Week 6

Section 4.1

1. If $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$, what is A^n. Find a matrix B such that $AB = I_2$.
2. If $A = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$, what are all the possible combinations of products of A and B?

Section 4.2

1. Use the definition of linear transformation to verifyy whether the given transformation T is linear. If liner, find the matrix A such that $T(\mathbf{v}) = A\mathbf{v}$ for each $\mathbf{v}$
   (a) $T(x_1) = |x_1|$
   (b) $T(x_1, x_2) = (x_2, x_1)$
   (c) $T(x_1, x_2) = 2x_1 + 3x_2$
   (d) $T(x_1, x_2, x_3) = (x_2 - x_3, x_1 + 1, x_2)$
2. Recall the rotation matrix introduced in class.
   (a) Use trigonometry to prove that for any two angles θ, ϕ,
   $$\mathrm{Rot}_\theta \circ \mathrm{Rot}_\phi = \mathrm{Rot}(\theta + \phi).$$
   Find a geometric interpretation of this fact.
   (b) Consider the matrix $A = \mathrm{Rot}_{\pi/3}$. Calculate A^6 and A^3,
3. Determine the standard matrices for the following linear transformations
   (a) Reflection along x-axis
   (b) Reflection along y-axis
   (c) Reflection across the line $x = y$
   (d) Reflection across the line $x = -y$