PAPER C140S=MC140S

LOGIC

Monday 24 August 2020, 11:00
Duration: 80 minutes
Post-processing time: 30 minutes
Answer TWO questions

While this time-limited remote assessment has not been designed to be open book, in the present circumstances it is being run as an open-book examination. We have worked hard to create exams that assesses synthesis of knowledge rather than factual recall. Thus, access to the internet, notes or other sources of factual information in the time provided will not be helpful and may well limit your time to successfully synthesise the answers required.
Where individual questions rely more on factual recall and may therefore be less discriminatory in an open book context, we may compare the performance on these questions to similar style questions in previous years and we may scale or ignore the marks associated with such questions or parts of the questions. In all examinations we will analyse exam performance against previous performance and against data from previous years and use an evidence-based approach to maintain a fair and robust examination. As with all exams, the best strategy is to read the question carefully and answer as fully as possible, taking account of the time and number of marks available.

Paper contains 2 questions

1 a Explain what it means to say that a propositional formula is

i) in conjunctive normal form,

ii) a tautology.

Give an illustrative example in each case.

b Using direct argument, determine whether the formula

$$\neg(((p \to q) \to \neg q) \to \neg q)$$

is satisfiable.

c Use (propositional) equivalences to show that the two formulas

$$p \vee \neg q, \qquad (p \to (\top \vee \neg q)) \wedge (\neg p \to (\bot \vee \neg q))$$

are logically equivalent. In each step of your proof, state the equivalence you use (e.g., $\neg\neg A \equiv A$). Do not use more than one equivalence in each step.

d Let p, q, r, s be propositional atoms. Prove by natural deduction that

$$\neg p \to q \vee r, \ \ \neg(q \vee s), \ \ \neg(s \to r \wedge s) \ \vdash \ p.$$

The four parts carry, respectively, 10%, 20%, 30%, and 40% of the marks.

2a i) Let A, B be first-order formulas. Explain what it means to say that A is *valid*, A is *unsatisfiable*, and A *logically entails* B.

ii) Give an example of a first-order formula S that is satisfiable but not valid.

iii) Let S be the formula you gave in part a(ii). Is $\neg S$ satisfiable? Explain your answer.

b Let $C(x)$ be a first-order formula and let D and F be first-order sentences. Draw the formation trees of the following three sentences:

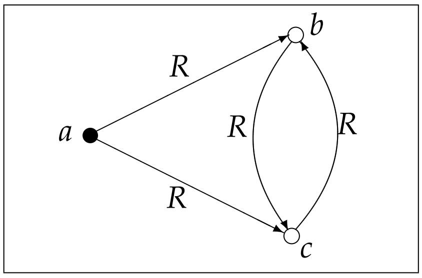
i) $(\forall x C(x)) \to (D \vee F)$

ii) $((\forall x C(x)) \to D) \vee F$

iii) $\forall x((C(x) \to D) \vee F)$

Which of your trees is the formation tree of $\forall x C(x) \to D \vee F$? Explain your answer.

c Let L be a signature consisting of a unary relation symbol P, a binary relation symbol R, and constants a, b, c. Let N be the L-structure shown below (only a satisfies P; $R(a,b)$ is true, $R(b,a)$ is false, etc).

N 

Which of the following sentences are true in N? Give brief reasons in each case.

i) $\forall x \exists y(R(x,y) \wedge R(y,x) \to P(x))$

ii) $\forall x(\exists y(R(x,y) \wedge R(y,x)) \to P(x))$

iii) $\forall x \forall y \forall z(R(x,y) \wedge R(y,z) \to P(x))$

d Draw a diagram of an L-structure K such that all the sentences in part c that you said were false in N are true in K.

e Use natural deduction to prove (ii) $\vdash$ (i), where (i) and (ii) denote the sentences in part c.

The five parts carry, respectively, 15%, 20%, 20%, 15%, and 30% of the marks.