SAGE CODE

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The following code was used to for calculating the proportion of *enemy primes* for elliptic curves with CM.

```
eList = ['27a2', '36a4' , '49a4', '64a2']

# Get all primes from 5 to 47
pList = prime_range(50)
pList = pList[2:]

# Get all primes below 200,000,000
qList = prime_range(2e8)

numQ = len(qList)

for eid in eList:
    print(f'\nElliptic curve: {eid}\n')
    for p in pList:
        E = EllipticCurve(eid)
        N = E.conductor()

        # qEnemy will contain all the enemy primes for p
        qEnemy = []

        numQModP = 0

        for q in qList:
            # Only proceed if q = 1 (mod p)
            if q % p == 1:
                numQModP += 1

                # q is an enemy prime if it either
                # a) q divides N and the AP value is 1, or
                if N % q == 0:
```

```
            if E.ap(q)==1:
                qEnemy.append(q)
        # b) q does not divide N and
        # p divides the size of the F_q points on the reduced curve.
        else:
            if (q + 1 - E.ap(q)) % p == 0:
                qEnemy.append(q)


    print(f'p={p} {len(qEnemy)/numQ} {len(qEnemy)/numQModP}')
```

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