```
In[•]:= getBoundaries[nSites_, nDim_] :=
          Module[{ForwardBoundary, BackwardBoundary, d, n, n1},
           ForwardBoundary = {};
           For[d = 1, d ≤ nDim, d = d + 1,
            AppendTo[ForwardBoundary, {}];
            For[n = 1, n ≤ nSites^(nDim - d), n = n + 1,
             For[n1 = 0, n1 < nSites^(d - 1), n1 = n1 + 1,
              AppendTo[ForwardBoundary[[d]], n * nSites^d - n1 - 1]
             ]
            ]
           ];
           BackwardBoundary = {};
           For[d = 1, d ≤ nDim, d = d + 1,
            AppendTo[BackwardBoundary, {}];
            Do[AppendTo[BackwardBoundary[[d]], elem - (nSites^(d - 1)) (nSites - 1)],
             {elem, ForwardBoundary[[d]]}
            ]
           ];
           {ForwardBoundary, BackwardBoundary}
          ];

      getNeighbours[nSites_, nDim_] := Module[
          {boundaries, ForwardBoundary, BackwardBoundary, neighbour, nTot, n, d, nNeigh},
          boundaries = getBoundaries[nSites, nDim];
          ForwardBoundary = boundaries[[1]];
          BackwardBoundary = boundaries[[2]];
          nTot = nSites^nDim;
          neighbour = <||>;
          For[n = 0, n < nTot, n = n + 1,
           nNeigh = {};
           For[d = 0, d < nDim, d = d + 1,
            (* Forward *)
            If[Not[MemberQ[ForwardBoundary[[d + 1]], n]],
             AppendTo[nNeigh, n + nSites^d],
             AppendTo[nNeigh, n - (nSites^(d)) (nSites - 1)]];
            (* Backward *)
            If[Not[MemberQ[BackwardBoundary[[d + 1]], n]],
             AppendTo[nNeigh, n - nSites^d],
             AppendTo[nNeigh, n + (nSites^(d)) (nSites - 1)]]
           ];
           AppendTo[neighbour, n → nNeigh];
          ];
          neighbour];

      numberOfSpanningTrees[nSites_, nDim_] := Module[
          {adj, neighbour, nTot, deg, Q},
          adj = {};
          neighbour = getNeighbours[nSites, nDim];
          nTot = nSites^nDim;
          Do[
```

```
    AppendTo[adj, {}];
    Do[
     AppendTo[adj[[i + 1]], If[MemberQ[neighbour[i], j], 1, 0]], {j, Range[0, nTot - 1]}
    ],
    {i, Range[0, nTot - 1]}];
   deg = 2 * nDim * IdentityMatrix[nSites^nDim];
   Q = adj - deg;
   Det[Drop[Q, {1}, {1}]]
  ];
```

In[•]:=

```
(*For a 3x3 lattice*)
nSites = 3;
nDim = 2;
neighbour = getNeighbours[nSites, nDim]
Print["Number of Spanning Trees: "]
numberOfSpanningTrees[nSites, nDim]
```

Out[•]=

```
⟨|0 → {1, 2, 3, 6}, 1 → {2, 0, 4, 7}, 2 → {0, 1, 5, 8}, 3 → {4, 5, 6, 0},
 4 → {5, 3, 7, 1}, 5 → {3, 4, 8, 2}, 6 → {7, 8, 0, 3}, 7 → {8, 6, 1, 4}, 8 → {6, 7, 2, 5}|⟩
```

```
Number of Spanning Trees:
```

Out[•]=

```
11 664
```

In[•]:=

```
(*For a 4x4 lattice*)
nSites = 4;
nDim = 2;
neighbour = getNeighbours[nSites, nDim]
Print["Number of Spanning Trees: "]
numberOfSpanningTrees[nSites, nDim]
```

Out[•]=

```
⟨|0 → {1, 3, 4, 12}, 1 → {2, 0, 5, 13}, 2 → {3, 1, 6, 14}, 3 → {0, 2, 7, 15},
 4 → {5, 7, 8, 0}, 5 → {6, 4, 9, 1}, 6 → {7, 5, 10, 2}, 7 → {4, 6, 11, 3},
 8 → {9, 11, 12, 4}, 9 → {10, 8, 13, 5}, 10 → {11, 9, 14, 6}, 11 → {8, 10, 15, 7},
 12 → {13, 15, 0, 8}, 13 → {14, 12, 1, 9}, 14 → {15, 13, 2, 10}, 15 → {12, 14, 3, 11}|⟩
```

```
Number of Spanning Trees:
```

Out[•]=

```
-42 467 328
```

In[•]:=
```
(*For a 3x3x3 lattice*)
nSites = 3;
nDim = 3;
neighbour = getNeighbours[nSites, nDim]
Print["Number of Spanning Trees: "]
numberOfSpanningTrees[nSites, nDim]
```

Out[•]=
```
<|0 → {1, 2, 3, 6, 9, 18}, 1 → {2, 0, 4, 7, 10, 19}, 2 → {0, 1, 5, 8, 11, 20},
 3 → {4, 5, 6, 0, 12, 21}, 4 → {5, 3, 7, 1, 13, 22}, 5 → {3, 4, 8, 2, 14, 23},
 6 → {7, 8, 0, 3, 15, 24}, 7 → {8, 6, 1, 4, 16, 25}, 8 → {6, 7, 2, 5, 17, 26},
 9 → {10, 11, 12, 15, 18, 0}, 10 → {11, 9, 13, 16, 19, 1}, 11 → {9, 10, 14, 17, 20, 2},
 12 → {13, 14, 15, 9, 21, 3}, 13 → {14, 12, 16, 10, 22, 4}, 14 → {12, 13, 17, 11, 23, 5},
 15 → {16, 17, 9, 12, 24, 6}, 16 → {17, 15, 10, 13, 25, 7}, 17 → {15, 16, 11, 14, 26, 8},
 18 → {19, 20, 21, 24, 0, 9}, 19 → {20, 18, 22, 25, 1, 10}, 20 → {18, 19, 23, 26, 2, 11},
 21 → {22, 23, 24, 18, 3, 12}, 22 → {23, 21, 25, 19, 4, 13}, 23 → {21, 22, 26, 20, 5, 14},
 24 → {25, 26, 18, 21, 6, 15}, 25 → {26, 24, 19, 22, 7, 16}, 26 → {24, 25, 20, 23, 8, 17}|>

Number of Spanning Trees:
```

Out[•]=
```
2 529 990 231 179 046 912
```