IMPERIAL COLLEGE LONDON

TIMED REMOTE ASSESSMENTS 2020-2021

BEng Honours Degree in Computing Part I
MEng Honours Degrees in Computing Part I
for Internal Students of the Imperial College of Science, Technology and Medicine

This paper is also taken for the relevant assessments for the Associateship of the City and Guilds of London Institute

PAPER COMP40004

DISCRETE MATHEMATICS

Monday 10 May 2021, 10:00
Duration: 95 minutes
Includes 15 minutes for access and submission

Answer ALL TWO questions
Open book assessment

Paper contains 2 questions

1 a Let $V = \{\{1\},\{2\}\}$ and $W = \{\{1\},1,2\}$. Determine $V \cap W$, $V \cup W$, $\wp W$, $V \cap \wp W$, $V \times W$, and $V \bigtriangleup W$.

b Let $A = \{a,b,c,d,e,f\}$ and let R be a binary relation on A. If $\langle a,b\rangle$, $\langle a,f\rangle$, $\langle d,c\rangle$, $\langle e,f\rangle \in R$ and R is reflexive, symmetric, and transitive, what (other) ordered pairs must belong to R?

c Prove that, for all sets A, B, C, and D, we have:

$$(A \cup B) \cap (C \cup D) \subseteq (A \cup C) \cup (B \cap D).$$

d Show that $(\overline{R \cup S})^{-1} = \overline{R^{-1}} \cap \overline{S^{-1}}$ and that $\overline{R^{-1}} \cap \overline{S^{-1}} = \overline{(R \cup S)^{-1}}$.

Write clearly when a step follows by definition, or when a logical reasoning, or when information from the notes is used.

e Let R denote a binary relation on $I\!N^2$, the set of pairs of natural numbers, defined by $\langle p,q\rangle \; R \; \langle r,s\rangle \triangleq \exists\, n,m \in I\!N \setminus \{0\} \; (n \times p = r \wedge m \times q = s)$. Prove that R is a partial order.

The five parts carry equal marks.

2 a Determine which of the following relations are (partial) functions. For those which are functions, determine whether they are injective and/or surjective. Also, give the inverse function when it exists. Justify your answer throughout.

i) $\{\langle x,y\rangle \in \mathbb{Z}^2 \mid x = y \wedge x = -y\}$;

ii) $\{\langle x,y\rangle \in (\mathbb{R}\setminus\{2\})\times(\mathbb{R}\setminus\{0\}) \mid y = 1/(x^3-8)\}$;

iii) $\{\langle x,y\rangle \in \mathbb{R}^2 \mid x^2 = y^3\}$.

iv) $\{\langle x,y\rangle \in \mathbb{Q}^2 \mid y = x^3\}$.

b Let A and B be arbitrary sets with $|A| = m$ and $|B| = n$. How many functions are there from $\wp A$ to $\wp B$ and how many partial functions? Motivate your answer.

c Consider the set $F \triangleq \{V \in \wp\, \mathbb{N} \mid \exists n \in \mathbb{N}\ \big(|V| = n\big)\}$ of finite subsets of $\mathbb{N}$, and consider the relation R on F defined by: $V\,R\,W \triangleq V \neq \emptyset \wedge V \subseteq W$.

Does (F,R) have minimal, least, maximal, or greatest elements? If yes, are these unique? If no, argue why. Give examples, and show why each satisfies the criterion.

d Assume that $A_1 \approx A_2$ and $B_1 \approx B_2$; show that $A_1 \times B_1 \approx B_2 \times A_2$.

e For your answers below, clearly state any result from the course that you may use in your proof.

i) Show that the set of all functions from $\mathbb{N}$ to $\{1\}$ is countable.

ii) Show that the set of all partial functions from $\mathbb{N}$ to $\{1\}$ is not countable.

The five parts carry equal marks.