1.) a) $6x_1 + 2x_2 + 2x_3 + 4x_4 + 5x_5 \le 9 \qquad x_i \in \{0,1\}$

1 $x_1 + x_4 \le 1$
2 $x_1 + x_5 \le 1$
3 $x_1 + x_2 + x_3 \le 2$
4 $x_2 + x_4 + x_5 \le 2$
5 $x_3 + x_4 + x_5 \le 2$

there are no more pairwise cover cuts
can't choose any other coeffs that have a sum > 9
for the cuts of 3 variables : in each cut, the variables not present are already in another constraint with some of the variables present
e.g. x_4 missing from (3) but x_4 already in constrain with x_1

1. b) $x_1 \le 4$
$(x_2 \le 2)$
$x_2 \geq x_1 - 2$
$x_2 \le \frac{x_1}{2}$

$x_1, x_2 \geq 0$

1. c) constraints are as follows

poss. values of x_1:
$x_1 \in [0, 4]$
$x_2 \in [0, 2]$
criteria on M :

1 $x_1 \geq 0$
2 $x_2 \geq 0$
3 $x_1 \le 4$
4 $x_2 \le 2$
5 $x_1 \le 2 + M\gamma_1$ — $4 \le 2 + M \rightarrow M \geq 2$
6 $x_2 \le 0 + M\gamma_1$ — $2 \le 0 + M \quad M \geq 2$
7 $x_1 \geq 2 - M\gamma_2$ — $0 \geq 2 - M \quad M \geq 2$
8 $x_2 \le x_1 - 2 + M\gamma_2$ — $2 \le 0 - 2 + M \rightarrow M \geq 4$
9 $x_2 \geq x_1 - 2 - M\gamma_2$ — $0 \geq 4 - 2 - M \quad M \geq 2$
10 $\gamma_1 + \gamma_2 = 1$

$\Rightarrow M \geq 4$ so $M = 4$ is a sufficiently big M-value in order to disable all constraints

1. d) by looking at the graphs of the two feasible sets we can see that the left feasible set is a superset of the feas. set on the right which means we have less constraints
if we remove some constraints from the second one we will be able to obtain all the feasible points of the set on the left

if we relax the right feasible set:

so only keep: $\{8, \cancel{9}, 10, \cancel{6}, 1, 2, 3, 4\}$ is equivalent to the original LP so we get: FS'

$x_1 \geq 0$

$x_2 \geq 0$

$x_1 \leq 4$

$x_2 \leq 2$

$x_2 \geq x_1 - 2 - M\gamma_2$

$\gamma_1 + \gamma_2 = 1$

but they're not equivalent because for example $(1,2)$ is in the feasible set of FS' but not in the left feasible set

1. e) $\max\ 3x_1 + 2x_2 + 4x_3$

s.t. $2x_1 + x_2 + 4x_3 \leq 4$

$x_i \leq 1 \quad \forall i \in \{1,2,3\}$

$x_i \geq 0 \quad \forall i \in \{1,2,3\}$

$x_i \in \{0,1\}$

soln to LP relax.: $x_1 = 1,\ x_2 = 1,\ x_3 = 0.5 \qquad z_{LP} = 6$

(~~add cover cuts:~~

$x_1 + x_3 \leq 1$

$x_1 + x_2 \leq 1$)

Step 1: initialisation:

$J = \{x_1, x_2, x_3\}$

$P_0 = [\,]$

Branch and bound:

Step 2: Node selection

OPT	P	$c^T x^*(P)$	$c^T x^*(P) > OPT$
∞	$P_0 = [\,]$	6	yes

choose P_0 because $c^T x^*(P_0) > OPT$

list variables

Step 3 branching rule

$x_j \in J$	x^*_j	$x^*_j \in \mathbb{N}_0$
x_1	1	yes
x_2	1	yes
x_3	0.25	no

Branching

→ select $x_3 \qquad x_3 \leq$

Node selection

OPT	P	$c^T x^*(P)$	$c^T x^*(P) > OPT$
∞	$P_1 = x_3 \leq 0$	5	yes
	$P_2 = x_3 \geq 1$	4	yes

$x_1 = 1 \quad x_2 = 1 \quad x_3 = 0$

$x_3 = 1 \quad x_1 = 0 \quad x_2 = 0$

select P_1 because biggest $c^T x^*(P_1)$

Branching rule list variables:

$x_j \in J$	x_j^*	$x^*_j \in \mathbb{N}$
x_1	1	yes
x_2	1	yes
x_3	0	no

all x_i $i \in \{1,2,3\}$ is integer

so update OPT:

$$OPT \leftarrow \min\{OPT, c^T x^*(P_1)\} \rightarrow OPT \leftarrow 5$$

Node selection

OPT	P	$c^T x^*(P)$	$c^T x^*(P) > OPT$
5	$P_{\cancel{1}} = x_3 \leq 0$	5	no
	$P_2 = x_3 \geq 1$	4	no

Termination

$c^T x^*(P) > OPT$ for both P_0, P_1 so we can terminate

Optimum of IP is $Z^*_{IP} = 5$ with $x_1 = 1$, $x_2 = 1$, $x_3 = 0$