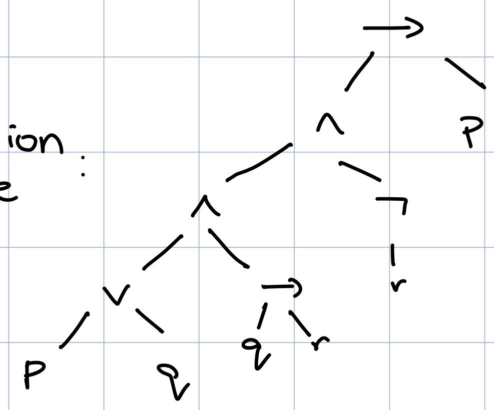
1a. $((p \vee q) \wedge (q \to r)) \wedge \neg r \to p$

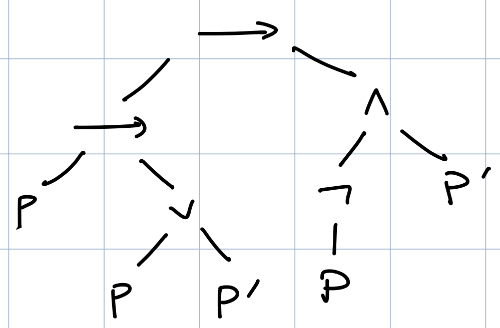
Formation tree:



Literals: $p, q, q, r, r, p, \neg r$

Clauses: $p, q, q, r, r, p, \neg r, p \vee q$

1b. $((p \to (p \vee p')) \to (\neg p \wedge p'))$



Literals: $p, p, p', p, p', \neg p$

Clauses: $p, p, p', p, p', \neg p, p \vee p'$

2a. Let ϕ be p_0 and ψ be $p_2 \vee \neg p_0 \vee p_1$

Overall form: $\phi \to \psi$

It is neither a conjunction or disjunction.

2b. Let Φ be $r \wedge r'$ and ψ be r''

Overall form: $\Phi \vee \psi$

It is a disjunction. $r \wedge r'$, r'' are the disjuncts.

2c Let ϕ be $\neg\neg p$ and ψ be p'

$\Phi \wedge \psi$

It is a conjunction. $\neg\neg p$, p' are the conjuncts.

3a. $((\neg(p \wedge \neg q)) \to (p \to q))$

3b. $(((p \vee q) \wedge (p \to r)) \to (r \vee q))$

4a.

p	q	$(p \to q)$	$((p \to q) \to p)$	$(((p \to q) \to p) \to p)$
tt	tt	tt	tt	tt
tt	ff	ff	tt	tt
ff	tt	tt	ff	tt
ff	ff	tt	ff	tt

The formula $(((p \to q) \to p) \to p)$ is true in every situation and hence valid.

4b.

p	$p \to p$	$\neg(p \to p)$
tt	tt	ff
ff	tt	ff

The formula $\neg(p \to p)$ is a contradiction, as it is false in every situation

4c.

p	$\neg p$	$\neg p \to p$
tt	ff	tt
ff	tt	ff

The formula $\neg p \to p$ is true in at least one situation, hence it is satisfiable

5. Take any situation. For $\neg\neg\phi \models \phi$ to be valid, $\neg\neg\phi \models \phi$ must be true in this situation.

For $\neg\neg\phi \models \phi$ to be true, $\neg\neg\phi \to \phi$ must be true, (by semantics of $\models$)

For the formula $\neg\neg\phi \to \phi$ to be true, either $\neg\neg\phi$ is false or ϕ is true. (by semantics of $\to$).

For $\neg\neg\phi$ to be false, $\neg\phi$ must be true, (by semantics of $\neg$)

For $\neg\phi$ to be true, ϕ must be false, (by semantics of $\neg$).

Hence, either ϕ is true for $\neg\neg\phi \models \phi$ to be true or ϕ is false for $\neg\neg\phi$ to be false, making $\neg\neg\phi \models \phi$ true

Hence, in any situation where ϕ is true or false, $\neg\neg\phi \models \phi$ is true

Hence, by the definition of propositional validity, $\neg\neg\phi \models \phi$ is propositionally valid.

6. $((\neg p \wedge q) \vee r) \rightarrow (\neg p \vee r)$

$\equiv \neg((\neg p \wedge q) \vee r) \vee (\forall p \vee r)$ $[\phi \rightarrow \psi \equiv \neg\phi \vee \psi]$

$\equiv \neg((\neg p \vee r) \wedge (q \vee r)) \vee (\neg p \vee r)$ [By distributivity of $\vee$]

$\equiv \neg(\neg p \vee r) \vee \neg(q \vee r) \vee (\neg p \vee r)$ [By DeMorgan laws]

$\equiv (\neg p \vee r) \vee \neg(\neg p \vee r) \vee \neg(q \vee r)$ [By commutativity of $\vee$]

$\equiv \top \vee \neg(q \vee r)$ $[\phi \vee \neg\phi \equiv \top]$

$\equiv \top$ $[\top \vee \neg\phi \equiv \top]$