I don't have permission to edit this file name but let this be the 2019 Models solutions!! 😊

Please, can everyone collaborate so we can get a decent set of answers

Hello to people in this doc atm hope you are doing well. Gl tomorrow <3

1)a)i)

a i) Give the derivation tree for the command

$$\texttt{for } x \texttt{ in } [a,b] \texttt{ do } sum := sum + x$$

in the store $s = [a \mapsto 1; b \mapsto 2; sum \mapsto 0]$. You may evaluate expressions and booleans directly, without showing their derivation trees. ← let this be rule K

$s' = s[x \mapsto 1]$
$s'' = s'[sum \mapsto 1]$
$s''' = s''[x \mapsto 2]$
$s^{iv} = s'''[sum \mapsto 3]$
$s^{v} = s^{iv}[x \mapsto 3]$

rules are labelled A → J for convenience

$$F\ \frac{K\,\dfrac{}{\langle a,s\rangle \Downarrow_e 1} \quad \dfrac{}{s[x\mapsto 1]=s'} \quad K\,\dfrac{}{\langle x>b,s'\rangle \Downarrow_b false} \quad C\,\dfrac{K\,\dfrac{}{\langle sum+x,s'\rangle \Downarrow_e 1} \quad \dfrac{}{s'[sum\mapsto 1]=s''}}{\langle sum := sum+x, s'\rangle \Downarrow_c \langle \mathcal{N}, s''\rangle} \quad \boxed{\text{GOTO TREE A}}}{\langle \text{for } x \text{ in } [a,b] \text{ do } sum := sum+x, s\rangle \Downarrow_c \langle \mathcal{N}, s^{v}\rangle}$$

$\boxed{\text{TREE A}}$ =

$$F\ \frac{K\,\dfrac{}{\langle 2,s''\rangle \Downarrow_e 2} \quad \dfrac{}{s''[x\mapsto 2]=s'''} \quad \dfrac{}{\langle x>b,s'''\rangle \Downarrow_b false} \quad C\,\dfrac{K\,\dfrac{}{\langle sum+x,s'''\rangle \Downarrow_e 3} \quad \dfrac{}{s'''[sum\mapsto 3]=s^{iv}}}{\langle sum := sum+x, s'''\rangle \Downarrow_c \langle \mathcal{N}, s^{iv}\rangle} \quad \boxed{\text{GOTO TREE B}}}{\langle \text{for } x \text{ in } [2,b] \text{ do } sum := sum+x, s''\rangle \Downarrow_c \langle \mathcal{N}, s^{v}\rangle}$$

these probably not needed

$\boxed{\text{TREE B}}$ =

$$H\ \frac{K\,\dfrac{}{\langle 3,s^{iv}\rangle \Downarrow_e 3} \quad \dfrac{}{s^{iv}[x\mapsto 3]=s^{v}} \quad \dfrac{}{\langle x>b,s^{v}\rangle \Downarrow_b true}}{\langle \text{for } x \text{ in } [3,b] \text{ do } sum := sum+x, s^{iv}\rangle \Downarrow_c \langle \mathcal{N}, s^{v}\rangle}$$

1)a)ii)

$s''' = s''[x \mapsto 4]$
$s' = s[x \mapsto 3]$
$s'' = s'[sum \mapsto 6]$

$$F\ \frac{\langle x+1,s\rangle \Downarrow_e 3 \quad s[x\mapsto 3]=s' \quad \langle x>b,s'\rangle \Downarrow_B false \quad E\,\dfrac{\langle x>c,s'\rangle \Downarrow_b false \quad C\,\dfrac{\langle sum+x,s'\rangle \Downarrow_e 6 \quad s'[sum\mapsto 6]=s''}{\langle sum := sum+x, s'\rangle \Downarrow_c (\mathcal{N}, s'')}}{\langle C, s'\rangle \Downarrow_c \langle \mathcal{N}, s''\rangle} \quad \boxed{\text{GOTO TREE C}}}{\langle \text{for } x \text{ in } [x+1,b] \text{ do } C, s\rangle \Downarrow_c (B, s''')}$$

$\boxed{\text{TREE C}}$

$$\frac{\langle x+1,s''\rangle \Downarrow_e 4 \quad s''[x\mapsto 4]=s''' \quad \langle x>b,s'''\rangle \Downarrow_B false \quad D\,\dfrac{\langle x>c,s'''\rangle \Downarrow_b true \quad B\,\dfrac{}{\langle break, s'''\rangle \Downarrow (B, s''')}}{\langle C, s'''\rangle \Downarrow_c (B, s''')}}{\langle \text{for } x \text{ in } [x+1,b] \text{ do } C, s''\rangle \Downarrow_c (B, s''')}$$

1)a) iii)

The for command iterates from x = E1 by incrementing E1 by using x during each iteration and terminates normally when E1 > E2. The break command also terminates the loop, but before x = E1 reaches the normal termination state.

1)b)i) Ah yes just merge 4 questions into one and make it worth 20%

~~A - Copies the value of R_1 into R_2 without changing value of R_1~~

A – Increments R_2 by R_1, I.e. R_2 = R_2 + R_1, without changing the value of R_1

L0 : R4- -> L0, L1

L1: R1- -> L2, L4

L2: R4+ -> L3

L3: R2+ -> L1

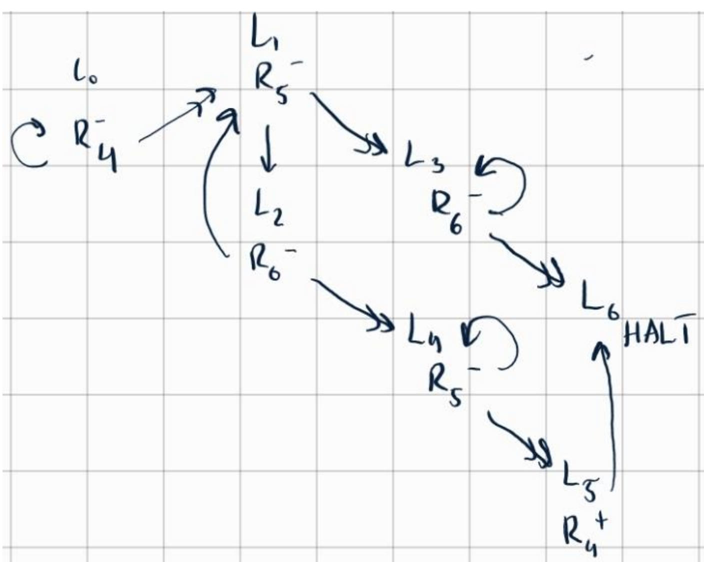
L4: R4- -> L5, L6

L5: R1+ -> L4

L6 -> HALT

~~B – if R_6 <= R_5 then R_4 = 1, else R_4 = 0 whilst not restoring their original contents~~

B – if R_6 < R_5 then R_4 = 1, else R_4 = 0 whilst not restoring their original contents



1)b)ii)

~~Let check(r_5, r_6) test if r_5 >= r_6 and if so returns in register r_4~~

REGS:
R_0 = sum
R_1 = a
R_2 = b
R_3 = x

GADGETS

$\boxed{\text{Inc}}$: $R_2 = R_2 + R_1$

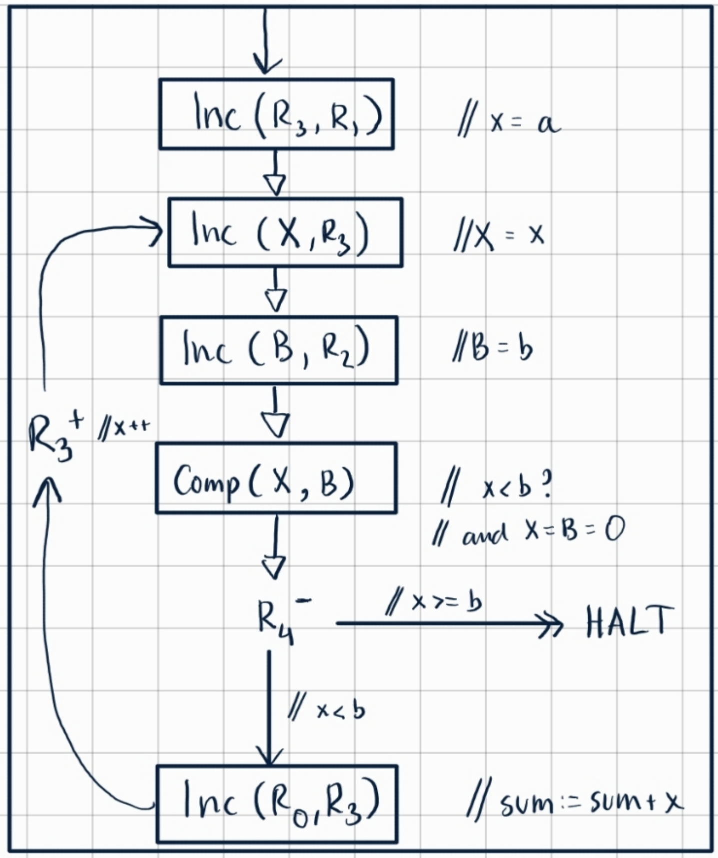
$\boxed{\text{Comp}}$: $\begin{cases} R_4 = 1 & R_6 < R_5 \\ R_4 = 0 & \text{otherwise} \end{cases}$

for x **in** $[a, b]$ **do** $sum := sum + x$

Assume that registers can be replaced in Inc and Comp for convenience, i.e.

Inc (x, y) = x := x + y

$\text{Comp}(x, y) = \begin{cases} R_4 = 1 & x < y \\ R_4 = 0 & \text{otherwise} \end{cases}$



2

a.

2a)

$BL\ (BLL) =_\beta \lambda bl.b\ (L\ bl)\ ((BLL)\ bl)$

$=_\beta \lambda bl.b\ ((\lambda bl.l)\ bl)\ ((BLL)\ bl)$

$\to_\beta \lambda bl.b\ l\ ((BLL)\ bl)$

$=_\beta \lambda bl.\ b\,l\ ((\lambda bl.b\ (L\ bl)\ (L\ bl))\ bl)$

$=_\beta \lambda bl.\ b\,l\ ((\lambda bl.\ b\ ((\lambda bl.l)\ bl)\ ((\lambda bl.l)\ bl))\ bl)$

$\to_\beta \lambda bl.\ b\,l\ ((\lambda bl.\ b\,l\,l)\ bl)$

$\to_\beta \lambda bl.\ bl\ (bll)$

b. (unsure about the last part but please correct if needed)

b) $M\,(BL\,(BLL))$

$=_\beta M(\lambda bl.\ bl\,(bll))$

$=_\beta (\lambda tbl.\ tb'l)\ (\lambda bl.\ bl\,(bll))$ $\qquad b' = \lambda t_1 t_2.\ b t_2 t_1$

$\to_\beta \lambda bl.\ (\lambda bl.\ bl\,(bll))\ b'l$

$\twoheadrightarrow_\beta \lambda bl.\ b'l\,(b'll)$

$=_\beta \lambda bl.\ (\lambda t_1 t_2.\ b t_2 t_1)\, l\ ((\lambda t_1 t_2.\ b t_2 t_1)\, l\, l)$

$\to_\beta \lambda bl.\ (\lambda t_1 t_2.\ b t_2 t_1)\, l\ (b\,l\,l)$

$\to_\beta \lambda bl.\ b\,(bll)\,l$

$=_\beta B\ (\lambda bl_2.\ bll)\ (\lambda bl.\ l)$ $\quad$ (inversion)

$=_\beta B\,(B\,(\lambda bl.\ l)\ (\lambda bl.\ l))\ L$ $\quad$ (inversion

$=_\beta B\,(BLL)\ L$

c) <u>Base case</u>: $t = L$. Prove $M\,L =_\beta \mathcal{M}(L)$

$$M\,L \overset{def}{=}_\beta (\lambda t b l.\, t\, b' l)\,(\lambda b l.\, l) \qquad b' = \lambda t_1 t_2.\, b\, t_2\, t_1$$

$$\to_\beta \lambda b l.\, (\lambda b l.\, l)\, b' l$$

$$\to_\beta \lambda b l.\, l =_\beta L = \mathcal{M}(L)$$

<u>Ind case</u>: take $t = B\, t_1 t_2$ such that

(IH_1): $M\,t_1 =_\beta \mathcal{M}(t_1)$ $\qquad$ prove $M\,t = \mathcal{M}(t)$

(IH_2): $M\,t_2 =_\beta \mathcal{M}(t_2)$

$$M\,t =_\beta M\,(B\,t_1\,t_2) =$$

$$=_\beta (\lambda t b l.\, t\, b' l)\,(B\, t_1\, t_2)$$

$$\to_\beta (\lambda b l.\, (B\, t_1\, t_2)\, b' l)$$

$$=_\beta (\lambda b l.\, (\lambda b l.\, b\,(t_1\, b l)\,(t_2\, b l))\, b' l)$$

$$\to_\beta (\lambda b l.\, b'\,(t_1\, b' l)\,(t_2\, b' l))$$

$$=_\beta (\lambda b l.\, ((\lambda t_1 t_2.\, b\, t_2\, t_1)\,(t_1\, b' l)\,(t_2\, b' l))$$

$$\to_\beta (\lambda b l.\, \quad .\, b\,(t_2\, b' l)\,(t_1\, b' l)) \quad (1)$$

$$\mathcal{M}(B t_1 t_2) =_\beta B\, \mathcal{M}(t_2)\, \mathcal{M}(t_1)$$

$$=_\beta B\, (M t_2)\, (M t_1)$$

$$=_\beta \lambda b l.\, b\, (M t_2\, b l)\, (M t_1\, b l)$$

$$=_\beta \lambda b l.\, b\, ((\lambda t b l.\, t b' l)\, t_2\, b l)\, ((\lambda t b l.\, t b' l)\, t_1\, b l)$$

$$\to_\beta \lambda b l.\, b\, (t_2\, b' l)\, (t_1\, b' l) \quad (2)$$

from (1) (2) we get $M t = \mathcal{M}(t)$

TODO: add proper conclusion for induction (Reasoning about programs flashbacks 🙁)