Roll No Dept. Page 1 Instructions: This question paper contains 1 page (2 sides of paper). Please verify. Write your name, roll number, department above in block letters neatly with ink. Write your final answers neatly with a blue/black pen. Pencil marks may get smudged. Don't overwrite/scratch answers especially in MCQ – such cases may get straight 0 marks. Do not rush to fill in answers. You have enough time to solve this quiz. Q1. (True-False) Write T or F for True/False in the box on the right and a brief justification pace below. Note: $L \in \mathbb{R}^{2 \times 2}$ is not necessarily positive semidefinite. (3 x (1+2) = 9) For any $\mathbf{w} \in \mathbb{R}^2$, $b \in \mathbb{R}$, $L \in \mathbb{R}^{2 \times 2}$, the set $\{\mathbf{x} \in \mathbb{R}^2 : \mathbf{w}^\top (L\mathbf{x}) + b = 0\}$ is either a line or the entire \mathbb{R}^2 or else empty. If T, give a brief proof. If F, give a counterexample.	CS 771A:	Intro to Machine Le	earning, IIT Ka	anpur	Quiz I	(28 Jan 202
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This question paper contains 1 page (2 sides of paper). Please verify. Write your name, roll number, department above in block letters neatly with ink . Write your final answers neatly with a blue/black pen . Pencil marks may get smudged. Don't overwrite/scratch answers especially in MCQ – such cases may get straight 0 marks. Do not rush to fill in answers. You have enough time to solve this quiz. 11. (True-False) Write T or F for True/False in the box on the right and a brief justification pace below. Note : $L \in \mathbb{R}^{2\times 2}$ is not necessarily positive semidefinite. (3 x (1+2) = 9) 12. For any $\mathbf{w} \in \mathbb{R}^2$, $b \in \mathbb{R}$, $b \in \mathbb$	KOII NO		Dept.			rage 1012
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or the entire \mathbb{R}^2 or else empty. If T , give a brief proof. If F , give a counterexample. For any $L \in \mathbb{R}^{2 \times 2}$ and any convex set $\mathcal{C} \subset \mathbb{R}^2$, if we define $\mathcal{D} \stackrel{\text{def}}{=} \{\mathbf{x} \in \mathbb{R}^2 : L\mathbf{x} \in \mathcal{C}\}$,	pace below	v. Note : $L \in \mathbb{R}^{2 \times 2}$ is no	ot necessarily p	oositive semidefii	nite. (3 x	(1+2) = 9 marl
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then \mathcal{D} is always convex or empty. If T , give a brief proof. If F , give a counterexample.	2 For ar	ny $L \in \mathbb{R}^{2 imes 2}$ and any \mathcal{L} is always convex or e	convex set $\mathcal{C} \subset$ mpty. If T , give	\mathbb{R}^2 , if we define a brief proof. If F	$\mathcal{D} \stackrel{ ext{ iny def}}{=} \{\mathbf{x} \in \mathbb{R}^2 \colon I_{\mathbf{x}}\}$, give a countere	$L\mathbf{x} \in \mathcal{C}$ }, xample.

If circles are sets of points of the form $\{\mathbf{x} \in \mathbb{R}^2 : \|\mathbf{x}\|_2 = r\}$ for some $r \geq 0$, then for any $L \in \mathbb{R}^{2 \times 2}$, the set $\{\mathbf{x} \in \mathbb{R}^2 : \|L\mathbf{x}\|_2 = 1\}$ is either a circle or else empty. If **T**, give

a brief proof. If **F**, give a counterexample (where it is non-empty but not a circle).

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Q2. (Subcalculus) Melba came across a function $f: \mathbb{R} \to \mathbb{R}$ described on the right and wants to analyse its properties. For parts a,b,c,d, fill only one circle. For parts d,e, answer

$$f(x) = \begin{cases} -x & x \le 0 \\ -\ln(1+x) & 0 < x \le 1 \\ -\ln(2) & 1 < x \end{cases}$$

in the space provided. No proofs/derivations needed in any part. Note: the subdifferential at a point is a set in general (singleton set if the func. is differentiable at that point). $(1 \times 6 = 6 \text{ marks})$

		_ True raise	
a.	Is f a continuous function over all of \mathbb{R} ?	\circ	
b.	Is f a convex function over all of \mathbb{R} ?	\circ	
c.	Is f differentiable at $x = 0$?		
d.	Is f differentiable at $x = 1$?		
e.	What is the subdifferential of f at $x = 0$?		
f.	What is the subdifferential of f at $x = 1$?		

Q4. (Too many prototypes) Melbu has a learning-with-prototypes (LwP) model for a binary problem with two labels + and - and 2D features. Every point on the circle $\{\mathbf{x} \in \mathbb{R}^2 : ||\mathbf{x}||_2 = 1\}$ is a - prototype and every point on the circle $\{\mathbf{x} \in \mathbb{R}^2 : ||\mathbf{x}||_2 = 2\}$ is a + prototype. Write down the equation for the decision boundary of this classifier and give justification below. (2 + 3 = 5 marks)

Write equation of decision boundary here	

Give justification here