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The graph of a relation might not satisfy the vertical line test.

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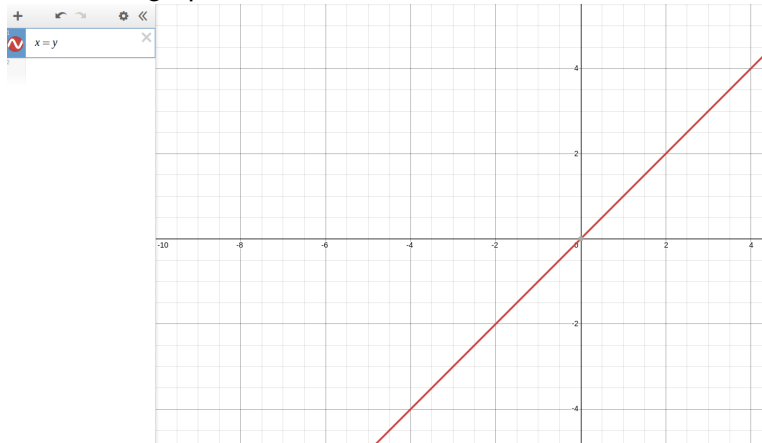
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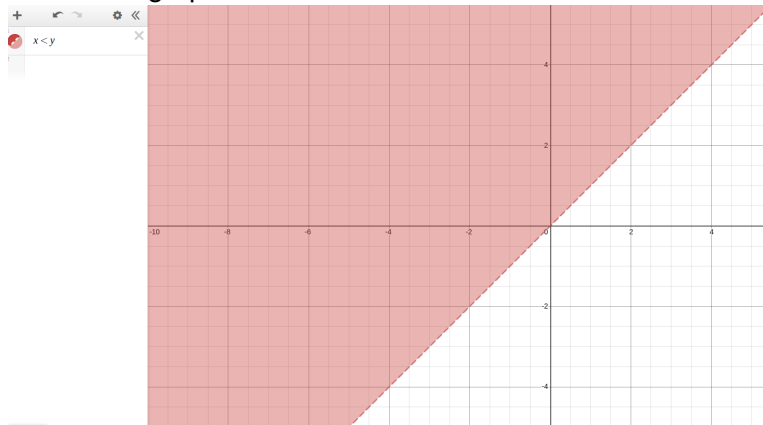
Here’s another question. “Draw the graph of  $\sin(x)$ ”. What is the associated predicate?

$<$  and  $=$  are binary relations on  $\mathbb{R}$ , and hence predicates on  $\mathbb{R}^2$ . Can you draw their graphs?

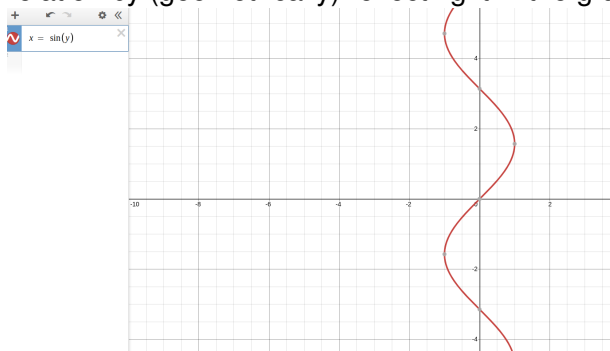
Here's the graph of  $=$ :



Here's the graph of  $<$ :



Last thing: If a function isn't bijective, it doesn't have a two-sided inverse. But you can take the “inverse” of a binary relation by (geometrically) reflecting it in the graph of  $id_X$ .



Algebraically, given the predicate  $P(x, y)$ , we can create the “transpose” predicate  $P(y, x)$ .