

4a. $((P \rightarrow Q) \rightarrow P) \rightarrow P$ is valid

$$\begin{aligned} &\equiv \neg((P \rightarrow Q) \rightarrow P) \vee P && (\neg x \vee y \Leftrightarrow x \rightarrow y) \\ &\equiv \neg(\neg(P \rightarrow Q) \vee P) \vee P && (\neg x \vee y \Leftrightarrow x \rightarrow y) \\ &\equiv \neg(\neg(\neg P \vee Q) \vee P) \vee P && (\neg x \vee y \Leftrightarrow x \rightarrow y) \\ &\equiv \neg((\neg \neg P \wedge \neg Q) \vee P) \vee P && (\text{De Morgan law}) \\ &\equiv \neg((P \wedge \neg Q) \vee P) \vee P && (\neg \neg x \Leftrightarrow x) \\ &\equiv (\neg(P \wedge \neg Q) \wedge \neg P) \vee P && (\text{De Morgan Laws}) \\ &\equiv ((\neg P \vee \neg \neg Q) \wedge \neg P) \vee P && (\text{De Morgan laws}) \\ &\equiv ((\neg P \vee Q) \wedge \neg P) \vee P && (\neg \neg x \Leftrightarrow x) \\ &\equiv (\neg P \vee Q \vee P) \wedge (\neg P \vee P) && (\text{Dist. of } \vee \text{ over } \wedge) \\ &\equiv (\neg P \vee Q \vee P) \wedge T && (\neg P \vee P \Leftrightarrow T) \\ &\equiv (T \vee Q) \wedge T && (\neg P \vee P \Leftrightarrow T) \\ &\equiv T \wedge T && (T \vee x \Leftrightarrow T) \\ &\equiv T && (T \wedge T \Leftrightarrow T) \end{aligned}$$

5d. $P \wedge \neg(P \vee r)$

No. ^{Not in CNF} \neg can be simplified to $P \wedge \neg P \wedge \neg r$, which is $\perp \wedge \neg r$, which is \perp

5j. $(P \vee q) \wedge s.$

Yes, it is in CNF form

8.

P	q	r	$\neg q$	$\neg r$	$P \rightarrow \neg q$	$(P \rightarrow \neg q) \wedge \neg r$	$((P \rightarrow \neg q) \wedge \neg r) \rightarrow q$	$((P \rightarrow \neg q) \wedge \neg r) \rightarrow q \wedge r$
1	1	1	0	0	0	0	1	1
1	1	0	0	1	0	0	1	0
1	0	1	1	0	1	0	1	1
1	0	0	1	1	1	1	0	0
0	1	1	0	0	1	0	1	1
0	1	0	0	1	1	1	1	0
0	0	1	1	0	1	0	1	1
0	0	0	1	1	1	1	0	0

$$(\neg p \vee \neg q \vee r) \wedge (\neg p \vee q \vee r) \wedge (p \vee \neg q \vee r) \wedge (p \vee q \vee r)$$

9f. $\vdash P \wedge q \rightarrow P$

1. $P \wedge q$	asm
2. P	$\wedge E(1)$

$$3. P \wedge q \rightarrow P \rightarrow I(1,2)$$

$$9: P \rightarrow (q \rightarrow r) \vdash (P \rightarrow q) \rightarrow (P \rightarrow r)$$

$$1. P \rightarrow (q \rightarrow r) \quad \text{Prem.}$$

2. $P \rightarrow q$	ass								
<table border="1" style="border-collapse: collapse; width: 100%;"> <tr> <td style="padding: 5px;">3. P</td> <td style="padding: 5px; text-align: right;">ass</td> </tr> <tr> <td style="padding: 5px;">4. $q \rightarrow r$</td> <td style="padding: 5px; text-align: right;">$\rightarrow E(2,1)$</td> </tr> <tr> <td style="padding: 5px;">5. q</td> <td style="padding: 5px; text-align: right;">$\rightarrow E(3,2)$</td> </tr> <tr> <td style="padding: 5px;">6. r</td> <td style="padding: 5px; text-align: right;">$\rightarrow E(5,4)$</td> </tr> </table>		3. P	ass	4. $q \rightarrow r$	$\rightarrow E(2,1)$	5. q	$\rightarrow E(3,2)$	6. r	$\rightarrow E(5,4)$
3. P	ass								
4. $q \rightarrow r$	$\rightarrow E(2,1)$								
5. q	$\rightarrow E(3,2)$								
6. r	$\rightarrow E(5,4)$								
7. $P \rightarrow r$	$\rightarrow I(3,6)$								

$$8. (P \rightarrow q) \rightarrow (P \rightarrow r) \rightarrow I(2,7)$$