COMP245: Probability and Statistics 2016 - Problem Sheet 4

Further Probability

Q1) Show that if three events A, B, and C are independent, then A and $B \cup C$ are independent.

Q2) The sample space S of a random experiment is given by $S = \{a, b, c, d\}$, with probabilities $\mathrm{P}(\{a\}) = 0.2$, $\mathrm{P}(\{b\}) = 0.3$, $\mathrm{P}(\{c\}) = 0.4$, $\mathrm{P}(\{d\}) = 0.1$. Let A denote event $\{a, b\}$, and B the event $\{b, c, d\}$. Determine the following probabilities:

(a) $\mathrm{P}(A)$

(b) $\mathrm{P}(B)$

(c) $\mathrm{P}(\overline{A})$

(d) $\mathrm{P}(A \cup B)$

(e) $\mathrm{P}(A \cap B)$

Q3) Two factories produce similar parts. Factory 1 produces 1000 parts, 100 of which are defective. Factory 2 produces 2000 parts, 150 of which are defective. A part is selected at random and found to be defective; what is the probability that it came from factory 1?

Q4) In an experiment in which two fair dice are thrown, let A be the event that the first die is odd, let B be the event that the second die is odd, and let C be the event that the sum is odd. Show that events A, B, and C are pairwise independent, but A, B, and C are not jointly independent.

Q5) A company producing mobile phones has three manufacturing plants, producing 50, 30, and 20 percent respectively of its product. Suppose that the probabilities that a phone manufactured by these plants is defective are 0.02, 0.05, and 0.01 respectively.

(a) If a phone is selected at random from the output of the company, what is the probability that it is defective?

(b) If a phone selected at random is found to be defective, what is the probability that it was manufactured by the second plant?

Q6) In a gambling game called craps, a pair of dice is rolled and the outcome is the sum of the dice. The player wins on the first roll if the sum is 7 or 11 and loses if the sum is 2, 3, or 12. If the sum is 4, 5, 6, 8, 9, or 10, that number is called the players 'point'. Once the point is established, the rule is: If the player rolls a 7 before the point, the player loses; but if the point is rolled before a 7, the player wins. Compute the probability of winning in the game of craps.

Q7) Suppose a coin has probability p of landing heads up. If we flip the coin many times, we would expect the proportion of heads to be close to p. Take $p = 0.3$ and $n = 1,000$ and simulate n coin flips. Plot the proportion of heads as a function of n. Repeat for $p = 0.03$. What do you notice about the two plots?

Q8) Consider throwing a fair die. Let $A = \{2, 4, 6\}$ and $B = \{1, 2, 3, 4\}$.

(a) Show that the events A and B are independent.

(b) Simulate draws from the sample space and verify that

$$\widehat{P}(A \cap B) = \widehat{P}(A)\widehat{P}(B)$$

where $\widehat{P}(A)$ is the proportion of times A occurred in the simulation and similarly for $\widehat{P}(B)$ and $\widehat{P}(A \cap B)$.

(c) Now find two events A and B that are not independent. Compute $\widehat{P}(A)$, $\widehat{P}(B)$ and $\widehat{P}(A \cap B)$. Compare the calculated values to their theoretical values. Report your results and interpret.