

## 2016 Reasoning

1)

$$a) \text{ let } P(is) \equiv \text{rev}(\text{flip } is) = \text{flip}(\text{rev } is)$$

inductive principle:

$$\begin{aligned} & [P([]) \wedge \forall x: \text{Int}, \forall xs: [\text{Int}] (P(xs) \rightarrow P(x:xs))] \\ & \rightarrow \forall xs: [\text{Int}]. P(xs) \end{aligned}$$

Base case:

to show  $P([])$  holds

$$\begin{aligned} P([]) & \equiv \text{rev}(\text{flip } []) = \text{flip}(\text{rev } []) && \text{by def.} \\ & \text{rev } [] = \text{flip } [] && \text{by def.} \\ & [] = [] && \\ & \text{True} \end{aligned}$$

shown.

Inductive case

Inductive hypothesis:  $P(xs)$  assume  
to show:  $P(x:xs)$

$$\begin{aligned} & \text{rev}(\text{flip } x:xs) \quad \text{by hyp.} \\ & = \text{rev}([x] ++ \text{flip } xs) = \text{rev}(\text{flip } xs) ++ \text{rev } [x] \\ & = \text{flip}(\text{rev } xs) ++ \text{rev } [x] \end{aligned}$$

$$\begin{aligned} & \text{rev}(\text{flip } x:xs) = \text{rev}(\text{flip } [x] ++ \text{flip } xs) \\ & = \text{rev}(\text{flip } xs) ++ \text{rev}(\text{flip } [x]) \\ & = \text{flip}(\text{rev } xs) ++ \text{flip}(\text{rev } [x]) && \text{by hyp.} \\ & = \text{flip}(\text{rev } xs ++ \text{rev } [x]) \\ & = \text{flip}(\text{rev } x:xs) \end{aligned}$$