40018 Natural deduction for Propositional Logic using Pandora

Have a go *with Pandora* at some of the natural deductions below. For instructions on downloading Pandora visit: `https://www.doc.ic.ac.uk/~da04/teaching/logic40018/pandora.html`

1. See if you can do the following basic proofs, using natural deduction.

   (a) $p \wedge q \vdash p$

   (b) $\vdash p \wedge q \rightarrow p$

   (c) $p \vdash q \rightarrow p \wedge q$

   (d) $p \rightarrow (q \rightarrow r) \vdash (p \rightarrow q) \rightarrow (p \rightarrow r)$

   (e) $p \vdash (p \wedge q) \vee (p \wedge \neg q)$

   (f) $\vdash p \rightarrow (q \rightarrow p)$

   (g) $(p \wedge q) \rightarrow r \vdash p \rightarrow (q \rightarrow r)$

2. Some slightly harder ones, often based on the equivalences in the notes:

   (a) $p,\ \ q \vee r \ \ \vdash \ \ (p \wedge q) \vee (p \wedge r)$

   (b) $\neg p,\ p \vee q \vdash q$ Hint: Use the rule $\perp E$.

   (c) $\neg(p \vee q) \vdash \neg p \wedge \neg q$

   (d) $\neg p \rightarrow \neg q \vdash q \rightarrow p$

   (e) $\neg(\neg p \vee q) \vdash p \wedge \neg q$

   (f) $\neg(p \rightarrow q) \vdash p \wedge \neg q$

   (g) $\neg p \wedge \neg q \vdash \neg(p \vee q)$

   (h) $\vdash (p \rightarrow q) \vee (q \rightarrow p)$.

   (i) $p \vee q \vdash \neg(\neg p \wedge \neg q)$

   (j) $p \rightarrow q \vdash \neg p \vee q$

   (k) $p \wedge \neg q \vdash \neg(p \rightarrow q)$

3. And some more challenging ones... Note that Pandora uses capital letters to represent arbitrary formulas (instead of greek letters as we did in the lecture notes).

   (a) $P \rightarrow Q,\ \neg P \rightarrow R,\ Q \rightarrow S,\ R \rightarrow S \vdash S$

   (b) $R \rightarrow \neg I,\ \ I \vee F,\ \ \neg F \vdash \neg R$

   (c) $D \vee B,\ \ \neg(D \wedge \neg C),\ \ B \rightarrow C \vdash C$

   (d) $F \rightarrow (B \vee W),\ \neg(B \vee P),\ W \rightarrow P \vdash \neg F$

   (e) $G \wedge B \rightarrow C,\ \neg D \rightarrow \neg(L \rightarrow F),\ C \rightarrow (L \rightarrow F) \vdash G \rightarrow (B \rightarrow D)$

   (f) $\neg P,\ \ (B \wedge W) \rightarrow P,\ \ \neg I \rightarrow B,\ \ \neg W \rightarrow M,\ \ L \rightarrow (\neg I \wedge \neg M) \vdash \neg L$

   (g) $R \rightarrow (B \rightarrow (D \vee L)),\ \ \neg(D \vee G),\ \ (L \wedge B) \rightarrow G \vdash B \rightarrow \neg R$

   (h) $\neg T,\ \ P \rightarrow \neg(R \wedge Q),\ \ P \rightarrow (R \vee T) \vdash P \rightarrow \neg Q$

   (i) $(C \wedge N) \rightarrow T,\ H \wedge \neg S,\ (H \wedge \neg(S \vee C)) \rightarrow P \vdash (N \wedge \neg T) \rightarrow P$

   (j) $K \leftrightarrow \neg B \vdash \neg(K \leftrightarrow B)$