

COMP245: Probability and Statistics 2016 - Problem Sheet 7

Solutions

Jointly Distributed Random Variables

S1) (a) We require

$$\begin{aligned}
 1 &= \int_{y=-\infty}^{\infty} \int_{x=-\infty}^{\infty} f_{XY}(x, y) dx dy = \int_{y=0}^2 \int_{x=0}^2 k(x+y) dx dy \\
 &= k \int_{y=0}^2 \left[\frac{x^2}{2} + yx \right]_{x=0}^2 dy = 2k \int_{y=0}^2 (y+1) dy = 2k \left[\frac{y^2}{2} + y \right]_0^2 = 8k \\
 \Rightarrow k &= \frac{1}{8}.
 \end{aligned}$$

(b)

$$\begin{aligned}
 f_X(x) &= \int_{y=-\infty}^{\infty} f_{XY}(x, y) dy = \frac{1}{8} \int_{y=0}^2 (x+y) dy = \frac{1}{8} \left[xy + \frac{y^2}{2} \right]_{y=0}^2 \\
 &= \frac{1}{4}(x+1),
 \end{aligned}$$

for $0 < x < 2$, and 0 otherwise. Identically for $f_Y(y)$.

(c) Since $f(x, y) \neq f(x)f(y)$, X and Y are not independent.

S2)

$$\begin{aligned}
 P(X > Y) &= \int_{y=-\infty}^{\infty} \int_{x=y}^{\infty} f_{XY}(x, y) dx dy = \int_{y=0}^{\infty} \int_{x=y}^{\infty} abe^{-(ax+by)} dx dy \\
 &= ab \int_{y=0}^{\infty} e^{-by} \int_{x=y}^{\infty} e^{-ax} dx dy = ab \int_{y=0}^{\infty} e^{-by} \left[\frac{-e^{-ax}}{a} \right]_{x=y}^{\infty} dy \\
 &= b \int_{y=0}^{\infty} e^{-by} e^{-ay} dy = b \int_{y=0}^{\infty} e^{-(a+b)y} dy \\
 &= b \left[\frac{-e^{-(a+b)y}}{a+b} \right]_{y=0}^{\infty} \\
 &= \frac{b}{a+b} \\
 &= \frac{1.2 \times 10^{-4}}{10^{-4} + 1.2 \times 10^{-4}} = \frac{12}{22} = 0.545.
 \end{aligned}$$

S3) (a) $\frac{1}{36}$.
 (b) $\frac{1}{6}$.

(c) $\frac{1}{4}$.
 (d) $\frac{5}{6}$.

(e) $\frac{1}{6}$.
 (f) $\frac{1}{6}$.

(g) $\frac{1}{2}$.

- S4) (a) $\frac{3}{2}$. (b) $\frac{1}{4}$. (c) $\frac{29}{64}$. (d) $\frac{5}{16}$.
(e) Not independent, $f(x, y) \neq f(x)f(y)$.