

1. a.i. $S = \text{"success"}$

$P = \text{"picture"}$

$A = \text{"audio"}$

$$P(S|P) = 1-q \quad P(S|A) = 1-r$$

$$P(S|A) = \frac{P(A|S) P(S)}{P(A)} \quad \text{bayes}$$

$$\frac{P(S|A) \cdot P(A)}{P(S)} = P(A|S)$$

$$P(S|A) = 1-r \quad P(A) = 1-p$$

$$\begin{aligned} P(S) &= P(S|A) P(A) + P(S|P) P(P) \\ &= (1-r)(1-p) + (1-q)p \end{aligned}$$

$$P(A|S) = \frac{(1-r) \cdot (1-p)}{(1-r)(1-p) + (1-q)p}$$

ii. $P(S|A) = 1-r$

$$P(S|P) = 1-q$$

$SSS = \text{"new succ"}$

$$\begin{aligned} P(SSS|A) &= {}^3C_2 \cdot P(S'|A) \cdot P(S|A)^2 + {}^3C_3 \cdot P(S|A)^3 \\ &= 3 \cdot r \cdot (1-r)^2 + 1 \cdot (1-r)^3 \\ &= (\cancel{3} - 6r^2 + 3r^3) + (1 - \cancel{3r} + 3r^2 - r^3) \end{aligned}$$

$$= 1 - 3r^2 + 2r^3$$

$$P(SSS|p) = \dots$$

$$= 3 \cdot q \cdot (1-q)^2 + 1 \cdot (1-q)^3$$

$$= 1 - 3q^2 + 2q^3$$

$$P(SSS) = P(SSS|A) P(A) + P(SSS|p) P(p)$$

$$= (1 - 3r^2 + 2r^3) (1-p) + (1 - 3q^2 + 2q^3) (p)$$