e) Compute the limits of the critical region.

(f) Draw a conclusion about whether or not the null hypothesis can be rejected.

Q5) A survey of 320 families with 5 children each, gave the distribution shown below. Is this table consistent with the hypothesis that male and female children are equally probable? Obtain results for both the 1% and 5% levels. Work through the details of the test - don't just hit the chi-squared button on a statistical calculator.

| | | | | | | |
|---|---|---|---|---|---|---|
| Boys/girls | 5/0 | 4/1 | 3/2 | 2/3 | 1/4 | 0/5 |
| Number of families | 18 | 56 | 110 | 88 | 40 | 8 |

- You may wish to use the table extract from Q1.

Q6) As part of a telephone interview, a sample of 500 executives and a sample of 250 MBA students were asked to respond to the question 'Should corporations become more directly involved with social problems such as homelessness, education, and drugs?' The results are shown below. Test the hypothesis that the patterns of response for the two groups are the same.

| | More involved | Not more involved | Note sure |
|---|---|---|---|
| Executives | 345 | 135 | 20 |
| MBA students | 222 | 20 | 8 |

- You may wish to use the table extract from Q1.

Q7) (a) For a test at a fixed significance level, and with given null and alternative hypotheses, what will happen to the power as the sample size increases?

(b) For a test of a given null hypothesis against a given alternative hypothesis, and with a given sample size, describe what would happen to the power of the test if the significance level was changed from 5% to 1%.

(c) A test of a given null hypothesis against a given alternative hypothesis, with a sample of size $n$ and significance level of , has power of 80%. What change could I make to the test to increase my chance of rejecting a false null hypothesis?

(d) How can we attain a test which has a very low probability of Type I error and also a very low probability of Type II error?

Q8) The data below show the frequency with which each of the balls numbered 1-49 have appeared in the main draw in the National Lottery between its inception in November 1994 until December 2008. This table shows that there are substantial differences between the numbers of times different balls have appeared - for an extreme example, number 20 has been drawn just 134 times, whereas 38 has appeared 197 times, almost 50% more often.

Should we conclude from this table that the balls have different probabilities of appearing?

| Ball | Freq. | Ball | Freq. | Ball | Freq. | Ball | Freq. |
|---|---|---|---|---|---|---|---|
| 1 | 160 | 11 | 185 | 21 | 148 | 31 | 180 |
| 2 | 168 | 12 | 178 | 22 | 169 | 32 | 170 |
| 3 | 163 | 13 | 147 | 23 | 183 | 33 | 176 |
| 4 | 163 | 14 | 160 | 24 | 164 | 34 | 159 |
| 5 | 155 | 15 | 157 | 25 | 182 | 35 | 173 |
| 6 | 174 | 16 | 147 | 26 | 161 | 36 | 148 |
| 7 | 168 | 17 | 158 | 27 | 173 | 37 | 150 |
| 8 | 158 | 18 | 164 | 28 | 165 | 38 | 197 |
| 9 | 176 | 19 | 166 | 29 | 164 | 39 | 166 |
| 10 | 171 | 20 | 134 | 30 | 177 | 40 | 178 |

| Ball | Freq. |
|---|---|
| 41 | 137 |
| 42 | 167 |
| 43 | 182 |
| 44 | 184 |
| 45 | 162 |
| 46 | 164 |
| 47 | 177 |
| 48 | 177 |
| 49 | 169 |