

# Summary of previous lectures

- computer architecture  
 = instruction set architecture  
 +  
 machine organisation
- $CPI = \sum (CPI_i \times \text{instr. count}_i) / (\sum \text{instr. count}_i)$
- minimise:  $\text{exe. time} = \text{instr. count} \times CPI \times \text{cycle time}$
- CISC:  $\downarrow \text{instr. count} \quad \downarrow \text{code size} \quad \uparrow CPI \quad \uparrow \text{cycle time}$
- RISC:  $\uparrow \text{instr. count} \quad \uparrow \text{code size} \quad \downarrow CPI \quad \downarrow \text{cycle time}$

# Computer arithmetic

(3rd Ed: p.160-175, Apx. B; 4th Ed: p.224-229, Apx. C.5; 5th Ed: p.178-182, Apx. B)

- two's complement: signed integer representation
- e.g.  $1011_{2C} = (1 \times -2^3) + (0 \times 2^2) + (1 \times 2^1) + (1 \times 2^0) = -5_{\text{ten}}$
- n-bit: range  $(-2^{n-1}) \dots (2^{n-1}-1)$
- sign extension:  $1011_{2C} = 1111011_{2C}$
- overflow:  $A, B > 0, \quad A+B \leq 0$   
 $A, B < 0, \quad A+B \geq 0$
- in MIPS: slt, slti work with two's complement  
sltu, sltiu work with unsigned representation  
(do not cause exception when overflow)

# Logical operations

- shift left logical

- **sll \$10, \$16, 8**    # reg10 = reg16 « 8 bits

- reg16 0 .. 0 0000 0000 1101

- reg10 0 .. 0 1101 0000 0000

20 bits
introduce zeros

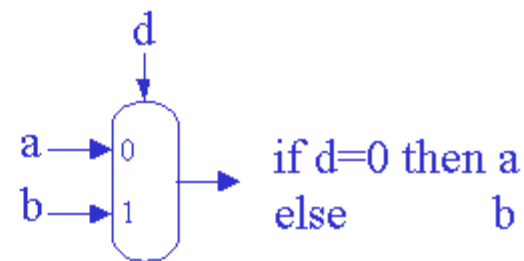
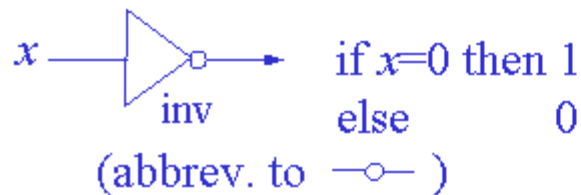
- format

0	0	16	10	8	0
R type	source1	source2	dest.	shamt	funct

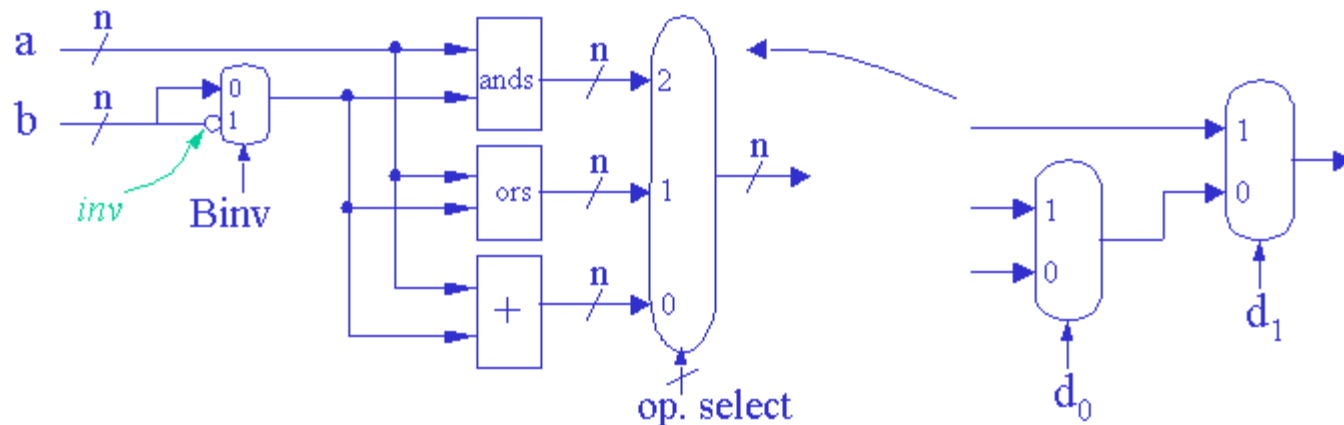
- shift left logical variable (sllv): shamt in register source1
- right shifts: srl, srlv, sra (sign-extend high order bits)
- bitwise: or, and, ori, andi

# ALU building blocks

- and, or, inv, mux



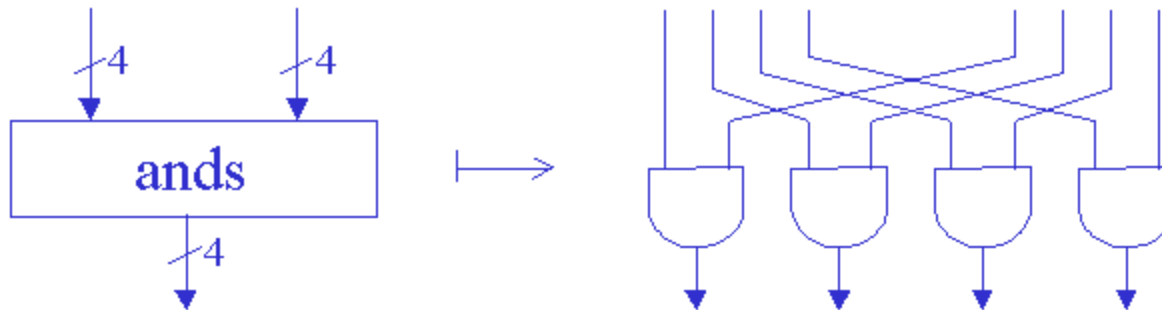
- ALU: Arithmetic Logic Unit  $n=32$  for MIPS



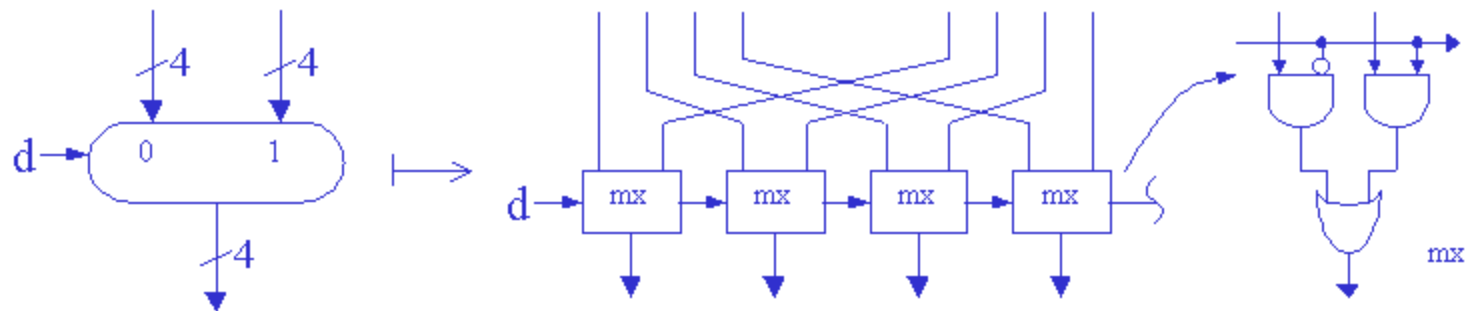
- bit-level realisation: hierarchical, regular structure

# Bit-wise logical / selection operations

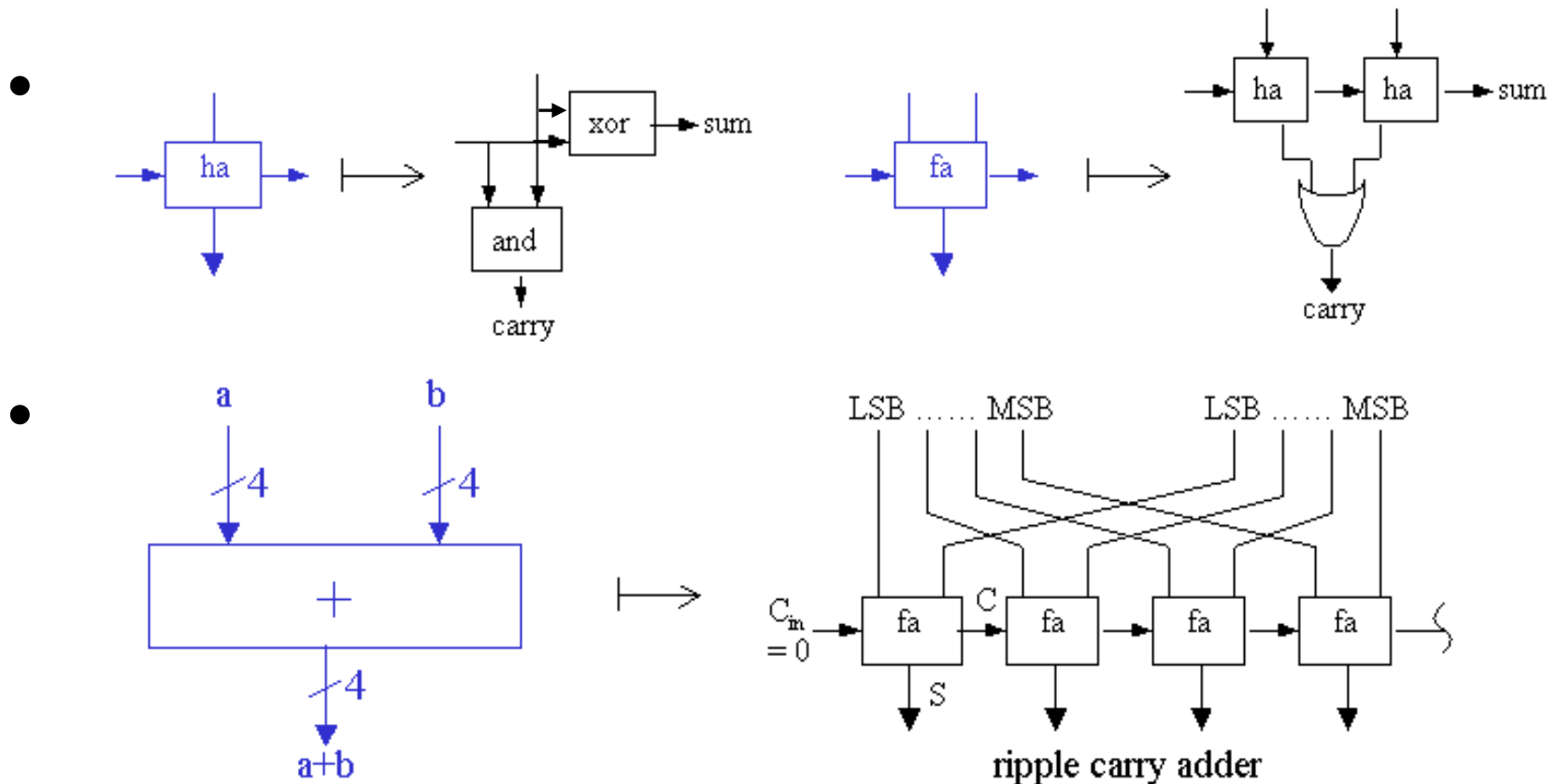
- and, or, ...



- selector / multiplexor



# Add / subtract



- subtractor:  $a - b$

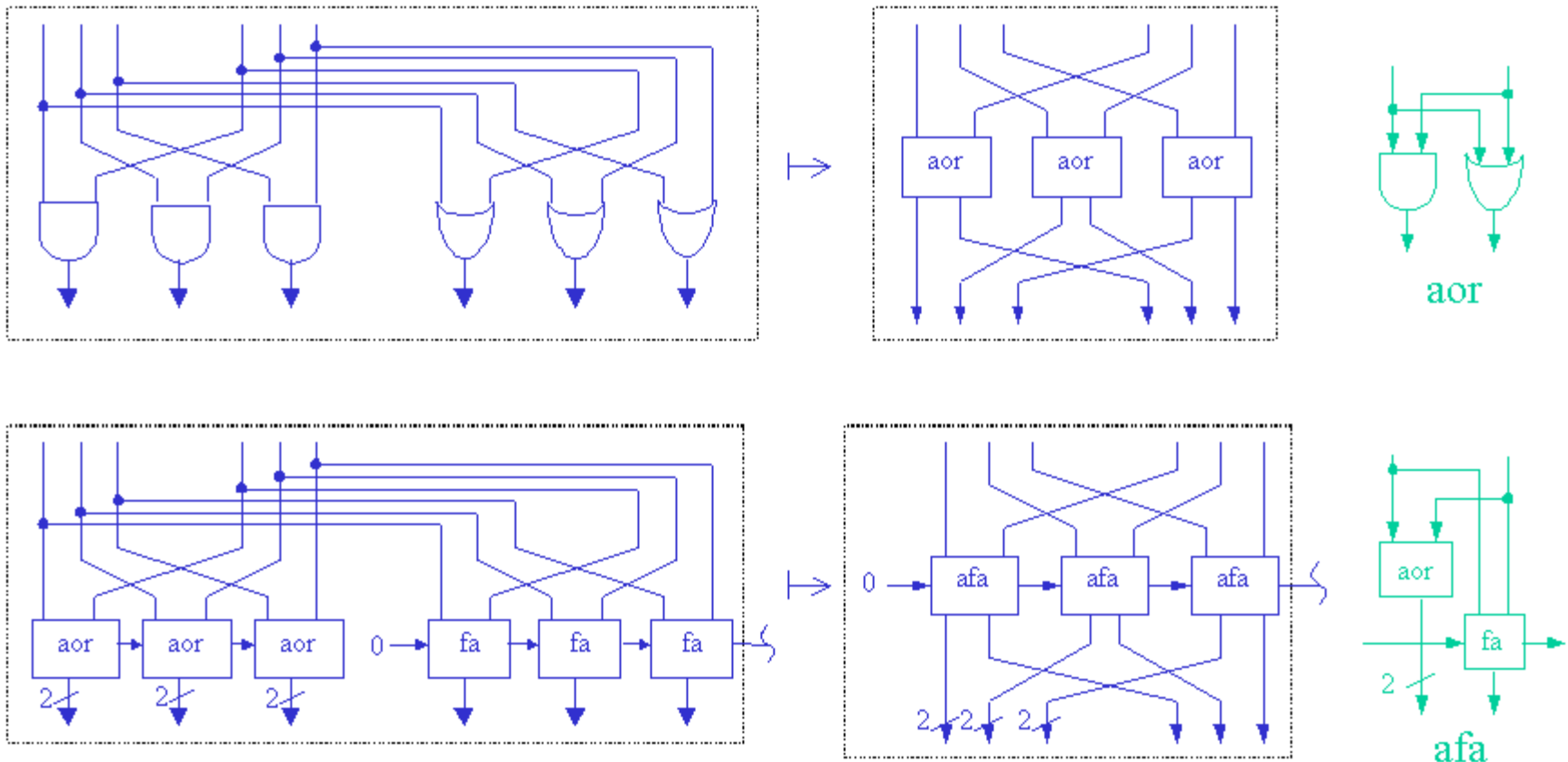
- $$- (\sum 2^i \times b_i) + (\sum 2^i \times \bar{b}_i) = -1 \quad \text{e.g. } 0101_2 \oplus 1010_2 = 1111_2 = -1_{10}$$

- $$- (\sum 2^i \times a_i) - (\sum 2^i \times b_i) = (\sum 2^i \times a_i) + (\sum 2^i \times \bar{b}_i) + 1$$

i.e. invert  $b$  bitwise and set  $C_{in} = 1$  for adder

# Deriving ALU cell by interleaving components

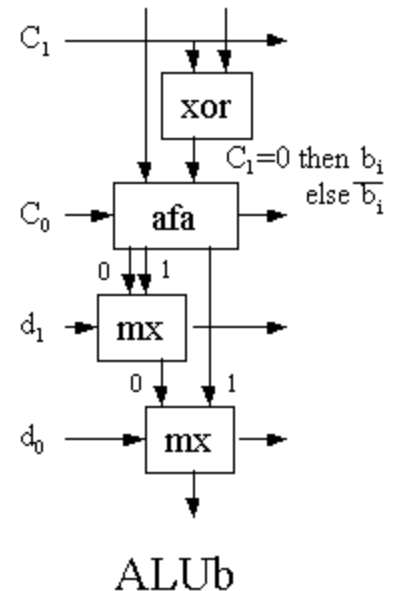
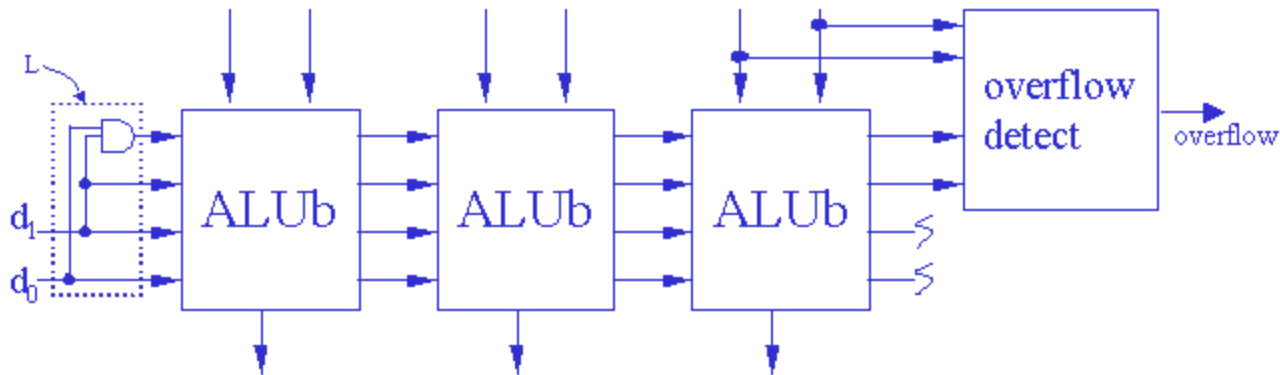
- group components together to form larger repeated unit



- the dotted boxes have the same function and interface

# Selecting ALU operation

- programmable inverter for  $b_i$  (using xor)
- connecting mux in series

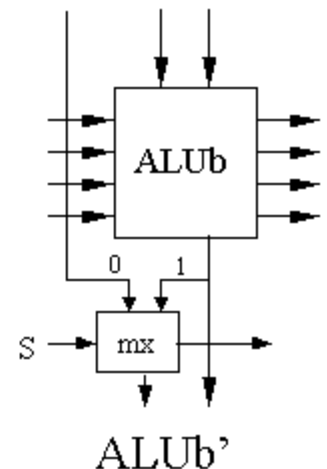
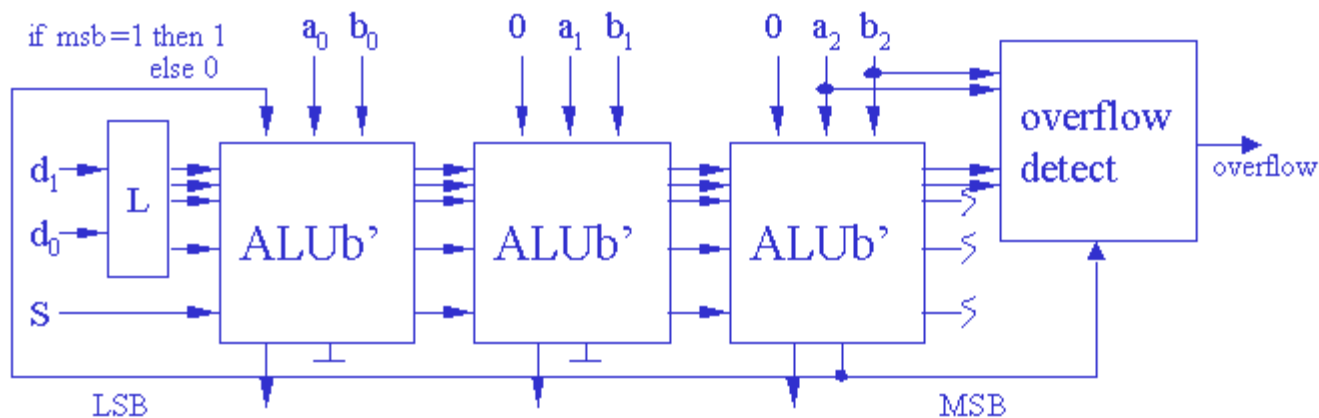


- $d_0d_1$ : 00 and, 01 or,  
10 add, 11 subtract
- detecting overflow: exercise



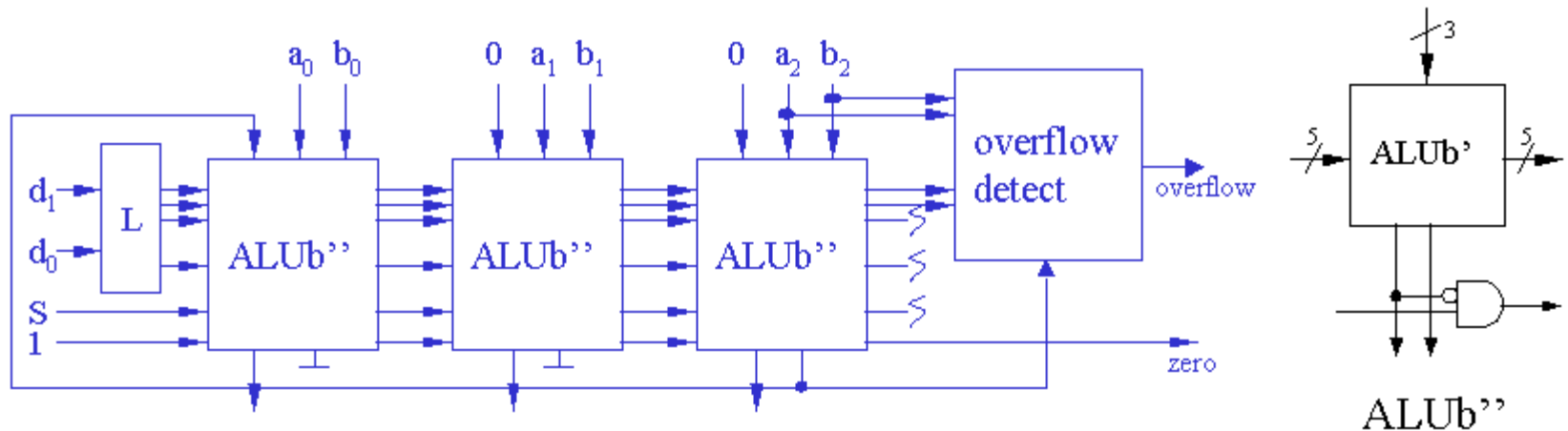
# Comparison operations

- slt: set on less than, if  $a < b$  then 1 else 0
- if  $a < b$ ,  $a - b < 0$ , so MSB of  $(a - b)$  is 1
- implementation
  - provide additional input to each cell
  - LSB input from MSB ALUb output, other inputs set to 0
  - include additional mux in cell for selection
  - to select slt,  $s=0$ ,  $d_0=1$ ,  $d_1=1$



# Zero detector

- beq, bne: test  $a=b$  or  $a-b=0$
- include another gate to test if output zero



- summary:
 

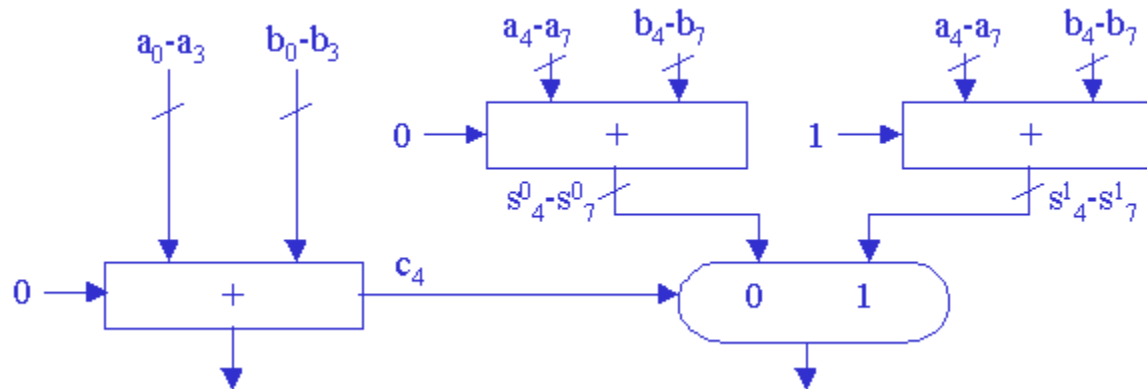
$s$	$d_0$	$d_1$	011	100	101	110	111
function:			set on	and	or	add	subtract
			less than				

# Performance estimation

- clocked circuit: no combinational loops
- speed limited by propagation delay through the slowest combinational path
- slowest path: usually carry path
- clock rate: approx.  $1/(\text{delay of slowest path})$   
assuming
  - edge-triggered design
  - flip-flop propagation delay, set-up time, clock skew etc. negligible (see 3<sup>rd</sup> Ed: B.7, B.11; 4<sup>th</sup> Ed: C.7, C.11)

# Fast addition

- carry select
  - compute both zero-carry-in and one-carry-in after  $n$  stages
  - e.g. 8 bits: use three 4-bit ripple carry adders



- other possibilities
  - carry-lookahead adder
  - conditional-sum adder