Logic exercises 1: Propositional logic syntax and semantics

Please submit your solutions for questions marked (**PMT**) electronically in scientia by Monday 20 Nov. 2023, 7pm GMT.

Questions marked as (**PMT**) are to be discussed in the PMT session following the submission deadline. These questions are intended to be an opportunity to get feedback.

1. Which of the following are propositional formulas *strictly according to Definition 1.1 on slide 21?* For those that are not, say why not.

   (a) $(\leftrightarrow p \vee \neg q \; \neg r)$ (b) p' (c) $p \wedge q \wedge r \wedge \top$ (d) $((p_0 \to p_1) \vee ((\neg p_0) \to p_2))$
   (e) $4 \wedge 3 > 5$ (f) p implies q (g) $\neg r$
   (h) **PMT:** $(\neg r \to (p \to ((q \vee r) \to (p \wedge \neg s))))$
   (i) $(\neg(p))$ (j) **PMT:** $(\neg\neg\top)$ (k) $)$ (l) $(\neg(\neg(\neg(\neg(\neg p)))))$
   (m) $()$ (n) (ϕ) (o) (p) (p) $(\phi \to r)$

2. Apply the bracket-removing conventions in slides $24 - 25$ (dropping outer brackets, and the binding conventions) to remove all possible brackets from the following formulas without changing their reading.

   E.g., we can abbreviate $(p \to (q \leftrightarrow (\neg r)))$ to $p \to (q \leftrightarrow \neg r)$. But $p \to q \leftrightarrow \neg r$ is going too far: the binding conventions force it to be read as $(p \to q) \leftrightarrow \neg r$.

   (a) $(p \vee (q \to r))$ (b) $((p \vee q) \wedge (\neg r))$ (c) $(\neg(p \leftrightarrow q))$
   (d)**PMT** : $(p \to (\neg(q \to r)))$ (e)**PMT** : $(((\neg p) \wedge q) \to r)$ (f) $((\neg(p \wedge q)) \to r)$
   (g) $(\neg(\neg p'))$
   (h) $((\neg p) \wedge (\neg(q \to (s \leftrightarrow (\top \vee r)))))$

3. Consider an alternative binding convention with the following rules:

   - $\neg$ is stronger than both $\wedge$ and $\vee$ (no order between $\wedge$ and $\vee$) which in turn are stronger than both $\to$ and $\leftrightarrow$ (no order between $\to$ and $\leftrightarrow$);
   - all binary connectives are *right-associative*, i.e., $p \wedge q \wedge r$ abbreviates $(p \wedge (q \wedge r))$.

   Apply these conventions to eliminate all possible brackets from the formulas in Q.2 without changing their reading.

   PMT: (a), (b), (f).

4. By adding brackets, indicate all possible ways that the following could be read if we had no binding conventions at all. E.g., $\neg\phi \to \psi$ could be read as $\neg(\phi \to \psi)$ or $(\neg\phi) \to \psi$.

   (a) $p' \leftrightarrow p \wedge p''$

(b) $p_1 \to \neg p_2 \wedge p_3 \leftrightarrow \neg p$

(c) $s \to \neg\neg s' \vee r$

5. By adding brackets, indicate all the possible ways that the items in Question 4 can be read, *subject to the binding conventions given in lecture slides.* E.g., $\neg\phi \to \psi$ can only be read as $((\neg\phi) \to \psi)$.

6. Draw the formation trees *and* list *all* subformulas of the following formulas. (Use the binding conventions in the slides to disambiguate them. Normally I'd put more brackets in (e).)

   $(a)\ p \to q \wedge r$ $\quad (b)\ \neg p \wedge q \leftrightarrow r \vee s$ $\quad (c)\ p \wedge q \vee r \to \neg(p \to r)$

   $(d)\mathbf{PMT}: \neg p \to (p \to r \vee s)$ $\quad (e)\ \neg\neg\top \leftrightarrow \neg\neg\bot \wedge \top \to \neg p$ $\quad (f)\mathbf{PMT}: \neg\neg\neg\neg\bot$

7. Using the symbols ϕ, ψ, ..., give the overall logical forms of the formulas in Q.6

   PMT: (a), (b), (e).

8. Which of the subformulas of the formulas in Q.6 are literals, and which are clauses?

   PMT: (d), (e).

9. Suppose $v(p) = \mathsf{tt}$, $v(q) = \mathsf{tt}$ and $v(s) = \mathsf{ff}$ in a certain situation. Which of the following evaluate to true and which to false in this situation? Show your working using Definition 1.5 (in the style shown in slide 39 of the lecture notes).

   (a) $\neg p \vee \neg s$

   (b) **PMT:** $\neg\neg(p \to q \leftrightarrow p)$

   (c) $\neg p \wedge (p \to (q \to p))$

10. Construct a truth table for each of the following formulas.

   (a) $p \to p \leftrightarrow q$

   (b) **PMT:** $\neg(p \vee \neg q)$

   (c) $p' \to ((p'' \vee (\neg p)) \to p')$

   (d) $(((p \vee \neg r) \wedge \neg q) \to p)$