Linear Regression

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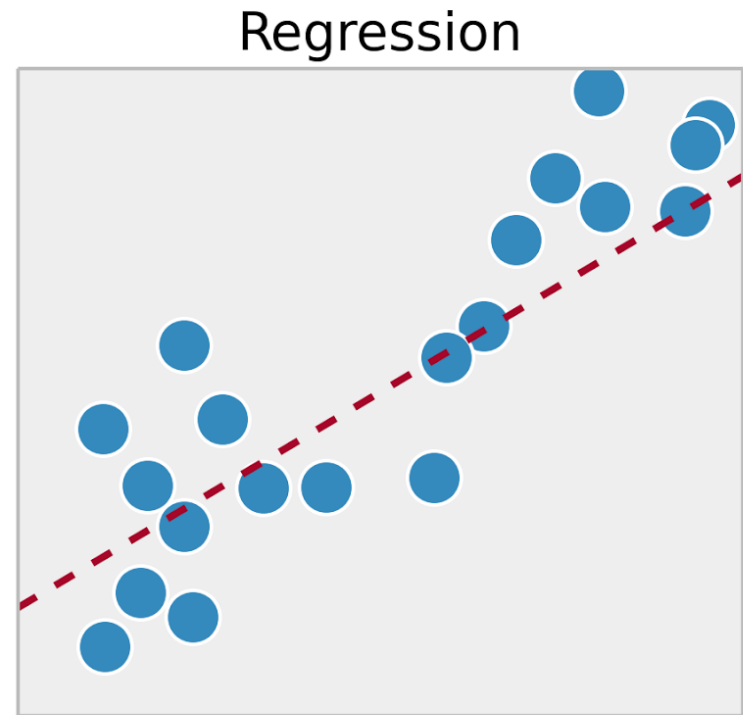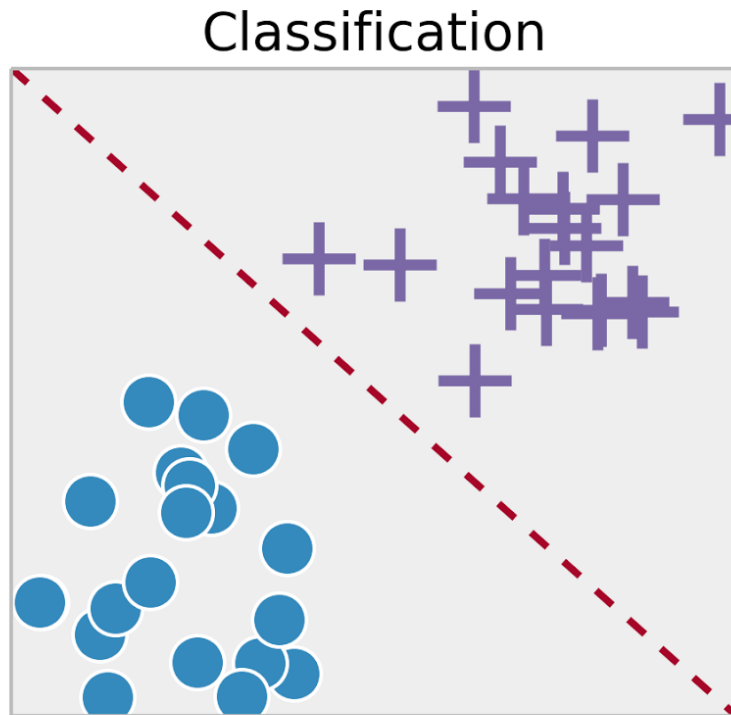
Centre for Excellence in Basic Sciences

Topics

- Supervised learning – Regression model
- Linear regression
- Least squares
- R^2
- Adjusted R^2
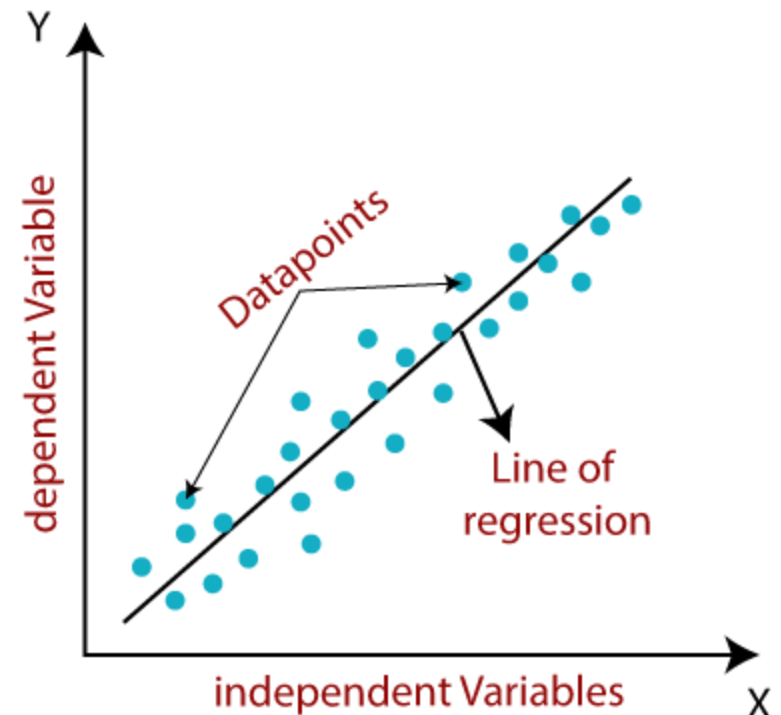- Applications
- Advantages
- Disadvantages

Supervised Learning

- Training set – { $(x^{(1)}, y^{(1)})$, $(x^{(2)}, y^{(2)})$, ..., $(x^{(m)}, y^{(m)})$}
- Labeled dataset
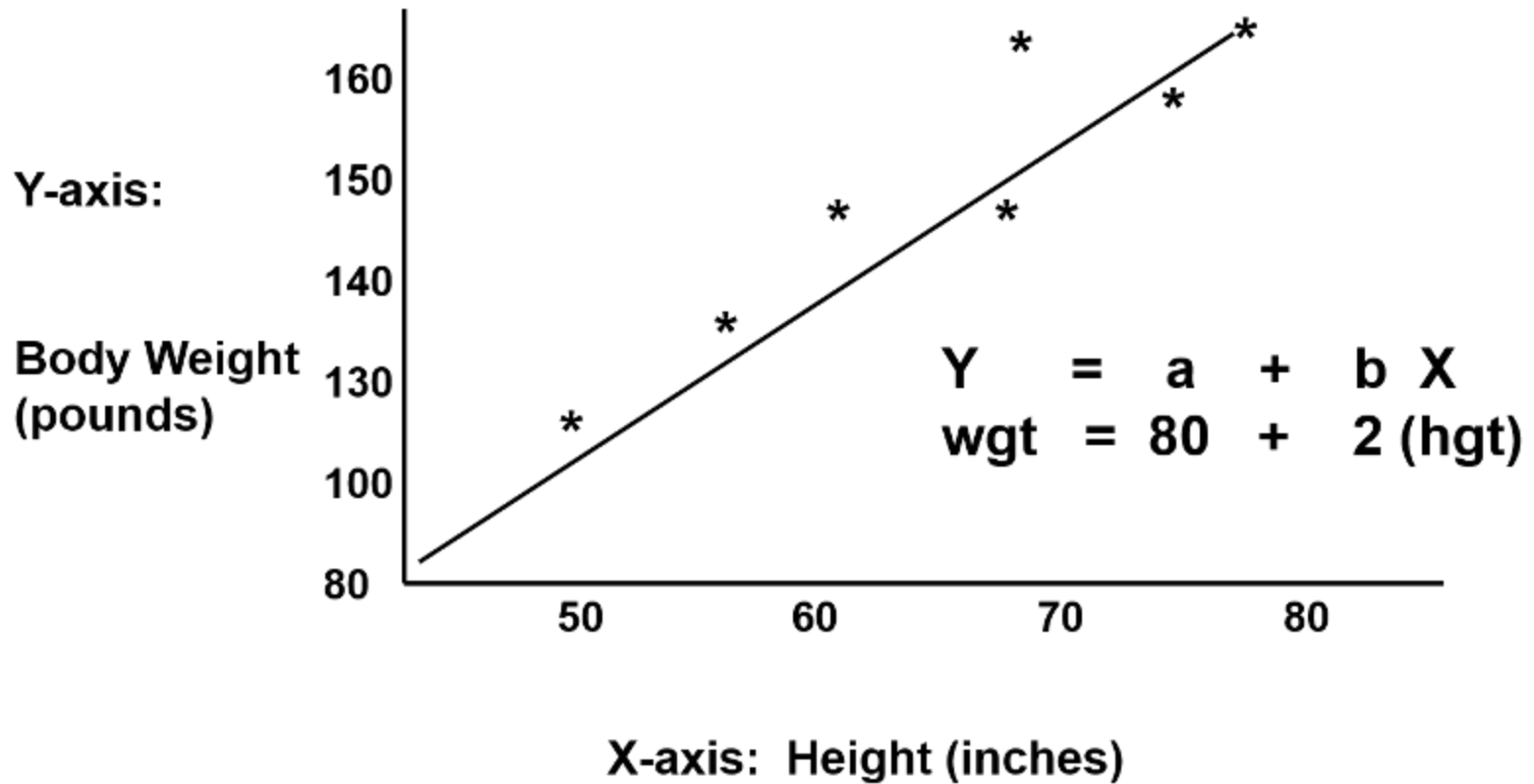

Classification   Regression

Linear Regression

- Independent variable – X axis

- Dependent variable – Y axis

- Data points – Samples

- Relationship

- Line of regression

Linear Regression

Linear Regression

- Input – X
  - Vector
  - $X \in R$
 - Dimension – n_X
- Output – y
  - Scalar
  - $y \in R$

Linear Regression

- Weights – W – $(W_1, W_2, W_3, ..., W_{nx})$
  - Vector
  - $W \varepsilon R$
  - Dimension – n_X
- Bias – b – W_0
  - Scalar
  - $b \varepsilon R$
- $y = W^T X + b$

Linear Regression

- Positive correlation
  - Independent variable increases
  - Dependent variable increases
- Negative correlation
  - Independent variable increases
  - Dependent variable decreases
- No correlation
  - Independent variable increases
  - Dependent variable no change

Linear Regression
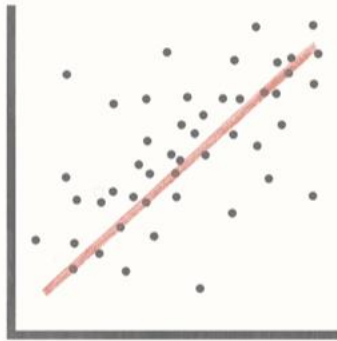
Positive Correlation
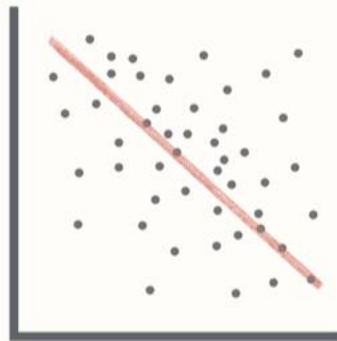
Negative Correlation
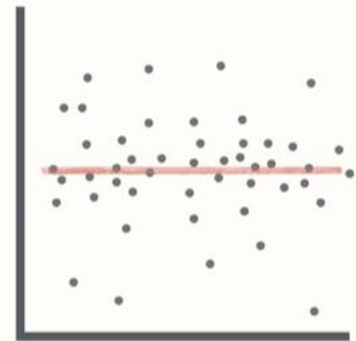
No Correlation

Linear Regression – Types

- Simple linear regression
  - Single independent variable
- Multiple linear regression
  - More than one independent variable

Least Squares

- Linear regression – Least squares
- Actual value – Observed value
- Estimated value – Predicted value
- Error value
- Sum of squared residuals
- Mean Squared Error – MSE
- Minimize error value
- Generic equation of line

Least Squares

$$\widehat{y}_i = W_0 + W_1 \times X_i$$

$$y_i = \widehat{y}_i + \varepsilon_i$$

$$J = \frac{1}{n_x} \sum_{i=1}^{n_x} \varepsilon_i^2$$

- X_i – Independent variable
- y_i – Dependent variable
- W_0 – Y axis intercept
- W_1 – Slope of line
- $\hat{y}_i$ – Estimated value
- ε_i – Random error
- Mean Squared Error – MSE
- J – Cost function – Minimize cost function

Loss Function

- Loss function
  - One sample – i^{th} sample
  - $L(y^{(i)}, \hat{y}^{(i)}) = (y^{(i)} - \hat{y}^{(i)})^2$
- Cost function
  - Average of loss function for all samples

$$J(W, b) = \frac{1}{n_x} \sum_{i=1}^{n_x} L(y^{(i)}, \hat{y}^{(i)})$$

$$J(W, b) = \frac{1}{n_x} \sum_{i=1}^{n_x} \left(y^{(i)} - \hat{y}^{(i)} \right)^2$$

Gradient Descent

- Input dataset – $\{ (x^{(1)}, y^{(1)}), (x^{(2)}, y^{(2)}), ..., (x^{(m)}, y^{(m)}) \}$

- Equations – $\hat{y} = W^T X + b$

- Loss function – $L(\hat{y}, y) = (y - \hat{y})^2$

- Cost function –
$$J(W, b) = \frac{1}{n_x} \sum_{i=1}^{n_x} L(y^{(i)}, \hat{y}^{(i)})$$

$$J(W, b) = \frac{1}{n_x} \sum_{i=1}^{n_x} \left(y^{(i)} - \hat{y}^{(i)} \right)^2$$

- Output
  - $\hat{y}^{(i)} \sim y^{(i)}$
  - W and b – Minimize $J(W, b)$

Gradient Descent

- Convex function
- Global optimum

$$J(w, b)$$
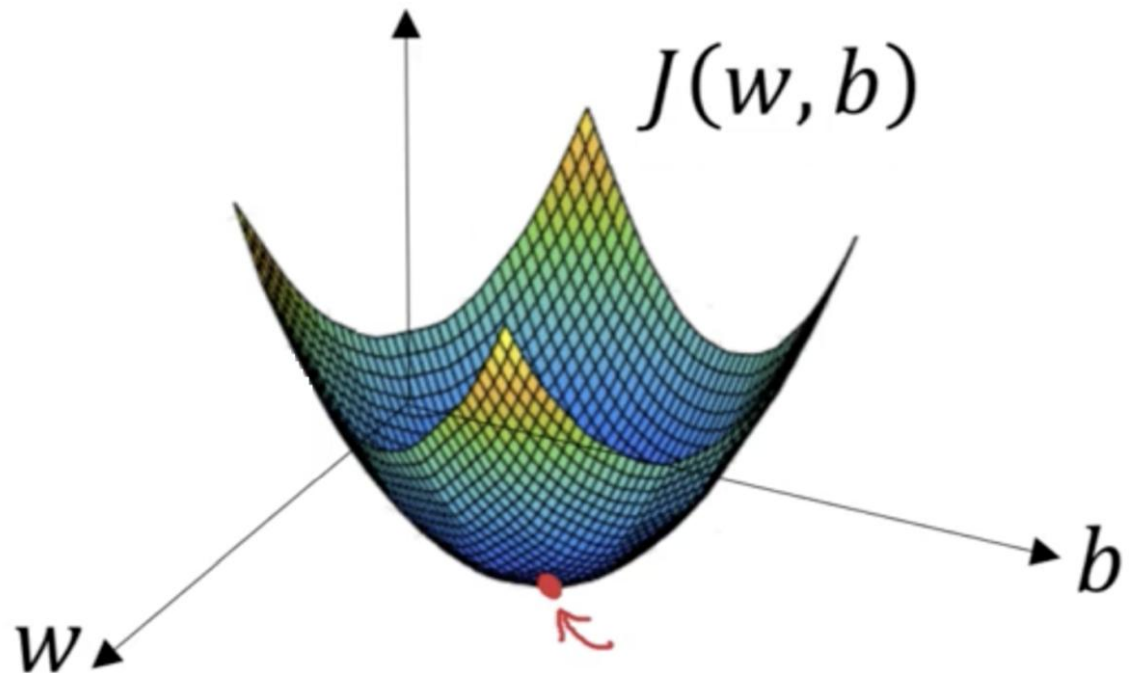
Gradient Descent

- Forward pass
  - $\hat{y} = W^{\mathsf{T}}X + b$
  - $L(y^{(i)}, \hat{y}^{(i)})$
  - $J(W, b)$

- Backward pass

$$dW = \frac{\partial J}{\partial W}$$

$$db = \frac{\partial J}{\partial b}$$

Gradient Descent

- Forward pass
  - $\hat{y} = W^T X + b$
  - $L(y^{(i)}, \hat{y}^{(i)})$
  - $J(W, b)$

- Backward pass

$$dW = \frac{\partial J}{\partial W}$$

$$db = \frac{\partial J}{\partial b}$$

$$W = W - \alpha * dW$$

$$b = b - \alpha * db$$

Gradient Descent

- Forward pass
  - $\hat{y} = W^TX + b$
  - $L(y^{(i)}, \hat{y}^{(i)})$
  - $J(W, b)$

- Backward pass

$$db = \frac{\partial J}{\partial b} = \frac{\partial J}{\partial \hat{y}}\frac{\partial \hat{y}}{\partial b}$$

$$\frac{\partial J}{\partial \hat{y}} = \frac{2}{n_x} \times (y - \hat{y})$$

$$\frac{\partial \hat{y}}{\partial b} = 1$$

Gradient Descent

- Forward pass
  - $\hat{y} = W^TX + b$
  - $L(y^{(i)}, \hat{y}^{(i)})$
  - $J(W, b)$

- Backward pass

$$db = \frac{2}{n_x} \times (y - \hat{y}) \times (1)$$

$$db = \frac{2}{n_x} \times (y - \hat{y})$$

Gradient Descent

- Forward pass
  - $\hat{y} = W^T X + b$
  - $L(y^{(i)}, \hat{y}^{(i)})$
  - $J(W, b)$

- Backward pass

$$dW = \frac{\partial L}{\partial W} = \frac{\partial L}{\partial \hat{y}} \frac{\partial \hat{y}}{\partial W}$$

$$\frac{\partial L}{\partial \hat{y}} = \frac{2}{n_x} \times (y - \hat{y})$$

$$\frac{\partial \hat{y}}{\partial W} = X$$

Gradient Descent

- Forward pass
  - $\hat{y} = W^T X + b$
  - $L(y^{(i)}, \hat{y}^{(i)})$
  - $J(W, b)$

- Backward pass

$$dW = \frac{2}{n_x} \times (y - \hat{y}) \times (X)$$

R^2

- Coefficient of determination
- Coefficient of multiple determination
- Strength of relationship
- Value between 0.0 – 1.0
- Percentage value
- Independent variable explains p percent of variation in dependent variable
- Independent variable reduces p percent of variation in dependent variable

Adjusted R^2

- R^2
  - Increase independent variables – Increase R^2
  - Increase independent variables – Constant R^2
- Adjusted R^2
  - Increase independent variables (then)
  - Increase model accuracy (then only)
  - Increase Adjusted R2

Applications

- Inputs to response mapping

- Error reduction

- Prediction
  - House price prediction from observed dataset

- Forecasting
  - Weather forecasting from observed dataset

Regularization

- Lasso regression

- Ridge regression

- Elastic net regression

Lasso Regression

- Least Absolute Shrinkage Selector Operator
- L1 regularization technique
- Reduce coefficients
- Feature selection
  - Select important features
  - Reduce coefficients of others to zero
- Suitable for more number of features

Ridge Regression

- L2 regularization technique
- Reduce coefficients
- Reduce model complexity
- Prevent multicollinearity

Elastic Net Regression

- L1 and L2 regularization technique

Advantages

- Good for linearly separable data
- Easier to implement and interpret
- Efficient to train
- Handle over-fitting
  - Dimensionality reduction techniques
  - Cross validation
  - Regularization
- Extrapolation of dataset

Disadvantages

- Assumption of linearity
  - Independent variables
  - Dependent variables
- Prone to noise
- Sensitive to outliers
- Prone to multicollinearity

Multicollinearity

- Correlated independent variables

- Example

  - Independent variables

    - Radius of a circle

    - Circumference of a circle

  - Radius and circumference – Correlated

Questions?

Thank you