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Adversarial Search

(Game Search)

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Overview

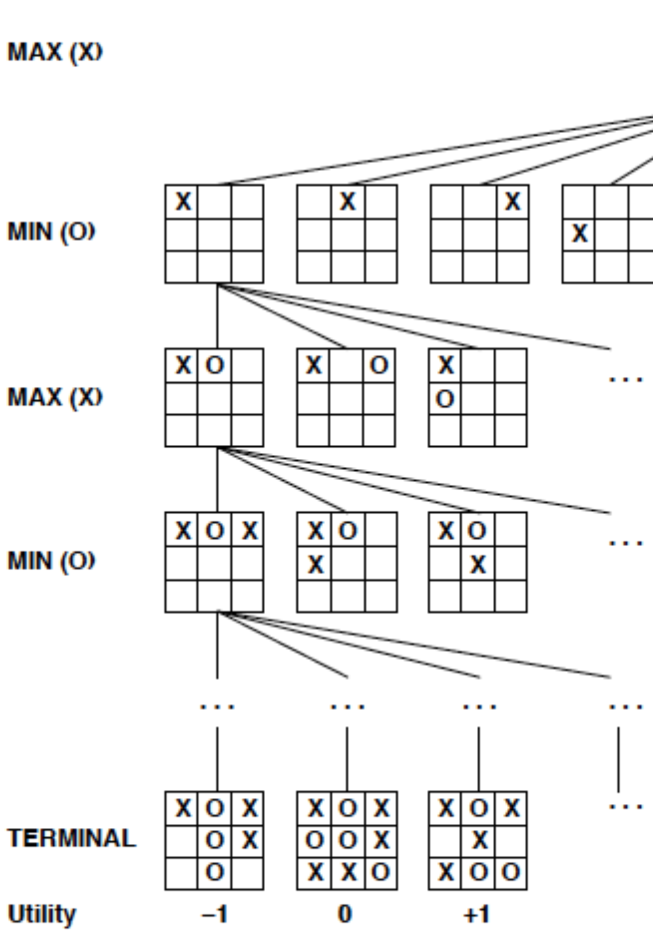
- Types of games
- Mimimax
- α-β pruning
- Monte Carlo tree search

Types of Games

- Games search involves an unpredictable opponent
- Games can be classified along several dimensions
- We'll be looking at two-player deterministic games where there is perfect information, such as chess

	Deterministic	Chance
Perfect information	chess, drafts, go, othello	backgammon, monopoly
Imperfect information	battleships	bridge, poker, scrabble

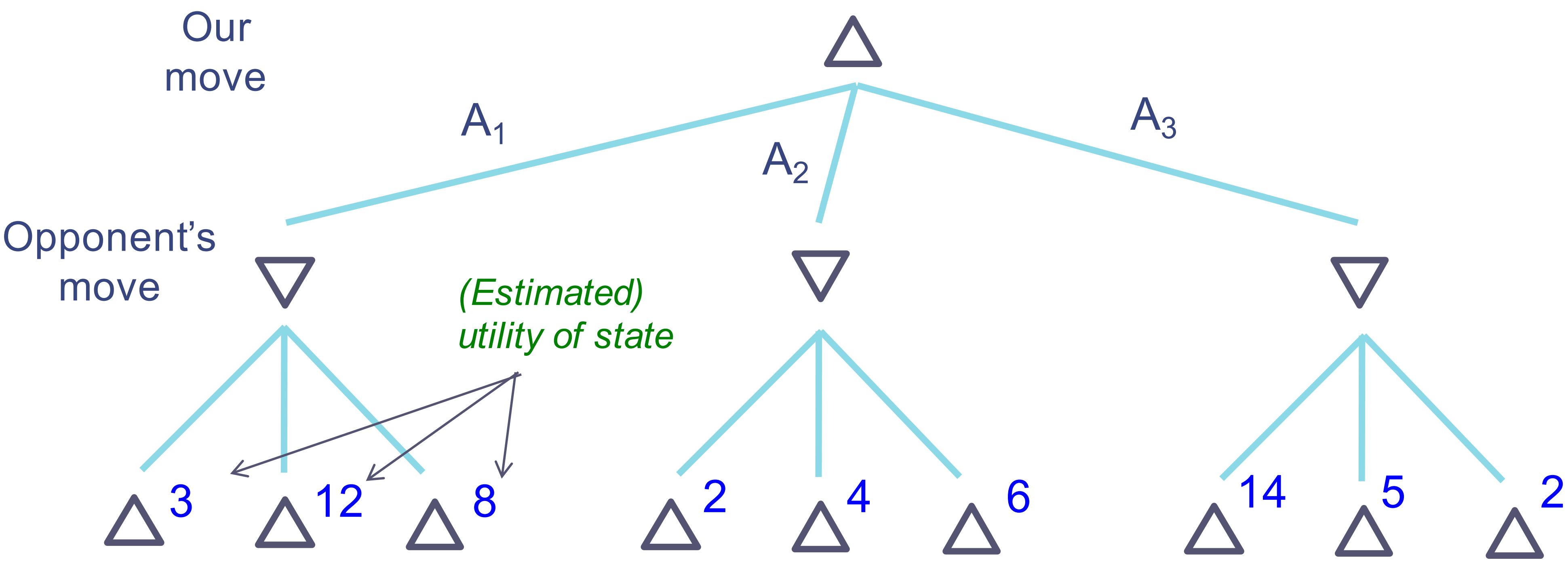
Game Trees



- The task is to search a game tree like this for the best move
- The search can go all the way to the game's terminal states
- Or it can finish after a given depth (an *n*-ply search is a search to depth *n*)
- An evaluation function is needed to estimate the utility of any given state

Choosing the Best Move

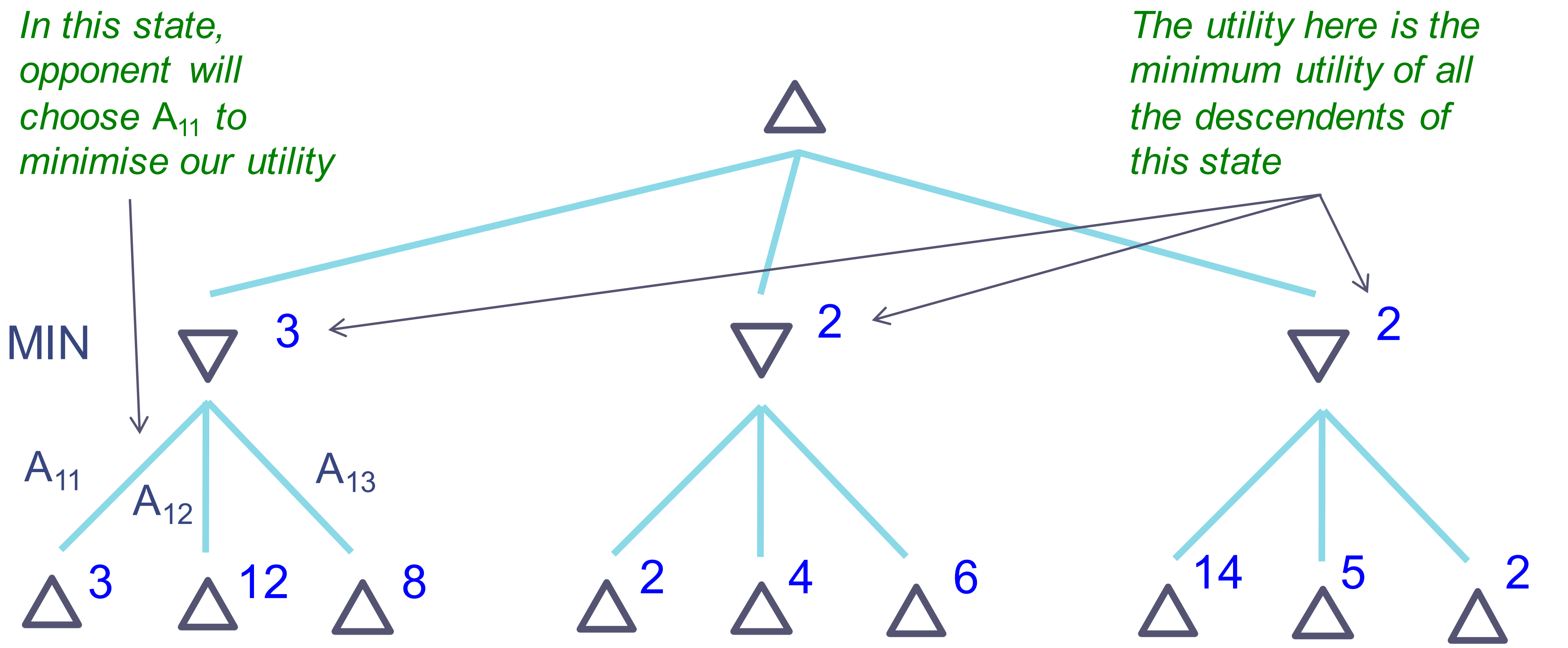
- Suppose we have the following search tree
- Should we select action A_1, A_2, or A_3?



The Minimax Principle

- Assume that the opponent will always make the worst move for us
  - This is the action with the lowest estimated utility
- But we will always make the best move
- So utilities can be propagated up the tree, alternating between minimizing utility (opponent’s move) and maximizing utility (our move)
- The move we make is the one with the maximum utility at the root

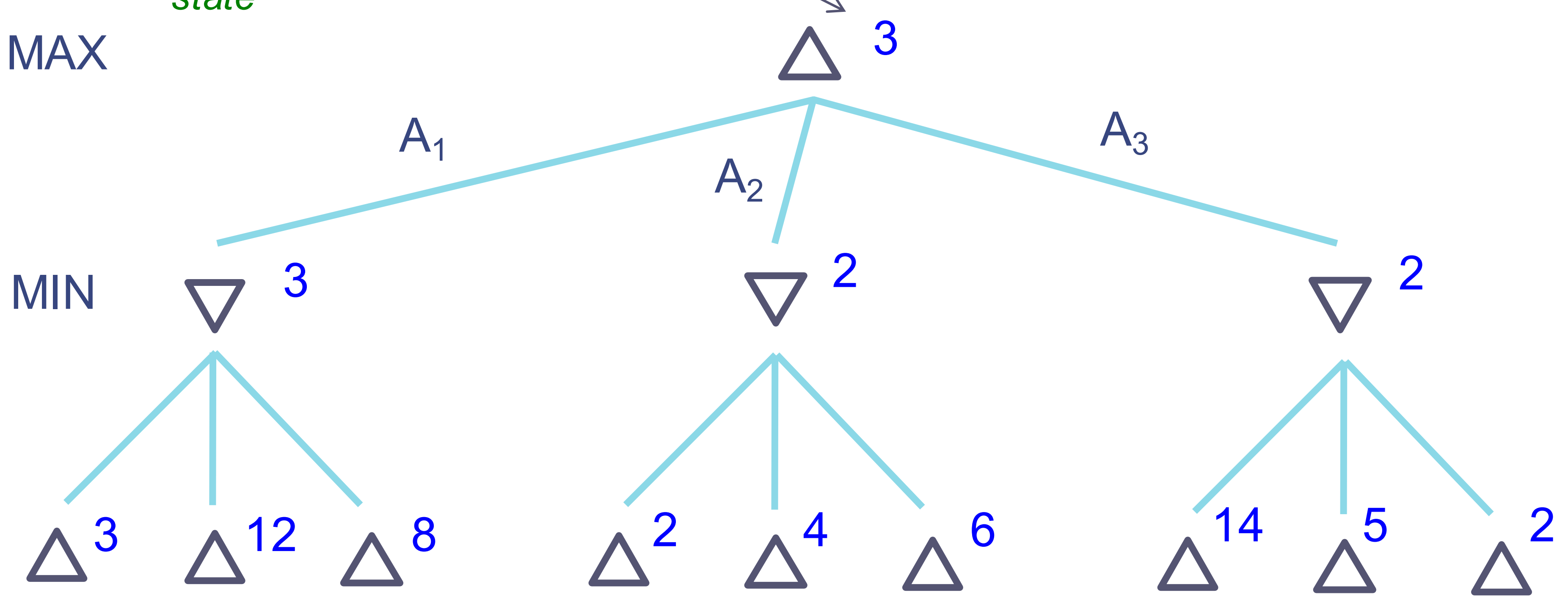
The Minimising Phase



The Maximising Phase

Now we're maximising. The utility is the maximum utility of all the descendents of this state

So the action to choose is A_1



The Minimax Algorithm 1

- The algorithm is expressed as a pair of mutually recursive functions `MinValue` and `MaxValue`
- `Eval(s)` is the evaluation function. It yields an estimate of the utility of state `s`

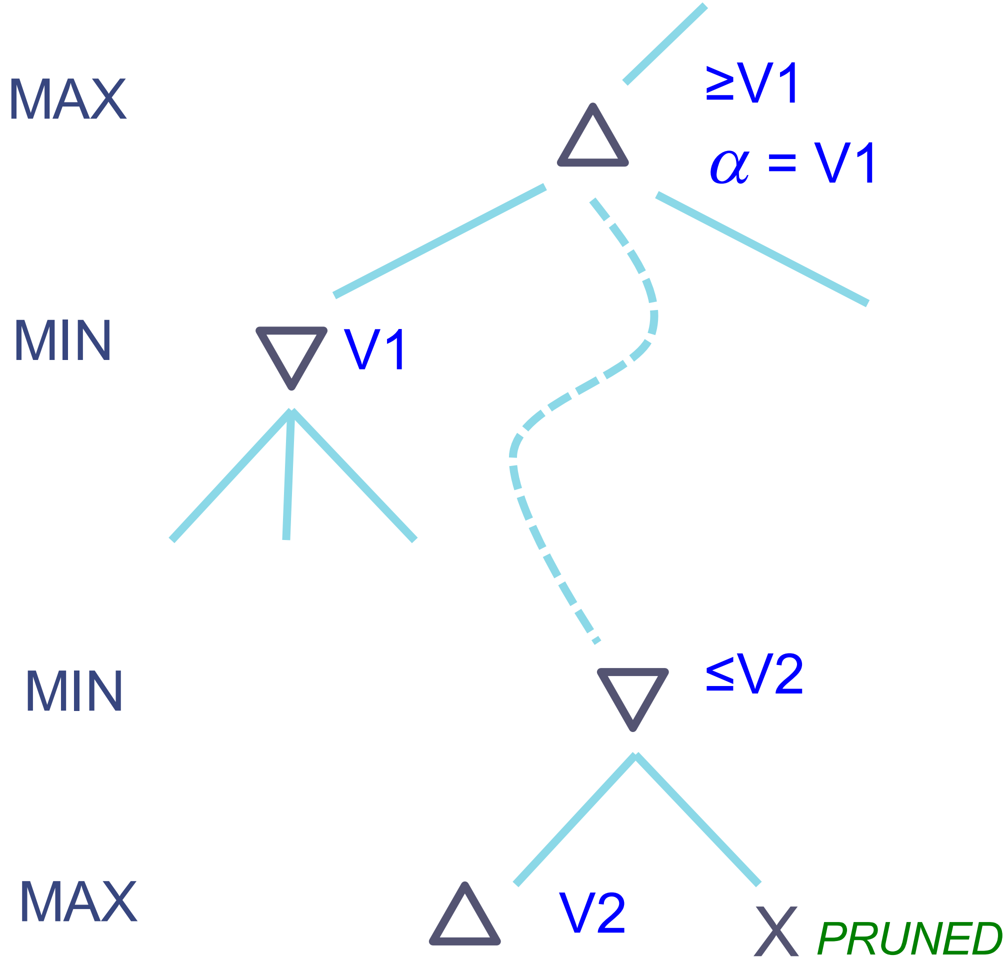
```
function MinValue(s,d)
      if s is a terminal state or d = MaxDepth
            return Eval(s)
      else
            v := ∞
            for each action a possible in s
                  v := Min(v,MaxValue(Result(a,s),d+1)
            return v
```

The Minimax Algorithm 2

- The best move is the action a that maximises `MinValue(Result(a,S0),1)` where `S0` is the current state of the game

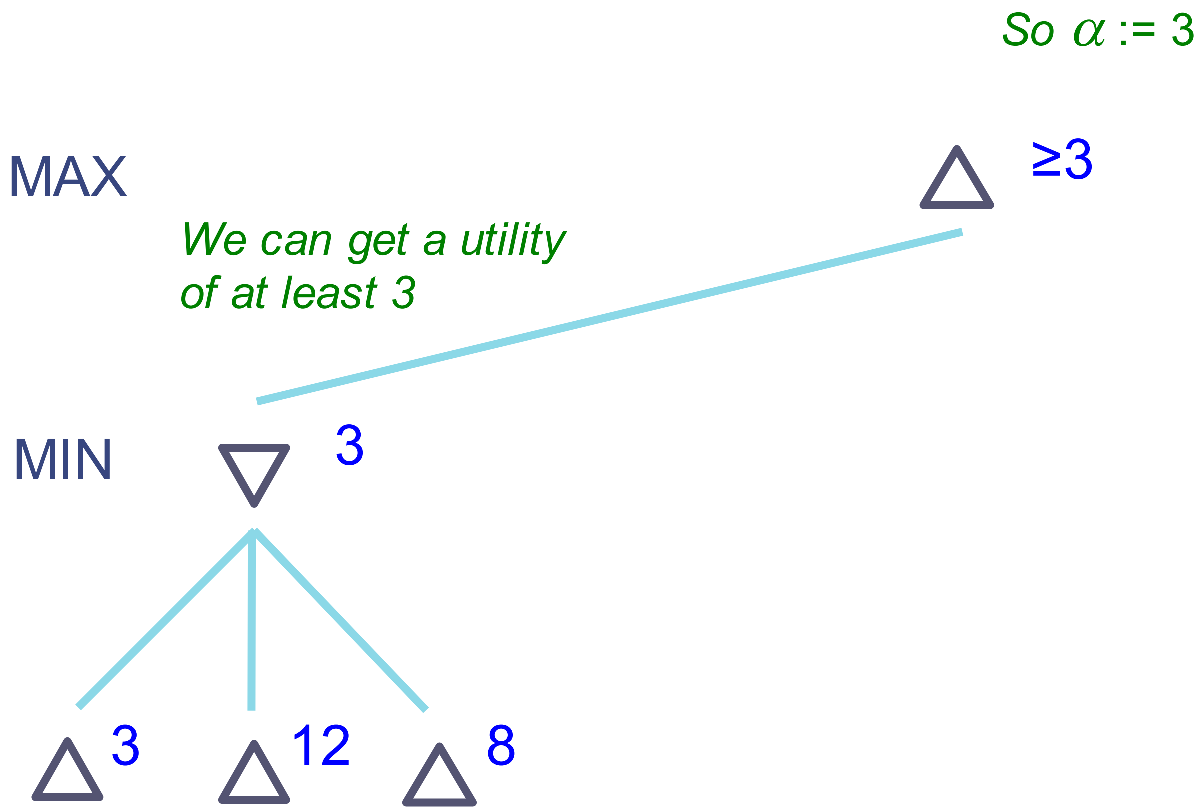
```
function MaxValue(s,d)
      if s is a terminal state or d = MaxDepth
            return Eval(s)
      else
            v := −∞
            for each action a possible in s
                  v := Max(v,MinValue(Result(a,s),d+1)
            return v
```

α-β Pruning

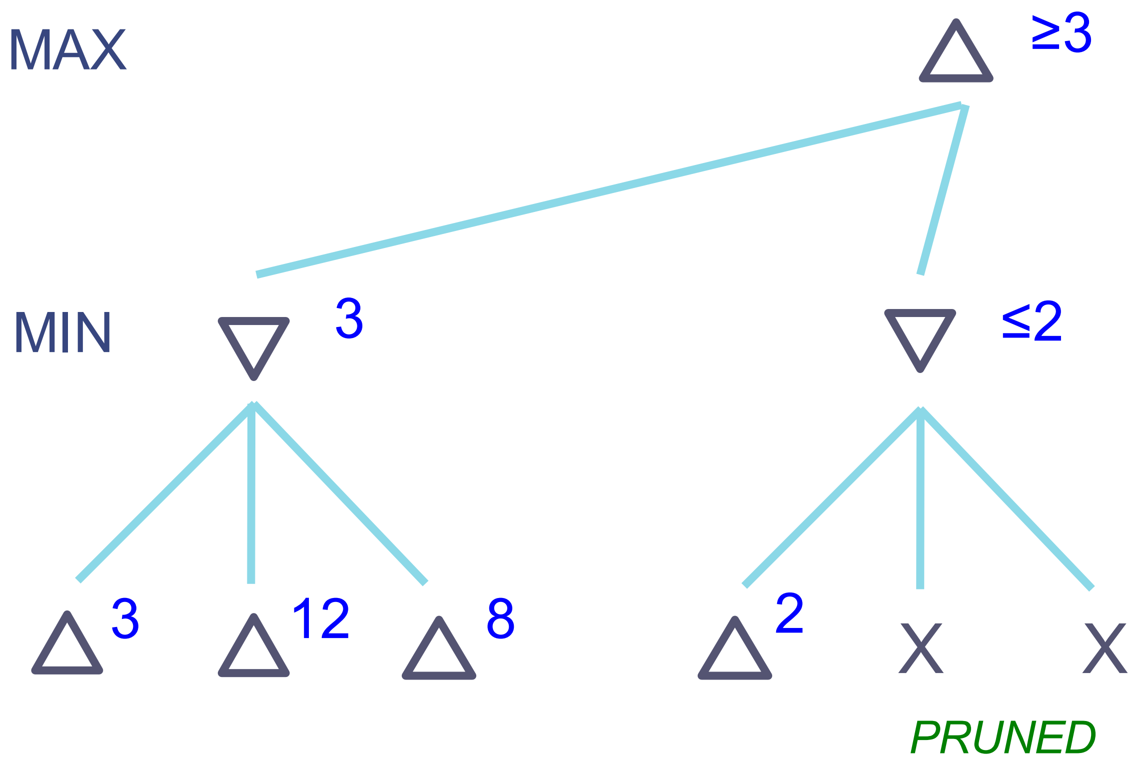


- Minimax performs a lot of redundant search
- It can be improved by keeping track of the best MAX value found so far (α) and the worst MIN value (β)
- There is no point in exploring MIN branches worse than α or MAX branches better than β
- Here, if we have V2 < α there is no need for MIN to explore more branches for that node

α-β Pruning Example

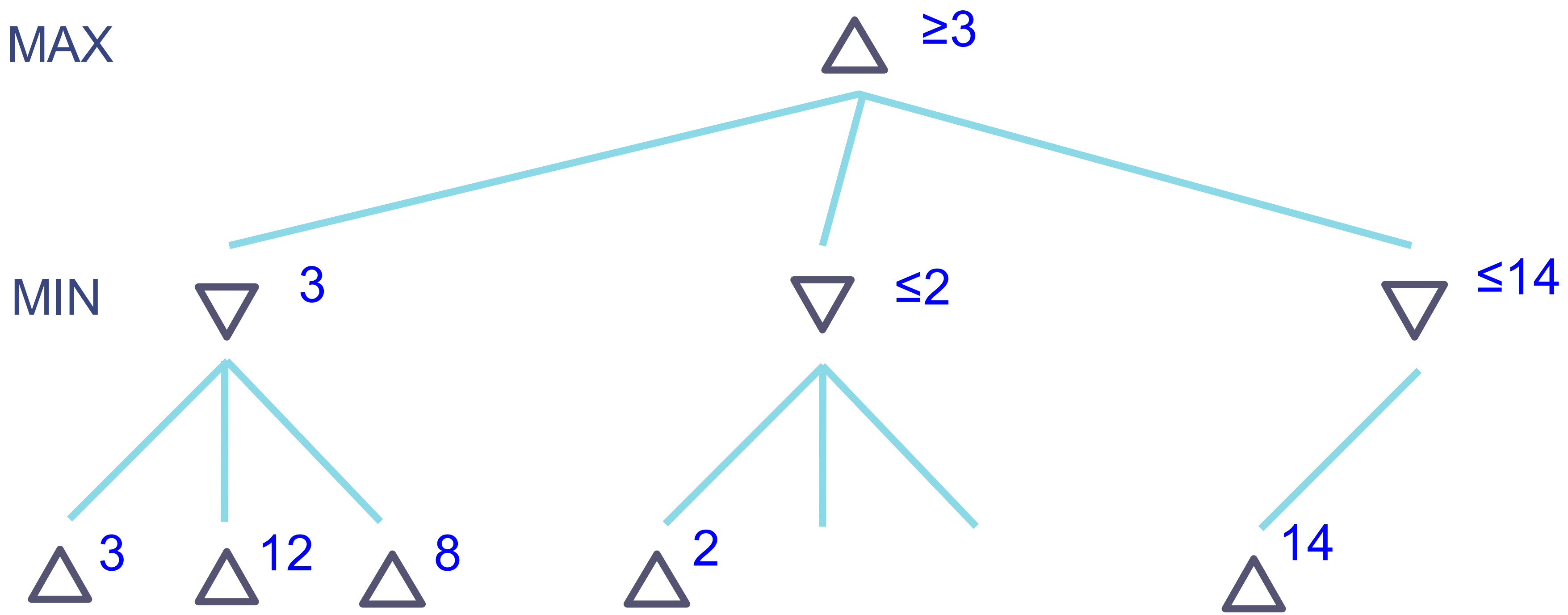


α-β Pruning Example

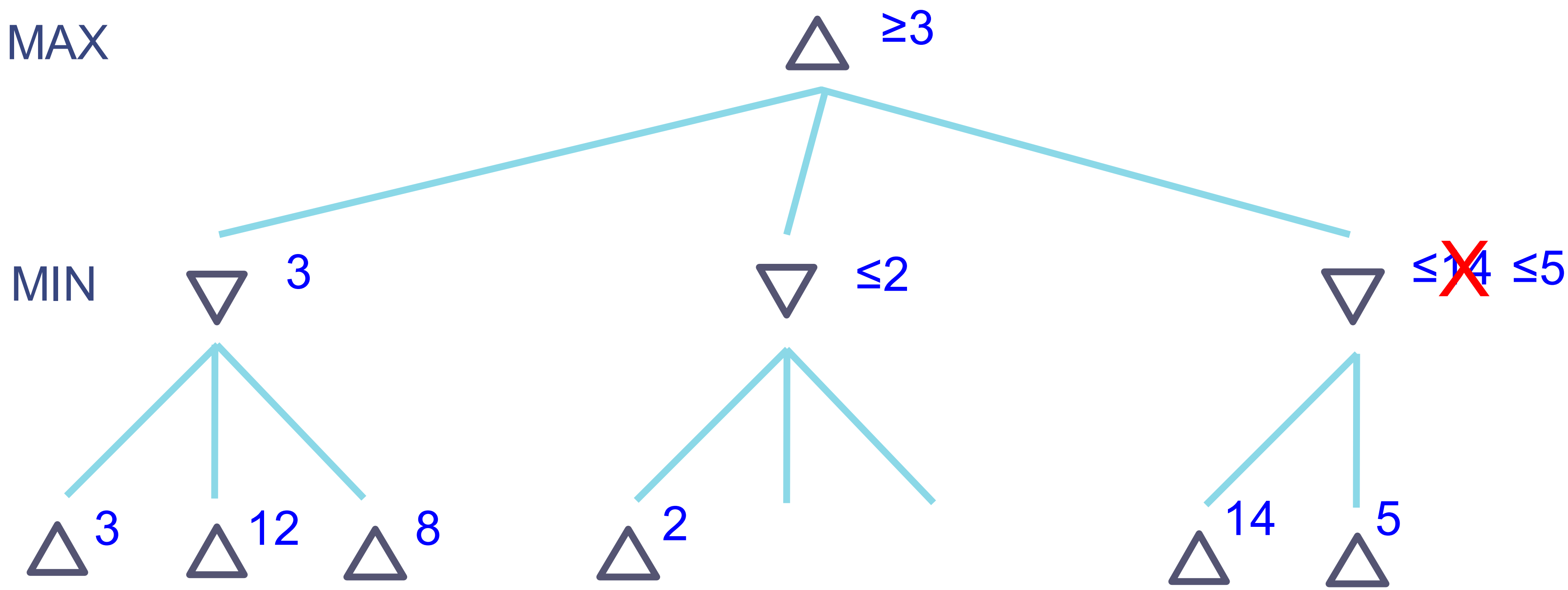


The opponent can force a utility of 2 or worse down this branch. But we already have α = 3 So there's no need to look further here

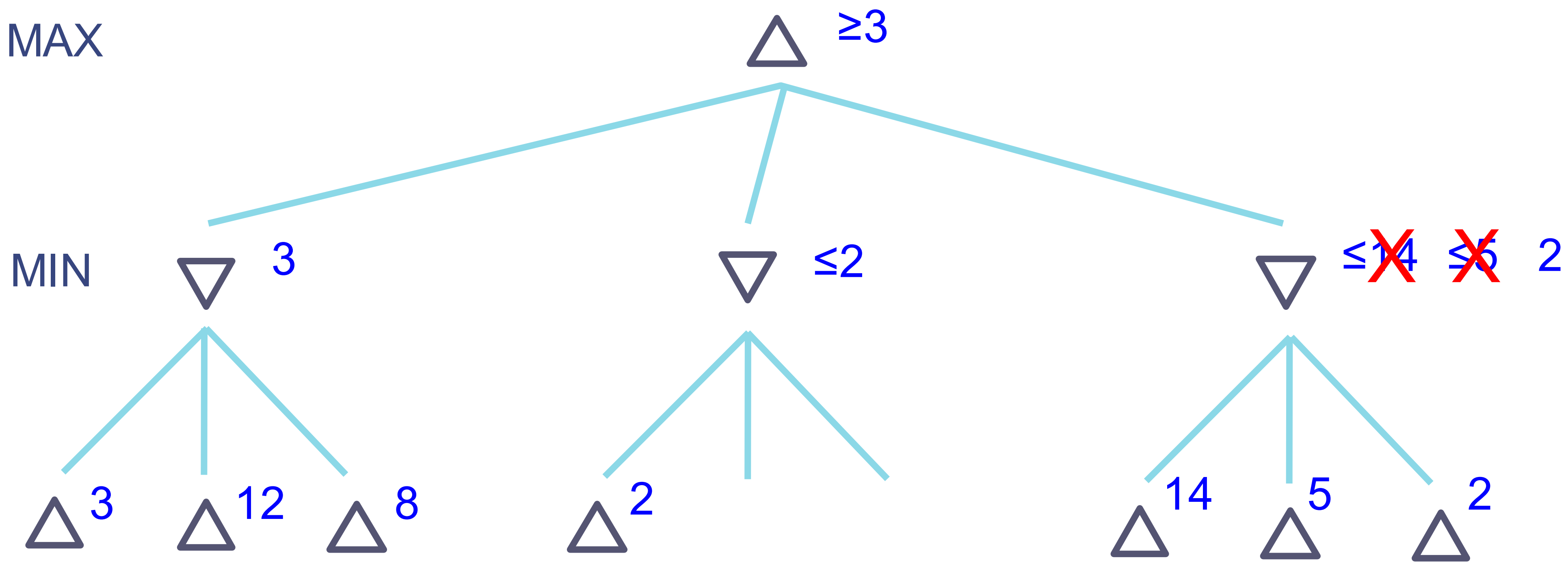
α-β Pruning Example



α-β Pruning Example



α-β Pruning Example



The Alpha-Beta Algorithm

- The Minimax algorithm is extended. Here's the new `MinValue`. `MaxValue` is analogous with roles of α and β reversed
- Need to maximise `MinValue(Result(a,S0),-∞,∞,1)`

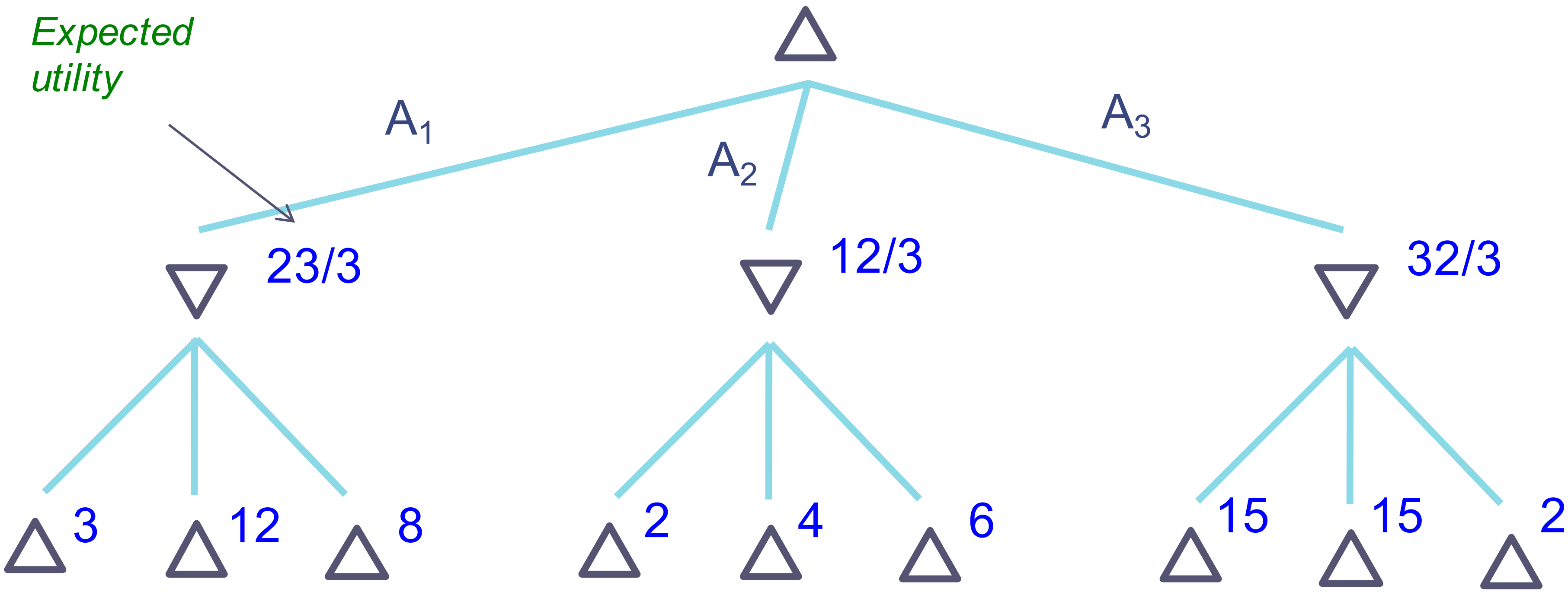
```
function MinValue(s,α,β,d)
     if s is a terminal state or d = MaxDepth
          return Eval(s)
     else
          v := ∞
          for each action a possible in s
               v := Min(v,MaxValue(Result(a,s),α,β,d+1)
               if v ≤ α return v
               else β := Min(β,v)
          return v
```

Optimality

- If there is no depth limit, minimax is guaranteed to find the optimal move against an optimal opponent
- Alpha-beta will find the same move as minimax (but faster)
- If the opponent is not optimal …
  - Consider an opponent that picks random moves. Then minimax might not be the best strategy for maximising expected reward
- If there is a depth limit, then minimax finds the optimal move for the given limit and evaluation function

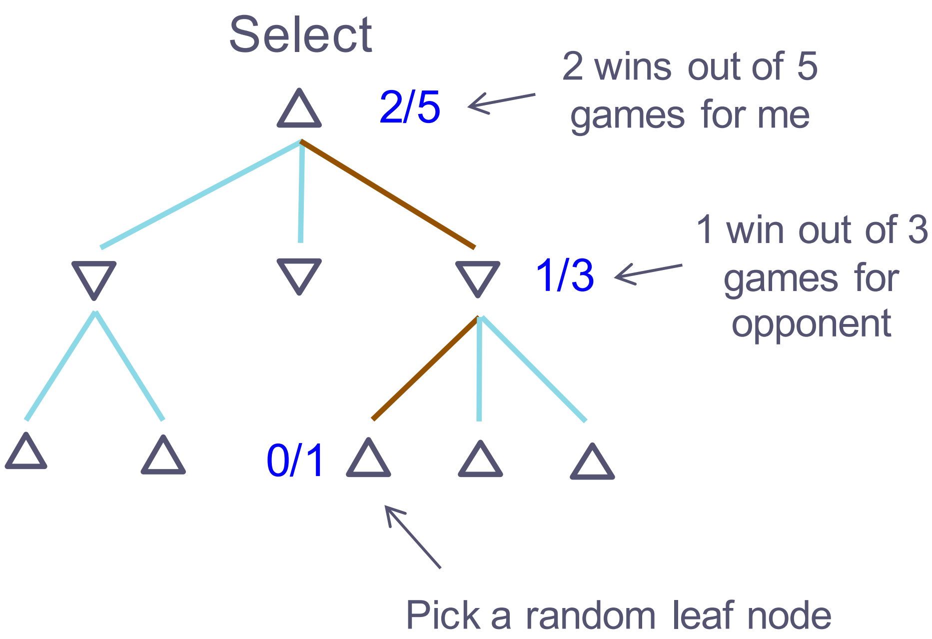
Expected Utility

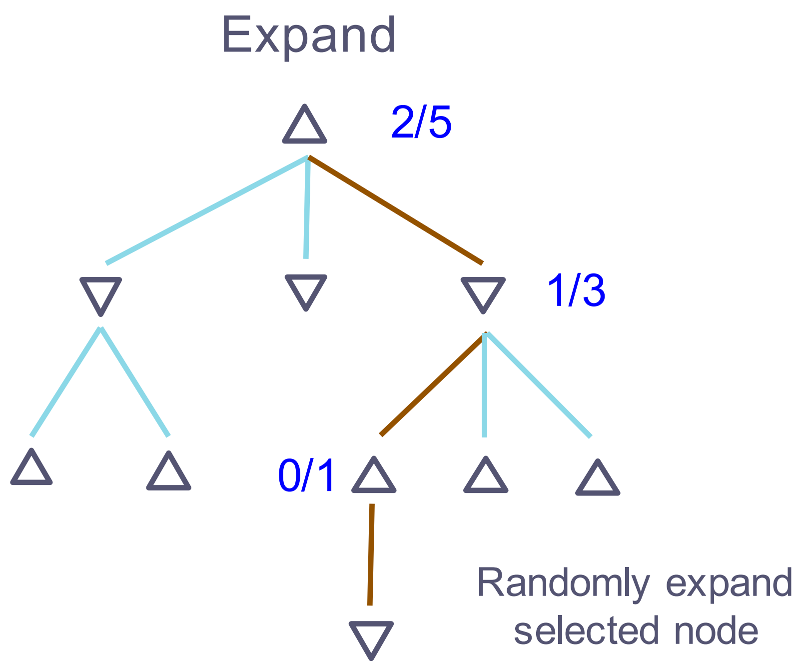
- If the opponent picks random moves, we should pick the move with maximum *expected* utility
- Here, mimimax would choose A_1, but the best move is A_3



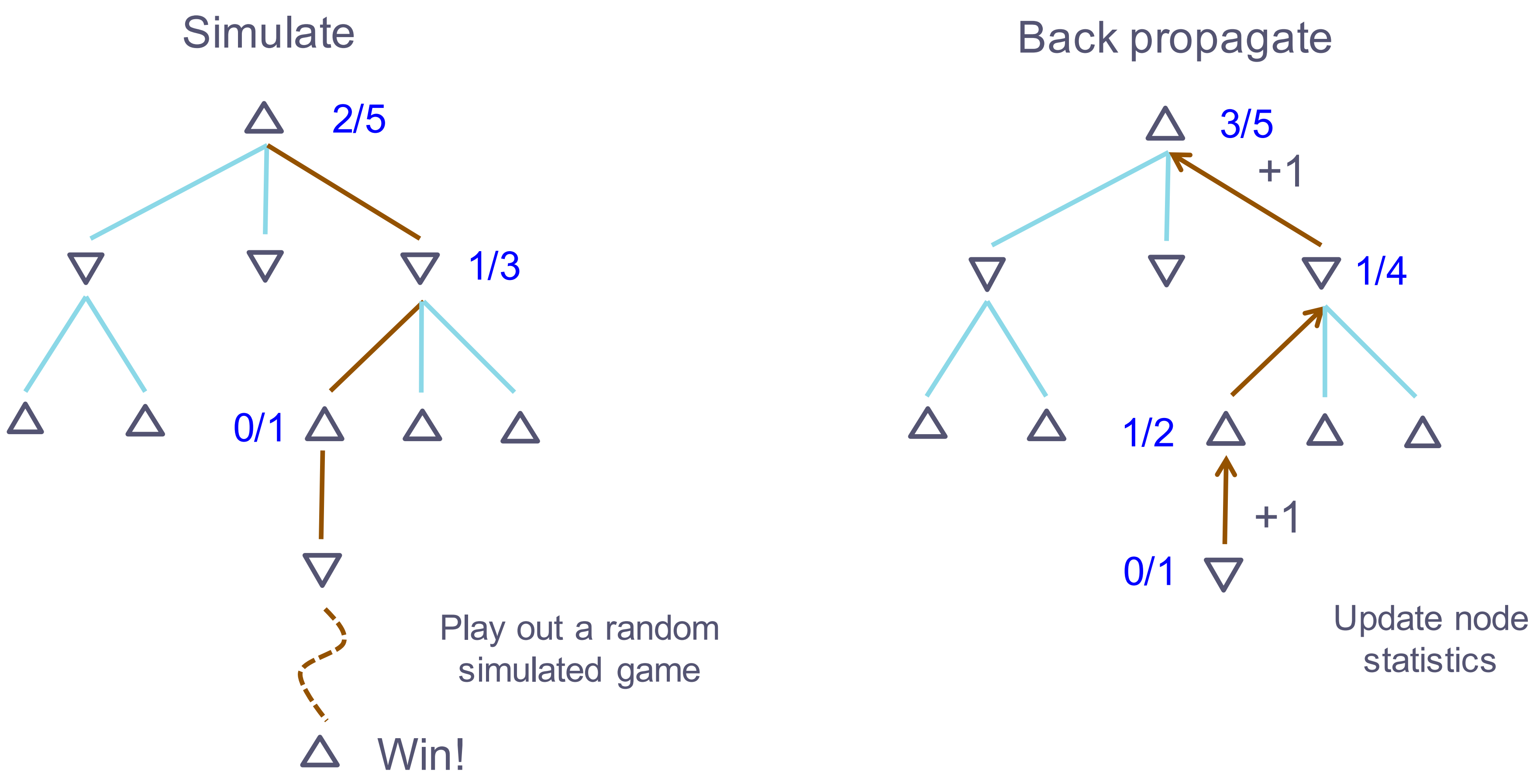
Monte Carlo Tree Search 1

- A statistical approach is taken in Monte Carlo tree search
- Random plays build up statistical picture of value of each node
- Four steps repeated: select, expand, simulate, back propagate





Monte Carlo Tree Search 2



Monte Carlo Tree Search 3

- Choice of leaf node to expand is crucial
- There is an exploitation / exploration trade-off. Do you pick most promising nodes or those that have been least explored?
- Can use upper confidence bound (UCB) — pick node i to maximise

$$\frac{w_i}{n_i} + C\sqrt{\frac{\ln(N)}{n_i}}$$

where w_i is number of wins, n_i is number of games with i, N is total number of games played, and C is a parameter

- Note that Monte Carlo tree search doesn't need an evaluation function
- Monte Carlo tree search is effective in games such as Go
- Useful resource here

http://mcts.ai/about/index.html