IMPERIAL COLLEGE OF SCIENCE, TECHNOLOGY AND MEDICINE

EXAMINATIONS 2016

BEng Honours Degree in Computing Part I
MEng Honours Degrees in Computing Part I
for Internal Students of the Imperial College of Science, Technology and Medicine

This paper is also taken for the relevant examinations for the Associateship of the City and Guilds of London Institute

PAPER C145

MATHEMATICAL METHODS

Wednesday 11 May 2016, 10:00
Duration: 80 minutes

Answer ALL TWO questions

Paper contains 2 questions
Calculators required

Always provide justifications and show any intermediate work for your answers. A correct but unsupported answer may not receive any marks.

1 a Consider the sequence

$$a_n = \sqrt{n^2 + 2n} - n.$$

Determine whether the sequence converges, and if it does, to what value. Show your work.

Hint: Consider multiplying and dividing the expression by $\sqrt{n^2 + 2n} + n$.

b Use an appropriate test to determine whether the series

$$\sum_{n=1}^{\infty} \frac{(2n)!}{(n!)^2}.$$

converges. Explain which test you use and show your work.

c Compute the Maclaurin series for the following function and find its radius of convergence:

$$f(x) = \int_0^x e^{-t^2} dt.$$

Hint: Use the fact that power series may be integrated term by term.

d Calculate the general solution for the differential equation

$$\frac{d^4 y}{dx^4} - 16y = 0.$$

The four parts carry equal marks.

2a i) Name two algorithms in Computer Science where eigenvalues and eigenvectors play a central role.

ii) Which of the following matrices are diagonalizable? Justify your answer.

$$\text{A)} \quad A_1 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \qquad\qquad \text{B)} \quad A_2 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$$

iii) Consider an endomorphism $\Phi : \mathbb{R}^4 \to \mathbb{R}^4$ with transformation matrix

$$A = \begin{bmatrix} 0 & 10 & -25 & 0 \\ 0 & 5 & -14 & 0 \\ 0 & 0 & -2 & 0 \\ 8 & -17 & 11 & 4 \end{bmatrix}$$

and a vector $x = [1, 0, 0, 0]^\top$.

A) Determine all eigenvalues of A

B) Determine the diagonal form D of A, such that the diagonal elements are ordered: $d_{11} \geq d_{22} \geq d_{33} \geq d_{44}$.

C) Using the standard scalar product in $\mathbb{R}^4$, determine the distance of x from its orthogonal projection $\pi_U(x)$ onto the subspace U spanned by the eigenvector associated with the largest eigenvalue of A.

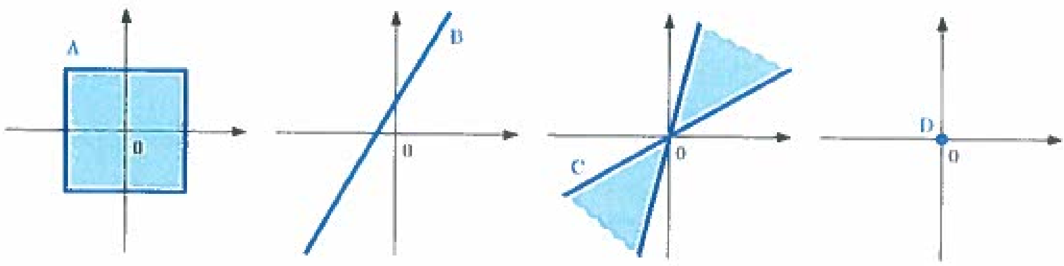

Fig. 1: Subsets in $\mathbb{R}^2$, see Question 2b(i).

b i) Are the blue sets in Figure 1 vector-subspaces of $\mathbb{R}^2$? Justify your answer.

ii) Find a basis of the intersection $L_1 \cap L_2$, where L_1 and L_2 are affine spaces defined as

$$L_1 := \underbrace{\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}}_{=:p_1} + [\underbrace{\begin{bmatrix} -3 \\ -2 \\ 1 \end{bmatrix}}_{=:U_1}], \qquad L_2 := \underbrace{\begin{bmatrix} 10 \\ 6 \\ -2 \end{bmatrix}}_{=:p_2} + [\underbrace{\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 5 \\ 4 \\ 1 \end{bmatrix}}_{=:U_2}].$$

The two parts carry equal marks.