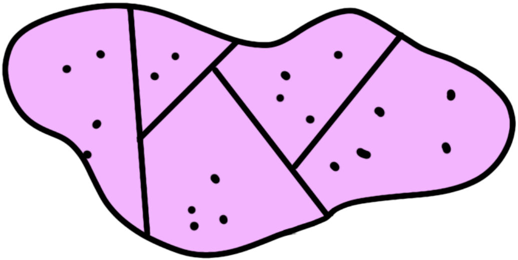
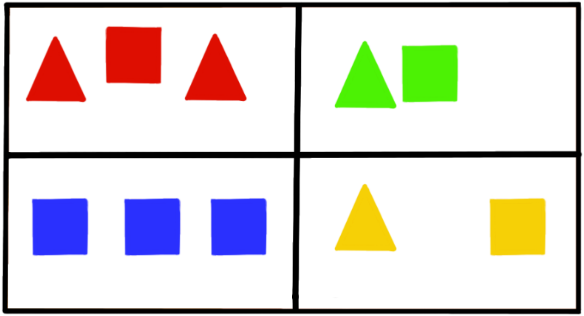
Partitions

To *partition* a set is to break it up into disjoint nonempty subsets.



Let me show you an important example of a partition on my set of shapes, which will show up later.



Here's the formal definition.

Definition. Let α be a set. A *partition* of α is a set $\mathcal{C} = \{X_1, X_2, X_3, \ldots\}$ of subsets of α called *blocks*, satisfying the following axioms:

1. Every $X_i \in \mathcal{C}$ is non-empty;
2. The union of all the X_i's is α;
3. Distinct X_i are disjoint. That is if $X_i \cap X_j \neq \emptyset$ then $X_i = X_j$.

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Now let me explain why I have introduced this notion.

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It turns out that equivalence relations and partitions are two ways of thinking about the same idea.

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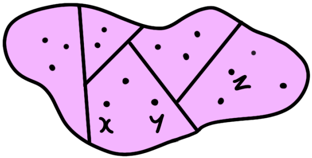

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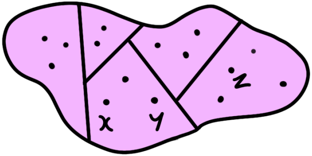

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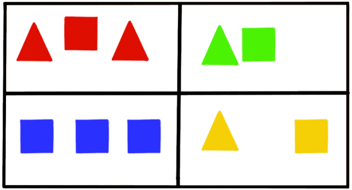
What are the three things to check to ensure this is an equivalence relation? Can you check them? Pause the video and have a go.

Example: if we consider the partition of my plastic shapes, we see that the corresponding equivalence relation is my favourite equivalence relation! Two shapes are related iff they're the same colour.



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In other words, if they have the same remainder when you divide them by 2.

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These constructions are the way we pass between our two different ways of understanding the idea of an equivalence relation.