CS 771A: Intro to Machine Learning, IIT Kanpur				**Quiz I** (28 Jan 2025)
Name				**20 marks** Page **1** of **2**
Roll No		Dept.		

Instructions:

1. This question paper contains 1 page (2 sides of paper). Please verify.
2. Write your name, roll number, department above in **block letters neatly with ink**.
3. Write your final answers neatly **with a blue/black pen**. Pencil marks may get smudged.
4. Don't overwrite/scratch answers especially in MCQ – such cases may get straight 0 marks.
5. Do not rush to fill in answers. You have enough time to solve this quiz.

Q1. (**True-False**) Write **T** or **F** for True/False in the **box on the right** and a **brief justification** in the space below. **Note**: $L \in \mathbb{R}^{2\times 2}$ is not necessarily positive semidefinite. (**3 x (1+2) = 9 marks**)

1	For any $\mathbf{w} \in \mathbb{R}^2, b \in \mathbb{R}, L \in \mathbb{R}^{2\times 2}$, the set $\{\mathbf{x} \in \mathbb{R}^2 : \mathbf{w}^\top (L\mathbf{x}) + b = 0\}$ is either a line or the entire $\mathbb{R}^2$ or else empty. If **T**, give a brief proof. If **F**, give a counterexample.	
2	For any $L \in \mathbb{R}^{2\times 2}$ and any convex set $\mathcal{C} \subset \mathbb{R}^2$, if we define $\mathcal{D} \stackrel{\text{def}}{=} \{\mathbf{x} \in \mathbb{R}^2 : L\mathbf{x} \in \mathcal{C}\}$, then $\mathcal{D}$ is always convex or empty. If **T**, give a brief proof. If **F**, give a counterexample.	
3	If circles are sets of points of the form $\{\mathbf{x} \in \mathbb{R}^2 : \|\mathbf{x}\|_2 = r\}$ for some $r \geq 0$, then for any $L \in \mathbb{R}^{2\times 2}$, the set $\{\mathbf{x} \in \mathbb{R}^2 : \|L\mathbf{x}\|_2 = 1\}$ is either a circle or else empty. If **T**, give a brief proof. If **F**, give a counterexample (where it is non-empty but not a circle).	

Q2. (Subcalculus) Melba came across a function $f: \mathbb{R} \to \mathbb{R}$ described on the right and wants to analyse its properties. For parts a,b,c,d, **fill only one circle**. For parts d,e, **answer in the space provided**. No proofs/derivations needed in any part. **Note**: the subdifferential at a point is a set in general (singleton set if the func. is differentiable at that point). (**1 x 6 = 6 marks**)

$$f(x) = \begin{cases} -x & x \leq 0 \\ -\ln(1+x) & 0 < x \leq 1 \\ -\ln(2) & 1 < x \end{cases}$$

		True	False
a.	Is f a continuous function over all of $\mathbb{R}$?	◯	◯
b.	Is f a convex function over all of $\mathbb{R}$?	◯	◯
c.	Is f differentiable at $x = 0$?	◯	◯
d.	Is f differentiable at $x = 1$?	◯	◯
e.	What is the subdifferential of f at $x = 0$?		
f.	What is the subdifferential of f at $x = 1$?		

Q4. (Too many prototypes) Melbu has a learning-with-prototypes (LwP) model for a binary problem with two labels $+$ and $-$ and 2D features. Every point on the circle $\{\mathbf{x} \in \mathbb{R}^2 : \|\mathbf{x}\|_2 = 1\}$ is a $-$ prototype and every point on the circle $\{\mathbf{x} \in \mathbb{R}^2 : \|\mathbf{x}\|_2 = 2\}$ is a $+$ prototype. Write down the equation for the decision boundary of this classifier and give justification below. (**2 + 3 = 5 marks**)

Write equation of decision boundary here

Give justification here