

UNIT 3

2D Geometric Transformations

In many applications, changes in orientations, size, and shape are accomplished with geometric transformations that alter the coordinate descriptions of objects.

- 2D Transformations take place in a two-dimensional plane.
- Transformations are helpful in changing the position, size, orientation, shape etc of the object.

Basic geometric transformations are:

Translation

Rotation

Scaling

Reflection

Shear

Translation

It is the straight line movement of an object from one position to another is called Translation. Here the object is positioned from one coordinate location to another.

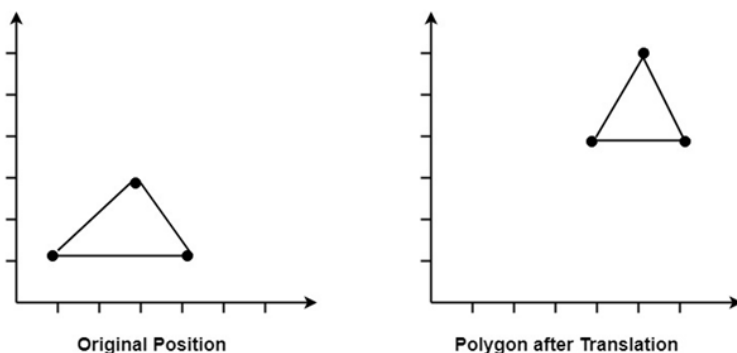
Translation of point:

To translate a point from coordinate position (x, y) to another (x_1, y_1) , we add algebraically the translation distances T_x and T_y to original coordinate.

The translation pair (T_x, T_y) is called as shift vector.

Translation is a movement of objects without deformation. Every position or point is translated by the same amount. When the straight line is translated, then it will be drawn using endpoints.

Translation of Polygon



Rotation:

It is a process of changing the angle of the object. Rotation can be clockwise or anticlockwise. For rotation, we have to specify the angle of rotation and rotation point. Rotation point is also called a pivot point. It is point about which object is rotated.

Types of Rotation:

1. Anticlockwise
2. Counterclockwise

The positive value of the pivot point (rotation angle) rotates an object in a counter-clockwise (anti-clockwise) direction.

The negative value of the pivot point (rotation angle) rotates an object in a clockwise direction.

When the object is rotated, then every point of the object is rotated by the same angle.

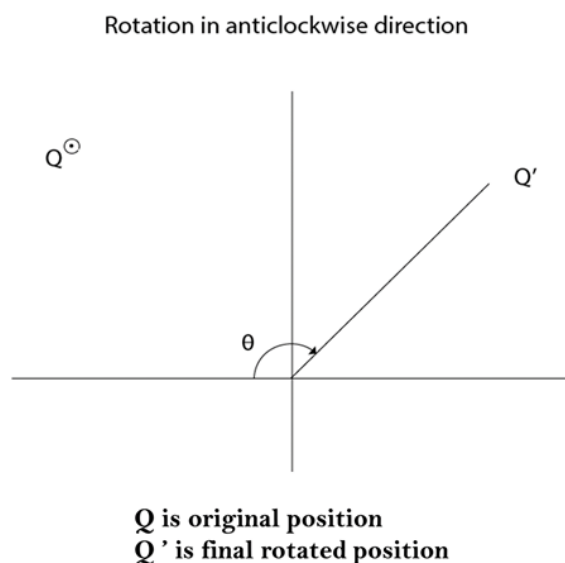
Straight Line: Straight Line is rotated by the endpoints with the same angle and redrawing the line between new endpoints.

Polygon: Polygon is rotated by shifting every vertex using the same rotational angle.

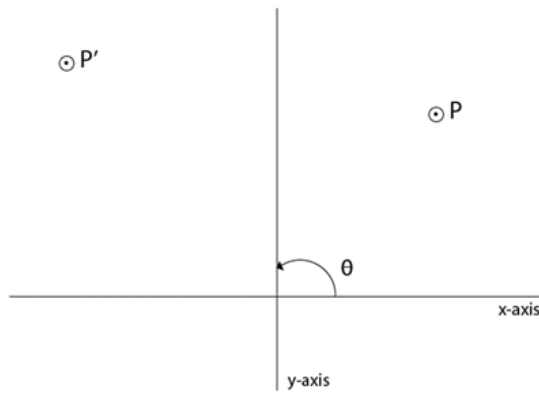
Curved Lines: Curved Lines are rotated by repositioning of all points and drawing of the curve at new positions.

Circle: It can be obtained by center position by the specified angle.

Ellipse: Its rotation can be obtained by rotating major and minor axis of an ellipse by the desired angle.



Rotation of P in clockwise direction



P is original Position

P' is final position or position after rotation

where θ is angle of rotation

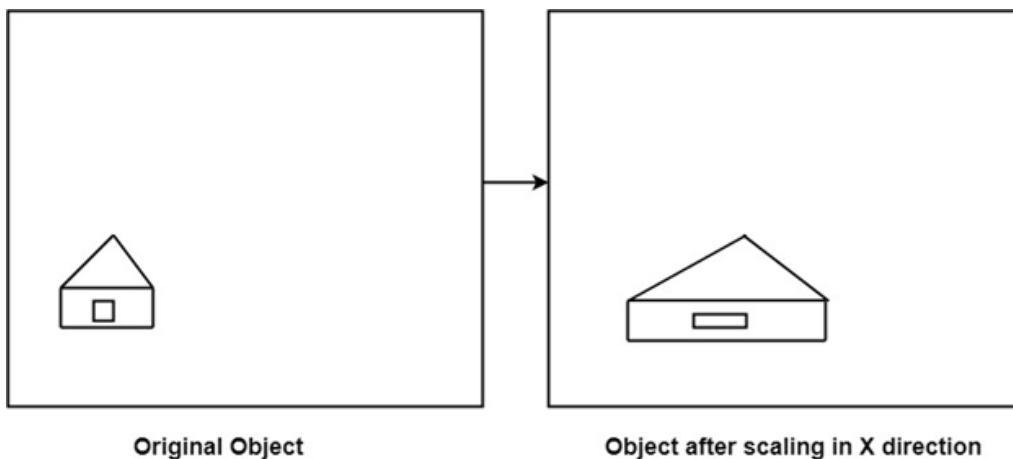
Scaling:

It is used to alter or change the size of objects. The change is done using scaling factors. There are two scaling factors, i.e. S_x in x direction S_y in y-direction. If the original position is x and y. Scaling factors are S_x and S_y then the value of coordinates after scaling will be x^1 and y_1 .

If the picture to be enlarged to twice its original size then $S_x = S_y = 2$. If S_x and S_y are not equal then scaling will occur but it will elongate or distort the picture.

If scaling factors are less than one, then the size of the object will be reduced. If scaling factors are higher than one, then the size of the object will be enlarged.

If S_x and S_y are equal it is also called as Uniform Scaling. If not equal then called as Differential Scaling. If scaling factors with values less than one will move the object closer to coordinate origin, while a value higher than one will move coordinate position farther from origin.



Reflection:

It is a transformation which produces a mirror image of an object. The mirror image can be either about x-axis or y-axis. The object is rotated by 180° .

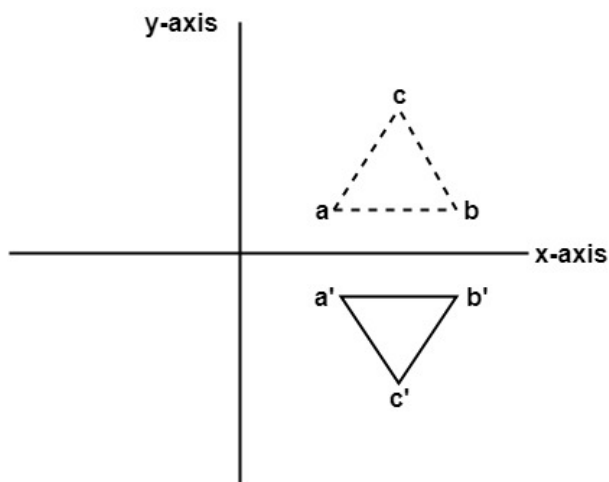
Types of Reflection:

1. Reflection about the x-axis
2. Reflection about the y-axis
3. Reflection about an axis perpendicular to xy plane and passing through the origin
4. Reflection about line $y=x$

1. Reflection about x-axis: The object can be reflected about x-axis with the help of the following matrix

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

In this transformation value of x will remain same whereas the value of y will become negative. Following figures shows the reflection of the object axis. The object will lie another side of the x-axis.

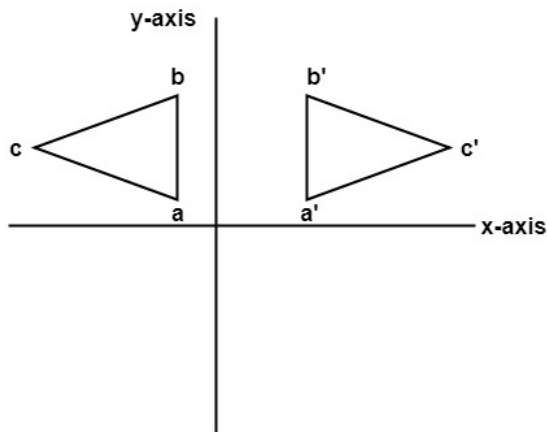


2. Reflection about y-axis: The object can be reflected about y-axis with the help of following transformation matrix

$$\begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Here the values of x will be reversed, whereas the value of y will remain the same. The object will lie another side of the y-axis.

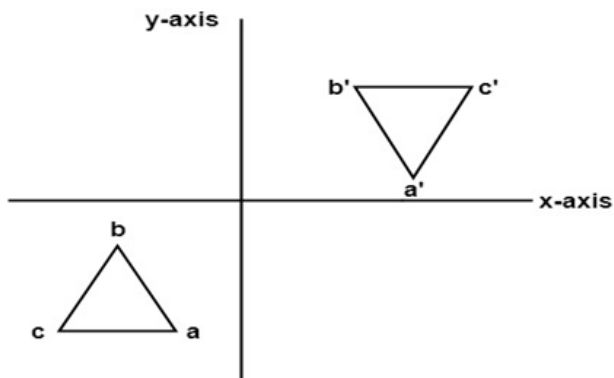
The following figure shows the reflection about the y-axis



3. Reflection about an axis perpendicular to xy plane and passing through origin:

In the matrix of this transformation is given below

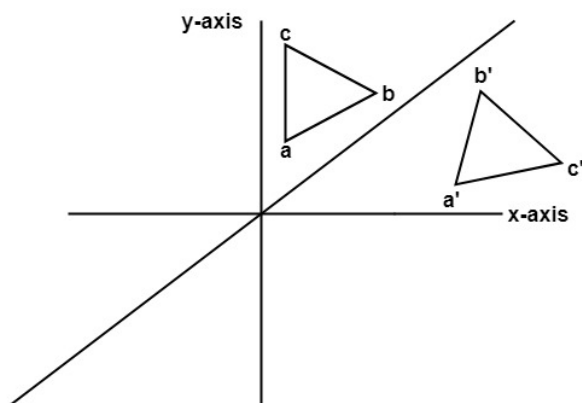
$$\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



In this value of x and y both will be reversed. This is also called as half revolution about the origin.

4. Reflection about line $y=x$: The object may be reflected about line $y = x$ with the help of following transformation matrix

$$\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



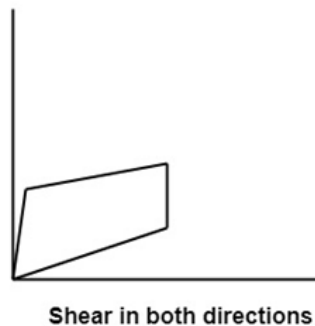
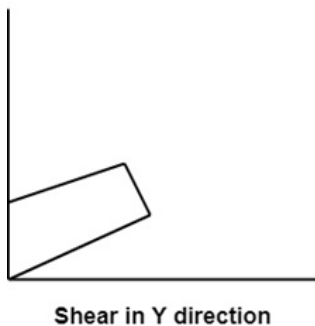
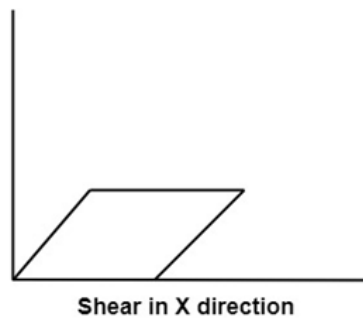
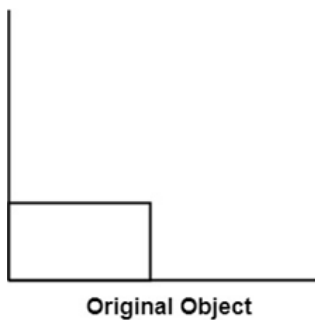
First of all, the object is rotated at 45° . The direction of rotation is clockwise. After it reflection is done concerning x-axis. The last step is the rotation of $y=x$ back to its original position that is counterclockwise at 45° .

Shearing:

It is transformation which changes the shape of object. The sliding of layers of object occur. The shear can be in one direction or in two directions.

Shearing in the X-direction: In this horizontal shearing sliding of layers occur. The homogeneous matrix for shearing in the x-direction is shown below:

$$\begin{bmatrix} 1 & 0 & 0 \\ Sh_x & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



Shearing in the Y-direction: Here shearing is done by sliding along vertical or y-axis.

$$\begin{bmatrix} 1 & Sh_y & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Shearing in X-Y directions: Here layers will be slid in both x as well as y direction. The sliding will be in horizontal as well as vertical direction. The shape of the object will be distorted. The matrix of shear in both directions is given by:

$$\begin{bmatrix} 1 & Sh_y & 0 \\ Sh_x & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$