

COMP245: Probability and Statistics 2016 - Problem Sheet 7

Jointly Distributed Random Variables

Q1) Suppose the joint pdf of a pair of continuous random variables is given by

$$f(x, y) = \begin{cases} k(x + y), & 0 < x < 2, 0 < y < 2 \\ 0, & \text{otherwise.} \end{cases}$$

- (a) Find the constant k .
- (b) Find the marginal pdfs of X and Y .
- (c) Find if X and Y are independent.

Q2) A manufacturer has been using two different manufacturing processes to make computer memory chips. Let X and Y be two continuous random variables, where X denotes the time to failure of chips made by process A and Y denotes the time to failure of chips made by process B . Assuming that the joint pdf of (X, Y) is

$$f(x, y) = \begin{cases} abe^{-(ax+by)}, & x, y > 0 \\ 0, & \text{otherwise.} \end{cases}$$

where $a = 10^{-4}$ and $b = 1.2 \times 10^{-4}$, determine $P(X > Y)$.

Q3) The joint probability mass function of two discrete random variables X and Y is given by $p(x, y) = cxy$ for $x = 1, 2, 3$ and $y = 1, 2, 3$, and zero otherwise. Find

- (a) the constant c ;
- (b) $P(X = 2, Y = 3)$;
- (c) $P(X \leq 2, Y \leq 2)$;
- (d) $P(X \geq 2)$;
- (e) $P(Y < 2)$;
- (f) $P(X = 1)$;
- (g) $P(Y = 3)$.

Q4) Let X and Y be continuous random variables having joint density function $f(x, y) = c(x^2 + y^2)$ when $0 \leq x \leq 1$ and $0 \leq y \leq 1$, and $f(x, y) = 0$ otherwise. Determine

- (a) the constant c ;
- (b) $P(X < 1/2, Y > 1/2)$;
- (c) $P(1/4 < X < 3/4)$;
- (d) $P(Y < 1/2)$;
- (e) whether X and Y are independent.