COMP245: Probability and Statistics 2016 - Problem Sheet 5
Solutions

Discrete Random Variables

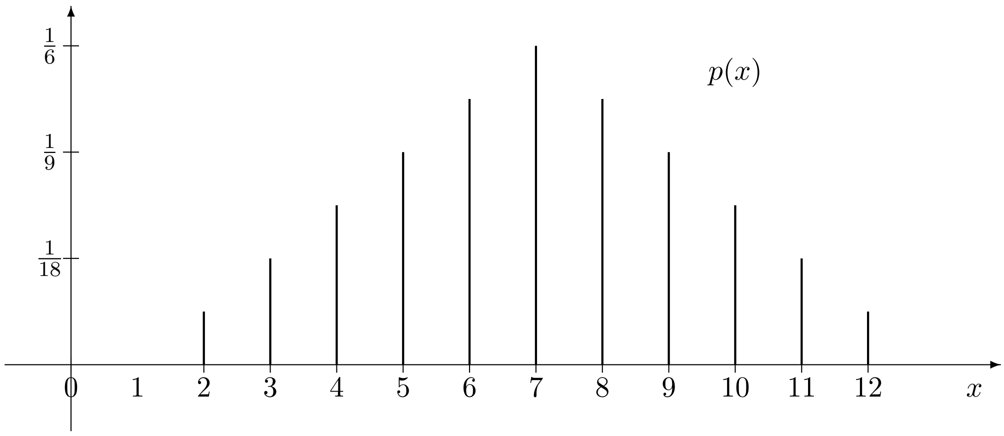
S1) (a) $S = \{\mathrm{HH}, \mathrm{HT}, \mathrm{TH}, \mathrm{TT}\}$.

(b) $p_X(0) = \frac{1}{4}$, $p_X(1) = \frac{1}{2}$, $p_X(2) = \frac{1}{4}$.

(c) $p_Y(1) = \frac{1}{4}$, $p_Y(3) = \frac{3}{4}$.

S2)

x	2	3	4	5	6	7	8	9	10	11	12
$p(x)$	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	$\frac{5}{36}$	$\frac{6}{36}$	$\frac{5}{36}$	$\frac{4}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$



S3) Let X be a random variable giving the number of heads obtained. Then $X \sim$Binomial$(4,\frac{1}{2})$

(a) $p(4) = \left(\frac{1}{2}\right)^4 = \frac{1}{16}$.

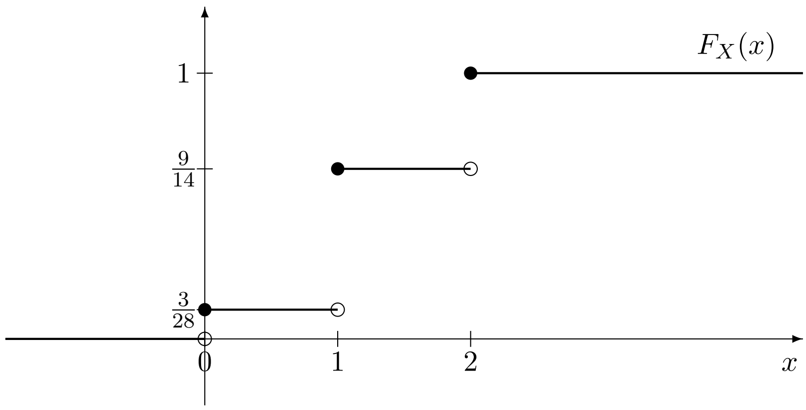
(b) $p(3) = \binom{4}{1}\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^1 = \frac{4}{16} = \frac{1}{4}$.

(c) $p(2) + p(3) + p(4) = \binom{4}{2}\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^2 + p(3) + p(4) = \frac{11}{16}$.

(d) By symmetry (switch heads with tails) $p(0) + p(1) = p(4) + p(3) = \frac{5}{16}$.

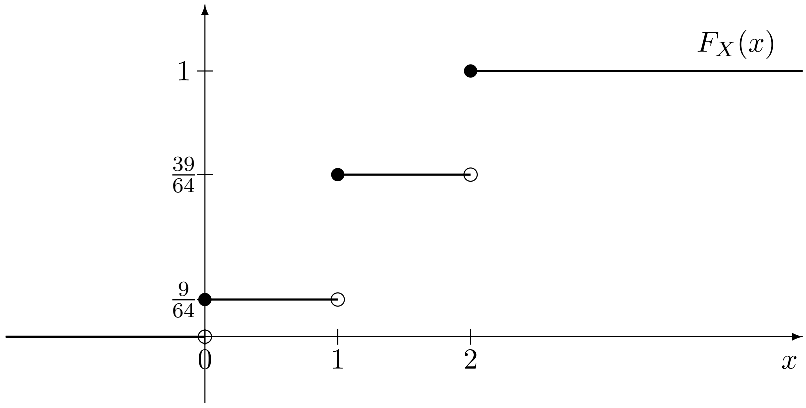
S4) (a) i. $p(0) = \frac{3}{8} \times \frac{2}{7} = \frac{3}{28}$; $p(1) = \frac{3}{8} \times \frac{5}{7} + \frac{5}{8} \times \frac{3}{7} = \frac{15}{28}$; $p(2) = \frac{5}{8} \times \frac{4}{7} = \frac{5}{14}$.

ii.


$F_X(x)$
1
$\frac{9}{14}$
$\frac{3}{28}$
0
1
2
x

(b) i. $p(0) = \frac{3}{8} \times \frac{3}{8} = \frac{9}{64}$; $p(1) = \frac{3}{8} \times \frac{5}{8} + \frac{5}{8} \times \frac{3}{8} = \frac{15}{32}$; $p(2) = \frac{5}{8} \times \frac{5}{8} = \frac{25}{64}$.

ii.



S5) (a) $\binom{5}{0}0.4^0 0.6^5 = 0.078$.

(b) $\binom{5}{1}0.4^1 0.6^4 = 0.26$.

(c) $1 - \text{P(none pass)} = 0.922$.

S6) (a) The distribution is Binomial(110,0.8) so

i. Mean $= 110 \times 0.8 = 88$.

ii. Standard deviation $= \sqrt{110 \times 0.8 \times (1 - 0.8)} = 4.195$.

(b) The distribution is Binomial(11000,0.8) so

i. Mean $= 11000 \times 0.8 = 8800$.

ii. Standard deviation $= \sqrt{11000 \times 0.8 \times (1 - 0.8)} = 41.95$.

S7) The distribution is Binomial$(5, \frac{1}{5})$ so

(a) 0.2^5.

(b) $\binom{5}{3}0.2^3 0.8^2 + \binom{5}{4}0.2^4 0.8^1 + \binom{5}{5}0.2^5 0.8^0$.

(c) $\binom{5}{2}0.2^2 0.8^3$.

(d) $1 - 0.2^5$.

S8) For Binomial(n, p), mean$=np$, standard deviation$=\sqrt{np(1-p)}$, skewness$=\dfrac{1-2p}{\sqrt{np(1-p)}}$.

(a) 90, 3, -0.2667.

(b) 70, 4.58, -0.0873.

(c) 50, 5, 0.

(d) 900, 9.49, -0.0843.

(e) 700, 14.49, -0.0276.

(f) 500, 15.81, 0.

Absolute value of skewness decreases as p gets closer to $\frac{1}{2}$and as sample size n increases.

S9) Let X be the total number of passes. This is a random variable formed as the sum of five Binomial random variables, one Binomial(2,0.4), one Binomial(4,0.6), etc.

(a) $\mathrm{E}(X) = 2 \times 0.4 + 4 \times 0.6 + 5 \times 0.7 + 7 \times 0.8 + 2 \times 0.9 = 14.1$.

(b) $\mathrm{Var}(X) = 2\times0.4\times0.6+4\times0.6\times0.4+5\times0.7\times0.3+7\times0.8\times0.2+2\times0.9\times0.1 = 3.79$, so $\mathrm{sd}(X) = 1.95$.

S10) (a) 1/0.4=2.5.

(b) 0.4.

(c) 0.4^3=0.064.

S11) (a) np, $np(1-p)$.

(b) $\sum_{i=1}^{n} p_i$, $\sum_{i=1}^{n} p_i(1-p_i)$.

(c) The mean is unaffected, but further information would be needed to calculate the variance.

S12) Write

$$E(N) = \sum_{n=1}^{\infty} P(N=n) \sum_{r=1}^{n} 1 = \sum_{n=1}^{\infty} \sum_{r=n}^{\infty} P(N=r) = \sum_{n=1}^{\infty} P(N \geq n) = \sum_{j=0}^{\infty} P(N > j).$$