COMP245: Probability and Statistics 2016 - Problem Sheet 6

Continuous Random Variables

Q1) Suppose X is a continuous random variable with density function f which is symmetric around zero, so $\forall x \in \mathbb{R}$, $f(-x) = f(x)$.

Show that the cdf satisfies $F(-x) = 1 - F(x)$.

Q2) Electrons hit a circular plate with unit radius. Let X be the random variable representing the distance of a particle strike from the centre of the plate. Assuming that a particle is equally likely to strike anywhere on the plate,

(a) for $0 < r < 1$ find $\mathrm{P}(X < r)$, and hence write down the full the cumulative distribution function of X, F_X;

(b) find $\mathrm{P}(r < X < s)$, where $r < s$;

(c) find the probability density function for X, f_X.

(d) calculate the mean distance of a particle strike from the origin.

Q3) Prove that the mean and variance of an $\mathrm{Exp}(\lambda)$ random variable are $\frac{1}{\lambda}$ and $\frac{1}{\lambda^2}$ respectively.

Q4) Let $X \sim \mathrm{U}(0, 1)$. Find the cdf and hence the pdf of the transformed variable $Y = e^X$.

Q5) Let $X \sim \mathrm{N}(\mu, \sigma^2)$, and let $Y = \dfrac{X - \mu}{\sigma}$. Using the results on transformations of variables, validate the claim in lectures that $Y \sim \mathrm{N}(0, 1)$.

Q6) Let X be a continuous random variable, with cdf $F_X(x)$ and pdf $f_X(x)$. Let $Y = aX+b$, where $a \neq 0, b \in \mathbb{R}$ are constants.

(a) Considering in turn the two cases $a > 0$ and $a < 0$, use the definition of a cdf to find expressions for the cdf of Y, $F_Y(y)$, in terms of F_X.

(b) Using the relationship between a pdf and its cdf, show that the pdf for Y is given by

$$f_Y(y) = \frac{1}{|a|} f_X\left(\frac{y - b}{a}\right).$$

Q7) If 'area' refers to the area under the curve of the standard normal probability density function ϕ, find the value or values of z such that

(a) the area between 0 and z is 0.3770;

(b) the area to the left of z is 0.8621;

(c) the area between -1.5 and z is 0.0217.

Q8) Find the area under the standard normal curve

(a) between $z = 0$ and $z = 1.2$;

(b) between z = -0.68 and $z = 0$;

(c) between z = -0.46 and z = 2.21;

(d) between z = 0.81 and z = 1.94;

(e) to the right of z = -1.28.