Week 2

Section 2.4

1. Consider the two vectors: $\mathbf{a} = [1, 2, 3]$ and $\mathbf{b} = [4, 5, 6]$. What is the area of the parallelogram P spanned by these two vectors?

2. Convince yourself geometrically (no need to write out a formal proof) that

$$\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c}) = (\mathbf{a} \times \mathbf{b}) \cdot \mathbf{c} = \det(\mathbf{a}\ \mathbf{b}\ \mathbf{c}).$$

3. Evaluate

   (a) $\mathbf{a} \times \mathbf{a}$

   (b) $(\hat{\mathbf{i}} + \hat{\mathbf{j}}) \times [(\hat{\mathbf{i}} - \hat{\mathbf{j}}) \times (\hat{\mathbf{i}} + \hat{\mathbf{k}})]$

   (c) $(\hat{\mathbf{i}} \times \hat{\mathbf{j}}) \times \hat{\mathbf{k}}$

   (d) $\hat{\mathbf{i}} \times (\hat{\mathbf{j}} \times \hat{\mathbf{k}})$

   (e) $(\hat{\mathbf{i}} \times \hat{\mathbf{j}}) \times \hat{\mathbf{j}}$

   (f) $\hat{\mathbf{i}} \times (\hat{\mathbf{j}} \times \hat{\mathbf{j}})$

4. Find the unit vectors that are perpendicular to both $[1, 2, 1]$ and $[3, -4, 2]$

5. Find a vector $\mathbf{n}$ that is perpendicular to the plane determined by the points $P(1, 2, 3)$, $Q(-1, 3, 2)$, $R(3, -1, 2)$, and find the area of the triangle.

6. Given the points $O(0, 0, 0)$, $P(1, 2, 3)$, $Q(1, 1, 2)$, $R(2, 1, 1)$, find the volume of the parallelepiped with edges $\overline{OP}$, $\overline{OQ}$, and $\overline{OR}$.

7. Mark True or False

   (a) Dot Product of two unit vectors is again a unit vector.

   (b) Cross Product of two unit vectors is again a unit vector.

   (c) Dot Product of a vector with itself is equal to the square of its length.

   (d) Cross Product of a vector with itself is equal to the square of the same vector.

   (e) Cross product of two vectors $\mathbf{a}$ and $\mathbf{b}$ is equal to the determinant of the vector $\mathbf{a}$ and $\mathbf{b}$.

(f) For any two vectors **a** and **b**,

$$|\mathbf{a} \times \mathbf{b}| = |\mathbf{b} \times \mathbf{a}|.$$

(g) For any two vectors **a** and **b**,

$$|\mathbf{a} \times \mathbf{b}| \cdot \mathbf{a} = 0.$$

(h) For any three vectors **a**, **b** and **c**,

$$(\mathbf{a} \times \mathbf{b}) \cdot \mathbf{c} = \mathbf{a} \times (\mathbf{b} \times \mathbf{c}).$$

(i) For any three vectors **a**, **b** and **c**,

$$(\mathbf{a} \times \mathbf{b}) \cdot \mathbf{c} = \mathbf{a} \cdot (\mathbf{b} \times \mathbf{c}) = (\mathbf{c} \times \mathbf{a}) \cdot \mathbf{b}.$$

(j) If $\mathbf{a} \cdot \mathbf{b} = 0$, then either $\mathbf{a} = 0$ or $\mathbf{b} = 0$.

(k) If **a** and **b** are parallel vectors, then their cross product is 0.

(l) If **a** and **b** are parallel vectors, then

$$\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}||\mathbf{b}|.$$

(m) If **a** and **b** are parallel vectors, then

$$|\mathbf{a} \times \mathbf{b}| = |\mathbf{a}||\mathbf{b}|.$$

(n) If **a**, **b**, and **c** are three non-zero vectors, such that $\mathbf{a} \times \mathbf{b} = \mathbf{a} \times \mathbf{c}$, then $\mathbf{b} = \mathbf{c}$.

(o) If **a**, **b**, and **c** are three non-zero vectors, such that $\mathbf{a} \cdot \mathbf{b} = \mathbf{a} \cdot \mathbf{c}$, then $\mathbf{b} = \mathbf{c}$.

Section 2.5

8. Find equation for the plane which cuts the x-axis at 4, y-axis at 5, and z-axis at 3.

9. Find the parametric form of the line passing through $\langle 2, -4, 3\rangle$ and perpendicular to the plane $x + 4y - 2z = 5$.

10. Find an equation for the plane passing through $\langle 1, 1, 1\rangle$ and parallel to the plane $2x + 3y + z = 5$.

11. Find the equation of a plane passing through $P(1, 1, 5)$ and perpendicular to the vector $(2, 0, 1)$.

12. Consider the plane W and a point Q on the plane. Denote the vector normal to the plane by **n**. Let P be some point in space. We will find the distance of the point P from the plane W using the following steps.

    (a) Draw a picture for this question.

    (b) Write the vector $\mathbf{v} = P - Q$.

    (c) Notice that the distance of the point P from the plane *is the same* as the *length* of the projection of the vector **v** along the vector **n**.

    (d) Use the projection formula to get the final answer.