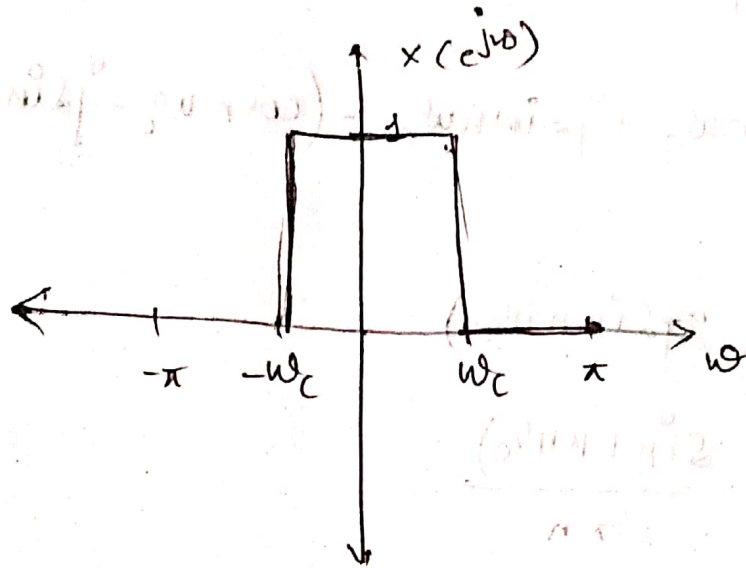6.2

a) $\quad X(e^{j\omega}) = \begin{cases} 1 & |\omega| \leq \omega_c \\ 0 & \omega_c < |\omega| < \pi \end{cases}$

$$x[n] = \frac{1}{2\pi} \int_{-\pi}^{\pi} X(e^{j\omega}) e^{j\omega n} \, d\omega$$

$$x[n] = \int_{-\omega_c}^{\omega_c} \frac{1}{2\pi} \cdot 1 \cdot e^{j\omega n} + 0$$

$$= \frac{1}{2\pi n j} \left[e^{j\omega n} \right]_{-\omega_c}^{\omega_c}$$

$$x[n] = \frac{1}{2\pi n j} \left[e^{jn\omega_c} - e^{-jn\omega_c} \right]$$

$$= \frac{1}{2\pi n j} \left(\cos n\omega_c + j\sin n\omega_c - (\cos n\omega_c - j\sin n\omega_c) \right)$$

$$= \frac{1}{2\pi n j} \left(2j\sin n\omega_c \right)$$

$$= \frac{\sin(n\omega_c)}{\pi n}$$

So $x[n]$ is a sinc function and is purely real.