Lecture - 06

Thursday, August 27, 2020 2:01 PM

Last class:

* LTI systems — time domain characterization using impulse response
* Frequency analysis of LTI systems - Frequency response

$$x(t) \rightarrow \boxed{h(t)} \rightarrow y(t) \qquad h(t) \xleftrightarrow{FT} H(\omega)$$

* $e^{j\omega t} \longrightarrow H(\omega)\, e^{j\omega t}$ eigen functions

* Convolution property : $x(t) * h(t) \xleftrightarrow{FT} X(\omega)\, H(\omega)$

* Use convolution property to find convolution (via FT)

Today's Class :

Cascade of LTI systems :

$$x(t) \rightarrow \boxed{h_1(t)} \rightarrow \boxed{h_2(t)} \rightarrow y(t)$$

Is this LTI ?

Equivalent system ?

$$x(t) \rightarrow \boxed{h(t)} \rightarrow y(t)$$

$$h(t) = h_1(t) * h_2(t)$$

FT

$$H(\omega) = H_1(\omega)\, H_2(\omega)$$

$$H(\omega) = H_2(\omega)\, H_1(\omega)$$

$$h(t) = h_2(t) * h_1(t)$$

$$x(t) \rightarrow \boxed{h_2(t)} \rightarrow \boxed{h_1(t)} \rightarrow y(t)$$

remark: order of cascade does not matter for LTI.

Ex.

$$x(t) \rightarrow \boxed{h(t)} \rightarrow y(t)$$

$$x(t) = \cos(\omega_0 t)$$

$$h(t) = \frac{\sin(\omega_c t)}{\pi t}$$

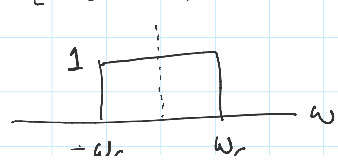
$\omega_0, \omega_c > 0$

Find $y(t) = x(t) * h(t)$

$\longrightarrow$

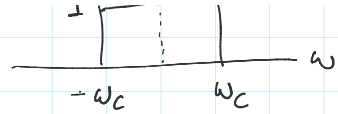
$$x(t) \xleftrightarrow{FT} X(\omega) = \pi\left[\delta(\omega-\omega_0) + \delta(\omega+\omega_0)\right]$$

$$h(t) \xleftrightarrow{FT} H(\omega) =$$



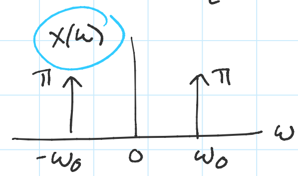
$$= \begin{cases} 1, & |\omega| \le \omega_c \\ 0, & |\omega| > \omega_c \end{cases}$$

$h(t) \xleftrightarrow{FT} H(\omega) =$

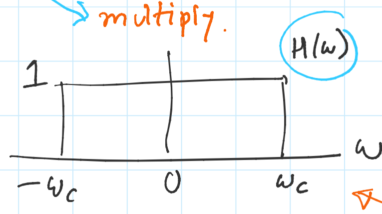


$= \begin{cases} \ldots \\ 0, & |\omega| > \omega_c \end{cases}$

$Y(\omega) = X(\omega)\,H(\omega)$

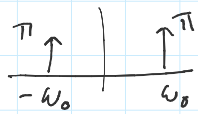


multiply.



If $\omega_0 < \omega_c$

$Y(\omega) =$

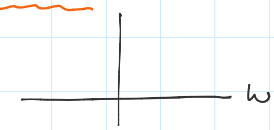


$= X(\omega)$

$\Rightarrow y(t) = \cos(\omega_0 t)$

If $\omega_0 > \omega_c$

$Y(\omega) = 0$



$\Rightarrow y(t) = 0$

Ideal Low pass filter with cutoff ω_c

$$y(t) = \begin{cases} \cos(\omega_0 t), & \omega_0 < \omega_c \\ 0, & \omega_0 > \omega_c \end{cases}$$

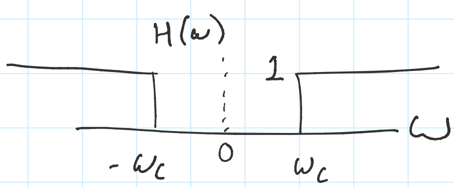
remark: new frequencies cannot be added (in the output) by an LTI system

LTI systems can be represented by $h(t) \xleftrightarrow{FT} H(\omega)$

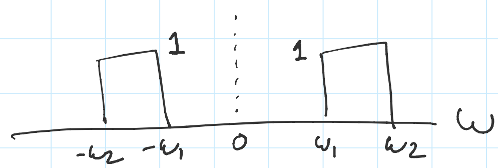
↪ can be thought of as filter

Ideal LPF – not practical.

Ideal HPF



Ideal BPF

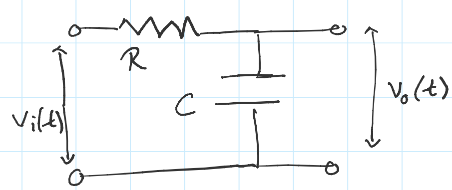


RC – low pass filter.

System.

input $v_i(t)$

output $v_o(t)$



Low pass filter.

$$V_i(t) = V_o(t) + R \cdot C \frac{dV_o(t)}{dt}$$

Take FT both sides.

$$V_i(\omega) = V_o(\omega) + R \cdot C\, j\omega V_o(\omega) = (1 + j\omega RC)\, V_o(\omega)$$

Take FT on both sides.

$$V_i(\omega) = V_o(\omega) + R \cdot C j\omega V_o(\omega) = (1 + j\omega RC)\, V_o(\omega)$$

compare with

$$V_o(\omega) = \frac{1}{1 + j\omega RC} \cdot V_i(\omega)$$

$$v_i(t) \longrightarrow \boxed{H(\omega)} \longrightarrow v_o(t)$$

$$V_o(\omega) = H(\omega) V_i(\omega)$$

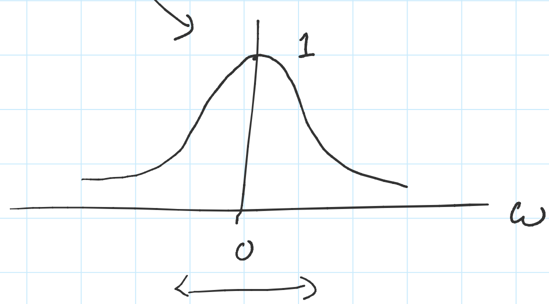
Freq. response $\Rightarrow H(\omega) = \dfrac{1}{1 + j\omega RC}$

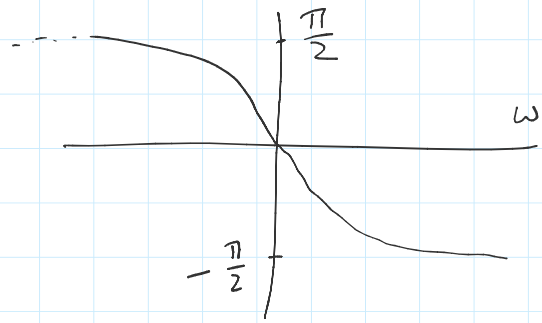
why is this low pass ?

plot $|H(\omega)|$ & $\measuredangle H(\omega)$

$$|H(\omega)| = \frac{1}{\sqrt{1 + \omega^2 R^2 C^2}} \quad \& \quad \measuredangle H(\omega) = -\tan^{-1}(\omega RC)$$

Analog non-ideal Low-pass filter.





what is impulse response $h(t) \xleftrightarrow{FT} H(\omega)$ of this RC system.

$$e^{-at} u(t) \xleftrightarrow{FT} \frac{1}{a + j\omega} \Longrightarrow \frac{1}{1 + j\omega RC} \xleftrightarrow{FT} ?\ \frac{1}{RC} \exp\left(\frac{-t}{RC}\right) u(t)$$

Complex freq. response $\equiv$ phase change (in freq. domain) $\Big\}$ F.T.

$\equiv$ time-delay or shift in time.

HW. For above RC-LPF, with input $x(t) = v_i(t) = \cos(\omega_0 t)$

find the output $y(t) = v_o(t)$ using FT.

Ex. Find Convolution : $x(t) = \dfrac{\sin(\omega_0 t)}{\pi t}$ & $h(t) = \dfrac{\sin(\omega_c t)}{\pi t}$

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find $y(t) = x(t) * h(t)$

$\rightarrow$ $X(\omega) = \begin{cases} 1, & |\omega| \leq \omega_0 \\ 0, & |\omega| > \omega_0 \end{cases}$ & $H(\omega) = \begin{cases} 1, & |\omega| \leq \omega_c \\ 0, & |\omega| > \omega_c \end{cases}$

$$Y(\omega) = X(\omega)H(\omega) = \begin{cases} 1, & |\omega| \leq \min(\omega_0, \omega_c) \\ 0, & |\omega| > \text{—''—} \end{cases}$$

$$\Rightarrow y(t) = \frac{\sin(\omega_1 t)}{\pi t}, \quad \omega_1 = \min(\omega_0, \omega_c).$$

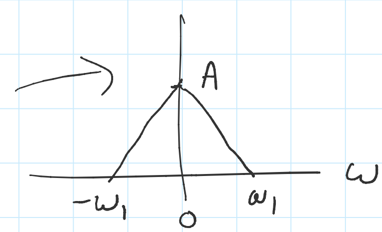
(11) Multiplication property:

$$x(t) \xleftrightarrow{FT} X(\omega) \qquad y(t) \xleftrightarrow{FT} Y(\omega)$$

$$\Rightarrow x(t)y(t) \xleftrightarrow{FT} \frac{1}{2\pi}\left[X(\omega) * Y(\omega)\right]$$

proof: HW

Ex. $s(t) = \cos(\omega_0 t)$ & $p(t) \xleftrightarrow{FT} P(\omega)$



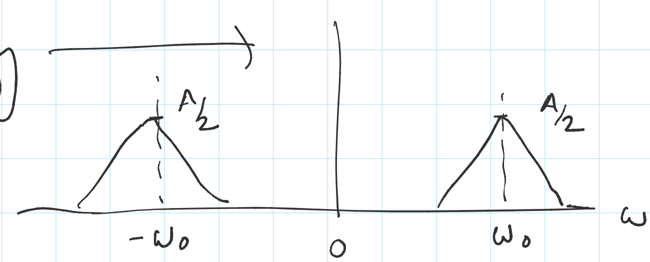
find FT of $s(t)p(t) = r(t)$

$\omega_0 > \omega_1$

$\rightarrow$ $S(\omega) = \pi\left[\delta(\omega - \omega_0) + \delta(\omega + \omega_0)\right]$

$$R(\omega) = \frac{1}{2\pi}\left[S(\omega) * P(\omega)\right] = \frac{1}{2\pi} \cdot \pi\left[\delta(\omega - \omega_0) + \delta(\omega + \omega_0)\right] * P(\omega)$$

$$R(\omega) = \frac{1}{2}\left[P(\omega - \omega_0) + P(\omega + \omega_0)\right]$$



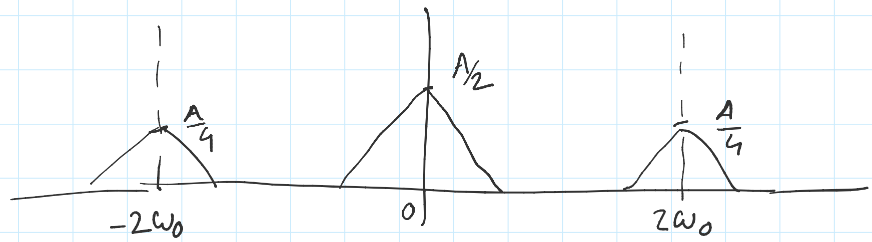
HW: $s(t) = \sin(\omega_0 t)$

$s(t) = e^{j\omega_0 t}$

Ex. find FT. of $s(t)\,r(t)$, $s(t) = \cos(\omega_0 t)$

$$s(t)\,r(t) \longleftrightarrow \frac{1}{2\pi}\,\pi\left[\delta(\omega-\omega_0)+\delta(\omega+\omega_0)\right] * R(\omega)$$

$$= \frac{1}{2}\left[R(\omega-\omega_0)+R(\omega+\omega_0)\right]$$



useful in CT for modulation & demodulation.

Ex. Find FT of $x(t) = t\left(\frac{\sin t}{\pi t}\right)^2$ using FT properties

Ex. Find FT of $\cos^2(\omega_0 t)$ (a) using multiplication property

(b) using $\cos^2(\omega_0 t) = \frac{1+\cos(2\omega_0 t)}{2}$