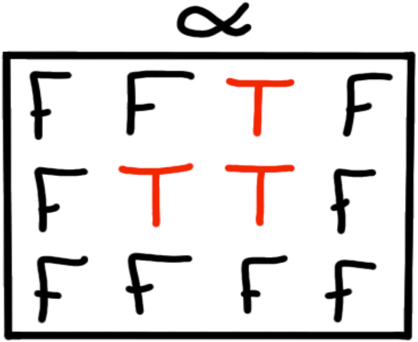
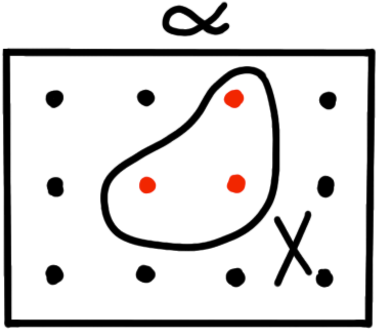
Subsets and predicates

Let α be a “base set”, and let P be a predicate on α. Recall what this is – it's a family of true/false statements, one for each element of α.

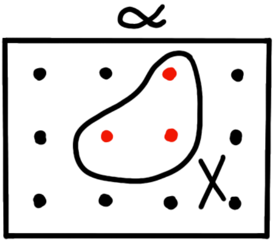


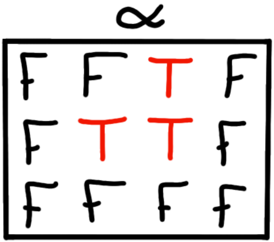
Using this predicate *P*, we can make a subset *X* of α, consisting of the elements of α where this predicate is true.



Let's call that construction "Construction 1". It takes a predicate on α, and constructs a subset of α.

In mathematical notation, Construction 1 takes the predicate $P(a)$ on α and returns the subset $X = \{a \in \alpha \mid P(a)\}$. Here's the picture:





We can go the other way too.

If X is a subset of α, then we can make a predicate on α.

How should we define the predicate?

Want $P(a)$ to be true if and only if $a \in X$.

So let's just define $P(a)$ to mean $a \in X$.

Let's call this "Construction 2" – it takes a subset X of α, and it spits out a predicate on α.

Fundamental fact: these constructions give us *two ways of thinking about the same idea*.

Interesting exercise: check that these constructions are inverse to each other.

Given a predicate Q, use construction 1 to make a subset Y, then use construction 2 to make a predicate R. Can you prove that $Q = R$, i.e., that $Q(a) \iff R(a)$ for all a?

Similarly, given a subset Y, use construction 2 to make a predicate R, and then use construction 1 to make a subset Z. Can you prove that $Y = Z$?

I will show you how to do this in the optional Lean video.

But what is much more important is that you begin to understand that *there can be more than one way of looking at the same idea*.

Later in the intro module: equivalence relations are the same as partitions. Later in the course: $m \times n$ matrices are linear maps $\mathbb{R}^n \to \mathbb{R}^m$.