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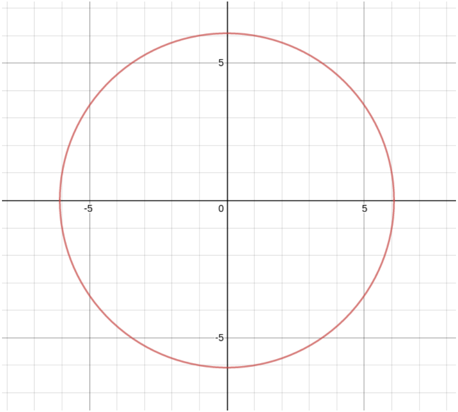

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My answer:

C	✓	X	X
B	✓	X	X
A	X	X	X
	A	B	C

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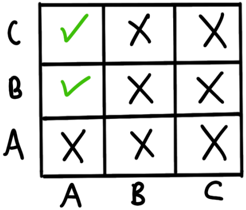

We'll be seeing more of this random relation later.

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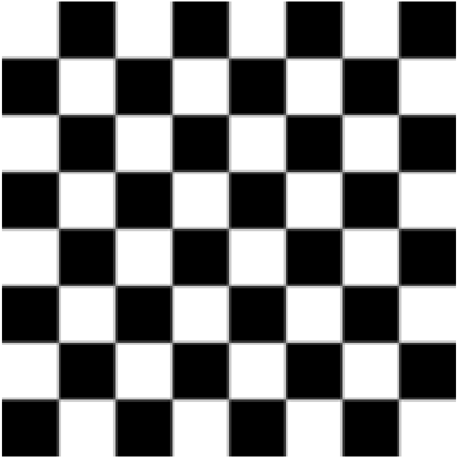

If instead we had said $R(a, b)$ was true if and only if a and b have equal remainders when you divide them by 37, what would the graph look like?

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Our even/odd example was like that, because $f : \mathbb{N} \to \mathbb{N}$ can be the function that sends a natural number to its remainder mod 2.

If α is a set, then $\subseteq$ is a binary relation on the subsets of α, because if X and Y are subsets of α then $X \subseteq Y$ is a true-false statement.

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In the next video I will define four predicates on binary relations and then we can try and figure out which ones are satisfied by these and other examples.