Welcome to Part 1 of the intro module!

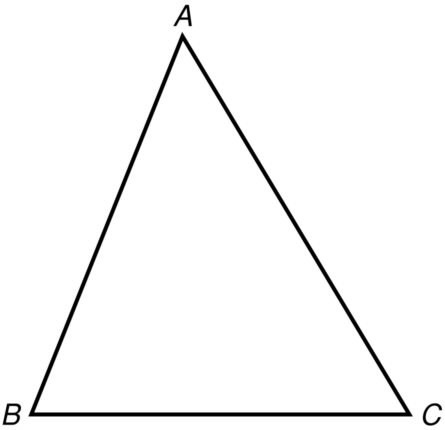
Maths students: M40001

Joint Maths and Computing (JMC) students: M40009

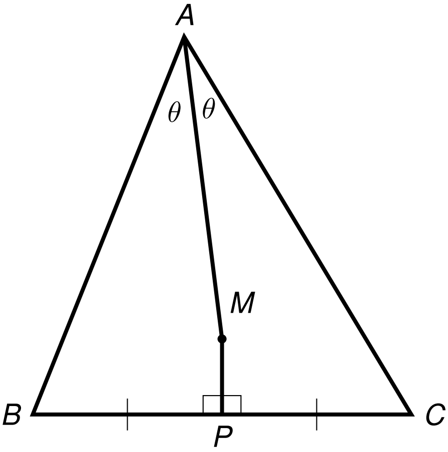
In this part of the course, we will think about the foundations of mathematical argument.

In this video we will see:

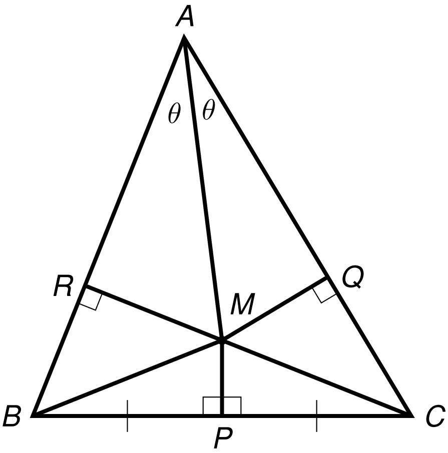
- A proof;
- A discussion about how university mathematics differs from school mathematics.



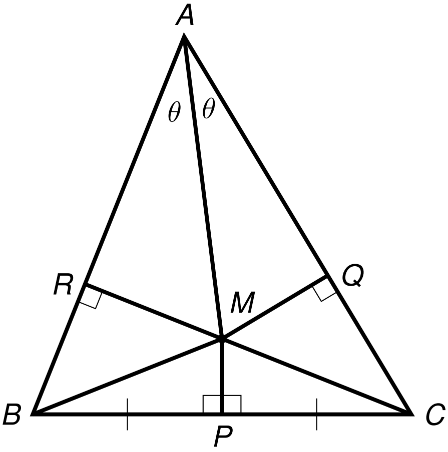
Let ABC be any triangle.



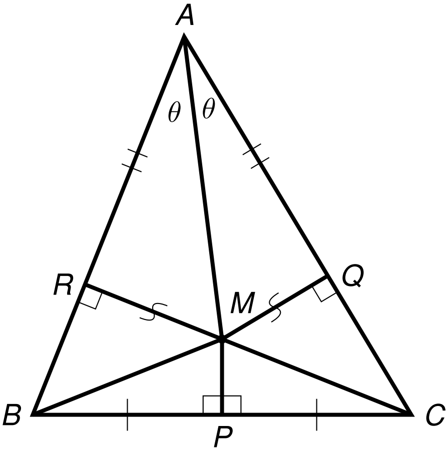
Let M be the point where the angle bisector AM of angle A meets the perpendicular bisector MP of BC.



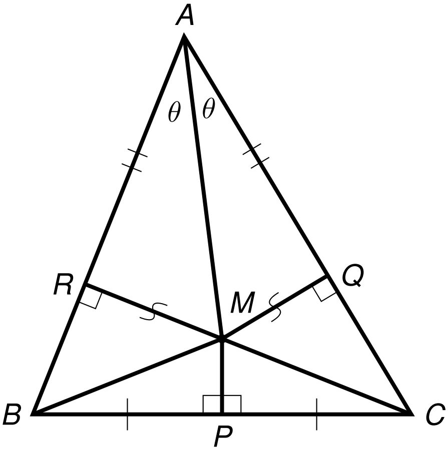
From this middle point *M*, drop perpendiculars *MQ* and *MR* to the other two sides, and also draw *MB* and *MC*. Now watch closely!



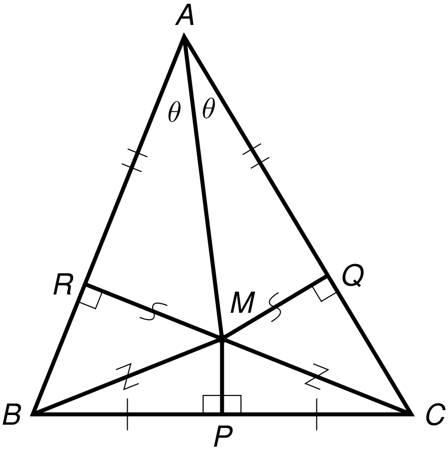
The triangles ARM and AQM are congruent, because all their angles are equal, and they share the side AM.



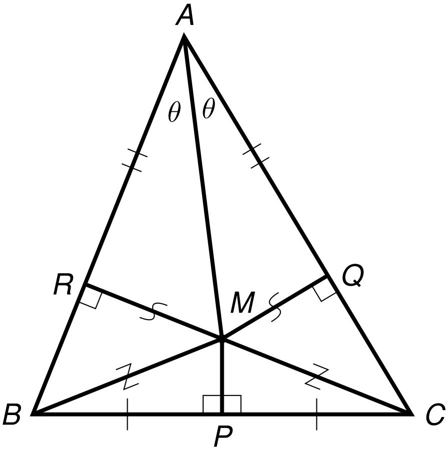
The triangles ARM and AQM are congruent, because all their angles are equal, and they share the side AM. Hence AR=AQ and RM=QM.



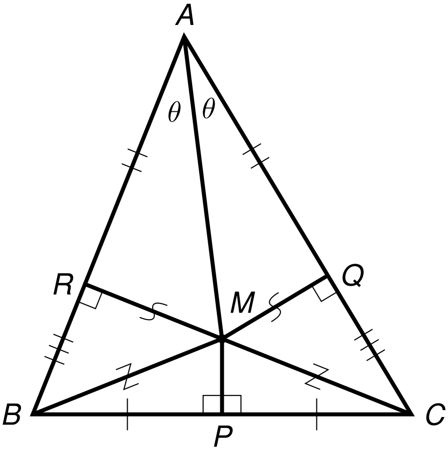
The triangles BPM and CPM are congruent, by two sides and included angle.



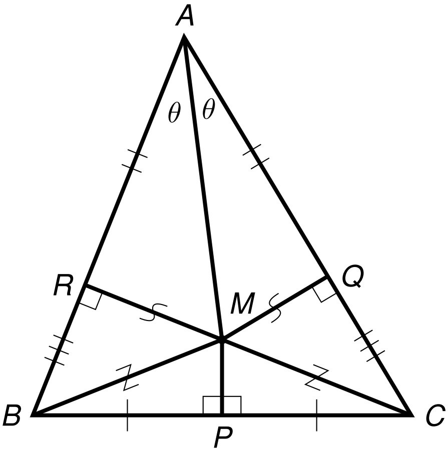
The triangles BPM and CPM are congruent, by two sides and included angle. Hence $BM = CM$.



Finally, the triangles BRM and CQM are congruent, by “right angle, hypotenuse, and one other side”.



Finally, the triangles BRM and CQM are congruent, by “right angle, hypotenuse, and one other side”. Hence $BR = CQ$.



This means that $AB = AR + RB$ and $AC = AQ + QC$ are equal! But ABC is an arbitrary triangle! Hence all triangles are isosceles.

Corollary: all triangles are equilateral.

There is probably something wrong with that proof.

We were *not as careful as we should have been* in that proof.

Here is a link to an article by Frank Quinn.

Quinn explains (on the last page) that there is a *disconnect between school mathematics and higher education* (i.e., what you just started).

At school, we can get away with intuitively reasonable arguments.

We are told that e is a magic number such that the derivative of e^x is e^x. Everything seems to work really well.

But now we need to start worrying about what is *actually going on*.

Examples of questions left unanswered at school:

- What is e^x? What does raising something to a real power *mean*?
- How do we know $\frac{1}{0!} + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \cdots$ converges?
- Why is there a real number whose square is 2?
- What is the *definition* of the real numbers?

My job in Part I of the intro module: to begin to show you *rigorous mathematics*.

We will do mathematics *carefully*, and begin to build it from first principles.

So we cannot start with triangles. Triangles are too hard.

And we cannot start with real numbers. Real numbers are too hard.

We will start by trying to find some easier things to do mathematics with. Because, as we will learn in the next video, there are many things you can do mathematics with.