Coursework: Propositional logic

Please submit your solutions electronically in Scientia by Thursday Nov. 30, 2023, 7pm GMT.

1. Draw the formation trees and list all literals and clauses for each formula below.

   (a) $((p \vee q) \wedge (q \to r)) \wedge \neg r \to p$

   (b) $((p \to (p \vee p')) \to (\neg p \wedge p'))$

   [Total marks (6)]

2. For each formula below, give its *overall logic form* and state whether it is a conjunction, a disjunction or neither. In case of a conjunction (respectively disjunction), give the conjuncts (respectively disjuncts).

   (a) $p_0 \to p_2 \vee \neg p_0 \vee p_1$

   (b) $r \wedge r' \vee r''$

   (c) $\neg\neg p \wedge p'$

   [Total marks (8)]

3. Give the corresponding implication formula for each of the arguments below.

   (a) $\neg(p \wedge \neg q) \models p \to q$

   (b) $p \vee q,\ p \to r \models r \vee q$

   [Total marks (2)]

4. Show which of the formulas below are valid, satisfiable or a contradiction using truth tables. **Justify** your answer.

   (a) $(((p \to q) \to p) \to p)$

   (b) $\neg(p \to p)$

   (c) $\neg p \to p$

   [Total marks (7)]

5. Let ϕ be an arbitrary propositional formula. Show, using **direct argument**, that the argument $\neg\neg\phi \models \phi$ is propositionally valid.

   [Total marks (3)]

6. Rewrite the formula below into DNF using equivalences.

$$((\neg p \wedge q) \vee r) \rightarrow (\neg p \vee r)$$

[Total marks (4)]