Estimating the Age of Universe via Interacting Tachyonic Scalar Field

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Introduction

The concept of dark energy has been proposed to explain the observed accelerated expansion of the universe. One popular and trivial choice of source of dark energy is the scalar field. We choose the tachyonic field, one of the scalar fields, as a candidate for dark energy and discuss it in a model where matter and dark energy (tachyonic scalar field) are allowed to interact with each other. The interaction term is considered to be linear in the energy density of matter and Hubble's parameter. With this interacting picture, we obtain the Age of Universe(AOU) for the different values of the coupling parameter and also obtain the possible constraints on the coupling parameter.

Background

The dynamics of a spatially flat universe can be obtained by considering the cosmological principle which states that the universe on large scales is homogenous and isotropic. From such an assumption follows the Robertson-Walker metric for the universe. The Friedmann equation arises when the equations of General Relativity are applied to such a metric,

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3c^2}\rho(t)$$

Conservation of energy leads to another equation relating the pressure of the components of the universe and their corresponding time variation of the energy density,

$$\dot{\rho} + 3\frac{\dot{a}}{a}(\rho + p) = 0$$

The equation of state gives a direct relation between the pressure of the components of the universe and their energy densities,

$$p = \omega\rho$$

These three equations together specify the complete dynamics of the universe.

Tachyonic Scalar Field

We consider a spatially homogenous tachyonic scalar field (TSF) with lagrangian

$$\mathcal{L} = -V(\phi)\sqrt{1 - \partial^\mu \partial_\mu \phi}$$

This gives the stress-energy tensor as

$$T^{\mu\nu} = \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)}\partial^\nu \phi - g^{\mu\nu}\mathcal{L}$$

From the spatial and temporal parts of the diagonal terms, we get for such a scalar field,

$$\rho = \frac{V(\phi)}{\sqrt{1 - \partial^\mu \partial_\mu \phi}}$$

and

$$p = -V(\phi)\sqrt{1 - \partial^\mu \partial_\mu \phi}$$

Thus from these, it follows that for a spatially homogenous TSF

$$p = -(1 - \dot{\phi}^2)\rho$$

For $\dot{\phi} = 0$, the equation becomes $p = -\rho$ which mimics the behaviour of Einstein's cosmological constant.

Interaction term

Here we consider the interaction in the form of energy exchange between matter and dark energy. This exchange can be modeled by modifying the continuity equations such that the two components violate the conservation of energy individually, but satisfy the conservation when considered together. The modification is of the form

$$\dot{\rho_\phi} + 3\frac{\dot{a}}{a}(1 + \omega_\phi)\rho_\phi = -Q$$

$$\dot{\rho_m} + 3\frac{\dot{a}}{a}(1 + \omega_m)\rho_m = Q$$

where $\omega = p/\rho$. Based on phenomenological arguments, we fix the exchange term $\mathcal{Q}$ as linear in ρ_m and to fix the dimensionality Hubble's parameter is included. Thus

$$\mathcal{Q} = \beta H \rho_m$$

where β is a dimensionless coupling parameter.

Evolution of Energy Densities

The first of the above two differential equations can be solved to give the evolution of ρ_m with scale factor as

$$\frac{\rho_m}{\rho_m^0} = \left(\frac{a}{a^0}\right)^{-3(1+\omega_m)+\beta} = \left(\frac{a}{a^0}\right)^{-\gamma}$$

Using this relation, the second differential equation also can be solved to relate ρ_ϕ to the scale factor as

$$\frac{\rho_\phi}{\rho_\phi^0} = \beta\frac{\rho_m^0}{\rho_\phi^0}\frac{1}{3(1+\omega_m) - \beta}\left(\left(\frac{a}{a^0}\right)^{-3(1+\omega_m)+\beta} - 1\right) + 1$$

These two relations are crucial as they are vital to solve for the Age of the universe.

Age of the Universe

Knowing the relation between ρ_m, ρ_ϕ and the scale factor a, the Friedmann equation gives a differential equation of a differentiated with respect to time.

$$\frac{\dot{x}}{x} = \sqrt{\frac{8\pi G \rho_m^0}{3c^2}}\sqrt{x^{-\gamma}\left(1 + \frac{\beta}{\gamma}\right) + \left(\frac{\rho_\phi^0}{\rho_m^0} - \frac{\beta}{\gamma}\right)}$$

This can be solved to obtain the Age of the Universe which is the time difference between the events in the history of the universe where $a = 0$ (big bang) and $a = 1$ (the present time).

$$t_{AOU} = \sqrt{\frac{3c^2}{8\pi G \rho_m^0}}\int_0^1 \frac{dx}{x\sqrt{\frac{1}{x^\gamma}\left(1 + \frac{\beta}{\gamma}\right) + \left(\frac{\rho_\phi^0}{\rho_m^0} - \frac{\beta}{\gamma}\right)}}$$

Using the values of the constants, we obtain

$$t_{AOU}(\beta) = \frac{2.09567\,{}_2F_1(0.5, 0.5; 1.5; 1.111\beta - 2.333)}{\sqrt{3. - 1.\beta}}$$

if $\beta < 3$

For $\beta \geq 3$ the integral does not converge and is therefore indeterminate. Thus such would not be a case of a real universe. Therefore for any real universe to have its dark energy modeled by an interacting tachyonic scalar field, then the coupling constant cannot be greater than or equal to 3.

Age of the Universe as a function of the coupling parameter.

The above-obtained equation can be plotted to visualize the Age of the Universe as a function of the coupling constant.

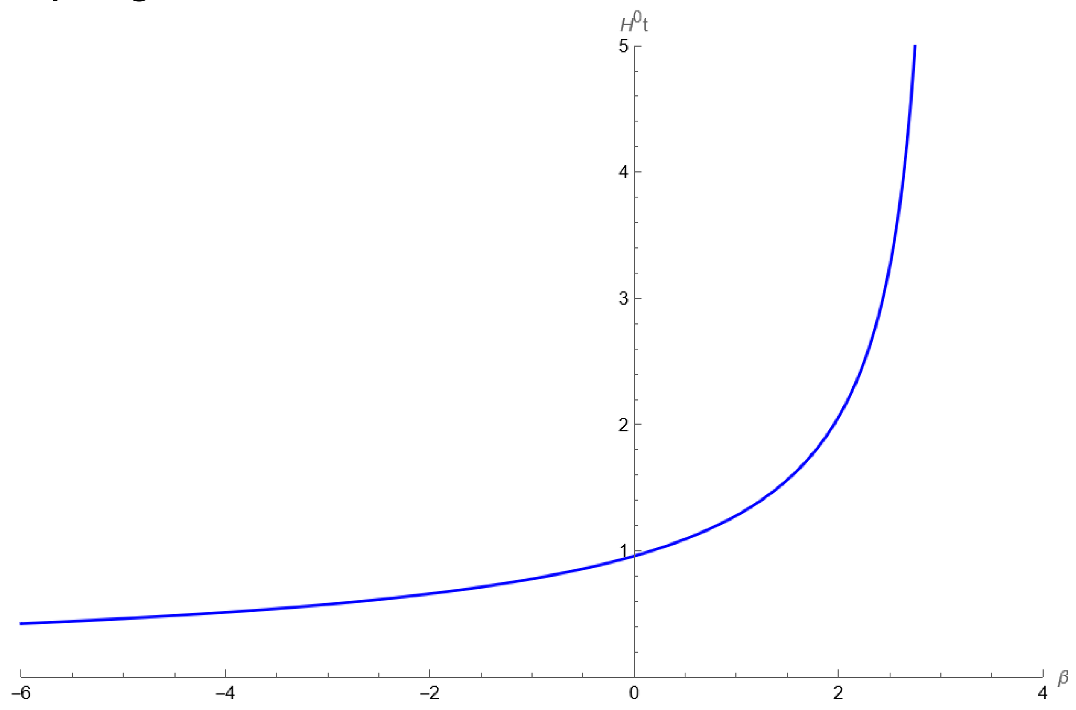

Figure: The variation of the Age of the Universe with the coupling constant.

The Age of the Universe for different values of the coupling parameter β is tabulated.

β	AOU	β	AOU
-2.	0.659605	0.5	1.09209
-1.75	0.685162	0.75	1.17692
-1.5	0.713089	1.	1.27883
-1.25	0.743752	1.25	1.40403
-1.	0.777598	1.5	1.56232
-0.75	0.815184	1.75	1.77029
-0.5	0.857206	2.	2.05868
-0.25	0.904552	2.25	2.49269
0.	0.958375	2.5	3.24407
0.25	1.0202	2.75	5.00924

For a universe with no interaction $\beta = 0$, we recover the Age of the Universe we live in, i.e. $0.958375 H_0^{-1}$

Conclusion

In this work, the Age of the Universe is calculated as a function of the coupling parameter β. It is also found that for $\beta \geq 3$ the age of the universe does not converge and hence becomes indeterminate, thus putting an upper limit on the possible values of coupling parameters for real universes.

References

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