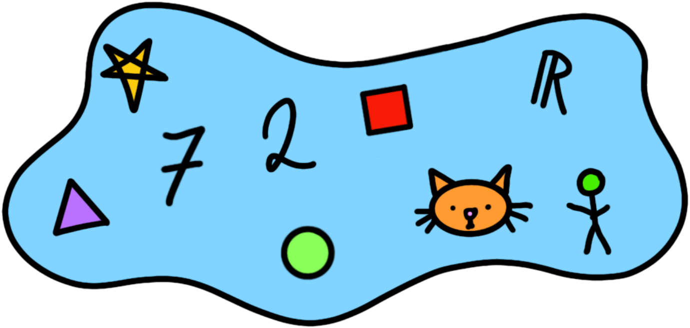
Let's talk about sets.

A set is a *collection of stuff*.



This is not on the exam

Of course that is not rigorous!

Here is a *nonexaminable* slide about the way it all works.

The *actual* definition of a set depends on what foundational system you use to do mathematics.

Example (ZFC set theory): A set has no definition in ZFC set theory.

Example (NBG set theory): a set is an element of a class. But a class has no definition.

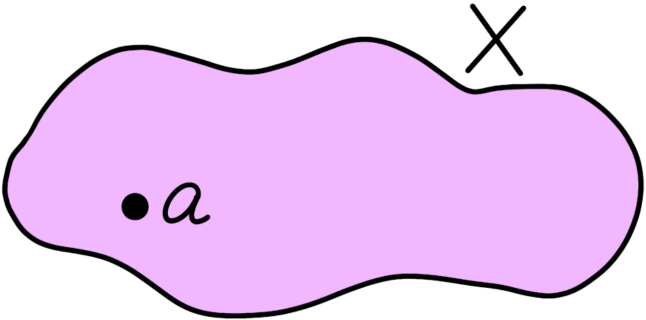
Example (Dependent Type Theory): A set is a subtype of a type. But a type has no definition.

Any way you look at it – we are *starting at the bottom*.

A set is a *collection of stuff*.

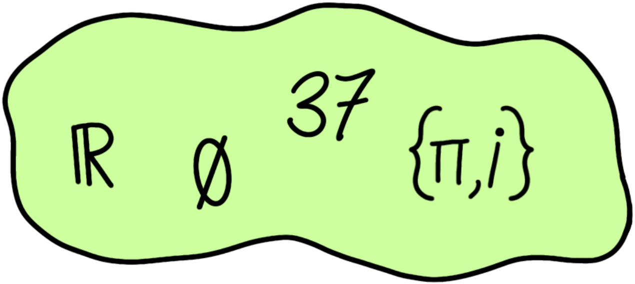
An *element* of a set is one of the things in the collection.

Set notation: Let X be a set. Let a be a thing. We write $a \in X$ to mean a is an element of X.



To better understand questions about sets, it is important that you *understand the picture* and *understand the notation*.

A set.



Set notation: This set is $\{\mathbb{R}, \emptyset, 37, \{\pi, i\}\}$.

Extensionality

To better understand questions about sets, it is important that you *understand when two sets are equal*.

Sets are extensional objects (Wikipedia).

In other words, sets are determined by their elements.

In other words, two sets with the same elements are *equal*.

For example, $\{\mathbb{R}, \emptyset, 37, \{\pi, i\}\} = \{37, \mathbb{R}, \emptyset, \{i, \pi\}\}$.

In the next video, we will learn about the $\forall$ notation, and will be able to write this fact as follows:

$$X = Y \iff (\forall a, a \in X \iff a \in Y).$$