Pricing: Auto-callable Multi-Barrier Reverse Convertible

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1 Product Description

The auto-callable barrier reverse convertible (AMBRC) security offers periodic coupon payments on quarterly basis until maturity. Specifically, the AMBRC in question pays an annual coupon of 2.50% on 1000 CHF of principal. Therefore, the issuing party makes a payment of 6.25 CHF quarterly to the buyer of the contract. The contract has five observation dates where the contract could be early redeemed based on the barrier events with the underlying assets. The AMBRC has three underlying assets, EURO STOXX 50 (SX5E), S&P 500 (SPX), and Swiss Market Index (SMI). The following table illustrates the final barrier levels and initial fixing levels of the respective underlying assets.

Bloomberg Ticker	Initial Fixing Level	Barrier Level
EURO STOXX 50 (SX5E)	3857.07 (EUR)	2275.67(EUR)
S&P 500 (SPX)	3917.45 (USD)	2311.30 (USD)
Swiss Market Index (SMI)	6566.87 (CHF)	11130.28 (CHF)

The following table shows the coupon payment dates in which the coupon amount is paid to the buyer in the settlement currency of Swiss Franks (CHF).

Coupon Payment dates
19/07/2021
19/10/2021
19/01/2022
21/04/2022
19/07/2022
19/10/2022
19/01/2023
19/04/2023

1.1 Autocallable Observations and Trigger Levels

The following table illustrates the autocall observation date, the consequent trigger levels per each observation date, and the early redemption dates following the observation date.

	Observation Date	Trigger Level	Early Redemption date
1	12/04/2022	95.00%	21/04/2022
2	12/07/2022	85.00%	19/07/2022
3	12/10/2022	75.00%	19/10/2022
4	12/01/2023	65.00%	19/01/2023
5	12/04/2023	59.00%	19/04/2023

Note that the Trigger levels are expressed as a percentage of the initial Fixing level, the last Observation Date is the Final Fixing Date, and last Early redemption is the Redemption Date.

1.2 Redemption and Barrier Events

On the autocall observation day, we observe the closing price of the underlying assets. If all underlying assets are above their respective barrier levels for the autocall observation day, and Early Redemption will occur and the product will expire immediately but the issuer pays the coupon and the Denomination of 1000 (CHF) on the consequent Redemption date. After Early Redemption, no futher payments will be made by the issuer. If no barrier event happens at the last autocall observation day, the investor will receive the denomination and coupon payment. If a barrier event occurs on the last autocall observation day, the investor will receive the coupon and the worst performance times the denomination. The worst performance is determining the underlying asset with the smallest ratio of final fixing level divided by the initial fixing level.

1.3 Pricing Formulation

Let C be the quarterly coupon payment amount, r be the Swiss risk free rate, t_i be the time in years between coupon payment dates and the initial fixing day. Let B_i represent a event that no underlying asset goes below its respective barrier level on the autocall observation day. Let $I(\cdot)$ be the indicator function that returns 1 if a event occurs and 0 if the event doesn't occur. Note that if early redemption occurs, or B_i is True for some i, no further payments are made on the subsequent coupons dates. Thus let E_i represent the event that no early redemption happened before the $(i-3)^{th}$ observation day. Let W be the lowest ratio of the final fixing level divided by initial fixing level of all underlying assets. Below, we have the payoff of this derivative as shown in (1).

$$P = \left(\sum_{i=1}^{3} C \right) + \left(\sum_{i=4}^{7} I(E_{i-3}) \left(C + I(B_i) D \right) \right) + I(E_5) \left(C + D \cdot W + I(B_8)(D - D \cdot W) \right) \tag{1}$$

After discounting the cash flows, we get a fair evaluation of the price of the derivative as shown in shown in (2).

$$V = \left(\sum_{i=1}^{3} C e^{-rt_i} \right) + \left(\sum_{i=4}^{7} I(E_{i-3}) e^{-rt_i} \left(C + I(B_i) D \right) \right) + I(E_5) e^{-rt_8} \left(C + D \cdot W + I(B_8)(D - D \cdot W) \right) \tag{2}$$

2 Historical Estimates

We estimate the parameters for the geometric brownian motion by pulling stocker data of the underlying assets of the AMCR. Thus, we pull ticker symbols, EURO STOXX 50 (STOXX50E), S&P 500 (GSPC), and Swiss Market Index (SSMI) from the pandas datareader package in Python. Specifically, we extract the closing prices of 3 year time horizon starting from January 1^{st}, 2018 to April 9^{th}, 2021 for each underlying asset.

First, we consider the closing prices and calculate the log returns as shown in (3).

$$R_{i+1} = log \left(1 + \frac{X_{i+1} - X_i}{Xi} \right) \tag{3}$$

We calculate one-day returns, μ by calculating mean and one-day standard deviations, σ for each asset. Thereafter, the returns are annualized by multiplying μ by 252 and multiplying σ by $\sqrt{252}$. The annualized dividend yields are pulled from various sources as referenced in citation section. Furthermore, we calculate the correlation matrix, Σ, of the returns from the underlying assets.

Underlying Asset	Log Returns	Volatility
EURO STOXX 50 (SX5E)	3.86%	21.24%
S&P 500 (SPX)	13.069%	22.92%
Swiss Market Index (SMI)	5.284%	16.99%

3 Monte Carlo Simulation

3.1 Stochastic Differential Equations

To price this contract, we will utilize the Monte Carlo method to simulate the price movements of the underlying assets for a 2 year time frame. We model the underlying assets with correlated geometric brownian motion as shown in the stochastic differential equation (SDE) below.

$$\frac{dS_t^1}{S_t^1} = (\mu_1 - \delta_1)dt + \sigma_1 W_t^1 \quad ; \quad \frac{dS_t^2}{S_t^2} = (\mu_2 - \delta_2)dt + \sigma_2 W_t^2 \quad ; \quad \frac{dS_t^3}{S_t^3} = (\mu_3 - \delta_3)dt + \sigma_3 W_t^3 \tag{4}$$

The solution to the SDEs of the three price movements are shown below:

$$S_t^1 = S_0^1 e^{(\mu_1 - \delta_1 - \frac{\sigma_1^2}{2})t + \sigma_1 W_t} \quad ; \quad S_t^2 = S_0^2 e^{(\mu_2 - \delta_2 - \frac{\sigma_2^2}{2})t + \sigma_2 W_t} \quad ; \quad S_t^3 = S_0^3 e^{(\mu_3 - \delta_3 - \frac{\sigma_3^2}{2})t + \sigma_3 W_t} \tag{5}$$

$$Z := [X_1, X_2, X_3] \text{ where } X_i \sim Normal(0,1) \text{ are independent normal samples}$$

$$\Sigma = CC^T \text{ where } \Sigma \text{ is the correlation matrix}$$

$$W = CZ \text{ where } W \text{ has a correlation matrix } \Sigma$$

Note that the brownian motions are correlated given the historical correlations. Now, we perform the simulations and calculate the discounted payoffs for each. The mean of the results will be a estimate of the fair price. The table below shows the fair price calculations and standard deviation of the price estimator.

Number of Simulations	Fair Price	Standard Error of Estimator
10^3	1007.15	0.05868
10^4	1008.50	0.005557
10^5	1008.30	0.00056858

3.2 Coupon Rate and Price

We cross compare the fair price of AMBRC for different coupon rates. As the coupon rate increases, the fair price increases linearly as shown in figure (1). Note that the price for each coupon rate is calculated on the same set of simulations of the underlying assets.

4 Variation Reduction Method

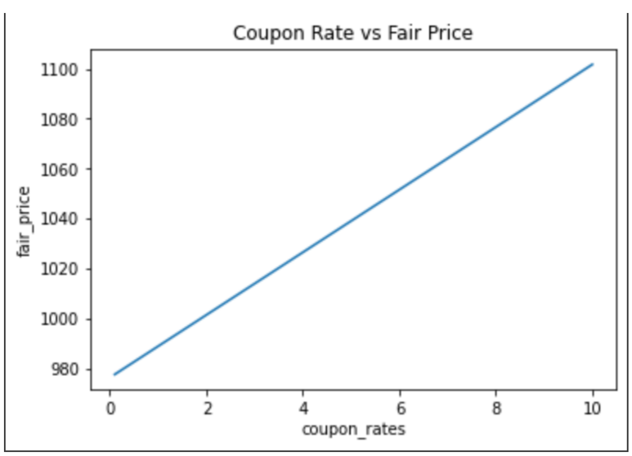

Figure 1: Graph of Coupon rate versus Price

To reduce variance, we use the antithetic variates method. First, we simulate the normal matrix with independent random samples in each columns. Provided the correlation matrix, we perform the cholesky decomposition and multiply with the normal matrix to get the correlated normal matrix. Note that with this method, we generate the normal matrix with half the simulation size, $\frac{N}{2}$. We take the conjugate of this new correlated normal matrix and append to old one. Now, we have correlated normal matrix with simulation size, N. The table down below holds the fair price and standard error of the estimator using the antithetic variates method.

Number of Simulations	Fair Price	Standard Error of Estimator
10^3	1008.61	0.05567
10^4	1009.39	0.005362
10^5	1007.59	0.000587

Appendix A References

EURO STOXX 50 dividend yield Hyperlink

S&P 500 dividend yield Hyperlink

SMI dividend yield Hyperlink