IMPERIAL COLLEGE LONDON

TIMED REMOTE ASSESSMENTS 2021-2022

BEng Honours Degree in Computing Part I
MEng Honours Degrees in Computing Part I
for Internal Students of the Imperial College of Science, Technology and Medicine

This paper is also taken for the relevant assessments for the Associateship of the City and Guilds of London Institute

PAPER COMP40017

LINEAR ALGEBRA

Wednesday 18 May 2022, 10:00
Writing time: 80 minutes
Upload time: 25 minutes

Answer ALL TWO questions
Open book assessment

Paper contains 2 questions

1 Let $A\vec{x} = \vec{b}$ be a system of linear equations where,

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \end{bmatrix} \quad \vec{b} = \begin{bmatrix} 8 \\ 4 \\ 0 \end{bmatrix}$$

a i) Perform elementary row operations on the augmented matrix $[A|\vec{b}]$ to reduce A to its Reduced Row Echelon Form.

ii) State the free variables.

iii) Find the general solution (Solution set S) of $A\vec{x} = \vec{b}$ and describe it geometrically.

b Find the solution of $A\vec{x} = \vec{b}$ closest (lowest squared Euclidean distance) to the origin $\vec{0}$.

c i) Find the rank and nullity of A.

ii) Find a basis for the image space $im(A)$.

iii) Find a basis for the kernel $ker(A)$.

d i) Find the rank and nullity of $A^\top$.

ii) Find a basis for the image space $im(A^\top)$.

iii) Find a basis for the kernel $ker(A^\top)$.

The four parts carry equal marks.

2 a Let $\vec{a}_1, \vec{a}_2, \vec{a}_3, \vec{a}_4, \vec{b}_1, \vec{b}_2, \vec{b}_3, \vec{b}_4 \in \mathbb{R}^4$ such that,

$$\vec{a}_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \vec{a}_2 = \begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \end{bmatrix}, \vec{a}_3 = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \vec{a}_4 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\vec{b}_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \vec{b}_2 = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \vec{b}_3 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \end{bmatrix}, \vec{b}_4 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

Let $A = (\vec{a}_1, \vec{a}_2, \vec{a}_3, \vec{a}_4)$ and $B = (\vec{b}_1, \vec{b}_2, \vec{b}_3, \vec{b}_4)$.

i) Show that A and B are two bases of $\mathbb{R}^4$.

ii) Compute the matrix I_{AB} that performs a basis change from B to A.

iii) Also, compute the matrix I_{BA} that performs a basis change from A to B.

b Find the intersection $S \cap U$ where,

$$U = \text{span}\left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}, \begin{bmatrix} 5 \\ 6 \\ 7 \\ 8 \end{bmatrix}, \begin{bmatrix} 9 \\ 10 \\ 11 \\ 12 \end{bmatrix} \right\} \text{ and } S = \begin{bmatrix} -10 \\ 9 \\ 0 \\ 0 \end{bmatrix} + \text{span}\left\{ \begin{bmatrix} 1 \\ -2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ -3 \\ 0 \\ 1 \end{bmatrix} \right\}$$

.

c Let $M = \begin{bmatrix} -2 & 1 \\ 12 & -3 \end{bmatrix}$ and $M^{13} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$.

i) Compute the Eigenvalues and Eigenspaces of M.

ii) Determine a transformation matrix B such that $B^{-1}MB$ is a diagonal matrix and provide this diagonal matrix.

iii) Compute M^{13}, i.e., compute a, b, c, d.

The three parts carry, respectively, 30%, 35%, and 35% of the marks.