Imperial College London

Introduction to University Mathematics   **MATH40001/MATH40009**

Diagnostic Test

Language of Mathematics

1. What is $1 + 2 + 3 + \cdots + 999$?

   (a) 499500
   (b) 500500
   (c) 1001000
   (d) Something else.

2. Which of the numbers $1, 2, 3, 4, 5$ are in the closed interval $[1, 4]$ but not in the open interval $(1, 3)$?

   (a) All of them
   (b) 4 only
   (c) 1, 3 and 4 only
   (d) Something else.

3. Which one is the odd one out:

   (a) Theorem
   (b) Definition
   (c) Lemma
   (d) Corollary

Real Functions

4. The function $f(x) = \frac{3x^3 + x}{x^3 - 1}$ has

   (a) a vertical asymptote at $x = 1$ and horizontal asymptote at $y = -3$.
   (b) a vertical asymptote at $x = \frac{1}{3}$ and horizontal asymptote at $y = -3$
   (c) a vertical asymptote at $x = 1$ and horizontal asymptote at $y = 3$.
   (d) none of the above

5. The function $f : \mathbb{N} \to \mathbb{N}$, $f(n) = \begin{cases} n + 1, & \text{if } n = 0 \\ n - 1, & \text{if } n \in \mathbb{N} - \{0\} \end{cases}$ is

   (a) injective but not surjective
   (b) surjective but not injective
   (c) bijective
   (d) none of them

6. The function $f : (-\frac{\pi}{2}, \frac{\pi}{2}) \to \mathbb{R}$, $f(x) = x + \sin(x)$ is

   (a) discontinuous
   (b) periodic
   (c) odd
   (d) none of the above

Solving Equations

7. A quadratic polynomial whose roots α and β have sum -3 and product 2 is

   (a) $x^2 - 3x + 6$

   (b) $x^2 + 3x + 6$

   (c) $2x^2 + 6x + 4$

   (d) None of the above

8. Determine the partial fraction decomposition of

$$f(x) = \frac{10x^3}{(x^2 + 1)(x^2 - 4)}$$

   (a) the partial fraction decomposition of $f(x)$ is given by

$$f(x) = \frac{1}{x - 1} + \frac{1}{x + 1} + \frac{4}{x - 2} + \frac{4}{x + 2}$$

   (b) the partial fraction decomposition of $f(x)$ is given by

$$f(x) = \frac{1}{x - i} + \frac{1}{x + i} + \frac{4}{x - 2} + \frac{4}{x + 2}$$

   (c) the partial fraction decomposition of $f(x)$ is given by

$$f(x) = \frac{1}{x - i} - \frac{1}{x + i} + \frac{4}{x - 2} - \frac{4}{x + 2}$$

   (d) None of the above

9. Solve the following equation $2e^{2-x} + e^{2x-4} = 0$ for $x \in \mathbb{R}$

   (a) $x = 2 + \log(-2)/3$

   (b) This equation does not have a solution

   (c) $x = 2 + \ln(-2)/3$

   (d) None of the above

Trigonometric and Hyperbolic Functions

10. Complete the following hyperbolic identity $\cosh(x + y) = \cdots$

    (a) $\cosh(x + y) = \sinh(x)\cosh(y) + \cosh(x)\sinh(y)$

    (b) $\cosh(x + y) = \cosh(x)\cosh(y) + \sinh(x)\sinh(y)$

    (c) $\cosh(x + y) = \sinh(x)\cosh(y) - \cosh(x)\sinh(y)$

    (d) None of the above

11. Find all the solutions to the following equation: $3\cos(3\theta/7)\sin(\theta/2) + 14\cos(3\theta/7) = 0$

    (a) $x = 7\pi/6 + 2\pi n$ or $x = 2\arcsin(-14/3) + 2\pi n,\ n \in \mathbb{Z}$

    (b) $x = 7\pi/6 + 2\pi n,\ n \in \mathbb{Z}$

    (c) $x = 7\pi/6 + 7\pi n/3,\ n \in \mathbb{Z}$

    (d) None of the above

12. Find the derivative of

$$f(x) = \frac{x^2 + 1}{\tan(2x + 1)}$$

(a)
$$f'(x) = \frac{2x}{\tan(2x+1)} + \frac{2(x^2+1)}{\sin^2(2x+1)}$$

(b)
$$f'(x) = \frac{2x}{\tan(2x+1)} - \frac{2(x^2+1)}{\sin^2(2x+1)}$$

(c)
$$f'(x) = \frac{2x}{\tan(2x+1)} + \frac{2(x^2+1)}{\csc^2(2x+1)}$$

(d)
$$f'(x) = \frac{2x}{\tan(2x+1)} - \frac{2(x^2+1)}{\csc^2(2x+1)}$$

Complex Numbers

13. What is $e^{4\pi i/3}$ in cartesian form?

    (a) $\frac{1}{2} + \frac{\sqrt{3}}{2}i$
    (b) $-\frac{1}{2} + \frac{\sqrt{3}}{2}i$
    (c) $-\frac{1}{2} - \frac{\sqrt{3}}{2}i$
    (d) Something else.

14. Which description below best describes the set of complex solutions to $5|z-1| = 6$?

    (a) One point
    (b) Two points
    (c) One circle
    (d) Two circles

15. What is $(1+i)^7$?

    (a) $1 - i$
    (b) $16 - 16i$
    (c) $16 + 16i$
    (d) Something else

16. Let $z(t)$ be a smoothly varying complex function of time t. Say $z(0) = 1$ and $z'(t) = \pi i z(t)$ for all t. What is $z(1)$?

    (a) e^i
    (b) -1
    (c) 1
    (d) Something else

Sequences and Series

17. If 1 is the first odd number, and 3 is the second odd number, what is the 37th odd number?

    (a) 73
    (b) 74
    (c) 75
    (d) Something else

18. What is $1 + (3/4) + (3/4)^2 + \cdots + (3/4)^{100}$?

    (a) $4 - 3(\frac{3}{4})^{100}$
    (b) $\left(1 - (\frac{3}{4})^{100}\right) / \left(1 - (3/4)\right)$
    (c) 3
    (d) Something else

19. What is the value of the infinite sum $1 + (-1) + 1 + (-1) + 1 + \cdots$?

    (a) 0
    (b) 1/2
    (c) 1
    (d) Doesn't exist.

Linear Algebra

20. Let $A = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$ and $B = \begin{pmatrix} 3 & 1 \end{pmatrix}$, then $\det(AB) =$

    (a) 0
    (b) 1
    (c) The determinant does not exist
    (d) None of the above

21. If A is a 3×3-matrix. Which of the following statement is false?

    (a) $A + A^T$ is symmetric.
    (b) $A^T A$ is symmetric.
    (c) $A - A^T$ is symmetric.
    (d) none of the above.

22. Which of the following functions are linear?

    (a) $f : \mathbb{R}^2 \to \mathbb{R}^2$, $f\left(\begin{pmatrix} x \\ y \end{pmatrix}\right) = \begin{pmatrix} x + y \\ y \end{pmatrix}$
    (b) $f : \mathbb{R}^2 \to \mathbb{R}^2$, $f\left(\begin{pmatrix} x \\ y \end{pmatrix}\right) = \begin{pmatrix} x + 2 \\ y + 1 \end{pmatrix}$
    (c) $f : \mathbb{R} \to \mathbb{R}$, $f(x) = |x|$
    (d) None of the above