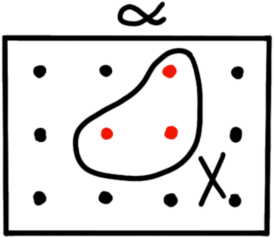
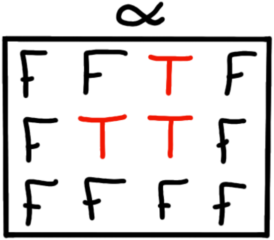
In the last video we saw that there was a correspondence between the following two things:

1) The collection of subsets of α.

2) The collection of predicates on α.





Say we have two predicates $P(a)$ and $Q(a)$ on α. We can think of these predicates as functions from α to `Prop`.

We can “and” those predicates together, and get a new predicate which sends a to $P(a) \wedge Q(a)$.

Because predicates on α are the same thing as subsets of α, we should be able to translate this construction into a story about subsets of α. What is that story?

If X is a subset of α, then the corresponding predicate $P(a)$ is $a \in X$.

Say Y is a subset with corresponding predicate $Q(a)$.

Which set has predicate sending a to $P(a) \wedge Q(a)$?

In other words, which set does the predicate $a \in X \wedge a \in Y$ describe? Pause the video and decide on an answer.

The set corresponding to $a \in X \wedge a \in Y$ is $X \cap Y$, because $a \in X \cap Y \iff a \in X \wedge a \in Y$.

If we "or" the predicates together, it correponds to union $X \cup Y$.

The relation $\subseteq$ on subsets is related to $\implies$ on predicates, because $X \subseteq Y$ just means $\forall a \in \alpha, a \in X \implies a \in Y$.

The relation $=$ on subsets is related to $\iff$ on predicates, because set extensionality says $X = Y$ if and only if $\forall a \in \alpha, a \in X \iff a \in Y$.

What set does the constant predicate `false` (i.e. $P(a)$ is `false` for all a) correspond to?

The corresponding set is the set X such that $a \in X$ is false for all $a \in \alpha$. In other words, this set has no elements.

This set is called the *empty set*.

Set notation: $\emptyset$ is the empty set.

Why don't you figure out the arithmetic of the empty set? If X is a subset of α, and $\emptyset$ is the empty set, what is $X \cap \emptyset$, and what is $X \cup \emptyset$? Pause the video and see if you can figure this out.

We showed in “logic” than $P \wedge \texttt{false} \iff \texttt{false}$, and $P \vee \texttt{false} \iff P$.

The translation of this statement into set language: $X \cap \emptyset = \emptyset$ and $X \cup \emptyset = X$.

In the next video I will talk about how $\forall$ and $\exists$ (language for Propositions) translate over into the language for sets.