Aditi Banerji
38 – F

Tutorial - 4

Q1. Solve using Master Theorem :-

1) $T(n) = 3T(n/2) + n^2$

$T(n) \Rightarrow aT(n/b) + f(n)$ where $a \geq 1$, $b > 1$

on Comparing,

$a = 3$, $b = 2$, $f(n) = n^2$

Now,

$c = \log_b a$ ie $\log_2 3$ ie 1.584

and n^c ie $n^{1.584} < n^2$

$\therefore f(n) > n^c$, $\therefore T(n) = \Theta(n^2)$

2) $T(n) = 4T(n/2) + n^2$

$a \geq 1$, $b > 1$ and $a = 4$, $b = 2$, $f(n) = n^2$

$c = \log_2 4$ ie 2

$\therefore n^c = n^2 = f(n) = n^2$

$\therefore T(n) = \Theta(n^2 \log_2 n)$

3) $T(n) = T(n/2) + 2^n$

$a = 1$, $b = 2$, $f(n) = 2^n$

$c = \log_b a \Rightarrow \log_2 1$ ie 0

Now n^c ie $n^0 \Rightarrow 1$ and $f(n) > n^c$

$T(n) = \Theta(2^n)$

4) $T(n) = 2^n T(n/2) + n^n$

$a = 2^n$, $b = 2$, $f(n) = n^n$

$c = \log_2 2^n \Rightarrow n$ and $n^c = n^n$, $f(n) = n^c$

$\therefore T(n) \Rightarrow \Theta(n^n \log_2 n)$

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Q5. $T(n) = 16T(n/4)+n$

$a=16,\ b=4,\ f(n)=n$

$c = \log_4^{16} \Rightarrow \log_4(4)^2$ ie $2\log_4^4$ ie 2

Now, $n^c \Rightarrow n^2$ and $f(n) < n^c$

$\therefore\ T(n) = \Theta(n^2)$

Q6. $T(n) = 2T(n/2) + n\log n$

$a=2,\ b=2,\ f(n) = n\log n$

$c = \log_2^2$ ie 1 and $n^c \Rightarrow n^1 \Rightarrow n$

Since $n\log n > n$ ie $f(n) > n^c$

So, $T(n) = \Theta(n\log n)$

7) $T(n) = 2T(n/2) + n/\log n$

$a=2,\ b=2,\ f(n) = n/\log n$

$c = \log_2^2$ ie 1 and $n^c \Rightarrow n^1$

Since $\frac{n}{\log n} < n$ $\therefore f(n) < n^c$

$\therefore\ T(n) = \Theta(n)$

8) $T(n) = 2T(n/4) + n^{0.51}$

$a=2,\ b=4,\ f(n) = n^{0.51}$

$c = \log_b^a$ ie $\log_4^2$ ie 0.5 and $n^c \Rightarrow n^{0.5}$

So, $n^{0.5} < n^{0.51}$ ie $f(n) > n^c$

$\therefore\ T(n) = \Theta(n^{0.51})$

9) $T(n) = 0.5T(n/2) + 1/n$ (3)

$a = 0.5$, $b = 2$, $f(n) = 1/n$

since Acc to Master theorem, $a \geq 1$ but here a is 0.5

so we can't apply Master Theorem.

10) $T(n) = 16T(n/4) + n!$

$a = 16$, $b = 4$, $f(n) = n!$

$c = \log_4 16$ ie 2 Now, $n^c = n^2$

As $n! > n^2$ $\therefore T(n) = \Theta(n!)$

11) $4T(n/2) + \log n$

$a = 4$, $b = 2$, $f(n) = \log n$

$c = \log_2 4$ ie 2 Now, $n^c = n^2$

Since $\log n < n^2$ $\therefore f(n) < n^c$

$\therefore T(n) = \Theta(n^2)$

12) $T(n) = \text{Sqrt}(n)T(n/2) + \log n$

$a = \sqrt{n}$, $b = 2$, $f(n) = \log n$

Now $c = \log_2 \sqrt{n}$ ie $\frac{1}{2}\log_2 n$

Now, $\frac{1}{2}\log_2 n < b\log(n)$ $\therefore f(n) > n^c$

So, $T(n) = \Theta(f(n))$ ie $\Theta(\log(n))$

13) $T(n) = 3T(n/2) + n$

$a = 3$, $b = 2$, $f(n) = n$

Now, $c = \log_2 3$ ie 1.5849 Now, $n^c = n^{1.5849}$

So, $n < n^{1.58489}$

ie $f(n) < n^c$

$\therefore T(n) = \Theta(n^{1.5849})$

14. $T(n) = 3T(n/3) + sqrt(n)$

$a=3,\ b=3,\ f(n) = \sqrt{n}$

Now $c = \log_3 3$ ie 1 $\quad \therefore n^c = n^1$

as $\sqrt{n} < n$ so $f(n) < n^c$

$\therefore T(n) = \Theta(n)$

15. $T(n) = 4T(n/2) + cn$

$a=4,\ b=2,\ f(n) = cn$

Now,

$c = \log_2 4$ ie 2 So, $n^c = n^2$

$\therefore cn < n^2$ (for any constant)

$\therefore f(n) < n^c$

$\therefore T(n) = \Theta(n^2)$

16. $T(n) = 3T(n/4) + n\log n$

$a=3,\ b=4,\ f(n) = n\log n$

Now, $c = \log_4 3$ ie 0.792, $n^c \Rightarrow n^{0.792}$

Now, $n^{0.792} < n\log n$

$\therefore T(n) = \Theta(n\log n)$

17. $T(n) = 6T(n/3) + n^2\log n$

$a=6,\ b=3,\ f(n) = n^2\log n$

Now, $c = \log_3 6 \Rightarrow 1.63903$

Now, $n^c \Rightarrow n^{1.63}$

So, $n^{1.63} < n^2\log n$

So, $T(n) = \Theta(n^2\log n)$

18. $T(n) = 3T(n/3) + n/2$

$a=3,\ b=3,\ f(n)=n/2$

So, $c = \log_3^3$ ie 1 Now, $n^c \Rightarrow n^1$

$\therefore\ n^c > n/2$ ie $f(n) < n^c$

So, $T(n) = \Theta(n)$

19. $T(n) = 4T(n/2) + n/\log n$

$a=4,\ b=2,\ f(n) = n\log n$

ie $c = \log_2^4$ ie 2 So, $n^c = n^2$

Now, $\dfrac{n}{\log n} < n^2$ ie $f(n) < n^c$

So, $T(n) = \Theta(n^2)$

20. $T(n) = 64T(n/8) - n^2\log n$

$a=64,\ b=8,\ f(n) = n^2 \log n$

ie $c = \log_8^{64}$ ie $2\log_8^8$ ie 2

Now $n^c = n^2$

So, $n^2\log n > n^2$ ie $f(n) > n^c$

So, $T(n) = \Theta(n^2 \log n)$

21. $T(n) = 7T(n/3) + n^2$

$a=7,\ b=3,\ f(n) = n^2$

ie $c = \log_3^7$ ie 1.7712

Now, $n^c = n^{1.7712}$

Since $n^{1.7712} < n^2$

$\therefore\ T(n) = \Theta(n^2)$

22 $T(n) = T(n/2) + n(2-\cos n)$

$a=1, b=2, \quad f(n) = n(2-\cos n)$

ie $c = \log_2 1$ ie 0 so, $n^c = n^0$

$\therefore \quad n(2-\cos n) > n^c$

$\therefore$

$T(n) = \Theta(n(2-\cos n))$.

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