# Rejection Sampling and Metropolis Algorithm

Fall 2022, MATH8050: Homework 6
**Your Name, Section XXX**

Due October 19, 12:00 PM

***General instructions for homeworks***: Please follow the uploading file instructions according to the syllabus. Each answer must be supported by written statements as well as any code used. Your code must be completely reproducible and must compile. For writing mathematical expressions in R Markdown, refer to the homework template posted on Canvas, a 30-minute tutorial, or LaTeX/Mathematics.

***Advice***: Start early on the homeworks and it is advised that you not wait until the last day. While the professor and the TA's check emails, they will be answered in the order they are received and last minute help will not be given.

**No late homeworks will be accepted.**

### *R Working Environment*

Please load all the packages used in the following R chunk before the function `sessionInfo()`

```
# load packages


sessionInfo()
```

Total points on assignment: 10 (reproducibility) + 90 (Q1)

Reproducibility component: 10 points.

1. (90pts total, equally weighted) We write $X \sim \mathcal{B}e(\alpha, \beta)$ if $X$ has the beta distribution with parameters $\alpha > 0$ and $\beta > 0$, that is, its pdf is

$$p(x \mid \alpha, \beta) = \mathcal{B}e(x \mid \alpha, \beta) = \frac{1}{B(\alpha, \beta)} x^{\alpha-1}(1-x)^{\beta-1},$$

where $B(\alpha, \beta)$ is the beta function. Suppose that we want to generate samples from the following target density (known up to a constant)

$$f(x; \alpha, \beta) \propto x^{\alpha-1}(1-x)^{\beta-1},$$

with $\alpha = 2.7, \beta = 6.3$. Work on the following questions.

(a) Plot the densities of $f(x; \alpha = 2.7, \beta = 6.3)$ and the Uniform distribution $U(0, 1)$. According to the rejection sampling approach, sample from the beta distribution using the $U(0, 1)$ pdf as an enveloping function.

(b) Plot the histogram of the points that fall in the acceptance region. Do this for a simulation size of $10^2$ and $10^5$ and report your acceptance ratio. Compare the ratios and histograms.

(c) Instead of using the uniform distribution, using $\mathcal{B}e(2, 6)$ as the enveloping function. Then repeat the tasks (a) and (b)

(d) Provide the four histograms from Tasks (2) and (3) using the $U(0, 1)$ and $\mathcal{B}e(2, 6)$ enveloping proposals. Provide the acceptance ratios. Provide commentary.

(e) Use a uniform distribution as a symmetric proposal, and implement the Metropolis algorithm. You need to tune the width of the interval around the current value to achieve good performance. Plot the traceplot and histogram.

(f) Use a normal distribution as a symmetric proposal, and implement the Metropolis algorithm. You need to tune the width of the interval around the current value to achieve good performance. Plot the traceplot and histogram.