Lecture - 10

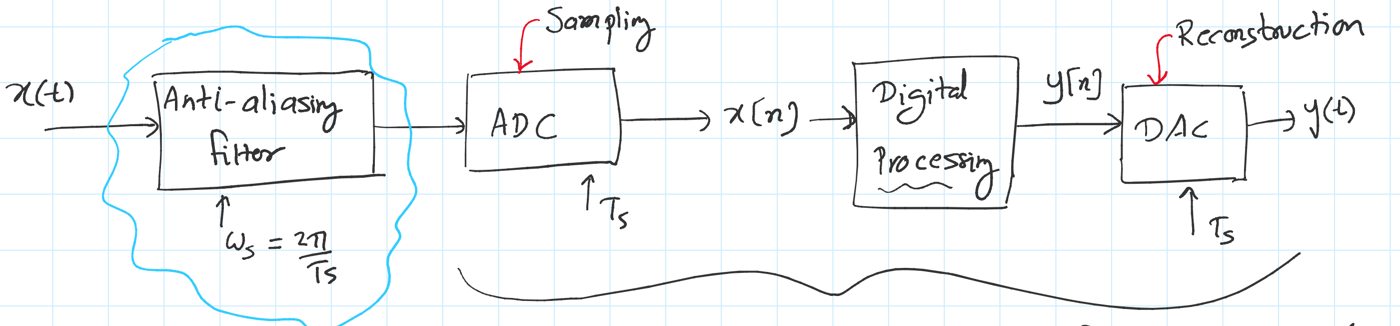
Thursday, September 10, 2020 2:08 PM

Last class:

- reconstruction from samples : ideal & non-ideal methods
- Aliasing : undersampling (sampling freq. < Nyquist rate)

Today's class:

Anti-aliasing filter :



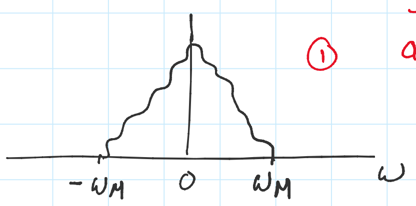
System is designed for some T_s (ω_s)

low pass filter with $\omega_c = \frac{\omega_s}{2}$

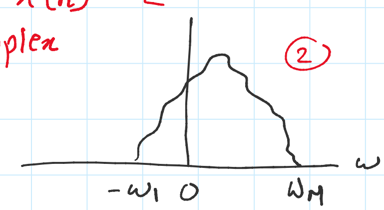
* Remarks on Sampling Theorem :

(A) Valid in general for complex signals (band-limited) → $|X(\omega)| = 0, \ |\omega| > \omega_M$

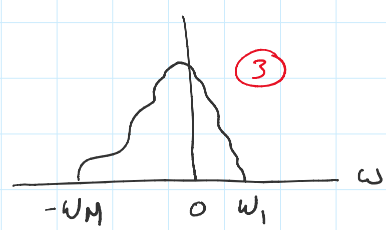
Samples $x[n]$ are complex



$\omega_{Ny} = 2\omega_M$



$\omega_{Ny} = 2\omega_M$



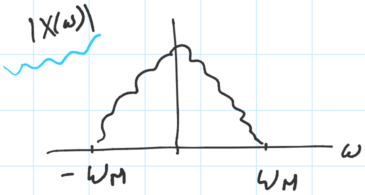
$\omega_{Ny} = 2\omega_M$

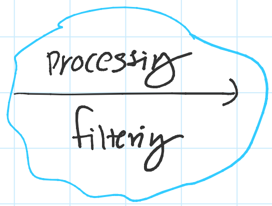
$$x(t) = e^{j\omega_0 t} = \cos(\omega_0 t) + j\sin(\omega_0 t)$$

(B) For some special cases (i.e. with some additional constraints on the signal) we could sample at rates below Nyquist rate without aliasing (but will need some extra processing)

Ex. real signals: $x(t) \xleftrightarrow{FT} X(\omega) = X^*(-\omega) \Rightarrow X(-\omega) = X^*(\omega)$

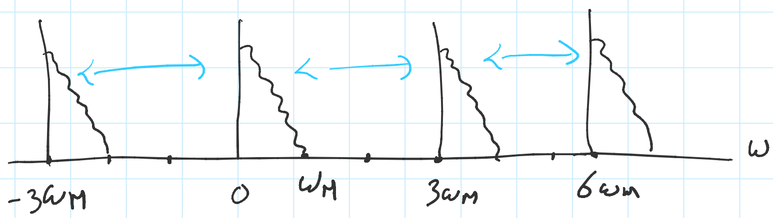
$\Rightarrow$ magnitude is even & phase/angle is odd





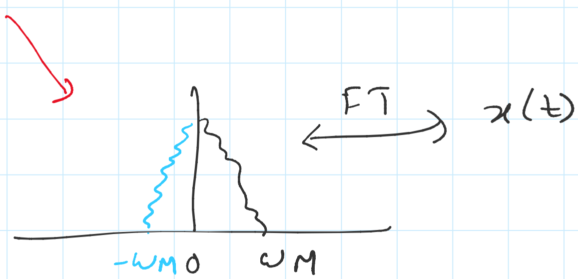
$\omega_{Ny} = 2\omega_M$ $\omega_{Ny} = 2\omega_M$

$\omega_s = 3\omega_M$



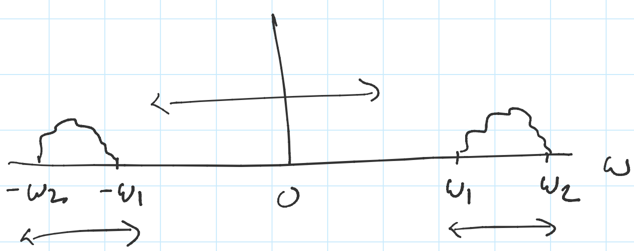
$\omega_s > \omega_M$

Sufficient for this case
(this is lower than Nyquist rate).



Ex. Band pass signals

There are ways to sample this without aliasing at lower than Nyquis rate.

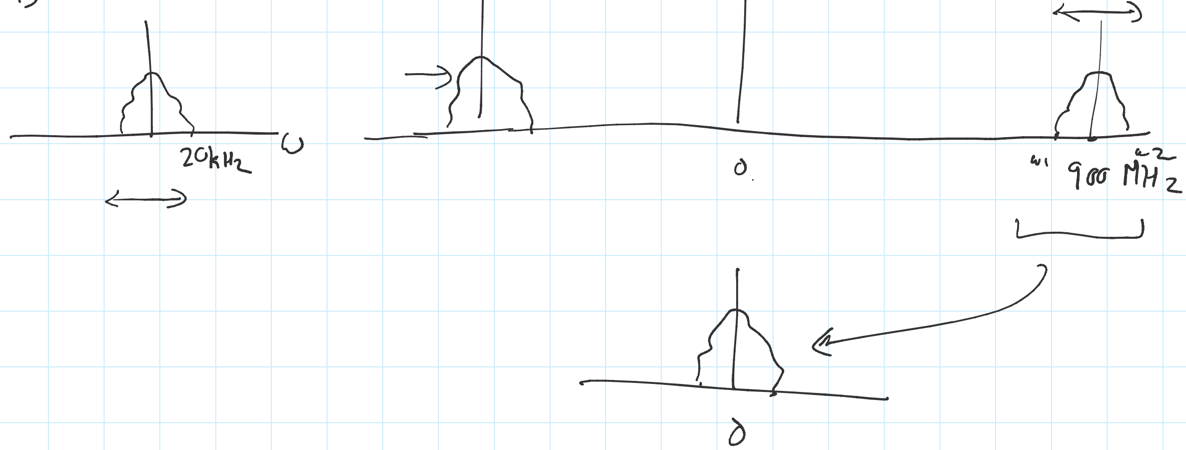


$\omega_{Ny} = 2\omega_2$

Ex.

Baseband

Ex.



(c) $x(t) = \sin(5\pi t)$ $\qquad \omega_M = 5\pi \Rightarrow \omega_{Ny} = 2\omega_M = 10\pi$

$\omega_s = \frac{2\pi}{T_s}$ & $\omega_s > \omega_{Ny} = 10\pi$

$\frac{2\pi}{T_s} > 10\pi \Rightarrow T_s < 0.2s.$

$x[n] = x(nT_s) = \sin(5\pi n T_s)$

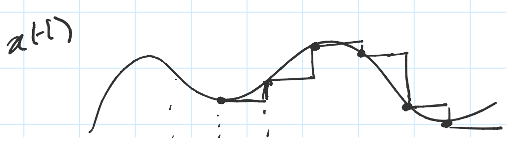
If $T_s = 0.2 \Rightarrow x[n] = \sin(n\pi) = 0$

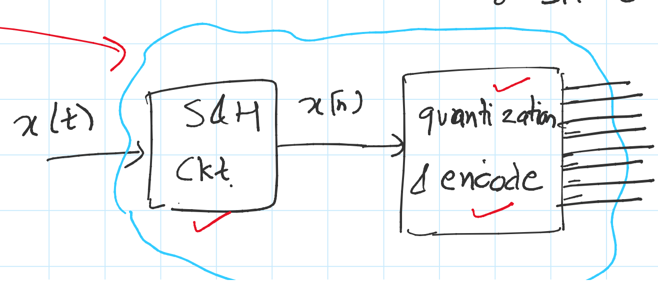
Strictly select $\omega_s > \omega_{Ny}$ to avoid aliasing.

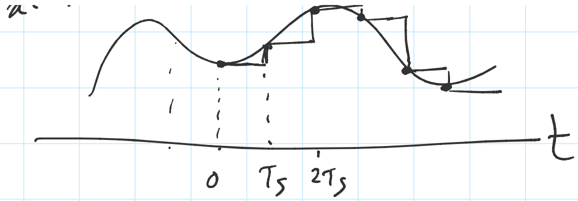
(D) $x(t) \rightarrow \boxed{ADC} \rightarrow$ digital signals

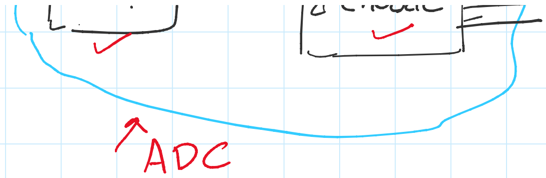
8-bit output.

Sample & Hold. circuit.









* Quantization:

continuous time → Analog Signals ← continuous-valued

DT Signals

discrete time → Digital Signals ← discrete-valued.

Quantization : continuous-valued signals → discrete-value signal

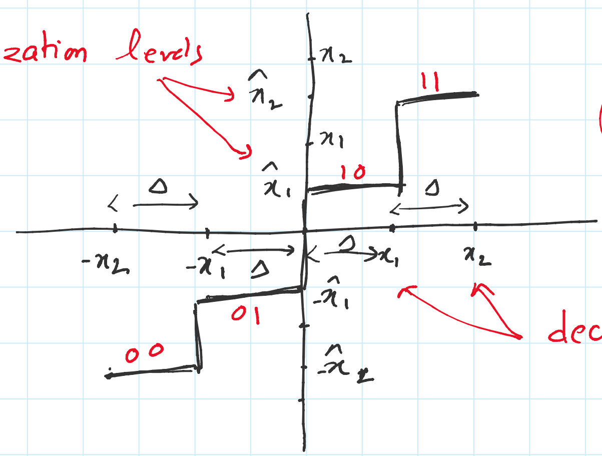
$$x_q[n] = Q(x[n]) \quad ; \quad Q() \text{ - quantization function.}$$

error: $e_q[n] = x[n] - x_q[n]$

Uniform Quantization: Ex. 2-bit quantizer $\equiv$ 4 levels

$x_1 = \Delta$; $\hat{x}_1 = \frac{\Delta}{2}$

$x_2 = 2\Delta$; $\hat{x}_2 = \frac{3\Delta}{2}$

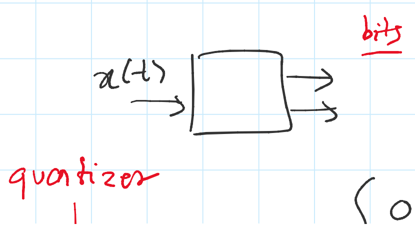


Δ — resolution / quantization step size

$$-\frac{\Delta}{2} < e_q[n] < \frac{\Delta}{2}$$

If $x[n] \in [0, x_1]$

$Q(x[n]) = \hat{x}_1$



Reconstruction

$(00 \rightarrow -\hat{x}_2)$, T_s

quantizer

$$n \longrightarrow x[n] \longrightarrow x_q[n] \longrightarrow \left\{ \begin{array}{ccc} 00 & \longrightarrow & -\hat{x}_2 \\ 01 & \longrightarrow & -\hat{x}_1 \\ 10 & \longrightarrow & \hat{x}_1 \\ 11 & \longrightarrow & \hat{x}_2 \end{array} \right\} \xrightarrow{T_s} x_r(t)$$

$$x[n] \longrightarrow e_q[n]$$

encoder

reconstruction levels

B-bit quantizer $\equiv 2^B$ levels

If signal values are in $[-x_{Max}, x_{Max}]$

$$2\,x_{Max} = 2^B \cdot \Delta$$

$$\Rightarrow \quad B = \log_2\left(\frac{2\,x_{Max}}{\Delta}\right) \quad \& \quad \Delta = \frac{2\,x_{Max}}{2^B}.$$

* *

as B increases, resolution Δ decreases

$\Rightarrow$ respresentation accuracy improves

Sampling rate : $\omega_s = 2\pi f_s$

samples per second.

$B \cdot f_s$ – bit rate / bits per second.

.wav files.

In practice : non-uniform quantization

log-spaced —"—