Binary relations

✓	X	X
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If P is a predicate on $X \times X$, and if $a, b \in X$, then we say a and b are *related* by the relation if $P(a, b)$ is `true`.
Let's look at some examples.

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Let's think a little about why this is the case.

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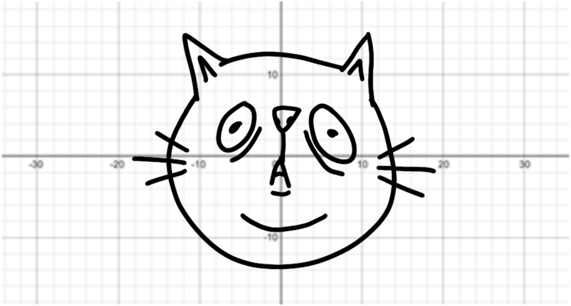
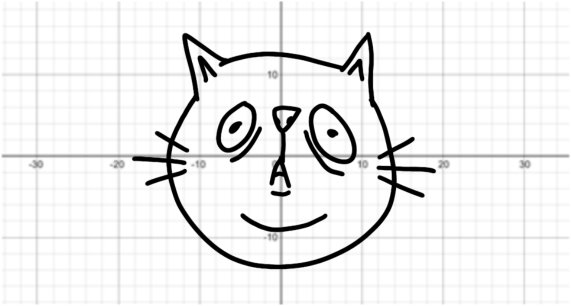
So by definition they're binary relations on $\mathbb{R}$.



The picture represents a subset of $\mathbb{R}^2$.



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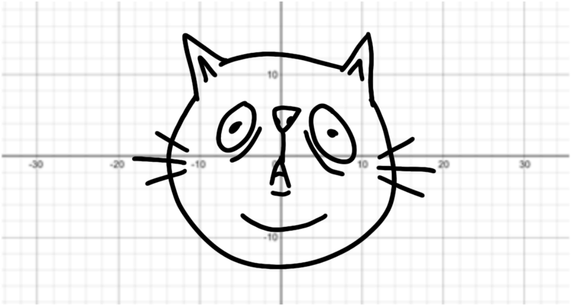

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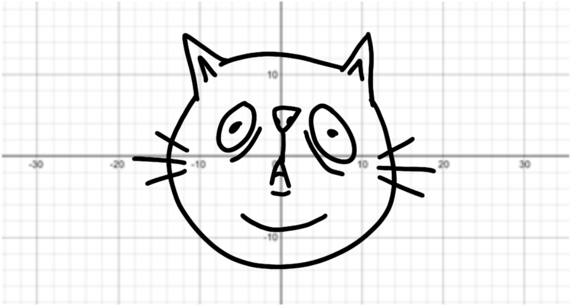
Let $P(a, b)$ be `true` if (a, b) is part of the cat, and `false` otherwise.
In other words, let $P(a, b)$ be the true-false statement "(a, b) is part of the cat".



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Then P is a predicate on $\mathbb{R}^2$ so it's a binary relation on $\mathbb{R}$.

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The *graph* of a binary relation is the subset of X^2 associated to this predicate.

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But that's OK – relations are more general than functions. Click here to see a mathoverflow post where someone puts in a lot of work to come up with a formula for a nice pictorial relation.

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NB does that mean $2^{(n^2)}$ or $(2^n)^2$?