IMPERIAL COLLEGE OF SCIENCE, TECHNOLOGY AND MEDICINE

EXAMINATIONS 2018

BEng Honours Degree in Mathematics and Computer Science Part I
MEng Honours Degree in Mathematics and Computer Science Part I
BEng Honours Degree in Computing Part I
MEng Honours Degrees in Computing Part I
for Internal Students of the Imperial College of Science, Technology and Medicine

This paper is also taken for the relevant examinations for the Associateship of the City and Guilds of London Institute

PAPER C150=MC150

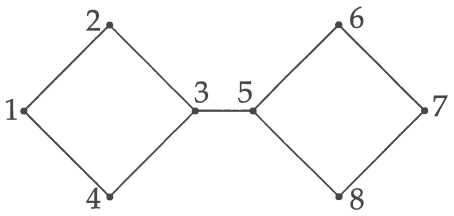
GRAPHS AND ALGORITHMS

Tuesday 8th May 2018, 14:00
Duration: 80 minutes

Answer ALL TWO questions

Paper contains 2 questions
Calculators not required

1 a Graph G is as shown in the diagram. How many isomorphisms are there from G to itself (including the identity)? Justify your answer briefly.



b i) Let G be an undirected connected graph.

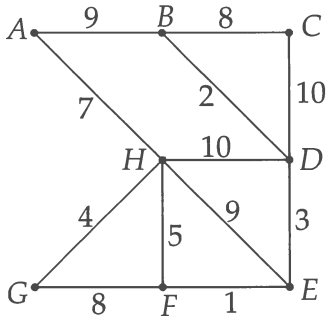
What does it mean for T to be a *spanning tree* for G?

ii) Let G be an undirected connected graph, and let T be a spanning tree for G. Let a be an arc of G (with endpoints x and y) which does not belong to T.

Show that if we form graph T' by adding a to T then T' has a *unique* cycle.

c i) Use Kruskal's algorithm to find a minimum spanning tree (MST) for the following weighted graph.

Give the MST as a diagram and also state the order in which the arcs are added.



ii) Does the graph from part (i) have a unique MST?

Justify your answer briefly.

d Let (G, W) be a weighted connected graph with natural number-valued weights and suppose that (G, W) has a *unique* MST T.

Let weight function W' be got by squaring all the arc weights given by W.

Is T necessarily also an MST for (G, W')? Justify your answer.

The four parts carry, respectively, 20%, 25%, 35%, and 20% of the marks.

2a The Binary Search algorithm is to be applied to ordered lists of length five, with elements indexed from 0 to 4.

i) Give the decision tree. Assume that the algorithm chooses the element with lower index at any point where there is a choice.

Also state the worst-case number of comparisons.

ii) Calculate the average number of comparisons, on the basis that the element being searched for is in the list, and all positions are equally likely.

b i) State the recurrence relation for the worst-case number of comparisons $W(n)$ for MergeSort applied to a list of n elements.

Do not solve the recurrence relation.

ii) Let us say that a list L of even length is *pair-swapped* if by swapping all adjacent pairs of elements we obtain a sorted list. As an example, for $n = 8$ the list $[2,1,4,3,6,5,8,7]$ is pair-swapped.

State a recurrence relation for the number of comparisons $S(n)$ performed by MergeSort applied to a pair-swapped list of length $n \geq 2$. Assume that n is a power of two and that all elements are distinct. Do not solve the recurrence relation.

c i) Let D and D' be decision problems. What does it mean for D to reduce to D' ($D \leq D'$)?

ii) Let $k \geq 1$. What does it mean for a graph to be k-colourable?

iii) The problem 3COL is defined as follows: given a graph G, is G 3-colourable?

Explain why 3COL belongs to the complexity class NP.

iv) The problem 4COL is defined as follows: given a graph G, is G 4-colourable?

Show that if 3COL is NP-complete then 4COL is NP-complete.

The three parts carry, respectively, 25%, 25%, and 50% of the marks.