PH354 HW 5:

(6) Given $w(x) = x^{-1/2}$ $\quad I = \int_0^1 \frac{x^{-1/2}\,dx}{e^x+1}$

(I) my importance sampling formula:

$$I \approx \left\langle \frac{f}{w} \right\rangle_w \int_0^1 w(x)\,dx$$

$$= \sum_i \frac{f}{w}(x_i)\, p(x_i)$$

where $p(x_i) = \dfrac{(w(x_i))}{\int_0^1 w(x)\,dx}$

$$\Rightarrow p(x) = \frac{x^{-1/2}}{\int_0^1 x'^{-1/2}\,dx'} = \frac{x^{-1/2}}{(2)(\sqrt{1}-\sqrt{0})} = \frac{1}{2\sqrt{x}}$$

II TRANSFn. formula:

suppose z drawn from unif[0,1]:

$$x = x(z) \text{ sth}$$

drawing x from p(x) $\cong$ drawing z from unif[0,1]

$$\Rightarrow p(x')\,dx' = (1)\,dz$$

$$\int_0^{x(z)} p(x')\,dx' = (z-0)$$

$$\frac{(2)}{2}\left(\sqrt{x(z)} - 0\right) = z \Rightarrow x(z) = z^2$$

(Q10) (a) θ = polar angle $\in [0, \pi]$
ϕ = azimuthal angle $\in [0, 2\pi)$

checking normalisation:

$$\int_0^{\pi} p(\theta)\, d\theta = \int_0^{\pi} \frac{\sin\theta\, d\theta}{2} = \frac{1}{2}\left[-\cos\theta\right]_0^{\pi} = \frac{1}{2}\left((1)+(1)\right) = 1$$

$$\int_0^{2\pi} p(\phi)\, d\phi = \int_0^{2\pi} \left(\frac{1}{2\pi}\right) d\phi = \frac{1}{2\pi}(2\pi) = 1$$

(ii) TRANSFORMATION formula:
for $z \in$ unif $[0,1]$: $p(z) = 1$

I for ϕ:
$$p(\phi)\, d\phi = p(z)\, dz$$
$$\frac{1}{2\pi} . d\phi = 1 . dz$$
$$\Rightarrow \frac{1}{2\pi}\int_0^{\phi(z)} d\phi = (z-0)$$
$$\phi(z) = 2\pi z$$

II for θ:
$$p(\theta)\, d\theta = p(z)\, dz$$

$c(\theta(z))$ cumulative frequency

$$\int_0^{\theta(z)} \frac{1}{2}\sin\theta\, d\theta = \int_0^{z} 1 . dz$$
$$\frac{1}{2} . (-\cos(\theta(z))+1) = z$$

$\cos\theta(z) = 1-2z$

$\theta(z) = \cos^{-1}(1-2z)$

$\cancel{\cos(\theta(z)) = (1-2z)}$

$\cancel{\theta(z) = \cos^{-1}(1-2z)}$

(now as $\sin\theta \geq 0$ $\theta \in [0, \pi]$
and range of $\cos^{-1}\theta$ is also $[0, \pi]$
we can work with this distribution

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