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The equivalence class of a is the things related to a.

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So it's the set of all the green shapes.

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So $cl(37)$ is all the odd natural numbers.

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So $cl(37)$ is all the odd natural numbers. As is $cl(43)$.

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I claim that the equivalence classes form a partition of X.

I prove this claim carefully in the notes. Let me step through the ideas here.

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To prove that distinct equivalence classes are disjoint, we show that if $cl(x)$ and $cl(y)$ have an element in common, then $x \sim y$ (this uses symmetry and transitivity), and then that $cl(x) = cl(y)$.

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If you're interested in Lean you could now watch the final Lean video, where I'll prove some things are transitive and then talk more about this challenge.