Numerical Analysis II

Simpson 1/3rd Method

1. The total mass of a variable density rod is given by

$$m = \int_0^L \rho(x)\, A_c(x)\, dx$$

where m = mass, ρ (x) = density, $Ac(x)$ = cross-sectional area, x = along the rod and L = the total length of the rod. The following data has been measured for a 10-m length rod. Determine the mass in kilograms to the best possible accuracy.

x, m	0	2	3	4	6	8	10
ρ, g/cm^3	4.00	3.95	3.89	3.80	3.60	3.41	3.30
A_c, cm^2	100	103	106	110	120	133	150

2. An 11-m beam is subjected to a load, and the shear force follows the equation

$$V(x) = 5 + 0.25\, x^2$$

where V is the shear force and x is length in distance along the beam. We know that $V = dM/dx$, and M is the bending moment. Integration yields the relationship

$$M = M_o + \int_0^x V\, dx$$

If Mo is zero and x = 11, calculate M using Simpson's rules. Take 1-m increments for this calculation.

3. Evaluate the following triple integral using Simpson's 1/3 rule and compute the percent relative error (εt).

$$\int_{-2}^{2} \int_{0}^{2} \int_{-3}^{1} (x^3 - 3yz)\, dx\, dy\, dz$$

1. Simpson 1/3rd Formula

$$I \cong \underbrace{(b-a)}_{\text{Width}} \underbrace{\frac{f(x_0) + 4\sum_{i=1,3,5}^{n-1} f(x_i) + 2\sum_{j=2,4,6}^{n-2} f(x_j) + f(x_n)}{3n}}_{\text{Average height}}$$

2. Error Formula for Simpson 1/3rd Formula

$$E_t = -\frac{1}{90}h^5 f^{(4)}(\xi)$$