FINAL

IMPERIAL COLLEGE OF SCIENCE, TECHNOLOGY AND MEDICINE

EXAMINATIONS 2018

BEng Honours Degree in Computing Part I
MEng Honours Degrees in Computing Part I
for Internal Students of the Imperial College of Science, Technology and Medicine

This paper is also taken for the relevant examinations for the Associateship of the City and Guilds of London Institute

PAPER C142

DISCRETE STRUCTURES

Monday 14th May 2018, 14:00
Duration: 80 minutes

Answer ALL TWO questions

Paper contains 2 questions
Calculators not required

1a Let $A = \{1,4,5,7\}$ and $B = \{1,3,7\}$. Write down explicit sets for

i) $A \cup B$ and $A \cap B$;

ii) $A \setminus B$ and $B \setminus A$;

iii) $A \triangle B$;

iv) $A \times \emptyset$ and $A \times (B \setminus A)$;

b i) Let R be a binary relation on A. State the formal property that R should satisfy in order to be called: *reflexive*, *symmetric*, or *transitive*.

For the following questions, either give a counter example, or show the result.

ii) Is a symmetric relation always reflexive?

iii) Is the union of two symmetric relations always symmetric?

iv) Is the intersection of two transitive relations always a transitive relation?

v) Is the union of two transitive relations always a transitive relation?

c Let $R = \{\langle 1,2\rangle, \langle 2,3\rangle, \langle 3,1\rangle, \langle 4,5\rangle, \langle 5,1\rangle\}$.

i) Give R^+.

ii) Illustrate both R and its transitive closure R^+ as directed graphs.

d Let R be a binary relation on A. Reason why if R is symmetric, also R^+ is symmetric.

e i) Give the properties a relation R should satisfy in order to be an *equivalence* relation.

ii) A binary relation R on a set A is called *circular* on A if

$$\forall a,c \in A \ (\exists b \ (a\,R\,b \wedge b\,R\,c) \Rightarrow c\,R\,a)$$

Prove that a relation R is an equivalence relation on A if and only if it is reflexive and circular on A.

The five parts carry, respectively, 20%, 30%, 15%, 10%, and 25% of the marks.

2 a Let R be a binary relation on a set A. Give the definitions of the following different notions of orders:

i) pre-order;

ii) anti-symmetric;

iii) partial order;

iv) irreflexive;

v) strict partial order;

vi) total order.

b Consider the set $F \triangleq \{2,3,4,6,10,12,15,20,30,40,60\}$ and consider the binary relation R on F defined by: $n\,R\,m \triangleq \exists k \in F\ (k \times n = m)$.

i) Give the Hasse diagram for $\langle F, R \rangle$.

ii) Argue if R is either

A) a pre-order;

B) anti-symmetric;

C) a partial order;

D) irreflexive;

E) a strict partial order;

F) a total order;

remark that more than one of these might be true.

c i) Give the definition of the relation $\approx$ between sets.

ii) Show that $\{0,1\}^V \approx \wp(V)$, for any set V.

d Using the (Dual) Cantor-Bernstein Theorem, show that the union of two countable infinite sets V and W is countable.

The four parts carry, respectively, 30%, 25%, 20%, and 25% of the marks.