Lecture – 09

Monday, September 7, 2020 2:04 PM

Last class :

* Sampling Theorem – conditions for exact reconstruction of a signal from samples

  (A) $\omega_s > 2\omega_M$ i.e. sampling frequency > Nyquist rate of signal

  (B) use Ideal LPF with $\omega_M < \omega_c < \omega_s - \omega_M$ and gain of T_s in passband

  } Analysis was done in FT domain

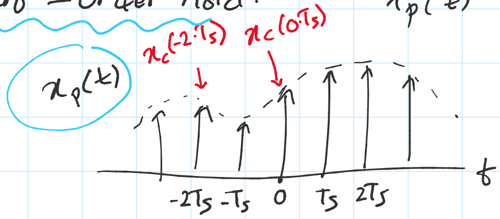
* Exact reconstruction – linear combination of shifted sinc shapes

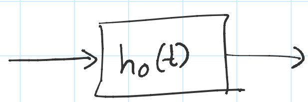
$$x_r(t) = T_s \sum_{n=-\infty}^{\infty} x_c(nT_s) \frac{\sin[\omega_c(t-nT_s)]}{\pi(t-nT_s)}$$

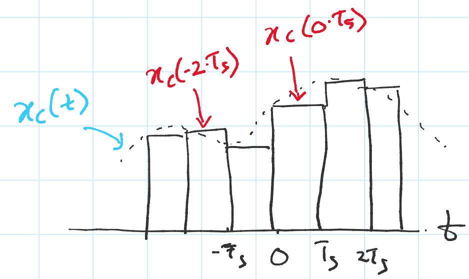
* All samples $x[n] = x_c(nT_s)$, $n = 0, \pm1, \pm2, \cdots$ required for reconstruction at any given t.

Todays class : non-ideal reconstruction , Aliasing , Quantization.

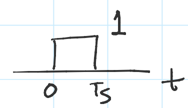
(A) Zero-order hold : $x_p(t) = x_c(t)p(t)$







$h_0(t) \equiv$

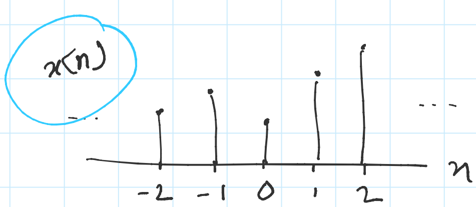


piecewise constant approximation of $x_c(t)$

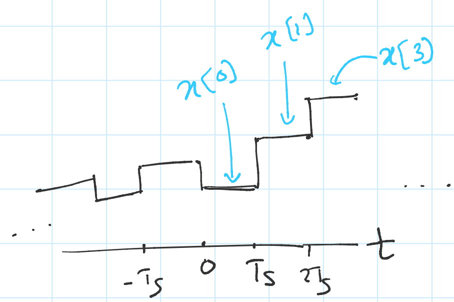
$x[n] = x_c(nT_s)$

$x[0] = x_c(0\cdot T_s)$ $\qquad x[2] = x_c(2\cdot T_s)$

$x[-2] = x_c(-2\cdot T_s)$



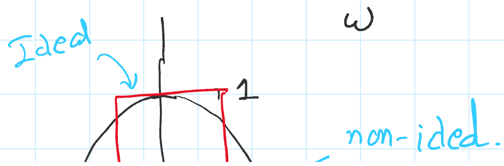
$\xrightarrow{T_s}$



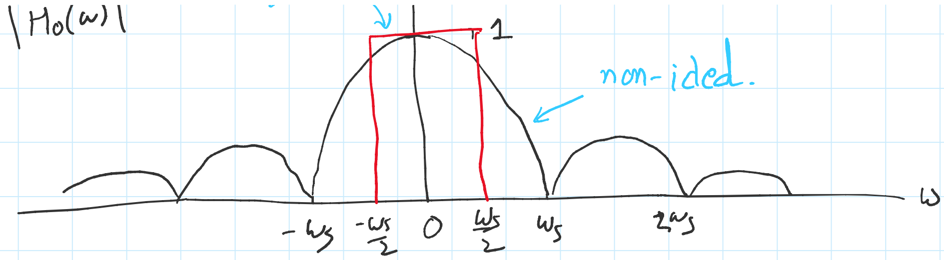
$$h_0(t) \xleftrightarrow{FT} H_0(\omega) = \frac{2\sin\left(\frac{\omega}{2}T_s\right)}{\omega} e^{-j\omega T_s/2}$$

$\omega_s = \frac{2\pi}{T_s}$

$|H_0(\omega)|$



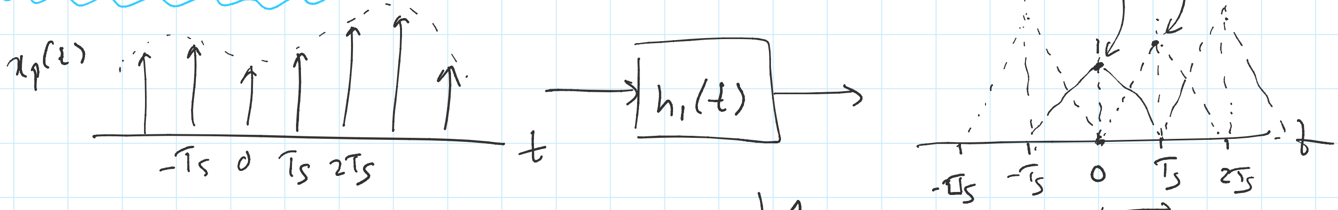
$\frac{\omega}{2} T_s = \pi k$

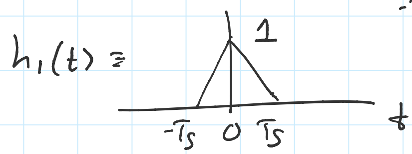


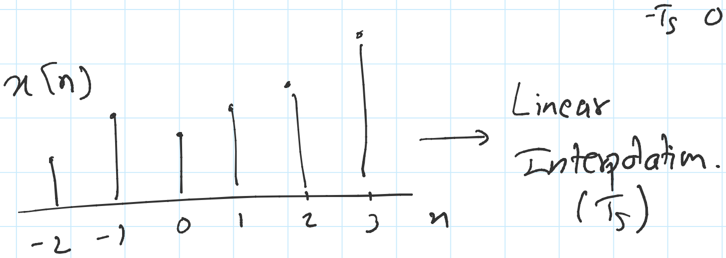
$$\frac{\omega}{2} T_s = \pi k$$

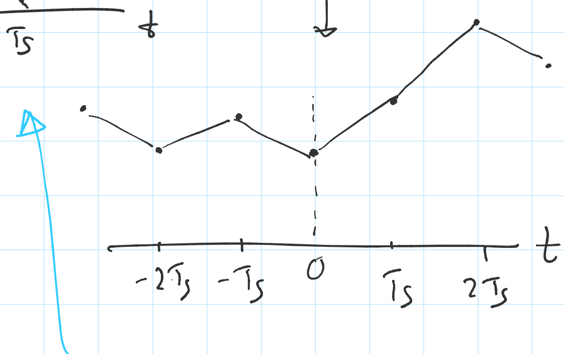
$$\Rightarrow \omega = \frac{2\pi}{T_s} k = \omega_s k$$

(B) Linear Interpolation:





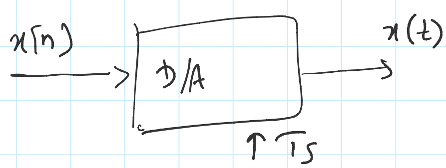




$$h_1(t) \xleftrightarrow{FT} H_1(\omega) = \frac{1}{T_s}\left[\frac{\sin\left(\frac{\omega T_s}{2}\right)}{\omega/2}\right]^2$$

First-order hold

Higher-order hold & more complicated interpolations.



Ex. Find Nyquist rate of following signals.

		ω_{Ny}
(a)	$\sin(40\pi t)$ $\longrightarrow$	80π
(b)	$\sin^2(40\pi t)$ $\longrightarrow$	160π
(c)	$1 + \cos(40\pi t)$ $\longrightarrow$	80π

Ex. $x(t)$ has Nyquist rate ω_0. find ω_{Ny} of following

		ω_{Ny}
(a)	$x(t-1)$ $\longrightarrow$	ω_0
(b)	$x(t) + x(t-1)$ $\longrightarrow$	ω_0
(c)	dx/dt $\longrightarrow$	ω_0

(c) $1 + \cos(40\pi t)$ $\longrightarrow$ 80π

(d) $\dfrac{\sin(40\pi t)}{\pi t}$ $\longrightarrow$ 80π

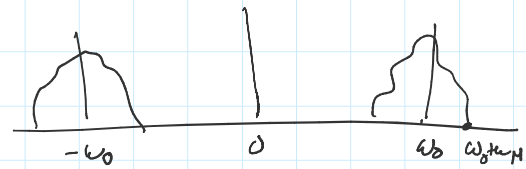
(e) $\left[\dfrac{\sin(40\pi t)}{\pi t}\right]^2$ $\longrightarrow$ 160π

(d) $x^2(t)$ $\longrightarrow$ $2\omega_0$

(e) $x(t)\cos(\omega_0 t)$ $\longrightarrow$ $3\omega_0$

(f) $x(t) * x(t)$ $\longrightarrow$ ω_0

$\omega_0 = 2\omega_M$



$\omega_{Ny} = 2(\omega_0 + \omega_M) = 2\omega_0 + \omega_0$
$= 3\omega_0$

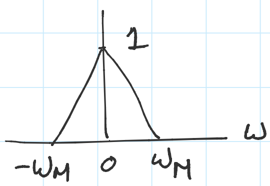
★ Aliasing:

Aliasing is said to happen when sampling frequency is less than the Nyquist rate. (under sampling)

In the spectrum of $X_p(\omega)$ the copies of $X_c(\omega)$ will overlap
$\Rightarrow$ exact reconstruction is not possible

$$X_p(\omega) = \frac{1}{T_s}\sum_{-\infty}^{\infty} X_c(\omega - k\omega_s)$$

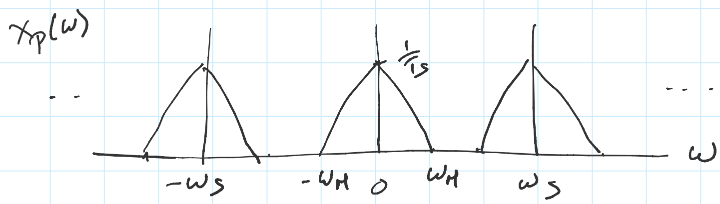
Ex. $X_c(\omega) =$



nyquist rate $= 2\omega_M$.

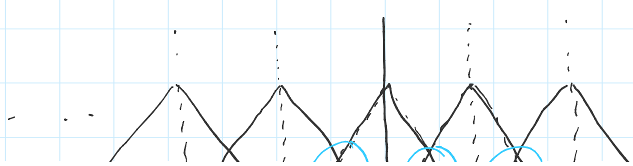
Plot $X_p(\omega)$ for :

(a) $\omega_s = 3\omega_M$

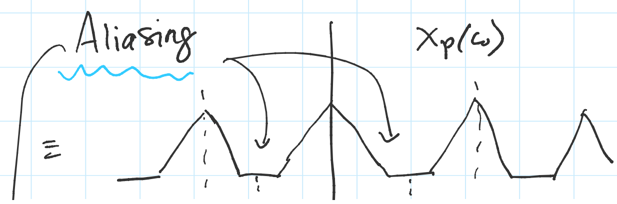


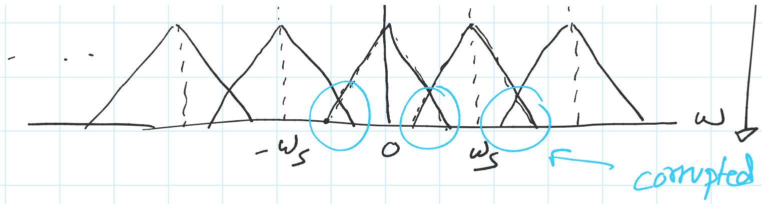
No Aliasing.

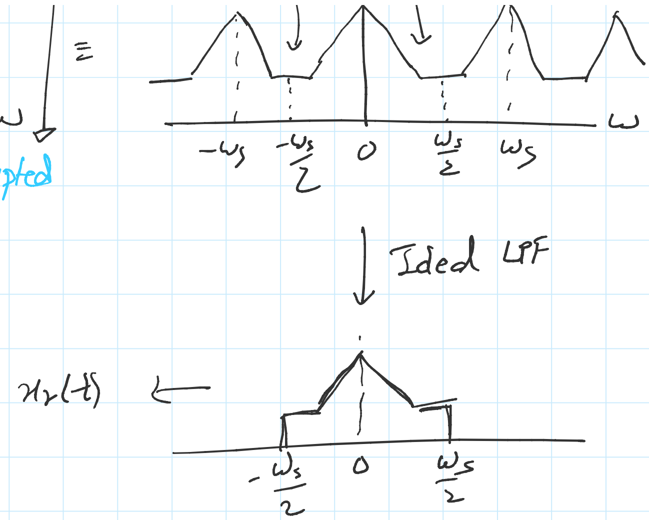
(b) $\omega_s = \dfrac{3\omega_M}{2}$



Aliasing





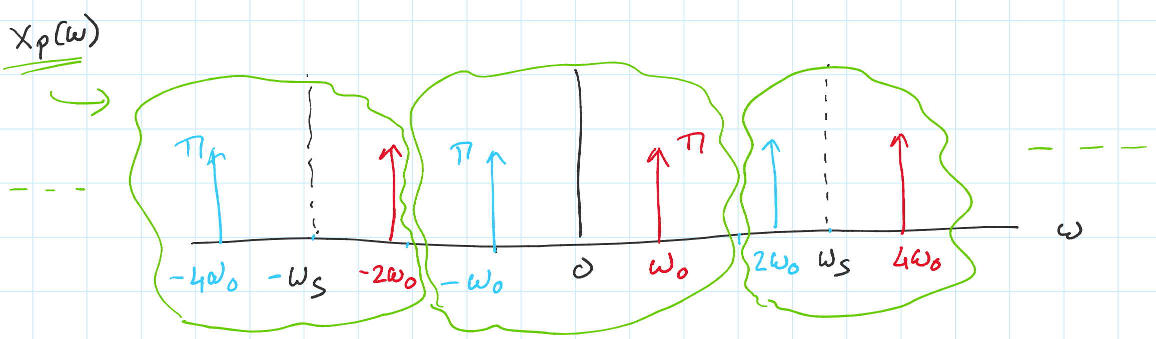


(*) No amount of LPF will give back original $x_c(t)$.

<u>Ex</u> $x(t) = \cos(\omega_0 t)$

sampling & reconstruction: use Ideal LPF with $\omega_c = \frac{\omega_s}{2}$.

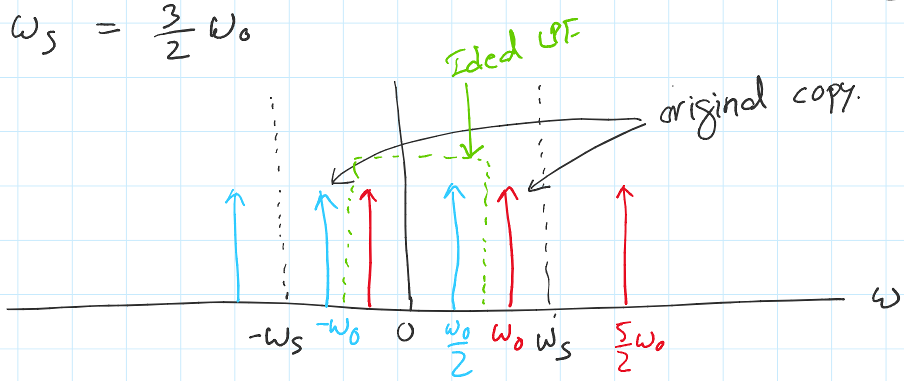
ⓐ $\omega_s = 3\omega_0$

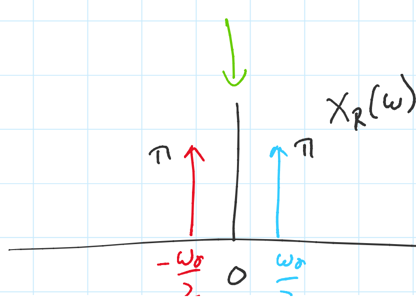


LPF gives me original signal back.

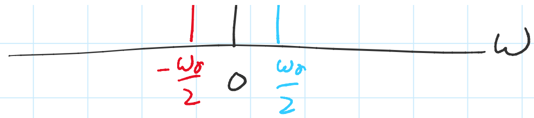
ⓑ $\omega_s = \frac{3}{2}\omega_0$

LPF, $\omega_c = \frac{\omega_s}{2} = \frac{3}{4}\omega_0$





$X_R(\omega) \overset{FT}{\longleftrightarrow} x_r(t) = \cos\left(\frac{\omega_0 t}{2}\right)$



<u>Remark</u>: when aliasing happens, the high frequency components will corrupt/appear as low frequency components.

<u>Remark</u>: Stroboscopic effect / Wagon-wheel effect.