

9/3/23

How do you do optimization on a computer?

It is slightly different from what we do with our hand

How to write it as a program?

Forward methods, central difference

~~Given a point, whether it is~~

- Find min of  $J(w)$
- First, a crude approach to find bounds (2 values where you select to get a minimum)

This is called bracketing approach

- <sup>use a more</sup> sophisticated method to find min

In general:

- ID initial guess & function values  $\#$
- Make appropriate guess for  $w$  & change its deriv.

- Bracketing method

- Exhaustive search

- Bounding plane

- Region elimination approach

- interval halving

- Fibonacci search

- golden search ratio

- Gradient based ones

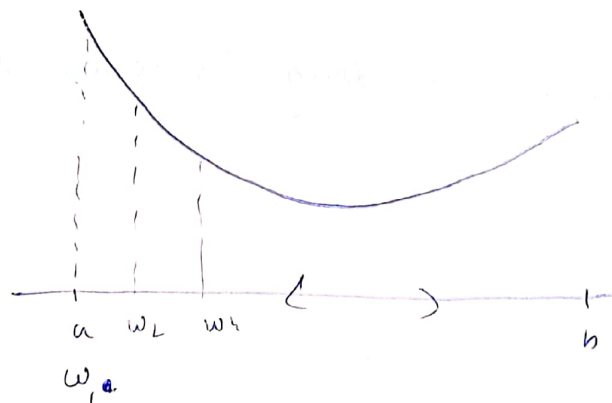
- Newton Rayleighson

- Bisection

- Secant

## Exhaustive search method

- Let  $n$  be no. of intermediate points



Take  $a$  &  $b$  &  
~~the~~ divide into  
 $n$  equal parts

We hope that  
 between some  
 2 values, we get  
 a minimum

Step 1:  $\Delta w_0 = \frac{b-a}{n}$

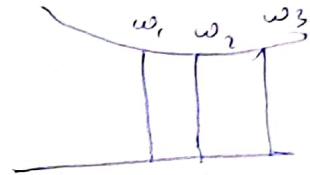
$$w_1 = a, w_L = w_1 + \Delta w, w_H = w_L + \Delta w$$

$a$  &  $b$  are the initial guesses. We are doing a coarse approximation. We aren't even looking at  $10^{-5}$  or  $10^{-6}$  precision. Once we bracket, we will perform some other method.

• Step 2:

$J(w_1) \leq J(w_2) \leq J(w_3)$ , then

min lies between  $w_1$  &  $w_3$ .  
We don't take  $w_2$  as it can be on either side of min.



• Else:

•  $w_1 = w_2$ ,  $w_2 = w_3$ ,  $w_3 = w_2 + \Delta w$

• Go to step 3

• Step 3:

• If  $w_3 \leq b$ , then go to step 2

• otherwise, no min between  $a$  &  $b$

• Min could be the boundary point to  $a$

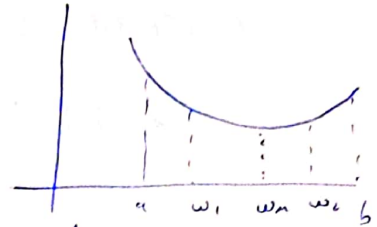
We can approach minima by doing this recursively.

For

## Region Elimination method.

### Overall idea

- $w_1 \leq w_2$ , if  $J(w_1) \geq J(w_2)$
- Remove region  $(a, w_1)$



we get  $a$  &  $b$  from bounds of exhaustive search / some initial values of  $a$  &  $b$

### Step 1

- choose  $a, b$ , ~~and~~  $w_m = \frac{a+b}{2}$ ,  ~~$J(w_m)$~~   ~~$J(w_1)$~~   ~~$J(w_2)$~~

Step 2:

Set:  $w_1 = a + \frac{L}{4}$ ,  $w_2 = b - \frac{L}{4}$  [ 4 parts ]

Step 3 • If  $J(w_1) < J(w_m)$ , then it is within  $(a, w_m)$ .

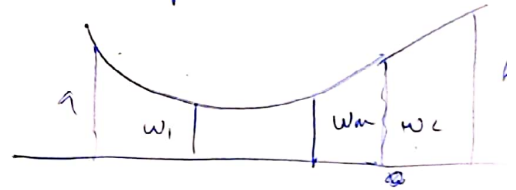
Remove  $(w_m, b)$

if Set  ~~$b \rightarrow w_m$~~   ~~$w_m \rightarrow w_1$~~ , go to step 5.

~~$w_m \rightarrow w_1$~~   
 $w_m \rightarrow w_1$   
 $a \rightarrow a$

• Else

• go to step 4,



Step 4:

- If  $J(w_2) < J(w_m)$

Definitely not between  $a < w_m$

• Set

$a \rightarrow w_m$

$w_m \rightarrow w_2$

$b \rightarrow b$

go to step 5

• Else:

- $a = w_1$ ,  $b = w_2$ ,

go to step 5

Step 5:

- Calculate  $L = b - a$

- If  $|L| < \epsilon$

- Terminate

- Else

- goto step 2