IMPERIAL COLLEGE OF SCIENCE, TECHNOLOGY AND MEDICINE

EXAMINATIONS 2019

BEng Honours Degree in Computing Part I
MEng Honours Degrees in Computing Part I
BEng Honours Degree in Mathematics and Computer Science Part I
MEng Honours Degree in Mathematics and Computer Science Part I
for Internal Students of the Imperial College of Science, Technology and Medicine

This paper is also taken for the relevant examinations for the Associateship of the City and Guilds of London Institute

PAPER C140=MC140

LOGIC

Friday 3rd May 2019, 14:00
Duration: 80 minutes

Answer ALL TWO questions

Paper contains 2 questions
Calculators not required

1 a Determine the truth value of the formula

$$\neg p \to p \leftrightarrow p \vee \neg p \wedge q$$

in a situation in which p is false and q is true. Show all working.

b Using equivalences, show that the following formula is valid:

$$(\neg p \vee q \to p) \to p \vee q.$$

In each step, state the equivalence used (for example, $\neg\neg A \equiv A$). Use only one equivalence per step.

c Prove using natural deduction that $\neg(A \wedge C),\ \neg C \to B\ \vdash\ A \vee \neg C \to B$.

d Let $*$ be a new unary connective. The 'monotonicity' natural deduction rule for $*$ is as follows:

1	$*A$	got this somehow…
2	$A \to B$	…and this…
3	$*B$	monot(1,2)

i) Assuming that the rule is sound, what are the possible truth tables for $*$? Briefly justify your answer.

ii) Using this rule and the standard rules for the other connectives, show that:

A) $*A \vdash **A$

B) $**A \vdash *A$

The four parts carry, respectively, 10%, 20%, 25%, and 45% of the marks.

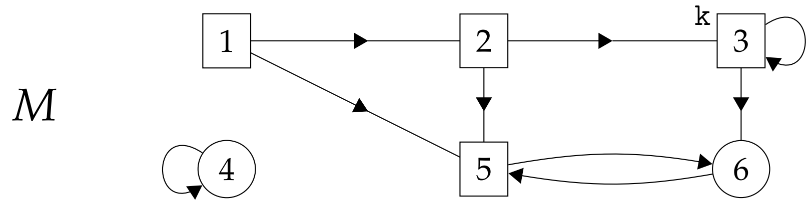
2 In this question, L is a signature containing a constant $\mathtt{k}$, unary relation symbols Q and S, and a binary relation symbol R.

a For each of the following L-sentences, either prove using equivalences that it is valid (use only one equivalence per step), or explain briefly why it is not valid.

i) $\forall x \exists y (Q(x) \to Q(y))$

ii) $\forall x \forall y (Q(x) \to Q(y))$

b Below is a diagram of an L-structure M with six objects in its domain, and in which S is true at precisely the square objects (1, 2, 3, 5); $\mathtt{k}$ is interpreted as object 3; and the arrows indicate the interpretation of R (for example, $M \models R(1,2) \wedge \neg R(2,1)$). The interpretation of Q is not shown.



The formula $R(x,x)$ is true in M for $x = 3$ and $x = 4$ only. Similarly, list all x for which each of the following formulas is true in M. You do not need to justify your answers.

i) $\exists y (R(x,y) \wedge R(y,\mathtt{k}))$

ii) $\forall y (R(y,x) \to S(y))$

iii) $\exists y \forall z (R(x,z) \to z = x \vee z = y)$

iv) $\forall y \forall z (R(y,x) \wedge R(x,z) \to (S(y) \leftrightarrow S(z)))$

v) $\exists y \forall z (R(x,z) \leftrightarrow R(z,y))$

c Let M be as in part b, and suppose that

$$M \models \forall x (\forall y [R(y,x) \wedge S(y) \to Q(y)] \to Q(x)).$$

Write down all possible interpretations of Q in M. Briefly justify your answer.

d Using natural deduction, show that

$$\forall x \exists y R(x,y), \quad \forall y (\exists x R(x,y) \to y = \mathtt{k}) \quad \vdash \quad \exists y \forall x R(x,y).$$

The four parts carry, respectively, 25%, 25%, 20%, and 30% of the marks.