Symbo 2023

Meaning of ***symbol*** *in English*

symbol

noun [C]

UK /ˈsɪm.bəl/ **US** /ˈsɪm.bəl/

Add to word list

Kevinop/iStock/Getty Image Plus/GettyImages

B2

a sign, shape, or object that is used to represent something else:

- *A heart shape is the symbol of love.*
- *The wheel in the Indian flag is a symbol* ***of*** *peace.*

Compare

emblem

Q1

a) (i)

This one is a freebie.

$$
\begin{aligned}
ground(K) = \{\, & p(a,a) \leftarrow q(a,a).,\\
& p(a,b) \leftarrow q(b,a).,\\
& p(b,a) \leftarrow q(a,b).,\\
& p(b,b) \leftarrow q(b,b).,\\
& p(a,a) \leftarrow q(a,a), p(a,a).,\\
& p(a,a) \leftarrow q(a,b), p(b,a).,\\
& p(a,b) \leftarrow q(a,a), p(a,b).,
\end{aligned}
$$

$p(a,b) \leftarrow q(a,b), p(b,b).,$

$p(b,a) \leftarrow q(b,a), p(a,a).,$

$p(b,a) \leftarrow q(b,b), p(b,a).,$

$p(b,b) \leftarrow q(b,a), p(a,b).,$

$p(b,b) \leftarrow q(b,b), p(b,b).,$

$q(a,b).,$

$q(b,b).\}$

(ii)

M_1 is supported but not a model

e.g. because $q(b,b) \in M_1$, and

$p(b,b) \leftarrow q(b,b) \in ground(K)$, but

$p(b,b) \notin M_1$.

[NB: this is (iii)]

At this point it's easiest to proceed by just finding the least model of K.

$T_P \uparrow 0 = \emptyset$

$T_P \uparrow 1 = \{q(a,b), q(b,b)\}$

$T_P \uparrow 2 = \{q(a,b), q(b,b),$
$p(b,a), p(b,b)\}$

$$T_P \uparrow^3 = \{ q(a,b),\ q(b,b),\ p(b,a),\ p(b,b),\ p(a,a),\ p(a,b) \}$$

$$T_P \uparrow^4 = T_P \uparrow^3$$

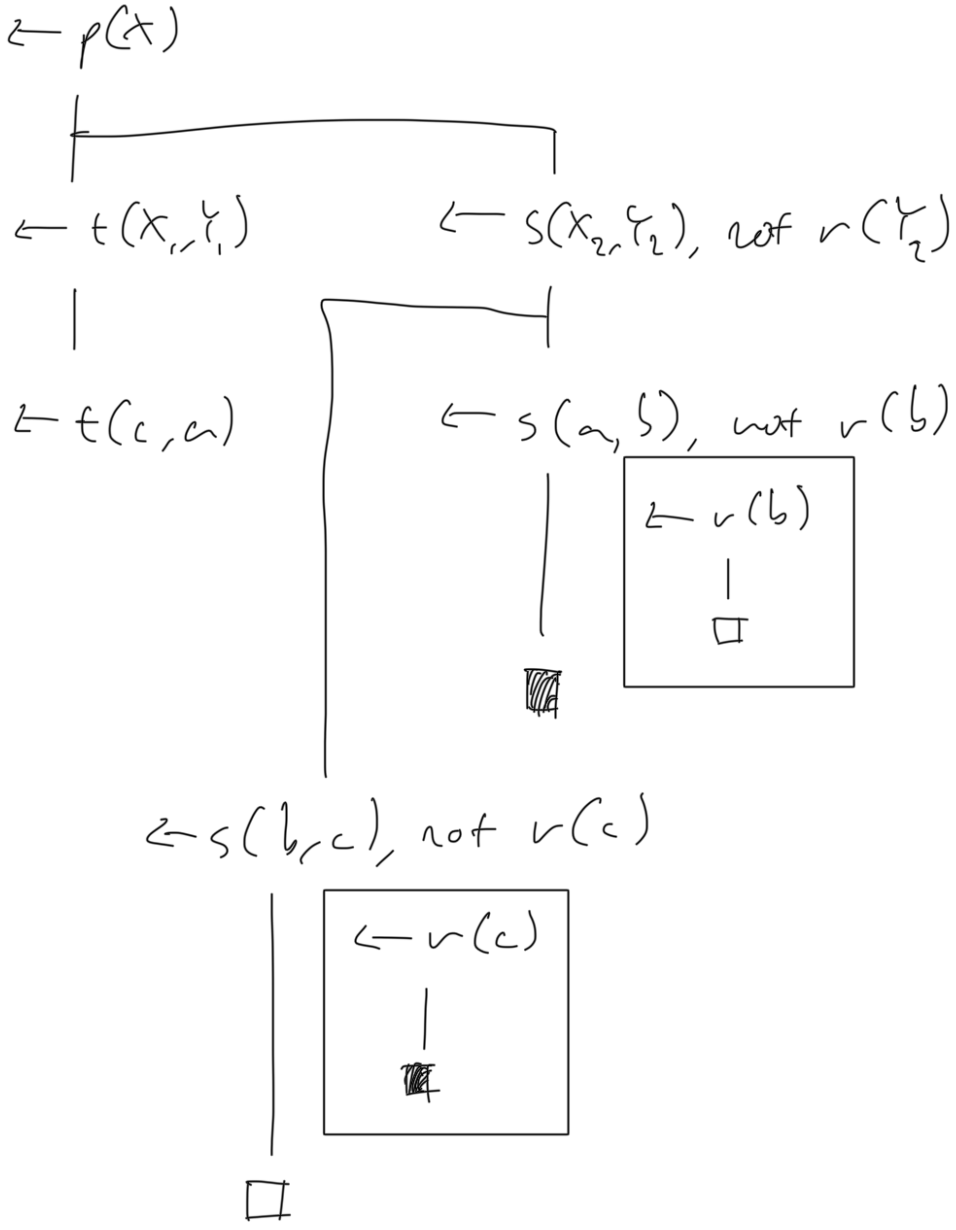
So $M(K) = T_P \uparrow^3$

Observe that $M(K) \subseteq M_2$.
M_2 is a model of K and is neither minimal nor supported e.g. because $q(a,a) \in M_2$

We see $M_3 = M(K)$ so M_3 is a minimal model and is supported.

(iii)
Did this already, see above.

b) (i)
Omitting the remaining substitutions and mgus because I'm lazy and they're obvious



Answer substitutions are

$\theta = \{X/a\}$

$\theta = \{X/b\}$

(ii)

Constants: a, b, c

Predicates: p(X), q(X, Y), r(X), s(X, Y), t(X, Y)

$U_L = \{a, b, c\}$

$B_L = \{p(a), p(b), p(c),$
$q(a,a), q(a,b), q(b,a), q(b,b),$
$r(a), r(b), r(c),$
$s(a,a), s(a,b), s(b,a), s(b,b),$
$t(a,a), t(a,b), t(b,a), t(b,b)\}$

c) (i)

r(a).

q(a,a).

q(a,b).

$\{p(a,a)\} \leftarrow r(a),$ not $q(a,a)$.

$\leftarrow p(a,a), q(a,a)$.

(ii)

r(a).

q(a,a).

q(a,b).

$\leftarrow p(a,a), q(a,a)$.

Q2.

a) (i)

We introduce:

$p \longleftrightarrow x > 0$

$q \longleftrightarrow y > 2$

$r \longleftrightarrow y < 2$

$B(F) = (p \vee (q \wedge \neg p)) \rightarrow r$

(ii)

We introduce:

$x_1 : (p \vee (q \wedge \neg p)) \rightarrow r$

$x_2 : p \vee (q \wedge \neg p)$

$x_3 : q \wedge \neg p$

$x_4 : \neg p$

$CNF(x_4 \leftrightarrow \neg p) = (x_4 \vee p) \wedge (\neg x_4 \vee \neg p)$

$CNF(x_3 \leftrightarrow q \wedge x_4)$

$= (\neg q \vee x_3 \vee \neg x_4) \wedge (\neg x_3 \vee q \vee \neg x_4)$

$\wedge (\neg x_3 \vee \neg q \vee x_4) \wedge (x_3 \vee q \vee x_4)$

$CNF(x_2 \leftrightarrow p \vee x_3)$

$= (\neg p \vee x_2) \wedge (\neg x_3 \vee x_2) \wedge (\neg x_2 \vee p \vee x_3)$

$CWF(x_1 \leftrightarrow \neg x_2 \rightarrow r)$

$=$

$$Tseitin(F) = (x_4 \vee p) \wedge (\neg x_4 \vee \neg p)$$
$$\wedge (\neg q \vee x_3 \vee \neg x_4) \wedge (\neg x_3 \vee q \vee \neg x_4)$$
$$\wedge (\neg x_3 \vee \neg q \vee x_4) \wedge (x_3 \vee q \vee x_4)$$
$$\wedge (\neg p \vee x_2) \wedge (\neg x_3 \vee x_2)$$
$$\wedge (\neg x_2 \vee p \vee x_3)$$
$$\wedge (x_2 \vee x_1) \wedge (\neg r \vee x_1)$$
$$\wedge (\neg x_1 \vee \neg x_2 \vee r)$$
$$\wedge x_1$$

(iii)

Clauses:

$(x_4 \vee p)$

$(\neg x_4 \vee \neg p)$

$(\neg q \vee x_3 \vee \neg x_4)$

$(\neg x_3 \vee q \vee \neg x_4)$

$(\neg x_3 \vee \neg q \vee x_4)$

$(x_3 \vee q \vee x_4)$

$(\neg p \vee x_2)$

$(\neg x_3 \vee x_2)$

$(\neg x_2 \vee p \vee x_3)$

$\cancel{(x_2 \vee x_1)}$

$\cancel{(\neg r \vee x_1)}$ unit clause : $x_1 \longmapsto$ true

$(\cancel{\neg x_1} \vee \neg x_2 \vee r)$

x_1

Simplified formula :

$$(x_4 \vee p) \wedge (\neg x_4 \vee \neg p) \wedge (\neg q \vee x_3 \vee \neg x_4)$$
$$\wedge (\neg x_3 \vee q \vee \neg x_4) \wedge (\neg x_3 \vee \neg q \vee x_4)$$
$$\wedge (x_3 \vee q \vee x_4) \wedge (\neg p \vee x_2) \wedge (\neg x_3 \vee x_2)$$
$$\wedge (\neg x_2 \vee p \vee x_3) \wedge (\neg x_2 \vee r)$$

Note: no unit clauses so no further assignments.

Assignments : $x_1 \mapsto$ true

(iv)

Scores:

p : 2	$\neg p$: 2
q : 2	$\neg q$: 2
r : 1	$\neg r$: 0

x_2: 2 $\qquad$ $\neg x_2$: 2

x_3 : 3 $\qquad$ $\neg x_3$: 3

x_4 : 2 $\qquad$ $\neg x_4$: 3

x_3, $\neg x_3$, $\neg x_4$ have the highest DLIS scores

b) (i) maintains soundness and termination

(ii) maintains soundness but not termination

(iii) neither sound nor terminating unless you disallow deleting of clauses that subsumed an original clause (in which case it is sound but not terminating)

c) For a given clause with four literals, we can find an equisatisfiable conjunction of three-literal clauses as follows:

$$(p \vee q \vee r \vee s)$$

$$\downarrow$$

$$(p \vee q \vee x) \wedge (\neg x \vee r \vee s)$$

where x is a fresh literal.

Note: see pages 92 - 96 of Algs's slides
for the reasoning behind this.

$\therefore$

We can reduce any 4-SAT problem to a 3-SAT problem, so 4-SAT is NP-complete.