IMPERIAL COLLEGE OF SCIENCE, TECHNOLOGY AND MEDICINE

EXAMINATIONS 2019

BEng Honours Degree in Computing Part I
MEng Honours Degrees in Computing Part I
for Internal Students of the Imperial College of Science, Technology and Medicine

This paper is also taken for the relevant examinations for the Associateship of the City and Guilds of London Institute

PAPER C142

DISCRETE STRUCTURES

Monday 13th May 2019, 14:00
Duration: 80 minutes

Answer ALL TWO questions

Paper contains 2 questions
Calculators not required

1 a Let $A = \{1,4,5,7\}$ and $B = \{1,3,7\}$. Write down explicit sets for

i) $A \setminus B$, $B \setminus A$, and $A \triangle B$;

ii) $\emptyset \times A$ and $A \times (A \setminus B)$;

iii) Give the number of elements of the sets $\wp A$, $\wp B$, $\wp(A \cup B)$, and $\wp\{A, B\}$.

b i) Let R be a binary relation on A. State the formal property that R should satisfy in order to be called: *reflexive*, *symmetric*, or *transitive*.

ii) Give the definition of the union of two relations R and S and the inverse of a relation R.

iii) Show that $(R \cup S)^{-1} = R^{-1} \cup S^{-1}$.

iv) Is the union of two symmetric relations always symmetric? Give a proof or a counterexample.

c i) Let A and B be arbitrary sets with $|A| = m$ and $|B| = n$. How many functions are there from A to B, and how many partial functions?

ii) Let $f : A \to B$ and $g : B \to C$ be functions.

A) State the properties that f has to satisfy to be called *injective*, respectively *surjective*.

B) Prove that if $g \circ f$ is injective then so is f.

C) Give a specific example of f and g such that $g \circ f$ is injective but g is not.

iii) A) Give the definition of the relation $\approx$ between sets.

B) Assume that $A_1 \approx A_2$ and $B_1 \approx B_2$; show that $A_1 \times B_1 \approx A_2 \times B_2$.

The three parts carry, respectively, 20%, 35%, and 45% of the marks.

2 a Let R be a binary relation on a set A. Give the definitions of the following different notions of orders:

i) pre-order;

ii) anti-symmetric;

iii) partial order;

iv) irreflexive;

v) strict partial order;

vi) total order.

b Consider the set $F \triangleq \{2,3,4,5,6,8,12,15,24,30,60\}$ and consider the binary relation R on F defined by: $n\,R\,m \triangleq \exists\, k \in F\ (k \times n = m)$.

i) Give the Hasse diagram for $\langle F, R \rangle$.

ii) Argue if R is either (remark that more than one of these might be true.)

A) a pre-order;

B) anti-symmetric;

C) a partial order;

D) irreflexive;

E) a strict partial order;

F) a total order;

c i) Prove that $\{0,1\} \times \mathbb{N}$ is countable.

ii) Use this property to prove that if the disjoint sets X and Y are countable, then so is $X \cup Y$.

iii) Now prove that if X and Y are countable then so is $X \cup Y$.

d i) Prove Cantor's theorem: for any set A, $A \not\approx \wp A$.

ii) Using this result, show that $\wp \mathbb{N}$ is not countable.

The four parts carry, respectively, 15%, 25%, 35%, and 25% of the marks.

