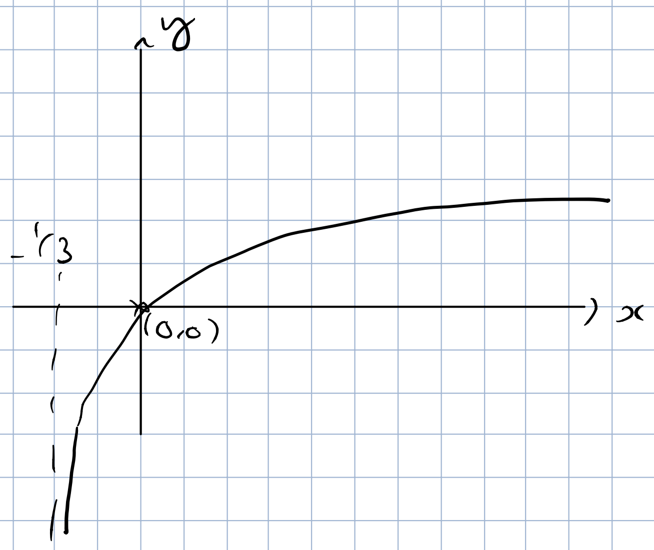
① ⓐ $f(x) = \ln(3x+1)$



ⓑ $f(x) = \ln(3x+1)$

$f'(x) = \dfrac{3}{3x+1} = 3(3x+1)^{-1}$

$f''(x) = (-1)(3)^2(3x+1)^{-2}$

$f'''(x) = (-1)(-2)(3)^3(3x+1)^{-3}$

$f^n(x) = (-1)^{n+1}(n-1)!\,3^n(3x+1)^{-n}$

$f^n(1) = \dfrac{(-1)^{n+1}(n-1)!\,3^n}{4^n}$

$f(x) = \ln 4 + \dfrac{(-1)^{n+1}(n-1)!\;3^n}{n!\;4^n}(x-1)^n$

$= \ln(4) + \sum_{n=1}^{\infty} \dfrac{(-1)^{n+1}}{n}\left(\dfrac{3}{4}\right)^n (x-1)^n$

Radius of Convergence

$$\lim \frac{a_{n+1}}{a_n} = \lim \left| \frac{(-1)^{n+2} \left(\frac{3}{4}\right)^{n+1} (x-1)^{n+1} \quad (n)}{(n+1) \quad (-1)^{n+1} \left(\frac{3}{4}\right)^{n} (x-1)^{n}} \right|$$

$$= \lim \left| \frac{(-1)\left(\frac{3}{4}\right)(x-1) \quad n}{n+1} \right|$$

$$= \left| \frac{3}{4}(x-1) \right| \quad \lim \frac{n}{n+1}$$

$$= \left| \frac{3}{4}(x-1) \right|$$

$$\left| \frac{3}{4}(x-1) \right| < 1 \qquad \text{(convergence condition.)}$$

$$|x-1| < \frac{4}{3}$$

$$x-1 < 4/3 \qquad \text{or} \qquad 1-x < \frac{4}{3}$$

$$x < \frac{7}{3} \qquad\qquad -\frac{1}{3} < x$$

$$-\frac{1}{3} < x < 7/3$$

(b) (i)

$$\left[\begin{array}{ccc|ccc} 1 & 2 & -3 & 1 & 0 & 0 \\ 0 & 1 & -1 & 0 & 1 & 0 \\ 1 & -1 & 1 & 0 & 0 & 1 \end{array}\right] -R_1 \rightsquigarrow \left[\begin{array}{ccc|ccc} 1 & 2 & -3 & 1 & 0 & 0 \\ 0 & 1 & -1 & 0 & 1 & 0 \\ 0 & -3 & 4 & -1 & 0 & 1 \end{array}\right] \begin{array}{l} -2R_2 \\ \\ +3R_2 \end{array}$$

$$\rightsquigarrow \left[\begin{array}{ccc|ccc} 1 & 0 & -1 & 1 & -2 & 0 \\ 0 & 1 & -1 & 0 & 1 & 0 \\ 0 & 0 & 1 & -1 & 3 & 1 \end{array}\right] \begin{array}{l} +R_3 \\ +R_3 \\ \end{array} \rightsquigarrow \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & -1 & 4 & 1 \\ 0 & 0 & 1 & -1 & 3 & 1 \end{array}\right]$$

$$M^{-1} = \begin{bmatrix} 0 & 1 & 1 \\ -1 & 4 & 1 \\ -1 & 3 & 1 \end{bmatrix}$$

(ii) $\left[\begin{array}{ccc|c} 1 & 2 & -3 & 1 \\ 0 & 1 & -1 & 2 \\ 1 & -1 & 1 & 4 \end{array}\right] \begin{array}{l} \\ \\ -R_1 \end{array} \rightsquigarrow \left[\begin{array}{ccc|c} 1 & 2 & -3 & 1 \\ 0 & 1 & -1 & 2 \\ 0 & -3 & 4 & 3 \end{array}\right] \begin{array}{l} -2R_2 \\ \\ +3R_2 \end{array}$

$\rightsquigarrow \left[\begin{array}{ccc|c} 1 & 0 & -1 & -3 \\ 0 & 1 & -1 & 2 \\ 0 & 0 & 1 & 9 \end{array}\right] \begin{array}{l} +R_3 \\ +R_3 \\ \\ \end{array} \rightsquigarrow \left[\begin{array}{ccc|c} 1 & 0 & 0 & 6 \\ 0 & 1 & 0 & 11 \\ 0 & 0 & 1 & 9 \end{array}\right]$

Point of intersection: $\begin{bmatrix} 6 \\ 11 \\ 9 \end{bmatrix}$

(iii) Rank of M is 3 as all columns are linearly independent as RREF was achieved.

Nullity is 0 by rank-nullity theorem.