# Equivalence relations

An *equivalence relation* on a set $X$ is a binary relation which is all three of

1. reflexive;
2. symmetric;
3. transitive.

A famous example, going back to Euclid – equality.

# Equivalence relations

An *equivalence relation* on a set $X$ is a binary relation which is all three of

1. reflexive;
2. symmetric;
3. transitive.

A famous example, going back to Euclid – equality. Pause the video and state the three claims I (or Euclid) am making about equality.

$$\forall x, x = x;$$

$$\forall x\ y, x = y \implies y = x;$$

$$\forall x\ y\ z, (x = y \land y = z) \implies x = z.$$

In Lean the first fact is an axiom and you can prove the other two by induction.

$$\forall x, x = x;$$

$$\forall x\ y, x = y \implies y = x;$$

$$\forall x\ y\ z, (x = y \land y = z) \implies x = z.$$

In Lean the first fact is an axiom and you can prove the other two by induction. I'll show you how to do that in the Lean video.

$$\forall x, x = x;$$

$$\forall x\ y, x = y \implies y = x;$$

$$\forall x\ y\ z, (x = y \land y = z) \implies x = z.$$

In Lean the first fact is an axiom and you can prove the other two by induction. I'll show you how to do that in the Lean video.

Next I'll talk about my favourite example: plastic squares and triangles.

In this example, $X$ is the set of plastic squares and triangles I'm holding.

In this example, $X$ is the set of plastic squares and triangles I'm holding.

The binary relation: if $a, b$ are plastic shapes then we say $R(a, b)$ is true if and only if $a$ and $b$ are the same colour.

In this example, $X$ is the set of plastic squares and triangles I'm holding.

The binary relation: if $a, b$ are plastic shapes then we say $R(a, b)$ is true if and only if $a$ and $b$ are the same colour.

If you like, $R(a, b)$ is "$a$ is the same colour as $b$".
Is this an equivalence relation? What does this mean? What do we have to do? What will the next slide start with a proof of?

**Reflexivity.**
Need to check that if $a$ is a plastic shape, then $a$ is the same colour as $a$.

**Reflexivity.**

Need to check that if $a$ is a plastic shape, then $a$ is the same colour as $a$.

`true`

**Reflexivity.**
Need to check that if *a* is a plastic shape, then *a* is the same colour as *a*.

`true`

**Symmetry.**
Need to check that if *a* and *b* are plastic shapes, and *a* is the same colour as *b*, then *b* is the same colour as *a*.

**Reflexivity.**
Need to check that if *a* is a plastic shape, then *a* is the same colour as *a*.

`true`

**Symmetry.**
Need to check that if *a* and *b* are plastic shapes, and *a* is the same colour as *b*, then *b* is the same colour as *a*.

`true`

**Transitivity.**
$a, b, c$: plastic shapes.

**Transitivity.**
$a, b, c$: plastic shapes.

$a$ is the same colour as $b$.

**Transitivity.**
$a, b, c$: plastic shapes.

$a$ is the same colour as $b$.

$b$ is the same colour as $c$.

**Transitivity.**
$a, b, c$: plastic shapes.

$a$ is the same colour as $b$.

$b$ is the same colour as $c$.

Want:

**Transitivity.**

$a, b, c$: plastic shapes.

$a$ is the same colour as $b$.

$b$ is the same colour as $c$.

Want: $a$ is the same colour as $c$.

**Transitivity.**
$a, b, c$: plastic shapes.

$a$ is the same colour as $b$.

$b$ is the same colour as $c$.

Want: $a$ is the same colour as $c$.

True because they're all the same colour. True by Euclid's first common notion!

**Transitivity.**
$a, b, c$: plastic shapes.

$a$ is the same colour as $b$.

$b$ is the same colour as $c$.

Want: $a$ is the same colour as $c$.

True because they're all the same colour. True by Euclid's first common notion!

We will come back to this example later on.

A good way to think about equivalence relations is to imagine that two things are related if some aspect of them is the same (for example their colour, or their remainder when you divide them by two). This is a good mental model for an equivalence relation.

A good way to think about equivalence relations is to imagine that two things are related if some aspect of them is the same (for example their colour, or their remainder when you divide them by two). This is a good mental model for an equivalence relation.

We will learn more about this idea in the next video, where we will learn a new way of thinking about equivalence relations.