① A-

$A = \{\{1\}, \{2\}\}$ $B = \{1, 2, \{2\}\}$

$C = \{1, \{2\}\}$.

$A \triangle B = \{\{1\}, 1, 2\}$

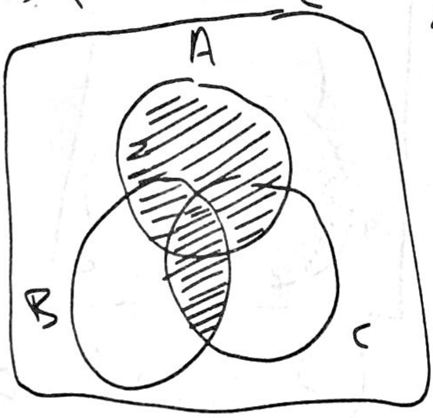
$B \triangle C = \{2\}$

$A \cap (B \cup C) = \{\{1\}, \{2\}\} \cap \{1, 2, \{2\}\}$

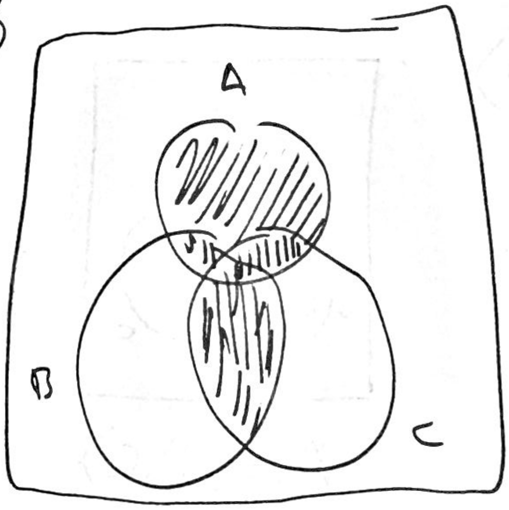
$= \{\{2\}\}$

$(A \cap B) \setminus C = \{\} = \emptyset$

B- i)



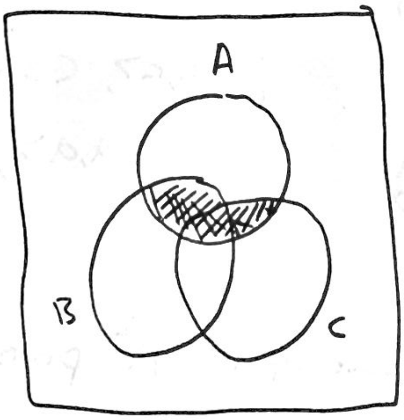
$A \cup (B \cap C)$



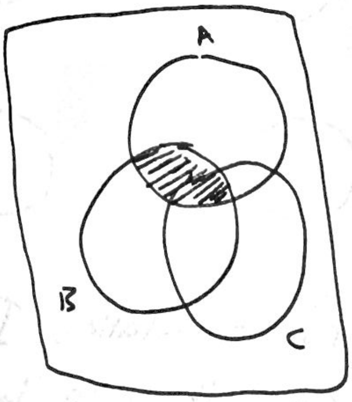
$(A \cup B) \cap (A \cup C)$

$\Rightarrow$ TRUE.

ii)



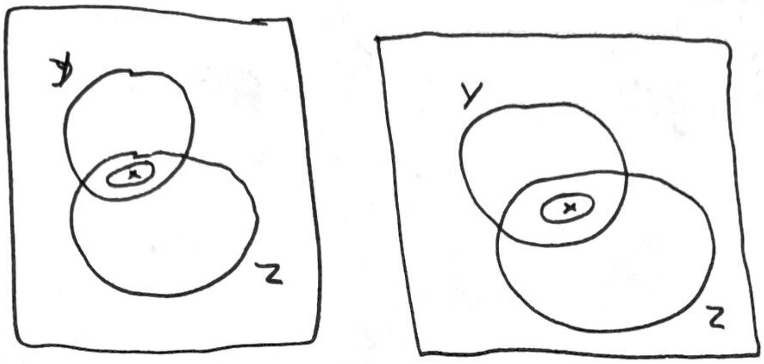
$A \cap (B \cup C)$



$(A \cap B) \cap (B \cup C)$

$\Rightarrow$ FALSE.

iii)

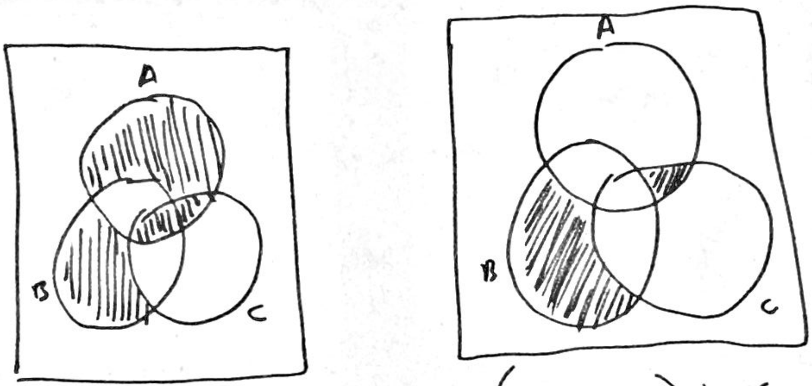


$\Longrightarrow$ TRUE.

iv) let $A = \{1, 2, 3\}$, $B = \{2, 3, 4\}$

$A \setminus B = \{1\}$, $B \setminus A = \{4\} \Longrightarrow$ FALSE.

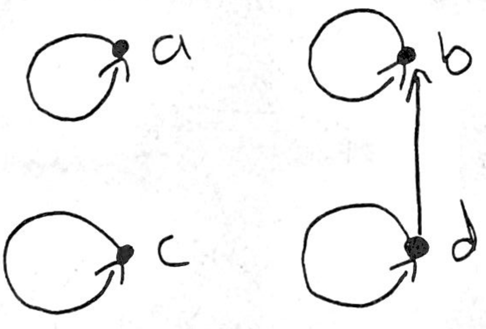
v)



$\Longrightarrow$ FALSE.

$A \triangle (B \setminus C)$ $\quad$ $(A \triangle B) \setminus (A \triangle C)$

c- i)



$\{\langle a,a\rangle, \langle b,b\rangle, \langle c,c\rangle, \langle d,d\rangle, \langle d,b\rangle\}$

ii) For a relation, I can have up to n^2 pairs. For each pair, it could be either in the relation, or not in the relation. $\Longrightarrow 2^{n^2}$ possible relations.

Of the n^2 pairs, n of them must be in the relation to be reflexive. Therefore we can only choose between $n^2 - n$ other pairs. $\Longrightarrow 2^{n^2-n}$ pairs.

D- i) $R \subseteq S \vdash R^{-1} \subseteq S^{-1}$

1. $R \subseteq S$ given.

2.	$\langle x,y \rangle \in R^{-1}$	ass.
3.	$\langle y,x \rangle \in R$	Definition of inverse.
4.	$\langle y,x \rangle \in S$	Definition of Subset
5.	$\langle x,y \rangle \in S^{-1}$	Definition of inverse

6. $\forall \langle x,y \rangle [\langle x,y \rangle \in R^{-1} \to \langle x,y \rangle \in S^{-1}]$ $\to I(2,5)$

7. $R^{-1} \subseteq S^{-1}$

ii) $(R \cap S)^{-1} = R^{-1} \cap S^{-1}$

1.	$\langle x,y \rangle \in (R \cap S)^{-1}$	ass.
2.	$\langle y,x \rangle \in S \cap R$	definition of inverse.
3.	$\langle y,x \rangle \in S$	
4.	$\langle y,x \rangle \in R$	
5.	$\langle x,y \rangle \in S^{-1}$	Def inverse
6.	$\langle x,y \rangle \in R^{-1}$	Def inverse
7.	$\langle x,y \rangle \in R^{-1} \cap S^{-1}$	

8. $\forall \langle x,y \rangle . \langle x,y \rangle \in (R \cap S)^{-1} \to \langle x,y \rangle \in R^{-1} \cap S^{-1}$

9. $(R \cap S)^{-1} \subseteq R^{-1} \cap S^{-1}$

10. $\langle x,y \rangle \in R^{-1} \cap S^{-1}$ ass.

11. $\langle x,y \rangle \in R^{-1}$

12. $\langle y,x \rangle \in R$

13. $\langle x,y \rangle \in S^{-1}$

14. $\langle y,x \rangle \in S$

15. $\langle y,x \rangle \in R \cap S$

16. $\langle x,y \rangle \in S \cap R$

17. $\forall \langle x,y \rangle . \langle x,y \rangle \in R^{-1} \cap S^{-1} \rightarrow \langle x,y \rangle \in (R \cap S)^{-1}$

18. $\cancel{\langle x,y \rangle}$ $R^{-1} \cap S^{-1} \subseteq (R \cap S)^{-1}$

19. $R^{-1} \cap S^{-1} = (R \cap S)^{-1}$ By (18) and (9)

iii) $(\overline{R})^{-1} = \overline{R^{-1}}$

take arbitrary $\langle x,y \rangle \in (\overline{R})^{-1}$

$\iff \langle y,x \rangle \in \overline{R}$ Definition of inverse

$\iff \langle y,x \rangle \notin R$ Def comp

$\iff \langle x,y \rangle \notin R^{-1}$ Def inv

$\iff \langle x,y \rangle \in \overline{R^{-1}}$ Def comp

Reasoning works both ways $\Rightarrow \overline{R^{-1}} \subseteq \overline{R}^{-1} \wedge \overline{R}^{-1} \subseteq \overline{R^{-1}}$

$\Rightarrow \overline{R^{-1}} = \overline{R}^{-1}$

iv) $(R \circ S)^{-1} = S^{-1} \circ R^{-1}$

Take arbitrary $\langle x, y \rangle \in (R \circ S)^{-1}$

$\iff \langle y, x \rangle \in R \circ S$ By inverse

$\iff \exists \alpha .\ \langle y, \alpha \rangle \in R$ By composition

$\iff \langle \alpha, y \rangle \in R^{-1}$ By inv

$\iff \langle \alpha, x \rangle \in S$ By comp

$\iff \langle x, \alpha \rangle \in S^{-1}$ By inv

$\iff \langle x, y \rangle \in R^{-1} \circ S^{-1}$ By comp

Reasoning works both ways $\Rightarrow (R \circ S)^{-1} \subseteq S^{-1} \circ R^{-1}$

$\wedge\ S^{-1} \circ R^{-1} \subseteq (R \circ S)^{-1}$

$\Rightarrow (R \circ S)^{-1} = S^{-1} \circ R^{-1}$.

v) R is Symmetric $\Rightarrow$ NO IDEA M8 PLS HELP.

For this part, I did the following (seems legit but I cant guarentee it is right):
Where you see 'in' replace with the member sign, (I most likely will never learn latex)

Take arbitrary <a,b> in R
Since R is symmetric, we have <b,a> in R.
By using composition, we have <a,a> in R and <b,b> in RoR
Both <a,a> and <b,b> are symmetric, so RoR is symmetric for any arbitrary <a,b> in R.

Since we have shown this for arbitrary <a,b> in R, this holds for all pairs in R.
So R is symmetric implies RoR is symmetric.

E- i) $g \circ f$ is one to one $\Rightarrow$

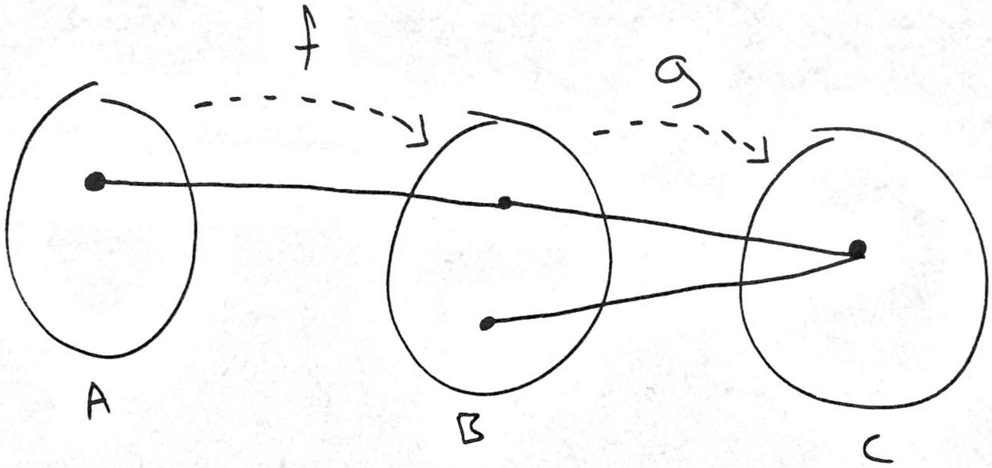
$\forall a, a' \in A. [g \circ f(a) = g \circ f(a') \to a = a']$

$\Rightarrow$ ~~Let $f(a) = b \in B$~~ $g \circ f(a) = g \circ f(a')$

$\Rightarrow \forall a, a' \in A\, [f(a) = f(a') \leftrightarrow f(a) = f(a') \to a = a']$

$\Rightarrow$ f is one to one.

ii)



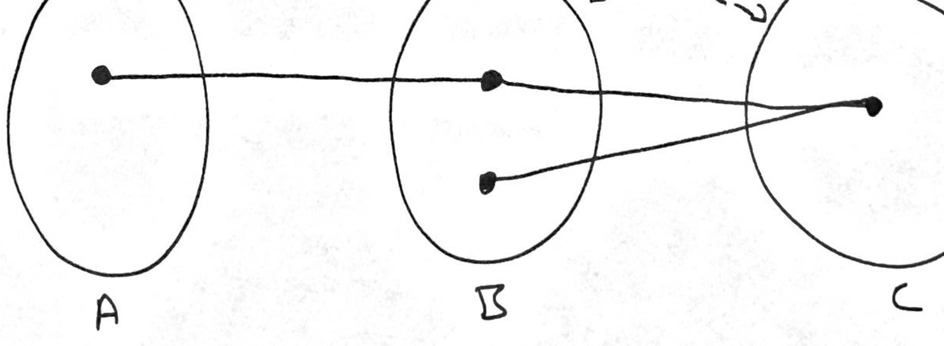
iii) $g \circ f$ is onto $\Rightarrow \forall c \in C, \exists a \in A. [g \circ f(a) = c]$

$\Rightarrow$ Let $f(a) = b \in B$

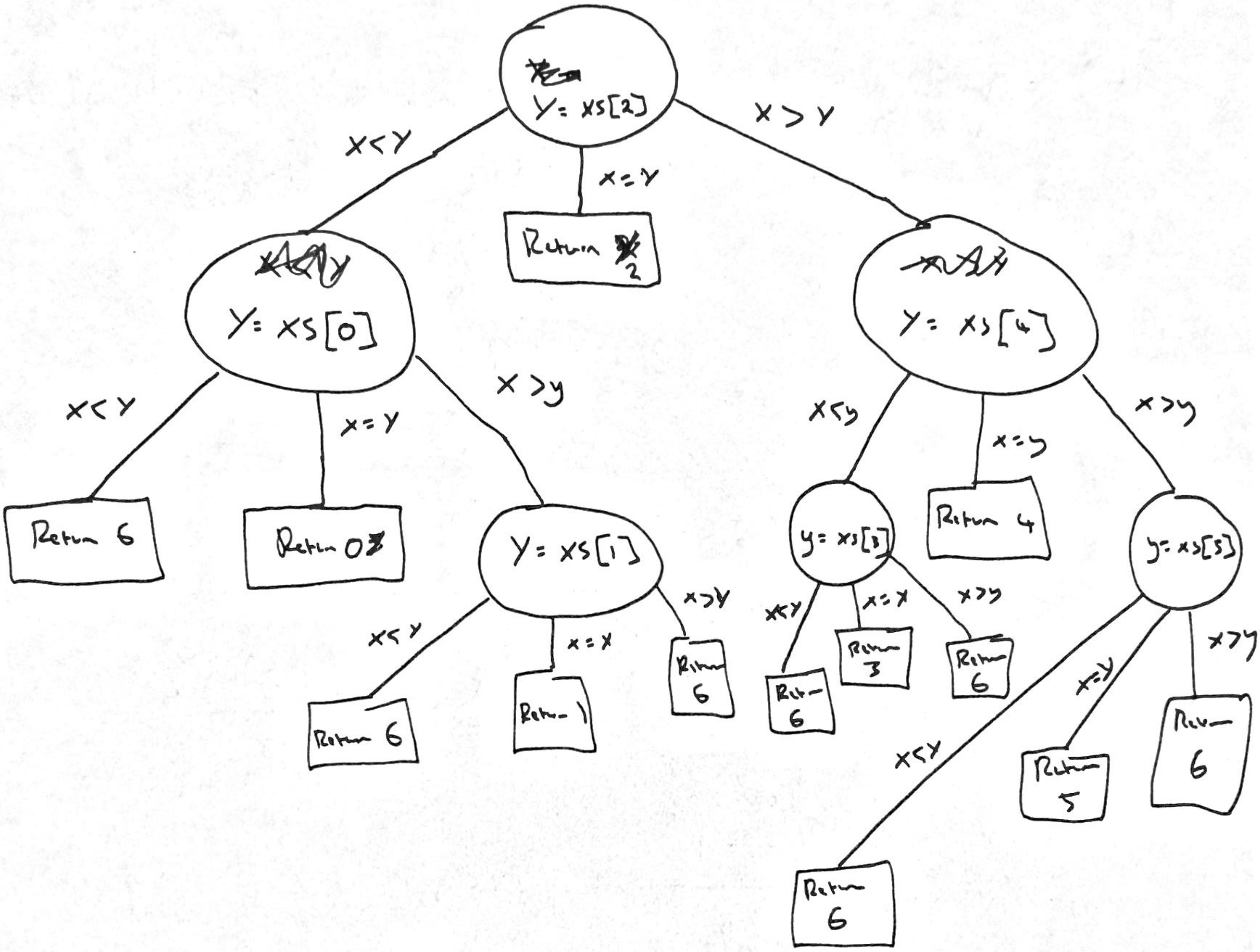
$\Rightarrow \forall c \in C, \exists b \in B. [g(b) = c]$

$\Rightarrow$ g is onto.

iv.



② A- Searching for X in XS.



- Worst case = 3 comparisons

B- i) Order n^2. For each element we are inserting, if the list is reverse ordered, it will need to be compared with every other element before we find it's position.

ii) Compares up to 3 elements for each element, therefore $3n$.

C- i) $W(1) = 0$

$W(n) = n-1 + W(n/2) + W(n/2)$

ii) $= 1 + 2W'(n/2)$

iii) $= \cancel{1+1+2(W(n/4))} = 1 + 2(1 + 2W(n/4)) = \overset{1+2}{\cancel{3}} + 4W(n/4)$

$= 1 + 2(1 + 2(1 + 2W(n/8)))$

$= 1 + 2 + 4 + 8W(n/8)$

$$= \sum_{n=0}^{\log(n)} 2^n$$

$$S_n = a + ar + ar^2 + \ldots + ar^{n-1}$$

$$rS_n = ar + ar^2 + \ldots + ar^{n-1} + ar^n$$

$$rS_n - S_n = ar^n - a \implies S_n = \frac{a(r^n - 1)}{r - 1}$$

$$\therefore = \frac{2^{\log n} - 1}{2 - 1} = n - 1$$