Imperial College London

DEPARTMENT OF MATHEMATICS
IMPERIAL COLLEGE LONDON
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MATH40001/MATH40009
Introduction to University Mathematics

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Administrative details:

- **Instructors:**
  - Dr. Marie-Amélie Lawn (Part I)
  - Prof. Kevin Buzzard (Part II)
  - Dr. Thibault Bertrand (Part III)
- **Graduate Teaching Assistants (GTA):**
  - Diego Artacho de Obeso (SGTA)
  - Ines Garcia Redondo (SGTA)
  - Bernat Bassols Cornudella
  - Hugo Chu
  - Tiziano Gaibisso
  - Albert Gevorgyan
  - Anton Grishechkin
  - Dan Leonte
  - Rishindra Melanathuru
  - Jossy Russell
  - Ananya Satoskar
- **Lectures:** This module consists of three parts (with equivalent weights). During your first two weeks, you will focus on Part I, while Part II will be the focus of weeks 3–4. Part III will be taught in parallel of the other two parts in weeks 1–4. **Note that the Joint Maths & Computing students only follow Part I and Part II of this module**. Each part has a different lecturer. Lectures are in general two hours long and should be in your online timetable.
- **Problems classes and tutorials:** You will have a total of around 20 hours of problems classes during weeks 1–4 of Autumn term. These are timetabled live events and will run on-campus; the information should be in your timetable.

  All three lecturers (as well as a team of graduate teaching assistants) will be involved in your problem classes. The problem sessions will take place in several rooms in Huxley – rooms 340, 341, 342 and 408. Not all rooms are used for each problems class! *Check your timetable* to see which rooms are being used for each problems class session.

  For each part of the module, you will have three scheduled problem classes. Along with the lectures, we will release problem sheets corresponding to each of the problem classes. Each of the problem sheets will form the basis of your work in one of your problem classes. Shortly after a problem class, solutions to the problems will be released via the module Blackboard site. You will need to check your timetable carefully as the timing and location of the classes may vary from day to day. Here is a list of problem classes and the problem sheet that is associated with them:
  - Wednesday 4 October – Problem Sheet 0
  - Friday 6 October – Problem Sheet 1 of Part III

  - Monday 9 October – Problem Sheet 1 of Part I
  - Tuesday 10 October – Problem Sheet 2 of Part I
  - Thursday 12 October – Problem Sheet 3 of Part I
  - Monday 16 October – Problem Sheet 2 of Part III
  - Tuesday 17 October – Problem Sheet 1 of Part II
  - Thursday 19 October – Problem Sheet 2 of Part II
  - Monday 23 October – Problem Sheet 3 of Part III
  - Tuesday 24 October – Problem Sheet 3 of Part II

  Note that the problem sheets will always be released a couple of days before the problem class associated with it takes place for you to have time to attempt the problems. You may be asked about your solutions during the class. The problems classes will also be a chance for you to ask questions about the problems you are stuck on.

- **Assessment:** There are 3 main assessments during the module (there will also be a number of smaller assessment tasks, designed to ensure that you are keeping up with the material – see below). The first of these is a piece of written coursework; the remaining two are written exams.
  - **Coursework** – Released on Tuesday 10 October (5pm) due Friday 13 October (5pm).
  - **Mid-module Test** – Friday 20 October, starting 9am
  - **Final Exam** – Friday 27 October, starting 9am

  Both exams will be in person; more details will be provided in due course.

- **Discussion forum:** For this module, we will be using an EdStem discussion forum which can be accessed through the blackboard site. To log in to the EdStem portal, you must use your Imperial College email address.

  This forum will allow you to answer questions about lecture material as well as problems classes. The forum will be monitored by the teaching team but we are hoping that it will also become a place of collaboration in between students. To keep things organized, we have arranged the discussion threads in thematic folders. EdStem allows you to write maths easily using a typesetting system called LaTeX. It is a very useful tool to learn for a mathematician; all of your coursenotes and problem sheets have been typeset using LaTeX.

Module overview:

This 5 ECTS module provides a transition towards the way you will be thinking about, and doing, Mathematics during your degree at Imperial College London. You will:

- develop further your computational and problem-solving skills;
- study and understand familiar concepts in greater depth; and
- extend and generalise these concepts into new topics and applications.

In particular, this module will provide an introduction to the notation and language of university mathematics. It will stress the importance of precise definitions and rigorous proofs, but also discuss their relationship to more informal styles of reasoning which are often encountered in applications of Mathematics. Topics to be covered will include an introduction to elementary set theory, common proof strategies as well as common functions and elementary vector operations and geometry.

Intended Learning Outcomes:

On completion of this module, you will be better able to:

- understand the role of precise definitions in Mathematics;

- write Mathematics in a precise, clear and unambiguous way, showing the logical development of an argument or calculation;
- understand and construct proofs and check them for accuracy and completeness;
- analyse problems and develop strategies for distinguishing between appropriate methods and unproductive ones;
- work informally with concepts from calculus and geometry and use them in applications;
- work with peers to understand concepts and solve problems.

Module contents:

Sets and Logic

- Propositions are true-false statements.
- Logical notation: $\wedge$, $\neg$, $\implies$ etc. De Morgan.
- Truth tables for basic logical connectives.
- Notation and basic results for sets: $\in$, $\forall$, $\subseteq$ etc.
- Basic proof strategies: direct, induction, contradiction, contrapositive.
- Basic examples of proofs ($\sqrt{2}$ irrational etc).
- Negation of ”for all x, there exists y such that ...” etc.

Functions and Equivalence relations

- Functions
- Injections, surjections and bijections
- Binary relations
- Equivalence relations and equivalence classes
- An equivalence relation ”is” a partition

Integers

- Natural numbers and Peano axioms
- Induction
- Division with remainder; Euclidean algorithm
- Prime numbers, infinitude of primes
- Fundamental Theorem of Arithmetic
- Modular arithmetic

Real Numbers

- Rationals
- Theory of inequalities, built from an axiomatic viewpoint
- Fields from an axiomatic point of view

Vectors and Geometry

- Vector algebra
- Definition of the dot-product
- Definition of the cross-product
- 2D and 3D elementary geometry: equations of lines and planes
- Relations with distances, areas and volumes
- Space curves
- Kinematics

Prerequisites:

The module will assume a good level of familiarity with topics found in UK Mathematics and Further Mathematics A-levels. These include: trigonometric, exponential and hyperbolic functions; complex numbers; proof by induction; differentiation and integration; use of vectors in 2 and 3-dimensional geometry. But much of the material will be brand new and explained from first principles.

What depends on this module?

Everything! This module has been designed to provide you with the necessary foundational knowledge to pursue a university degree in mathematics as well as teach you best practices and the way mathematics at a higher level is actually done.

Assessment:

First, remember that this is a Pass/Fail module. To pass this module, you need to obtain a final mark of 50% or more. The assessment for the module will consist of:

- Coursework (Released on Tuesday 10 October and due Friday 13 October at 5pm): this is a group coursework, students should turn in a collaborative piece of work produced in equal parts by a group of maximum 3 students. We recommend that every student in a group attempts all questions on the coursework first. Then the students in a group should organize to meet and discuss their work and converge on their answers. Details about how to submit the coursework will follow later.
- Mid-module Test (Friday 20 October, starting 9am) and Final Exam (Friday 27 October, starting 9am): these will be individual in-person exams.
- Throughout the module, you will also be asked to undertake short MCQs online quizzes which will count small percentage towards your final mark. There will be a total of 3 short MCQs online quizzes for each part of the module. They are to be found in the Quizzes folder on the Blackboard site. These quizzes are intended to test your direct understanding of the material. You can attempt them as many times as you wish, but **only the highest mark you obtain will be recorded**. Each quiz will be worth 0.5% of your total mark for the module but you will only obtain these 0.5% if you obtain full marks at the quiz (remember, you can attempt the quiz as many times as you wish!). Note that JMC students will only take the quizzes associated to part I–II and each quiz will be worth 0.75% of the final mark for JMC students.
- Throughout the first 4 weeks, we will also conduct small activities which will count towards towards your final mark. More details will be given in due time.

The detail of the weighting of each assessment in your final mark is given as follows:

- Coursework - 20%
- Mid-module Test - 20%
- Final exam - 50%
- Others (e.g. online quizzes, problem classes activities) - 10%

Feedback:

Even though this is a very large class, we aim to give you feedback on any work you hand in via Blackboard, within a few days. If you have any question about your marking and/or the feedback you receive, the problem classes are a perfect moment to ask!

Reading list:

“A concise introduction to pure mathematics” by M. Liebeck. This book covers most of the material in Parts I and II of the course very thoroughly.
“How to think like a mathematician : a companion to undergraduate mathematics”, by K. Houston. An introduction to rigorous mathematical thinking.
“What is mathematics? An elementary approach to ideas and methods”, by R. Courant and H. Robbins. A classic introduction to modern mathematics.
Other books of interest:
“Bridging the gap to university mathematics”, by M. Gould and E. Hurst.
“Proofs and refutations : the logic of mathematical discovery” by I. Lakatos.
“How to study for a mathematics degree” by L. Alcock.