IMPERIAL COLLEGE OF SCIENCE, TECHNOLOGY AND MEDICINE

EXAMINATIONS 2023

BEng Honours Degree in Computing Part II
MEng Honours Degrees in Computing Part II
BEng Honours Degree in Mathematics and Computer Science Part II
MEng Honours Degree in Mathematics and Computer Science Part II
BEng Honours Degree in Mathematics and Computer Science Part III
MEng Honours Degree in Mathematics and Computer Science Part III
for Internal Students of the Imperial College of Science, Technology and Medicine

This paper is also taken for the relevant examinations for the Associateship of the City and Guilds of London Institute

PAPER COMP50009

SYMBOLIC REASONING

Wednesday 10th May 2023, 10:00
Duration: 90 minutes

Answer ALL TWO questions

Paper contains 2 questions
Calculators not required

1 a Let K be the following definite logic program:

$p(Y,X) \leftarrow q(X,Y).$
$p(X,Y) \leftarrow q(X,Z), p(Z,Y).$
$q(a,b).$
$q(b,b).$

i) Give the ground instantiation of the program K.

ii) For each of the following Herbrand structures, state whether or not

* $*$ it is a model of the definite logic program above, and if it is a model, whether or not it is minimal;

* $*$ it is a supported Herbrand structure;

Justify your answers using the immediate consequence operator.

$$M_1 = \{q(a,b), q(b,b)\}$$

$$M_2 = \{q(a,b), q(b,b), p(b,a), p(a,a), p(a,b), p(b,b), q(a,a)\}$$

$$M_3 = \{q(a,b), q(b,b), p(b,a), p(a,a), p(a,b), p(b,b)\}$$

iii) Find the least Herbrand Model of the program K, $T_P\uparrow^{\omega}$.

b Consider the following normal logic program P:

$p(X) \leftarrow s(X,Y), \text{not } r(Y).$
$p(a) \leftarrow t(X,Y).$
$q(X,Y) \leftarrow t(Y,X), s(b,Y).$
$t(c,a).$
$r(b).$
$s(a,b).$
$s(b,c).$

i) Apply SLDNF resolution (using the Prolog selection rule) to find the computed answer(s) to the query $\leftarrow p(X)$. Give the computed answer substitution(s), if applicable.

Be sure to show *all* renaming substitutions and most-general unifiers (mgus) at each step where relevant.

ii) Define the signature $\mathcal{L}$ of the above program, then write down the Herbrand universe and Herbrand base for $\mathcal{L}$.

c Consider the following ASP program:

$$
\begin{aligned}
\{p(X,Y) : r(X)\} &\leftarrow r(Y), \mathtt{not}\ q(Y,X).\\
&\leftarrow p(X,X), q(X,X).\\
r(a)&.\\
q(a,a)&.\\
q(a,b)&.
\end{aligned}
$$

i) Give its relevant ground instantiation.

ii) Find the reduct of this program w.r.t. the Herbrand structure M specified below and state whether M is a stable model. Make sure you explain how the reduct was derived.

$$M = \{r(a), q(a,a), q(a,b)\}$$

The three parts carry, respectively, 45%, 35%, and 20% of the marks.

2a Consider the following formula F in the theory of linear integer arithmetic:

$$((x > 0) \vee ((y > 2) \wedge \neg(x > 0))) \rightarrow (y < 2)$$

i) Apply the boolean abstraction function B to produce a propositional logic formula $B(F)$ corresponding to F. When introducing names for propositional variables, start with the letter p and proceed alphabetically.

ii) Apply the Tseitin transformation to turn $B(F)$ into an equisatisfiable CNF formula. Use propositional variable x_1 to represent the largest subformula of $B(F)$, x_2 the next-largest subformula, etc.

In each clause of the CNF formula, order literals alphabetically according to the names of their underlying propositional variables (so that e.g. a literal involving p occurs before a literal involving q, and a literal involving x_1, occurs before a literal involving x_2).

iii) Simplify the CNF formula of ii) using boolean constraint propagation (BCP). Write down the simplified formula and any assignments to literals that are forced by BCP.

iv) Which literal(s) occurring in the simplified CNF formula of iii) would be given the highest score by the dynamic largest individual sum (DLIS) heuristic? Briefly justify your answer.

b A problem with the CDCL algorithm is that it can lead to a large number of conflict clauses being learned, which can lead to unacceptable space requirements.

Comment on the impact of the following strategies for managing conflict clauses, arguing whether they have the potential to affect the *soundness* of the algorithm (i.e., whether it correctly computes SAT or UNSAT), and the *termination* of the algorithm (whether the algorithm is theoretically guaranteed to terminate).

i) Removing subsumed clauses: when a conflict clause K is learned, remove every clause C such that the literals occurring in K form a subset of the literals occurring in C

ii) Deleting conflict clauses: periodically delete half of the conflict clauses that have been learned, selecting the clauses to be deleted at random.

iii) Removing subsumed clauses *and* deleting conflict clauses (i.e., using the techniques from i and ii in combination).

c A 4-SAT instance is a CNF formula in which every clause contains exactly four literals. The 4-SAT problem is the problem of determining the satisfiability of

4-SAT instances. Show that the 4-SAT problem is NP-complete, based on the fact that the related 3-SAT problem is known to be NP-complete.

The three parts carry, respectively, 50%, 30%, and 20% of the marks.