**MP - 409**

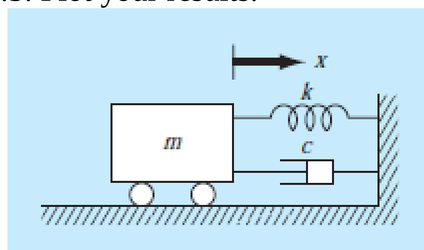
**Runge-Kutta Methods: PROBLEMS**

**1** Solve the following problem with the fourth-order RK method:

$$\frac{d^2y}{dx^2} + 0.5\frac{dy}{dx} + 7y = 0$$

where $y(0) = 4$ and $y\_(0) = 0$. Solve from $x = 0$ to 5 with $h = 0.5$. Plot your results.

**2.** The motion of a damped spring-mass system (See Fig. ) is described by the following ordinary differential equation:

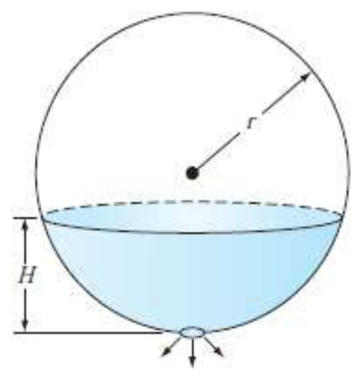



$$m\frac{d^2x}{dt^2} + c\frac{dx}{dt} + kx = 0$$

where $x$ = displacement from equilibrium position (m), $t$ = time(s), $m$ = 20-kg mass, and $c$ = the damping coefficient (N · s/m). The damping coefficient $c$ takes on three values of 5 (under damped), 40 (critically damped), and 200 (over damped). The spring constant $k$ = 20 N/m. The initial velocity is zero, and the initial displacement $x$ = 1 m. Solve this equation using a numerical method over the time period $0 \leq t \leq 15$ s. Plot the displacement versus time for each of the three values of the damping coefficient on the same curve.

**3.** A spherical tank has a circular orifice in its bottom through which the liquid flows out (See Fig.). The flow rate through the hole can be estimated as

$$Q_{out} = CA\sqrt{2gH}$$




Where $Q_{out}$ = outflow ($m^3$/s), $C$ = an empirically-derived coefficient, $A$ = the area of the orifice ($m^2$), $g$ = the gravitational constant (= 9.81 m/$s^2$), and $H$ = the depth of liquid in the tank. Use Runge Kutta methods to determine how long it will take for the water to flow out of a 3-m diameter tank with an initial height of 2.75 m. Note that the orifice has a diameter of 3 cm and $C$ = 0.55.

**Fourth-Order Runge-Kutta Methods Formulla:**

$$y_{i+1} = y_i + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)h$$

$$k_1 = f(x_i, y_i)$$

$$k_2 = f\left(x_i + \frac{1}{2}h,\ y_i + \frac{1}{2}k_1 h\right)$$

$$k_3 = f\left(x_i + \frac{1}{2}h,\ y_i + \frac{1}{2}k_2 h\right)$$

$$k_4 = f(x_i + h,\ y_i + k_3 h)$$