Signal Synchronization Optimization for Rapid Transit

Using Linear Programming

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Motivation

- Signal Synchronization of Traffic Lights - NP Hard problem

- Currently signals at intersections are sequentially assigned values

- Peak hours cause more transit time and more emissions

- Synchronization can prioritize vehicles
    - Emergency and first responders
    - Public Transit

Model Discussed in Paper: Fitness Function

Base Paper : Traffic lights synchronization for Bus Rapid Transit using parallel evolutionary algorithm

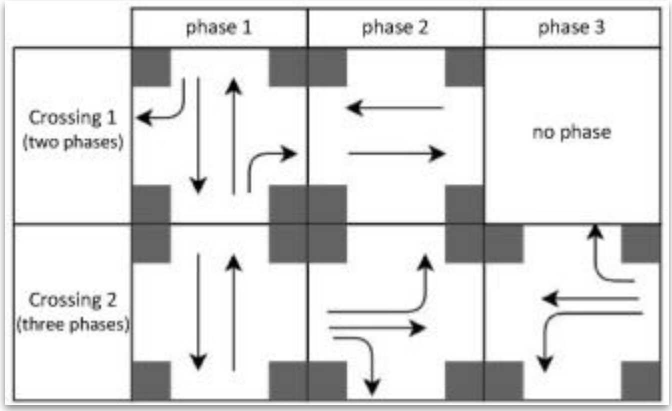
Methodology

- Map
  - Designed a map of the area of study.
- Field research
  - Gathered real data from traffic lights, buses, and vehicles.
- Traffic simulator
  - Candidate solutions (i.e., traffic-light configurations) are evaluated using SUMO

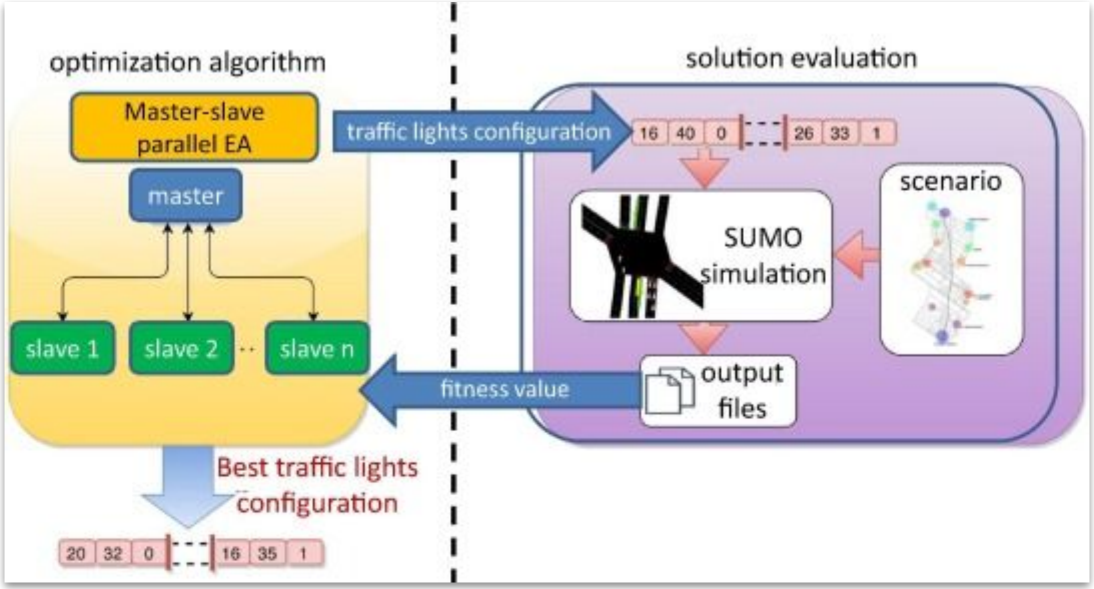
Genetic Algorithm

- Optimization/Fitness criteria: $f = W_b*S_b + W_o*S_o$
  - To optimize on road public transit
- S_b - Average speed of bus, S_o - Average speed of other vehicles
- W_b - Priority of bus, W_o - Priority of other vehicles

Optimizing Phases at Intersections



Combinations of optimal traffic flow patterns



The 2 Components of Resolution: Optimization Strategy (Left) and Solution Evaluation on Simulated (Right)

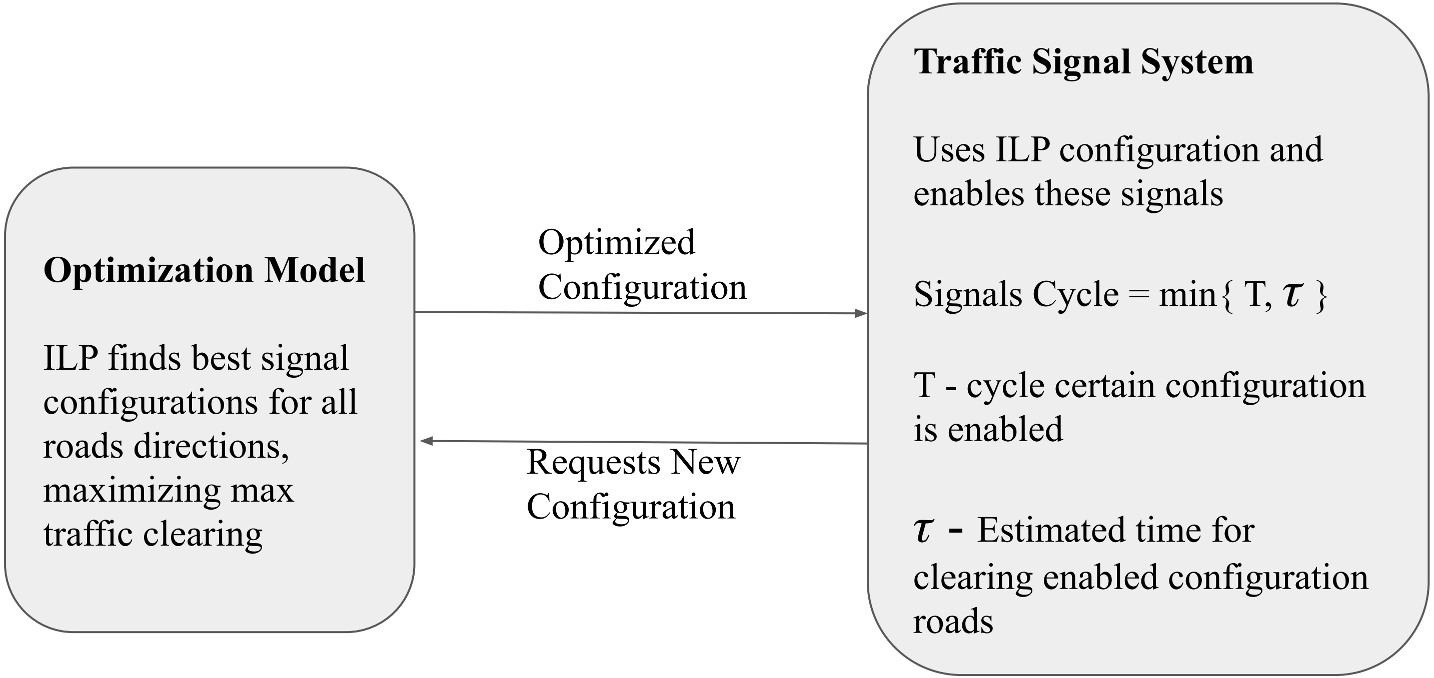
Problem Statement

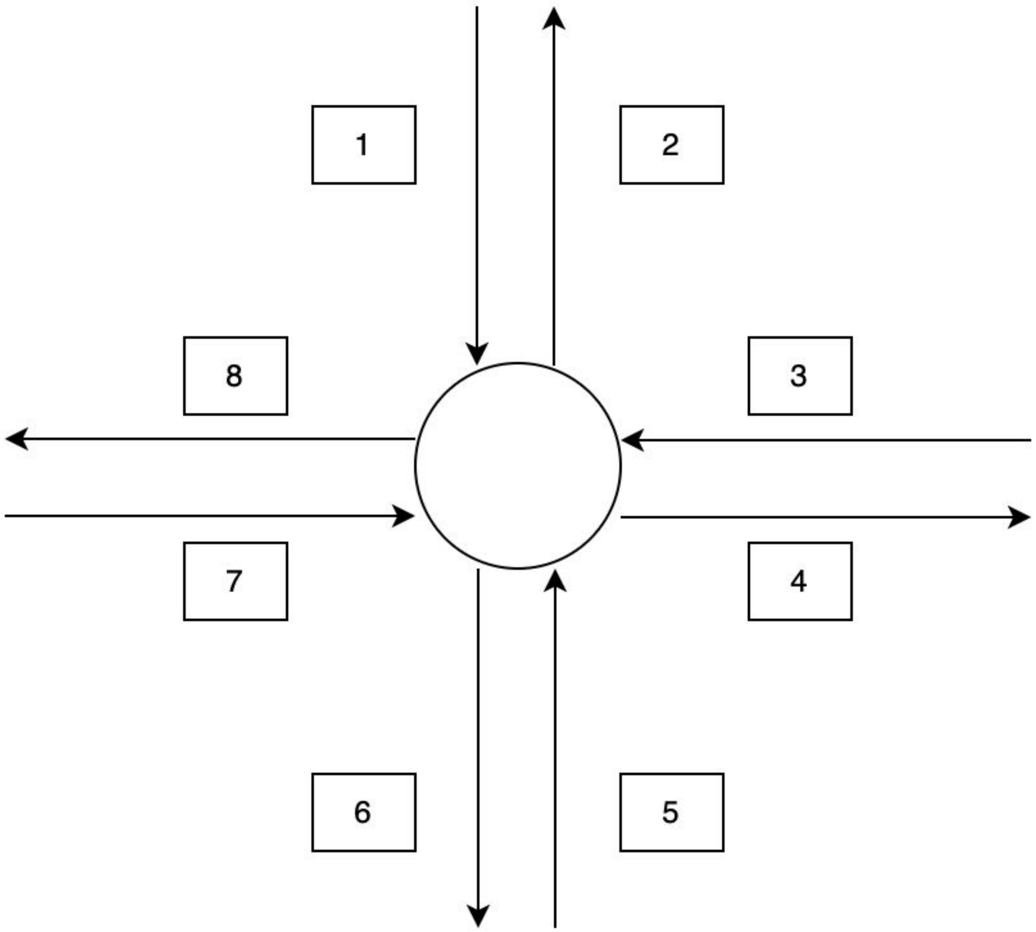
Given a directed graph of road network $G = (V, E, W)$
with vertices as signals at intersection,
edges as directional paths between vertices,
weights as traffic aggregation on directed edge

Find subset of edge activations across different signal vertices
To clear maximum possible aggregated traffic

maximizing lanes with most traffic $\propto$ minimizing vehicle wait time on road
maximizing aggregated traffic $\propto$ maximizing vehicle average speed

System Design





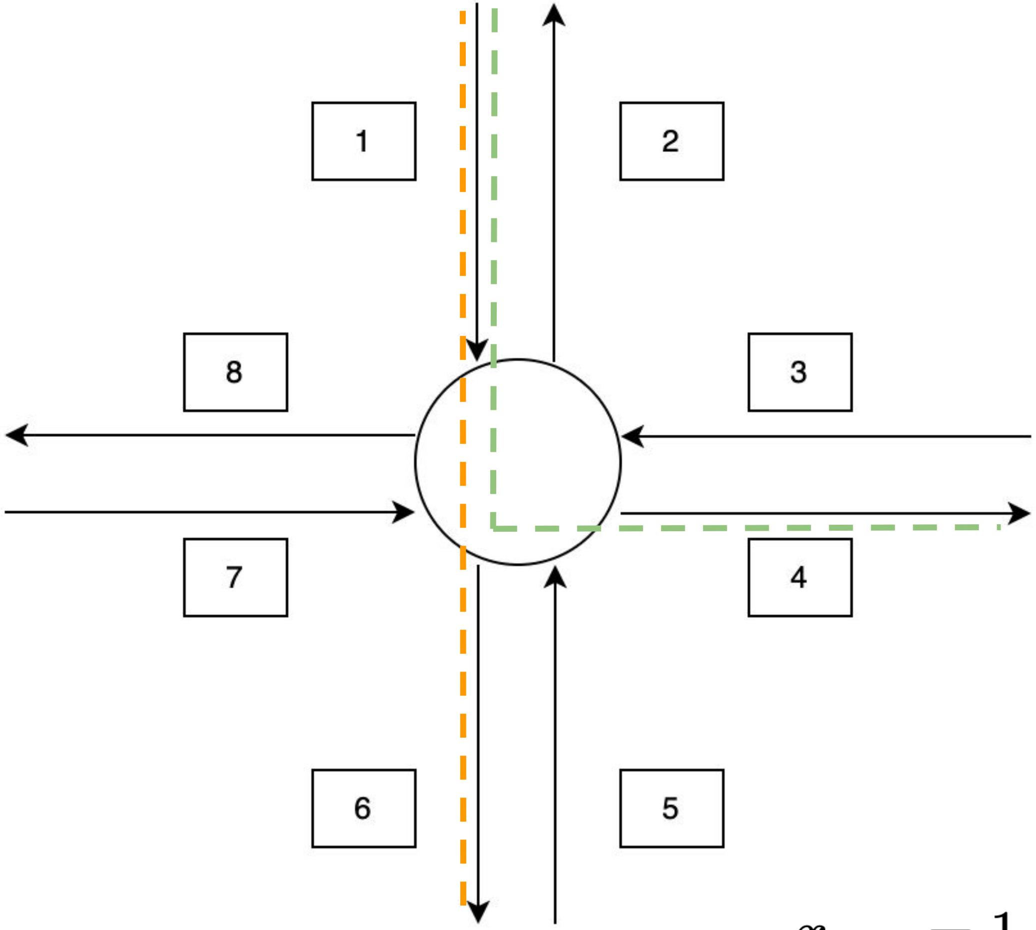
Assumption 1: Assume all cars are placed on the road at the same time interval, move at the same speed, and are separated by the same distance.

Assumption 2: Time τ for vehicles to clear the road $\propto$ (d / road speed limit), d = 0.01 mile of intersection gap

Assumption 3: Odd numbered rows represent lanes that are entering the intersection. Even numbered rows represent lanes that are exiting the intersection.

Assumption 4: Signal changes at a period of $\min\{T, \tau\}$. There is a new matrix $\mathbf{X}$ at each time $t = T$ where T is the period, or light cycle and τ is time for road traffic to be cleared

Assumption 5: Roads are separated by a median so cars traveling on parallel roads can enter the intersection simultaneously but not perpendicular.



Intersection only has 4 waiting lanes
x_i, $i \in \{1, 3, 5, 7\}$

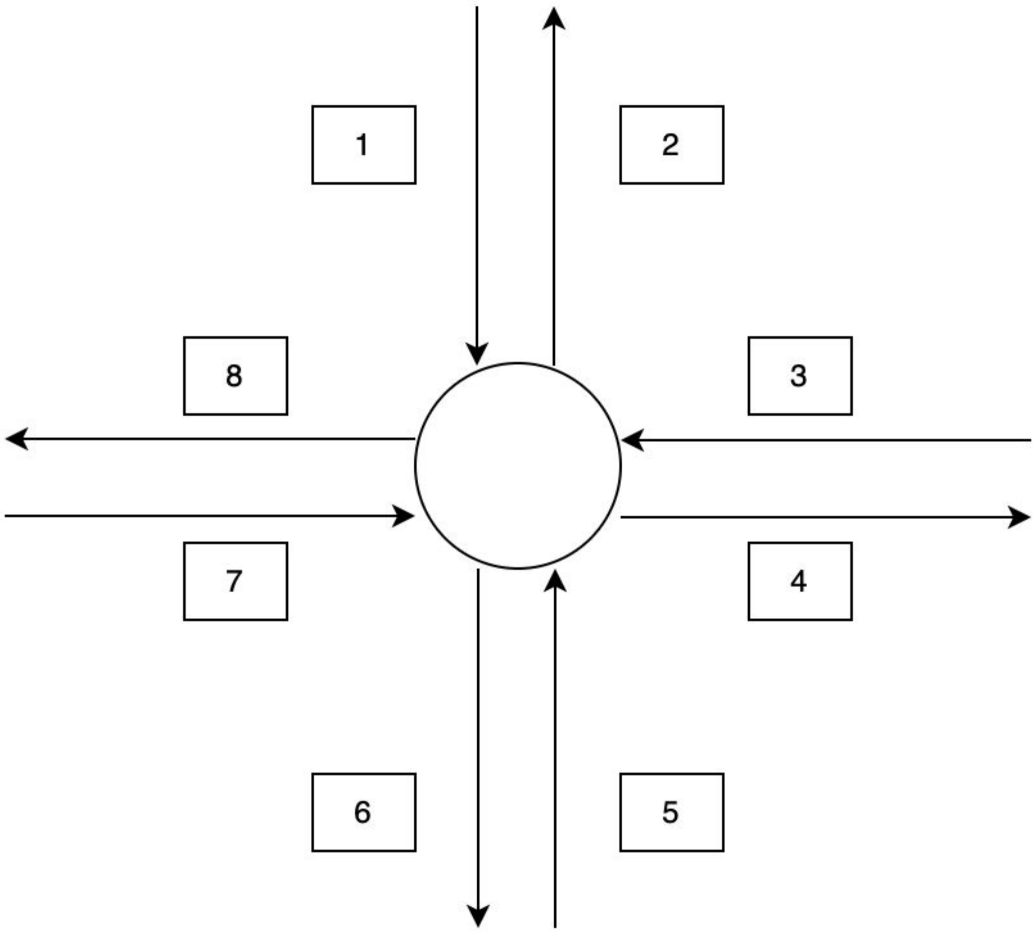
Why additional 4 lane variables?
x_i, $i \in \{2, 4, 6, 8\}$

Assumption 6: While the 4 odd variables indicate the inbound bandwidth of the road, these additional 4 even variables indicate outbound bandwidth that needs to be activated, to create a outflow passage for the inflow

Example: For travelling straight from road 1 to road 6, inbound bandwidth of 1, flows out to road 6, creating a path 1-6 to travel straight

For travelling from road 1 and taking a left. The inbound bandwidth of 1, to flow left needs edge 4 activation to travel from 1-4

$$x_{2k,i} = 1 \text{ for all k} \in \{1, 2, 3, 4\}, i \in \{1, 2, 3\}$$



Assumption on variables

Given graph of an intersection with 8 roads as edges, and each edge has 3 lanes {left, center and right}

$x_{i,j}$ = [0/1 0/1 0/1], is a 8*3 matrix, value 1 indicates road enabled with green in specific lanes

For i ∈ [1,8] and j ∈ [1,3]
i indicating one of 8 roads, and j indicating the lanes

Inflow roads i ∈ {1, 3, 5, 7} has weight as traffic on roads that needs to be cleared with variable configuration

Outflow roads i ∈ {2, 4, 6, 8} has weight = 1 for all three lanes, and its configuration is $x_{i,j}$= [1, 1, 1] when enabled

Signals Assumptions for 4x4 Intersection

$$\mathbf{X}_i = \begin{bmatrix} x_{1l} & x_{1c} & x_{1r} \\ x_{2l} & x_{2c} & x_{2r} \\ x_{3l} & x_{3c} & x_{3r} \\ x_{4l} & x_{4c} & x_{4r} \\ x_{5l} & x_{5c} & x_{5r} \\ x_{6l} & x_{6c} & x_{6r} \\ x_{7l} & x_{7c} & x_{7r} \\ x_{8l} & x_{8c} & x_{8r} \end{bmatrix}_{8\times 3}$$

1.) $\mathbf{X}$ is a matrix containing the signal values for each lane.

2.) Each lane vector $\mathbf{x}_n$ denotes if a car can turn left, proceed straight, or turn right for each directed road.

3.) Lane vector $\mathbf{x}_n$ contains binary variables 0/1, signalling red/green for roads with left, center, and right lanes.

4.) Let $\mathbf{X}_i$ denote the signal matrix $\mathbf{X}$ for intersection i.

$$\mathbf{x}_1 = \begin{bmatrix} x_{1l} & x_{1c} & x_{1r} \end{bmatrix}$$
$$\mathbf{x}_2 = \begin{bmatrix} x_{2l} & x_{2c} & x_{2r} \end{bmatrix}$$
$$\vdots$$
$$\mathbf{x}_8 = \begin{bmatrix} x_{8l} & x_{8c} & x_{8r} \end{bmatrix}$$

$$x_{nl} \in \{0,1\}, 1 \text{ iff left turn allowed}$$
$$x_{nc} \in \{0,1\}, 1 \text{ iff center travel allowed}$$
$$x_{nr} \in \{0,1\}, 1 \text{ iff right turn allowed}$$
$$n \in [1,8]$$

Constraints to Avoid Collisions

Let $x_{i,j}$ denote the element of the matrix $\mathbf{X}$ at row i and column j.

1. Two or more cars in different lanes attempt to make a left turn at the same time, and collide in the intersection:

$$x_{2k-1,1} + x_{2l-1,1} \leq 1, \text{ for all } k, l \in \{1,2,3,4\} \text{ and } k \neq l.$$

2. A car making a left turn collides with a car going straight through the intersection:

$$x_{2k-1,1} + x_{2l-1,2} \leq 0, \text{ for all } k, l \in \{1,2,3,4\} \text{ and } k \neq l.$$

3. A car making a right turn collides with a car going straight through the intersection from the opposite direction:

$$x_{2k-1,3} + x_{2l-1,2} \leq 0, \text{ for all } k, l \in \{1,2,3,4\} \text{ and } k \neq l.$$

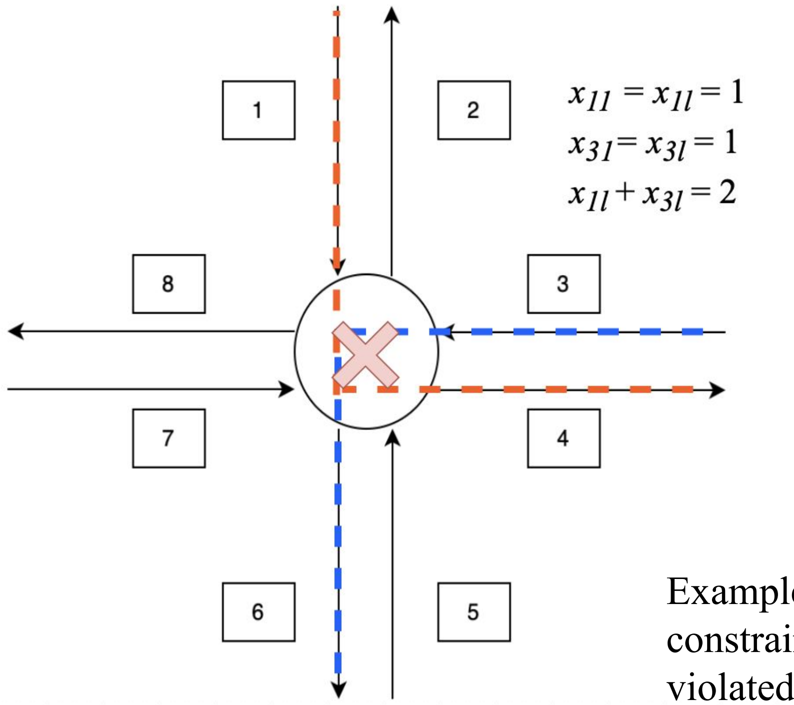
4. Two or more cars in different lanes attempt to go straight through the intersection at the same time, and collide, when the two roads are perpendicular to each other and both using the center lane:

$$\sum_{i=1}^{3} x_{2k-1,i} + x_{2l-1,i} \leq 1, \text{ for all } k, l \in \{1,2,3,4\} \text{ and } k \neq l \text{ such that } \{k,l\} = \{1,3\} \text{ or } \{k,l\} = \{2,4\}$$

Constraints to avoid left hand turn collisions.

$$x_{2k-1,1} + x_{2l-1,1} \leq 1, \text{ for all } k, l \in \{1, 2, 3, 4\} \text{ and } k \neq l.$$

$$k = 1, l = 2 \rightarrow x_{11} + x_{31} \leq 1$$
$$k = 1, l = 3 \rightarrow x_{11} + x_{51} \leq 1$$
$$k = 1, l = 4 \rightarrow x_{11} + x_{71} \leq 1$$
$$k = 2, l = 1 \rightarrow x_{31} + x_{11} \leq 1$$
$$k = 2, l = 3 \rightarrow x_{31} + x_{51} \leq 1$$
$$k = 2, l = 4 \rightarrow x_{31} + x_{71} \leq 1$$
$$k = 3, l = 1 \rightarrow x_{51} + x_{11} \leq 1$$
$$k = 3, l = 2 \rightarrow x_{51} + x_{31} \leq 1$$
$$k = 3, l = 4 \rightarrow x_{51} + x_{71} \leq 1$$
$$k = 4, l = 1 \rightarrow x_{71} + x_{11} \leq 1$$
$$k = 4, l = 2 \rightarrow x_{71} + x_{31} \leq 1$$
$$k = 4, l = 3 \rightarrow x_{71} + x_{51} \leq 1$$



Example of constraints being violated resulting in a collision.

Traffic Assumptions for 4x4 Intersection

$$\mathbf{W}_i = \begin{bmatrix} w_{1l} & w_{1c} & w_{1r} \\ w_{2l} & w_{2c} & w_{2r} \\ w_{3l} & w_{3c} & w_{3r} \\ w_{4l} & w_{4c} & w_{4r} \\ w_{5l} & w_{5c} & w_{5r} \\ w_{6l} & w_{6c} & w_{6r} \\ w_{7l} & w_{7c} & w_{7r} \\ w_{8l} & w_{8c} & w_{8r} \end{bmatrix}_{8\times 3}$$

1.) **W** is a matrix containing traffic amounts for each lane.

2.) Lane vector $\mathbf{w}_n$ contains integer values for road *n* left, center, and right lanes.

3.) The values in each lane vector denotes the number of cars in each lane. The number of cars should be a natural number from 0 to the traffic limit, $\boldsymbol{\alpha}$.

4.) Let $\mathbf{W}_i$ denote the traffic matrix **W** for intersection *i*.

$$\begin{aligned} \mathbf{w}_1 &= \begin{bmatrix} w_{1l} & w_{1c} & w_{1r} \end{bmatrix} \\ \mathbf{w}_2 &= \begin{bmatrix} w_{2l} & w_{2c} & w_{2r} \end{bmatrix} \\ &\vdots \\ \mathbf{w}_8 &= \begin{bmatrix} w_{8l} & w_{8c} & w_{8r} \end{bmatrix} \end{aligned}$$

$$\begin{aligned} w_{nl} &\in [0, \alpha], \quad w_{nl} \in \mathbb{N} \\ w_{nc} &\in [0, \alpha], \quad w_{nc} \in \mathbb{N} \\ w_{nr} &\in [0, \alpha], \quad w_{nr} \in \mathbb{N} \end{aligned}$$

Where α is the traffic limit and $n \in [1, 8]$.

Priority/Anti-Stalling Values at 4x4 Intersection

$$\mathbf{K}_i = \begin{bmatrix} k_{1l} & k_{1c} & k_{1r} \\ k_{2l} & k_{2c} & k_{2r} \\ k_{3l} & k_{3c} & k_{3r} \\ k_{4l} & k_{4c} & k_{4r} \\ k_{5l} & k_{5c} & k_{5r} \\ k_{6l} & k_{6c} & k_{6r} \\ k_{7l} & k_{7c} & k_{7r} \\ k_{8l} & k_{8c} & k_{8r} \end{bmatrix}_{8\times 3}$$

$$\mathbf{k}_1 = \begin{bmatrix} k_{1,l} & k_{1,c} & k_{1,r} \end{bmatrix}$$
$$\mathbf{k}_2 = \begin{bmatrix} k_{2,l} & k_{2,c} & k_{2,r} \end{bmatrix}$$
$$\vdots$$
$$\mathbf{k}_8 = \begin{bmatrix} k_{8,l} & k_{8,c} & k_{8,r} \end{bmatrix}$$

1.) **K** is a matrix containing the scalar anti-stalling values.

2.) Entries in **K** are applied via the dot product to all weighted lanes in the matrix $\mathbf{W}^T\mathbf{X}$.

3.) Entries in **K** denote which lanes should receive priority.

4.) **K** values reset to zero at each signal period, or, time $t = T$, for the lanes that were active in the last period (light cycle).

5.) Let $\mathbf{K}_i$ denote the priority/anti-stalling matrix **K** for intersection i.

$$k_n = t_e, n \in [1, 8]$$
Where t_e is time elapsed since road activated.

Applying Priority to Weighted Edges

$$\mathbf{W}^T\mathbf{X} = \begin{bmatrix} w_{1l} & w_{2l} & w_{3l} & w_{4l} & w_{5l} & w_{6l} & w_{7l} \\ w_{1c} & w_{2c} & w_{3c} & w_{4c} & w_{5c} & w_{6c} & w_{7c} \\ w_{1r} & w_{2r} & w_{3r} & w_{4r} & w_{5r} & w_{6r} & w_{7r} \end{bmatrix}_{3\times 8} \begin{bmatrix} x_{1l} & x_{1c} & x_{1r} \\ x_{2l} & x_{2c} & x_{2r} \\ x_{3l} & x_{3c} & x_{3r} \\ x_{4l} & x_{4c} & x_{4r} \\ x_{5l} & x_{5c} & x_{5r} \\ x_{6l} & x_{6c} & x_{6r} \\ x_{7l} & x_{7c} & x_{7r} \\ x_{8l} & x_{8c} & x_{8r} \end{bmatrix}_{8\times 3} = \begin{bmatrix} x_{1l}w_{1l} & x_{1c}w_{1c} & x_{1r}w_{1c} \\ x_{2l}w_{2l} & x_{2c}w_{2c} & x_{2r}w_{2r} \\ x_{3l}w_{3l} & x_{3c}w_{3c} & x_{3r}w_{3r} \\ x_{4l}w_{4l} & x_{4c}w_{4c} & x_{4r}w_{4r} \\ x_{5l}w_{5l} & x_{5c}w_{5c} & x_{5r}w_{5r} \\ x_{6l}w_{6l} & x_{6c}w_{6c} & x_{6r}w_{6r} \\ x_{7l}w_{7l} & x_{7c}w_{7c} & x_{7r}w_{7r} \\ x_{8l}w_{8l} & x_{8c}w_{8c} & x_{8r}w_{8r} \end{bmatrix}_{8\times 3}$$

$$\mathbf{W}^T\mathbf{X} \odot \mathbf{K} = \begin{bmatrix} x_{1l}w_{1l} & x_{1c}w_{1c} & x_{1r}w_{1c} \\ x_{2l}w_{2l} & x_{2c}w_{2c} & x_{2r}w_{2r} \\ x_{3l}w_{3l} & x_{3c}w_{3c} & x_{3r}w_{3r} \\ x_{4l}w_{4l} & x_{4c}w_{4c} & x_{4r}w_{4r} \\ x_{5l}w_{5l} & x_{5c}w_{5c} & x_{5r}w_{5r} \\ x_{6l}w_{6l} & x_{6c}w_{6c} & x_{6r}w_{6r} \\ x_{7l}w_{7l} & x_{7c}w_{7c} & x_{7r}w_{7r} \\ x_{8l}w_{8l} & x_{8c}w_{8c} & x_{8r}w_{8r} \end{bmatrix}_{8\times 3} \odot \begin{bmatrix} k_{1l} & k_{1c} & k_{1r} \\ k_{2l} & k_{2c} & k_{2r} \\ k_{3l} & k_{3c} & k_{3r} \\ k_{4l} & k_{4c} & k_{4r} \\ k_{5l} & k_{5c} & k_{5r} \\ k_{6l} & k_{6c} & k_{6r} \\ k_{7l} & k_{7c} & k_{7r} \\ k_{8l} & k_{8c} & k_{8r} \end{bmatrix}_{8\times 3} = \begin{bmatrix} x_{1l}w_{1l}k_{1l} & x_{1c}w_{1c}k_{1c} & x_{1r}w_{1c}k_{1r} \\ x_{2l}w_{2l}k_{2l} & x_{2c}w_{2c}k_{2c} & x_{2r}w_{2r}k_{2r} \\ x_{3l}w_{3l}k_{3l} & x_{3c}w_{3c}k_{3c} & x_{3r}w_{3r}k_{3r} \\ x_{4l}w_{4l}k_{4l} & x_{4c}w_{4c}k_{4c} & x_{4r}w_{4r}k_{4r} \\ x_{5l}w_{5l}k_{5l} & x_{5c}w_{5c}k_{5c} & x_{5r}w_{5r}k_{5r} \\ x_{6l}w_{6l}k_{6l} & x_{6c}w_{6c}k_{6c} & x_{6r}w_{6r}k_{6r} \\ x_{7l}w_{7l}k_{7l} & x_{7c}w_{7c}k_{7c} & x_{7r}w_{7r}k_{7r} \\ x_{8l}w_{8l}k_{8l} & x_{8c}w_{8c}k_{8c} & x_{8r}w_{8r}k_{8r} \end{bmatrix}_{8\times 3}$$

Model: ILP Expression

Varibles: $x_{ij} \in \mathbf{X}, 1$ iff allowed to travel, 0 otherwise.

Objective: For all 4-way intersections, $\sum_{i=1}^{8} \sum_{j=1}^{3} x_{ij} w_{ij} k_{ij} \rightarrow$ maximize

Where $x_{ij} w_{ij} k_{ij} \in \mathbf{W}^T \mathbf{X} \odot \mathbf{K}$.

$x_{2k-1,1} + x_{2l-1,1} \leq 1$, for all $k, l \in \{1, 2, 3, 4\}$ and $k \neq l$.

$x_{2k-1,1} + x_{2l-1,2} \leq 0$, for all $k, l \in \{1, 2, 3, 4\}$ and $k \neq l$.

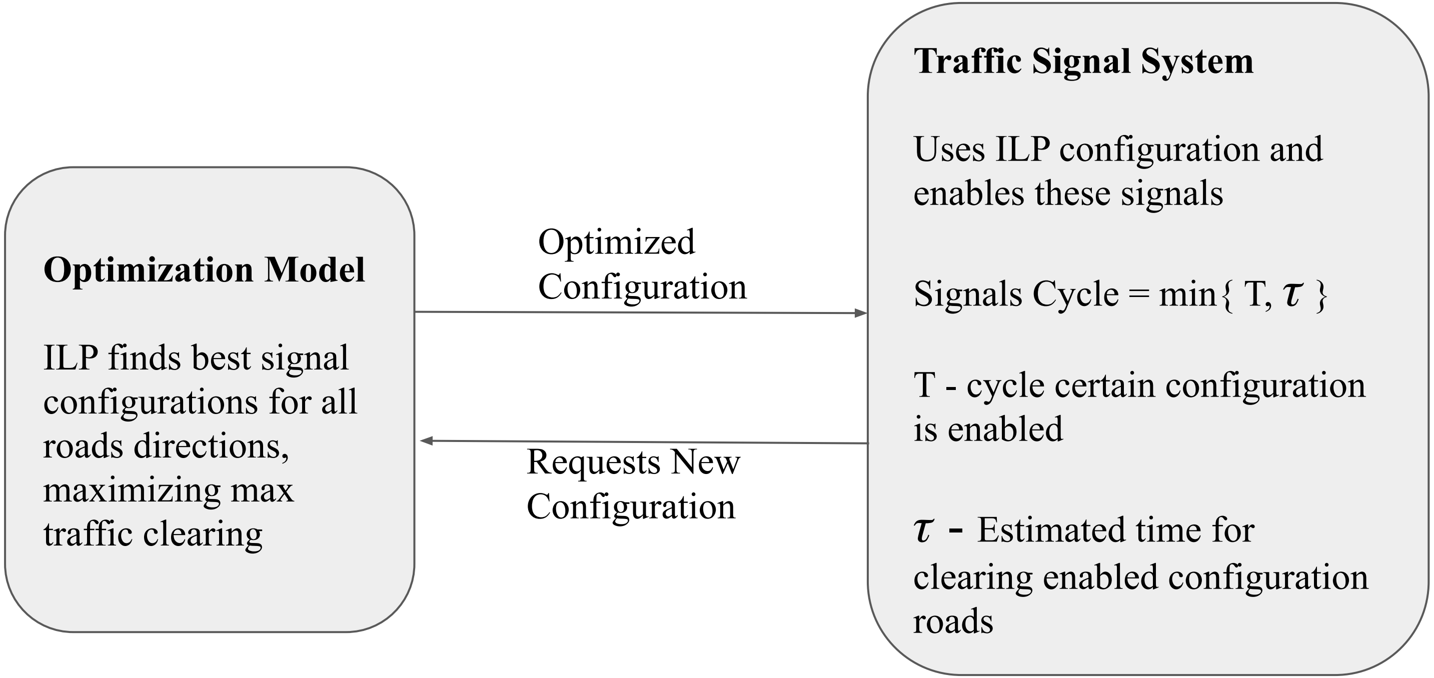
$x_{2k-1,3} + x_{2l-1,2} \leq 0$, for all $k, l \in \{1, 2, 3, 4\}$ and $k \neq l$.

$\sum_{i=1}^{3} x_{2k-1,i} + x_{2l-1,i} \leq 1$, for all $k, l \in \{1, 2, 3, 4\}$ and $k \neq l$

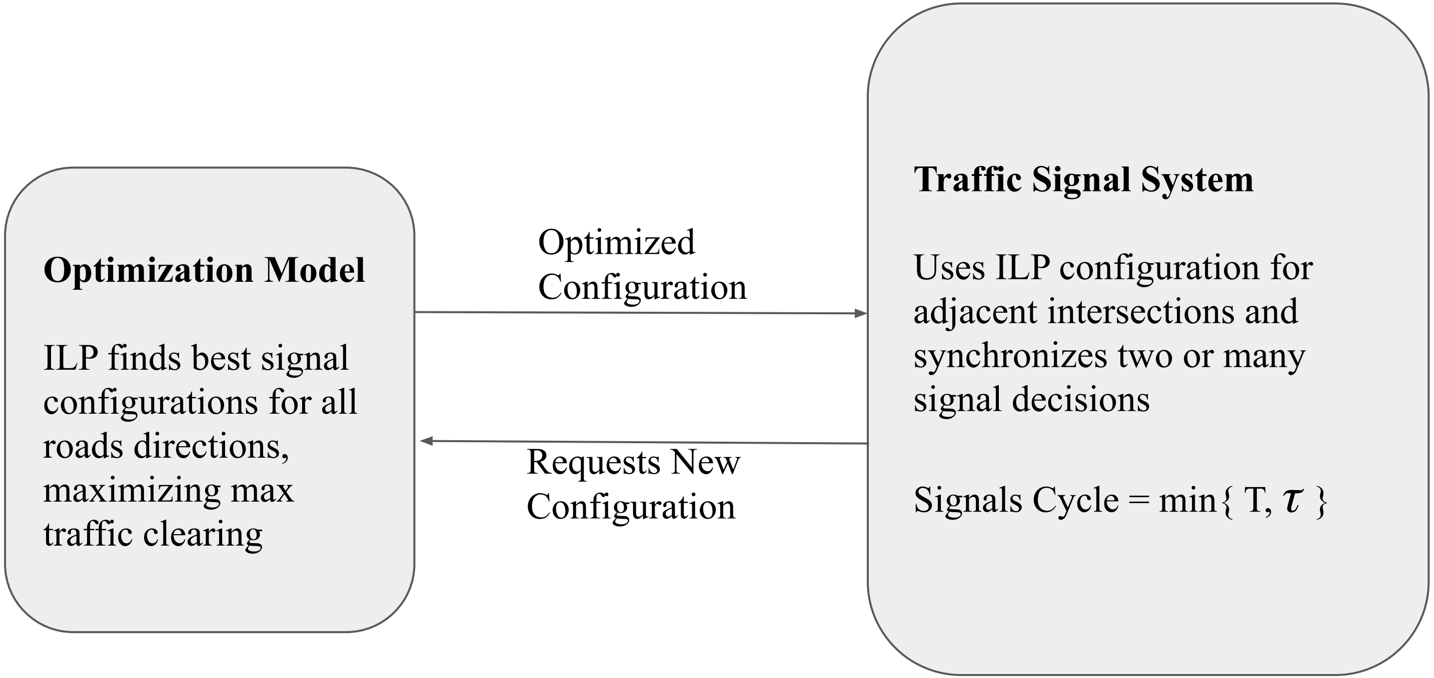
such that $\{k, l\} = \{1, 3\}$ or $\{k, l\} = \{2, 4\}$.

$x_{2k,i} = 1$ for all k $\in \{1, 2, 3, 4\}, i \in \{1, 2, 3\}$

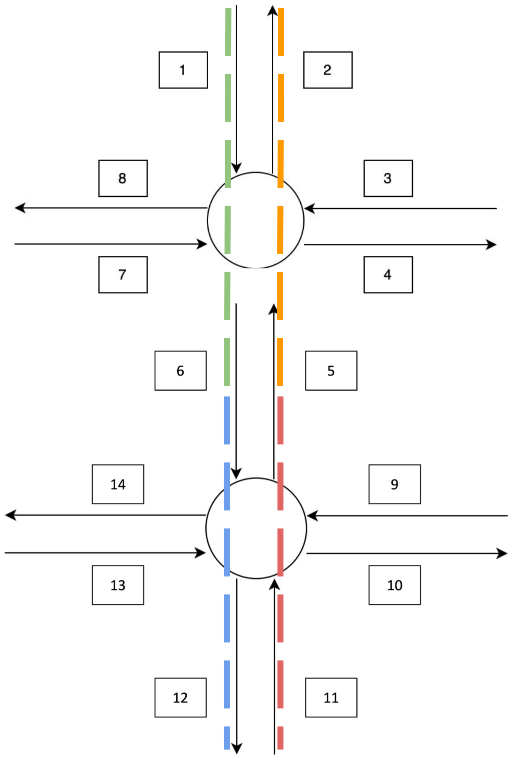
System Design



Synchronization System Design



Synchronization with 2 Intersections



Serial Synchronization

Optimal configuration at intersection 1
x1 = [0 1 0]
x6 = [1 1 1]
x5 = [0 1 0]
x2 = [1 1 1]

These above configuration can inform the ILP strictly enforcing constraints as below
x6 = [0 1 0] or [1 1 1]
x5 = [0 1 0]

Such that the intersection 2 can align with adjacent intersection 1's decisions and optimize its own traffic flow accordingly

Next steps

- ILP program implementation for 1 intersection
- Extend the program to a 2 intersection model
- Simulate with current sequential signal system
- Simulate with maximization ILP program