Gibbs Sampling and Metropolis-Hastings Algorithm

Fall 2022, MATH8050: Homework 7
Your Name, Section XXX

Due October 26, 12:00 PM

General instructions for homeworks: Please follow the uploading file instructions according to the syllabus. Each answer must be supported by written statements as well as any code used. Your code must be completely reproducible and must compile. For writing mathematical expressions in R Markdown, refer to the homework template posted on Canvas, a 30-minute tutorial, or LaTeX/Mathematics.

Advice: Start early on the homeworks and it is advised that you not wait until the last day. While the professor and the TA's check emails, they will be answered in the order they are received and last minute help will not be given.

No late homeworks will be accepted.

R Working Environment

Please load all the packages used in the following R chunk before the function `sessionInfo()`

```
# load packages


sessionInfo()
```

Total points on assignment: 10 (reproducibility) + 45 (Q1) + 45 (Q2)

Reproducibility component: 10 points.

1. (45pts in total, equally weighted) Suppose that we want to generate a truncated beta distribution Beta$(2.7, 6.3)$ restricted to the interval (c, d) with $c, d \in (0, 1)$. Assume that $c = 0.1$ and $d = 0.9$.

(a) Implement a Metropolis-Hastings algorithm based on a Beta$(2, 6)$ proposal, and provide convergence diagnostics and acceptance ratio

(b) Implement a Metropolis-Hastings algorithm based on a $U(c, d)$ proposal, and provide convergence diagnostics and acceptance ratio.

(c) Compute $P(X > 0.5)$ using the samples obtained in part (a) and (b).

2. (45pts in total, equally weighted) We call $X \sim \mathcal{T}_\nu$ is a Student's t random variable with ν degrees of freedom, that is, its pdf is given by

$$f(x|\nu) = \frac{\Gamma(\frac{\nu+1}{2})}{\Gamma(\frac{\nu}{2})} \frac{1}{\sqrt{\nu\pi}} \left(1 + \frac{x^2}{\nu}\right)^{-(\nu+1)/2}.$$

Assume that $\nu = 4$. Make sure that you need to perform convergence diagnostics.

(a) Implement a Metropolis-Hastings algorithm with normal distribution $\mathcal{N}(0, 1)$ as the proposal distribution.

(b) Implement a Metropolis-Hastings algorithm with t distribution $\mathcal{T}_2$ as the proposal distribution.

(c) Calculate $E(X)$ and the 95% credible interval for X using the MCMC samplers in part (a) and (b).