

IMPERIAL COLLEGE LONDON

TIMED REMOTE ASSESSMENTS 2021-2022

BEng Honours Degree in Computing Part I
MEng Honours Degrees in Computing Part I
for Internal Students of the Imperial College of Science, Technology and Medicine

*This paper is also taken for the relevant assessments for the
Associateship of the City and Guilds of London Institute*

PAPER COMP40017

LINEAR ALGEBRA

Wednesday 18 May 2022, 10:00

Writing time: 80 minutes

Upload time: 25 minutes

Answer ALL TWO questions

Open book assessment

This time-limited remote assessment has been designed to be open book. You may use resources which have been identified by the examiner to complete the assessment and are included in the instructions for the examination. You must not use any additional resources when completing this assessment.

The use of the work of another student, past or present, constitutes plagiarism. Giving your work to another student to use constitutes an offence. Collusion is a form of plagiarism and will be treated in a similar manner. This is an individual assessment and thus should be completed solely by you. The College will investigate all instances where an examination or assessment offence is reported or suspected, using plagiarism software, vivas and other tools, and apply appropriate penalties to students. In all examinations we will analyse exam performance against previous performance and against data from previous years and use an evidence-based approach to maintain a fair and robust examination. As with all exams, the best strategy is to read the question carefully and answer as fully as possible, taking account of the time and number of marks available.

Paper contains 2 questions

- 1 Let $A\vec{x} = \vec{b}$ be a system of linear equations where,

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \end{bmatrix} \quad \vec{b} = \begin{bmatrix} 8 \\ 4 \\ 0 \end{bmatrix}$$

- a
- i) Perform elementary row operations on the augmented matrix $[A|\vec{b}]$ to reduce A to its Reduced Row Echelon Form.
 - ii) State the free variables.
 - iii) Find the general solution (Solution set S) of $A\vec{x} = \vec{b}$ and describe it geometrically.
- b Find the solution of $A\vec{x} = \vec{b}$ closest (lowest squared Euclidean distance) to the origin $\vec{0}$.
- c
- i) Find the rank and nullity of A .
 - ii) Find a basis for the image space $im(A)$.
 - iii) Find a basis for the kernel $ker(A)$.
- d
- i) Find the rank and nullity of A^\top .
 - ii) Find a basis for the image space $im(A^\top)$.
 - iii) Find a basis for the kernel $ker(A^\top)$.

The four parts carry equal marks.

2a Let $\vec{a}_1, \vec{a}_2, \vec{a}_3, \vec{a}_4, \vec{b}_1, \vec{b}_2, \vec{b}_3, \vec{b}_4 \in \mathbb{R}^4$ such that,

$$\vec{a}_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \vec{a}_2 = \begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \end{bmatrix}, \vec{a}_3 = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \vec{a}_4 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\vec{b}_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \vec{b}_2 = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \vec{b}_3 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \end{bmatrix}, \vec{b}_4 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

Let $A = (\vec{a}_1, \vec{a}_2, \vec{a}_3, \vec{a}_4)$ and $B = (\vec{b}_1, \vec{b}_2, \vec{b}_3, \vec{b}_4)$.

- i) Show that A and B are two bases of \mathbb{R}^4 .
 - ii) Compute the matrix I_{AB} that performs a basis change from B to A .
 - iii) Also, compute the matrix I_{BA} that performs a basis change from A to B .
- b Find the intersection $S \cap U$ where,

$$U = \text{span} \left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}, \begin{bmatrix} 5 \\ 6 \\ 7 \\ 8 \end{bmatrix}, \begin{bmatrix} 9 \\ 10 \\ 11 \\ 12 \end{bmatrix} \right\} \text{ and } S = \begin{bmatrix} -10 \\ 9 \\ 0 \\ 0 \end{bmatrix} + \text{span} \left\{ \begin{bmatrix} 1 \\ -2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ -3 \\ 0 \\ 1 \end{bmatrix} \right\}$$

c Let $M = \begin{bmatrix} -2 & 1 \\ 12 & -3 \end{bmatrix}$ and $M^{13} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$.

- i) Compute the Eigenvalues and Eigenspaces of M .
- ii) Determine a transformation matrix B such that $B^{-1}MB$ is a diagonal matrix and provide this diagonal matrix.
- iii) Compute M^{13} , i.e., compute a, b, c, d .

The three parts carry, respectively, 30%, 35%, and 35% of the marks.