③ A- $A = \{x \in \mathbb{Z} \mid x^2 - 3x + 2 = 0\}$

$= \{x \in \mathbb{Z} \mid (x-2)(x-1) = 0\}$

$= \{2, 1\}$

$\Rightarrow$ Power set $A = \{\emptyset, \{1\}, \{2\}, \{1,2\}\}$.

B- $A = \{a, b, c\}$. $|A|^2$ possible pairs. For each pair, could be in relation or not.

$\Rightarrow 2^{|A|^2}$.

C- $f(x) = 3x \quad g(x) = 3x+1 \quad h(x) = 3x+2$

$f \circ g(x) = 3(3x+1) = 9x+3.$

$g \circ f(x) = 3(3x)+1 = 9x+1.$

$g \circ h(x) = 3(3x+2)+1 = 9x+7.$

$\cancel{g}$

$f \circ g \circ h(x) = 3(9x+7) = 27x+21.$

D- $f(a) = r$ for $\forall a \in \mathbb{Z}$ where $a = qm + r$ $0 \leq r < m$.

This function finds the remainder of a/m.

- Yes f is a function as for every number in the integers, it must have a remainder ~~of~~ when divided by an integer m.
- ~~NOT ONTO as for m<3, there can be no r~~
- IS ONTO ~~as r is~~ assuming the set containing r is only integers, for any given m, there is a remainder with a number for every $0 \leq r < m$.

- NOT ONE-TO-ONE for instance let $M=3$, $f(9)=0$ and $f(15)=0$. $\Rightarrow$ Two elements mapping to same value.

E- i) $P(A) = \{\emptyset, \{a\}, \{b\}, \{c\}, \{a,b\}, \{a,c\}, \{b,c\}, \{a,b,c\}\}$

$\Rightarrow$ Relation: If a set has the same number of elements as ~~{{{a},{b}}, {{~~ another set, they are contained in ~~the~~ the relation.

To prove equivalence relation, must be:

TRANSITIVE:

For any a, b, c, if $\langle a,b\rangle$ and $\langle b,c\rangle$ are in relation, then $|a| = |b|$ and $|b| = |c| \Rightarrow |a| = |c|$

$\Rightarrow \langle a,c\rangle$ in relation. (When $|a|$ is number of elements in a).

$\Rightarrow$ TRANSITIVE.

SYMMETRIC! ~~REFLEXIVE~~:

For a pair $\langle a,b\rangle \Leftrightarrow |a| = |b|$

$\Rightarrow |b| = |a| \Rightarrow \langle b,a\rangle$ is an element of relation too.

$\Rightarrow$ ~~REFLEXIVE~~. SYMMETRIC

~~Sym:~~

REFLEXIVE:

For an element a, $|a| = |a|$

$\Rightarrow \langle a,a\rangle$ is an element in relation for all a in binary relation

$\Rightarrow$ REFLEXIVE.

ii) $P(A)/\sim$ = Set of equivalence classes for each element.

$[\{a\}]_\sim = \{\{a\}, \{b\}, \{c\}\}$

$[\{b\}]_\sim =$ "

$[\{c\}]_\sim =$ "

$[\{a,b\}]_\sim = \{\{a,b\}, \{b,c\}, \{a,c\}\}$

$[\{a,c\}]_\sim =$ "

$[\{b,c\}]_\sim =$ "

$[\emptyset]_\sim = \{\emptyset\}$

$[\{a,b,c\}]_\sim = \{\{a,b,c\}\}$

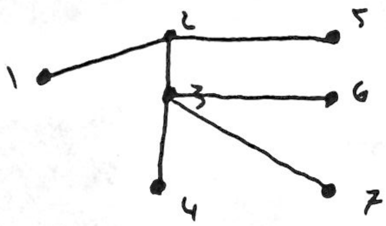
$\Rightarrow$ QUOTIENT SET $\{\{\{a\},\{b\},\{c\}\}, \{\{a,b\},\{b,c\},\{a,c\}\}, \{\emptyset\}, \{\{a,b,c\}\}\}$

F- Suppose A has elements, for every element, it could map to 2 different things. Therefore 2^n possible mappings, Therefore $|B| = 2^n$.

For $A \to B$ to be bijective, $|A| = |B|$ but $|B| = 2^{|A|}$.

$\Rightarrow$ NO BIJECTION.

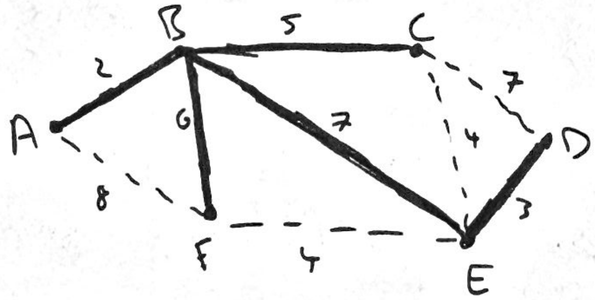
4. A- i) 1, 2, 3, 4, 3, 6, 3, 7, 3, 2, 5.



ii)

10. Suppose that when performing depth-first search we reach y before z. Then while executing the procedure call dfs(y) we will process z as it belongs to adj[y]. At this point we either add z to the tree as a child of y, or else z has already been processed during dfs(y) and is a descendant of y.
The case where we reach z before y is similar.

B- i)



A->D = 12.

ii) No, by Kruskal's algorithm, I must choose the smallest arcs
⟹ I would have chosen C->E with weight 4 instead of an arc like B->E, as this would produce a lower total weight.

iii) No, I could have chosen A->F rather than A->B->F as both had a total weight of 8, creating a different SPT.