2

Linear Programming Problem —Mathematical Formulation

"Resources are scarce and property of the society and their abuse is a social evil"

2:1. INTRODUCTION

Many business and economic situations are concerned with a problem of planning activity. In each case, there are limited *resources* at your disposal and your problem is to make such a use of these resources so as to yield the maximum production or to minimise the cost of production, or to give the maximum profit, etc. Such problems are referred to as the problems of constrained optimisation. *Linear programming* is a technique for determining an optimum schedule of interdependent activities in view of the available resources. *Programming* is just another word for 'planning' and refers to the process of determining a particular plan of action from amongst several alternatives. The word *linear* stands for indicating that all relationships involved in a particular problem are linear.

In the present chapter, some applications of linear programming problems and their mathematical formulations are discussed. The concepts are then extended to the general linear programming problem.

2:2. LINEAR PROGRAMMING PROBLEM

A Linear Programming Problem (LPP) consists of three *components*, namely the (*i*) decision variables (activities), (*ii*) the objective (goal), and (*iii*) the constraints (restrictions).

(*i*) The *decision variables* refer to the activities that are competing one another for sharing the resources available. These variables are usually inter-related in terms of utilisation of resources and need simultaneous solutions. All the decision variables are considered as continuous, controllable and non-negative.

(*ii*) A linear programming problem must have an objective which should be clearly identifiable and measurable in quantitative terms. It could be of profit (sales) maximisation, cost (time) minimisation, and so on. The relationship among the variables representing objective must be linear.

(*iii*) There are always certain limitations (or constraints) on the use of resources, such as labour, space, raw material, money, etc. that limit the degree to which an objective can be achieved.

Such constraints must be expressed as linear inequalities or equalities in terms of decision variables.

Basic assumptions. The following four basic assumptions are necessary for all linear programming problems:

(*a*) *Certainty.* In all LPP's, it is assumed that all the parameters; such as availability of resources, profit (or cost) contribution of a unit of decision variable and consumption of resources by a unit decision variable must be known and fixed. In other words, this assumption means that all the coefficients in the objective function as well as in the constraints are completely known with certainty and do not change during the period of study.

(*b*) *Divisibility (or continuity).* This implies that solution values of the decision variables and resources can take on any non-negative values, including fractional values of the decision variables. For instance, it is possible to produce 4.35 quintals of wheat or 17.35 thousand kilometers of cloth or 6.52 thousand kilolitres of milk, so these variables are divisible. But it is not possible to produce 2.6 refrigerators. Such variables are not divisible and hence are to be assigned integer values. When it is necessary to have integer variables, the integer programming problem is considered to attain the desired values.

(*c*) *Proportionality.* This requires the contribution of each decision variable in both the objective function and the constraints to be directly proportional to the value of the variable. For example, if production of one unit of a particular product uses 3 hours of a particular resource, then the production of 6 units of that product uses 3×6, *i.e.*, 18 hours of that resource.

(*d*) *Additivity.* The value of the objective function for the given values of decision variables and the total sum of resources used, must be equal to the sum of the contributions (profit or cost) earned from each decision variable and the sum of the resources used by each decision variable respectively. For example, the total profit earned by the sale of two products A and B must be equal to the sum of the profits earned separately from A and B. Similarly, the amount of a resource consumed by A and B must be equal to the sum of resources used for A and B individually.

2:3. MATHEMATICAL FORMULATION OF THE PROBLEM

The procedure for mathematical formulation of a linear programming problem consists of the following major steps :

Step 1. Study the given situation to find the *key decisions* to be made.

Step 2. Identify the *variables* involved and designate them by symbols x_j (j = 1, 2, ...).

Step 3. State the *feasible alternatives* which generally are : $x_j \geq 0$, for all j.

Step 4. Identify the constraints in the problem and express them as linear inequalities or equations, LHS of which are linear functions of the decision variables.

Step 5. Identify the objective function and express it as a linear function of the decision variables.

2:4. ILLUSTRATIONS ON MATHEMATICAL FORMULATION OF LPPs

Here are some problems from real life, which have been put in the mathematical format.

SAMPLE PROBLEMS

201. (Product Allocation Problem). *A company has three operational departments (weaving, processing and packing) with capacity to produce three different types of clothes namely suitings, shirtings and woollens yielding a profit of Rs. 2, Rs. 4 and Rs. 3 per metre respectively. One metre of suiting requires 3 minutes in weaving, 2 minutes in processing and 1 minute in packing. Similarly one metre of shirting requires 4 minutes in weaving, 1 minute in processing and 3 minutes in packing. One metre of woollen requires 3 minutes in each department. In a week, total run time of each department is 60, 40 and 80 hours for weaving, processing and packing respectively.*

Formulate the linear programming problem to find the product mix to maximize the profit.

Mathematical Formulation

The data of the problem is summarized below :

	Departments			Profit
	Weaving (in minutes)	*Processing (in minutes)*	*Packing (in minutes)*	*(Rs. per metre)*
Suitings	3	2	1	2
Shirtings	4	1	3	4
Woollens	3	3	3	3
Availability (minutes)	60×60	40×60	80×60	

Step 1. The key decision is to determine the weekly rate of production for the three types of clothes.

Step 2. Let us designate the weekly production of suitings, shirtings and woollens by x_1 metres, x_2 meters and x_3 metres respectively.

Step 3. Since it is not possible to produce negative quantities, feasible alternatives are sets of values of x_1, x_2 and x_3 satisfying $x_1 \geq 0$, $x_2 \geq 0$ and $x_3 \geq 0$.

Step 4. The constraints are the limited availability of three operational departments. One metre of suiting requires 3 minutes of weaving. The quantity being x_1 metres, the requirement for suiting alone will be $3x_1$ units. Similarly, x_2 metres of shirting and x_3 metres of woollen will require $4x_2$ and $3x_3$ minutes respectively. Thus, the total requirement of weaving will be $3x_1 + 4x_2 + 3x_3$, which should not exceed the available 3600 minutes. So, the labour constraint becomes $3x_1 + 4x_2 + 3x_3 \leq 3600$.

Similarly, the constraints for the processing department and packing departments are $2x_1 + x_2 + 3x_3 \leq 2400$ and $x_1 + 3x_2 + 3x_3 \leq 4800$ respectively.

Step 5. The objective is to maximize the total profit from sales. Assuming that whatever is produced is sold in the market, the total profit is given by the linear relation $z = 2x_1 + 4x_2 + 3x_3$.

The linear programming problem can thus be put in the following mathematical format :

Find x_1, x_2 and x_3 so as to maximize

$$z = 2x_1 + 4x_2 + 3x_3$$

subject to the constraints :

$$\begin{aligned} 3x_1 + 4x_2 + 3x_3 &\leq 3600 \\ 2x_1 + x_2 + 3x_3 &\leq 2400 \\ x_1 + 3x_2 + 3x_3 &\leq 4800 \\ x_1 \geq 0,\ x_2 \geq 0 \ \text{and} \ x_3 &\geq 0. \end{aligned}$$

202. (Product Mix Problem). *Consider the following problem faced by a production planner in a soft drink plant. He has two bottling machines A and B. A is designed for 8-ounce bottles and B for 16-ounce bottles. However, each can be used on both types with some loss of efficiency. The following data is available :*

Machine	*8-ounce bottles*	*16-ounce bottles*
A	*100/minute*	*40/minute*
B	*60/minute*	*75/minute*

Each machine can be run 8-hours per day, 5 days per week. Profit on a 8-ounce bottle is 25 paise and on a 16-ounce bottle is 35 paise. Weekly production of the drink cannot exceed 3,00,000 ounces and the market can absorb 25,000 8-ounce bottles and 7,000 16-ounce bottles per week. The planner wishes to maximize his profit subject, of course, to all the production and marketing restrictions. Formulate this as a linear programming problem. **[Meerut M.Sc. (Math.) 1998]**

Mathematical Formulation

The data of the problem is summarized as follows :

Resource/ constraint	*Production*		*Availability*
	8-ounce bottle	*16-ounce bottle*	
Machine A time	100/minute	40/minute	$8 \times 5 \times 60 = 2400$ minutes
Machine B time	60/minute	75/minute	$8 \times 5 \times 60 = 2400$ minutes
Production	1	1	3,00,000 ounces/week
Marketing	1	—	25,000 units/week
	—	1	7,000 units/week
Profit/unit (Rs.)	0.25	0.35	

Step 1. The key decision to be made is to determine the number of bottles (*8-ounce* and *16-ounce*) to be produced per week. Let x and y be the number of *8-ounce* and *16-ounce* bottles respectively, produced per week.

Step 2. Feasible alternatives are the sets of values $x \geq 0$, $y \geq 0$.

Step 3. Constraints are on the availability of machine time and production.

(*i*) *Machine-time constraints.* An *8-ounce* bottle takes 1/100 minutes on machine A and 1/60 minutes on machine B, while a *16-ounce* bottle takes 1/40 minutes on machine A and 1/75 minutes on machine B. Since both the machines can run 8 hours per day for 5 days per week, the time available on both the machines is 2,400 minutes per week individually. Thus, the two machine time constraints are :

$$\frac{x}{100} + \frac{y}{40} \leq 2{,}400 \quad (\textit{Machine } A)$$

$$\frac{x}{60} + \frac{y}{75} \leq 2{,}400 \quad (\textit{Machine } B)$$

(*ii*) *Production constraints.* It is given that the weekly production of the drink should not exceed 3,00,000 ounces and the market can absorb only up to 25,000 (*8-ounce* bottles) and 7,000 (*16-ounce* bottles) per week. Therefore, the two production constraints are :

$$8x + 16y \leq 3{,}00{,}000 \quad (\textit{Production})$$

$$x \leq 25{,}000 \text{ and } y \leq 7{,}000 \quad (\textit{Market})$$

Step 4. The objective is to maximize the total profit, *viz.*, $0.25x + 0.35y$.

The linear programming problem, therefore, can be put in the following mathematical format :

$$\begin{array}{l}
\text{Maximize} \quad z = 0.25x + 0.35y \\
\text{subject to the constraints :} \\
\quad 4x + 10y \leq 9{,}60{,}000 \\
\quad 15x + 12y \leq 21{,}60{,}000 \\
\quad 8x + 16y \leq 3{,}00{,}000 \\
\quad x \leq 25{,}000 \text{ and } y \leq 7{,}000 \\
\quad x \geq 0,\ y \geq 0.
\end{array}$$

203. (Production Problem). *An electronic company is engaged in the production of two components C_1 and C_2 used in T.V. sets. Each unit of C_1 costs the company Rs. 25 in wages and Rs. 25 in material, while each unit of C_2 costs the company Rs. 125 in wages and Rs. 75 in material. The company sells both products on one-period credit terms, but the company's labour and material expenses must be paid in cash. The selling price of C_1 is Rs. 150 per unit and of C_2 is Rs. 350 per unit. Because of the strong monopoly of the company for these components, it is assumed that the company can sell at the prevailing prices as many units as it produces. The company's production* *capacity is, however, limited by two considerations. First, at the beginning of period 1, the company has an initial balance of Rs. 20,000 (cash plus bank credit plus collections from past credit sales). Second, the company has available in each period 4,000 hours of machine time and 2,800 hours of assembly time. The production of each C_1 requires 6 hours of machine time and 4 hours of assembly time, whereas the production of each C_2 requires 4 hours of machine time and 6 hours of assembly time. Formulate this problem as an Linear Programming model so as to maximize the total profit to the company.*

Mathematical Formulation

The data of the problem is summarised as below :

Resource/constraint	*Components*		*Total availability*
	C_1	C_2	
Machine time (hours)	6	4	4,000 hours
Assembly time (hours)	4	6	2,800 hours
Budget (Rs.)	50	200	Rs. 20,000
Selling price (Rs.)	150	350	
Cost (= Wages + Material) price in Rs.	(25 + 25)	(125 + 75)	

Step 1. The key decision is to determine the number of units of C_1 and C_2 to be produced.

Step 2. Decision variables : Let x = number of units of C_1 and y = number of units of C_2.

Step 3. Feasible alternatives : $x \geq 0$ and $y \geq 0$.

Step 4. Constraints are on the availability of time and budget as under :

$$6x + 4y \leq 4{,}000 \quad \text{(Machine time)}$$

$$4x + 6y \leq 2{,}800 \quad \text{(Assembly time)}$$

$$50x + 200y \leq 20{,}000 \quad \text{(Budget)}$$

Step 5. The objective is to maximize the total profit from the sale of two type of components. Assuming that whatever is produced is sold in the market, the total profit is given by the relation

$$z = (150 - 50)x + (350 - 200)y \quad \text{or} \quad z = 100x + 150y$$

The LPP in mathematical format, therefore, is :

$$\boxed{\begin{array}{l} \text{Maximize } z = 100x + 150y \\ \text{subject to the constraints :} \\ 6x + 4y \leq 4{,}000, \quad 4x + 6y \leq 2{,}800; \\ 50x + 200y \leq 20{,}000; \quad \text{and} \quad x \geq 0, \; y \geq 0. \end{array}}$$

204. (Product Allocation Problem). *An Electronics Company produces three types of parts for automatic washing machine. It purchases casting of the parts from a local foundry and then finishes the part of drilling, shaping and polishing machines.*

The selling prices of part A, B and C respectively are Rs. 8, Rs. 10 and Rs. 14. All parts made can be sold. Castings for parts A, B and C respectively cost Rs. 5, Rs. 6 and Rs. 10.

The shop possesses only one of each type of machine. Costs per hour to run each of the three machines are Rs. 20 for drilling, Rs. 30 for shaping and Rs. 30 for polishing. The capacities (parts per hour) for each part on each machine are shown in the following table :

Machine	*Capacity per hour*		
	Part A	*Part B*	*Part C*
Drilling	25	40	25
Shaping	25	20	20
Polishing	40	30	40

The management of the shop wants to know how many parts of each type it should produce per hour in order to maximize profit for an hour's run. Formulate this problem as an Linear Programming model so as to maximize total profit to the company. **[Delhi M.B.A. (Nov.), 2003]**

Mathematical Formulation

Step 1. The key decision is to produce the three type of parts for the automatic washing machine.

Step 2. Decision variables : Let x_1, x_2 and x_3 = number of type *A*, *B* and *C* parts to be produced per hour, respectively.

Step 3. Feasible alternatives : $x_1 \geq 0$, $x_2 \geq 0$ and $x_3 \geq 0$.

Step 4. The constraints are on the time. All the three parts are to be processed on each of the three machines. On the drilling-machine, one type *A* part consumes 1/25th of the available hour, a type *B* part consumes 1/40th, and type *C* part consumes 1/25th of an hour. Thus the drilling-machine constraint is

$$\frac{x_1}{25} + \frac{x_2}{40} + \frac{x_3}{25} \leq 1, \text{ i.e., } 0.04x_1 + 0.025x_2 + 0.04x_3 \leq 1$$

Similarly, the other two constraints are :

$$\frac{1}{25}x_1 + \frac{1}{20}x_2 + \frac{1}{20}x_3 \leq 1, \text{ i.e., } 0.04x_1 + 0.05x_2 + 0.05x_3 \leq 1 \qquad \textit{(Shaping-machine)}$$

and
$$\frac{1}{40}x_1 + \frac{1}{30}x_2 + \frac{1}{40}x_3 \leq 1, \text{ i.e., } 0.025x_1 + 0.033x_2 + 0.025x_3 \leq 1 \qquad \textit{(Polishing-machine)}$$

Step 5. Profit must allow not only for the cost of the casting but for the cost of drilling, shaping, and polishing. Since, 25 type *A* parts per hour can be run on the drilling machine at a cost of Rs. 20, then Rs. $20 \times \frac{1}{25}$ = Re. 0.80 is the drilling cost per type *A* part. Similar reasoning for shaping and polishing gives

$$\text{Profit per type } A \text{ part} = (8-5) - \left(\frac{20}{25} + \frac{30}{25} + \frac{30}{40}\right) = 3 - 2.75 = 0.25$$

$$\text{Profit for type } B \text{ part} = (10-6) - \left(\frac{20}{40} + \frac{30}{20} + \frac{30}{30}\right) = 4 - 3.00 = 1.00$$

$$\text{Profit for type } C \text{ part} = (14-10) - \left(\frac{20}{25} + \frac{30}{20} + \frac{30}{40}\right) = 4 - 3.05 = 0.95$$

The objective is to maximize the total profit from sales, *viz.*, $0.25x_1 + x_2 + 0.95x_3$.

The linear programming problem, therefore, can be put in the following mathematical format :

$$\text{Maximize } z = 0.25x_1 + x_2 + 0.95x_3$$

subject to the constraints :

$$0.04x_1 + 0.025x_2 + 0.04x_3 \leq 1,$$
$$0.04x_1 + 0.05x_2 + 0.05x_3 \leq 1,$$
$$0.025x_1 + 0.033x_2 + 0.025x_3 \leq 1,$$

and $x_1 \geq 0,\ x_2 \geq 0,\ x_3 \geq 0.$

205. (Blending Problem). *The manager of an oil refinery must decide on the optimum mix of two possible blending processes of which the input and output production runs are as follows :*

Process	*Input*		*Output*	
	Crude A	*Crude B*	*Gasoline X*	*Gasoline Y*
1	6	4	6	9
2	5	6	5	5

The maximum amounts available of crudes A and B are 250 units and 200 units respectively. Market demand shows that at least 150 units of gasoline X and 130 units of gasoline Y must be produced. The profits per production run from process 1 and process 2 are Rs. 4 and Rs. 5 respectively. Formulate the problem for maximising the profit. **[Madras B.Com. 2005]**

Mathematical Formulation

Step 1. The key decision is to determine the number of units of gasoline produced from process *1* and process 2.

Step 2. Decision variables: Let x_1, x_2 = number of units of gasoline produced from process *1* and 2 respectively.

Step 3. Feasible alternatives: $x_1 \geq 0$ and $x_2 \geq 0$.

Step 4. The constraints are on the availability of crude oil and demand of crude oil, *viz.*,

$$6x_1 + 5x_2 \leq 250 \quad \textit{(Availability of crude A)}$$
$$4x_1 + 6x_2 \leq 200 \quad \textit{(Availability of crude B)}$$
$$6x_1 + 5x_2 \geq 150 \quad \textit{(Demand of gasoline X)}$$

and

$$9x_1 + 5x_2 \geq 130 \quad \textit{(Demand of gasoline Y)}$$

Step 5. The objective is to maximize the total profit from the production of gasoline, *viz.*, $4x_1 + 5x_2$.

The required linear programming problem, therefore, is

$$\boxed{\begin{array}{c} \text{Maximize } z = 4x_1 + 5x_2 \\ \text{subject to the constraints :} \\ 6x_1 + 5x_2 \leq 250,\ 4x_1 + 6x_2 \leq 200, \\ 6x_1 + 5x_2 \geq 150,\ 9x_1 + 5x_2 \geq 130, \\ x_1 \geq 0 \text{ and } x_2 \geq 0. \end{array}}$$

206. (Production Problem). *A complete unit of a certain product consists of four units of component A and three units of component B. The two components (A and B) are manufactured from two different raw materials of which 100 units and 200 units, respectively, are available. Three departments are engaged in the production process with each department using a different method for manufacturing the components per production run and the recoulting units of each component are given below :*

Department	*Input per run (units)*		*Output per run (units)*	
	Raw material I	*Raw material II*	*Component A*	*Component B*
1	7	5	6	4
2	4	8	5	8
3	2	7	7	3

Formulate this problem as a linear programming model so as to determine the number of production runs for each department which will maximize the total number of complete units of the final product. **[Himachal B.Tech. (Mech.) June 2007]**

Mathematical Formulation

Decision variables : Let x_1 = number of production runs for department *1*,
x_2 = number of production runs for department *2*, and
x_3 = number of production runs for department *3*.

Objective function :

Since each unit of the final product requires 4 units of component A and 3 units of component B, therefore, maximum number of units of the final product cannot exceed the smaller value of

$$\left\{ \frac{\text{Total number of units } A \text{ produced}}{4}, \frac{\text{Total number of units } B \text{ produced}}{3} \right\}$$

i.e., Minimum of $\left\{ \dfrac{6x_1 + 5x_2 + 7x_3}{4}, \dfrac{4x_1 + 8x_2 + 3x_3}{3} \right\}$.

Constraints : (*i*) If y is the number of component units of final product, then obviously, we have

$$\frac{6x_1 + 5x_2 + 7x_3}{4} \geq y \quad \text{and} \quad \frac{4x_1 + 8x_2 + 3x_3}{3} \geq y$$

(*ii*) Constraints on raw material are :

$$7x_1 + 4x_2 + 2x_3 \leq 100 \qquad \textit{(Raw material I)}$$

and

$$5x_1 + 8x_2 + 7x_3 \leq 200 \qquad \textit{(Raw material II)}$$

The LPP, therefore, is expressed as follows:

$$\text{Maximize } z = \text{Minimum of } \left\{ \frac{6x_1 + 5x_2 + 7x_3}{4}, \frac{4x_1 + 8x_2 + 3x_3}{3} \right\}$$

subject to the constraints :

$$\left.\begin{aligned} 6x_1 + 5x_2 + 7x_3 - 4y &\geq 0 \\ 4x_1 + 8x_2 + 3x_3 - 3y &\geq 0 \end{aligned}\right\} \qquad \textit{(Number of components of final product)}$$

$$7x_1 + 4x_2 + 2x_3 \leq 100 \qquad \textit{(Raw material I)}$$

$$5x_1 + 8x_2 + 7x_3 \leq 200 \qquad \textit{(Raw material II)}$$

$$x_1 \geq 0,\ x_2 \geq 0 \text{ and } x_3 \geq 0. \qquad \textit{(Non-negative restrictions)}$$

207. (Advertisement Problem). *The owner of Metro Sports wishes to determine how many advertisements to place in the selected three monthly magazines A, B and C. His objective is to advertise in such a way that total exposure to principal buyers of expensive sports good is maximized. Percentages of readers for each magazine are known. Exposure in any particular magazine is the number of advertisements placed multiplied by the number of principal buyers. The following data may be used :*

	Magazine		
	A	*B*	*C*
Readers	*1 lakh*	*0.6 lakh*	*0.4 lakh*
Principal Buyers	*20%*	*15%*	*8%*
Cost per Advertisement (Rs.)	*8000*	*6000*	*5000*

The budgeted amount is at most Rs. 1 lakh for the advertisements. The owner has already decided that magazine A should have no more than 15 advertisements and that B and C each have at least 80 advertisements. Formulate on LP model for the problem.

Mathematical Formulation

Decision variables : Let x_1 = number of insertions in magazine A,

x_2 = number of insertions in magazine B, and

x_3 = number of insertions in magazine C.

Objective function :

Maximize (total exposure)

$$z = (20\% \text{ of } 1{,}00{,}000)\, x_1 + (15\% \text{ of } 60{,}000)\, x_2 + (8\% \text{ of } 40{,}000)\, x_3$$

$$= 20{,}000\, x_1 + 9{,}000\, x_2 + 3{,}200\, x_3$$

Constraints : $8{,}000\,x_1 + 6{,}000\,x_2 + 5{,}000\,x_3 \le 1{,}00{,}000$ *(Budgeting)*

$x_1 \le 15,\ x_2 \ge 8,\ x_3 \ge 8$ *(Advertisement)*

Also, $x_1 \ge 0,\ x_2 \ge 0$ and $x_3 \ge 0.$ *(Non-negative restrictions)*

208. (Agriculturist Problem). *An agriculturist has a farm with 125 acres. He produces Radish, Muttar and Potato. Whatever he raises is fully sold in the market. He gets Rs. 5 for Radish per kg., Rs. 4 for Muttar per kg. and Rs. 5 for Potato per kg. The average yield is 1,500 kg. of Radish per acre, 1,800 kg. of Muttar per acre and 1,200 kg. of Potato per acre. To produce each 100 kg. of Radish and Muttar and to produce each 80 kg. of Potato, a sum of Rs. 12.50 has to be used for manure. Labour required for each acre to raise the crop is 6 man-days for Radish and Potato each and 5 man-days for Muttar. A total of 500 man-days of labour at a rate of Rs. 40 per man-day are available.*

Formulate this as a Linear Programming model to maximize the Agriculturist's total profit.

[Delhi M.B.A. (PT) 2005; Visvesvaraya M.B.A. (June) 2011]

Mathematical Formulation

Decision variables : Let x_1 = acreage for Raddish in kg.,

x_2 = acreage for Muttar in kg., and

x_3 = acreage for Potato in kg.

Objective function : We are given the following information :

	Radish	*Muttar*	*Potato*
Output (in kg.)	1,500	1,800	1,200
Cost of manure (in Rs. per kg.)	$\frac{12.50}{100} = 0.125$	$\frac{12.50}{100} = 0.125$	$\frac{12.50}{80} = 0.156$
Labour cost (in Rs.)	$6 \times 40 = 240$	$5 \times 40 = 200$	$6 \times 40 = 240$
Selling price (per kg.)	Rs. 5	Rs. 4	Rs. 5
Total cost	$0.125 \times 1{,}500 + 240 = 427.50$	$0.125 \times 1{,}800 + 200 = 425.00$	$0.156 \times 1{,}200 + 240 = 427.20$
Total Revenue	$5 \times 1{,}500 = 7{,}500$	$4 \times 1{,}800 = 7{,}200$	$5 \times 1{,}200 = 6{,}000$

Since, total profit = revenue − cost; the *objective function* is to maximize

$$z = (7{,}500 - 427.5)\,x_1 + (7{,}200 - 425)\,x_2 + (6{,}000 - 427.2)\,x_3$$

$$= 7{,}072.5x_1 + 6{,}775x_2 + 5{,}572.8x_3.$$

Constraints : Constraints on the availability of land and man-days are :

$x_1 + x_2 + x_3 \le 125$ *(Land)*

$6x_1 + 5x_2 + 6x_3 \le 500$ *(Man-days)*

Also, $x_1 \ge 0,\ x_2 \ge 0$ and $x_3 \ge 0$ *(Non-negative restrictions)*

209. (Crop and Livestock Problem). *A cooperative farm owns 100 acres of land and has Rs. 25,000 in funds available for investment. The farm members can produce a total of 3,500 man-hours worth of labour during the months September–May and 4,000 man-hours during June–August. If any of these man-hours are not needed, some members of the farm will use them to work on a neighbouring farm for Rs. 2/hour during September–May and Rs. 3/hour during June–August. Cash income can be obtained from the three main crops and two types of livestock : dairy cows and laying hens. No investment funds are needed for the crops. However, each cow will require an investment outlay of Rs. 3,200 and each hen will require Rs. 15.*

Moreover, each cow will require 1.5 acres of land, 100 man-hours of work during September–May and another 50 man-hours during the summer. Each cow will produce a net annual cash income of Rs. 3,500 for the farm. The corresponding figures for each hen are : no acreage, *0.6 man-hours during September–May, 0.4 man-hours during June–August, and an annual net cash income of Rs. 200. The chicken house can accommodate a maximum of 4,000 hens and the size of the cattle-shed limits the numbers to a maximum of 32 cows.*

Estimated man-hours and income per acre planted in each of the three crops are :

	Paddy	*Bajra*	*Jowar*
Man-hours:			
September–May	40	20	25
June–August	50	35	40
Net annual cash income (Rs.)	1,200	800	850

The cooperative farm wishes to determine how much acreage should be planted of each of the crops and how many cows and hens should be kept to maximise its net cash income.

Formulate this problem as LPP to maximize net annual cash income.

Mathematical Formulation

The data of the problem is summarised below :

Constraints	*Cows*	*Hens*	*Crop*			*Extra hours*		*Total availability*
			Paddy	*Bajra*	*Jowar*	*Sept.–May*	*June–Aug.*	
Man-hours:								
Sept.–May	100	0.6	40	20	25	1	—	3,500
June–Aug.	50	0.4	50	35	40	—	1	4,000
Land	1.5	—	1	1	1	—	—	100
Cow	1	—	—	—	—	—	—	32
Hens	—	1	—	—	—	—	—	4,000
Net annual cash income (Rs.)	3,500	200	1,200	800	850	2	3	

Decision variables : Let x_1 and x_2 = number of cows and hens respectively;

x_3, x_4 and x_5 = acreage planting of paddy, bajra and jowar respectively.

x_6 and x_7 = extra man-hours utilized during Sept.–May and May–June respectively.

Objective function : Maximization of net annual cash, *viz.,*

$$z = 3{,}500x_1 + 200x_2 + 1{,}200x_3 + 800x_4 + 850x_5 + 2x_6 + 3x_7$$

Constraints :

(*i*) $$\left.\begin{aligned} 100x_1 + 0.6x_2 + 40x_3 + 20x_4 + 25x_5 + x_6 &= 3{,}500 \\ 50x_1 + 0.4x_2 + 50x_3 + 35x_4 + 40x_5 + x_7 &= 4{,}000 \end{aligned}\right\} \quad \textit{(Man-hours)}$$

(*ii*) $$1.5x_1 + x_3 + x_4 + x_5 \le 100 \quad \textit{(Land availability)}$$

(*iii*) $$x_1 \le 32 \text{ and } x_2 \le 4{,}000 \quad \textit{(Livestock constraints)}$$

$$x_1 \ge 0,\ x_2 \ge 0,\ x_3 \ge 0,\ x_4 \ge 0,\ x_5 \ge 0,\ x_6 \ge 0 \text{ and } x_7 \ge 0 \quad \textit{(Non-negative restrictions)}$$

210. (Trim Loss Problem). *Rolls of paper having a fixed length and width of 180 cm are being manufactured by a paper mill. These rolls have to be cut to satisfy the following demand :*

Width :	*80 cm*	*45 cm*	*27 cm*
No. of rolls :	200	120	130

Obtain the linear programming formulation of the problem to determine the cutting pattern, so that the demand is satisfied and wastage of paper is a minimum.

Mathematical Formulation

Here, the key decision is to determine how the paper rolls be cut to the required width so that trim loss (wastage) is a minimum.

Various alternatives for the number of rolls are given below :

Feasible patterns of cutting	*No. of rolls cut*	*Wastage per roll*	Rolls obtained from each roll of width		
			80 cm	*45 cm*	*27 cm*
80 + 80	x_1	20	2	—	—
80 + 45 + 45	x_2	10	1	2	—
80 + 45 + 27 + 27	x_3	1	1	1	2
80 + 27 + 27 + 27	x_4	19	1	—	3
45 + 45 + 45 + 45	x_5	0	—	4	—
45 + 45 + 45 + 27	x_6	18	—	3	1
45 + 45 + 27 + 27 + 27	x_7	9	—	2	3
45 + 27 + 27 + 27 + 27 + 27	x_8	0	—	1	5
27 + 27 + 27 + 27 + 27 + 27	x_9	18	—	—	6

Decision variables : Let x_j ($j = 1, 2, \ldots, 9$) represent the number of times each cutting alternative is to be used.

Objective function : The objective is to minimize the wastage produced, *i.e.*,

$$\text{Minimize } z = 20x_1 + 10x_2 + x_3 + 19x_4 + 18x_6 + 9x_7 + 18x_9$$

Constraints : Constraints on the availability of rolls and the requirement of desired width of rolls are :

$$2x_1 + x_2 + x_3 + x_4 = 200 \quad \textit{(80 cm rolls)}$$

$$2x_2 + x_3 + 4x_5 + 3x_6 + 2x_7 + x_8 = 120 \quad \textit{(45 cm rolls)}$$

$$2x_3 + 3x_4 + x_6 + 3x_7 + 5x_8 + 6x_9 = 130 \quad \textit{(27 cm rolls)}$$

$$x_j \geq 0\,;\ j = 1, 2, 3, \ldots, 9. \quad \textit{(Non-negativity)}$$

211. (Marketing Problem). *The PQR Stone Company sells stone secured from any of the three adjacent quarries. The stone sold by the company must conform to the following specifications :*

Material X equal to 30 per cent ; Material Y equal to or less than 40 per cent ; Material Z between 30 per cent and 40 per cent.

Stone from quarry A costs Rs. 100 per tonne and has the following properties :

Material X : 20 per cent, material Y : 60 per cent, and material Z : 20 per cent.

Stone from quarry B costs Rs. 120 per tonne and has the following properties :

Material X : 40 per cent, material Y : 30 per cent, and material Z : 30 per cent.

Stone from quarry C costs Rs. 150 per tonne and has the following properties :

Material X : 10 per cent, material Y : 40 per cent, and material Z : 50 per cent.

Formulate the above as an LPP to minimise cost per tonne.

Mathematical Formulation

The data of the problem is summarised below :

Material	*Quarry*			*Specification*
	A	B	C	
X	20%	40%	10%	30%
Y	60%	30%	40%	less than or equal to 40%
Z	20%	30%	50%	between 30% and 40%
Cost per tonne (Rs.)	100	120	150	

Decision variables : Let x_1, x_2, x_3 represent the proportion of stone in tonne to be produced from quarries *A*, *B* and *C* respectively.

Objective function : The objective is to minimize the total cost, *i.e.*,

$$\text{Minimize } z = 100x_1 + 120x_2 + 150x_3$$

Constraints : The constraints are on the three types of material of the stone. These are :

$$20x_1 + 40x_2 + 10x_3 = 30 \quad \text{(Material } X\text{)}$$
$$60x_1 + 30x_2 + 40x_3 \leq 40 \quad \text{(Material } Y\text{)}$$
$$\left.\begin{aligned} 20x_1 + 30x_2 + 50x_3 \leq 40 \\ 20x_1 + 30x_2 + 50x_3 \geq 30 \end{aligned}\right\} \quad \text{(Material } Z\text{)}$$

Also, $x_1 \geq 0,\ x_2 \geq 0,\ x_3 \geq 0.$ *(Non-negative restrictions)*

212. (Blending Problem). *Three grades of coal A, B and C contain ash and phosphorus as impurities. In a particular industrial process a fuel obtained by blending the above grades containing not more than 25% ash and 0.03% phosphorus is required. The maximum demand of the fuel is 100 tons. Percentage impurities and costs of the various grades of coal are shown below. Assuming that there is an unlimited supply of each grade of coal and there is no loss in blending, formulate the blending problem to minimise the cost.*

Coal grade	*% ash*	*% phosphorus*	*Cost per ton (in Rs.)*
A	30	0.02	240
B	20	0.04	300
C	35	0.03	280

[Pune M.B.A. 2009]

Mathematical Formulation

Decision variables : Let x_1 = tons of grade *A* coal,

x_2 = tons of grade *B* coal, and x_3 = tons of grade *C* coal.

Objective function : Minimize $z = 240x_1 + 300x_2 + 280x_3$

Constraints : $0.3x_1 + 0.2x_2 + 0.35x_3 \leq 0.25(x_1 + x_2 + x_3)$

or $x_1 - x_2 + 2x_3 \leq 0$ *(ash)*

$$\frac{0.02}{100}x_1 + \frac{0.04}{100}x_2 + \frac{0.03}{100}x_3 \leq \frac{0.03}{100}(x_1 + x_2 + x_3)$$

or $-x_1 + x_2 \leq 0$ *(phosphorus)*

$x_1 + x_2 + x_3 \leq 100$ *(demand of fuel)*

$x_1 \geq 0,\ x_2 \geq 0$ and $x_3 \geq 0.$ *(Non-negative restrictions)*

213. (Capital Budgeting Problem). *An engineering company is planning to diversify its operations during the year 2006-07. The company has allocated capital expenditure budget equal to Rs. 5.15 crore in the year 2006 and Rs. 6.50 crore in the year 2007. The company has five investment projects under consideration. The estimated net returns at present value and expected cash expenditures of each project in the two years are as follows :*

Project	*Estimated net returns (in '000 Rs.)*	*Cash expenditure (in '000 Rs.)*	
		Year 2006	*Year 2007*
A	240	120	320
B	390	550	594
C	80	118	202
D	150	250	340
E	182	324	474

Assume that the returns from a particular project would be in direct proportion to the investment in it, so that, for example, if in a project, say A, 20% (of 120 in 2006 and of 320 in 2007) is invested, then the resulting net returns in it would be 20% (of 240). This assumption also implies that *individuality of the project should be ignored. Formulate this capital budgeting problem as a Linear Programming model to maximize the net returns.*

Mathematical Formulation

Decision variables : Let x_1, x_2, x_3, x_4 and x_5 represent the proportion of investment in project *A, B, C, D* and *E* respectively.

Objective function : The objective is to maximize the net returns. Thus, the objective function is :

$$\text{Maximize } z = 240x_1 + 390x_2 + 80x_3 + 150x_4 + 182x_5$$

Constraints : Capital expenditure budget constraints are :

$$120x_1 + 550x_2 + 118x_3 + 250x_4 + 324x_5 \le 515 \qquad \textit{(for 2006)}$$

$$320x_1 + 594x_2 + 202x_3 + 340x_4 + 474x_5 \le 650 \qquad \textit{(for 2007)}$$

Also, $x_1 \ge 0,\ x_2 \ge 0,\ x_3 \ge 0,\ x_4 \ge 0$ and $x_5 \ge 0$. *(Non-negative restrictions)*

In addition to the above, there is a requirement of 0–1 (*i.e.*, of integrality). So,

$$x_j = 1 \text{ or } 0 \quad (j = 1, 2, 3, 4, 5).$$

214. (Media Problem). *The Marketing Department of Everest Company has collected information on the problem of advertising for its products. This relates to the advertising media available, the number of families expected to be reached with each alternative, cost per advertisement, the maximum availability of each medium and the expected exposure of each one (Measured as the relative value of one advertisement in each of the media).*

The information is given as under :

Advertising media	*No. of families to cover*	*Cost/ad. (Rs.)*	*Maximum availability (No. of times)*	*Expected exposure (Units)*
TV (30 sec.)	*3,000*	*8,000*	*8*	*80*
Radio (15 sec.)	*7,000*	*3,000*	*30*	*20*
Sunday Edition (1/4 page)	*5,000*	*4,000*	*4*	*50*
Magazine (1 page)	*2,000*	*3,000*	*2*	*60*

Other information and requirements :

(*a*) *The advertising budget is Rs. 70,000.* (*b*) *At least 40,000 families should be covered.*

(*c*) *At least 2 insertions be given in Sunday Edition of Daily but not more than 4 advertisements should be given on the TV.*

Formulate this as a linear programming problem. The company's objective is to maximize the expected exposure.

Mathematical Formulation

The key decision to be made is to determine the number of advertisements to be brought in Television, Radio, Sunday Edition and Magazine.

Decision variables. Let x_1, x_2, x_3 and x_4 represent the number of advertisements in TV, Radio, Sunday Edition and Magazine respectively.

Using the given information, the appropriate linear programming problem is :

$$\text{Maximize (total expected exposure) } z = 80x_1 + 20x_2 + 50x_3 + 60x_3$$

subject to the constraints :

$$3{,}000x_1 + 7{,}000x_2 + 5{,}000x_3 + 2{,}000x_4 \ge 40{,}000 \qquad \textit{(Families)}$$

$$8{,}000x_1 + 3{,}000x_2 + 4{,}000x_3 + 3{,}000x_4 \ge 70{,}000 \qquad \textit{(Budget)}$$

$$x_1 \le 8,\ x_2 \le 30,\ x_3 \le 4,\ x_4 \le 2 \qquad \textit{(Availability)}$$

$$x_3 \ge 2 \text{ and } x_1 \le 4 \qquad \textit{(No. of advt.)}$$

Also, $x_1 \ge 0,\ x_2 \ge 0,\ x_3 \ge 0$ and $x_4 \ge 0$. *(Non-negative restrictions)*

215. (Investment Problem). *A person is interested in investing Rs. 5,00,000 in a mix of investments. The investment choices and expected rates of return on each one of them are :*

Investment	*Projected rate of return*
Mutual Fund A	0.12
Mutual Fund B	0.09
Money Market Fund	0.08
Government Bonds	0.085
Share Y	0.16
Share X	0.18

The investor wants at least 35 per cent of his investment in government bonds. Because of the higher perceived risk of the two shares, he has specified that the combined investment in these not to exceed Rs. 80,000. The investor has also specified that at least 20 per cent of the investment should be in the money market fund and that the amount of money invested in shares should not exceed the amount invested in mutual funds. His final investment condition is that the amount invested in mutual fund A should be no more than the amount invested in mutual fund B. The problem is to decide the amount of money to invest in each alternative so as to obtain the highest annual total return. Formulate the above as linear programming problem.

Mathematical Formulation

The *key decision* is to determine the amount to be invested in various options available to the investor.

Decision variables : Let the decision variables x_1, x_2, x_3, x_4, x_5 and x_6 represent the amount invested in mutual fund *A*, mutual fund *B*, money-market fund, government bonds, shares *Y* and shares *X* respectively.

Then, the appropriate L.P.P. is :

$$\text{Maximize } z = 0.12x_1 + 0.09x_2 + 0.08x_3 + 0.085x_4 + 0.16x_5 + 0.18x_6$$

subject to the constraints :

$$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 \le 5{,}00{,}000 \qquad \textit{(Availability of money)}$$

$$x_4 \ge 1{,}75{,}000 \qquad \textit{(Government bond)}$$

$$x_5 + x_6 \le 80{,}000 \qquad \textit{(Shares)}$$

$$x_3 \ge 1{,}00{,}000 \qquad \textit{(Money market fund)}$$

$$x_5 + x_6 \le x_1 + x_2 \qquad \textit{(Shares vers. Mutual fund)}$$

$$x_1 \le x_2 \qquad \textit{(Mutual fund)}$$

Also, $x_1 \ge 0,\ x_2 \ge 0,\ x_3 \ge 0,\ x_4 \ge 0,\ x_5 \ge 0$ and $x_6 \ge 0$. *(Non-negative restrictions)*

216. (Personnel-Manpower Problem). *The super bazar in a city daily needs between 32 and 40 workers depending on the time of day. The rush hours are between noon and 2 p.m. The table below indicates the number of workers needed at various hours when the bazar is open.*

Time period	*Number of workers needed*
9 a.m.—11 a.m.	32
11 a.m.— 1 p.m.	40
1 p.m.— 3 p.m.	35
3 p.m.— 5 p.m.	33

The super bazar now employs 34 full-time workers, but needs a few part-time workers also. A part-time worker must put in exactly 4 hours per day, but can start any time between 9 a.m. and 1 p.m. Full-time workers work from 9 a.m. to 5 p.m. but are allowed an hour for lunch (half of the full-timers eat at 12 noon, the other half at 1 p.m.).

The management of the super bazar limits part-time hours to a maximum of 50 per cent of the day's total requirement.

Part-timers earn Rs. 48 per day on the average, while full-timers earn Rs. 140 per day in salary and benefits on the average. The management wants to set a schedule that would minimize total manpower costs. Formulate this problem as an Linear Programming model to minimize total daily manpower cost.

Mathematical Formulation

The *key decision* is to determine the number of full-time and part-time workers needed in Super Bazar.

Decision variables : Let y represents the number of full-time workers and x_j represents the part-time workers starting at 9 a.m., 11 a.m. and 1 p.m. respectively. (j = 1, 2, 3).

Using the given information, the appropriate LPP is :

Minimize (total daily manpower cost) $z = 140y + 48(x_1 + x_2 + x_3)$

subject to the constraints :

$y + x_1 \geq 32$ (*9 a.m.—11 a.m. need*)

$\frac{1}{2}y + x_1 + x_2 \geq 40$ (*11 a.m.—1 p.m. need*)

$\frac{1}{2}y + x_2 + x_3 \geq 35$ (*1 p.m.—3 p.m. need*)

$y + x_3 \geq 33$ (*3 p.m.—5 p.m. need*)

$y \leq 34$ (*Full timers available*)

$4(x_1 + x_2 + x_3) \leq 0.50(32 + 40 + 35 + 33)$

Part timers' hour cannot exceed 50% of total hours required each day which is the sum of the workers needed each hour)

Also, $y \geq 0$ and $x_1 \geq 0,\ x_2 \geq 0,\ x_3 \geq 0.$ (*Non-negative restrictions*)

217. (Manpower Schedule Problem). *A city hospital has the following minimal daily requirements for nurses :*

Period	*Clock time (24 hour day)*	*Minimal number of nurses required*
1	*6 a.m.—10 a.m.*	2
2	*10 a.m.— 2 p.m.*	7
3	*2 p.m.— 6 p.m.*	*15*
4	*6 p.m.—10 p.m.*	*8*
5	*10 p.m.— 2 a.m.*	20
6	*2 a.m.— 6 a.m.*	6

Nurses report to the hospital at the beginning of each period and work for 8 consecutive hours. The hospital wants to determine the minimal number of nurses to be employed so that there will be sufficient number of nurses available for each period. Formulate this problem as a linear programming model by setting up appropriate constraints and objective function.

[Delhi M.B.A. (October) 2002, B.Com. (Hons.) 2006]

Mathematical Formulation

The *key decision* to be made is to determine the number of nurses to be employed in the city hospital.

Decision variables : Let x_j (j = 1, 2, 3, 4, 5, 6) be the number of nurses reporting to work on shift j.

Also, since negative values of x_j are meaningless, we must have $x_j \geq 0$ for all j.

Objective function : If the number of nurses reporting to work on shifts 1, 2, 3, ..., 6 is known, then the manpower schedule can be decided. Thus, the objective is to minimize the total number of nurses (manpower for all the six periods together). As such, the objective is to minimize the linear function : $z = x_1 + x_2 + x_3 + x_4 + x_5 + x_6$.

Constraints : In any 8-hour period, the number of nurses reporting at the beginning of the period and the number of nurses continuing from the earlier period should be at least equal to the minimum requirement.

Thus, we have the following constraints :

$$x_1 + x_2 \geq 7,\quad x_2 + x_3 \geq 15,\quad x_3 + x_4 \geq 8,$$
$$x_4 + x_5 \geq 20,\quad x_5 + x_6 \geq 6,\quad x_6 + x_1 \geq 2.$$

∴ The given *linear programming problem* is :

$$\text{Minimize } z = x_1 + x_2 + x_3 + x_4 + x_5 + x_6$$

subject to the constraints :

$$x_1 + x_2 \geq 7,\quad x_2 + x_3 \geq 15,\quad x_3 + x_4 \geq 8,$$
$$x_4 + x_5 \geq 20,\quad x_5 + x_6 \geq 6,\quad x_6 + x_1 \geq 2;$$

and $$x_1 \geq 0,\ x_2 \geq 0,\ x_3 \geq 0,\ x_4 \geq 0,\ x_5 \geq 0,\ x_6 \geq 0.$$

218. (Transportation Problem). *ABC manufacturing company wishes to develop a monthly production schedule for the next three months. Depending upon the sales commitments, the company can either keep the production constant, allowing fluctuation in inventory or inventories can be maintained at a constant level, with fluctuating production. Fluctuating production makes overtime work necessary, the cost of which is estimated to be double the normal production cost of Rs. 12 per unit. Fluctuating inventories result in an inventory carrying cost of Rs. 2 per unit/month. If the company fails to fulfil its sales commitment, it incurs a shortage cost of Rs. 4 per unit/month. The production capacities for the next three months are shown below :*

Month	*Production capacity*		*Sales*
	Regular	*Overtime*	
1	*50*	*30*	*60*
2	*50*	*0*	*120*
3	*60*	*30*	*40*

Formulate this problem as an LP model so as to minimize the total cost. **[Delhi M.B.A. 2000]**

Mathematical Formulation

We are given the following information :

Normal production cost : Rs. 12 per unit, Overtime cost : Rs. 24 per unit,

Inventory carrying cost : Rs. 2 per unit per month, Shortage cost : Rs. 4 per unit per month.

Let there be five sources of supply—three regular and two overtime (because second month overtime production is zero) production capacities. The demand for the three months will be the sales during these months.

It is assumed that all supplies against order have to be fulfilled and can be made in the subsequent month, if not possible during the current month of order with additional cost equivalent to shortage cost, *i.e.*, in month 2. The cumulative production of first two months in regular and overtime is of 130 units, while the orders are for 180 units. The balance can be supplied during month 3 at an additional production cost of Rs. 4.

The total cost (normal cost + inventory carrying cost + shortage cost, if any) is given below :

Month	*Cost (Rs.)*			*Overtime cost (Rs.)*		*Sales*
	Month 1	*Month 2*	*Month 3*	*Month 1*	*Month 2*	
Month 1	Rs. 12	Rs. 16	Rs. 20	Rs. 24	Rs. 32	60
Month 2	Rs. 14	Rs. 12	Rs. 16	Rs. 26	Rs. 28	120
Month 3	Rs. 16	Rs. 14	Rs. 12	Rs. 28	Rs. 24	40
Supply (units)	50	50	60	30	30	220

Decision variables : Let x_{ij} represents the number of units to be shipped from source of supply i ($i = 1, 2, 3$) to destination j ($j = 1, 2, 3, 4, 5$).

Objective function : The objective is to minimize the total cost involved, *i.e.*, to minimize the expression

$$z = \{(12x_{11} + 16x_{12} + 20x_{13} + 24x_{14} + 32x_{15}) + (14x_{21} + 12x_{22} + 16x_{23} + 26x_{24} + 28x_{25}) \\ + (16x_{31} + 14x_{32} + 12x_{33} + 28x_{34} + 24x_{35})\}.$$

Constraints : The constraints on demand and supply are :

(*i*) $$x_{11} + x_{12} + x_{13} + x_{14} + x_{15} = 60,$$
$$x_{21} + x_{22} + x_{23} + x_{24} + x_{25} = 120, \text{ and}$$
$$x_{31} + x_{32} + x_{33} + x_{34} + x_{35} = 40.$$

(*ii*) $$x_{11} + x_{21} + x_{31} = 50, \quad x_{12} + x_{22} + x_{32} = 50,$$
$$x_{13} + x_{23} + x_{33} = 60, \quad x_{14} + x_{24} + x_{34} = 30,$$
$$x_{15} + x_{25} + x_{35} = 30,$$

(*iii*) $$x_{ij} \ge 0 \text{ for all } i \text{ and } j$$ (*Non-negative restrictions*)

219. (Cargo Loading Problem). *A ship has three cargo holds, forward, aft and centre; the capacity limits are :*

Forward	*2,000 tonnes*	*100,000 m³*
Centre	*3,000 tonnes*	*135,000 m³*
Aft	*1,500 tonnes*	*30,000 m³*

The following cargoes are offered; the ship owners may accept all or any part of each commodity :

Commodity	*Amount (tonnes)*	*Volume per tonne* (m^3)	*Profit per tonne (Rs.)*
A	*6,000*	*60*	*60*
B	*4,000*	*50*	*80*
C	*2,000*	*25*	*50*

In order to preserve the trim of the ship, the weight in each hold must be proportional to the capacity in tonnes. The objective is to maximize the profit. Formulate the linear programming model for this problem.

Mathematical Formulation

Let x_{1A}, x_{2A}, x_{3A} be the weights (in kg.) of the commodity A to be accommodated in *forward, centre* and *aft* portions respectively. Similarly, let x_{1B}, x_{2B}, x_{3B} and x_{1C}, x_{2C}, x_{3C} be the corresponding weights (in kg.) of B and C. Then the *objective* is to maximize profit :

$$z = 60(x_{1A} + x_{2A} + x_{3A}) + 80(x_{1B} + x_{2B} + x_{3B}) + 50(x_{1C} + x_{2C} + x_{3C})$$

Various constraints are :

$$\left.\begin{array}{l} x_{1A} + x_{2A} + x_{3A} \le 6000 \\ x_{1B} + x_{2B} + x_{3B} \le 4000 \\ x_{1C} + x_{2C} + x_{3C} \le 2000 \end{array}\right\} \quad \textit{(Commodity cargo)}$$

$$\left.\begin{array}{l} x_{1A} + x_{1B} + x_{1C} \le 2000 \\ x_{2A} + x_{2B} + x_{2C} \le 3000 \\ x_{3A} + x_{3B} + x_{3C} \le 1500 \end{array}\right\} \quad \textit{(Weight capacity)}$$

$$\left.\begin{array}{l} 60x_{1A} + 50x_{1B} + 25x_{1C} \le 1,00,000 \\ 60x_{2A} + 50x_{2B} + 25x_{2C} \le 1,35,000 \\ 60x_{3A} + 50x_{3B} + 25x_{3C} \le 30,000 \end{array}\right\} \quad \textit{(Volume capacity)}$$

$$x_{iA} \ge 0, \; x_{iB} \ge 0 \text{ and } x_{iC} \ge 0 \text{ for } i = 1, 2, 3.$$ (*Non-negative restrictions*)

220. *WELLTYPE manufacturing company produces three types of typewriter: Manual typewriters, Electronic typewriters, and Deluxe Electronic typewriters. All the three models are required to be machined first and then assembled. The time required for the various models are as follows:*

Types	*Machine time (in hours)*	*Assembly time (in hours)*
Manual Typewriter	*15*	*4*
Electronic Typewriter	*12*	*3*
Deluxe Electronic Typewriter	*14*	*5*

The total available machine time and assembly time are 3,000 hours and 1,200 hours respectively. The data regarding the selling price and variable costs for the three types are:

	Manual	*Electronic*	*Deluxe*
Selling Price (Rs.)	*4,100*	*7,500*	*14,600*
Labour, material and other variable costs (Rs.)	*2,500*	*4,500*	*9,000*

The company sells all the three types on credit basis, but will collect the amounts on the first of next month. The labour, material and other variable expenses will have to be paid in cash. This company has taken a loan of Rs. 40,000 from a cooperative bank and this company will have to repay it to the bank on 1-4-2006. The TNC Bank from whom this company has borrowed Rs. 60,000 has expressed its approval to renew the loan.

The Balance Sheet of this Company as on 31-03-2006 is as follows:

Liabilities	Rs.	*Assets*	Rs.
Equity Share Capital	*1,50,000*	*Land*	*90,000*
Capital Reserve	*15,000*	*Building*	*70,000*
General Reserve	*1,10,000*	*Plant & Machinery*	*1,00,000*
Profit & Loss A/c	*25,000*	*Furniture & Fixtures*	*15,000*
Long-term Loan	*1,00,000*	*Vehicles*	*30,000*
Loan from TNC Bank	*60,000*	*Inventory*	*5,000*
Loan from Co-operative Bank	*40,000*	*Receivables*	*50,000*
		Cash	*1,40,000*
Total	*5,00,000*	*Total*	*5,00,000*

The company will have to pay a sum of Rs. 10,000 towards the salary for top management executives and other fixed overheads for the month. Interest on long-term loans is to be paid every month at 24% per annum. Interest on loan from TNC and Co-operative Banks may be taken to be Rs. 1,200 for the month. Also, this company has promised to deliver 2 Manual typewriters and 8 Deluxe Electronic typewriters to one of its valued customers next month. Also, make sure that the level of operations in this company is subject to the availability of cash next month. This company will also be able to sell all their types of typewriters in the market. The Senior Manager of this company desires to know as to how many units of each typewriter must be manufactured in the factory next month so as to maximise the profits of the company. Formulate this as a linear programming problem. The formulated problem need not be solved.

Mathematical Formulation

Let x_1, x_2 and x_3 denote the number of manual, electronic and deluxe electronic typewriters to be produced by the company next month. The profit contributions from the above three products are: Rs. 1,600, Rs. 3,000 and Rs. 5,600 respectively.

The objective, therefore, is to maximize $z = 1{,}600x_1 + 3{,}000x_2 + 5{,}600x_3$ subject to the constraints :

$$15x_1 + 12x_2 + 14x_3 \leq 3{,}000 \qquad \textit{(Machine time restriction)}$$

$$4x_1 + 3x_2 + 5x_3 \leq 1{,}200 \qquad \textit{(Assembly time restriction)}$$

From the balance-sheet, we observe that the cash availability

= Cash Balance + Receivables – Co-operative bank loan paid off – Top management salary *plus* fixed overheads – Interest on long-term loan – Interest on short-term loan.

= 1,40,000 + 50,000 – 40,000 – 10,000 – 2,000 – 1,200

= 1,36,800 (Interest on long-term loan is 24% p.a. paid every month, *viz.*, Rs. 2,000 p.m.)

Thus, $$2{,}500x_1 + 4{,}500x_2 + 9{,}000x_3 \leq 1{,}36{,}800 \qquad \textit{(Cash Requirement Restriction)}$$

Also, $$x_1 \geq 2,\ x_2 \geq 0 \text{ and } x_3 \geq 8.$$

All x_j $(j = 1, 2, 3)$ are also integral valued.

PROBLEMS

221. A firm manufactures headache pills in two sizes *A* and *B*. Size *A* contains 2 grains of aspirin, 5 grains of bicarbonate and 1 grain of codeine. Size *B* contains 1 grain of aspirin, 8 grains of bicarbonate and 6 grains of codeine. It is found by users that it requires at least 12 grains of aspirin, 74 grains of bicarbonate and 24 grains of codeine for providing immediate effect. It is required to determine the least number of pills a patient should take to get immediate relief. Formulate the problem as a standard LPP. ***[GGSIP Univ. B.B.A. 2011]***

222. Old hens can be bought for Rs. 2.00 each and young ones cost Rs. 5.00 each. The old hens lay 3 eggs per week and the young ones, 5 eggs per week, each being worth 30 paise. A hen costs Re. 1.00 per week to feed. If I have only Rs. 80.00 to spend for hens, how many of each kind should I buy to give a profit of more than Rs. 6.00 per week, assuming that I cannot house more than 20 hens? Write a mathematical model of the problem. ***[Pune M.B.A. 1999; Delhi B.F.I.A. 2008]***

223. An animal feed company must produce 200 lbs of a mixture containing the ingredients X_1 and X_2. X_1 costs Rs. 3 per lb. and X_2 costs Rs. 8 per lb. Not more than 80 lbs. of X_1 can be used and minimum quantity to be used for X_2 is 60 lbs. Find how much of each ingredient should be used if the company wants to minimise the cost. Formulate.

224. A factory engaged in the manufacturing of pistons, rings and valves for which the profits per unit are Rs. 10, 6 and 4 respectively wants to decide the most profitable mix. It takes one hour of preparatory work, ten hours of machining and two hours of packing and allied formalities for a piston. Corresponding time requirements for rings and valves are 1, 4 and 2 and 1, 5 and 6 hours respectively. The total number of hours available for preparatory work, machining and packing and allied formalities are 100, 600 and 300 respectively. Determine the most profitable mix, assuming that what all produced can be sold. Formulate the LPP.

225. A ship is to carry 3 types of liquid cargo—*X*, *Y* and *Z*. There are 3,000 litres of *X* available, 2,000 litres of *Y* available and 1,500 litres of *Z* available. Each litre of *X*, *Y* and *Z* sold fetches a profit of Rs. 20, Rs. 35 and Rs. 40 respectively. The ship has 3 cargo holds—*A*, *B* and *C*, of capacities 2,000, 2,500 and 3,000 litres respectively. From stability considerations, it is required that each hold be filled in the same proportion. Formulate the problem of loading the ship as a linear programming problem. State clearly all decision variables and constraints.

226. A firm produces three products *A*, *B* and *C*. It uses two types of raw materials *I* and *II* of which 5,000 and 7,500 units respectively are available. The raw material requirements per unit of the products are given below :

Raw material	*Requirement per unit of product*		
	A	*B*	*C*
I	3	4	5
II	5	3	5

The labour time for each unit of product *A* is twice that of product *B* and three times that of product *C*. The entire labour force of the firm can produce the equivalent of 3,000 units. The minimum demand of the three products is 600, 650 and 500 units respectively. Also the ratios of the number of units produced must be equal to 2 : 3 : 4. Assuming the profits per unit of *A*, *B* and *C* as Rs. 50, 50 and 80 respectively.

Formulate the problem, as a linear programming model in order to determine the number of units of each product which will maximize the profit.

[Madras MCA 1999]

227. A manufacturing company has four machine centres—machining, grinding, assembling and painting, to produce four products *A*, *B*, *C* and *D*. The available number of hours per month in each of these centres are 150, 40, 750 and 500 hours respectively. The number of hours required by each product in each of the centre are given below :

		A	*B*	*C*	*D*
Machining	...	1.5	1	2	1.5
Grinding	...	3	3	4	3
Assembling	...	6	5	6	7
Painting	...	3	4	4	3

The profit contribution for products *A*, *B*, *C* and *D* is Rs. 10, Rs. 15, Rs. 12 and Rs. 20 respectively. Assuming there is enough demand for these products, how much of each product should the company manufacture to maximise the total profit?

228. A firm buys castings of *P* and *Q* type of parts and sells them as finished product after machining, boring and polishing. The purchasing cost for castings are Rs. 3 and Rs. 4 each for parts *P* and *Q* and selling costs are Rs. 8 and Rs. 10 respectively. The per hour capacity of machines used for machining, boring and polishing for two products is given below :

Capacity (per hour)	*Parts*	
	P	*Q*
Machining	30	50
Boring	30	45
Polishing	45	30

The running costs for machining, boring and polishing are Rs. 30, Rs. 22.5 and Rs. 22.5 per hour respectively. Formulate the linear programming problem to find out the product mix to maximize the profit.

[Delhi M.B.A. (PT) 2002; Panjab B.Com. 2007]

229. A Mutual Fund Company has Rs. 20 lakhs available for investment in Government Bonds, blue chip stocks, speculative stocks and short-term bank deposits. The annual expected return and risk factor are given below :

Type of investment	*Annual expected return (%)*	*Risk factor (0-100)*
Government Bonds	14	12
Blue Chip Stocks	19	24
Speculative Stocks	23	48
Short-term Deposits	12	6

Mutual fund is required to keep at least Rs. 2 lakhs in short-term deposits and not to exceed an average risk factor of 42. Speculative stocks must be at most 20 per cent of the total amount invested. How should mutual fund invest the funds so as to maximize its total expected annual return? Formulate this as a Linear Programming Problem. Do not solve it.

[Himachal B.Tech. (Mech.) 2006]

230. An advertising company wishes to plan an advertising campaign in three different media—television, radio and magazines. The purpose of the advertising is to reach as many potential customers as possible. Results of a market study are given below (in Rs.) :

	Television		*Radio*	*Magazine*
	Prime day	*Prime time*		
Cost of an advertising unit	40,000	75,000	30,000	15,000
Number of potential customers reached per unit	4,00,000	9,00,000	5,00,000	2,00,000
Number of women customers reached per unit	3,00,000	4,00,000	2,00,000	1,00,000

The company does not want to spend more than Rs. 8 lakhs on advertising. It is further required that

(*i*) at least 2 million exposures take place among women;

(*ii*) advertising on television be limited to Rs. 5,00,000;

(*iii*) at least 3 advertising units be bought in prime day and two units during prime time; and

(*iv*) the number of advertising units on radio and magazine should each be between 5 and 10.

231. Formulate the following linear programming problems. Do not solve.

A publishing house publishes three weekly magazines—*Daily Life, Agriculture Today* and *Surf's Up*. Publication of one issue of each of the magazines requires the following amounts of production time and paper :

Magazine	*Production (hour)*	*Paper (kg)*
Daily Life	0.01	0.2
Agriculture Today	0.03	0.5
Surf's Up	0.02	0.3

Each week the publisher has available 120 hours of production time and 3,000 kg. of paper. Total circulation for all three magazines must exceed, 5,000 issues per week, if the company is to keep its advertisers. The selling price per issue is Rs. 22.50 for *Daily Life*, Rs. 40.00 for *Agriculture Today*, and Rs. 15.00 for *Surf's Up*. Based on past sales the publisher knows that the maximum weekly demand for *Daily Life* is 3,000 issues; for *Agriculture Today*, 2000 issues; and for *Surf's Up*, 6,000 issues. The production manager wants to know the number of issues of each magazine to produce weekly in order to maximize total sales revenue. ***[Delhi M.B.A. (Nov.) 2008]***

232. A media specialist plans to allocate advertising expenditure in three media whose unit costs of a message are Rs. 7,500, Rs. 6,250 and Rs. 5,000 respectively. The total advertising budget available for the year is Rs. 2,50,000. The first medium is monthly magazine and it is desired to advertise not more than once in one issue. At least five advertisements should appear in the second medium and the number of advertisements in the third medium should strictly lie between 6 and 10. The effective audience for unit advertisement in the three media is given below:

Medium	:	I	II	III
Expected affective audience	:	100,000	80,000	50,000

Formulate a linear programming problem to find the optimum allocation of advertisements in three media that would maximize the total effective audience. ***[Delhi PG Dip. in Glob. Bus. Oper. 2011]***

233. Evening shift resident doctors in a government hospital work five consecutive days and have two consecutive days off. Their five days of work can start on any day of the week and the schedule rotates indefinitely. The hospital requires the following minimum number of doctors working :

Sun.	*Mon.*	*Tues.*	*Wed.*	*Thurs.*	*Fri.*	*Sat.*
35	55	60	50	60	50	45

No more than 40 doctors can start their five working days on the same day. Formulate the general linear programming model to minimize the number of doctors employed by the hospital.

234. A Company manufactures three grades of paints—Venus, Diana and Aurora. The plant operates on a three-shift basis and the following data are available from the production records :

Requirement of resource		*Grade*		*Availability (capacity per month)*
	Venus	*Diana*	*Aurora*	
Special additive (kgs. per litre)	0.30	0.15	0.75	600 tonnes
Milling (kilolitres per machine shift)	2.0	3.0	5.0	100 machine shifts
Packing (kilolitres per shift)	12.0	12.0	12.0	80 shifts

There are no limitations on other resources. The particulars of sale forecasts and estimated contribution to overheads and profits are given below :

	Venus	*Diana*	*Aurora*
Maximum possible sales per month (in kilolitres)	100	400	600
Contribution (Rs. per kilolitre)	4,000	3,500	2,000

Due to commitments already made, a minimum of 200 kilolitres per month of *Aurora* has to be necessarily supplied during the next year.

Just as the company was able to finalise the monthly production programme for the next 12 months, an offer was received from a nearby competitor for hiring 40 machine shifts per month of milling capacity for grinding *Diana* paint, that can be spared for at least a year. However, due to additional handling and the profit margin of the competitor involved, by using this facility, the contribution from *Diana* will get reduced by Re. 1 per litre. Formulate (do not solve) the linear programming model for determining the monthly production programme to maximize contribution. *[Delhi M.B.A. (Nov.) 2003]*

235. Two alloys *A* and *B* are made from four different metals, *I, II, III* and *IV* according to the following specifications :

A :	at most 80% of *I*	*B* :	between 40% & 60% of *II*
	at most 30% of *II*		at least 30% of *III*
	at least 50% of *III*		at most 70% of *IV*

The four metals are extracted from three different ores whose constituents percentage of these metals, maximum available quantity and cost per tonne are as follows :

Ore	Maximum quantity (tonnes)	Constituent %					Price (Rs./tonne)
		I	*II*	*III*	*IV*	*Others*	
1	1,000	20	10	30	30	10	30
2	2,000	10	20	30	30	10	40
3	3,000	5	5	70	20	0	50

Assuming the selling prices of alloys *A* and *B* are Rs. 200 and Rs. 300 per tonne respectively, formulate the above as a linear programming problem selecting appropriate objective function and constraints. *[Poona M.B.A. 1998]*

236. A manufacturer of biscuits is considering four types of gift packs containing three types of biscuits– Orange cream (OC), Chocolate cream (CC), and Wafers (W). Market research study conducted recently to assess the preferences of the consumers shows the following types of assortments to be in good demand :

Assortments	*Contents*	*Selling price per kg. in Rs.*
A	Not less than 40% of OC Not more than 20% of CC Any quantity of W	20
B	Not less than 20% of OC Not more than 40% of CC Any quantity of W.	25
C	Not less than 50% of OC Not more than 10% of CC Any quantity of W	22
D	No restrictions	12

For the biscuits, the manufacturing capacity and costs are given below :

Type of biscuit	*Plant capacity (kg/day)*	*Manufacturing cost (Rs./kg)*
OC	200	8
CC	200	9
W	150	7

Formulate the LP model to find the production schedule which maximise the profit assuming that there are no market restrictions. *[Delhi M.B.A. April 1999]*

237. A 24-hour B.P.O. Company has the following minimal requirements for IT personnel and software engineers:

Time of day	Minimum number of IT personnel and software engineers required
Midnight — 4 A.M.	7
4 A.M. — 8 A.M.	20
8 A.M. — Noon	14
Noon — 4 P.M.	20
4 P.M. — 8 P.M.	10
8 P.M. — Midnight	5

Shift 1 follows immediately after shift 6. A B.P.O. employee works eight consecutive hours, starting at the beginning of one of the six periods. The company wants to determine how many IT experts should work each shift in order to minimize the total number of persons employed while still satisfying the staffing requirements.

Formulate the problem as a linear programming problem. *[Delhi PG Dip. in Glob. Bus. Oper. 2010]*

238. A company manufacturing laminated sheets is considering to adopt an optimum advertising strategy to advertise its product in South India for which it has an advertising budget of Rs. 5 lakh for the coming year. The company has decided to advertise in three magazines, say, M_1, M_2 and M_3. M_1 and M_3 are monthly and M_2 is a fortnightly. The company is targeting the audience having age 20—40 years, monthly income above Rs. 10,000, and education above SSC. These three characteristics have been assigned weights as 30 per cent, 50 per cent and 20 per cent respectively. The following table gives, for each of the three magazines, the percentage of readers having the above characteristics:

Characteristics	*Percentage of readers of magazine*		
	M_1	M_2	M_3
Age: 20—40 years	70	60	90
Monthly income: > Rs. 10,000	50	40	75
Education: above SSC	80	70	80

The advertisement may be in colour, or black and white. The efficiency index for colour advertisement may be taken to be 0.3 and that for black and white 0.2. The cost per insertion of colour, black and white advertisements and the readership for the three magazines are given below:

Magazine	*Cost (in Rs.) per insertion*		*Readership (in lakh)*
	Colour	*Black and white*	
M_1	18,000	12,000	4
M_2	16,000	10,000	3
M_3	19,000	15,000	2

To create the desired impact on the audience, it has been felt that during the coming year, at least five insertions are necessary in each of M_1 and M_3, and at least four in M_2. No issue will have more than one insertion.

Formulate the above as a linear programming problem to find the optimum advertising strategy for the coming year in order to maximize the expected effective exposure. (Do not solve the LPP.) *[Delhi M.I.B. (Apr.) 2001]*

239. Dr. Shilpa Soni, the head administrator at *XYZ* hospital must determine a schedule for nurses to make sure that there are enough nurses on duty throughout the day. During the day, the demand for nurses varies. Dr. Shilpa has broken the day into 12 two-hour periods. The slowest time of the day encompasses the three periods from 12:00 mid-night to 6:00 A.M., which beginning at midnight, require a minimum of 30, 20 and 40 nurses, respectively.

The demand for nurses steadily increases during the next four daytime periods. Beginning with the 6:00 A.M. to 8:00 A.M. period, a minimum of 50, 60, 80 and 80 nurses are required for these four periods, respectively. After 2:00 P.M. the demand for nurses decreases during the afternoon and evening hours. For the five two-hours periods beginning at 2:00 P.M. and ending at midnight 70, 70, 60, 50 and 50 nurses are required, respectively. A nurse reports for duty at the beginning of one of the two hour periods and works eight consecutive hours (which is required in the nurses contract). Dr. Shilpa Soni wants to determine a nursing schedule that will meet the hospital's minimum requirements throughout the day while using the minimum number of nurses.

[Delhi M.B.A. (Nov.) 2008]

240. In a chemical industry, two products *A* and *B* are made involving two operations. The production of *B* also results in a by-product *C*. The product *A* can be sold at a profit of Rs. 3 per unit and *B* at a profit of Rs. 8 per unit. The by-product *C* has a profit of Rs. 2 per unit. Forecasts show that up to 5 units of *C* can be sold. The company gets 3 units of *C* for each unit of *B* produced. The manufacturing times are 3 hours per unit for *A* on each of the operation one and two and 4 hours and 5 hours per unit for *B* on operation one and two respectively. Because the product *C* results from producing *B*, no time is used in producing *C*. The available times are 18 hours and 21 hours of operation one and two respectively. The company desires to know as to much *A* and *B* should be produced keeping *C* in mind to make the highest profit. Formulate LP model for this problem.

[C.A. Final (May) 2000]

241. A computer company produces three types of models, which are first required to be machined and then assembled. The time (in hours) for these operations for each model is given below:

Model	*Machine time*	*Assembly time*
P III	20	5
P II	15	4
Celeron	12	3

The total available machine-time and assembly-time are 1,000 hours and 1,500 hours, respectively. The selling price and other variable costs for three models are:

	P III	*P II*	*Celeron*
Selling price (Rs.)	3,000	5,000	15,000
Labour, material, and other variable costs (Rs.)	2,000	4,000	8,000

The company has taken a loan of Rs. 50,000 from a nationalised bank, which is required to be re-paid on 1.4.2001. In addition, the company has borrowed Rs. 1,00,000 from *XYZ* Cooperative Bank. However, this bank has given its consent to renew the loan.

The balance sheet of the company as on 31.3.2001 is as follows:

Liabilities	*(Rs.)*	*Assets*	*(Rs.)*
Equity share capital	1,00,000	Land	80,000
Capital reserve	20,000	Buildings	50,000
Profit and loss account	30,000	Plant and machinery	1,00,000
Long-term loan	2,00,000	Furniture, etc.	20,000
Loan from *XYZ* cooperative bank	1,00,000	Vehicles	40,000
Loan from nationalised bank	50,000	Cash	2,10,000
Total	5,00,000		5,00,000

The company is required to pay a sum of Rs. 15,000 towards the salary. Interest on long-term loan is to be paid every month at the rate of 18 per cent per annum. Interest on loan from *XYZ* cooperative and nationalised banks may be taken as Rs. 1,500 per month. The company has promised to deliver three *P* III, two *P* II, and five Celeron type of computers to M/s *ABC* Ltd. next month. The level of operation in the company is subject to the availability of cash next month.

The company manager is willing to know that how many units of each model must be manufactured next month, so as to maximise the profit.

Formulate a linear programming problem for the above. *[C.A. Final (May) 2001]*

242. A firm places an order for a particular product at the beginning of each month and the product is received at the end of the month. The firm sells the same product during the month from the stocks and it can sell any quantity.

The prices at which the firm buys and sells vary every month. The following table shows the projected buying and selling prices for the next four months :

Month	*Selling price (Rs.) (During the month)*	*Purchase price (Rs.) (Beginning of the month)*
April	—	75
May	90	75
June	60	60
July	75	—

The firm has no stock on hand as on April 1, and does not wish to have any stock at the end of July. The firm has a warehouse of limited size, which can hold a maximum of 150 units of the product.

Formulate this problem as an LP model to determine the number of units to buy and sell each month so as to maximize the profits from its operations.

[Delhi M.B.A. (Nov.) 2009]

MULTIPLE CHOICE QUESTIONS

1. Linear programming problem (LPP) must have an
   (*a*) Objective (goal) that we aim to maximize or minimize.
   (*b*) Constraints (restrictions) that we need to specify.
   (*c*) Decision variables (activities) that we need to determine.
   (*d*) all of the above.
2. Which of the following is not correct about LPP ?
   (*a*) All constraints must be linear relationships.
   (*b*) Objective function must be linear.
   (*c*) All the constraints and decision variables must be of either '≤' or '≥' type.
   (*d*) All decision variables must be non-negative.
3. Which of the following is not associated with an LPP ?
   (*a*) Proportionality (*b*) Uncertainty
   (*c*) Additivity (*d*) Divisibility.
4. Which of the following is correct ?
   (*a*) Linear programming takes into consideration the effect of time and uncertainty.
   (*b*) An LPP can have only two decision variables.
   (*c*) Decision variables in an LPP may be more or less than the number of constraints.
   (*d*) Linear programming deals with problems involving only a single objective.
5. A constraint in an LPP restricts
   (*a*) value of objective function (*b*) value of a decision variable
   (*c*) use of available resource (*d*) uncertainty of optimum value.
6. A constraint in an LPP is expressed as
   (*a*) an equation with = sign (*b*) inequality with ≥ sign
   (*c*) inequality with ≤ sign (*d*) any of the above.
7. Non-negativity condition is an important component of LPP, because
   (*a*) variables are interrelated in terms of limited resources
   (*b*) value of variables make sense and correspond to real world problems
   (*c*) value of variables should remain under the control of decision-maker
   (*d*) none of the above.
8. Minimization of objective function in LPP means
   (*a*) least value chosen among the allowable decisions
   (*b*) greatest value chosen among the allowable decisions
   (*c*) both (*a*) and (*b*)
   (*d*) none of the above.
9. Which of the following is not correct ?
   For the application of linear programming, the following requirements must hold :
   (*a*) There should be an objective which should be clearly identifiable and measurable in quantitative terms.
   (*b*) The activities to be included should be distinctly identifiable and measurable in quantitative terms.
   (*c*) The relationships representing the objective as well as the resource limitation considerations must be linear in nature.
   (*d*) There should be a series of infeasible alternative courses of action available to the decision-maker.
10. Which of the following is correct ?
   (*a*) Variables can be unrestricted in the context of an LPP.
   (*b*) For an LPP having *n*-decision variables, there must be an equal number of constraints.
   (*c*) Objective function specifies the dependent relationship between the decision variables and the objective fu
   (*d*) Linear programming is probabilistic in nature.

ANSWERS

1. (*d*)	2. (*c*)	3. (*b*)	4. (*c*)	5. (*c*)
6. (*d*)	7. (*b*)	8. (*a*)	9. (*d*)	10. (*c*)

REVIEW QUESTIONS

1. Explain the linear programming problem giving two examples. *[Madurai M.Com. 2002]*
2. What are the components of linear programming model ? Explain them in brief. *[Madras M.B.A. 2004]*
3. (*a*) What is linear programming? Explain by taking an example, what are its limitations? Discuss. *[Delhi M.B.A. (PT.) 2008]*

   (*b*) What is linear programming? Discuss its importance using one application to a business problem. *[Delhi M.B.A. (PT.) 2015]*
4. Two of the major limitations of linear programming are: assumptions of 'additivity' and 'single objective'. Elaborate by giving appropriate examples. *[Delhi M.B.A. 2008]*
5. Give the general form of a basic model of linear programming. *[Madurai M.Com. 2002]*
6. State any two applications of linear programming. *[Bharthidasan M.Com. 2007]*
7. Explain how the linear programming technique can be helpful in decision-making in the areas of marketing and finance. *[Delhi M.Com. 1998]*
8. Describe the major applications of linear programming in business and industry, pointing out limitations, if any. *[Delhi M.B.A. 2002]*
9. Explain the advantages of linear programming and its limitations. *[Pune M.B.A. 1999]*
10. Explain the major characteristics of a linear programming problem. *[Delhi M.Com. 2000]*
11. What are the major assumptions and limitations of linear programming ? *[Madurai M.Com. 2003; Madras M.B.A. 2004]*
12. Give the mathematical and economic structure of linear programming problems. What requirements should be met in order to apply linear programming ?
13. Describe the role of linear programming in Health Care Administration, bringing out limitations, if any. *[Delhi M.B.A. (HCA) PT 2003]*
14. Discuss the assumptions in linear programming. *[Madras M.B.A. (Nov.) 2006]*
15. "Linear Programming has no real application." Do you agree? Discuss. *[Delhi M.B.A. 2009]*
16. "Linear Programming is useful Management Science Technique, but it has some limitations." Discuss it and give appropriate examples. *[Delhi M.B.A. (Nov.) 2009]*