Imperial College London

DEPARTMENT OF MATHEMATICS
IMPERIAL COLLEGE LONDON
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How to study for MATH40001/40009 Introduction to University Mathematics

Welcome to Imperial! As you are embarking on a wonderful journey through mathematics and we are here to guide you. Over the next couple of weeks, students from all over the world will be gathering in the classrooms of Huxley. This incredibly diverse community is just one of the reasons why a Maths degree at Imperial College London is so enriching. This diversity of profiles also comes with a diversity of backgrounds. Some of you took the A-levels while others took other equivalent qualifications like the international baccalaureate. One of our goals during the **Introduction to University Mathematics** module is to help you make the transition to university independently of which programme you followed. Now, as the saying goes, **mathematics is not a spectator sport**, so you need to get your brain working again after a nice long summer break!

What to expect at the start of term?

The undergraduate programme in the Department of Mathematics starts with a single module called **Introduction to University Mathematics**, which runs for the **first four weeks of term**. In many aspects, this module is different from the other modules you will take during your time at Imperial. **Introduction to University Mathematics is fast-paced and demanding**, but it is also **rewarding** and will set you up well for the start of your university career.

Using a **problem-based approach to learning**, this module offers a transition towards the way you will need to think and do mathematics during your time at Imperial. In particular, this module introduces you to the notation and language of university mathematics. Not only do we stress the importance of precise definitions and rigorous proofs, but also discuss their relationship to more informal styles of reasoning which are often encountered in applications of mathematics. This module is the **springboard to kickstart your time studying mathematics at university**.

The module consists of three parts, with equivalent weights. Part I will run during weeks 1–2, Part II during weeks 3–4 and Part III will be taught in parallel throughout the four weeks. Each part has a different lecturer but **all three lecturers, as well as an incredible team of graduate teaching assistants, will be involved in your problem solving sessions**. **By default, these sessions will be held on campus in Huxley Building** (home to the Maths Department on the South Kensington campus). These sessions will run in the Huxley Building, in rooms 130, 340, 341 ans 342. In each room, lecturers and GTAs will animate problem solving sessions during which you will be split in smaller groups and encouraged to collaborate/discuss to solve mathematical problems.

They are a **staple of this module** and we really urge you to make the most out of them: **come ready to solve problems, work in small groups and present your work to the rest of the group**. In short – it is crucial that you work on the problems *before* the session, **to ensure you learn as much as you can during the session**.

Each week, we will release video material (replacing the traditional lectures) for you to review and study. Problem sheets will be released prior to the sessions and will form the basis of the work to be covered during the problem solving sessions. **Make sure you look at the problems in advance** – you will benefit much more from the sessions if you do!

At the end of the four weeks, there will be a **comprehensive pass/fail final exam** to check that you have understood the material covered in Parts I – III.

What do we expect from you?

You **should not expect to complete entire problem sheets during these problem sessions.** We will not cover all exercises in the associated problem session, but, do not worry, example solutions will be provided. Studying mathematics is hard work and practice is necessary to become a better mathematician. In this module, we will emphasize how to reason and write like a mathematician. Problem sessions will be a great occasion for you to practice your **"proof-writing"** and your **"proof-reading"**. So we expect you to:

- □ **come with an open mind**;
- □ **forget potential bad habits and "recipes" you learned at school to solve problems**;
- □ **to learn how to write Mathematics in a precise, clear and unambiguous way, showing the logical development of an argument or calculation**.

Throughout the first four weeks, you **will be assessed in various formats**: online quizzes, courseworks and remote tests. These will provide you with **multiple occasions to get feedback on your work**.
To summarize, what we expect from you is to **work hard, be engaged with your classmates, help each other** and **work together in groups**! But most of all, we expect you to **take it all in and enjoy your university experience**.

As mentioned earlier, while it is only the beginning of the term, this module is fast-paced. It is of paramount importance that your A-levels Mathematics/Further Mathematics foundations (or equivalent) are sound, as there **will be limited time to revise all this material during term time**. After these four weeks, you will be more comfortable with elements of logic and notations. Although this might seem like abstract nonsense at first, it will be relevant in your future career regardless of which branch of mathematics you end up choosing, starting with the four first year modules awaiting you in November!

How can you prepare for the autumn term?

Here, we include as a reminder a list of topics you should be in control of. Most will hopefully be very familiar, however, some may be new to you. We are aware that students come to Imperial with different educational backgrounds and we know that certain topics are not covered by everybody at school.

To help you prepare for your course, you were all invited to follow a Pre-arrival session held in September. We hope that this proved useful to many of you. To make the start of your course a lot easier, you are **urged to study anything unfamiliar on the Topics to Review** list and practice any which you are not confident with. To do so, the material that you were provided during the **Maths Pre-arrival Course** will remain available to you throughout the year. It can be found here! (Log in required)

We also provide at the end of this booklet a list of useful resources and sources of extra support to help you revise the topics that you have forgotten or may be less familiar with.

Topics to Review

In this section, we provide a detailed checklist of topics to review. You are expected to be comfortable with most of the topics below, but do not worry if you didn't cover one or two topics at school. In general, your first year modules will start from the basics, but for your own benefit we highly recommend that you review this material before the term starts.

Language of Mathematics

Make sure to review the following topics:

- ☐ Axioms of mathematics;
- ☐ Mathematical terminology (theorem/proposition/lemma/conjecture/proof);
- ☐ Mathematical notation: quantifiers, sets, ordered pairs, cartesian product (we provide a list of common notations at the end of this document);
- ☐ Example of logical arguments;
- ☐ Techniques for proofs (induction/proof by contradiction).

Real Functions

Make sure to review the following topics:

- ☐ Definition of a function and domain of a function (graph);
- ☐ Symmetries and periodicity;
- ☐ Injectivity, surjectivity, bijectivity, inverse of a function;
- ☐ Asymptote;
- ☐ Continuity;
- ☐ Derivation and higher derivatives (Slope, Minima/Maxima and inflection points);
- ☐ Curve sketching;
- ☐ Integration.

You should also revise elementary techniques in calculus and be comfortable with:

- ☐ Derivatives of standard functions such as $\sin x$, $\cos x$, $\tan x$, e^x, $\log x$, $\sin^{-1} x$, $\cos^{-1} x$, $\tan^{-1} x$, a^x (where a is a constant);
- ☐ Derivative of sum, product, quotient and composition of functions

$$f(x) + g(x), \quad f(x)g(x), \quad \frac{f(x)}{g(x)}, \quad f(g(x))$$

- ☐ Derivatives of inverse functions;
- ☐ Integration of standard functions x^n, $1/x$, e^x, $\sin(x)$, $\cos(x)$, including using simple substitutions;
- ☐ Integration using trigonometric substitution: e.g. integration of

$$\frac{1}{1+x^2}, \frac{1}{\sqrt{1-x^2}}$$

- ☐ Integration by parts;
- ☐ Partial fractions.

Solving Equations

Make sure to review the following topics:

- ☐ Solutions to linear and quadratic equations as well as inequalities;
- ☐ Polynomials (roots as zeros of a function);
- ☐ Rational functions;
- ☐ Decomposition of rational functions into partial fractions;
- ☐ Solution of simple first order and second order ODEs: e.g. solution of $\frac{dy}{dx} + \alpha y = 0$, where α is a constant and solution of $\frac{d^2y}{dx^2} \pm \omega^2 y = 0$ by the substitution $y = Ae^{mx}$, or otherwise (where ω, A and m are constants).
- ☐ Definition of exponential, logs etc.;
- ☐ Definition of trigonometric, hyperbolic functions;
- ☐ Inverse/derivative/integral of usual functions.

Trigonometric and Hyperbolic Functions

Make sure to review the following topics:

- ☐ Cartesian and polar coordinates;
- ☐ Trigonometric circle, definition of angle and cosine and sine functions;
- ☐ Trigonometric identities of the kind $\sin(A \pm B)$, $\cos(A \pm B)$, $\sin(2A)$, $\cos(2A)$, $\sin A \pm \sin B$, etc. and their geometric interpretation;
- ☐ Switch from polar coordinates to Cartesian coordinates and vice-versa, e.g. being able to rewrite $a\cos\theta + b\sin\theta$ in the form $r\cos(\theta + \alpha)$.
- ☐ Hyperbolic functions including their geometric interpretation (similarity with trigonometric functions);
- ☐ Inverse trigonometric and hyperbolic functions.

Complex Numbers

Make sure to review the following topics:

- ☐ Introduction of common sets of numbers ($\mathbb{N}$, $\mathbb{Z}$, $\mathbb{Q}$ and $\mathbb{R}$) and the necessity to define the complex unit i to define $\mathbb{C}$;
- ☐ Real part, imaginary part of a complex number;
- ☐ Elementary operations on complex numbers: addition, multiplication, conjugation;
- ☐ Exponential notation of a complex number (modulus and angle).
- ☐ Complex plane;
- ☐ Geometric interpretation for complex numbers (Cartesian and polar coordinates) and elementary operations on them;
- ☐ n^{th} root of a complex number;
- ☐ Euler - De Moivre Formulas.

Sequences and Series

Make sure to review the following topics:

- ☐ Definition of a sequence as a function from $\mathbb{N}$ to $\mathbb{R}$;
- ☐ Arithmetic and Geometric sequences and series;
- ☐ Partial sums and infinite series;
- ☐ Further examples of series such as the harmonic series;
- ☐ Definition of some function as infinite series: exponential, sine and cosine.

Linear algebra

Make sure to review the following topics:

- ☐ Elementary knowledge of vectors: addition, substraction, scalar product;
- ☐ Elementary knowledge of matrices: addition, substraction, multiplication by a scalar, multiplication of a matrix by a vector and matrix multiplication;
- ☐ Zoology of matrices: identity, zero, diagonal, upper-triangular, lower-triangular, (skew-)symmetric;
- ☐ Trace of a matrix;
- ☐ Determinants for 2×2 and 3×3 matrices;
- ☐ Linear transformations, matrix multiplication as a representation of a linear map;
- ☐ Examples of reflection matrices and rotation matrices.

Extra support

Something that you would like to revise and you cannot find in our **Maths Pre-arrival course**? There are lots of very good A-level Mathematics or Further Mathematics resources. Here is a selection.

A-levels Textbooks

There are numerous useful A-level Mathematics and Further Mathematics textbooks out there. You may wish to revisit your books from your A-levels/IB, etc or the following two references should be a good starting point:

- *Introducing Pure Mathematics*
  by Robert Smedley and Gary Wiseman, Oxford University Press (ISBN:0199148031)
- *Further Pure Mathematics*
  by Brian Gaulter and Mark Gaulter, Oxford University Press (ISBN:0199147353)

Online resources

There are many resources online that you may wish to consult. Do a quick search online or look at some of the resources below:

- ☐ **EdX** has online material under A-level Mathematics for Year 12 and Year 13 co-created by one of your year 1 maths lecturers, Phil Ramsden: https://www.edx.org/school/imperialx
- ☐ The **Advanced Mathematics Support Programme** has lots of great resources to support students including short videos, take a look at: https://amsp.org.uk/students/a-level-further/resources
- ☐ **METRIC (Mathematics Education Technology Research at Imperial College)** – Material and exercises about most of the topics can be found in the online resource METRIC. METRIC is an online tool developed at Imperial College which consists of self-test exercises, interactive explorations of concepts and mathematical tools. You can use METRIC to brush up on topics above that you have forgotten or may be less familiar with.

  For information about METRIC, please visit: https://www.imperial.ac.uk/engineering/staff/ed-tech/metric/.

Please don't try to look at all of the above resources, but identify sections from the topics to review that you feel that you may need some extra practice on prior to the start of the term.

Mathematical Notation

For our module, we will make great use of an **Ed Stem** discussion forum. It will allow you to ask questions about the material. It will be monitored by the teaching team but we are hoping for it to be a place of collaboration in between students. Ed Stem allows you to write maths easily using a language called LaTeX. It is a very useful tool to learn; all of our material has been typeset using LaTeX. Here is a cheatsheet gathering useful mathematical notations for our module:

Maths notation	Plain english	LaTeX syntax
Logic		
$A \implies B$	A implies B	`A \implies B`
$A \iff B$	A if and only if B	`A \iff B`
$\neg A$	not A	`\neg A`
$A \land B$	A and B	`A \land B`
$A \lor B$	A or B	`A \lor B`
$\forall x$	For all/any x	`\forall x`
$\exists x$	There exists at least one x	`\exists x`
$\exists! x$	There exists one and only one x	`\exists! x`
: or |	Such that	`:` or `|`
$\therefore$	Therefore	`\therefore`
$\because$	Because	`\because`
Important Sets		
$\emptyset$ or $\varnothing$	the empty set (no members)	`\O` or `\emptyset`
$\mathbb{N}$	the Natural numbers	`\mathbb{N}`
$\mathbb{Z}$	the Integers	`\mathbb{Z}`
$\mathbb{Q}$	the Rational numbers	`\mathbb{Q}`
$\mathbb{R}$	the Real numbers	`\mathbb{R}`
$\mathbb{C}$	the Complex numbers	`\mathbb{C}`
(a, b)	real numbers between a and b, **excluding** a and b	`(a,b)`
$[a, b]$	real numbers between a and b, **including** a and b	`[a,b]`
Numbers		
$\sum$	sum of/over...	`\sum`
$\prod$	product of/over...	`\prod`
$a = b$	a is equal to b	`a = b`
$a \neq b$	a is not equal to b	`a \neq b`
$a \approx b$	a is approximately equal to b	`a \approx b`
$a \propto b$	a is proportional to b	`a \propto b`
$a \equiv b$	a is identically equal to b	`a \equiv b`
$a \cong b$	a is isomorphic to b	`a \cong b`
$\langle a, b \rangle$	scalar product of a and b	`\langle a,b \rangle`
$a \mid b$	a divides b	`a \mid b`
Set theory		
$a \in A$	a is a member of set A	`a \in A`
$A \cup B$	union of A and B: elements that are in A **or** B (or both)	`A \cup B`
$A \cap B$	intersection of A and B: elements that are in A **and** B	`A \cap B`
$A \setminus B$	elements that are in set A **but not** in B	`A \setminus B`
$A \subseteq B$	A is a subset of B	`A \subseteq B`
$A \subset B$	A is a proper subset of B	`A \subset B`
Miscellaneous		
$\square$ or $\blacksquare$	"I have finished my proof"	`\square` - `\blacksquare`
$a := b$	a is defined equal to b	`a := b`
$\#$	contradiction spotted in the proof!	`\#`

There exists also a certain number of useful **abbreviations** including **iff** (if and only if), **wlog** (without loss of generality), **s.t.** (such that) or **QED** (Quod Erat Demonstrandum).