

# PH 354 HW3

classmate

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Q16)  $\hat{H} = -\frac{\hbar^2}{2m} \frac{d^2}{dx^2} + V(x)$

$$\psi(x) = \sum_{n=1}^{\infty} \psi_n \sin\left(\frac{n\pi x}{L}\right)$$

gn  $\int_0^L \sin \frac{n\pi x}{L} \sin \frac{m\pi x}{L} dx = \begin{cases} \frac{L}{2}, & m=n \\ 0, & \text{otherwise} \end{cases}$

Now:  $\hat{H}\psi(x) = E\psi(x)$  ~~consider this~~

$$\hat{H}\psi(x) = \hat{H} \left( \sum_{n=1}^{\infty} \psi_n \sin\left(\frac{n\pi x}{L}\right) \right)$$

$$\hat{H}\psi(x) = E\psi(x)$$

in matrix form

$$H\psi = E\psi$$

where  $H, \psi$  in the same basis of functions

$$\Rightarrow \begin{pmatrix} H_{11} & H_{12} & H_{13} & \dots \\ H_{21} & H_{22} & H_{23} & \dots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix} \begin{pmatrix} \psi_1 \\ \psi_2 \\ \vdots \end{pmatrix} = E \begin{pmatrix} \psi_1 \\ \psi_2 \\ \vdots \end{pmatrix} \quad \left\{ \sin \frac{n\pi x}{L} \mid n \in \mathbb{N} \right\}$$

$\Rightarrow$  for some ~~an~~ row index  $m$ :

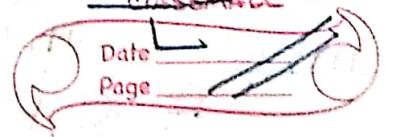
$$\text{LHS} = \sum_{n=1}^{\infty} H_{mn} \psi_n$$

$$\text{RHS} = E \psi_m$$

$$\Rightarrow \sum_{n=1}^{\infty} H_{mn} \psi_n = E \psi_m //$$

we know

$$H_{mn} = \frac{2}{L} \int_0^L \sin\left(\frac{m\pi x}{L}\right) \hat{H} \sin\left(\frac{n\pi x}{L}\right) dx$$



substituting:

$$\sum_{n=1}^{\infty} \left( \frac{2}{L} \int_0^L \sin\left(\frac{m\pi x}{L}\right) \hat{H} \sin\left(\frac{n\pi x}{L}\right) dx \right) \psi_n$$
$$= E \psi_m$$

$\Rightarrow$

$$\sum_{n=1}^{\infty} \left( \int_0^L \sin\left(\frac{m\pi x}{L}\right) \hat{H} \sin\left(\frac{n\pi x}{L}\right) dx \right) \psi_n$$

$$= \cancel{2} \frac{1}{2} L E \psi_m$$



(ii)  $H_{mn} = \frac{2}{L} \int_0^L \sin\left(\frac{m\pi x}{L}\right) \left[ -\frac{\hbar^2}{2m} \frac{d^2}{dx^2} + V(x) \right] \sin\left(\frac{n\pi x}{L}\right) dx$

$V(x) = \frac{ax}{L}$

$$= \frac{2}{L} \left\{ \int_0^L \sin\left(\frac{m\pi x}{L}\right) \left( \frac{\hbar^2}{2m} \right) \left( \frac{n\pi}{L} \right)^2 \times -\sin\left(\frac{n\pi x}{L}\right) dx \right.$$

$$+ \left. \int_0^L \sin\left(\frac{m\pi x}{L}\right) \cdot \left( \frac{ax}{L} \right) \sin\left(\frac{n\pi x}{L}\right) dx \right\}$$

$$= \frac{2}{L} \left[ \left( \frac{\hbar^2}{2m} \right) \left( \frac{n\pi}{L} \right)^2 \int_0^L \sin\left(\frac{m\pi x}{L}\right) \sin\left(\frac{n\pi x}{L}\right) dx \right.$$

$$+ \left. \left( \frac{a}{L} \right) \int_0^L x \sin\left(\frac{m\pi x}{L}\right) \sin\left(\frac{n\pi x}{L}\right) dx \right]$$

$\left. \begin{matrix} \frac{L}{2}, \\ m=n \\ =0 \end{matrix} \right\}$

$\left. \begin{matrix} 0, & m \neq n, & \text{both even or both odd} \\ - \left( \frac{2L}{\pi} \right)^2 \frac{mn}{(m^2 - n^2)^2} & & m \neq n, \text{ one even, other is odd} \\ \frac{L^2}{4} & \text{if } m=n & \end{matrix} \right\}$

$H_{mn} =$

$$= \frac{2}{L} \left[ \frac{\hbar^2}{2m} \left( \frac{n^2 \pi^2}{L^2} \right) \left( \frac{L}{2} \right) + \left( \frac{a}{L} \right) \left( \frac{L^2}{4} \right) \right] \quad m=n$$

$$= \frac{2}{L} \left[ - \left( \frac{a}{L} \right) \left( \frac{2L}{\pi} \right)^2 \frac{mn}{(m^2 - n^2)^2} \right] \quad m \neq n, \text{ one is odd, other is even}$$

$$= 0 \quad m \neq n, \text{ both are odd or both are even}$$

$\therefore H_{mn}$

$$= \left( \frac{\hbar^2}{2m_e} \left( \frac{n^2 \pi^2}{L^2} \right) + \left( \frac{a}{2} \right) \right) \quad m = n$$

$$- \frac{8}{\pi^2} \frac{mn}{(m^2 - n^2)^2} (a)$$

$$m \neq n$$

one is even, other is odd

( $m+n$  is odd)

0

( $m \neq n$   
( $m+n$  is even))