CS386 AI Lab 5

Markov Chains

categories / states = { Inactive, Active, Super Active }

user behaviour

Day 1 → Day 365

Inactive 188969 → Inactive - 168103
Inactive 188969 → Active - 18733
Inactive 188969 → Super Active - 2133

Active 81356 → Inactive - 61017
Active 81356 → Active - 17899
Active 81356 → Super Active - 2440

Super Active 14210 → Inactive - 6963
Super Active 14210 → Active - 6252
Super Active 14210 → Super active - 995

Calculating the transition Probability Matrix

Inactive → Inactive $= \frac{168103}{188969} \approx 0.89$

Inactive → active $= \frac{18733}{188969} \approx 0.10$

Inactive → super active $= \frac{2133}{188969} \approx 0.01$

Active → Inactive $= \frac{61017}{81356} \approx 0.75$

Active → active $= \frac{17899}{81356} \approx 0.22$

Active → super active $= \frac{2440}{81356} \approx 0.03$

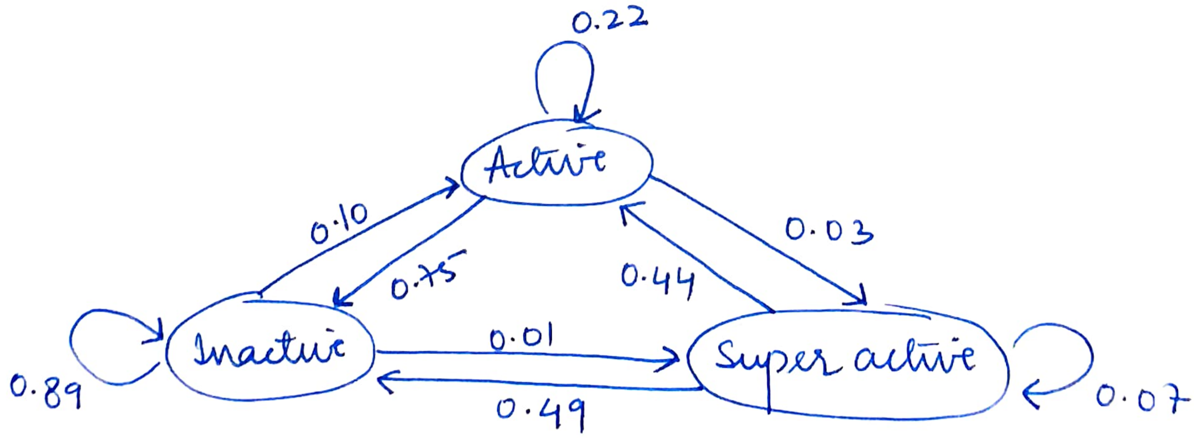
super active → inactive $= \frac{6963}{14210} \approx 0.49$

super active → active $= \frac{6252}{14210} \approx 0.44$

super active → super active $= \frac{995}{14210} \approx 0.07$

TPM -

	Inactive	Active	super active
Inactive	0.89	0.10	0.01
active	0.75	0.22	0.03
super active	0.49	0.44	0.07



Deriving Stationary Distribution

$$P_\infty(X) = P_{\infty-1}(X) = \sum_x P(X|x)\, P_\infty(x)$$

Here we have 3 states, so

$$\begin{aligned} P_\infty(\text{Inactive}) = {} & P(\text{Inactive}\,|\,\text{Inactive}) \cdot P_\infty(\text{Inactive}) + \\ & P(\text{Inactive}\,|\,\text{active}) \cdot P_\infty(\text{active}) + \\ & P(\text{Inactive}\,|\,\text{superactive}) \cdot P_\infty(\text{superactive}) \end{aligned}$$

$$= 0.89\, P_\infty(\text{Inactive}) + 0.75\, P_\infty(\text{active}) + 0.49\, P_\infty(\text{superactive})$$

$$\Rightarrow \quad 0.11\, P_\infty(\text{Inactive}) = 0.75\, P_\infty(\text{active}) + 0.49\, P_\infty(\text{superactive}) \tag{1}$$

$$\begin{aligned} P_\infty(\text{active}) = {} & P(\text{active}\,|\,\text{Inactive}) \cdot P_\infty(\text{Inactive}) + \\ & P(\text{active}\,|\,\text{active}) \cdot P_\infty(\text{active}) + \\ & P(\text{active}\,|\,\text{superactive}) \cdot P_\infty(\text{superactive}) \end{aligned}$$

$$= 0.10\, P_\infty(\text{Inactive}) + 0.22\, P_\infty(\text{active}) + 0.44\, P_\infty(\text{superactive})$$

$$\Rightarrow \quad 0.78\, P_\infty(\text{active}) = 0.10\, P_\infty(\text{Inactive}) + 0.44\, P_\infty(\text{superactive}) \tag{2}$$

$$\begin{aligned} P_\infty(\text{superactive}) = {} & P(\text{superactive}\,|\,\text{Inactive}) \cdot P_\infty(\text{Inactive}) + \\ & P(\text{superactive}\,|\,\text{active}) \cdot P_\infty(\text{active}) + \\ & P(\text{superactive}\,|\,\text{superactive}) \cdot P_\infty(\text{superactive}) \end{aligned}$$

$$= 0.01\, P_\infty(\text{Inactive}) + 0.03\, P_\infty(\text{active}) + 0.07\, P_\infty(\text{superactive})$$

$$\Rightarrow \quad 0.93\, P_\infty(\text{superactive}) = 0.01\, P_\infty(\text{Inactive}) + 0.03\, P_\infty(\text{active}) \tag{3}$$

$$P_\infty(\text{active}) + P_\infty(\text{Inactive}) + P_\infty(\text{superactive}) = 1 \tag{4}$$

on solving 1, 2 and 4 simultaneously,
we get

P_∞(Inactive) = 0.8681
P_∞(active) = 0.1187
P_∞(superactive) = 0.0132