2a.

i. $\bar{x} = \frac{\sum x_i}{n}$

$= 33$

ii. $t(\theta)$ is biased if $E(t(\theta)) \neq \theta$

TP. $E(s^2) \neq \sigma^2$

$s^2 = \frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2 \qquad \sigma^2 = \frac{1}{n}\sum_{i=1}^{n}(x_i - \mu)^2$

$E\left(\frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2\right)$

$= E\left(\frac{1}{n}\sum_{i=1}^{n}\left(x_i^2 - 2\bar{x}x_i + \bar{x}^2\right)\right)$

$= E\left(\frac{1}{n}\sum_{i=1}^{n}(x_i^2) - \frac{2\bar{x}}{n}\sum_{i=1}^{n}(x_i) + \frac{1}{n}\sum_{i=1}^{n}(\bar{x}^2)\right)$

$= E\left(\frac{1}{n}\sum_{i=1}^{n}(x_i^2) - 2\frac{\left(\sum_{i=1}^{n}(x_i)\right)^2}{n^2} + \frac{1}{n}\cdot n\cdot\frac{\left(\sum_{i=1}^{n}(x_i)\right)^2}{n^2}\right)$

$= E\left(\frac{1}{n}\sum_{i=1}^{n}(x_i^2) - \bar{x}^2\right)$

$= E\left[\frac{1}{n}\sum_{i=1}^{n}(x_i^2)\right] - E\left[\bar{x}^2\right]$

now $x_i - \bar{x} = x_i - \bar{x} - \mu + \mu$

$= (x_i - \mu) - (\bar{x} - \mu)$

so let $X_i = x_i - \mu$ and $\bar{X} = \bar{x} - \mu$

$$= E\left[\frac{1}{n}\sum(x_i-\mu)^2\right] - E\left[(\bar{X}-\mu)^2\right]$$

$$= \left[\frac{1}{n}\cdot\sum_{i=1}^{n} E(x_i-\mu)^2\right] - Var(\bar{X})$$

$$= \sigma^2 - var\left(\frac{1}{n}\sum_{i=1}^{n} x_i\right)$$

$$= \sigma^2 - \frac{1}{n^2}\sum_{i=1}^{n} var(x_i)$$

$$= \sigma^2 - \frac{1}{n^2}\cdot n\sigma^2$$

$$= \sigma^2 - \frac{\sigma^2}{n} \neq \sigma^2$$

bias detected

iii. $s^2_{13-1} = s^2_{12} = 177$

iv. assume $\mu \sim t_{12}(\bar{x}, var(\bar{x}))$

$t_{12,0.01} = 2.680998$

$Var(\bar{x}) = \frac{\sigma^2}{n}$ from above

$\approx s^2_{12}/n = 13.615385$

$\mu \sim t_{12}\left(\bar{x}, \frac{177}{13}\right)$

98% ... $\left[\bar{x} - t_{12, 0.01} \cdot \sqrt{\frac{177}{13}} \, , \, \bar{x} + t_{12, 0.01} \sqrt{\frac{177}{13}} \right]$

$= [\, 33 - 9.892623 \, , \; 33 + 9.892623 \,]$

$= [\, 23.107377 \, , \; 42.892623 \,]$