

“Again, [the Analytical Engine] might act upon other things besides number, were objects found whose mutual fundamental relations could be expressed by those of the abstract science of operations. . . .”

Ada Lovelace, 1842.

What is the “abstract science of operations”?

Operations on numbers: $+$ $-$ \times $/$ $=$ $<$ etc.

$2 + 2 = 4$, $2 \times 2 < 5$.

Question: Can we use these operations on other things, apart from numbers?

Let $f(x) = x^2 + 1$ and $g(x) = x^3 - 1$ be polynomials.

Then $f + g$ makes sense, and is another polynomial.

And f/g makes sense too, but is not a polynomial. What is it?

Does $f < g$ make sense?

$\frac{df}{dx}$ is another polynomial too. So (d/dx) is a new kind of operation.

If M and N are matrices, then $M + N$ might make sense. Or it might not.

And $M \times N$ might make sense, or it might not.

Matrices have transposes. The transpose is a new operation.

If A , B and C are numbers, then $A \times (B + C) = A \times B + A \times C$.

This is part of the abstract science of operations for numbers.

If A , B and C are polynomials, does

$$A \times (B + C) = A \times B + A \times C?$$

If A , B and C are matrices, does $A \times (B + C) = A \times B + A \times C$?

Did you ever check this?

If X and Y are numbers, then does $X \times Y = Y \times X$?

What about if X and Y are 2×2 matrices?

There are many collections of objects that we can do mathematics with.

Numbers, polynomials, matrices, triangles, groups, elements of groups, fields, elements of fields, vector spaces, perfectoid spaces and so on.

In the next video, we will begin to learn about how to do mathematics with *propositions*, which are true–false statements.

Mathematics with propositions

You perhaps know that if $x^2 = y^2$ then $x = \pm y$.

The mathematical notation for this: $x^2 = y^2 \implies x = \pm y$.

Here, $x^2 = y^2$ and $x = \pm y$ are propositions, and \implies is an operation on propositions.

In the next video, we will look at some more operations on propositions, and, following Lovelace's idea, we will try and figure out the science of these operations.