IMPERIAL COLLEGE LONDON

TIMED REMOTE ASSESSMENTS 2020-2021

BEng Honours Degree in Computing Part I
MEng Honours Degrees in Computing Part I
BEng Honours Degree in Mathematics and Computer Science Part I
MEng Honours Degree in Mathematics and Computer Science Part I
for Internal Students of the Imperial College of Science, Technology and Medicine

This paper is also taken for the relevant assessments for the Associateship of the City and Guilds of London Institute

PAPER COMP40008

GRAPHS AND ALGORITHMS

Friday 14 May 2021, 10:00
Duration: 95 minutes
Includes 15 minutes for access and submission

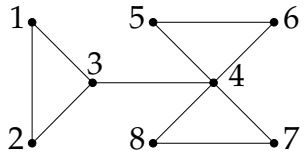
Answer ALL TWO questions
Open book assessment

This time-limited remote assessment has been designed to be open book. You may use resources which have been identified by the examiner to complete the assessment and are included in the instructions for the examination. You must not use any additional resources when completing this assessment.

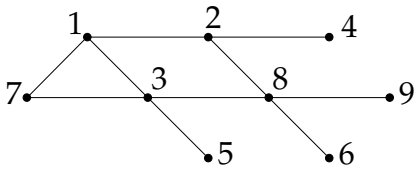
The use of the work of another student, past or present, constitutes plagiarism. Giving your work to another student to use constitutes an offence. Collusion is a form of plagiarism and will be treated in a similar manner. This is an individual assessment and thus should be completed solely by you. The College will investigate all instances where an examination or assessment offence is reported or suspected, using plagiarism software, vivas and other tools, and apply appropriate penalties to students. In all examinations we will analyse exam performance against previous performance and against data from previous years and use an evidence-based approach to maintain a fair and robust examination. As with all exams, the best strategy is to read the question carefully and answer as fully as possible, taking account of the time and number of marks available.

Paper contains 2 questions

1 a State the number of automorphisms (including the identity) for the undirected graph in the following diagram. Give a brief explanation for your answer.



b Consider the following undirected graph G:



i) State the order of traversal of the nodes in G starting from node 1 for *depth-first* search. Also draw the associated spanning tree. Assume that where there is a choice the numerically least node is chosen.

ii) State the order of traversal of the nodes in G starting from node 1 for *breadth-first* search. Also draw the associated spanning tree. Assume that where there is a choice the numerically least node is chosen.

c Show that an undirected graph has an Euler circuit if and only if it has a circuit which uses each arc exactly three times. You may use results from the lectures.

d A bakery can make a profit of $P[k]$ on a cake made with k units of mixture (for any $k \geq 1$). Write a program (in pseudocode or Java) to compute the maximum profit achievable if the bakery has a total of n units of mixture at its disposal. Here n and k are natural numbers and the total mixture is to be divided into whole number units.

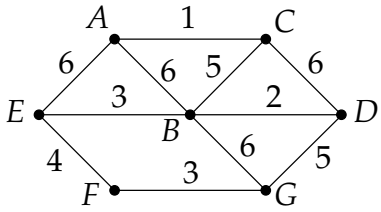
Your pseudocode should use a bottom-up (non-recursive) dynamic programming style.

If you wish, you can use the following outline and fill in the blanks ($\cdots$):

```
for i = 0 to n:
    M[i] = ···
for i = 1 to n:
    for j = 1 to i:
        ...
return M[n]
```

The four parts carry, respectively, 20%, 30%, 25%, and 25% of the marks.

2a i) Use Kruskal's algorithm to find a minimum spanning tree (MST) for the following weighted graph. Give the MST as a diagram and also state the order in which the arcs are added.



ii) Does the graph from part (i) have a *unique* MST? Explain your answer.

iii) Suppose that Kruskal's algorithm is implemented using union-find with weighted union (on size) but not path compression, and using a parent relation on nodes.

Show as a non-binary tree the final state of the parent relation resulting from applying Kruskal's algorithm with union-find to the graph in part (i). Assume that where the union is taken of two trees of equal size with leaders x and y respectively, the new leader of the combined trees is the alphabetically first node among x and y.

b Let E be an array of elements with keys $[2, 4, 1, 8, 6, 3, 7, 5]$.

i) Draw (as a tree) the (binary) heap structure corresponding to E.

ii) Use BuildMaxHeap (as defined in the lectures) to convert your answer from (i) into a max heap.

Give your answer both as a tree and as an array.

c i) The problem 3COLN is as follows: given an undirected simple graph G and three distinct nodes x, y, z of G (which may or may not be adjacent), can G be 3-coloured in such a way that x, y, z all receive different colours?

Explain why 3COLN belongs to the complexity class NP.

ii) Show that 3COLN is NP-complete.

You may assume that the following problem 3COL is NP-complete: given an undirected simple graph G, is G 3-colourable?

The three parts carry, respectively, 40%, 25%, and 35% of the marks.