IMPERIAL COLLEGE LONDON

TIMED REMOTE ASSESSMENTS 2020-2021

BEng Honours Degree in Computing Part I
MEng Honours Degrees in Computing Part I
for Internal Students of the Imperial College of Science, Technology and Medicine

This paper is also taken for the relevant assessments for the Associateship of the City and Guilds of London Institute

PAPER COMP40003

LOGIC

Friday 7 May 2021, 10:00
Duration: 95 minutes
Includes 15 minutes for access and submission

Answer ALL TWO questions
Open book assessment

Paper contains 2 questions

1 a Draw the formation tree for the following formula and list all its subformulas.

$$(p \land (q \to r) \lor \neg s \land \neg q \to t)$$

b Consider a propositional language with three propositional atoms *peloton*, *bike* and *sold_out*. Translate the following sentences into propositional logic.

i) All peloton bikes are sold out.

ii) Unless it is a peloton bike, it is not sold out.

iii) A bike is sold out only if it is a peloton.

iv) The bike is either not a peloton or sold out.

c Let P and Q be arbitrary propositional formulas.

i) Show, using direct argument, that $\top \models P$ if and only if $P \equiv \top$.

ii) Each of the formulas below is "not valid but satisfiable". For each one, give situations that demonstrate the phrase in quotes, and show your reasoning.

A) $(\neg P \lor Q) \land (P \lor \neg Q)$

B) $(\neg P \to \neg Q) \to (P \to Q)$

d Using truth tables, rewrite the following formula into an equivalent formula in disjunctive normal form. (Do not simplify the formula extracted from the table.)

$$(((p \to \neg q) \land \neg r) \to q) \land r$$

e Using natural deduction, prove that:

i) $(A \land B) \lor A \leftrightarrow A \land (A \lor B)$ is a theorem.

ii) $(p \land s \to \bot) \land (s \to p) \land (\top \land s)$ is a contradiction.

f Consider a different natural deduction system $\vdash^*$ which includes all the rules of $\vdash$ as well the additional inference rule:

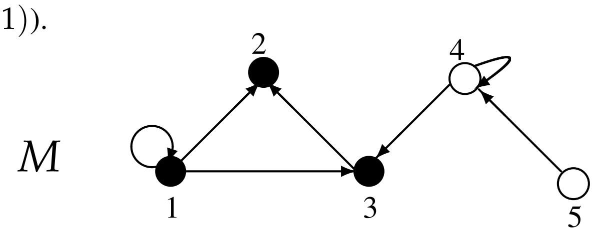
1	A	
2	$A \land \neg A$	$\land\neg I(1)$

Is $\vdash^*$ sound? Is it complete? Justify your answer in both cases.

The six parts carry, respectively, 10%, 10%, 25%, 15%, 25%, and 15% of the marks.

2a Prove using equivalences that $\forall x \exists y (P(x) \vee Q(y)) \to \exists y Q(y)$ is logically equivalent to $\forall x P(x) \to \exists y Q(y)$. In each step use only one equivalence and state the general form of the equivalence used.

b Let L be the signature consisting of a unary relation symbol P and a binary relation symbol R. Below is a diagram of an L-structure M. The arrows and the black circles denote respectively the interpretation of R and P (e.g., $M \models R(1,2) \wedge P(1)$).



The formula $R(x,x)$ is true in M for $x = 1$ and $x = 4$. In a similar way, list all objects x for which the following formulas are true in M. You do not need to justify your answers.

i) $\neg \exists y (R(x,y) \wedge R(y,x))$

ii) $\forall y \exists z (R(x,z) \wedge \neg (y = z))$

iii) $\exists y \forall z (R(x,z) \leftrightarrow (y = z \wedge \neg P(z))$

iv) $\forall y \forall z (R(y,x) \wedge R(z,x) \to P(y) \leftrightarrow P(z))$

c Let M be as in part 2b and let B be a new unary relation symbol. Assume that:

$$M \models \forall x (\forall y (R(y,x) \wedge \neg P(y) \to B(y)) \to B(x))$$

Give all possible interpretations of B in M. Briefly justify your answer.

d Now, let L be the 2-sorted signature with sorts `Nat` and `[Nat]`, containing the constants $\underline{0}, \underline{1}, \underline{2}, \ldots$: `Nat` and $[\,]$: `[Nat]`, the function symbol $\times$, and the relation symbol $=$. These symbols have the usual meaning over the sort `Nat`. Variables, x, y and n have sort `Nat` and xs, ys and zs have sort `[Nat]`. You are given an L-formula $\mathtt{in}(n, xs)$ expressing that the natural number n occurs in the list xs.

Write down L-formulas expressing the following properties of n, xs ad ys:

i) xs and ys have at least one common element that is equal to n.

ii) xs is not empty and all its elements are equal.

iii) xs has at most two elements that are different.

e Using natural deduction, show that

$$\forall y \neg \exists x (R(y,x) \wedge \neg P(x)) \vdash \forall x (\exists y R(y,x) \to P(x))$$

The five parts carry, respectively, 20%, 20%, 20%, 15%, and 25% of the marks.