COMP245: Probability and Statistics 2016 - Problem Sheet 8

Estimation

Q1) If $(X_1, \ldots, X_n)$ are a random sample from an exponential distribution with rate parameter λ, find the maximum likelihood estimate for λ.

Q2) Derive the maximum likelihood estimate for λ for n independent samples from Poisson(λ).

Q3) In a study of traffic congestion, data were collected on the number of occupants in private cars on a certain road. These data, collected for 1469 cars, are given below

Count	1	2	3	4	5	≥6
Frequency	902	403	106	38	16	4

One theory suggests that these data may have arisen from a modified geometric distribution, in which the probability that there are x occupants in a car is

$$p(x) = p(1-p)^{x-1}, x = 1, 2, \ldots.$$

(a) Find the maximum likelihood estimate of the parameter p of the geometric distribution for these data. (Note that $\mathrm{P}(X \geq x) = (1-p)^{x-1}$.)

(b) *[To be attempted after the lectures on hypothesis testing]* Describe how a hypothesis test could be carried out, at the 1% level, to see if these data do come from a geometric distribution.

Q4) (a) For a random sample of size n from a normal distribution with unknown mean μ and known variance σ^2, what is the confidence level for each of the following confidence limits for μ?

i. $\overline{x} \pm 1.96\frac{\sigma}{\sqrt{n}}$;

ii. $\overline{x} \pm 1.645\frac{\sigma}{\sqrt{n}}$;

iii. $\overline{x} \pm 2.575\frac{\sigma}{\sqrt{n}}$;

iv. $\overline{x} \pm 0.99\frac{\sigma}{\sqrt{n}}$;

(b) A random sample of 64 observations from a population produced the following summary statistics:

$$\sum_i x_i = 700, \qquad \sum_i (x_i - \overline{x})^2 = 4238.$$

Find a 95% confidence interval for μ, and interpret this interval.

Q5) Compute confidence intervals at the 95% level for the means of the distributions from which the following sample values were obtained:

(a) $n = 100, \sum_i x_i = 250, \sum_i x_i^2 = 725000$;

(b) $n = 100, \overline{x} = 83.2, s_{n-1} = 6.4$.

Q6) The following random sample was selected from a normal distribution:

$$7.53, \quad 4.35, \quad 7.66, \quad 7.54, \quad 5.83, \quad 1.92, \quad 3.14, \quad 4.41$$

(a) Construct a 90% confidence interval for the population mean.

(b) Construct a 99% confidence interval for the population mean.