IMPERIAL COLLEGE OF SCIENCE, TECHNOLOGY AND MEDICINE

EXAMINATIONS 2019

BEng Honours Degree in Computing Part I
MEng Honours Degrees in Computing Part I
BEng Honours Degree in Mathematics and Computer Science Part I
MEng Honours Degree in Mathematics and Computer Science Part I
for Internal Students of the Imperial College of Science, Technology and Medicine

This paper is also taken for the relevant examinations for the Associateship of the City and Guilds of London Institute

PAPER C150=MC150

GRAPHS AND ALGORITHMS

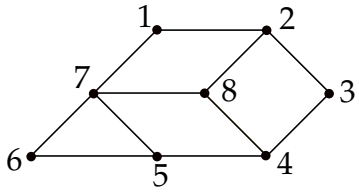
Tuesday 7th May 2019, 14:00
Duration: 80 minutes

Answer ALL TWO questions

Paper contains 2 questions
Calculators not required

1 a i) What is a *spanning tree* in an undirected connected graph G?

ii) State the order of traversal of the nodes in the graph in the diagram starting from node 1 for breadth-first search. Also draw the associated spanning tree. Assume that where there is a choice the numerically least node is chosen.



b A *sink node* in a directed graph is a node with no outgoing arcs. A *source node* is a node with no ingoing arcs.

Show that a finite directed acyclic graph must have at least one sink node and at least one source node.

c Here is a graph algorithm which iterates through the nodes of an undirected graph and their adjacency lists.

```
count = 0
for x in Nodes:
    count = count+2
    for y in adj[x]:
        count = count+3
return count
```

If this algorithm is applied to a simple graph with n nodes and m arcs, what is the value (of count) returned? Give both an exact answer and one using big-O notation.

d i) What is a *minimum* spanning tree in a connected weighted graph (G, W)?

ii) Describe an algorithm (in words or in pseudocode) to find a *maximum* spanning tree in a weighted graph (G, W).

Explain why your algorithm is correct. You can assume the correctness of standard algorithms for finding minimum spanning trees.

The four parts carry, respectively, 30%, 20%, 15%, and 35% of the marks.

2 a i) How many comparisons does Insertion Sort take when applied to the list $[1, 4, 3, 2, 5]$?

ii) Let us say that a list L of even length is *pair-swapped* if by swapping all adjacent pairs of elements we obtain a sorted list. As an example, for $n = 8$ the list $[2, 1, 4, 3, 6, 5, 8, 7]$ is pair-swapped.

Calculate the number of comparisons performed by Insertion Sort applied to a pair-swapped list of even length $n \geq 2$. Assume that all elements are distinct.

Give a brief explanation.

b Let E be an array of elements with keys $[3, 5, 16, 7, 2, 10, 9, 13]$.

i) Draw (as a tree) the (binary) heap structure corresponding to E.

ii) Use buildMaxHeap (as defined in the lectures) to convert your heap from (i) into a max heap.

Give your answer both as a tree and as an array.

c i) What does it mean for an undirected graph to have a *Hamiltonian path*?

ii) The problem HPN is defined as follows: given an undirected graph G and nodes x and y of G, is there a Hamiltonian path in G with endpoints x and y?

Explain why HPN belongs to the complexity class NP.

iii) Show that HPN is NP-complete.

You may assume that the following problem HAMPATH is NP-complete: given a graph G, does G have a Hamiltonian path?

The three parts carry, respectively, 30%, 30%, and 40% of the marks.