2022 Models Collaborative Solution

Her bert

Lmao

Question 2: 10:55:59 BST

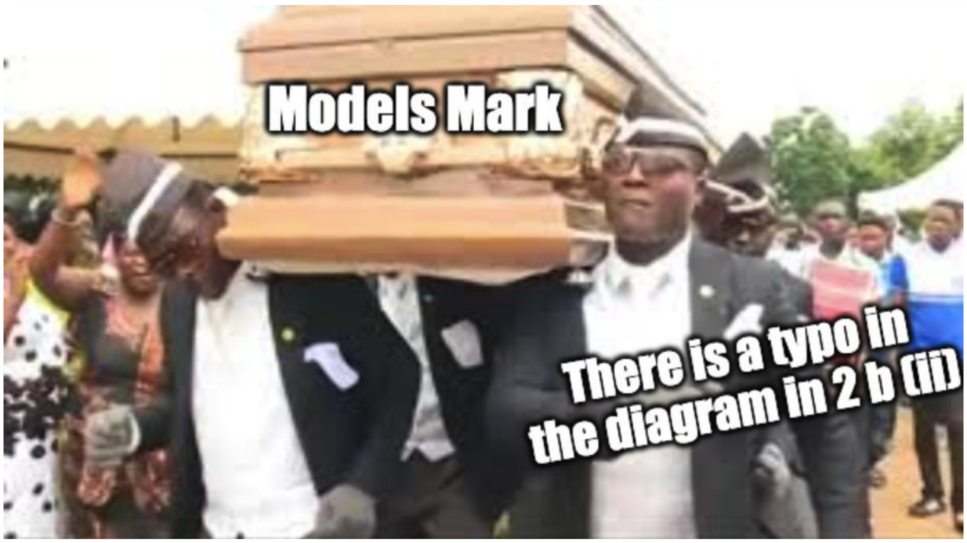
Sorry - There is another typo: The sum in 2c should be: "\sum_i x_i + n", i.e. the index is i - going from 1 to n.

herbert

Question 2: 10:28:06 BST

There is a typo in Q2bii. The last substitution/replacement should concern R_1^+, that is "replace R_1^+ by copy R_2 to R_1"

herbert



Q1

a.i.

a.i.
$$\dfrac{P(0) = x := x+1 \qquad \langle x, s_0\rangle \Downarrow 1 \qquad \dfrac{P(1) = \text{goto}(x>0, 2, 0) \qquad \langle x>0, s_0[x\to 1]\rangle \Downarrow \text{true} \qquad \dfrac{P(2) = \text{exit}}{\langle P, s_1, 2\rangle \Downarrow \langle s_1, 1\rangle}}{\langle P, s_0[x\to 1], 1\rangle \Downarrow \langle s_1, 2\rangle}}{\langle P, s_0, 0\rangle \Downarrow \langle s_1, 3\rangle}$$

a.ii. k is the number of instructions executed

a.iii. goto(i) is an immediate jump to command i and goto(B, i, j) is a conditional jump; when B is true it jumps to command I, otherwise it jumps to command j.

a.iv. goto(i) = goto(true, i, 0)

$$\frac{P(pc) = goto(B,i,j) \quad \langle B, s\rangle \Downarrow_b false \quad pc' = j}{P, s, pc \longrightarrow P, s, pc'}$$

$$\frac{P(pc) = goto(B,i,j) \quad \langle B, s\rangle \Downarrow_b true \quad pc' = i}{P, s, pc \longrightarrow P, s, pc'}$$

b. b.

$$\frac{P(pc) = goto(i) \qquad pc' = i}{P, s, pc \longrightarrow P, s, pc'}$$

c. c. Base Case: $k_1 = 1$. Take arb k_2, s_1, s_2, P, s and pc.

Assume $\langle P, s, pc\rangle \Downarrow \langle s_1, 1\rangle$ and $\langle P, s, pc\rangle \Downarrow \langle s_2, k_2\rangle$

now

$$\frac{P(pc) = exit}{\langle P, s, pc\rangle \Downarrow \langle s_1, 1\rangle}$$ by inversion.

so $P(pc) = exit$ and $s = s_1$.

and then this means it must be that

$$\frac{P(pc) = exit}{\langle P, s, pc\rangle \Downarrow \langle s_2, k_2\rangle}$$ so $s_2 = s_1$ and $k_2 = 1 = k_1$

so $s_1 = s_2 \wedge 1 = k_2 \; (= k_1)$

Inductive Case: Assume $k_1 = k$ and that $\forall k_1, k_2, s_1, s_2, P, s, pc$.

$\langle P, s, pc\rangle \Downarrow \langle s_1, k\rangle \wedge \langle P, s, pc\rangle \Downarrow \langle s_2, k_2\rangle \Rightarrow s_1 = s_2 \wedge k = k_2$ (IH)

Take k_2, s_1, s_2, P, s and pc arb

Assume also:

$\langle P, s, pc\rangle \Downarrow \langle s_1, k+1\rangle$ and $\langle P, s, pc\rangle \Downarrow \langle s_2, k_2\rangle$ To show $s_1 = s_2 \wedge k+1 = k_2$

By cases:

Case 1:

$$\frac{P(pc) = goto(i) \quad \langle P, s, i\rangle \Downarrow \langle s_1, k\rangle}{\langle P, s, pc\rangle \Downarrow \langle s_1, k+1\rangle}$$ then from the we know $P(pc) = goto(i)$ for some i.

So then also $$\frac{P(pc) = goto(i) \quad \langle P, s, i\rangle \Downarrow \langle s_2, k_2 - 1\rangle}{\langle P, s, pc\rangle \Downarrow \langle s_2, k_2\rangle}$$

But we know from the IH that $\langle P, s, i\rangle \Downarrow \langle s_1, k\rangle$ and $\langle P, s, i\rangle \Downarrow \langle s_2, k_2 - 1\rangle$

$\Rightarrow s_1 = s_2$ and $k = k_2 - 1$

so therefore adding one to both sides: $k+1 = k_2$ and $s_1 = s_2$ $(s_1 = s_2 \wedge k_1 = k_2)$

other cases similarly will try to finish

Case 2: $$\frac{P(pc) = x ::= E \quad \langle E, s\rangle \Downarrow n \quad \langle P, s[x \to n], pc+1\rangle \Downarrow \langle s_1, k\rangle}{\langle P, s, pc\rangle \Downarrow \langle s_1, k+1\rangle}$$

so $P(pc) = x ::= E$ for some E and $s_1 = s[x \to n]$

so then also $$\frac{P(pc) = x ::= E \quad \langle E, s\rangle \Downarrow n \quad \langle P, s[x \to n], pc+1\rangle \Downarrow \langle s_2, k_2 - 1\rangle}{\langle P, s, pc\rangle \Downarrow \langle s_2, k_2\rangle}$$

so $s_2 = s[x \to n] = s_1$ and from IH we know $\langle P, s, i\rangle \Downarrow \langle s_1, k\rangle$

and $\langle P, s, i\rangle \Downarrow \langle s_2, k_2 - 1\rangle$

so adding 1 both sides: $k+1 = k_2$

$s_1 = s_2 \wedge k+1 = k_2$

Case 3: No time.

Explanation: must enumerate remaining (rules) cases and prove inductive step for each to prove the rules are deterministic.

Q2 (The day of this exam is 6th May 2022) a.

2. a. $220506_{10} = 110101110101011010$

Pair $= \langle\langle 0, 925742 \rangle\rangle$

List: $\langle\langle 0, 925742 \rangle\rangle$

$925742 = \langle\langle 1, 231435 \rangle\rangle$

$231435 = \langle\langle 0, 115717 \rangle\rangle$

$115717 = \langle\langle 0, 57858 \rangle\rangle$

$57858 = \langle\langle 1, 14464 \rangle\rangle$

$14464 = \langle\langle 8, 56 \rangle\rangle$

$56 = \langle\langle 3, 3 \rangle\rangle$

$3 = \langle\langle 0, 1 \rangle\rangle$

$1 = \langle\langle 0, 0 \rangle\rangle$

List $= [0, 1, 0, 0, 1, 8, 3, 0, 0, 0]$

Instruction represented: $R_0^+ \rightarrow L_{925742}$

b. i. L_0: $R_0^- \rightarrow L_1, L_2$

L_1: $N^+ \rightarrow L_3$

L_2: HALT

L_3: $R_1^- \rightarrow L_4, L_5$

L_4: $R_2^- \rightarrow L_6, L_7$

L_5: $R_0^+ \rightarrow L_0$

L_6: $R_3^+ \rightarrow L_8$

L_7: $0^+ \rightarrow L_5$

L_8: $R_2^+ \rightarrow L_9$

L_9: $R_1^+ \rightarrow L_4$

b.i.

b.ii. typo again.

b.iii.

2.c.

δ	$\sqcup$	0	1
1	(2, 1, R)	(1, 0, R)	(2, 1, R)
2	(3, ~~$\sqcup$~~ 1, R)	(2, 0, R)	(3, 1, R)
3	(4, ~~$\sqcup$~~ 1, R)	(3, 0, R)	(4, 1, R)
4	(5, 1, R)	(4, 0, R)	(5, 1, R)
5	(6, 1, R)	(5, 0, R)	(6, 1, R)
⋮	⋮	⋮	⋮
p	(p+1, 1, R)	(p, 0, R)	(p+1, 1, R)
0	(1, 1, R)	(0, 0, R)	(1, 1, R)

(assume ~~put~~ s = 0 at top)

$\Sigma = \{\sqcup, 0, 1\}$

$Q = \{0, 1, 2, \dots \sum_{i=1}^{n} x_n + n\}$

$s = 0$

For list [1, 2, 3] the ~~ma~~ turing machine will start from the leftmost 0 (with state 0), move right ~~to~~ to find the first digit 1 of a series of 1s (x_1 such 1s) and it will continually move to the right, incrementing the state as it goes until it reaches a $\sqcup$. ~~when it reaches~~ In this case it will only move right once as $x_1 = 1$ in this examp

So:

(1)

... | ⊔ | ⊔ | 0 | 1 | ⊔ | 1 | 1 | ⊔ | 1 | 1 | 1 | 0 | ⊔ | ⊔ | ...

it will then flip the $\sqcup$ to make it a 1, ~~and~~ ~~set~~ incement the state, and repeat ~~until~~ Until we reach this:

(9)

... | ⊔ | ⊔ | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | ⊔ | ⊔ |

c.

d.

~~2d~~ $\text{next} \triangleq \lambda p.\ \text{Pair}\ ((\text{mult}$

2d $\text{nextt} \triangleq \lambda p.\ \text{add}\ (\text{mult}\ (\text{second}\ p)\ (\text{second}\ p))$
$(\text{mult}\ \underline{2}\ (\text{first}\ p))$

$\text{nextp} \triangleq \lambda p.\ \text{Pair}\ (\text{nextt}\ p)\ (\text{first}\ p)$

$f \triangleq \lambda n.\ \text{second}\ (n\ \text{nextp}\ (\text{Pair}\ \underline{1}\ \underline{1}))$

Using the fact that a number n is n application of a function, we get

$(x_1, x_0) \rightarrow (x_2, x_1) \rightarrow \ldots \rightarrow (x_{n+1}, x_n)$

$f\ \underline{2}$
$= (\lambda n.\ \text{Second}\ (n\ \text{nextp}\ (\text{Pair}\ \underline{1}\ \underline{1})))\ (\lambda f.\ \lambda x.\ f\ (f\ x))$
$\rightarrow_\beta \text{Second}\ ((\lambda f.\ \lambda x.\ f\ (f\ x))\ \text{nextp}\ (\text{Pair}\ \underline{1}\ \underline{1}))$
$\rightarrow_\beta^* \text{Second}\ (\text{nextp}\ (\text{nextp}\ (\text{Pair}\ \underline{1}\ \underline{1})))$
$\rightarrow_\beta^* \text{Second}\ (\text{nextp}\ (\text{Pair}\ (\text{nextt}\ (\text{Pair}\ \underline{1}\ \underline{1})\ \underline{1})))$
$\rightarrow_\beta^* \text{Second}\ (\text{nextp}\ (\text{Pair}\ (\text{add}\ \underline{1}\ \underline{2})\ \underline{1}))$
$\rightarrow_\beta^* \text{Second}\ (\text{nextp}\ (\text{Pair}\ \underline{3}\ \underline{1}))$
$\rightarrow_\beta^* \text{Second}\ (\cancel{\text{nextp}}\ \text{Pair}\ \underline{7}\ \underline{3})$
$\rightarrow_\beta^* \underline{3}$