Logic exercises 2: Propositional logic semantics, translation and validity

This sheet includes questions on propositional logic semantics, translation into and from propositional logic and validity in propositional logic.

Questions marked (**TUT**) will be covered during the tutorial session (Thursday 23/11) and not during PMT sessions.

Please submit your solutions for questions marked (**PMT**) **only** electronically in scientia by Monday 27th Nov. 2023, 7pm GMT.

1. Give a truth-table definition of the ternary Boolean connective $\mathsf{if}\ \phi\ \mathsf{then}\ \psi\ \mathsf{else}\ \rho$ in the following two cases.

   (a) inclusive: $\mathsf{if}\ \phi\ \mathsf{then}\ \psi$ *(and don't care about* ρ*)* $\mathsf{else}\ \rho$ *(and don't care about* ψ*)*

   (b) exclusive: if ϕ then ψ *(but not* ρ*)* else ρ *(but not* ψ *)*

ϕ	ψ	ρ	$\mathsf{if}\ \phi\ \mathsf{then}\ \psi\ \mathsf{else}\ \rho$
.	.	.	.

2. Similar to the Sheffer stroke connective $\uparrow$, which represents"not both" (nand), we can define the connective $\downarrow$ representing 'neither ... nor ...' (nor).

   (a) Write down the truth table for $\downarrow$.

   (b) Find a formula in propositional logic that defines $\downarrow$ using only the connectives $\{\neg, \vee\}$.

   (c) Give two distinct examples of truth-functional, binary connectives, other than $\wedge$, $\vee$, $\rightarrow$, $\leftrightarrow$, $\uparrow$ and $\downarrow$. Denote these as $\bowtie$ and $\smile$.

   (d) Let $\bowtie$ and $\smile$ denote the two connectives defined in 2c. Give the truth value of the followings formulas for $v(p) = \mathsf{ff}$, $v(s) = \mathsf{ff}$ and $v(r) = \mathsf{tt}$.

      i. $((s \rightarrow p) \bowtie (s \rightarrow r))$

      ii. $((s \rightarrow p) \smile (s \rightarrow r))$

3. The set $\{\wedge, \vee, \neg\}$ is functionally complete. Using truth tables, show that each of the following sets are also functionally complete.

   (a) **PMT:** $\{\vee, \neg\}$

(b) **TUT:** $\{\rightarrow, \bot\}$

4. Consider the following English sentence.

   'Sarah believes that the earth is flat'

   Is the connective 'Sarah believes that' truth functional? Justify your answer.

5. Consider the objects, elenor, cleopatra, mary and theodora, placed on a 4×4 grid, and the following atomic formulas about these objects where x, y can be replaced with elenor, cleopatra, mary or theodora.

   - 'x attacks y' means x and y share the same row or the same column, or diagonal.
   - 'x next-to y' means x and y are in adjacent cells (not diagonally).
   - 'x right-of y' means x is in a column to the right of the column of y'
   - 'x below y' means x is in a row below the row of y.

   (a) For the queens placed in the grid below, which of the following expressions evaluates to true and why?

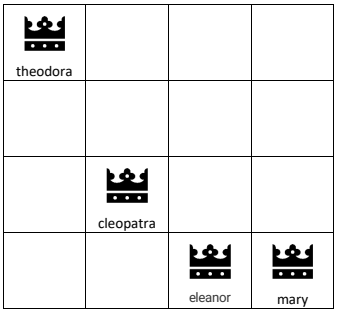
   (i) 'theodora attacks cleopatra' $\wedge$ ('mary attacks eleanor' $\rightarrow$ 'mary attacks theodora')

   (ii) 'cleopatra attacks cleopatra' $\leftrightarrow$ 'cleopatra attacks eleanor'

   (iii) $\neg$'eleanor next-to cleopatra' $\rightarrow$ 'eleanor attacks theodora'

   (iv) ($\neg$'eleanor below mary' $\vee$ $\neg$'theodora next-to mary') $\rightarrow$ 'cleopatra attacks theodora'

   (v) ($\neg\neg$'cleopatra right-of cleopatra' $\rightarrow$ 'eleanor attacks theodora') $\leftrightarrow$ ('mary below theodora' $\wedge$ 'mary attacks theodora')



   (b) Place the 4 queens in the grid so that all formulas in 5a are true. Justify your answers.

   (c) Place the 4 queens in the grid so that all formulas in 5a are false. Justify your answer by explaining why each formula is false.

6. Translate the following sentences into logic. First decide on the atoms to use (e.g., p to represent 'Jess bought ice-cream'), and then get the sentence structure correct. Provide your interpretation where alternative translations arise.

   (a) Ahmed and Nik are both artists.

(b) Beckham is a footballer who is English.

(c) Alice is a serious contender.

(d) If neither Sangita nor Rowan are available, then I'm not going.

(e) Adam sells stamps and either antiques or lamps.

(f) There's no annual fee provided that you pay with a credit card.

(g) In the case where I make it back by 2 p.m., I will bring the shopping only if it does not rain.

(h) I'm going to make him an offer he can't refuse.

(i) **PMT:**Who let the dogs out?

(j) **PMT:**A late application is accepted only if the class quota increases.

(k) **PMT:**Unless you pay now, I can't guarantee a ticket

7. Show that the following arguments are valid using truth tables:

(a) $p \to \neg q, q \models \neg p$

(b) **PMT:** $\models p \wedge q \leftrightarrow \neg(\neg p \vee \neg q)$

(c) $p \leftrightarrow \neg q \models \neg(p \leftrightarrow q)$