ex.1.

a) i) $S\_x = \begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix}$, $S\_y = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 0 & 0 \\ -1 & -2 & -1 \end{bmatrix}$

$S\_x = flip(S\_x) = \begin{bmatrix} 1 & 0 & -1 \\ 2 & 0 & -2 \\ 1 & 0 & -1 \end{bmatrix}$, $S\_y = flip(S\_y) = \begin{bmatrix} -1 & -2 & -1 \\ 0 & 0 & 0 \\ 1 & 2 & 1 \end{bmatrix}$

at the central pixel we have: $6 \cdot 1 + 7 \cdot 0 + 5 \cdot -1$ ;

$22 \cdot 2 + 0 \cdot 15 + 10 \cdot -2$

$32 \cdot 1 + 21 \cdot 0 + 18 \cdot -1$

$= 6 - 5 + 44 - 20 + 32 - 18 = \underline{39}$ w.r.t $x$

and $-6 - 7 \cdot 2 - 5 + 32 + 2 \cdot 21 + 18 = 67$ w.r.t $y$

$\Rightarrow$ edge strength $= \sqrt{38^2 + 67^2} = \sqrt{1521 + 4489} = \sqrt{6010} = \cancel{77.52}$

ii) I guess they are asking again for the central pixel, so $\theta = \arctan(g_y / g_x) = \arctan(67/38)$, this is the direction I guess. Not sure tbh.

b)

i) Estimated mask $\rightarrow$ $\begin{bmatrix} 0 & 1 & 0 \\ 0 & -2 & 0 \\ 0 & 1 & 0 \end{bmatrix}$

ii) $mask = flip\left(\begin{bmatrix} 0 & 1 & 0 \\ 0 & -2 & 0 \\ 0 & 1 & 0 \end{bmatrix}\right) = \begin{bmatrix} 0 & 1 & 0 \\ 0 & -2 & 0 \\ 0 & 1 & 0 \end{bmatrix}$

Convolve at the center of the image patch:

$-30 + 28 = -2$ $\rightarrow$ not an edge, so edges this should be 0

c) Horizontal $\rightarrow \begin{bmatrix} -1 & -1 & -1 \\ 2 & 2 & 2 \\ -1 & -1 & -1 \end{bmatrix}$, vertical - $\begin{bmatrix} -1 & 2 & -1 \\ -1 & 2 & -1 \\ -1 & 2 & -1 \end{bmatrix}$

Diagonal $\rightarrow \begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix}$, $\begin{bmatrix} -1 & -1 & 2 \\ -1 & 2 & -1 \\ 2 & -1 & -1 \end{bmatrix}$

ex.2

a) $\nabla^2 G(x,y) = 0 \Leftrightarrow \boxed{x^2+y^2 = 2\sigma^2}$

$$DG(x,y)=0 \Leftrightarrow \frac{1}{2\pi\sigma_1^2} e^{-\left(\frac{x^2+y^2}{2\sigma_1^2}\right)} = \frac{1}{2\pi\sigma_2^2} e^{-\left(\frac{x^2+y^2}{2\sigma_2^2}\right)}$$

$$\Leftrightarrow e^{\left(\frac{x^2+y^2}{2\sigma_2^2}\right)} e^{-\left(\frac{x^2+y^2}{2\sigma_1^2}\right)} = \frac{\sigma_1^2}{\sigma_2^2}$$

$$\Leftrightarrow e^{\left(\frac{x^2+y^2}{2\sigma_2^2} - \frac{x^2+y^2}{2\sigma_1^2}\right)} = \frac{\sigma_1^2}{\sigma_2^2}$$

$$\Leftrightarrow e^{\frac{\sigma_1^2(x^2+y^2) - \sigma_2^2(x^2+y^2)}{2\sigma_1^2\sigma_2^2}} = \frac{\sigma_1^2}{\sigma_2^2}$$

$$\Leftrightarrow (x^2+y^2)\frac{(\sigma_1^2-\sigma_2^2)}{2\sigma_1^2\sigma_2^2} = \ln\frac{\sigma_1^2}{\sigma_2^2}$$

$$\Leftrightarrow x^2+y^2 = \ln\frac{\sigma_1^2}{\sigma_2^2} \cdot \frac{2\sigma_1^2\sigma_2^2}{(\sigma_1^2-\sigma_2^2)}$$

$$\Rightarrow \sigma = \sqrt{\frac{\sigma_1^2\sigma_2^2}{(\sigma_1^2-\sigma_2^2)} \ln\frac{\sigma_1^2}{\sigma_2^2}}$$

b) step 1 $\rightarrow$ convert the images to grayscale (black and white)

step 2 $\rightarrow$ use SIFT to find the interesting points in both images

step 3) Match thees keypoint by using for example brute force matcher.

Reasoning: SIFT has a good performance.

c) i) I will track the balloons using edges. We can extract Edges with Sobel operator.

ii) The edges of the balloons will be a lot smoother

iii) we can use sift, which will detect more keypoints than just the edges, so the probability to lose the balloon for some frames will decrease

ex. 3

a)

i) We can use the face as a template and then slide this template through the picture, calculating cross-correlation.

ii) we can use discriminants such as area of the nose, perimeter of the nose.

iii) No, the results for continuous space doesn't generalise to discrete space very well

b)

i) we can use convolutional neural network and train it to classify each pixel of our image.

ii) area, perimeter, direction

iii) for each object we can create a periodic representation by plotting the radial distance w.r.t arc length. Then we find the frequency components of that periodic function. We can use this to classify the shape

iv) The only thing here that I can say is that for circular object the function r(arc_length) will be smoother and in general the deviation in the r value will be smaller than for a non-circular shape.