IMPERIAL COLLEGE OF SCIENCE, TECHNOLOGY AND MEDICINE

EXAMINATIONS 2015

BEng Honours Degree in Computing Part I
MEng Honours Degrees in Computing Part I
for Internal Students of the Imperial College of Science, Technology and Medicine

This paper is also taken for the relevant examinations for the Associateship of the City and Guilds of London Institute

PAPER C145

MATHEMATICAL METHODS

Wednesday 13 May 2015, 10:00
Duration: 80 minutes

Answer ALL TWO questions

Paper contains 2 questions
Calculators required

1a i) Using the formal definition for converging sequences, show that the sequence:

$$a_n = \begin{cases} 1 & : n < 100 \\ \frac{n^3-1}{2n^3} & : n \geq 100 \end{cases}$$

converges for $n > 0$.

ii) Given that the sequence $\lim_{n\to\infty} b_n = b$ and **using the formal definition for converging sequences**, prove that

$$\lim_{n\to\infty}(2b_n + c) = 2b + c$$

for some $c \in \mathbb{R}$.

b Show using a suitable comparison test that the series S either converges or diverges and **give full details of the test you are using.**

$$S = \sum_{n=0}^{\infty} \frac{n}{n^2 + 3n - 2}$$

c For which range of values of x does the following series converge:

$$S = \sum_{n=1}^{\infty} \frac{n^3}{(x-2)^n} \qquad : \text{for } x \in \mathbb{R}, x \neq 2$$

The three parts carry, respectively, 50%, 25%, and 25% of the marks.

2a The three points $\vec{p}_1 = \vec{i} + \vec{k}$, $\vec{p}_2 = 4\vec{i} + \vec{j}$ and $\vec{p}_3 = 2\vec{i} + \vec{j} + \vec{k}$ lie in the plane T. Find an equation for the plane T in the form:

$$\vec{r} \cdot \vec{n} = d$$

b A line is given by the equation $\vec{r} = (\vec{i} + \vec{j}) + \lambda(\vec{i} - \vec{j} - \vec{k})$. It intersects a sphere with equation $|\vec{s}| = a$ at two distinct points on the surface of the sphere. Find the range of values of a for which this true.

c For the matrix, M, defined as:

$$M = \begin{pmatrix} 1 & -1 & 0 \\ -2 & 2 & 0 \\ -3 & 3 & -6 \end{pmatrix}$$

i) Find the general solution of the following matrix–vector equation:

$$M \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

ii) Hence, or otherwise, find the rank of M and justify your answer.

iii) Find all the eigenvalues of M.

The three parts carry, respectively, 25%, 25%, and 50% of the marks.