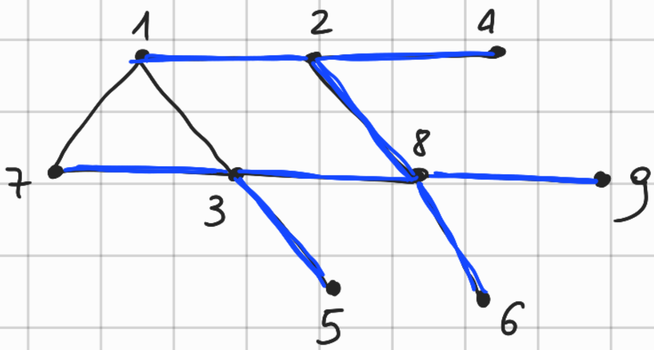
(1a) 3 is fixed (the only node of degree 3).
1 can be placed in 2 positions. Then 2 is fixed.
4 is fixed (the only node of degree 5).
6 can be placed in 4 positions. Then 5 is fixed.
7 can be placed in 2 positions. Then 8 is fixed.
Putting things together:

$$\# \text{ automorphisms} = 1 \cdot 2 \cdot 1 \cdot 1 \cdot 4 \cdot 1 \cdot 2 \cdot 1 = 16$$

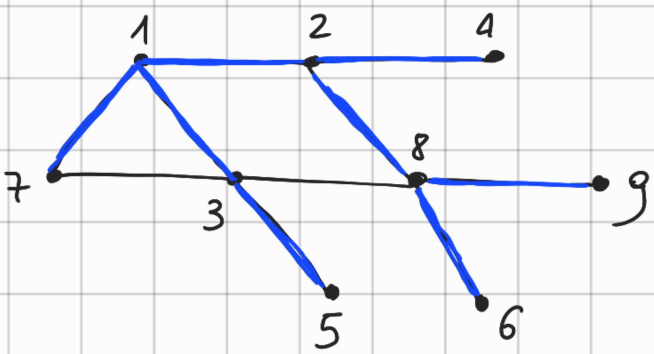
(1b) (i)



ORDER OF TRAVERSAL OF NODES USING DFS:

1, 2, 4, 8, 3, 5, 7, 6, 9

(ii)



ORDER OF TRAVERSAL OF NODES USING BFS:

1, 2, 3, 7, 4, 8, 5, 6, 9

(1d)

```
M[0]=0
for i=1 to n :
    M[i] = P[i]
for i=1 to n :
    for j=1 to i:
        M[i] = max(M[i], M[j] + M[i-j])
return M[n]
```

(1c) Therefore we have to show that "if an undirected graph G has an Euler circuit then there is a circuit in G that uses every arc three times" and "if there is a circuit in G that uses every arc three times, then there exist an Euler circuit.

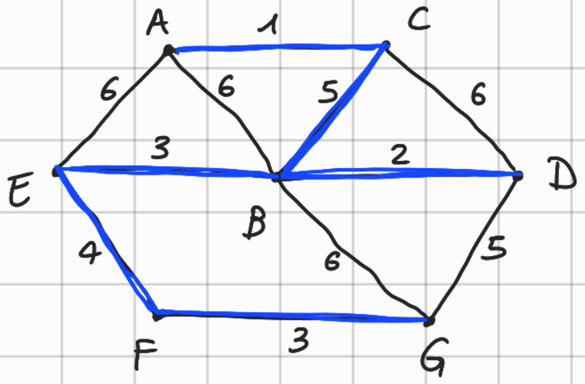
Assume G has an Euler circuit and call such circuit γ.
Consider a node in the circuit, call it x. Then if we travel γ three times starting from x, we end up with a circuit that uses each arc three times and returns to x.

Assume G has a circuit that uses every arc three times.
Imagine another graph G' with the same nodes, but every arc is repeated three times.

Then every node in G' has three times the degree of its correspondent node in G. Since there is an Euler circuit in G', then every node in G' has even degree. But then also the nodes in G have even degree because x is even iff 3x is even. Then we are guaranteed that G has an Euler circuit.

Since we proved both directions, we know that an undirected graph has an Euler circuit iff has a circuit that uses each arc exactly three times.

2a (i)



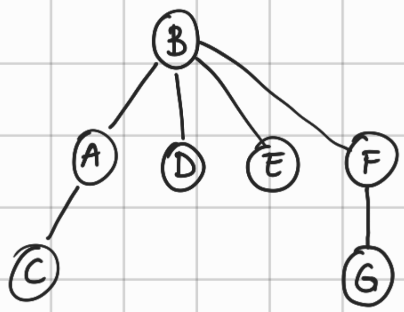
ORDER IN WHICH ARCS ARE ADDED TO THE MST:

(A,C), (B,D), (B,E), (F,G), (E,F)
(B,C)

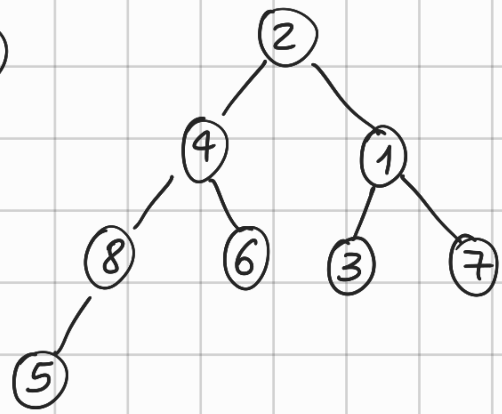
(ii) There is a unique MST in the graph. We used the arcs with minimum weights (same order as the one given in part i): 1, 2, 3, 3, 4, 5. There is only one non-used arc that has weight 5 (i.e. (D,G)) so if there exist another MST then it must include (D,G) instead of (B,C). But we can see that adding (D,G) and removing (B,C) will disconnect the graph (A,C are disconnected from the rest of the graph). Therefore the MST given in part (i) is unique.

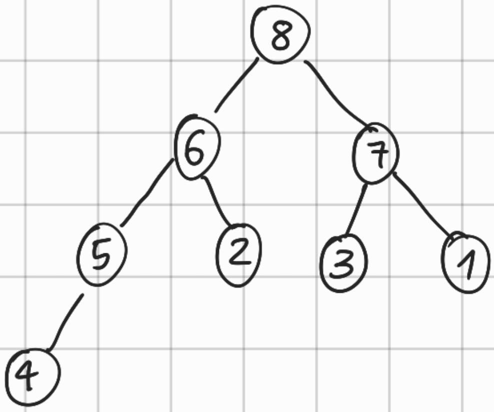
(iii) evolution of the leaders during the execution of the algorithm:

A	A	A	A	A	A	A	B
B	B	B	B	B	B	B	B
C	C	A	A	A	A	A	A
D	D	D	B	B	B	B	B
E	E	E	E	B	B	B	B
F	F	F	F	F	F	B	B
G	G	G	G	G	F	F	F



2b (i)



(ii)  $[8, 6, 7, 5, 2, 3, 1, 4]$

(2c) (i) 3ColN belongs to NP because there exist a problem Ver-3ColN(G, x, y, z, π) and a polynomial $p(n)$ such that:

- 3ColN(G, x, y, z) iff $\exists \pi$ Ver-3ColN(G, x, y, z, π)
- if Ver-3ColN(G, x, y, z, π) then $|\pi| \leq p(|G|)$.

Consider a graph G, 3 nodes x, y, z of G and a mapping π from nodes to colors. Define Ver-3ColN(G, x, y, z, π) iff π is a valid 3-color coloring of G and x, y, z have different colors.

Ver-3Col(G, π)

Then 3ColN(G, x, y, z) iff $\exists \pi$ Ver-3ColN(G, x, y, z, π).
Also, π is a mapping from G's nodes to their associated colors, so it is polynomially bounded by $p(|G|)$.

(ii) To show that 3ColN is NP complete we show that 3Col $\leq$ 3ColN (this implies that 3ColN is NP-Hard, and, since 3ColN $\in$ NP, we deduce that 3ColN is NP-Complete).
We need to find a p-time reduction function f such that 3Col(G) iff 3ColN$(f(G))$.
Define $f(G) = (G', x, y, z)$ as follows:

- nodes(G') = nodes$(G) \cup \{x, y, z\}$
- arcs(G') = arcs$(G) \cup \{\langle x, y\rangle, \langle y, z\rangle, \langle x, z\rangle, \langle x, v\rangle\}$ where $v \in$ nodes(G).

f is clearly polynomially bounded.
To show: 3Col(G) iff 3ColN$(f(G))$.
Assume 3Col(G). Consider $f(G) = (G', x, y, z)$ and $v \in$ nodes(G).
Then we can color x with a color different from the color of v, and we can color y and z with the remaining two colors. Then we have 3ColN(G', x, y, z) so 3ColN$(f(G))$.

Assume $3ColN(f(G))$. Consider $f(G) = (G', x, y, z)$. Then we know that G is 3-colorable because it an induced subgraph of a 3-colorable graph G' (in particular G is the induced subgraph of G' with $nodes(G) = nodes(G') \setminus \{x, y, z\}$).

Therefore $3Col(G)$ iff $3ColN(f(G))$.

Therefore $3Col \leq 3ColN$, so $3ColN$ is NP-hard and $3ColN \in NP$ (by part (i)). It follows that $3ColN$ is NP-Complete.