FINAL

IMPERIAL COLLEGE OF SCIENCE, TECHNOLOGY AND MEDICINE

EXAMINATIONS 2018

BEng Honours Degree in Computing Part I
MEng Honours Degrees in Computing Part I
for Internal Students of the Imperial College of Science, Technology and Medicine

This paper is also taken for the relevant examinations for the Associateship of the City and Guilds of London Institute

PAPER C145

MATHEMATICAL METHODS

Thursday 17th May 2018, 10:00
Duration: 80 minutes

Answer ALL TWO questions

Paper contains 2 questions
Calculators required

Always provide justifications and show any intermediate work for your answers. A correct but unsupported answer may not receive any marks. Pay attention to mathematical formalism.

1 a Consider the sequence

$$a_n = \frac{\sin n}{n}.$$

i) Find an upper bound of the sequence, or show that none exists.

ii) Find the limit $\lim_{n\to\infty} a_n$, or show that the limit does not exist.

Justify your answers.

b For what values of x does the series

$$S = \sum_{n=1}^{\infty} \frac{(4x-1)^n}{n^n}$$

converge? Show your work.

c Compute the Maclaurin series expansions for the following functions and find their radii of convergence:

$$f(x) := \frac{\ln(1+x^2)}{x^2},$$

$$g(x) := \int_0^x \frac{\sin t}{t}\, dt.$$

Show your work.

The three parts carry, respectively, 20%, 20%, and 60% of the marks.

2a Consider a vector $\boldsymbol{x} = \begin{bmatrix} 6 \\ 0 \\ 0 \end{bmatrix}$ and the subspace U spanned by the basis vectors

$$\boldsymbol{b}_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \boldsymbol{b}_2 = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix} .$$

i) Assuming the dot product as the inner product determine

A) The projection matrix

B) The projection $\pi_U(\boldsymbol{x})$ of $\boldsymbol{x}$ onto U.

C) The coordinates of $\pi_U(\boldsymbol{x})$ with respect to the basis $\boldsymbol{b}_1, \boldsymbol{b}_2$

b Let V, W be two vector spaces. Check whether the following mappings $\Phi : V \to W$ are linear (justify your answers):

i) $V = \mathbb{R}^n, W = \mathbb{R}, \Phi(x_1, \dots, x_n) = \sum_{k=1}^n kx_k$

ii) $V = \mathbb{R}^3, W = \mathbb{R}^2, \Phi(x_1, x_2, x_3) = \begin{bmatrix} x_1 - x_2 + 1 \\ 0 \end{bmatrix}$

c Determine all solutions of the inhomogeneous linear equation system $\boldsymbol{A}\boldsymbol{x} = \boldsymbol{b}$, where

$$\boldsymbol{A} = \begin{bmatrix} 1 & -1 & 0 & 0 & 1 \\ 1 & 1 & 0 & -3 & 0 \\ 2 & -1 & 0 & 1 & -1 \\ -1 & 2 & 0 & -2 & -1 \end{bmatrix}, \quad \boldsymbol{b} = \begin{bmatrix} 3 \\ 6 \\ 5 \\ -1 \end{bmatrix}$$

The three parts carry, respectively, 50%, 20%, and 30% of the marks.