COMP245: Probability and Statistics 2016 - Problem Sheet 1

Mathematical Methods Revision

Q1) Find the nth term and infinite sum of the following sequences, stating for which real values of x the infinite sums converge

(a) $\frac{1}{x}, \frac{1}{4x}, \frac{1}{16x}, \frac{1}{64x}, \ldots$ (b) $\frac{1}{x}, \frac{1}{x^2}, \frac{1}{x^3}, \frac{1}{x^4}, \ldots$ (c) $1, \frac{1}{x}, \frac{1}{x^2}, \frac{1}{x^3}, \frac{1}{x^4}, \ldots$

Q2) Using your answer from 1b, find the value of x such that $\sum_{i=1}^{\infty} x^{-i} = 1$.

Q3) Find $\frac{df}{dx}$ for the following functions $f(x)$:

(a) $f(x) = \sum_{i=0}^{n} a_i x^i \qquad (a_i \in \mathbb{R}, n \in \mathbb{Z}^+)$;

(b) $f(x) = x\log(x)$;

(c) $f(x) = e^{e^x}$;

Q4) Integrate the following functions $f(x)$ with respect to x:

(a) $f(x) = \sum_{i=0}^{n} a_i x^i \qquad (a_i \in \mathbb{R}, n \in \mathbb{Z}^+)$;

(b) $f(x) = x\log(x)$;

(c) $f(x) = e^{-ax} \qquad (a \in \mathbb{R},\ a \neq 0)$;

(d) $f(x) = xe^{-ax} \qquad (a \in \mathbb{R},\ a \neq 0)$;

Q5) Using your answer from 4c, find the value of a such that

$$\int_0^{\infty} e^{-ax} dx = 1.$$

Q6) Integrate the function $f(x, y) = xy$ over the interior of the quarter-ellipse which satisfies

$$\frac{x^2}{2} + y^2 = 1,\ x > 0, y > 0.$$

What would be the integral of the function $g(x, y) = |xy|$ over the interior of the entire ellipse?

Q7) For the function $f : \mathbb{R} \to \mathbb{R}$, $f(x) = x^2 + 1$, find the inverse image of $[1, 3)$.

Q8) Suppose a curve is known to pass through the following points (x, y):

$$(-1.4, 3.0), (-0.2, -1.6), (3.0, 0.9)$$

By linear interpolation, find approximate y-values of the curve at

(a) $x = -0.8$; (b) $x = 1.0$.