Lecture-07

Monday, August 31, 2020 1:54 PM

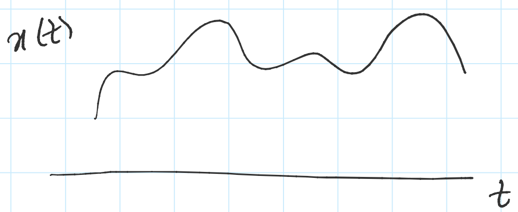
Last Class:

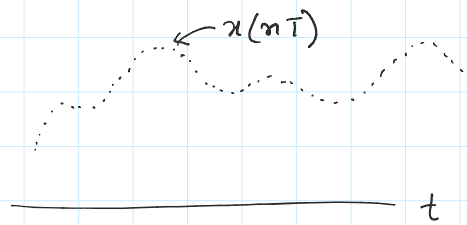
- ☆ Ideal Low Pass filter (LPF) – $\cos(\omega_0 t)$ input
- ☆ RC – low pass filter : non-ideal but practical

  frequency response $<$ magnitude / phase
- ☆ Multiplication property : $x(t)\,y(t) \xleftrightarrow{FT} \frac{1}{2\pi}[X(\omega) * Y(\omega)]$

Today's class : ☆ Sampling Theorem ☆

main-link between { Continuous-time signals & Discrete-time signals





samples (T) apart

For small T – "looks" continuous-time.

Ex. video i.e. motion picture

Q. when are samples sufficient to describe / capture / represent continuous-time signals ?

In general, not sufficient

why samples / sampling ?

- storage is difficult in Analog / compress
- analog processing is hard
- digital processing is easily available

☆ Remark – slightly unrelated examples :

Ex. sine wave : $A\sin(\omega_0 t) \equiv (A, \omega_0)$

Ex. sum of 3 sine waves : $\sum_{k=1}^{3} A_k \sin(\omega_k t) \equiv (3+3)$ parameters

∞ $jk\omega_0 t$

Ex. continuous-time periodic signals : $x(t) = \sum_{-\infty}^{\infty} a_k e^{jk\omega_0 t}$

Fourier Series : $x(t) \xleftrightarrow{FS} \{a_k\}$

Q. In general, for aperiodic continuous-time signals, when can we have a complete representation using a discrete-set of values ?

Periodic / uniform sampling :

$x_c(t)$ samples : $x[n] = x_c(t)\Big|_{t=nT_s} = x_c(nT)$

T_s – sampling period / interval

$f_s = \frac{1}{T_s}$ – sampling frequency (in Hz)

$\omega_s = \frac{2\pi}{T_s}$ – sampling freq. (in rad/s)

$$x_c(t) \longrightarrow \boxed{A/D} \longrightarrow x[n]$$

$\uparrow T_s$

Analog-to-Digital Converters (ADC)

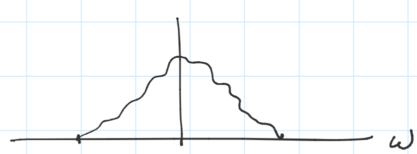
Analog continuous-time signals → discrete-time signals → digital signals

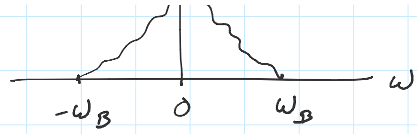
(continuous-valued) (continuous-valued) (discrete-valued)

★ Bandlimited Signals: A (continuous-time) signal $x_c(t)$ is said to be bandlimited if there is some frequency ω_B such that,

$x_c(t) \xleftrightarrow{FT} X_c(\omega)$ $X_c(\omega) = 0$ for $|\omega| > \omega_B$

Ex.



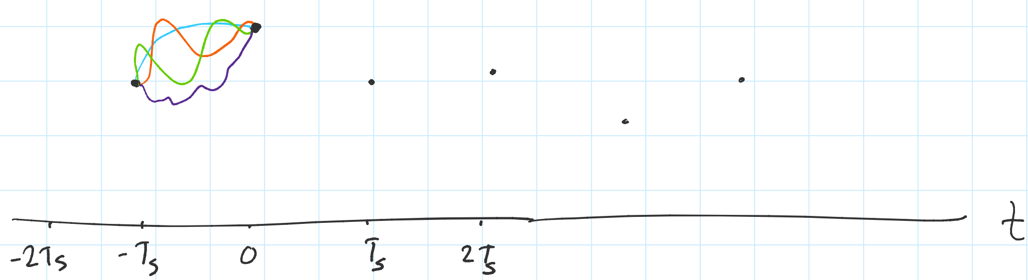


★ **Sampling Theorem**: A band limited signal with maximum frequency ω_B can be perfectly recovered / reconstructed from its samples if the sampling frequency ω_s satisfies

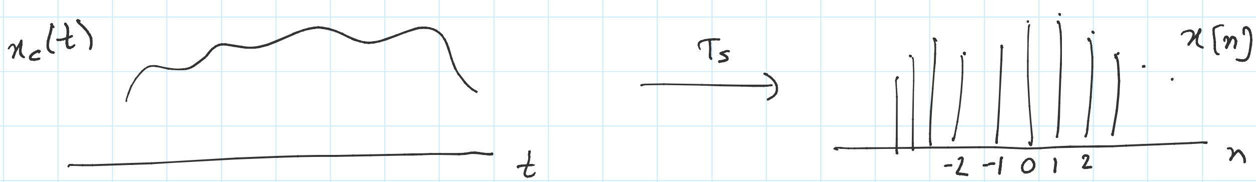
$$\omega_s > 2\omega_B .$$

$2\omega_B$ – Nyquist rate. ... { proposed by: Nyquist, Shannon.

intuition: large $\omega_B \Rightarrow$ large $\omega_s = \frac{2\pi}{T_s} \Rightarrow$ small T_s.



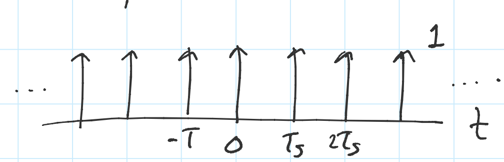
understanding of sampling theorem we use impulse-train sampling



n is always integer

impulse-train: $x_c(t) \longrightarrow x[n]$

$$p(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT_s)$$

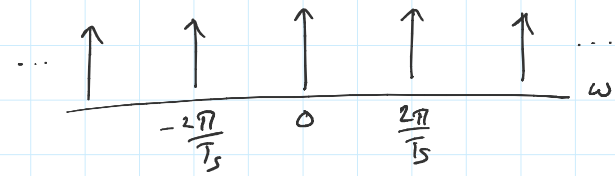


$$p(t) \xleftrightarrow{FT} P(\omega) = ?$$

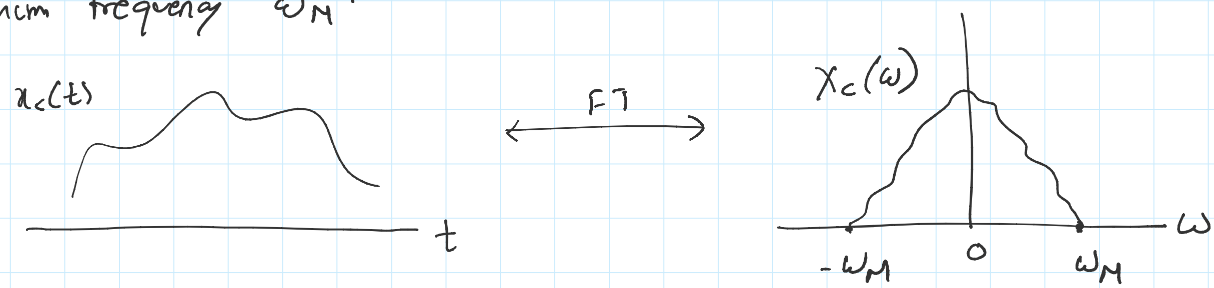
$p(t) \xleftrightarrow{FS} a_k = \frac{1}{} \quad \forall k \quad \Rightarrow P(\omega) = \sum^{\infty} 2\pi a_k \, \delta(\omega - k\omega_s) \quad ; \omega_s = \frac{2\pi}{}$

$$p(t) \xleftrightarrow{FS} a_k = \frac{1}{T_s} \quad \forall k \quad \Rightarrow \quad P(\omega) = \sum_{k=-\infty}^{\infty} 2\pi a_k \, \delta(\omega - k\omega_s) \quad ; \; \omega_s = \frac{2\pi}{T_s}$$

$$\Rightarrow P(\omega) = \frac{2\pi}{T_s} \sum_{-\infty}^{\infty} \delta\left(\omega - \frac{2\pi}{T_s} k\right)$$



let $x_c(t)$ be bandlimited signal with a maximum frequency ω_M.



consider : $x_p(t) = x_c(t)\, p(t)$

Find & Sketch Fourier Transform of $x_p(t)$.