E9 241 Digital Image Processing

Assignment 03

Due Date: October 26, 2025 - 11:59 pm **Total Marks:** 60

Instructions:

For all the questions, write your own functions. Use library functions for comparison only.

- Your function should take the specified parameters as inputs and output the specified results.
- Also provide the wrapper/demo code to run your functions. Your code should be self-contained, i.e., one should be able to run your code as is without any modifications.
- For Python, if you use any libraries other than numpy, scipy, scikit-image, opencv, pillow, matplotlib, pandas, and default modules, please specify the library that needs to be installed.
- Along with your code, also submit a PDF with all the **results** (images or numbers) and **inferences** (very important: you may not be explicitly asked to give inferences in each question. You should always include your inferences from what you have observed). Include answers to subjective questions, if any.
- Put all your files (code files and a report PDF) in a single zip file and submit the zip file. Name the zip file with your name.

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1. **Directional Filtering:**

   Directional filtering is used to emphasize or suppress frequency components along specific orientations in the frequency domain. For an $M \times N$ centered Discrete Fourier Transform (DFT) spectrum, the angle of each frequency component relative to the center of the spectrum is given by:

$$\theta(u,v) = \tan^{-1}\left(\frac{v - \frac{N}{2}}{u - \frac{M}{2}}\right)$$

   A directional filter $H(u,v)$ can then be defined as:

$$H(u,v;\theta_{\min},\theta_{\max}) = \begin{cases} 1, & \text{if } \theta_{\min} \le \theta(u,v) \le \theta_{\max} \\ 0, & \text{otherwise} \end{cases}$$

   Here, $\theta_{\min}$ and $\theta_{\max}$ specify the angular range of frequencies to be retained.

   (a) Generate an image x of size $M \times M$ ($M = 256$) using three sinusoidal components:

$$x_1(m,n) = \sin\left(\frac{2\pi \cdot 12\, m}{M}\right),$$
$$x_2(m,n) = \sin\left(\frac{2\pi \cdot 8\, n}{M}\right),$$
$$x_3(m,n) = \sin\left(\frac{2\pi (6\, m + 10\, n)}{M}\right),$$
$$x(m,n) = \frac{x_1(m,n) + x_2(m,n) + x_3(m,n)}{3}.$$

   Compute the centered 2D DFT of $x(m,n)$. Plot the results in a single figure showing:

   i. The image of the sinusoidal component $x_1(m,n)$

ii. The image of the sinusoidal component $x_2(m,n)$
iii. The image of the sinusoidal component $x_3(m,n)$
iv. The combined image $x(m,n)$
v. The magnitude of the 2D DFT of the combined image

Explain the contribution of each individual sinusoid to the combined image magnitude spectrum.

(b) Design directional filters of size $M \times M$ ($M = 256$) for given angular ranges using the expression above. Apply each filter to the DFT of the combined image ($X(u,v)$) reconstruct the filtered images using the inverse DFT, visualize the results and comment on the observations. For each directional filter, display the following in a single figure:

i. The original image.
ii. The original image magnitude spectrum.
iii. The directional filter magnitude spectrum.
iv. The filtered magnitude spectrum.
v. The reconstructed filtered image.

Use the following filter definitions in your implementation:

$$\begin{aligned}
H_1(u,v) &= H(u,v;-20^\circ,20^\circ),\\
H_2(u,v) &= H(u,v;70^\circ,110^\circ),\\
H_3(u,v) &= H(u,v;25^\circ,65^\circ),\\
H_4(u,v) &= \max\big(H_1(u,v),H_2(u,v),H_3(u,v)\big), \quad \text{element-wise max } \forall u < M,\ \forall v < M
\end{aligned}$$

(c) Compute the Mean Squared Error (MSE) between the original and each filtered output and comment on their values.

Note:

- For efficient 2D DFT/IDFT computation built-in functions can be used.
- Take care of unit conversions (degrees to radians) appropriately when necessary.
- **Visualization tip:** Since most images have frequency spectra where the majority of components have low magnitude (appearing very dark), the magnitude of the frequency response $H(u,v)$ can be better visualized (strictly only for plots) by applying an increasing non-linear transformation:

$$H_{\text{vis}}(u,v) = \log\left(1 + |H(u,v)|^{1/n}\right), \quad n \in \mathbb{N}.$$

(5+20+5=**30 Marks**)

Gaussian Blurring and Inverse Filtering:

Gaussian blurring is a common technique to smooth an image and attenuate high-frequency components. A Gaussian kernel of size $k \times k$ and standard deviation σ is defined as:

$$G(x,y) = K\exp\left(-\frac{x^2+y^2}{2\sigma^2}\right), \quad \text{with } K \text{ chosen such that } \sum_{x,y} G(x,y) = 1$$

(a) Read the image `buildings.jpg`. Design a Gaussian kernel of size 13×13 with standard deviation $\sigma = 2.5$. Apply the blur in the frequency domain and reconstruct the blurred image using the inverse DFT.

(b) For the Gaussian kernel compute and plot in a single figure and comment on your observations:

i. Its centered 2D DFT magnitude spectrum (13×13-point DFT).

ii. The inverse centered magnitude spectrum $1/(|H(u,v)| + \epsilon)$, with $\epsilon = 10^{-3}$.

iii. The centered 2D DFT magnitude spectrum (1036×1036-point DFT).

iv. The inverse centered magnitude spectrum $1/(|H(u,v)| + \epsilon)$.

(c) Gaussian frequency response fit:

i. The frequency-domain representation of a Gaussian kernel can be modeled as:

$$H_{\text{cont}}(u,v) = \exp\Big(-k\,(U^2 + V^2)\Big), \quad U = u - \frac{M-1}{2},\ V = v - \frac{N-1}{2}$$

where k is a parameter to be optimized to best fit the magnitude response of a given kernel, $H_{\text{cont}}(u,v)$ is the centered magnitude response and $M \times N$ are the dimensions of the DFT.

ii. Sweep k over the range 10^{-6} to 10^{-3} and find the value that minimizes the error:

$$\min_k \sum_{u,v} \Big| H_{\text{cont}}(u,v) - |H_{\text{DFT}}(u,v)| \Big|^2$$

where $H_{\text{DFT}}(u,v)$ is the previously computed 1036×1036-point centered DFT of the kernel.

iii. Report the optimized k_{opt} obtained from the previous sweep and plot in a single figure the magnitude spectrum of the Gaussian fit $|H_{\text{cont}}(u,v)|$ along with its inverse $1/(|H_{\text{cont}}(u,v)| + \epsilon)$.

(d) Restore the original image by applying the previously computed inverse responses (both from the direct kernel DFT and the Gaussian fit) and reconstruct the images using the inverse DFT. Plot in a single figure and compare:

i. The original image.

ii. The restored image using the direct kernel inverse response.

iii. The restored image using the Gaussian fit inverse response.

Compute and report the Mean Squared Error (MSE) between the original image and each reconstructed image and comment on which method gives better restoration and why.

Notes:

- Plot all magnitude responses in the frequency domain and ensure they are centered.
- Apply inverse filters only in the frequency domain to reconstruct images.
- Take care of proper padding to avoid circular convolution artifacts.

(5+10+10+5=**30 Marks**)