Lecture – 24

Thursday, November 5, 2020 1:54 PM

## Last Class:

- FIR filter design
- Linear phase & FIR filters – symmetry / anti-symmetry conditions on $h[n]$
  - $\hookrightarrow$ corresponds to delay / shift.
- FIR filter design using Windows

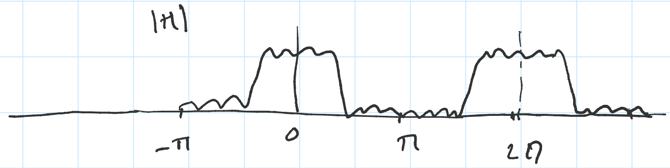
## Todays class:

- Linear phase and impulse response (Type I, Type II, etc.)
- FIR filter design using frequency sampling (least squares)
- Optimum Equiripple FIR filters (Chebyshev filters)
- Example of Analog signal processing using digital filter

**Ex.** Let $h[n]$ be a low-pass filter then $h_1[n] = (-1)^n h[n]$ behaves as a ______

$$h[n] \xleftrightarrow{DTFT} H(e^{j\omega})$$

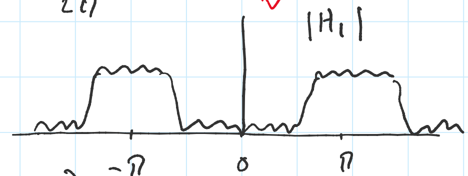
$|H|$ generic LPF



shift by $\pi$

$|H_1|$

$$h_1[n] = (-1)^n h[n] = (e^{j\pi})^n h[n]$$
$$= e^{j\pi n} h[n] \xleftrightarrow{DTFT} \underset{\substack{\uparrow \\ H_1(e^{j\omega})}}{H(e^{j(\omega-\pi)})} \longrightarrow$$



generic high pass filter

- Linear-phase & FIR impulse response.

Let $h[n]$, $n = 0, 1, \cdots N-1$ be an FIR impulse response.

Case: $N$ is odd, let $M = \frac{N-1}{2}$

$$H(e^{j\omega}) = \sum_{n=0}^{N-1} h[n] e^{-j\omega n} = e^{-j\omega M}\left(\sum_{0}^{N-1} h[n] e^{-j\omega n} e^{j\omega M}\right)$$
$$= e^{-j\omega M}\left(\sum_{n=0}^{N-1} h[n] e^{j\omega(M-n)}\right)$$

$n=0 \quad h[0] e^{j\omega M} \quad \& \quad h[N-1] e^{-j\omega M} \quad n = N-1$

$j\omega(M-1)$ $\qquad$ $-j\omega(M-1)$

$= e \quad \sum_{n=0}$

$n=0 \quad h[0]e^{j\omega M} \quad \& \quad h[N-1]e^{-j\omega M} \quad n=N-1$

$n=1 \quad h[1]e^{j\omega(M-1)} \quad \& \quad h[N-2]e^{-j\omega(M-1)} \quad n=N-2$

$\vdots \qquad\qquad \vdots$

$n=M-1 \quad h[M-1]e^{j\omega} \qquad h[M+1]e^{-j\omega}, \quad n=(N-1)-(M-1)$

$n=M \quad h[M]$

$$H(e^{j\omega}) = e^{-j\omega M}\left\{\left(h[0]e^{j\omega M} + h[N-1]e^{-j\omega M}\right) + \left(h[1]e^{j\omega(M-1)} + h[N-2]e^{-j\omega(M-1)}\right) + \cdots + h[M]\right\}$$

Euler's formula

$$\left(h[0]+h[N-1]\right)\cos(\omega M) + j\left(h[0]-h[N-1]\right)\sin(\omega M)$$

$$\Rightarrow H(e^{j\omega}) = e^{-j\omega M}\left\{h[M] + \sum_{n=0}^{M-1}\left(h[n]+h[N-1-n]\right)\cos(\omega(M-n)) + j\left(h[n]-h[N-1-n]\right)\sin(\omega(M-n))\right\}$$

we design filters with real coefficients i.e. $h[n]$ is real.

under what conditions the quantity in { } is real?

Selecting: $h[n] = h[N-1-n]$ , $n = 0,1,\cdots M-1$

$\Rightarrow$ { } will be real

$\Rightarrow H(e^{j\omega})$ will have linear phase (due to $e^{-j\omega M}$)

symmetric $h[n]$ $\Rightarrow$ linear phase of $H(e^{j\omega})$.

Type I FIR filters : $N$ − odd & $h[n]$ is symmetric.

Alternately selecting: $h[n] = -h[N-1-n]$ $\quad n = 0,1\cdots M-1$

& $h[M] = 0$

$\Rightarrow H(e^{j\omega})$ will have linear phase

anti-symmetric $h[n]$ $\Rightarrow$

Type III FIR filters : $N$ − odd & $h[n]$ is anti-symmetric.

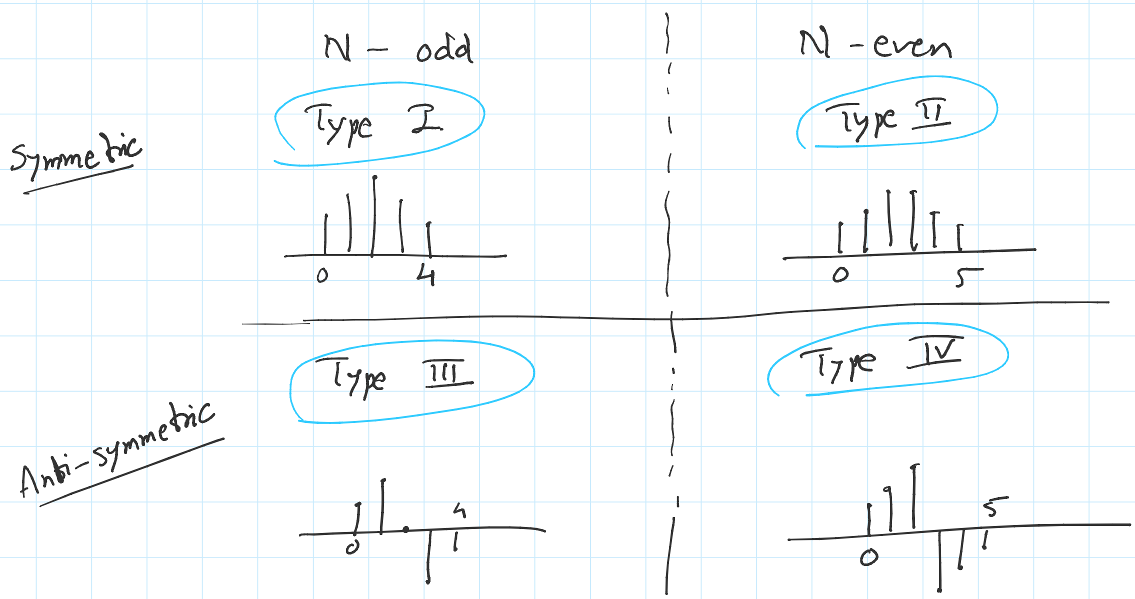
Case: N is even & $M = N/2$

we can show

$$H(e^{j\omega}) = e^{-j\omega M} \left\{ \sum_{n=0}^{M-1} \left(h[n] + h[N-1-n]\right) \cos(\omega(M-n)) + j\left(h[n] - h[N-1-n]\right) \sin(\omega(M-n)) \right\}$$

Type II FIR : N-even & $h[n]$ symmetric.

Type IV FIR : N-even & $h[n]$ anti-symmetric.




Type III - cannot make LPF & ( )

Type IV - cannot make LPF

$$H(e^{j\omega})\Big|_{\omega=0} = \sum_{n=0}^{N-1} h[n]$$

$= 0$ (for anti-symmetry)

$\therefore$ do not pass low frequencies.

H.W. show that for symmetric & anti-symetric filters $h[n]$

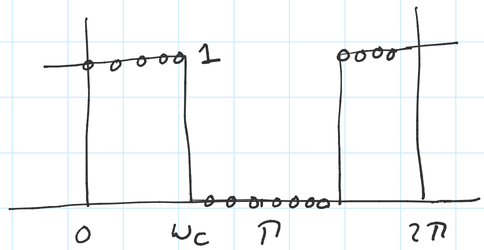
$$h[n] \longleftrightarrow H(z)$$

If $z_0$ is a zero of $H(z)$ then so is $\frac{1}{z_0}$.

★ FIR filter design using frequency sampling. (method of least squares)

↳ not proved here.

Ex. Ideal LPF




$|H_d(e^{j\omega})|$

$h_d[n]$ is sinc()

(equispaced) Sample the ideal spectrum. to get $H[k]$. , $k = 0, 1 \cdots N-1$

$$H[k] = H_d(e^{j\omega})\Big|_{\omega = \frac{2\pi}{N}k = \omega_k} \qquad M = \begin{cases} \frac{N-1}{2}, & N\text{-odd} \\ N/2, & N\text{-even} \end{cases}$$

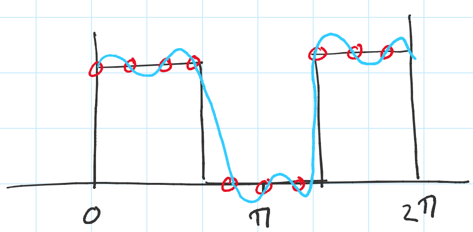
$\updownarrow$ DFT

get $h[n]$ from inverse DFT of $H[k]\,e^{-j\omega_k M}$

↳ for linear phase.

$\updownarrow$ DTFT

$H(e^{j\omega})$ — $H[k]$ samples this spectrum.




$H_d(e^{j\omega})$ – desired filter

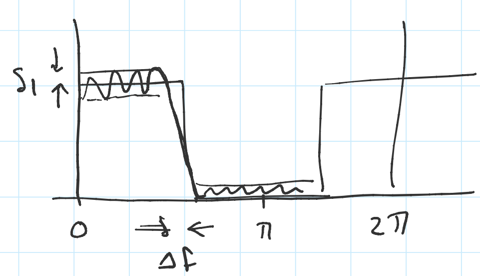
$H[k]$ – samples

$H(e^{j\omega})$ – designed filter

For real $h[n]$, select $H[k]$ symmetric.

Ex. In matlab, $N = 15$, design FIR filter (low-pass) using freq. sampling.

★ Optimum Equiripple FIR filters (Chebyshev)




– This method allows choosing various parameters i.e.

- $\delta_1$ – pass band ripple
- $\delta_2$ – stop band ripple
- $\Delta f$ – transition region
- $N$ – length of filter.

$\Delta f$ $\quad$ ( $N$ - length of filter.

This method minimizes the absolute error between desired filter & designed filter. $\Big\} \Rightarrow$ hence optimality.

$\Rightarrow$ This process gives a filter with equal size ripples in the passband. (also for stop band).

**Ex.** given speech signal (cont. time) with frequency content $< 8$ kHz.

we want to perform LPF on this digitally.

requirement : analog LPF with cutoff of 6 kHz

design : a digital LPF to achieve this when signal is sampled at (a) 16 kHz $\quad$ (b) 20 kHz.

Q. What should be the digital LPF cutoff $\omega_c$ ?