CO245 - Assessed Coursework

Due date: 24th November 2016 — 7pm
Submit solutions on CATe

Answer all questions.

Q1) (a) Suppose P is a probability measure of $(S, \mathcal{F})$, where S is the sample space and $\mathcal{F}$ is a σ-algebra on S. For a fixed event $F \subseteq S$, with $P(F) > 0$ show that the conditional probability $P(\cdot|F)$ is a probability measure.

(b) If the events E and F are independent, show that $\overline{E}$ and $\overline{F}$ are independent.

(c) To model the number of visits of a particular individual to my website during a five year period, I make the assumption that $P(E_{n+1}) = 0.3P(E_n)$, $n \geq 0$, where E_n is the event that the individual makes n visits during the period. Under this assumption, determine $P(E_0)$ and calculate the probability that the individual makes more than one visit to my website. 12 marks

Q2) Let $X \sim \text{Poisson}(\lambda)$, for some $\lambda > 0$.

(a) Verify that the Poisson distribution has a valid probability mass function.

(b) Find $E(X^2)$.

(c) Suppose Z is another discrete random variable with

$$P(Z = 16) = 0.2017$$

and for $z \in \mathbb{R} \backslash \{16\}$

$$P(Z = z) \propto P(X = z).$$

Find the probability mass function of Z. 10 marks

Q3) Three snooker players, Jimmy Whirlwind, Stevie Henny and Rocketto Silvain take turns at potting the black ball off its spot. They take a single shot in the order Jimmy, Stevie, Rocketto, Jimmy, Stevie, Rocketto, ... The first to pot the black wins. Jimmy Whirlwind has a probability p_{JW} chance of potting the black ball, Stevie Henny has p_{SH} and Rocketto Silvain has p_{RS}. The shot outcomes are independent. Calculate the probability of Jimmy Whirlwind, Stevie Henny and Rocketto Silvain winning.

Rocketto Silvain is upset as he believes he is at a disadvantage since he plays after Jimmy and Stevie. Is he correct? 13 marks

Q4) Every MacContent meal contains a plastic toy drawn at random from a set of k different toys. Let N be the number of meals bought in order to obtain a full set. Let $X_1 = 1$ be the number of meals required to get the first toy. For $i = 2, 3, \ldots,$ let X_i be the number of further meals required to obtain the i^{th} toy after the $(i-1)^{\text{th}}$ toy has been obtained.

(a) Write N in terms of the X_i.

(b) What is the distribution of each X_i?

(c) Find $E(N)$ and $\text{Var}(N)$ simplifying your answers as far as possible,

10 marks