IMPERIAL COLLEGE OF SCIENCE, TECHNOLOGY AND MEDICINE

EXAMINATIONS 2015

BEng Honours Degree in Computing Part I
MEng Honours Degrees in Computing Part I
BEng Honours Degree in Mathematics and Computer Science Part I
MEng Honours Degree in Mathematics and Computer Science Part I
for Internal Students of the Imperial College of Science, Technology and Medicine

This paper is also taken for the relevant examinations for the Associateship of the City and Guilds of London Institute

PAPER C140=MC140

LOGIC

Wednesday 29 April 2015, 14:00
Duration: 80 minutes

Answer ALL TWO questions

Paper contains 2 questions
Calculators not required

1 a Prove by natural deduction that

$$B, \quad \neg C \to \neg A \vee \neg B \quad \vdash \quad A \to C.$$

b Prove by equivalences that the following two sentences are logically equivalent:

$$\exists x \forall y (P(x,y) \to Q(x)),$$
$$\forall x \exists y P(x,y) \to \exists x Q(x).$$

In each step, state the general form of the equivalence you use (e.g., $\forall x(A \wedge B) \equiv \forall x A \wedge \forall x B$). Do not use more than one equivalence per step.

c Let A be a propositional formula. Suppose that A is not valid. For each of the following, state whether it *is definitely true*, *is definitely false*, or *may be true or false* (depending on A). You do not need to justify your answers.

i) There is no situation in which A is true.

ii) A is logically equivalent to $\top$.

iii) $\neg A$ is satisfiable.

iv) $\neg A$ is valid.

d For propositional formulas A, B, we write $A(B/p)$ for the formula obtained from A by replacing every occurrence of the atom p by B.

For example, if A is $\neg p \to p$ then $A(\bot/p)$ is $\neg\bot \to \bot$.

Let C be a propositional formula involving the atom p but no other atoms. Suppose that C is satisfiable. Let D be $C(\top/p)$. Explain why $C(D/p)$ is valid.

The four parts carry, respectively, 30%, 30%, 20%, and 20% of the marks.

2a In this part, you may use only the unary relation symbols `dragon`, `green`, `happy`, and `can_fly`, with the obvious meanings, and the binary relation symbol `child`, where $\texttt{child}(x, y)$ means that x is a child of y.

Translate the following into logic:

i) All dragons are green.

ii) Some dragon has a child that can fly.

iii) Every grandchild (that is, child of a child) of a dragon can fly.

iv) Every dragon with at least two green children is happy.

b Let L be a signature consisting of a unary function symbol f and a unary relation symbol P. Let E be the L-sentence

$$\Big(\forall x \forall y (f(x) = f(y) \to x = y)\Big) \wedge \Big(\forall x (P(x) \leftrightarrow \neg P(f(x)))\Big).$$

i) Let $n \geq 1$ be an integer. Describe (e.g., with a diagram) an L-structure M with exactly $2n$ objects in its domain, and such that $M \models E$.

ii) Let M be an L-structure whose domain is finite, and with $M \models E$. Explain why there are an *even number* of objects in the domain of M.

c Prove by natural deduction that

$$\left.\begin{array}{r} \exists x P(x) \\ \forall x (P(x) \to Q(x)) \\ \forall x \forall y (Q(x) \wedge Q(y) \to x = y) \end{array}\right\} \quad \vdash \quad \forall x (Q(x) \to P(x)).$$

The three parts carry, respectively, 35%, 30%, and 35% of the marks.