2a i). PCA

Pagerank

etc...

ii). A) $\det(A_1 - \lambda I) = (-\lambda)^2$

$\lambda_1 = \lambda_2 = 0$, algebraic complexity 2.

$A_1 - \lambda_1 I = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ $\quad rk(A - \lambda_1 I) = 0$

$nullity(A - \lambda_1 I) = 2$

A_1 diagonalizable.

B). $\det(A_2 - \lambda I) = \begin{vmatrix} -\lambda & 1 \\ 0 & -\lambda \end{vmatrix}$

$= \lambda^2$

$\lambda_1 = \lambda_2 = 0$, a.m is 2

$A_2 - \lambda I_2 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$ $\quad rk(A_1 - \lambda I_2) = 1$

$nullity(A_1 - \lambda I_2) = 2 - 1 = 1 \neq$ a.m.

A_2 not diagonalizable.

iii). A). $\det(A - \lambda I) = \begin{vmatrix} -\lambda & 10 & -25 & 0 \\ 0 & 5-\lambda & -14 & 0 \\ 0 & 0 & -2-\lambda & 0 \\ 8 & -17 & 11 & 4-\lambda \end{vmatrix}$

$= (4-\lambda) \begin{vmatrix} -\lambda & 10 & -25 \\ 0 & 5-\lambda & -14 \\ 0 & 0 & -2-\lambda \end{vmatrix}$

$= (4-\lambda)(-\lambda)(5-\lambda)(-2-\lambda)$..

$= (4-\lambda)(\lambda)(5-\lambda)(2+\lambda)$.

$\lambda_1 = 4$, $\lambda_2 = 0$ $\quad \lambda_3 = 5$ $\quad \lambda_4 = -2$ //.

B).

$$D = \begin{bmatrix} 5 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -2 \end{bmatrix}$$

c). for $\lambda = 5$.

$$A - \lambda I = \begin{bmatrix} -5 & 0 & -25 & 0 \\ 0 & 0 & -14 & 0 \\ 0 & 0 & -7 & 0 \\ 8 & -17 & 11 & -1 \end{bmatrix}$$

$$\sim \begin{bmatrix} 8 & -17 & 11 & -1 \\ -5 & 0 & -25 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 5 & 0 \\ 8 & -17 & 11 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 0 & 5 & 0 \\ 0 & -17 & -29 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Eigenspace is $E \cdot \left[\begin{bmatrix} 0 \\ 1/17 \\ 0 \\ -1 \end{bmatrix} \right]$.

P projection matrix $= E(E^T E)^{-1} E^T$

(to be continued)

2b.

i). No — addition of two vectors in A may escape A.

ii. Yes — hyperplane.

iii. Yes — ~~Conic~~ Cone.

Yes. — trivial subspace.

iii).

$v \in L_1 \cap L_2$

$$v = \vec{p_1} + \alpha_1 \begin{bmatrix} -3 \\ -2 \\ 1 \end{bmatrix} = \vec{p_2} + \beta_1 \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} + \beta_2 \begin{bmatrix} 5 \\ 4 \\ 1 \end{bmatrix}.$$

$$\alpha_1 \begin{bmatrix} -3 \\ -2 \\ 1 \end{bmatrix} - \beta_1 \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} - \beta_2 \begin{bmatrix} 5 \\ 4 \\ 1 \end{bmatrix} = \begin{bmatrix} 9 \\ 6 \\ -3 \end{bmatrix}$$

$$\begin{bmatrix} -3 & -1 & -5 \\ -2 & -1 & -4 \\ 1 & -1 & -1 \end{bmatrix} \begin{bmatrix} \alpha_1 \\ \beta_1 \\ \beta_2 \end{bmatrix} = \begin{bmatrix} 9 \\ 6 \\ -3 \end{bmatrix}.$$

$$\left[\begin{array}{ccc|c} -3 & -1 & -5 & 9 \\ -2 & -1 & -4 & 6 \\ 1 & -1 & -1 & 3 \end{array}\right] \sim \left[\begin{array}{ccc|c} 0 & -4 & -8 & 18. \\ 0 & -3 & -6 & 6 \\ 1 & -1 & -1 & 3 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} -3 & -1 & -5 & 9 \\ -2 & -1 & -4 & 6 \\ 1 & -1 & -1 & -3 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & -1 & -1 & -3 \\ 0 & -4 & -8 & 0 \\ 0 & -3 & -6 & 0. \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & -1 & -1 & 3 \\ 0 & 4 & -8 & 18 \\ 0 & 1 & 2 & -2 \end{array}\right].$$

$$\sim \left[\begin{array}{ccc|c} 1 & -1 & -1 & -3 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]. \sim \left[\begin{array}{ccc|c} 1 & -1 & -1 & 3 \\ 0 & 1 & 2 & -2 \\ 0 & 0 & 0 & 10 \end{array}\right].$$

$$\sim \left[\begin{array}{ccc|c} 1 & 0 & 1 & -3 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right].$$

$$\therefore \begin{bmatrix} \alpha_1 \\ \beta_1 \\ \beta_2 \end{bmatrix} = \begin{bmatrix} -3 \\ 0 \\ 0 \end{bmatrix} + \varphi \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$$

Thus, $L_1 \cap L_2 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} + \begin{bmatrix} -3 \\ -2 \\ 1 \end{bmatrix}(-3+\varphi). = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} + \begin{bmatrix} 9 \\ 6 \\ -3 \end{bmatrix} - \varphi \begin{bmatrix} 6 \\ 4 \\ -1 \end{bmatrix}.$

$$= \begin{bmatrix} 10 \\ 6 \\ -2 \end{bmatrix} + \begin{bmatrix} -3 \\ -2 \\ 1 \end{bmatrix}\varphi \quad = \begin{bmatrix} 10 \\ 6 \\ -2 \end{bmatrix} - \varphi \begin{bmatrix} 6 \\ 4 \\ -1 \end{bmatrix} \quad //.$$