Given:

(1) $str[..) \approx str[..)_{pre}$

(2) $0 \leq i \leq str.length$

(3) $0 \leq j \leq stack.length$

(4) $stack.length - j \leq i \leq j$

(5) $Wb(str[..i) : stack[j..))$

(6) $\forall d \in stack[j..).\ [Cb(d)]$

(7) $i < str.length$

(8) $c = str[i]$

(9) $Cb(c)$

(10) $j = stack.length \lor c \,!\!= stack[j]$

(11) $\underline{r} \longleftrightarrow \bot$

To show:

(α) $str[..) \approx str[..)_{pre}$

(β) $\underline{r} \leftrightarrow Wb(str[..))$

Proof

(α) from (1)

Case analysis on (10)

(case 1) $j = stack.length$

(12) $Wb(str[..i))$ by (5)(case 1)

(13) $Cb(str[i])$ by (8)(9)

(14) $\neg Wb(str[..i) : str[i] : str[i+1..))$ by (12)(13)(lemma 1)

(15) $\neg Wb(str[..))$ by (14)

(case 2) $j \,!\!= stack.length \land c \,!\!= stack[j]$

(16) $c \,!\!= stack[j]$ by (case 2)

(17) $Wb(str[..i) : stack[j] : stack[j+1..))$ by (5)(case 2)

(18) $\neg Wb(str[..i) : str[i] : str[i+1..))$ by (17)(lemma 2) (8)(9)(16)

(19) $\neg Wb(str[..))$

(20) $\neg Wb(str[..))$ by (10) (case 1)(case 2)(15)(19)

(21) $Wb(str[..)) \leftrightarrow \bot$ by (20)

(22) $\underline{r} \leftrightarrow Wb(str[..))$ by (21)(11)