

3.1.b)

$$x(t) = \begin{cases} \left(\frac{1}{4} - |t|\right), & -T_1 \leq t \leq T_1 \\ 0 & \text{elsewhere} \end{cases}$$

Given, $T = 1$, $T_1 = 1/4$.

Now,

$$x(t) = \begin{cases} \frac{1}{4} - t & 0 < t \leq 1/4 \\ \frac{1}{4} + t & -1/4 \leq t < 0 \\ 0 & T_1 < |t| < T/2 \end{cases}$$

$$a_0 = \int_0^{T/4} \left(\frac{1}{4} - t\right) dt + \int_{-T/4}^0 \left(\frac{1}{4} + t\right) dt$$

$$\left[\frac{t}{4} - t^2\right]_0^{T/4} + \left[\frac{t}{4} + t^2\right]_{-T/4}^0 = \frac{1}{8} - \left(\frac{1}{32} + \frac{1}{32}\right) = \frac{1}{8} - \frac{1}{16} \\ = \frac{1}{16} \quad (\because T=1)$$

$a_k \neq 0$.

$$a_k = \frac{1}{T} \left[\int_{-T/4}^0 \left(\frac{1}{4} + t\right) e^{-j\omega_0 t} dt + \int_0^{T/4} \left(\frac{1}{4} - t\right) e^{-j\omega_0 t} dt \right]$$

$$\frac{1}{T} \left[\frac{-1}{4jk\omega_0} + \frac{e^{jk\omega_0/4}}{4jk\omega_0} + \int_{-1/4}^0 t \cdot e^{-jk\omega_0 t} dt + \frac{1}{4jk\omega_0} - \frac{e^{-jk\omega_0/4}}{4jk\omega_0} - \int_0^{1/4} t \cdot e^{-jk\omega_0 t} dt \right]$$

$$= \frac{1}{T} \left[\frac{e^{-jk\omega_0/4}}{4jk\omega_0} - \int_0^{1/4} t e^{-jk\omega_0 t} dt + \frac{e^{jk\omega_0/4}}{4jk\omega_0} + \int_{-1/4}^0 t e^{-jk\omega_0 t} dt \right]$$

$$= \frac{1}{T} \left[\frac{e^{-jk\omega_0/4}}{4jk\omega_0} - \left[\frac{t e^{-jk\omega_0/4}}{j^2 k^2 \omega_0^2} - \frac{1}{j^2 k^2 \omega_0^2} \right] + \frac{e^{jk\omega_0/4}}{4jk\omega_0} - \left[\frac{1}{j^2 k^2 \omega_0^2} - \frac{e^{jk\omega_0/4}}{j^2 k^2 \omega_0^2} \right] \right]$$

$$\frac{1}{T} \left[\frac{e^{jk\omega_0/4} - e^{-jk\omega_0/4}}{4j\omega_0 k} \right] \quad (\because \omega_0 T = 2\pi)$$

$$= -\cos \pi$$

$$= \frac{1 - \cos(\omega_0 k/4)}{2 k^2 \pi^2}$$

$$= \frac{1 - \cos(\omega_0 k/4)}{4j\omega_0 k T \cdot k\pi}$$