1 a Consider the following program Π:

$$
\begin{aligned}
a &\leftarrow \text{not } c. \\
b &\leftarrow a. \\
c &\leftarrow \text{not } a. \\
d &\leftarrow c. \\
e &\leftarrow b. \\
e &\leftarrow d, \text{not } c.
\end{aligned}
$$

Consider the sets:

$$
\begin{aligned}
S_1 &= \{\} \\
S_2 &= \{a\} \\
S_3 &= \{a, b\} \\
S_4 &= \{c, d\} \\
S_5 &= \{a, b, c, d, e\}
\end{aligned}
$$

i) For each set S_i, state whether or not it is a *model* of Π. If S_i is not a model of Π, provide a rule in Π that is violated by S_i.

ii) Provide a *splitting set* for Π that is non-trivial (i.e. it is neither the empty set $\{\}$ nor the set of all atoms $\{a, b, c, d, e\}$).

iii) List all the stable models of Π.

b Consider the following program:

$$
\begin{aligned}
q &\leftarrow \text{not } s. \\
r &\leftarrow t, \text{not } p. \\
s &\leftarrow \text{not } r. \\
r &\leftarrow t, \text{not } p.
\end{aligned}
$$

Show that this program is stratifiable, and use the stratification to compute the (single) stable model.

c Let Π be a ground normal logic program, and let X be a set of ground atoms. Show that if X is not a model of Π, then X is not a stable model of Π.

The three parts carry, respectively, 45%, 25%, and 30% of the marks.

2 a The formulas $p \rightarrow q$ and $\neg q \rightarrow \neg p$ are equivalent in propositional logic. Compare the following extended logic programs:

$$q \leftarrow p.$$

and:

$$\neg p \leftarrow \neg q.$$

Are these programs equivalent, strongly equivalent, or neither? Justify your answer.

b Compare the following programs:

$$q \leftarrow \text{not } p.$$
$$q \leftarrow p.$$

and:

$$q \leftarrow .$$

(The second program is the single fact q.)
Are these programs equivalent, strongly equivalent, or neither? Justify your answer.

c Consider the following:

- r_1: birds can usually fly
- r_2: except for penguins, who usually cannot fly
- r_3: magical creatures can usually fly

Here, r_2 is an exception to r_1, and r_3 is an exception to r_2.

i) Express these defeasible rules, and the exceptions between them, as an extended logic program.

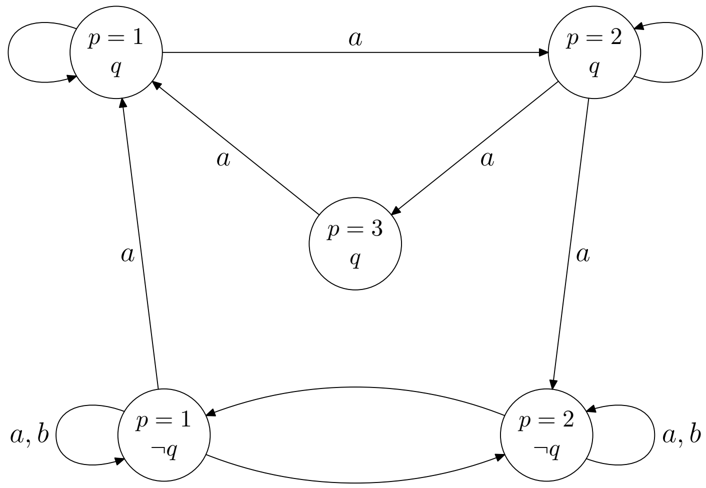
ii) Suppose Xavier is a penguin, and Yasmine is a magical penguin. What does your program conclude (in terms of *cautious* entailment) about whether Xavier and Yasmine can fly?

If you remove the exceptions between the rules, what does your program conclude (in terms of *cautious* entailment) about Xavier and Yasmine?

How many answer sets are there altogether, if you remove the exceptions between the rules?

The three parts carry, respectively, 20%, 30%, and 50% of the marks.

3 a Consider the following diagram:



This depicts a labelled transition system (LTS) defined by a $\mathcal{C}+$ action description, D.

i) Write down a $\mathcal{C}+$ action signature $(\sigma^{\mathrm{f}}, \sigma^{\mathrm{a}})$ for D.
(Be sure to specify the domains of constants.)

Suppose that the following causal laws are included in D:

inertial p
inertial q
exogenous a
exogenous b

Complete D, so that it defines the LTS depicted above, by writing down in order:

ii. Any static causal laws for D.

iii. Causal laws containing the keyword **nonexecutable** (to constrain which actions can be performed in which states).

iv. The remaining dynamic causal laws, to describe the effects of actions.

(Note that there are many possible correct answers.)

b Suppose a $\mathcal{C}+$ action signature in which all constants are Boolean, and

$$\sigma^{\mathrm{f}} = \{p, q\}$$
$$\sigma^{\mathrm{a}} = \{a, b\}$$

Consider the following $\mathcal{C}+$ action description, D', using this signature:

inertial p
inertial q
exogenous a
caused b **if** $b \wedge a$
caused $\neg b$ **if** $\neg b \wedge a$
caused $\perp$ **if** p
a **causes** q **if** $\neg q$
$a \wedge b$ **causes** $\neg q$ **if** q
nonexecutable $a \wedge b$ **if** $\neg q$

With respect to that action description:

i) Give the interpretations $s \in \mathbf{I}(\sigma^{\mathrm{f}})$ such that $s \models T_{static}(s)$.

ii) For $s = \{\neg p, q\}$ and every $\varepsilon \in \mathbf{I}(\sigma^{\mathrm{a}})$, write down the set $A(s,\varepsilon)$. Note, for this s, those ε such that $\varepsilon = A(s,\varepsilon)$.

iii) For $s = \{\neg p, q\}$, $\varepsilon = \{a, b\}$, and for every $s' \in \mathbf{I}(\sigma^{\mathrm{f}})$, find $E(s,\varepsilon,s')$. Thus find all triples (s,ε,s'), for the specific s and ε, in the labelled transition system defined by D'.

The two parts carry equal marks.

4 a Consider the following knowledge:

> Three cars, a, b, and c, can each be either driving or not. At any point, each of them is either at the *start*, *middle*, or *end* of the route. Driving when at the *start* takes a car to the *middle*; driving when at the *middle* takes a car to the *end*; and then, no further driving is possible. However, any number of cars which are currently driving may crash; if a car crashes, it cannot drive any more.

For this knowledge:

i) Write down a suitable $\mathcal{C}+$ action signature $(\sigma^{\mathrm{f}}, \sigma^{\mathrm{a}})$, being sure to include the domains for all constants.

ii) Write down a $\mathcal{C}+$ action descripton, using the signature from 4(a.i), to formalize the knowledge above.

Now consider the following knowledge:

> A barrier is placed between the *start* and *middle* of the route, which can be moved to be up or down. When up, things proceed as before. When down, then no cars can drive between the *start* and the *middle*.

This adds to the preceding knowledge.

iii) Make additions to, or otherwise modify, the signature and action description of (i) and (ii), in order to incorporate the new knowledge in your $\mathcal{C}+$.

b Let A and B be two formulas of propositional logic. Suppose that Cn is a consequence operator (not necessarily monotonic) satisfying $\mathrm{Th}(\mathrm{Cn}(W)) \subseteq \mathrm{Cn}(W)$, for any set W of formulas.

i) Suppose that $(A \rightarrow B) \in \mathrm{Cn}(W)$.

Prove that if $A \in \mathrm{Cn}(W)$, then $B \in \mathrm{Cn}(W)$.

ii) Suppose, instead, that if $A \in \mathrm{Cn}(W)$, then $B \in \mathrm{Cn}(W)$. Suppose, further, that $\mathrm{Cn}(W)$ is consistent—i.e., that there is no formula X such that $X \in \mathrm{Cn}(W)$ and $\neg X \in \mathrm{Cn}(W)$.

Prove, on these suppositions, that $(A \rightarrow B)$ is consistent with $\mathrm{Cn}(W)$—i.e., that $\neg(A \rightarrow B) \notin \mathrm{Cn}(W)$.

The two parts carry, respectively, 60% and 40% of the marks.

1 a Consider the following program Π:

$$\begin{aligned} a &\leftarrow \text{not } c. \\ b &\leftarrow a. \\ c &\leftarrow \text{not } a. \\ d &\leftarrow c. \\ e &\leftarrow b. \\ e &\leftarrow d, \text{not } c. \end{aligned}$$

Consider the sets:

$$\begin{aligned} S_1 &= \{\} \\ S_2 &= \{a\} \\ S_3 &= \{a, b\} \\ S_4 &= \{c, d\} \\ S_5 &= \{a, b, c, d, e\} \end{aligned}$$

i) For each set S_i, state whether or not it is a *model* of Π. If S_i is not a model of Π, provide a rule in Π that is violated by S_i.

S_1 is not a model, since it violates $a \leftarrow$ not c. S_2 is not a model, since it violates $b \leftarrow a$. S_3 is not a model, since it violates $e \leftarrow b$. S_4 is a model. S_5 is a model.

Marks: **3**

ii) Provide a *splitting set* for Π that is non-trivial (i.e. it is neither the empty set $\{\}$ nor the set of all atoms $\{a, b, c, d, e\}$).

$\{a, c\}$ is a suitable splitting set.

Marks: **2**

iii) List all the stable models of Π.

The stable models are $\{a, b, e\}$ and $\{c, d\}$.

Marks: **4**

b Consider the following program:

$$\begin{aligned} q &\leftarrow \text{not } s. \\ r &\leftarrow t, \text{not } p. \\ s &\leftarrow \text{not } r. \\ r &\leftarrow t, \text{not } p. \end{aligned}$$

Show that this program is stratifiable, and use the stratification to compute the (single) stable model.

The program can be divided into three strata. In the first stratum is $q \leftarrow \text{not } s$*. In the second stratum is* $s \leftarrow \text{not } r$*. In the third stratum is* $r \leftarrow t, \text{not } p$*. and* $r \leftarrow t, \text{not } p$*. The unique stable model is* $\{q, r, t\}$*.*

Marks: **5**

c Let Π be a ground normal logic program, and let X be a set of ground atoms. Show that if X is not a model of Π, then X is not a stable model of Π.

We prove by contraposition: if X *is a stable model of* Π*, then* X *is a model of* Π*. Since* X *is stable,* $X = M(P^X)$*. Consider any clause* r *of the form* $A_0 \leftarrow A_1, ..., A_m, \text{not } A_{m+1}, ..., \text{not } A_n$*. If* $X \not\models body(r)$ *then* $X \models r$*. Otherwise,* $X \models body(r)$*. In other words,* $\{A_1, ..., A_m\} \subseteq X$ *and* $\{A_{m+1}, ..., A_n\} \cap X = \emptyset$*. Since* $\{A_{m+1}, ..., A_n\} \cap X = \emptyset$*,* $A_0 \leftarrow A_1, ..., A_m$ *is in the reduct* P^X*. Since* $\{A_1, ..., A_m\} \subseteq X$*,* $A_0 \leftarrow A_1, ..., A_m \in P^X$*, and* $M(P^X)$ *is closed under* $T_{P^X}(.)$*,* $A_0 \in X$*. Hence* $X \models r$*.*

Marks: **6**

The three parts carry, respectively, 45%, 25%, and 30% of the marks.

2 a The formulas $p \rightarrow q$ and $\neg q \rightarrow \neg p$ are equivalent in propositional logic. Compare the following extended logic programs:

$$q \leftarrow p.$$

and:

$$\neg p \leftarrow \neg q.$$

Are these programs equivalent, strongly equivalent, or neither? Justify your answer.

The programs are equivalent but not strongly equivalent. They are equivalent since they both have the same single answer set $\{\}$. To see that they are not strongly equivalent, add the fact $p \leftarrow$. Adding this fact to the first program creates the single answer set $\{p, q\}$, while adding this fact to the second program produces the single answer set $\{\}$. Another way to see that they are not strongly equivalent is to use the logic of here and there (HT).

Marks: **4**

b Compare the following programs:

$$q \leftarrow \text{not } p.$$
$$q \leftarrow p.$$

and:

$$q \leftarrow .$$

(The second program is the single fact q.)
Are these programs equivalent, strongly equivalent, or neither? Justify your answer.

The programs are equivalent. They both have the single answer set $\{q\}$. They are not strongly equivalent. This can be shown in two ways. First, by adding a single rule to both programs: $p \leftarrow q$. Adding this rule to the first program makes it unsatisfiable, while adding this rule to the second program produces a program with the single answer set $\{p, q\}$.

Another way to see that they are not strongly equivalent is to use the logic of here and there (HT). The truth table should have 9 rows. The HT formula for the first program is $(q \leftarrow p) \wedge (q \leftarrow (\bot \leftarrow p))$. This formula has the same truth *value as q in 8 out of the 9 rows, but has a different value when $q = p = u$. In this case, the formula $(q \leftarrow p) \wedge (q \leftarrow (\perp \leftarrow p))$ gets the value 1 while $q = u$.*

Marks: **6**

c Consider the following:

- r_1: birds can usually fly
- r_2: except for penguins, who usually cannot fly
- r_3: magical creatures can usually fly

Here, r_2 is an exception to r_1, and r_3 is an exception to r_2.

i) Express these defeasible rules, and the exceptions between them, as an extended logic program.

$$sat(r_1(X)) \leftarrow bird(X).$$
$$fires(r_1(X)) \leftarrow sat(r_1(X)), \text{not } \neg flies(X), \text{not } \neg fires(r_1(X)).$$
$$flies(X) \leftarrow fires(r_1(X)).$$

$$sat(r_2(X)) \leftarrow penguin(X).$$
$$fires(r_2(X)) \leftarrow sat(r_2(X)), \text{not } flies(X), \text{not } \neg fires(r_2(X)).$$
$$\neg flies(X) \leftarrow fires(r_2(X)).$$

$$sat(r_3(X)) \leftarrow magic(X).$$
$$fires(r_3(X)) \leftarrow sat(r_3(X)), \text{not } \neg flies(X), \text{not } \neg fires(r_3(X)).$$
$$flies(X) \leftarrow fires(r_3(X)).$$

$$\neg fires(r_1(X)) \leftarrow sat(r_2(X))$$
$$\neg fires(r_2(X)) \leftarrow sat(r_3(X))$$

Marks: **8**

ii) Suppose Xavier is a penguin, and Yasmine is a magical penguin. What does your program conclude (in terms of *cautious* entailment) about whether Xavier and Yasmine can fly?

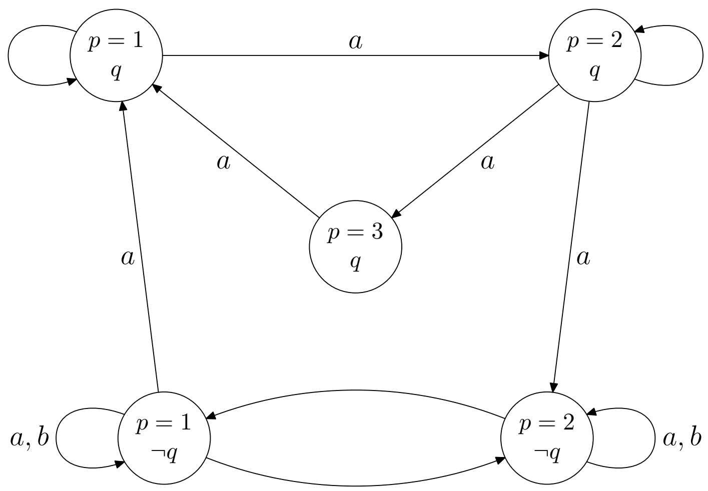
If you remove the exceptions between the rules, what does your program conclude (in terms of *cautious* entailment) about Xavier and Yasmine? How many answer sets are there altogether, if you remove the exceptions between the rules?

Initially, it concludes that Xavier cannot fly, and that Yasmine can fly. If you remove the exceptions between the rules, the program concludes nothing about. Without the exceptions between the rules, there are four answer sets.

Marks: **2**

The three parts carry, respectively, 20%, 30%, and 50% of the marks.

3 a Consider the following diagram:



This depicts a labelled transition system (LTS) defined by a $\mathcal{C}+$ action description, D.

i) Write down a $\mathcal{C}+$ action signature $(\sigma^{\mathrm{f}}, \sigma^{\mathrm{a}})$ for D. (Be sure to specify the domains of constants.)

Suppose that the following causal laws are included in D:

inertial p
inertial q
exogenous a
exogenous b

Complete D, so that it defines the LTS depicted above, by writing down in order:

ii. Any static causal laws for D.

iii. Causal laws containing the keyword **nonexecutable** (to constrain which actions can be performed in which states).

iv. The remaining dynamic causal laws, to describe the effects of actions.

(Note that there are many possible correct answers.)

i)

$$\sigma^{\mathrm{f}} = \{p, q\}$$
$$\sigma^{\mathrm{a}} = \{a, b\}$$
$$dom(p) = \{1, 2, 3\}$$

All other domains Boolean.

ii) ***caused*** $\perp$ ***if*** $p{=}3 \wedge \neg q$

iii) *We need something like:*

nonexecutable b ***if*** $p = 1 \wedge q$
nonexecutable b ***if*** $p = 3 \wedge q$
nonexecutable $a \wedge b$ ***if*** q
nonexecutable $\neg a \wedge \neg b$ ***if*** $\neg q$
nonexecutable $b \wedge \neg a$ ***if*** $p = 2 \wedge \neg q$

iv)

a ***causes*** $p = 2$ ***if*** $q \wedge p = 1$
a ***causes*** $p = 3$ ***if*** $q \wedge p = 2$
a ***causes*** $p = 1$ ***if*** $q \wedge p = 3$
$a \wedge \neg b$ ***causes*** $p = 2$ ***if*** $\neg q \wedge p = 1$
$a \wedge \neg b$ ***causes*** $p = 1$ ***if*** $\neg q \wedge p = 2$
b ***causes*** $\neg q$ ***if*** $p = 2 \wedge q$
$b \wedge \neg a$ ***causes*** q ***if*** $p = 1 \wedge \neg q$

*[*Marking scheme*: (i), 2 marks; (ii), 1 mark; (iii), 3 marks; (iv), 4 marks.]*

Marks: **10**

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b Suppose a $\mathcal{C}+$ action signature in which all constants are Boolean, and

$\sigma^{\mathrm{f}} = \{p, q\}$
$\sigma^{\mathrm{a}} = \{a, b\}$

Consider the following $\mathcal{C}+$ action description, D', using this signature:

inertial p
inertial q
exogenous a
caused b **if** $b \wedge a$
caused $\neg b$ **if** $\neg b \wedge a$
caused $\perp$ **if** p
a **causes** q **if** $\neg q$
$a \wedge b$ **causes** $\neg q$ **if** q
nonexecutable $a \wedge b$ **if** $\neg q$

With respect to that action description:

i) Give the interpretations $s \in \mathbf{I}(\sigma^{\mathrm{f}})$ such that $s \models T_{static}(s)$.

ii) For $s = \{\neg p, q\}$ and every $\varepsilon \in \mathbf{I}(\sigma^{\mathrm{a}})$, write down the set $A(s, \varepsilon)$. Note, for this s, those ε such that $\varepsilon = A(s, \varepsilon)$.

iii) For $s = \{\neg p, q\}$, $\varepsilon = \{a, b\}$, and for every $s' \in \mathbf{I}(\sigma^{\mathrm{f}})$, find $E(s, \varepsilon, s')$. Thus find all triples (s, ε, s'), for the specific s and ε, in the labelled transition system defined by D'.

---

i) *$\mathbf{I}(\sigma^f)$ can be represented as $\{\{p,q\},\{p,\neg q\},\{\neg p,q\},\{\neg p,\neg q\}\}$, where each member here represents those fluent atoms made true by the relevent $I \in \mathbf{I}(\sigma^f)$.*

Recall the definition of $T_{static}(s)$:

$$\{F \mid F \textbf{ if } G \text{ is in } D,\ s \models G\}$$

So,

$$\begin{aligned}
\{p,q\} &\not\models \{\bot\} \quad (= T_{static}(\{p,q\})) \\
\{p,\neg q\} &\not\models \{\bot\} \quad (= T_{static}(\{p,\neg q\})) \\
\{\neg p,q\} &\models \{\} \quad (= T_{static}(\{\neg p,q\})) \\
\{\neg p,\neg q\} &\models \{\} \quad (= T_{static}(\{\neg p,\neg q\}))
\end{aligned}$$

We thus have two states, $\{\neg p, q\}$ and $\{\neg p, \neg q\}$.

ii) *We can see $\mathbf{I}(\sigma^a)$ as $\{\{a,b\},\{a,\neg b\},\{\neg a,b\},\{\neg a,\neg b\}\}$. Now, the definition of $A(s,\varepsilon)$ is:*

$$\{A \mid A \textbf{ if } \psi \text{ is in } D,\ s \cup \varepsilon \models \psi\}$$

We therefore have:

$$\begin{aligned}
A(\{\neg p,q\},\{a,b\}) &= \{a,b\} \\
A(\{\neg p,q\},\{a,\neg b\}) &= \{a,\neg b\} \\
A(\{\neg p,q\},\{\neg a,b\}) &= \{\neg a\} \\
A(\{\neg p,q\},\{\neg a,\neg b\}) &= \{\neg a\}
\end{aligned}$$

The relevant ε are $\{a,b\}$ and $\{a,\neg b\}$.

iii) *The definition of $E(s,\varepsilon,s')$ is:*

$$\{F \mid F \textbf{ if } G \textbf{ after } \psi \text{ is in } D,\ s' \models G,\ s \cup \varepsilon \models \psi\}$$

We have:

$$
\begin{aligned}
E(\{\neg p, q\}, \{a, b\}, \{p, q\}) &= \{q, \neg q\} \\
E(\{\neg p, q\}, \{a, b\}, \{p, \neg q\}) &= \{\neg q\} \\
E(\{\neg p, q\}, \{a, b\}, \{\neg p, q\}) &= \{\neg p, q, \neg q\} \\
E(\{\neg p, q\}, \{a, b\}, \{\neg p, \neg q\}) &= \{\neg p, \neg q\}
\end{aligned}
$$

We also have:

$$
\begin{aligned}
T_{static}(\{p, q\}) &= \{\bot\} \\
T_{static}(\{p, \neg q\}) &= \{\bot\} \\
T_{static}(\{\neg p, q\}) &= \{\} \\
T_{static}(\{\neg p, \neg q\}) &= \{\}
\end{aligned}
$$

Thus the only (s, ε, s') *satisfying the requirements on a transition is* $(\{\neg p, q\}, \{a, b\}, \{\neg p, \neg q\})$.

*[*Marking scheme*: (i), 3 marks; (ii), 3 marks; (iii), 4 marks.]*

Marks: **10**

The two parts carry equal marks.

4 a Consider the following knowledge:

> Three cars, a, b, and c, can each be either driving or not. At any point, each of them is either at the *start*, *middle*, or *end* of the route. Driving when at the *start* takes a car to the *middle*; driving when at the *middle* takes a car to the *end*; and then, no further driving is possible. However, any number of cars which are currently driving may crash; if a car crashes, it cannot drive any more.

For this knowledge:

i) Write down a suitable $\mathcal{C}+$ action signature $(\sigma^{\mathrm{f}}, \sigma^{\mathrm{a}})$, being sure to include the domains for all constants.

ii) Write down a $\mathcal{C}+$ action descripton, using the signature from 4(a.i), to formalize the knowledge above.

Now consider the following knowledge:

> A barrier is placed between the *start* and *middle* of the route, which can be moved to be up or down. When up, things proceed as before. When down, then no cars can drive between the *start* and the *middle*.

This adds to the preceding knowledge.

iii) Make additions to, or otherwise modify, the signature and action description of (i) and (ii), in order to incorporate the new knowledge in your $\mathcal{C}+$.

Here, many different answers are possible.

i) *Use:*

$$\begin{aligned}
\sigma^f &= \{loc(C), broken(C)\} && C \in \{a, b, c\} \\
\sigma^a &= \{drive(C), crash(C)\} && C \in \{a, b, c\} \\
dom(loc(C)) &= \{start, middle, end\}
\end{aligned}$$

ii) *For the action description (and where this is for all $C \in \{a, b, c\}$):*

> ***inertial*** $f \qquad (f \in \sigma^f)$
> ***exogenous*** $\alpha \qquad (\alpha \in \sigma^a)$
> $drive(C)$ ***causes*** $loc(C) = middle$ ***if*** $loc(C) = start \wedge \neg crash(C)$
> $drive(C)$ ***causes*** $loc(C) = end$ ***if*** $loc(C) = middle \wedge \neg crash(C)$
> ***nonexecutable*** $drive(C)$ ***if*** $broken(C)$
> ***nonexecutable*** $drive(C)$ ***if*** $loc(C) = end$
> $crash(C)$ ***causes*** $broken(C)$
> ***nonexecutable*** $crash(C)$ ***if*** $\neg drive(C)$

iii) *We add Boolean fluent constant $barrierUp$, and the Boolean action constant $toggleBarrier$.*

We can simply add, for all $C \in \{a, b, c\}$:

> ***inertial*** $barrierUp$
> ***exogenous*** $toggleBarrier$
> ***nonexecutable*** $drive(C)$ ***if*** $\neg barrierUp(C) \wedge loc(C) = start$
> $toggleBarrier$ ***causes*** $barrierUp$ ***if*** $\neg barrierUp$
> $toggleBarrier$ ***causes*** $\neg barrierUp$ ***if*** $barrierUp$

*[*Marking scheme*: 3 marks for (i), 6 marks for (ii), 3 marks for (iii).]*

Marks: **12**

---

b Let A and B be two formulas of propositional logic. Suppose that Cn is a consequence operator (not necessarily monotonic) satisfying $\mathrm{Th}(\mathrm{Cn}(W)) \subseteq \mathrm{Cn}(W)$, for any set W of formulas.

i) Suppose that $(A \rightarrow B) \in \mathrm{Cn}(W)$.

Prove that if $A \in \mathrm{Cn}(W)$, then $B \in \mathrm{Cn}(W)$.

ii) Suppose, instead, that if $A \in \mathrm{Cn}(W)$, then $B \in \mathrm{Cn}(W)$. Suppose, further, that $\mathrm{Cn}(W)$ is consistent—i.e., that there is no formula X such that $X \in \mathrm{Cn}(W)$ and $\neg X \in \mathrm{Cn}(W)$.

Prove, on these suppositions, that $(A \rightarrow B)$ is consistent with $\mathrm{Cn}(W)$—i.e., that $\neg(A \rightarrow B) \notin \mathrm{Cn}(W)$.

---

i) *For any W, assume $(A \rightarrow B) \in Cn(W)$. Now suppose that $A \in Cn(W)$. Since $A, A \rightarrow B \in Cn(W)$, then $Cn(W) \models_{PL} B$, so by definition,*

$B \in Th(Cn(W))$. But then since $Th(Cn(W)) \subseteq Cn(W)$, we have $B \in Cn(W)$, as desired.

ii) *Suppose, first: if $A \in Cn(W)$, then $B \in Cn(W)$. Suppose, secondly: $Cn(W)$ is consistent. We must show that $\neg(A \to B) \notin Cn(W)$.*

Assume that $\neg(A \to B) \in Cn(W)$, for contradiction. Since $Th(Cn(W)) \subseteq Cn(W)$, then as $\neg(A \to B) \equiv (A \wedge \neg B)$, and $(A \wedge \neg B) \models A$ and $(A \wedge \neg B) \models \neg B$, we have that $A \in Cn(W)$ and $\neg B \in Cn(W)$. But by our supposition, $B \in Cn(W)$. So $Cn(W)$ is inconsistent. Contradiction.

Therefore, $\neg(A \to B) \notin Cn(W)$, as desired.

*[*Marking scheme*: 3 marks for (i), 5 marks for (ii).]*

Marks: **$\overline{8}$**

The two parts carry, respectively, 60% and 40% of the marks.