PAPER C140=MC140

LOGIC

Friday 1 May 2020, 11:00
Duration: 80 minutes
Post-processing time: 30 minutes
Answer TWO questions

While this time-limited remote assessment has not been designed to be open book, in the present circumstances it is being run as an open-book examination. We have worked hard to create exams that assesses synthesis of knowledge rather than factual recall. Thus, access to the internet, notes or other sources of factual information in the time provided will not be helpful and may well limit your time to successfully synthesise the answers required.
Where individual questions rely more on factual recall and may therefore be less discriminatory in an open book context, we may compare the performance on these questions to similar style questions in previous years and we may scale or ignore the marks associated with such questions or parts of the questions. In all examinations we will analyse exam performance against previous performance and against data from previous years and use an evidence-based approach to maintain a fair and robust examination. As with all exams, the best strategy is to read the question carefully and answer as fully as possible, taking account of the time and number of marks available.

Paper contains 2 questions

1 a Remove as many brackets as possible from the following formula by applying the bracketing conventions.

$$\neg((\neg(p \wedge (q \to (\neg s)))) \vee (s \wedge p))$$

b Let A and B be arbitrary propositional formulas. Is it necessarily true that if $A \models B$ and $\neg A \not\models B$, then $A \equiv B$? Justify your answer.

c Consider the following argument stated in English.

If David's plant is healthy, it's also yellow provided it's not blooming. David's plant is blooming only if it's healthy. The plant is either not overwatered, not yellow, or unhealthy. So, David's plant isn't healthy.

i) Formalize the above sentences in propositional logic. Give the meaning for each propositional atom used in your formalization.

ii) Identify the premises and the conclusion in the above argument and determine whether it is "propositionally valid" using direct argument. If it is not, then give a situation demonstrating this.

d Using equivalences, rewrite the following formula into an equivalent formula in conjunctive normal form.

$$(p \to \neg q) \vee \neg(s \leftrightarrow r)$$

In each step, use only one equivalence rule, and state the general form of the equivalence rule used.

e Consider a new natural deduction rule $\vee^+ I$ that says if we can prove a formula B on the assumption $\neg A$ then we can deduce $A \vee B$. This is written in natural deduction style as:

1	$\neg A$	ass
	$\vdots$	
2	B	
3	$A \vee B$	$\vee^+ I$ (1,2)

Show that $\vee^+ I$ is a derived rule of natural deduction.

f Consider a different natural deduction system $\vdash^*$ in which the rule for $\to E$ (arrow elimination) is replaced by the elimination rule $\to^* E$ given below.

$$\begin{array}{lll} 1 & A \to B & \\ & \vdots & \\ 2 & \neg A & \\ 3 & \neg B & \to^{*} E(1,2) \end{array}$$

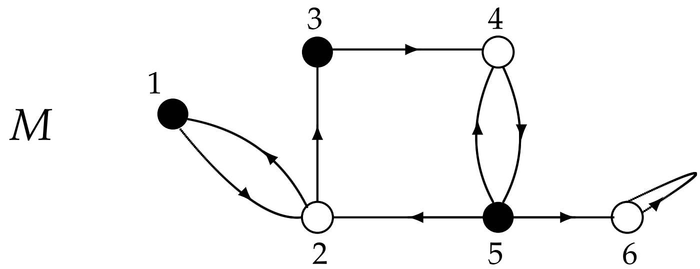
Is $\vdash^{*}$ sound? Justify your answer.

The six parts carry, respectively, 5%, 10%, 35%, 20%, 15%, and 15% of the marks.

2 a Prove using equivalences that the following argument is valid. In each step, use only one equivalence and state the general form of the equivalence used (for example, '$\exists x(A \wedge B) \equiv A \wedge \exists x B$ if x is not free in A').

$$\forall x[P(x) \wedge \exists y Q(x,y)] \models \exists x \neg P(x) \rightarrow \forall x \exists y Q(x,y)$$

b Let L be a signature consisting of a unary relation symbol P, a binary relation symbol Q and a constant c. Below is a diagram of an L-structure M. The arrows and the filled circles denote respectively the interpretation of Q and P (e.g., $M \models Q(2,3) \wedge P(3)$). The interpretation of c is not shown.



i) Give an interpretation of the constant c in M that makes the L-sentence $\exists x[Q(x,c) \wedge x = c]$ true in M.

ii) Let $Q(x,y)$ represent *x sees y*, or equivalently *y is seen by x*. Translate the following into L-sentences.

A) Everything that sees itself is seen by something else.

B) Some object sees only itself.

C) No object sees every other object

iii) The formula $Q(x,x)$ is true in M for $x = 6$. Similarly, list all objects x for which the following formulas are true in M. You do not need to justify your answers.

1. $\exists y[Q(x,y) \wedge Q(y,x)]$
2. $\exists y \forall z(Q(y,x) \wedge Q(z,x) \leftrightarrow z = y)$
3. $\exists y \exists z(Q(y,x) \wedge Q(z,x) \wedge y \neq z \wedge P(y) \wedge P(z))$

iv) Write down an L-formula $A(x)$ that is true in M for $x = 5$ only. (Your formula should have a single free variable, x).

c Using natural deduction, show that

$$\exists y \forall x(P(x) \leftrightarrow x = y), \quad \forall x,y(Q(x,y) \rightarrow \neg P(y)) \quad \vdash \quad \exists x(P(x) \wedge \forall y \neg Q(y,x))$$

The three parts carry, respectively, 20%, 45%, and 35% of the marks.