Interval estimation

In point estimation, a single value of a statistic is used as an estimate of the popular parameter But even the best possible point estimate may deviate enough from the true parameter value to make the estimate unsatisfactory.

For example, in point estimation to estimate the population mean we draw a sample and find the sample mean and it is taken as an estimate of the population mean. But there is a possibility of population mean to be either less or more than this estimate. If we can find out the lowest and the highest values within which the population mean is expected to lie, they set up two limits. These two limits give an interval. This interval within which population mean is expected to lie is called interval estimate.

It cannot be expected that the population mean will always lie within this interval. So find out the probability for the population mean to lie in this interval. This probability is denoted by $1-\alpha$. If $1-\alpha = .95$; then we are 95% confident that the population mean to lie outside this interval is only .05. ie $\alpha = .05$ (α is the level of significance)

Thus, the interval estimation is a statistical technique which consists in the determination

of two constants a and b such that $P\{a < \theta < b, \text{ for a given value of } t\} = 1-\alpha$ where α is the level of significance. The interval (a, b) within which the unknown value of the parameter θ is expected to lie, is known as confidence interval or fiducial interval. The limits 'a' and 'b' are known as confidence limits. $1-\alpha$ is called confidence coefficient.

Steps for finding confidence limit for the population mean

Let $\bar{x}$ be the sample mean

(SE - standard error)

$SE = \frac{\sigma}{\sqrt{n}}$ pop SD (dof = ∞) σ: pop SD

$\frac{s}{\sqrt{n}}$ sample SD, n large (dof = ∞) s: sample SD, n: sample size

$\frac{s}{\sqrt{n-1}}$ sample SD, n small

α = level of significance

t = table value dof = ∞ (large)

Confidence limit for population mean = $\bar{x} \pm (t \times SE)$

$\underbrace{a, b}_{\text{confidence limit}}$

Theory of Estimation

Types of estimate
- Point estimate
- interval estimate

→ Point estimate

Any statistic suggested as an estimate of an unknown parameter is called a point estimate of that parameter.

So under point estimation, we determine a value which maybe taken as an estimate of the population parameter.

for example if we suggest the mean of a random sample taken from the population as an estimate of the population mean it is point estimation.

The best estimate is one which falls nearer to the true value of the parameter to be estimated.

Estimator and Estimates

A sample statistic that is used to estimate a population parameter is called an estimator

eg if a sample mean is an estimator of population mean

An estimate is a specific observed value of a statistic.

To find an estimate we select a sample and compute the value of the estimater from that sample.
This computed value of the estimater is the estimate

for eg if sample mean is the estimater the particular value of the sample mean obtained from the sample is the estimate and the population mean is the parameter.

Criterias for a good estimater

I. An estimator should be unbiased.

A statistic 't' is said to be an unbiased estimater of population parameter 'θ' if

$$E(t) = \theta$$

for example; if 't' follows a sampling distribution and mean of that distribution is the value of the parameter θ then t is an unbiased estimater of θ.

result

Sample mean is an unbiased estimator of population mean

proof

Consider a population with mean μ

IMPORTANT

ie $E(x) = \mu$

Consider a random sample $x_1, x_2 \ldots x_n$ drawn from the population.

$E(x_1) = \mu \quad E(x_2) = \mu \quad \ldots E(x_n) = \mu$

$$\underbrace{\text{sample mean}}_{\bar{x}} = \frac{x_1 + x_2 \ldots + x_n}{n}$$

We have to prove $E(\bar{x}) = \mu$,

$$E(\bar{x}) = E\left(\frac{x_1 + x_2 + \ldots x_n}{n}\right)$$

$$= \frac{1}{n} E(x_1 + x_2 + x_3 + \ldots x_n)$$

$$= \frac{1}{n}\left[E(x_1) + E(x_2) + \ldots E(x_n)\right]$$

$$= \frac{1}{n}\left[\underbrace{\mu + \mu + \mu + \ldots \mu}_{n \text{ times}}\right]$$

$$= \frac{1}{n} \times n\mu$$

$$= \mu$$

II. The estimator should be consistent

A statistic 't' based on a sample of size n is said to be consistent estimater of the parameter θ, if it converges in probability to θ.

ie if $P(t \to \theta) \to 1$ as $n \to \infty$,

then t is a consistent estimator of θ

for eg sample mean is a consistent estimator of population mean

Proof since for large values of n sample mean tends to population mean

III. An estimator should be efficient
property is called efficiency

If t_1 and t_2 are two consistent estimators of parameter θ and if variance of t_1 is less than variance of t_2 for all n, then t_1 is more efficient than t_2.

ie An estimator with lesser variability is said to be more efficient and consequently more reliable than the other.

for eg Sample mean is more efficient than sample median as an estimator of population mean.

IV. An estimator should be sufficient

A statistic t is said to be sufficient estimator of parameter θ if it contains all the information in the sample regarding the parameter.

In other words a sufficient statistic utilizes all the information that a given sample can furnist about the ~~part~~ parameter.

NOTE

→ A sufficient estimator is more ~~eff~~ efficient when an efficient estimator exist.

→ It is always a consistent estimator.
→ It may or may not be unbiased

eg sample mean is a sufficient estimator of population mean.

Desirable properties

- → Unbiased →
- → Consistency
- → ~~Sufficiency~~
- → ~~Eff~~
- → Efficiency
- → Sufficiency

Methods used for point estimation

1) Method of maximum likelihood
2) Method of moments
3) Method of minimum Variance
4) Method of Least squares
5) Method of Minimum Chi-Square
6) Method of Inverse Probability

$1-\alpha$

Interval Estimation

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to make the estimate satisfactory.

For example in point estimation to estimate the population mean, we draw a sample and find the sample mean and it is taken as an estimate of the population mean.
But there is a possibility of population mean to be either less or more than this estimate.

If we can find out the lowest and highest values within which the population mean is expected to lie, they set up two limits.
These two limits give an interval.
This interval within ~~with~~ which population mean is expected to lie is called interval estimate.

It cannot be expected that the population mean will always lie within this interval.
So find out the probability for the population mean to lie in this interval.

This probability is denoted by $1-\alpha$.
If $1-\alpha = 0.95$, then we are 95% confident that the population ~~mean will~~ lie in this interval so that the probability for the population mean to lie outside this interval is only 0.05 ie $\alpha = \cancel{0.5}\ .05$ ('α' is the level of significance.)

Thus the interval estimation is a statistical technique which consists in the determination of two constants a and b such that $P\{a < \theta < b$, for a given value of $t\} = 1 - \alpha$

where α is the level of significance.

The interval (a, b) within which the unknown value of the parameter θ is expected to lie, is known as confidence interval or fiducial interval.

The limits 'a' and 'b' are known as confidence limits. $1 - \alpha$ is called confidence coefficient.

STEPS FOR FINDING CONFIDENCE LIMIT

confidence limit for population mean

$= \bar{x} \pm (t \times SE)$

[table of z-test is required]

Q) A random sample of size 100 has mean 45 and SD 15. find the range within which the population mean may lie.

confidence limit : $\bar{x} \pm (t \times SE)$

$n = 100$ $\quad s = 15$

$\bar{x} = 45$

$SE = \frac{s}{\sqrt{n}} = \frac{15}{\sqrt{100}} = \frac{15}{10}$

$SE = 1.5$

$\alpha = 0.05$ $\quad$ degree of freedom $= \infty$

$t = 1.960$

$45 \pm (1.96 \times 1.5)$

$= 45 \pm 2.94$

$(42.06 , 47.94)$

Q2 Random sample of size 16 has 53 as mean. The sum of the squares of deviations taken from mean is 150. Obtain 95% and 99% confidence limits for population mean

$n = 16$

$\bar{x} = 53$

$s = 3.061$

$SD = \sqrt{\frac{150}{16}}$

$= \sqrt{9.375}$

$SD = 3.061$

Confidence limit $= \bar{x} \pm (t \times SE)$

$SE = \frac{s}{\sqrt{n-1}} = \frac{3.061}{\sqrt{16-1}}$

$= \frac{3.061}{\sqrt{15}} = 3.0$

$SE = 0.79$

$\alpha = 0.05$ dof $\rightarrow n-1 \rightarrow 15$

$t = 2.131$

conf limit $= \bar{x} \pm (t \times SE)$

$= 53 \pm (2.131 \times 0.79)$

$= 53 \pm 1.6834$

$(51.3166, 54.6834)$

$\alpha = 0.01$ $\quad$ dof $\rightarrow n-1 = 15$

$t = 2.947$

$CI = 53 \pm (2.947 \times 0.79)$

$= 53 \pm 2.328$

$(50.672, 55.328)$

Q.3 With a sample size of 400, the calculated standard error of mean is 2 with mean of 120. What sample size would be required so that we could be 95% confident that the population mean is within ± 3.5 of the sample mean?

$n = ?$ $\quad$ $n' = 400$ $\quad$ ~~z~~ value at a 5% confidence level is 1.96

$SE = 2$

$\bar{x} = 120$ $\quad$ $\alpha = 0.05$

$\pm (t \times SE) = \pm 3.5$

$t = 1.96$ $\quad$ ~~$SE = \frac{s}{\sqrt{n'}}$~~ $\quad$ ~~$SE = \frac{5}{\sqrt{400}}$~~

$SE = \frac{s}{\sqrt{n'}}$

$2 = \frac{s}{\sqrt{400}}$

$s = 40$

$SE = \frac{s}{\sqrt{n}} = \frac{40}{\sqrt{n}}$

$1.96 \times \frac{40}{\sqrt{n}} = \pm 3.5$

$\frac{1.96 \times 40}{3.5} = \sqrt{n}$

$0.28 \times 80 = \sqrt{n}$

$\sqrt{n} = 501.76$

$n = \underline{\underline{502}}$

<u>Lecture notes</u>

[2, 5] ← closed interval

point → 2.58 (population mean)
sample mean may differ from actual population mean (a high / low can be)

Methods used for Point estimation

Commonly used methods for obtaining the Point estimation are

1. Method of Maximum likelihood
2. Method of moments
3. Method of minimum variance.
4. Method of least squares.
5. Method of minimum chi - square
6. Method of inverse probability

(1) Method of maximum likelihood

This is the most commonly used method of estimating the population parameter.

Maximum likelihood estimation is based on the assumption that different populations geneate different samples and that any given sample is more likely to have come from the given population than others. Let $x_1, x_2, \ldots\ldots\ldots x_n$ be a sample independently drawn from the population with parameter θ. Then $f(x_1, x_2, \ldots\ldots\ldots x_n, \theta)$ is called the likelihood function and denoted by L.

$\therefore$ Likelihood function L is the probability for selecting the sample $x_1, x_2, \ldots\ldots x_n$. That is, L is the probability for the sample $(x_1, x_2, \ldots x_n)$ to belong to the population. $\therefore L = P(x_1, x_2, \ldots\ldots x_n, \theta) = P(x_1, \theta) \times P(x_2, \theta), \ldots\ldots\ldots\ldots\ldots \times P(x_n, \theta)$ since $x_1, x_2, \ldots\ldots x_n$ are independent.

Then the principle of Maximum likelihood states that the statistic which maximises the likelihood function, L is the maximum likelihood estimator of the parameter. This means, that the statistic computed as estimator of the population parameter must maximise the likelihood of drawing the given sample.

Hence the maximum likelihood estimator of the parameter 'θ' is that value of θ which maximises the likelihood function, L. Therefore among all the possible estimators of θ, that which maximises L is the MLE.

By calculus method, L is maximum when $\frac{\partial L}{\partial \theta} = 0$ and $\frac{\partial^2 L}{\partial \theta^2}$ is negative.

$\frac{\partial L}{\partial \theta} = 0$ is called the likelihood equation and solving it we get the value of θ. The value of θ which makes $\frac{\partial^2 L}{\partial \theta^2}$ negative is the M. L. E.

Note: If L is maximum, log L is also maximum.

$\therefore$ To find the MLE, it is enough if we solve the equation $\frac{\partial \log L}{\partial \theta} = 0$.

Properties of MLE

(i) MLE are not necessarily be unbiased. But for large values of n, MLE may be unbiased estimators.

(ii) MLE are consistent.

(iii) MLE are most efficient.

(iv) MLE are sufficient, if sufficient estimtors exist.

(v) MLE have the invariance property [ie if 't' is the MLE of θ then f(t) is the MLE of $f(\theta)$].

(vi) The distribution of MLE tends to normal for large values of n.

(2) Methods of Moments

This is a simple method. Under this method, we find moments from the sample and equate them with the corresponding moments of the population. Then we get equations containing the parameters. Solving these equations we get the estimates of the population parameters.

Let $\theta_1, \theta_2, \ldots\ldots\ldots\ldots \theta_k$ be k parameters of the population. Let $\mu'_1, \mu'_2, \ldots\ldots \mu'_k$ be the first k raw moments of the population and let $m'_1, m'_2, \ldots\ldots m'_k$ be the first k raw moments of the sample. Then take $\mu'_1 = m'_1, \mu'_2 = m'_2$ and so on. Solving these equations we get the estimates of the parameters.

For example: μ'_1 is the population mean and m'_1 is the sample mean. Equating them we get the sample mean as the estimator of population mean.

(3) Method of least squares estimation

Consider a population with r^{th} moment about zero as μ'_r. Let

$x_1, x_2, \ldots\ldots x_n$ be a sample drawn from that population. Then $\sum [x^r - \mu'_r]^2 = E$, is the sum of the squares of the difference between the rth power of the observations in the sample and the rth moment of the population.

Then the estimate of μ'_r is obtained by minimising this sum E. By calculus method $\frac{\partial E}{\partial \mu'_r} = 0$. Solving this equation we get an estimate of μ'_r.

For example: We know that first moment about zero is the mean ie population mean $(\mu) = \mu'_1$.

$\therefore \quad \sum [x^1 - \mu'_1]^2 = E$, should be minimum

$$\therefore \quad \frac{\partial E}{\partial \mu} = 2\sum (x - \mu)(-1) = 0$$

$$\therefore \quad \sum (x - \mu) = 0 \qquad \therefore \sum x - n\mu = 0.$$

$$\therefore \quad \mu = \frac{\sum x}{n} \qquad \therefore \mu = \bar{x} \qquad \therefore \bar{x} \text{ is the estimate of } \mu$$

ie sample mean is the least square estimate of population mean.

This method is called least square, since we make the sum of the squares of the difference between r^{th} power of the observation and r^{th} moment, least.

(4) Method of minimum variance

Let θ be the population parameter, L be the likelihood function, 'c' be any constant, and 't' be a function of sample observation. Suppose $\frac{\partial \log L}{\partial \theta}$ can be expressed in the form $c(t - \theta)$, then 't' is the minimum variance unbiased estimator of θ.

In that case, variance of 't' is less than the variance of any other estimator of θ. Therefore 't' can be called minimum variance unbiased estimator. Then the Quantity $\frac{1}{c}$ is called the minimum variance.

<u>Rao - Crammer inequality (Rao - Cramer lower bound)</u>

The most efficient estimator is that which has the least variance. But there is a lower limit for the variance of any estimator. This is given by Rao - Crammer inequality.

Let $P(x, \theta)$ be the probability density function of the population, with parameter 'θ'. Let the range of variation of x be independent of θ.

Let $x_1, x_2, \ldots\ldots\ldots x_n$ be a sample independently drawn from the population. Let 'L' be the likelihood function. Let 't' be an unbiased estimator of θ.

Then the Rao Crammer inequality states that the variance of the estimator 't' is not less than the reciprocal of $E\left(\frac{\partial \log L}{\partial \theta}\right)^2$

$$\therefore \quad v(t) \geq \frac{1}{E\left[\left(\frac{\partial \log L}{\partial \theta}\right)^2\right]}$$

$$\therefore \quad \frac{1}{E\left[\left(\frac{\partial \log L}{\partial \theta}\right)^2\right]}$$ is the minimum variance.

If $\frac{\partial \log L}{\partial \theta}$ can be expressed in the form of $c\,(t - \theta)$ where 'c' is a constant or a function of θ and 't' is a function of $x_1, x_2, \ldots\ldots\ldots x_n$ then

$$c = \frac{1}{E\left[\left(\frac{\partial \log L}{\partial \theta}\right)^2\right]}$$

MODULE - IV

Statistical Inference - (Testing of Hypothesis)

Statistics and Parameter

A parameter is a function of the population values. It is a statistical measure derived from the population.

For example: population mean is a parameter. A function of the sample values is a sample statistic. It is a statistical measure derived from the sample. For example sample mean is a statistic

Thus measures obtained from population are population parameters while measures obtained from sample are sample statistics.

Statistical Inference

The primary objective of sample study is to draw inferences (or conclusions) about the population by examining only a part of the population.

Such inferences drawn are called statistical inferences. Statistical inference is therefore a process by which we draw conclusions about the population based on samples drawn from that population.

Ho: not influenced by

Two main branches of statistical inference are
1. Test of hypothesis 2. Estimation

1. Test of Hypothesis (statistical tests)

Statistical test of hypothesis is a process or procedure under which a statistical hypothesis is laid down and it is accepted or rejected on the basis of a random sample drawn from the population.
The tests conducted to accept or to reject the hypothesis are known as statistical tests of hypothesis.

commonly used statistical tests are Z-test, t-test, χ^2 test, F-test.

Procedure for testing hypothesis:

The steps followed in a test of hypothesis are
1. set up null hypothesis (H_0) and an alternative hypothesis (H_1) appropriate to the test to be conducted.

2. Decide the appropriate test criterion (such as Z-test, t-test, χ^2 test, F-test. etc)

3. Specify level of significance desired. usually level of significance specified is 5% (or 1%) Determine suitable level of significance. In the absence of any specific instruction, it should be normally 5%.

4. Calculate the value of the test statistic using appropriate formula.

5. Then obtain the table value of the test statistic corresponding to the level of significance and the degree of freedom.

6. Make decision about accepting or rejecting the null hypothesis. The decision depends on whether the computed value of the test statistic falls in the acceptance region or rejection region.
When the computed value of the test statistic is numerically less than the table value of the test statistic it falls in the acceptance region. Otherwise in the rejection region.

Statistical Hypothesis

In any test of hypothesis we begin with some assumptions about the population from which the sample is drawn. This assumption may be about the form of the population or about the parameters of the population. Such an assumption should be logically drawn. This assumption is called Hypothesis

A statistical hypothesis may therefore be defined as a tentative conclusion logically drawn concerning the parameter of the form of the distribution of the population.

For example, the assumption "the sample is drawn from a normal population with mean = 40 and S.D = 10" is a hypothesis.

Simple and Composite Hypothesis

A hypothesis may be simple or composite. If a hypothesis is concerning the population completely such as functional form and the parameter, it is called simple hypothesis. If a hypothesis is not simple, then it is a composite hypothesis.

For example: the hypothesis "Population is normal with mean 25 and SD = 10" is a simple hypothesis while the hypothesis "Population follows normal distribution with mean = 25" is a composite hypothesis.

Parametric and Non Parametric Hypothesis

A hypothesis which specifies only the parameters of the probability density function is called a parametric hypothesis.
If a hypothesis specifies only the form of the density function in the population, it is called a non-parametric hypothesis.

For example: the hypothesis "Mean of the population is 25" is parametric ~~an~~ while the hypothesis "population is normal" is non parametric.

Null and alternative hypotheses

A null hypothesis can be defined as a statistical hypothesis which is stated for the purpose of possible acceptance.

A null hypothesis is original hypothesis. Any hypothesis other than null hypothesis is called an alternative hypothesis. So when the null hypothesis is rejected we accept the other hypothesis known as alternative hypothesis.

The null hypothesis is denoted by H_0 and the alternative hypothesis by H_1.

for example: When we want to test whether the population mean is 65, the null hypothesis is 'Population mean is 65' and alternative hypothesis is 'population mean is not 65'.

Test Statistic

The decision to accept or to reject the null hypothesis made on the basis of a statistic computed from the sample. Such a statistic is called test statistic. It follows a sampling distribution. The commonly used test statistics are t, Z, F, χ^2 etc.

Sampling Distributions

Select a number of random samples of definite size from a given population and calculate some statistic from each of these samples. We find they are different in value.

So sample statistic is a variable and therefore it should have a distribution. The distrubution thus obtained for a sample statistic is called a sampling distribution It is a probability distribution.

A sample statistic is random variable and it has a probability distribution called sampling distribution of that statistic.

Sampling distribution of sample mean

Sample mean is a random variable, as it changes from one sample to another with a particular probability. Therefore sample mean has a probability distribution known as sampling distribution of sample mean.

Mean and SD of such a sampling distribution (when the sample is large) are respectively μ and $\frac{\sigma}{\sqrt{n}}$ where μ is the population mean and σ is the population standard deviation and 'n' is the size of the sample.

Standard Error

The standard deviation of a sampling distribution of a statistic is called Standard error (SE) of that statistic.

For example: standard deviation of the sampling distributions of the sample mean is $\sigma/\sqrt{n}$ where σ is the population SD and n is the sample size.

Therefore SE of sample mean $= \sigma/\sqrt{n}$

Uses of Standard Error:-

Standard error plays a very important role in the large sample theory and forms the basis of the testing of hypothesis.

~~(1) SE. is us~~

(1) SE is used for testing a given hypothesis

(2) SE gives an idea about the reliability of a sample. The reciprocal of SE is a measure of reliability of the sample.

(3) S.E. can be used to determine the confidence limits for population values like mean, proportion and standard deviation

Difference between Standard deviation and Standard error

1. Standard deviation is a measure of dispersion of statistical data (or a probability distribution). Standard error

is a measure of dispersion of a sampling distribution.

2. Standard deviation measures the variability or consistency of a statistical series. Standard error determines the precisions or reliability of an estimated value.

3. Standard deviation is calculated in relation to the mean of a series. Standard error can be calculated in relation to mean, median, SD, correlation coefficient. etc.

4. Standard deviation can be used to find out SE. Standard error can be used for estimation and testing of hypothesis.

Level of Significance :

Confidence with which a null hypothesis is accepted or rejected depends on what is called a significant level. The probability with which we may reject a null hypothesis when it is true, is called the level of significance

So when the level of significance is .05, it means that in the long run the statistician is rejecting true null hypothesis 5 times out of every 100 times.
Therefore the level of significance is the risk, a statistician is running in his decisions.

The level of significance is denoted by α.
Then, α = probability for rejecting H_0 when it is true.
The level of significance is usually determined before conducting the test of hypothesis.

Type I and Type II errors

In any test of hypothesis the decision is to accept or to reject a null hypothesis. This decision is based on the information supplied by the sample data. The four possibilities of the decision are:

(1) Accepting a null hypothesis when it is true
(2) Rejecting a null hypothesis when it is false
(3) Rejecting a null hypothesis when it is true
(4) Accepting a null hypothesis when it is false.

(1) and (2) are correct while (3) and (4) are errors. The last two cases are respectively known as type I and type II errors.

In the absence of the proper and adequate sample information it is likely that errors may occur.

Type I error is committed by rejecting null hypothesis even when that hypothesis is true.

Similarly Type II error is committed by

accepting a hypothesis when it is false.

Type I error → Rejecting Ho when Ho is true

Type II error → Accepting Ho when Ho is false.

For example: Suppose the test conducted on the basis of a sample drawn from the population leads to the conclusion that population mean ≠ 125 even though population mean is 125. This is type I error.
Similarly on the basis of the sample drawn, if the test conducted leads to the conclusion that the population mean is 125 even though population mean is not 125, it is type II error.

Degree of freedom

Degree of freedom is defined as the number of independent observations which is obtained by subtracting the number of constraints from the total number of observations.

For example, in testing the population mean, suppose the test statistic follows t-distribution, then degree of freedom = n − 1

Critical Region

In a test procedure we calculate a test statistic on which we base our decision. The range of variation of this statistic is divided into two regions, acceptance region and rejection region.

If the computed value of the test statistic falls in the rejection region we reject the null hypothesis. The rejection region is also known as critical region.

Critical region corresponds to a predetermined level of significance (say α) and the acceptance region corresponds to the region, ~~$1-\alpha$~~ $1-\alpha$

Size of the critical region

Probability for a selected sample to belong to the critical region is called the size of the critical region. Therefore size of the critical region = P [rejecting H_0 when H_0 is true]

Size of the critical region is also known as level of significance.

Best Critical Region

While fixing the size of the critical region our aim is to minimise both type I and type II errors. But decreasing one type of

error causes increase in the other type of error. This is undesirable. So we have to keep ~~up~~ the critical region as low as possible without allowing type II error to go up very much.

In testing of hypothesis, we can think of a number of critical regions each having same level of significance. Of all these critical regions that which has least type II error is called best critical region

Two tailed and one tailed tests

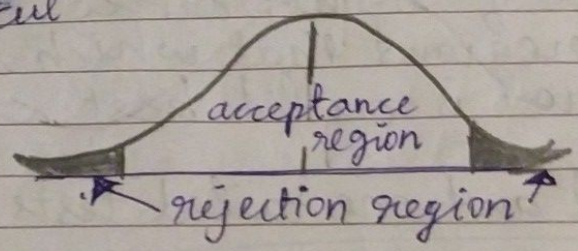
A two tailed test is one in which we reject the null hypothesis if the computed value of the test statistic is significantly greater than or lower than the critical value of the test statistic.

Thus in two tailed tests the critical region is represented by bothe tails. If we are testing hypothesis at 5% level of significance, the size of the acceptance region is 0.95 and the size of the rejection region is 0.05 on both sides together.

So if the computed value of test statistic falls either in the left tail or in the right tail, hypothesis is rejected. For example, if we are interested in testing the null hypothesis that the average

height of people is 156 cm. Then the rejection would be on both sides since the null hypothesis is rejected if the average height in the sample is much more than 156 or much less than 156.

In the following diagram, the rejection region is marked in left tail and in right tail



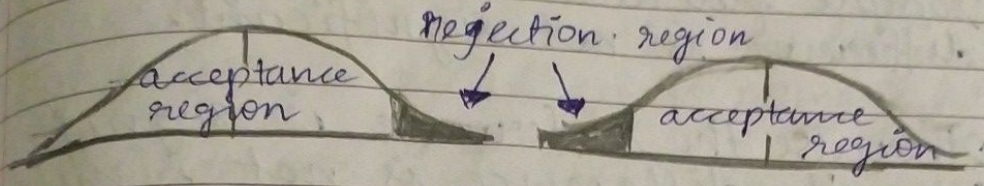
In one tailed test, the rejection region will be located in only one tail which may be either left or right, depending on the alternative hypothesis.

Thus in one tailed test critical region is represented by only one tail. For example, if we are interested in testing whether the average height of the people in the population is more than 156 cm, then we place the rejection completely on the right side and the test will therefore be one sided. Similarly if we want to test whether the average height is less than 156 cm the rejection region is completely on the left side.

Suppose if the level of significance is 0.5 then in the case of one tailed test the

size of the rejection region is 0.5 either falling in the left side only or in the right side only.

See the following diagram for one tailed test



Critical Value (Table Value)

The value of the test statistic which separates the rejection region from the acceptance region is called the critical value.

It depends on the level of significance and the alternative hypothesis

When Z test is applied (ie the test statistic follows normal distribution) the critical values for different levels of significance are

Level of Significance	Critical Value Two Tailed	One tailed
1%	2.58	2.33
5%	1.96	1.645

When the calculated value of the test statistic is numerically less than the critical value, we accept the null hypothesis.

Significant Difference

If the difference between a sample statistic and the corresponding population parameter is attributable ~~to~~ only to sample fluctuations, we say the difference is not significant.

Otherwise the difference is significant. When the difference is not significant we conclude that the sample statistic and population parameter are equal

Power of a test

Probability for rejecting the null hypothesis when the alternative hypothesis is true is called power of a test.

$\therefore$ Power of a test = 1 − P [type II error]

Most Powerful Test

The test based on the most powerful critical region is called most powerful test.

Neyman - Pearson Lemma

For a given hypothesis, any number of tests can be suggested. The best of them is that for which level of significance is less than ~~a~~ preassigned small number

and the power is maximum.

Therefore according to Neyman-Pearson, a level of significance should be fixed. Then consider all tests whose level of significance is less than this. Of these tests, select that which has the maximum power. Such a test is called most powerful test.

Large Sample and Small Sample tests

It is difficult to draw a clear-cut line of demarcation between large and small samples. It is generally agreed among statisticians that a sample is to be considered large if its size exceeds 30.

The statistical tests used for large samples are different from those used for small samples since the assumptions made in the case of large samples are not true for small samples.

For large samples the sampling distribution of the test statistic is normal distribution and therefore the statistical test applied is Z-test.

For small samples, the sampling distribution of the test statistic may be (1) Normal distribution (2) t-distribution (3) χ^2 distribution (4) F-distribution.

Therefore in the case of small samples statistical test applied may be Z-test, t-test, χ^2-test, F-test etc.

Z-test

Z test is applied when the test statistic follows normal distribution.

Uses of Z-test are

1. To test the given population mean when the sample is large or when the population SD is known

2. To test the equality of the two sample means when the samples are large or when the population SD is known

3. To test the population proportion
4. To test the equality of two sample proportions

5. To test the population SD when the sample is large
6. To test the equality of two sample standard deviations when the samples are large or when population standard deviations are known.

7. To test the equality of ~~co~~ correlation coefficients.

Assumptions in Z-test are:-

1) Sampling distribution of test statistic

is normal
Sample statistics are close to the population parameter and therefore for finding standard error, sample statistics are used in places where population parameters are to be used.

t - test

t test is applied when test statistic follows t-distribution

Applications (uses) of t-test are:-

1. To test the given population mean when the sample is small and the population SD is not known

2. To test the equality of two sample means when the samples are small and population S.D. is unknown

3. To test the difference in values of two dependent samples.

4. To test the significance of correlation coefficients.

Assumptions in t-test:-
1) The parent population from which the sample drawn is normal.
2) The sample observations are independent

3) The population SD σ is unknown

4) when the equality of two population means is tested, the samples are assumed to be independent and the population variances are assumed to be equal and unknown.

I Testing the given population mean

Problems

① It is claimed that a random sample of 100 tyres with mean life of 15269 km is drawn from a population of tyres which has a mean life of 15200 km and SD of 1248 km. Test the validity of the claim

two tailed test $\mu \neq \mu'$

$H_0 : \mu = 15200$ null hypothesis

$H_1 : \mu' \neq 15200$ alternative hypothesis

~~Z test~~ $\sigma = 1248$ km (population S.D.)

μ - population mean = 15200 km

$\bar{x}$ - sample mean = 15269 km

$n = 100$ tyres (sample size) large sample

$$Z = \frac{\bar{x} - \mu}{SE} = \frac{15269 - 15200}{124.8}$$

$$Z = \frac{69}{124.8} \quad Z = 0.552 \text{ (computed value)}$$

2 tailed test → 0.05

table value ⇒ 1.960

$SE = \frac{\sigma}{\sqrt{n}} = \frac{1248}{\sqrt{100}} = \frac{1248}{10}$

$SE = 124.8$

since calculated value ie 0.552 is less than table value ie 1.960 then accept null hypothesis

ie $0.552 < 1.960$

$\therefore$ Accept H_0

ie $\mu = 15200$

The claim is valid.

NOTE

	large sample (n)	small sample (n < 30)
σ (Known)	(z test) $SE = \sigma/\sqrt{n}$	(z test) $SE = \sigma/\sqrt{n}$
σ (unknown)	(z test) $SE = s/\sqrt{n}$	(t test) $SE = \frac{s}{\sqrt{n-1}}$

The average life of 26 electric bulbs were found to be 1200 hours with a standard deviation of 150 hours. Test whether these bulbs would be considered as a random sample from a normal population with mean 1300 hours

$n = 26$ (small sample size) $\bar{x} = 1200$

$s - 150$ hrs (sample SD)

$\mu = 1300$ (population mean)

$H_0 \Rightarrow \mu = 1300$ σ unknown

$H_1 \Rightarrow \mu' \neq 1300$

$SE = s/\sqrt{n-1} \rightarrow$ t test

$= \frac{150}{\sqrt{26-1}} = \frac{150}{\sqrt{25}}$

$SE = \frac{150}{5} = \underline{\underline{30}}$

$z = \frac{\bar{x} - \mu}{SE}$

$z = \frac{1200 - 1300}{30} = \frac{100}{30}$

$z = \underline{\underline{-10/3}} = \underline{\underline{-3.3}}$

t test

dof $\rightarrow n-1$

$\rightarrow 25$

level of significance (not specified)

$\therefore \rightarrow 0.05$

$\mu \neq \mu'$

2 tailed test

$\rightarrow$ bold 0.05

2.060

2 tailed test $\rightarrow 0.05$

table value $\rightarrow \underline{\underline{2.060}}$

~~Since~~ calculated value $\not>$ table value

ie $+3.3 \not> 2.060$

Accept H_0 (null hypothesis)

ie $\mu \neq \underline{\underline{1300}}$

A stenographer claims that she can take dictations at the rate of more than 120 words per minute. Of the 12 tests given to her she could perform an average of 135 words with a SD of 40. Is her claim valid? ($\alpha = 0.01$)

$n = 12$
$\bar{x} = 135$
$s = 40$
} t-test

$H_0 : \mu = 120$
$H_1 : \mu > 120$
} one tailed test

$SE = \frac{s}{\sqrt{n-1}}$

$SE = \frac{40}{\sqrt{12-1}} = \frac{40}{\sqrt{11}} = 12.06$

$t = \frac{\bar{x} - \mu}{SE} = \frac{135 - 120}{12.06} = 1.243$

Table value $\Rightarrow$ 2.718
[since dof $\rightarrow$ $n-1 \rightarrow 11$
and $\alpha = 0.01$)
(not bold)]

calculated value $\leq$ table value
ie $1.243 < 2.718$
Accept H_0 (null hypothesis)
ie $\mu = 120$

Her claim is not valid.

(t test because $n < 30$, sample sample and σ is unknown)

④ Prices of shares of a com

$\mu = 2000$
$n = 25$
$\bar{x} = 2200$
$\sigma = 300$

→ z test since σ is is known and n is small

$H_0 : \mu = 2000$
$H_1 : \mu > 2000$ } one tailed test

$$SE = \frac{\sigma}{\sqrt{n}} = \frac{300}{\sqrt{25}} = \frac{300}{5} = 60$$

$$z = \frac{\bar{x} - \mu}{SE} = \frac{2200 - 2000}{60} = \frac{200}{60}$$

$$z = 3.33$$

Table value · degree of freedom → $n - 1 = 25 - 1$
$\alpha = .05$ $= 24$

table value = 1.645

Reject H_0

Testing Significance of difference between two population means
OR Testing Equality of two population means

Procedure

1. Set up a null hypothesis that there is no significant difference between the two means ie $\mu_1 = \mu_2$

2. When population SD is known or when the samples are large the test applied is Z-test; otherwise t-test

3. Test statistic $= \frac{\bar{x}_1 - \bar{x}_2}{SE}$ where $\bar{x}_1$ and $\bar{x}_2$ are sample means.

SE can be obtained by one of the following formulae:

(a) When the two samples are drawn from populations with same standard deviation say (say 'σ') then $SE = \sigma\sqrt{\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}$

(b) When the two samples are drawn from populations with different standard deviations (say 'σ_1' and 'σ_2') then $SE = \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$

(c) When population standard deviations are not known and the samples are large then assuming population S.D are different $SE = \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$

(d) When population standard deviations are not known and samples are small, assuming population SD is the same for both populations,

$$SE = \sqrt{\frac{n_1 s_1^2 + n_2 s_2^2}{n_1 + n_2 - 2}\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}$$ where n_1 and n_2 are sample sizes and s_1 and s_2 are sample standard deviations

(4) Degree of freedom is $(n_1 + n_2 - 2)$ for t-test and degree of freedom $= \infty$ for z-test

(5) Obtain the table value

(6) When the calculated value is less than the table value numerically the test statistic falls in the acceptance region and so we accept the null hypothesis. Otherwise reject it.

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(5) The mean yield of wheat from a district A was 210 lbs with standard deviation = 10 lbs per acre from a sample of 100 plots. In another district B, the mean yield was 200 lbs, standard deviation = 12 lbs from a sample of 150 plots. Assuming that the standard deviation of yield in the entire state was 11 lbs, test whether there is any significant difference between the mean yield of the crops in the two districts.

A	B
$\bar{x}_1 = 210$	$\bar{x}_2 = 200$
$s_1 = 10$	$s_2 = 12$
$n_1 = 100$	$n_2 = 150$

$\sigma = 11$

$H_0 : \mu_1 = \mu_2$ } (null hypothesis)
$H_1 : \mu_1 \neq \mu_2$ } - two tailed test

Large sample size → z test

$$z = \frac{\bar{x}_1 - \bar{x}_2}{SE}$$

$$SE = \sigma\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$$

$$= 11\sqrt{\frac{1}{100} + \frac{1}{150}}$$

$$= 11\sqrt{0.01 + 0.007}$$

$$= 11\sqrt{0.017}$$

$$= 1.43$$

$$z = \frac{\bar{x}_1 - \bar{x}_2}{SE}$$

$$= \frac{210 - 200}{1.43}$$

$$= \frac{10}{1.43}$$

$$= 6.99$$ ie 7 (computed value)

~~z = ie 6.~~

Degree of freedom - infinity (∞) [∵ z test]
Level of significance - not specified = 0.05

∴ Table value = 1.960

Calculated value 7 is greater than table value 1.960 ie $7 > 1.960$
∴ we have to reject H_0
Accept H_1
ie $\mu_1 \neq \mu_2$

50 children were given special diet for a certain period and control group of 50 other children were given normal diet. Their average gain in weight were found

to be 7.2 lbs and 5.7 lbs respectively and the common standard deviation for gain in weight was 2 lbs. Assuming normality of the distribution would you conclude that the special diet really promoted weight?

special diet

$n_1 = 50$

$\bar{x}_1 = 7.2$

normal diet

$n_2 = 50$

$\bar{x}_2 = 5.7$

$\sigma = 2$

$\mu_1 > \mu_2$

$H_0 : \mu_1 = \mu_2$
$H_1 : \mu_1 > \mu_2$ } one tailed test

Sample size is large so Z-test

$$\text{Test statistic} = \frac{\bar{x}_1 - \bar{x}_2}{SE}$$

$$SE = \sigma\sqrt{\frac{1}{n_1} + \frac{1}{n_2}} = 2 \times \sqrt{\frac{1}{50} + \frac{1}{50}}$$

$$= 2 \times \sqrt{\frac{2}{50}} = 2\sqrt{0.04}$$

$$= 2 \times 0.2$$

$$SE = 0.4$$

$$TS = \frac{\bar{x}_1 - \bar{x}_2}{SE} = \frac{7.2 - 5.7}{0.4} = \frac{1.5}{0.4}$$

TS = 3.75

Table value $\alpha = 0.05$

degree of freedom = ∞

(one tailed test)

$\therefore$ Table value = 1.645

Since calculated value is greater than table value i.e $3.75 > 1.645$

we reject null hypothesis and accept alternative hypothesis

ie $\mu_1 > \mu_2$ (Accept H_1)

$\therefore$ The special diet promotes weight

Electric bulbs manufactured by X and Y companies gave the following result.

	X company	Y company
No of bulbs used	100	100
Mean life in hours	1300	1248
Standard deviation	82	93

Using standard error of the difference between mean, state whether there is any significant differences in the life of two makes

X

$n_1 = 100$

$\bar{x}_1 = 1300$

$s_1 = 82$

Y

$n_2 = 100$

$\bar{x}_2 = 1248$

$s_2 = 93$

$\sigma = ?$

Sample size is large $\rightarrow$ z test

$$SE = \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} = \sqrt{\frac{82^2}{100} + \frac{92^2}{100}}$$

$SE = \sqrt{67.24 + 86.49} = \sqrt{153.73}$

$SE = 12.4$

Test statistic $= \frac{\bar{x}_1 - \bar{x}_2}{SE}$

$= \frac{1300 - 1248}{12.4} = 4.19$

table value $\alpha = 0.05$

degree of fredom $= \infty$

TV $= 1.960$

$H_0: \mu_1 = \mu_2$
$H_1: \mu_1 \neq \mu_2$ } two tailed test

Since calculated value is greater than table value ie $4.19 > 1.960$
so we should reject null hypothesis
Reject H_0 ie Accept H_1
ie $\mu_1 \neq \mu_2$

Difference ~~betw~~ is highly significant

(8) A random sample of 200 villages was taken from district A and average population per village was 485 with SD 50 Another random sample of 250 villages from the same district gave an average population of 510 per village with SD of 40 Is the difference between the averages of the two samples statistically significant?

$n_1 = 200$ $\quad$ $n_2 = 250$

$\bar{x}_1 = 485$ $\quad$ $\bar{x}_2 = 510$

$s_1 = 50$ $\quad$ $s_2 = 40$

$H_0 : \mu_1 = \mu_2$
$H_1 : \mu_1 \neq \mu_2$ } two tailed test

(z test)

$$SE = \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} = \sqrt{\frac{50^2}{200} + \frac{40^2}{250}}$$

$$SE = \sqrt{\frac{2500}{200} + \frac{1600}{250}} = \underline{4.35}$$

Test statistic (z) $= \dfrac{\bar{x}_1 - \bar{x}_2}{SE} = \dfrac{200 - 250}{4.35}$

$= \dfrac{-25}{4.35} = -5.75$ ie $\underline{\underline{5.75}}$

Table value $\Rightarrow$ Two tailed

$\alpha = 0.05$

degree of freedom $= \infty$

$\therefore$ Table value $= \underline{\underline{1.960}}$

Since calculated value / calculate value is greater than table value. we reject H_0 and accept H_1

ie $\underline{\underline{\mu_1 \neq \mu_2}}$

The difference between the averages of the two samples is significant.

(9) The average number of articles produced by two machines per day are 200 and 250 with standard deviation 20 and 25 respectively on the basis of 25 day's production. Can you regard both the machines equally efficient at 1% level of significance?

$\alpha = 0.01$ (1%)

t test $\begin{cases} \text{sample size is small} \\ \sigma = \text{not given} \end{cases}$

$n_1 = 25$ $\quad$ $n_2 = 25$

$s_1 = 20$ $\quad$ $s = 25$

$\bar{x}_1 = 200$ $\quad$ $\bar{x}_2 = 250$

$$\left.\begin{array}{l} H_0 : \mu_1 = \mu_2 \\ H_1 : \mu_1 \neq \mu_2 \end{array}\right\} \text{two-tailed test}$$

$$\text{Test statistic (t test)} = \frac{\bar{x}_1 - \bar{x}_2}{SE}$$

$$SE = \sqrt{\frac{n_1 s_1^2 + n_2 s_2^2}{n_1 + n_2 - 2}\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}$$

$$SE = \sqrt{\frac{25 \times 400 + 25 \times 625}{25 + 25 - 2}\left(\frac{1}{25} + \frac{1}{25}\right)}$$

$$SE = \sqrt{\frac{10000 + 15625}{48}\left(\frac{1}{25}\right)}$$

$$= \sqrt{\frac{25625}{48} \times 0.08}$$

$$= \sqrt{533.8 \times 0.08} = \sqrt{42.704}$$

$SE = 6.535$

Test statistic $= \frac{\bar{x}_1 - \bar{x}_2}{SE} = \frac{200 - 250}{6.535}$

$= \frac{-50}{6.535} = -7.65 = 7.65$

Table value two tailed test

$\alpha = 0.01$

degree of freedom $= n_1 + n_2 - 2$

dof $= 48$

Table value $= 2.58$

$(7.65 > 2.58)$

Computed value is greater than table value ie Reject null hypothesis and accept alternative hypothesis

ie (Accept H_1) $\mu_1 \neq \mu_2$

The machines are not equally efficient.

(10) For a sample of 100 labourers from Kerala the average daily wages is 10.50 ~~and~~ with SD Rs 1.50. For a sample of 150 labourers from Tamil Nadu the corresponding wages is ₹ 8 and ₹ 1 respectively. Can you conclude that the average wages of workers in Kerala are more than that of workers in Tamil Nadu?

Kerala	Tamil Nadu
$n_1 = 100$	$n_2 = 150$
$\bar{x}_1 = 10.50$	$\bar{x}_2 = 8$
$s_1 = 1.50$	$s_2 = 1$

$H_0 : \mu_1 = \mu_2$
$H_1 : \mu_1 > \mu_2$ } one tailed test

Large sample so z test

test statistic $z = \dfrac{\bar{x}_1 - \bar{x}_2}{SE}$

$SE = \sqrt{\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}}$

$z = \dfrac{10.50 - 8}{0.17}$

$SE = \sqrt{\dfrac{(1.5)^2}{100} + \dfrac{1^2}{150}}$

$= \dfrac{2.5}{0.17}$

$SE = \sqrt{\dfrac{2.25}{100} + \dfrac{1}{150}}$

$= 14.705$

$SE = \sqrt{0.0291}$

(computed value)

$SE = 0.17$

Table value $\alpha = 0.05$

degree $= \infty$

Table value $= 1.645$

Since computed value greater than table value ie $14.705 > 1.645$
Reject H_0 and Accept H_1
ie $\mu_1 > \mu_2$

Hence, the average wages of workers in Kerala are more than that of workers in Tamil Nadu

III Testing Significance of difference in the case of dependent samples (Paired Observations)

We shall discuss two samples where the observation are not independent. ie observations in one sample are some way related to those in the other.

for eg:- We may impart some special coaching to a set of students. Their performance before and after will form two dependent samples.

Their performance before and after will form two dependent samples. The students in the two samples are the same.
A value from one sample with the corresponding value in the other sample are together taken and formed a pair. Then we get pair of observations

Here the hypothesis is "There is no difference between the two sets of observations".

Here we apply 't' test and 't' is computed by the formula.

$t = \frac{\bar{d}}{SE}$ where $\bar{d}$ is the mean of the difference between the values.

$SE = \frac{s}{\sqrt{n-1}}$ Degree of freedom = $n-1$

(11) ~~For the~~ Ten soldiers visit a rifle range for two consecutive weeks. For the first week, their scores are 67, 24, 57, 55, 63, 54, 56, 68, 33, 43 and during their second week, they score in the same order 70, 38, 58, 58, 56, 67, 68, 72, 42, 38. Examine if there is significant difference in their performance.

Ho : There is no difference (two-tailed test)

I	II	d	d^2
67	70	-3	9
24	38	-14	196
57	58	-1	1
55	58	-3	9
63	56	7	49
54	67	-13	169
56	68	12	144
68	72	-4	16
33	42	-9	81
43	38	5	25
		-47	699
		Σd	Σd^2

Dependent

$$t = \frac{\bar{d}}{SE}$$

$$\bar{d} = \frac{\Sigma d}{n} = \frac{-47}{10} = -4.7$$

$$SE = \frac{s}{\sqrt{n-1}} = \frac{6.91}{\sqrt{9}} = 2.3$$

$$s = \sqrt{\frac{\Sigma d^2}{n} - \left(\frac{\Sigma d}{n}\right)^2} = \sqrt{\frac{699}{10} - \left(\frac{-47}{10}\right)^2}$$

$$= \sqrt{69.9 - 22.09}$$

$s = 6.91$

$$t = \frac{\bar{d}}{SE} = \frac{-4.7}{2.3} = -2.04$$

Table value degree of freedom → $n-1$
$= 9$

$\alpha = 0.05$

Table value $= 2.262$

Since computed value is lesser than table value ie $2.04 < 2.262$
Accept null hypothesis ie Accept H_0

ie There is no significant difference (since we accept null hypothesis) in their performance

(12) A certain stimulus administered to each of the 12 patients resulted in the following increase of blood pressure 5, 2, 8, −1, 3, 0, −2, 1, 5, 0, 4, 6. Can it be concluded that the stimulus will in general be accomplished by an increase in blood pressure?

H_0: There is no increase (One tailed test)
alternative- there is increase!

Dependent samples ∴ $t = \frac{\bar{d}}{SE}$

d	d^2
5	25
2	4
8	64
-1	1
3	9
0	0
-2	4
1	1
5	25
0	0
4	16
6	36
31	185

$$\bar{d} = \frac{\Sigma d}{n} = \frac{31}{12}$$

$$\bar{d} = 2.58$$

One tailed test

so $SE = \frac{S}{\sqrt{n-1}}$

$$S = \sqrt{\frac{\Sigma d^2}{n} - \left(\frac{\Sigma d}{n}\right)^2}$$

$$= \sqrt{\frac{185}{12} - (2.58)^2}$$

$$S = 2.96$$

$$SE = \frac{S}{\sqrt{n-1}} = \frac{2.96}{\sqrt{11}} = 0.89$$

$$t = \frac{\bar{d}}{SE} = \frac{2.58}{0.89} = 2.9$$ Computed value

Table value degree of freedom $\rightarrow n-1 = 11$

level of significance $= 0.05$

one tailed test

Table Value $= 1.796$

$\Rightarrow$ Since computed value is greater than table value ie $2.9 > 1.796$

Reject H_0 ie Accept H_1 $\Rightarrow$ There is an increase in blood pressure

(13) In a test given to two groups of students, the marks obtained were as follows

Group 1: 18 20 36 50 49 36 34 49 41

Group 2: 29 26 28 35 30 44 46

Assuming that the group SD are same and that the marks normally distributed, test the hypothesis that the group means are equal.

$H_0 : \mu_1 = \mu_2$
$H_1 : \mu_1 \neq \mu_2$ } Two tailed test

$d = x - a$

DC	d	d^2
18	-19	361
20	-17	289
36	-1	1
50	13	169
49	12	144
36	-1	1
34	-3	9
49	12	144
41	4	16
333	0	1134

$\bar{x} = \frac{333}{9} = 37$

$S_1 = \sqrt{\frac{\Sigma d^2}{n} - \left(\frac{\Sigma d}{n}\right)^2}$

$= \sqrt{\frac{1134}{9} - \left(\frac{0}{9}\right)^2}$

$\therefore S_1 = \sqrt{126 - 0} = \sqrt{126}$

$S_1 = 11.23$

x	d	d^2
29	-5	25
26	-8	64
28	-6	36
35	1	1
30	-4	16
34	10	100
46	12	144
238	0	386

$$\bar{x}_2 = \frac{238}{7} = 34$$

$$s_2 = \sqrt{\frac{\Sigma d^2}{n} - \left(\frac{\Sigma d}{n}\right)^2}$$

$$s_2 = \sqrt{\frac{386}{7} - \left(\frac{0}{7}\right)^2} = \sqrt{55.14}$$

$$s_2 = 7.426$$

$$SE = \sqrt{\frac{n_1 s_1^2 + n_2 s_2^2}{n_1 + n_2 - 2}\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}$$

$$SE = \sqrt{\frac{9 \times 126 + 7 \times 55.14}{9 + 7 - 2}\left(\frac{1}{9} + \frac{1}{7}\right)}$$

$$SE = \sqrt{\frac{1134 + 385.98}{14} \times (0.254)} = \underline{\underline{5.25}}$$

$$T = \frac{\bar{x}_1 - \bar{x}_2}{SE} = \frac{37 - 34}{5.25} = \frac{3}{5.25} = 0.57$$

degree of freedom $= n_1 + n_2 - 2 = 9 + 7 - 2 = 14$

α, level of significance $= 0.05$

$\therefore$ Table value $= 2.145$

Since calculated value is less than table value ie $0.57 < 2.145$, we accept null hypothesis ie Accept H_0

ie $\mu_1 = \mu_2$

Group means are equal

prices of shares of a company on the different days in a month were found to be
x : 66 65 69 70 69 71 70 63 64 68
Discuss whether mean price of shares in a month is 65.

$a = 70$

x	$d = x - a$	d^2
66	-4	16
65	-5	25
69	1	1
70	0	0
69	-1	1
71	1	1
70	0	0
63	-7	49
64	-6	36
68	-2	4
675	-25	133

$$\bar{x} = \frac{675}{10} \quad \left(\frac{\Sigma x}{N}\right)$$
$$= 67.5$$

$$s = \sqrt{\frac{\Sigma d^2}{n} - \left(\frac{\Sigma d}{n}\right)^2}$$

$$s = \sqrt{\frac{133}{10} - \left(\frac{-25}{10}\right)^2}$$

$$= \sqrt{13.3 - 6.25}$$
$$s = \sqrt{7.05}$$
$$s = 2.655$$

$H_0 : \mu = 65$
$H_1 : \mu \neq 65$ } two tailed

$n = 10$ (sample is small)

Here the sample is small and σ is unknown
It follow T- test and it is a two - tailed test.

$$SE = \frac{s}{\sqrt{n-1}} = \frac{2.655}{\sqrt{10-1}} = \frac{2.655}{\sqrt{9}} = \frac{2.655}{3}$$

$$SE = 0.885$$

$$T = \frac{\bar{x} - \mu}{SE} = \frac{67.5 - 65}{0.885} = 2.824$$

Degree of freedom $= n - 1 = 10 - 1 = 9$

$\alpha = 0.05$

$\therefore$ Table value for T test with degree of freedom 9 and level of significance 0.05 is 2.262

ie Table value $= 2.262$

Here table value is less than computed value so we reject null hypothesis H_0

ie Accept H_1 ie $\mu \neq 65$

$\therefore$ The mean price of shares in month is not 65.

(15) From the following data obtained from a sample of persons calculate the standard error of the mean

Earnings:	0-10	10-20	20-30	30-40	40-50	50-60	60-70	70-80
No of persons:	50	100	150	200	200	100	100	100

If the average of the population were Rs 42, what conclusion can you arrive about the reliability of sample?

$a = 45 \qquad c = 10$

$$\left.\begin{aligned} H_0 &: \mu = 42 \\ H_1 &: \mu \neq 42 \end{aligned}\right\} \text{Two-tailed test}$$

Earning	f	x	$d' = \frac{x-45}{10}$	fd'	fd'²
0-10	50	5	-4	-200	800
10-20	100	15	-3	-300	900
20-30	150	25	-2	-300	600
30-40	200	35	-1	-200	200
40-50	200	45	0	0	0
50-60	100	55	1	100	100
60-70	100	65	2	200	400
70-80	100	75	3	300	900
	1000			-400	3900

Now

$\Sigma f = 1000 \quad \Sigma fd' = -400 \quad \Sigma fd'^2 = 3900$

Mean, $\bar{x} = a + \frac{\Sigma fd'}{n} \times c$

$= 45 + \frac{-400}{1000} \times 10$

$\bar{x} = 41$

$s = \sqrt{\frac{\Sigma fd'^2}{n} - \left(\frac{\Sigma fd'}{n}\right)^2} \times c$

$= \sqrt{\frac{3900}{1000} - \left(\frac{-400}{1000}\right)^2} \times c$

$s = 19.34$

Here the sample is large and σ is unknown so it follows z test.

$SE = \frac{s}{\sqrt{n}} = \frac{19.34}{\sqrt{1000}} = \frac{19.34}{31.62} = 0.61$

$$z = \frac{\bar{x} - \mu}{SE} = \frac{41 - 42}{0.61} = -1.64$$

The table value for z test with degree of freedom infinity and level of significance 0.05 is 1.960

Here the computed value is less than table value ie $1.64 < 1.960$
So we accept null hypothesis, H_0

ie Accept H_0 $\mu = 42$

Problems

(18) A sample of size ~~400 was~~ 900 items is taken from a population with SD 15. The mean of the sample is 25. Test whether the sample has come from a population with mean 26.8. (20)

$n = 900$

~~$\bar{x} = 15$~~ $\bar{x} = 25$ z test since large simple

$\sigma = 15$

$\mu = 26.8$

$H_0 : \mu_1 = 26.8$
$H_1 : \mu_1 \neq 26.8$ } two tailed test

z test so test statistic $z = \frac{\bar{x} - \mu}{SE}$

$SE = \sigma / \sqrt{n}$

$$SE = \frac{15}{\sqrt{900}} = \frac{15}{30} = 0.5$$

$$z = \frac{\bar{x} - \mu}{SE} = \frac{25 - 26.8}{0.5} = \frac{-1.8}{0.5}$$

$z = -3.6$ ie $z = 3.6$

table value

degree of freedom $= \infty$

level of significance $\alpha = 0.05$

Table value $= 1.960$

since computed value greater than table value

ie $3.6 > 1.960$

Accept alternative hypothesis ie Accept H_1

ie $\mu_1 \neq \mu_2$

The sample has not come from a population with mean 26.8.

(20) A random sample of 200 tins of coconut oil gives an average weight of 4.95 kg with a SD of 0.21 kg. Do we accept the hypothesis of net weight of 5 kgs per tin at 1% level?

$n = 200$

$\bar{x} = 4.95$

$\sigma = 0.21$

$\mu = 5$

$\alpha = 1\% = \frac{1}{100} = 0.01$

$$\left.\begin{array}{l} H_0 : \mu = 5 \\ H_1 : \mu \neq 5 \end{array}\right\} \text{two tailed test}$$

z test so test statistic $z = \frac{\bar{x} - \mu}{SE}$

$$SE = \frac{\sigma}{\sqrt{n}} = \frac{0.21}{\sqrt{200}}$$

$$SE = \frac{0.21}{14.14} = 0.0148$$

$$z = \frac{\bar{x} - \mu}{SE} = \frac{4.95 - 5}{0.0148} = \frac{-0.05}{0.0148}$$

$$z = -3.378 \quad \text{ie } 3.37$$

Tabla value

degree of freedom = ∞ $\quad \alpha = 0.05$

Table value = 1.960

since computed value greater than table value we Accept alternative hypothesis ie Accept H_1 ie $\mu \neq 5$. ∴ We donot accept the hypothesis of net weight of 5 kgs.

(22) A machine is designed to produce insulating washers for electric devices of average thickness of 0.25 cm. A random sample of 10 washers was to have an average thickness of 0.024 cm with a standard deviation of 0.002 cm. Test the significance of the deviation. Value of t for 9 degrees of freedom at 5% level is 2.262.

$n = 10$

$\mu = 0.002$

$\bar{x} = 0.024$

$\sigma = 0.25$

A college conducts both day and night classes intended to be identical. A sample of 100 day-students yield examination results as under $\bar{x} = 72.4$ $\sigma = 14.8$
A sample of 200 night-students yields examination result as under
$\bar{x} = 73.9$, $\sigma = 17.9$
Are the two means statistically equal at 10% level.

Day
$n = 100$
$\sigma_1 = 14.8$
$\bar{x} = 72.4$

Night
$n = 200$
$\sigma_2 = 17.9$
$\bar{x} = 73.9$

Sample size is large → z test

$$SE = \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} = \sqrt{\frac{(14.8)^2}{100} + \frac{(17.9)^2}{200}}$$

$$SE = \sqrt{\frac{219.04}{100} + \frac{320.41}{200}} = \sqrt{2.1904 + 1.60205}$$

$$SE = \sqrt{3.792} = 1.947$$

$$SE = 1.947$$

$$T = \frac{\bar{x}_1 - \bar{x}_2}{SE} = \frac{72.4 - 73.9}{1.947}$$

$$= \frac{-1.5}{1.947} = -0.77$$

$$= -0.8$$

$z = 0.8$

Degree of freedom $= \infty$
Level of significance $= 0.10$
Table value is $\underline{1.645}$

$H_0: \mu_1 = \mu_2$
$H_1: \mu_1 \neq \mu_2$ } two tailed test

Since the computed value is less than table value ie $0.8 < 1.645$,
we Accept H_0

ie $\underline{H_0 : \mu_1 = \mu_2}$

The two means <u>are</u> statistically equal

Q8 Of the two salesman, X claims that he has made larger sales than Y. For the accounts examined with which were comparable for the two men, results were

	No of sales	Average size	SD
X	10	6200	690
Y	17	5600	600

Do these average size of sales figures differ significantly? Explain your result.

X

$n_1 = 10$

$\bar{x}_1 = 6200$

$s_1 = 690$

Y

$n_2 = 17$

$\bar{x}_2 = 5600$

$s_2 = 600$

sample size small → t-test
and σ unknown

$$SE = \sqrt{\frac{n_1 s_1^2 + n_2 s_2^2}{n_1 + n_2 - 2}\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}$$

$$SE = \sqrt{\frac{10 \times 690 \times 690 + 17 \times 600 \times 600}{10 + 17 - 2}\left(\frac{1}{10} + \frac{1}{17}\right)}$$

$$SE = \sqrt{\frac{4761000 + 6120000}{25}(0.1589)}$$

$$SE = \sqrt{\frac{10881000}{25}(0.1589)} = \sqrt{435240 \times 0.1589}$$

$$SE = \sqrt{69159.636} \qquad SE = 262.98$$

$$t = \frac{\bar{x}_1 - \bar{x}_2}{SE} = \frac{6200 - 5600}{262.98} = \frac{600}{262.98}$$

$$t = 2.28$$

$H_0 : \mu_1 = \mu_2$
$H_1 : \mu_1 \neq \mu_2$ } Two tailed test

Computed value = 2.28

Table value

degree of freedom $= n_1 + n_2 - 2 = 25$

level of significance, $\alpha =$

Table value =

Since