

IMPERIAL COLLEGE LONDON

TIMED REMOTE ASSESSMENTS 2020-2021

BEng Honours Degree in Computing Part I
MEng Honours Degrees in Computing Part I
for Internal Students of the Imperial College of Science, Technology and Medicine

*This paper is also taken for the relevant assessments for the
Associateship of the City and Guilds of London Institute*

PAPER COMP40004

DISCRETE MATHEMATICS

Monday 10 May 2021, 10:00

Duration: 95 minutes

Includes 15 minutes for access and submission

Answer ALL TWO questions

Open book assessment

This time-limited remote assessment has been designed to be open book. You may use resources which have been identified by the examiner to complete the assessment and are included in the instructions for the examination. You must not use any additional resources when completing this assessment.

The use of the work of another student, past or present, constitutes plagiarism. Giving your work to another student to use constitutes an offence. Collusion is a form of plagiarism and will be treated in a similar manner. This is an individual assessment and thus should be completed solely by you. The College will investigate all instances where an examination or assessment offence is reported or suspected, using plagiarism software, vivas and other tools, and apply appropriate penalties to students. In all examinations we will analyse exam performance against previous performance and against data from previous years and use an evidence-based approach to maintain a fair and robust examination. As with all exams, the best strategy is to read the question carefully and answer as fully as possible, taking account of the time and number of marks available.

Paper contains 2 questions

1 a Let $V = \{\{1\}, \{2\}\}$ and $W = \{\{1\}, 1, 2\}$. Determine $V \cap W$, $V \cup W$, $\varnothing W$, $V \cap \varnothing W$, $V \times W$, and $V \triangle W$.

b Let $A = \{a, b, c, d, e, f\}$ and let R be a binary relation on A . If $\langle a, b \rangle, \langle a, f \rangle, \langle d, c \rangle, \langle e, f \rangle \in R$ and R is reflexive, symmetric, and transitive, what (other) ordered pairs must belong to R ?

c Prove that, for all sets A, B, C , and D , we have:

$$(A \cup B) \cap (C \cup D) \subseteq (A \cup C) \cup (B \cap D).$$

d Show that $(\overline{R \cup S})^{-1} = \overline{R^{-1}} \cap \overline{S^{-1}}$ and that $\overline{R^{-1}} \cap \overline{S^{-1}} = \overline{(R \cup S)^{-1}}$.

Write clearly when a step follows by definition, or when a logical reasoning, or when information from the notes is used.

e Let R denote a binary relation on \mathbb{N}^2 , the set of pairs of natural numbers, defined by $\langle p, q \rangle R \langle r, s \rangle \triangleq \exists n, m \in \mathbb{N} \setminus \{0\} (n \times p = r \wedge m \times q = s)$. Prove that R is a partial order.

The five parts carry equal marks.

- 2a Determine which of the following relations are (partial) functions. For those which are functions, determine whether they are injective and/or surjective. Also, give the inverse function when it exists. Justify your answer throughout.
- i) $\{\langle x, y \rangle \in \mathbb{Z}^2 \mid x = y \wedge x = -y\}$;
 - ii) $\{\langle x, y \rangle \in (\mathbb{R} \setminus \{2\}) \times (\mathbb{R} \setminus \{0\}) \mid y = 1/(x^3 - 8)\}$;
 - iii) $\{\langle x, y \rangle \in \mathbb{R}^2 \mid x^2 = y^3\}$.
 - iv) $\{\langle x, y \rangle \in \mathbb{Q}^2 \mid y = x^3\}$.
- b Let A and B be arbitrary sets with $|A| = m$ and $|B| = n$. How many functions are there from $\wp A$ to $\wp B$ and how many partial functions? Motivate your answer.
- c Consider the set $F \triangleq \{V \in \wp \mathbb{N} \mid \exists n \in \mathbb{N} (|V| = n)\}$ of finite subsets of \mathbb{N} , and consider the relation R on F defined by: $V R W \triangleq V \neq \emptyset \wedge V \subseteq W$. Does (F, R) have minimal, least, maximal, or greatest elements? If yes, are these unique? If no, argue why. Give examples, and show why each satisfies the criterion.
- d Assume that $A_1 \approx A_2$ and $B_1 \approx B_2$; show that $A_1 \times B_1 \approx B_2 \times A_2$.
- e For your answers below, clearly state any result from the course that you may use in your proof.
- i) Show that the set of all functions from \mathbb{N} to $\{1\}$ is countable.
 - ii) Show that the set of all partial functions from \mathbb{N} to $\{1\}$ is not countable.

The five parts carry equal marks.