Naive Bayes

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Topics

- Supervised learning
- Conditional probability
- Bayes' rule
- Naive Bayes
- Laplacian smoothing
- Prior ratio
- Log likelihood
- Applications
- Advantages
- Disadvantages

Supervised Learning

- Training set – { $(x^{(1)},y^{(1)})$, $(x^{(2)},y^{(2)})$, …, $(x^{(m)},y^{(m)})$}
- Labeled dataset

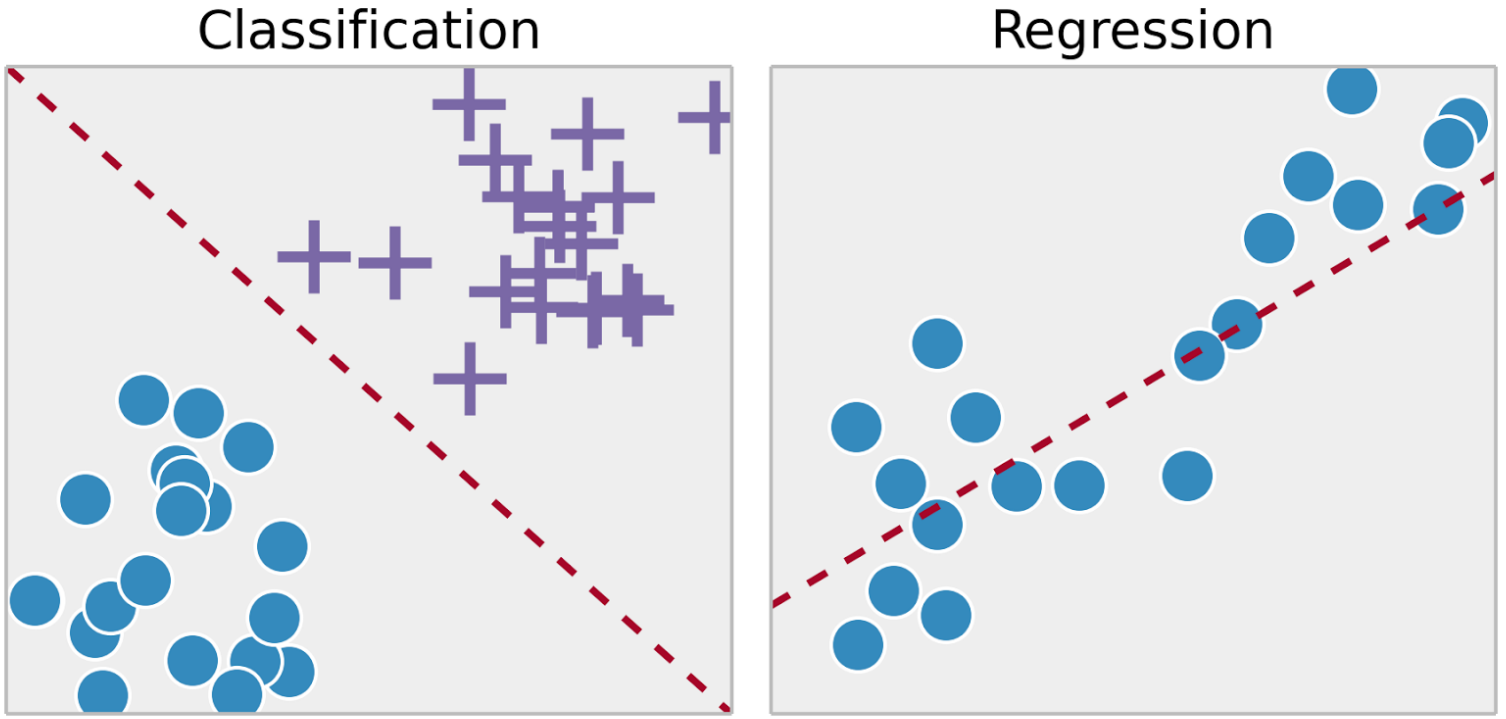


Image ack – Google

Probability

- SMS spam detection
- Corpus of SMSs
- Events – Ham SMS or Spam SMS

$$P(\text{Spam}) = \frac{N_{\text{Spam}}}{N_{\text{Total}}} = 1 - P(\text{Ham})$$

$$P(\text{Ham}) = \frac{N_{\text{Ham}}}{N_{\text{Total}}} = 1 - P(\text{Spam})$$

Name	Value
N_{Total}	1000
N_{Ham}	800
N_{Spam}	200
P(Ham)	0.8 (80%)
P(Spam)	0.2 (20 %)

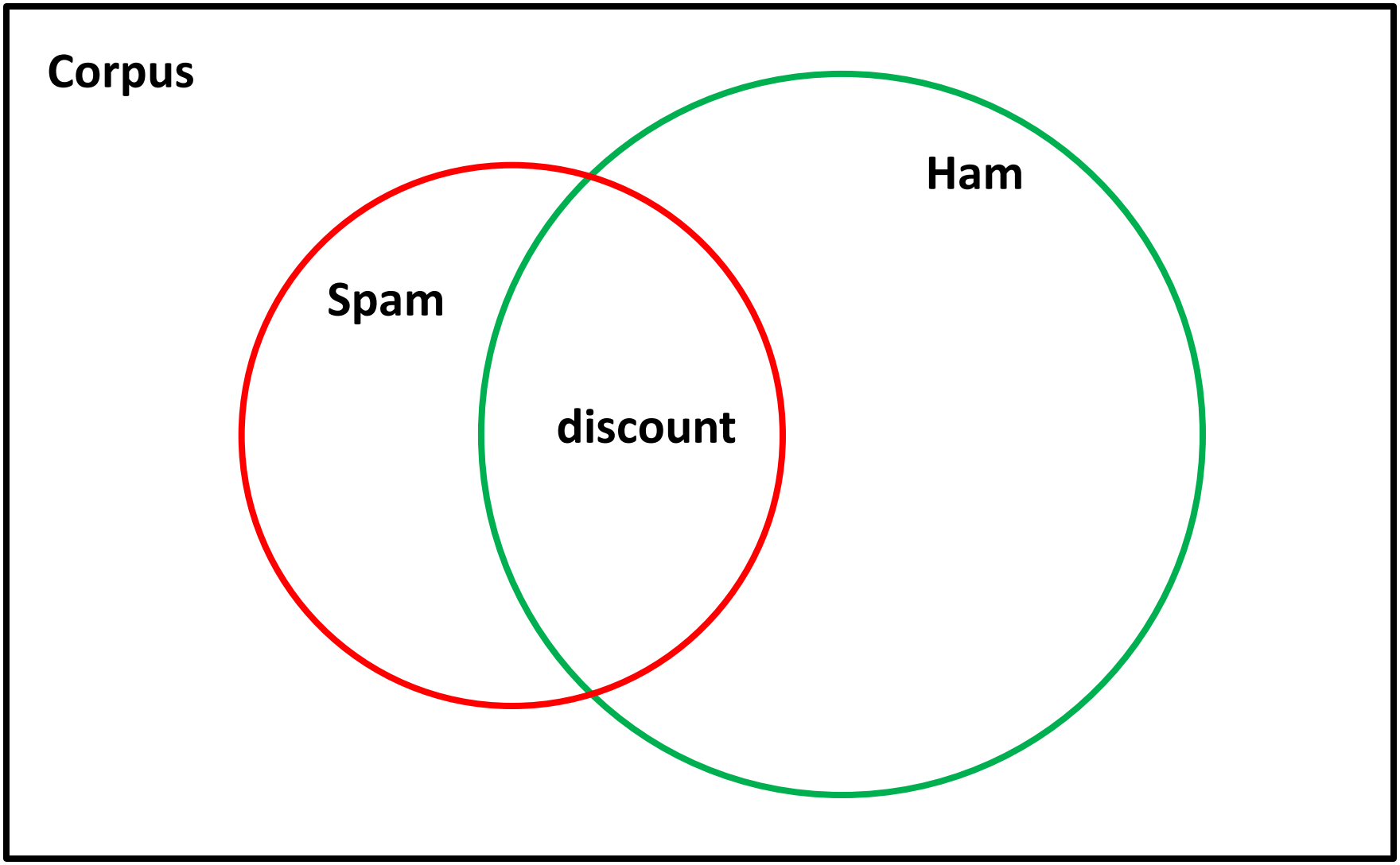
Probability

- SMS spam detection
- Corpus of SMSs
- Event – SMS contains word ‘discount’

$$P(\text{discount}) = \frac{N_{\text{discount}}}{N_{\text{Total}}}$$

Name	Value
N_{Total}	1000
N_{discount}	20
P(discount)	0.02 (2 %)

Conditional Probability



Conditional Probability

- Event – SMS – Spam
- Event – SMS – ‘discount’
- Intersection – Spam and ‘discount’

$$P(\text{Spam} \mid \text{discount}) = \frac{P(\text{Spam} \cap \text{discount})}{P(\text{discount})}$$

- Probability that SMS is Spam, given SMS contains word ‘discount’.
- Probability that SMS is Spam and SMS contains word ‘discount’.

Bayes' Rule

- Event – SMS – Spam
- Event – SMS – 'discount'
- Intersection – Spam and 'discount'

$$P(\text{Spam} \mid \text{discount}) = \frac{P(\text{Spam} \cap \text{discount})}{P(\text{discount})}$$

$$P(\text{discount} \mid \text{Spam}) = \frac{P(\text{discount} \cap \text{Spam})}{P(\text{Spam})}$$

Bayes' Rule

- Event – SMS – Spam
- Event – SMS – 'discount'
- Intersection – Spam and 'discount'

$$P(\text{Spam} \mid \text{discount}) = P(\text{discount} \mid \text{Spam}) \times \frac{P(\text{Spam})}{P(\text{discount})}$$

Bayes' Rule

- Event – SMS – Spam
- Event – SMS – 'discount'
- Intersection – Spam and 'discount'

$$P(\text{Spam} \mid \text{discount}) = P(\text{discount} \mid \text{Spam}) \times \frac{P(\text{Spam})}{P(\text{discount})}$$

$$P(A \mid B) = P(B \mid A) \times \frac{P(A)}{P(B)}$$

Naive Bayes

- SMS spam detection
- Corpus
  - Ham corpus
  - Spam corpus
- Vocabulary
  - All words
  - Ham and Spam corpus

Word	Ham	Spam
I	400	400
am	400	400
because	50	0
meeting	200	100
company	350	100
discount	100	200
lottery	100	400
	1600	**1600**

Naive Bayes

- SMS spam detection
- Corpus
  - Ham corpus
  - Spam corpus
- Vocabulary
  - All words
  - Ham and Spam corpus

Word	Ham	Spam
I	0.25	0.25
am	0.25	0.25
because	0.03125	0
meeting	0.125	0.0625
company	0.21875	0.0625
discount	0.0625	0.125
lottery	0.0625	0.25
	1600	1600

$$P(\text{Word}_i \mid \text{Class})$$

Naive Bayes

- SMS spam detection
- Corpus
  - Ham corpus
  - Spam corpus
- Vocabulary
  - All words
  - Ham and Spam corpus

Word	Ham	Spam
I	400	400
am	400	400
because	50	0
meeting	200	100
company	350	100
discount	100	200
lottery	100	400
	1600	1600

$$P(\text{I} \mid \text{Ham}) = \frac{400}{1600} = 0.25$$

Naive Bayes

- SMS spam detection
- Corpus
  - Ham corpus
  - Spam corpus
- Vocabulary
  - All words
  - Ham and Spam corpus

Word	Ham	Spam
I	0.25	0.25
am	0.25	0.25
because	0.03125	0
meeting	0.125	0.0625
company	0.21875	0.0625
discount	0.0625	0.125
lottery	0.0625	0.25

Naive Bayes

- Identical probabilities
  - Neutral words
  - I, am
- Significant words
  - Power words
  - meeting, company
  - discount, lottery
- Table of probabilities

Word	Ham	Spam
I	0.25	0.25
am	0.25	0.25
because	0.03125	0
meeting	0.125	0.0625
company	0.21875	0.0625
discount	0.0625	0.125
lottery	0.0625	0.25

Naive Bayes

$$\prod_{i=1}^{m} \frac{P(word_i \mid Ham)}{P(word_i \mid Spam)} > 1$$

Word	Ham	Spam
I	0.25	0.25
am	0.25	0.25
because	0.03125	0
meeting	0.125	0.0625
company	0.21875	0.0625
discount	0.0625	0.125
lottery	0.0625	0.25

I am in meeting.

$$\frac{0.25}{0.25} \times \frac{0.25}{0.25} \times \frac{0.125}{0.0625}$$

$$= 2 > 1$$

Ham

Naive Bayes

$$\prod_{i=1}^{m} \frac{P(word_i \mid Ham)}{P(word_i \mid Spam)} > 1$$

You got discount.

$$\frac{0.0625}{0.125}$$

$$= 0.5 < 1$$

Spam

Word	Ham	Spam
I	0.25	0.25
am	0.25	0.25
because	0.03125	0
meeting	0.125	0.0625
company	0.21875	0.0625
discount	0.0625	0.125
lottery	0.0625	0.25

Laplacian Smoothing

- Word count – 0
- Probability of word – 0
- Probability of sentence – 0
- e.g.
  - Word ‘because’ – Spam

Laplacian Smoothing

$$P(\text{Word}_i \mid \text{Class}) = \frac{C(\text{Word}_i, \text{Class})}{N_{\text{Class}}}$$

$$P(\text{Word}_i \mid \text{Class}) = \frac{C(\text{Word}_i, \text{Class}) + 1}{N_{\text{Class}} + U_{\text{Class}}}$$

$C(\text{Word}_i, \text{Class}) = \text{Number of } \text{Word}_i \text{ words in Class}$

$N_{\text{Class}} = \text{Number of words in Class}$

$U_{\text{Class}} = \text{Number of unique words in Class}$

Word	Ham	Spam
I	0.2495	0.2495
am	0.2495	0.2495
because	0.0317	6.2e-4
meeting	0.125	0.0628
company	0.2184	0.0628
discount	0.0628	0.125
lottery	0.0628	0.2495

Prior Ratio

- Number of Ham (positive) SMSs
- Number of Spam (negative) SMSs
- Ratio of number of Ham and Spam SMSs

Likelihood

- Ratio of probabilities
- Neutral words
  - ~1.
- Positive words
  - > 1 – Larger than 1
- Negative words
  - < 1 – Less than 1
- Multiplication
  - Numerical underflow

Log Likelihood

- Logarithm of ratio of probability
  - Log likelihood
- Addition of logarithms
  - No numerical underflow
- Neutral words – ~0
- Positive words – > 0
- Negative words – < 0
- Logprior

Applications

- Spam detection
- Text classification
- Author identification
- Word disambiguation
- Sentiment analysis

Advantages

- Simple
- Easy to implement
- Real-time predictions
- Work with less training dataset
- Continuous and discrete data
- Not sensitive to irrelevant features

Disadvantages

- Assume independent features

Questions?

Thank you