structure1a)

1	B	given
2	$\neg C \to \neg A \vee \neg B$	given
3	A	ass
4	$\neg C$	ass
5	$\neg A \vee \neg B$	$\to E$ (2,4)
6	$\neg A$	ass
7	$\perp$	$\neg I$ (3,6)
8	$\neg B$	ass
9	$\perp$	$\neg I$ (1,8)
10	$\perp$	$\vee E$ (5,6,7,8,9)
11	C	PC (4,10)
12	$A \to C$	$\to I$ (3,11)

1b)

$\exists x \forall y(P(x,y) \to Q(x))$

$\equiv \quad \exists x(\exists y(P(x,y) \to Q(x))$ by $\forall x(A \to B) \equiv \exists xA \to B$ (when x not free in B)

$\equiv \quad \exists x(\neg \exists y(P(x,y) \vee Q(x))$ by $A \to B \equiv \neg A \vee B$

$\equiv \quad \exists x \neg \exists y P(x,y) \vee \exists x Q(x)$ by $\exists x(A \vee B) \equiv \exists xA \vee \exists xB$

$\equiv \quad \neg \forall x \exists y P(x,y) \vee \exists x Q(x)$ by $\neg \forall xA \equiv \exists x \neg A$

$\equiv \quad \forall x \exists y P(x,y) \to \exists x Q(x)$ by $A \to B \equiv \neg A \vee B$

1c)i) may be true or false

1c)ii) false

1c)iii) true
1c)iv) may be true or false

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1d)
C is satisfiable and has p as its only atom
So two cases, either $C \leftrightarrow p$ (case 1), or $C \leftrightarrow \neg p$ (case 2)
D = "C with T replacing p"
First case: D is valid so equivalent to true
Second case: $\neg D$ is valid so D equivalent to false

Now consider C with D replacing p
In the first case: this is equivalent to C with T replacing p, so this is valid
Int the second case: this is equivalent to C with $\perp$ replacing p, so this is also valid

In either case, C(D/p) is valid

Alternate answers on next page

Alternate answer:

d.

C is satisfiable $\therefore$ it's satisfiable for <u>some</u> L-structure M and assignment h $M, h \models C$

By setting D to be $C(\top/p)$, we are essentially reducing down the formula / evaluating it / collapsing it into one of two values: $\top$ or $\bot$

Let's look at the two possible cases:

<u>Case 1: D is $\top$</u>

In that case $C(D/p)$ becomes $C(\top/p)$.

Again, be setting p to be D (i.e. $\top$) we are evaluating C / collapsing it down to one of two possibilities: $\top$ or $\bot$

Well we know that C is satisfiable, so it is true in <u>at least</u> one L-structure it definitely can't be $\bot$. Hence it must be $\top$ $\therefore$ it's valid because it's true in every situation / every L-structure

<u>Case 2: D is $\bot$</u>

In that case $C(D/p)$ becomes $C(\bot/p)$

Again, setting p to be D (i.e. $\bot$) we are evaluating C / collapsing it down to one of two possibilities: $\top$ or $\bot$

// Same reasoning as case 1

$\therefore$ In conclusion $C(D/p)$ is valid

Alternative on next page

alternative :

may be true or false

d) C is satisfiable so $C \Leftrightarrow p$ or $C \Leftrightarrow \neg p$.

Case $C \Leftrightarrow p$:

D is valid because p is replaced with T and $(C \Leftrightarrow T) \rightarrow C$.

D is valid meaning $D \equiv T$.

$C(D/p)$ means $C(T/p)$ which is valid.

Case $C \Leftrightarrow \neg p$:

$C(T/p)$ is false as $((C \Leftrightarrow \neg T) \rightarrow \neg C$.

$\therefore D \equiv \bot$

$C(D/p) \equiv C(\bot/p)$ which is valid because every p is replaced with $\bot$ and $((C \Leftrightarrow \neg\bot) \rightarrow C$.

2a)i) ∀x(dragon(x) -> green(x))

2a)ii) ∃x(dragon(x) ∧ ∃y(child(y,x) ∧ can_fly(y)))

2a)iii) Possible answers:

∀x∀y∀z[(child(x, z) ∧ child(z, y) ∧ dragon(y)) -> can_fly(x)]

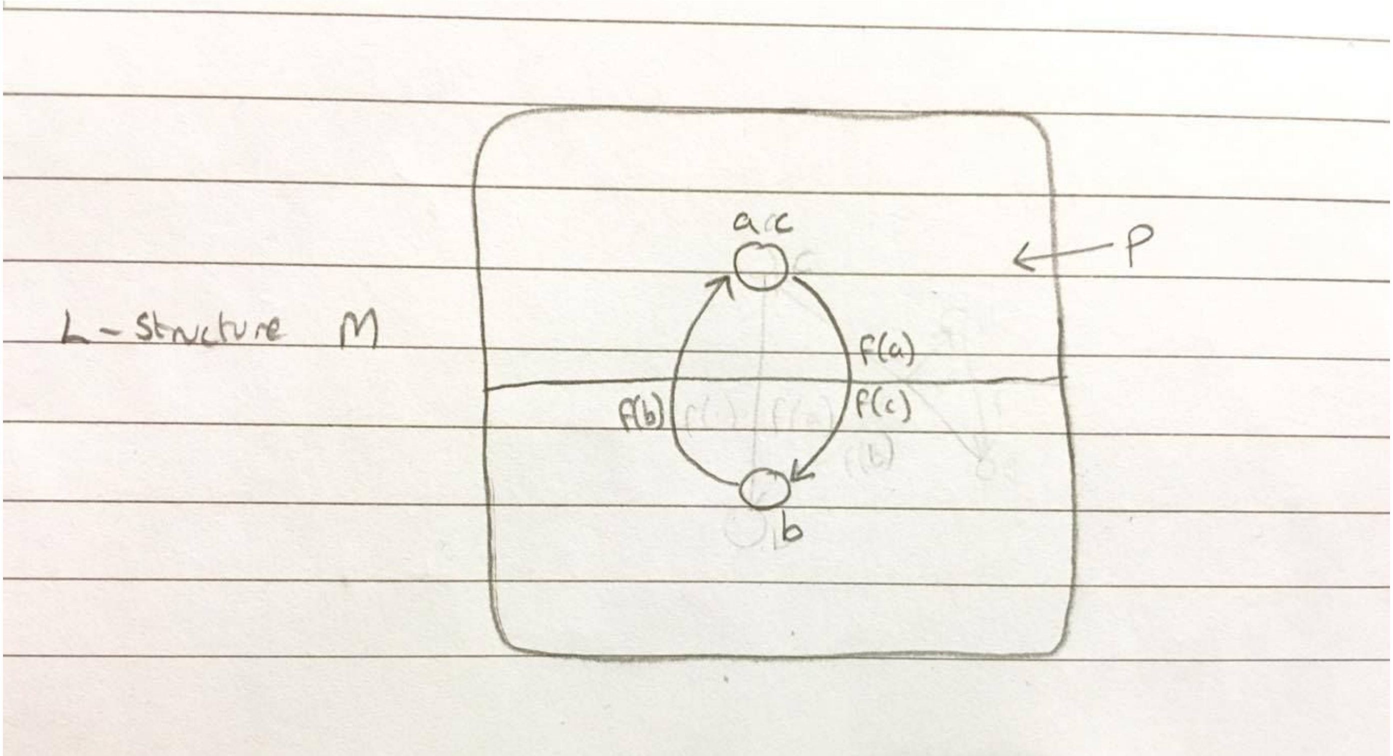
∀z[∃y∃x[dragon(x) ∧ child(y, x) ∧ child(z, y) ∧ x≠z ∧ x≠y] -> can_fly(z)]/.

(A)x (dragon(x) -> (A)y (child(y, x) -> (A)z (child(z, y) -> fly(z))))?

“All dragons’ all childrens’ all children can fly”

2a)iv) ∀x[(dragon(x) ∧ ∃y∃z(child(y, x) ∧ child(z, x) ∧ y≠z ∧ green(y) ∧ green(z)) -> happy(x)]

2b)i)



The following is a diagram for n=1, so it can be repeated n times for it to have exactly 2n objects in its domain

2b)ii)
f must must map every object to exactly one object in M because of the first conjunct
Every object x in M has either P(x) or ¬P(x), so can divide M into two sets of objects that have P(x) or ¬P(x)
f must map every object in either of these sets to exactly one object in the other set because of the second conjunct
Therefore the sets are the same size
So an even number of objects.

Alternate:

ii. For the L-sentence to be true, both $(\forall x \forall y (f(x) = f(y) \rightarrow x = y))$ AND $(\forall x (P(x) \leftrightarrow \neg P(f(x))))$ have to be true. For $\forall x (P(x) \leftrightarrow \neg P(f(x)))$ to be true, we can see that x HAS to be mapped by function f to an $f(x)$. So if every object has to have a mapping then if we have n objects, we will also have another n objects which are mapped to, hence a total of $2n$ objects (and $2n$ where $n \geq 1$ is always even $\therefore$ always an even number of objects in the domain of M).

Additionally, since it's a function (i.e. bijective and total in this case) then every element in the domain is mapped to one element in the codomain of the function. However in the case of the L-structure M, the domain & codomain of the function both make up the domain of M.

2c)

c) $\exists x\, P(x),\ \forall x(P(x) \to Q(x)),\ \forall x \forall y(Q(x) \land Q(y) \to x = y)$
$\vdash \forall x\,(Q(x) \to P(x))$

1	$\exists x\, P(x)$	given
2	$\forall x(P(x) \to Q(x))$	given
3	$\forall x \forall y(Q(x) \land Q(y) \to x = y)$	given
4	c	$\forall$I const
5	$Q(c)$	ass
6	$P(d)$	ass
7	$P(d) \to Q(d)$	$\forall$E(2)
8	$Q(d)$	$\to$E(6, 7)
~~9~~	$\cancel{Q(c) \land Q(d) \to}$	
9	$\forall y(Q(c) \land Q(y) \to c = y)$	$\forall$E(3)
10	$(Q(c) \land Q(d) \to c = d)$	$\forall$E(9)
11	$Q(c) \land Q(d)$	$\land$I(5, 8)
12	$c = d$	$\to$E(11, 10)
13	$P(c)$	=sub(6, 12)
14	$Q(c) \to P(c)$	$\to$I(5, 13)
15	$\forall x\,(Q(x) \to P(x))$	$\forall$I(4, 14)

Alternate (only slightly different)

1	$\exists x[P(x)]$	given
2	$\forall x[P(x) \rightarrow Q(x)]$	given
3	$\forall x[\forall y[Q(x) \wedge Q(y) \rightarrow x = y]]$	given
4 - 13		
4	sk1	$\forall$Iconst
5 - 12		
5	$Q(sk1)$	ass
6 - 11		
6	sk2	$\exists$Econst
7	$P(sk2)$	ass
8	$Q(sk2)$	$\forall\rightarrow$E(2,7)
9	$Q(sk1) \wedge Q(sk2)$	$\wedge$I(5,8)
10	$sk1 = sk2$	$\forall\rightarrow$E(3,9)
11	$P(sk1)$	=Sub(7,10)
12	$P(sk1)$	$\exists$E(1,7,11)
13	$Q(sk1) \rightarrow P(sk1)$	$\rightarrow$I(5,12)
14	$\forall x[Q(x) \rightarrow P(x)]$	$\forall$I(4,13)