PH354 : Assignment 2

Q10) (a) $E = \frac{1}{2}m\left(\frac{dx}{dt}\right)^2 + V(x) = V(a)$

$$\frac{dx}{dt} = \left[\left(\frac{2}{m}\right)(V(a)-V(x))\right]^{1/2}$$

$$\int_a^0 \frac{dx}{\sqrt{\left(\frac{2}{m}\right)(V(a)-V(x))}} = \int_0^{T/4} dt$$

$$\Rightarrow \frac{T}{4} = \int_a^0 \frac{dx}{\sqrt{\left(\frac{2}{m}\right)(V(a)-V(x))}}$$

$$T = 4\int_a^0 \frac{dx}{\sqrt{\frac{2}{m}(V(a)-V(x))}}$$

(c) as amplitude goes to 0, the values of x in range $(0,a)$ also go to 0.

~~We know there is singularity in I around x=a~~
~~Moreover for $V(x) = x^4$: $\lim_{a\to 0}(V(a)-V(0))$~~
~~as $a \to 0$: very low velocity imparted to oscillator~~

We know there is singularity in I around $x=a$
Thus, as $a \to 0$, all the values $x \in (0,a)$ are very close to this singularity, causing T to blow up.

Q12) (a) $I(\omega) = \frac{\pi}{\hbar\pi^2c^2} \frac{\omega^3}{(e^{\hbar\omega/k_BT} - 1)}$

Total energy per unit area per unit time radiated by black body:

$$\frac{E}{\text{per s / per m}^2} = \int_0^\infty I(\omega)d\omega = \int_0^\infty \frac{\pi}{\hbar\pi^2c^2} \frac{\omega^3}{e^{\hbar\omega/k_BT} - 1} d\omega$$

$$x = \frac{\hbar\omega}{k_BT} \qquad d\omega = \frac{k_BT}{\hbar}\, dx$$

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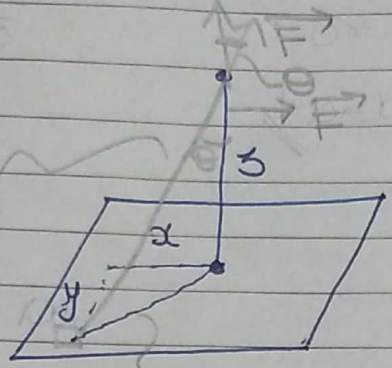
$$= \frac{\pi}{\hbar^3 c^2} \left(\frac{RT}{\pi}\right)^4 \int_0^\infty \frac{x^3 dx}{e^x - 1}$$

$$= \frac{R^4 T^4}{\hbar^3 c^2 \pi^3} \int_0^\infty \frac{x^3 dx}{e^x - 1}$$

(g)



$(x^2+y^2+z^2)^{1/2}$

$(x^2+y^2)^{1/2}$

where $\sigma = \frac{m}{A}$

$$|d\vec{F}| = \frac{G\sigma\, dx\, dy}{(x^2+y^2+z^2)}$$

But as the axis is in middle of sheet
$\Rightarrow$ components of $\vec{F}$ ⊥ll to XY plane cancel out due to symmetry.

$$\Rightarrow d\vec{F}_z = |d\vec{F}| \cos\theta$$

$$= |\vec{F}| \frac{z}{\sqrt{x^2+y^2+z^2}}$$

$$\Rightarrow \vec{F_z} = \int_{-\frac{L}{2}}^{\frac{L}{2}} \int_{-\frac{L}{2}}^{\frac{L}{2}} \frac{G\sigma\, dx\, dy \cdot z}{(x^2+y^2+z^2)^{3/2}}$$

$$= G\sigma\, dx\, dy \frac{z}{(x^2+y^2+z^2)\sqrt{x^2+y^2+z^2}}$$

(c) this artifact seen because:
though the numerator decreases linearly: had denominator decreased more rapidly we would get it to be increasing, but we are calculating such that denominator does not decrease rapidly enough.

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Q15) (a) $I = x^{a-1} e^{-x}$

$$\frac{dI}{dx} = 0 = (a-1)x^{a-2} e^{-x} + x^{a-1}(-1)e^{-x} = 0$$

$$\implies \frac{(a-1)}{x} - 1 = 0$$

$$x = (a-1)$$

(b) gn $z = \frac{x}{c+x}$

$$z = \frac{1}{2} = \frac{x}{c+x}$$

$$\implies c+x = 2x$$

$$\boxed{x = c}$$

Now, if we want maximum at $z = \frac{1}{2}$

$\implies x = (a-1)$ at $z = \frac{1}{2}$

$$\implies \boxed{c = (a-1)}$$

(c) attempting to solve using form $\exp((a-1)\ln x - x)$ which

☆ NOTE: those ~~ttt~~ files with "_" ~~#~~ ~~hello~~ towards the end of their name do not work and are merely attempts